Embedding Generalized Parsing in Haskell

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Why parser combinators?

For the parser generator crowd

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- Language integration

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- Directly parse into a semantic value

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numbers = (+) <$> number <* char ' ' <*> numbers <|> pure 0

> parse numbers "2 31 9"
Just 42
```

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maybe p = Just <$> p <|> pure Nothing
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Just 42
```

- Abstract over common patterns in your grammar

```
maybe p = Just <$> p <|> pure Nothing
```

- Monadic parsers enable data-dependent disambiguation (1)

```
ndots 0 = pure ()
ndots n = char '.' *> ndots (n - 1)

> parse (number >>= ndots) "5....."
Just ()
```

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- Left-recursion

$$\mathrm{D} ::= 0 \mid 1 \mid ... \mid 9$$
$$\mathrm{N} ::= \mathrm{N}\ \mathrm{D} \mid \mathrm{D}$$

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$$\mathrm{N} ::= \mathrm{N}\ \mathrm{D} \mid \mathrm{D}$$

- Compositionality

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$$\mathrm{D} ::= 0 \mid 1 \mid ... \mid 9$$
$$\mathrm{N} ::= \mathrm{N}\ \mathrm{D} \mid \mathrm{D}$$

- Compositionality

  "… it can be quite difficult to determine what language is defined by a TDPL program." ~ Aho and Ullman (2, p466)

- Disambiguation through annotation rather than deformation

Aside: eliminating left-recursion

“Can’t we just…”

- $\mathrm{N} ::= \mathrm{N}\ \mathrm{D} \mid \mathrm{D}$
  becomes
  $\mathrm{N} ::= \mathrm{D}\ \mathrm{N'}$
  $\mathrm{N'} ::= \mathrm{D}\ \mathrm{N'} \mid \varepsilon$
- Complicated for hidden left recursion and semantic values
- Grammar size can grow exponentially

GLL (3)

- Slots, Extended Packed Nodes, Descriptors, Commencements, Continuations
- Essentially building up big set of intermediate results
- $O(n^3)$ time and space

Generalized Parser Combinators

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Generalized Parser Combinators

- Partial normalization up front (Free MonadPlus)
- Then simple driver
- Stack of continuations
- Actions
  - Descend (Push)
  - Loop (Append)
  - Continue (Read)
  - Ascend (Pop)

Stack and continuations

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  - nonterminal name and pivot

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  - list of loop continuations

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Descend & Loop

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- If it is **not** on the stack at the current pivot: Descend
  - Push it to the stack with an empty list of loop continuations and the current continuation
  - Enter the right hand side

Descend & Loop

- When a nonterminal is encountered
- If it is **not** on the stack at the current pivot: Descend
  - Push it to the stack with an empty list of loop continuations and the current continuation
  - Enter the right hand side
- If it is on the stack at the current pivot: Loop

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- If it is **not** on the stack at the current pivot: Descend
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  - Enter the right hand side
- If it is on the stack at the current pivot: Loop
  - Capture a slice up to that occurrence

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  - Capture a slice up to that occurrence
  - Append that slice along and current continuation to the loop continuations

Descend & Loop

- When a nonterminal is encountered
- If it is **not** on the stack at the current pivot: Descend
  - Push it to the stack with an empty list of loop continuations and the current continuation
  - Enter the right hand side
- If it is on the stack at the current pivot: Loop
  - Capture a slice up to that occurrence
  - Append that slice along and current continuation to the loop continuations
  - Bail out

Continue & Ascend

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Continue & Ascend

- When a nonterminal is fully parsed
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  - Append its slice to the current stack
  - Continue with its parser

Continue & Ascend

- When a nonterminal is fully parsed
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  - Choose one
  - Append its slice to the current stack
  - Continue with its parser
- Ascend

Continue & Ascend

- When a nonterminal is fully parsed
- Do either of
- Continue
  - Peek at the list of loop continuations
  - Choose one
  - Append its slice to the current stack
  - Continue with its parser
- Ascend
  - Pop the stack

Continue & Ascend

- When a nonterminal is fully parsed
- Do either of
- Continue
  - Peek at the list of loop continuations
  - Choose one
  - Append its slice to the current stack
  - Continue with its parser
- Ascend
  - Pop the stack
  - Continue with the final continuation

Parsing $5 + 3 + 7$ with $\mathrm{X} ::= \mathrm{X} + \mathrm{X} \mid \mathbb{N}$

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$\mathrm{X}_0[+\ \mathrm{X}]$	$\mathbb{N}$	$5 + 3 + 7$	parse $\mathbb{N}$

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$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]

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$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
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$\mathrm{X}_0[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 3 + 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]

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$\mathrm{X}_0[+\ \mathrm{X}]$	ε	$+ 3 + 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 3 + 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +

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$\mathrm{X}_0[+\ \mathrm{X}]$	$\mathbb{N}$	$5 + 3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}]$	ε	$+ 3 + 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 3 + 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	X	7	descend X

Parsing $5 + 3 + 7$ with $\mathrm{X} ::= \mathrm{X} + \mathrm{X} \mid \mathbb{N}$

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ε	X	$5 + 3 + 7$	descend X
X_0	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$5 + 3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$\mathbb{N}$	$5 + 3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}]$	ε	$+ 3 + 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 3 + 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	X	7	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	7	fail*

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ε	X	$5 + 3 + 7$	descend X
X_0	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$5 + 3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$\mathbb{N}$	$5 + 3 + 7$	parse $\mathbb{N}$
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$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	X	7	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	7	fail*
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathbb{N}$	7	parse $\mathbb{N}$

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$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	X	7	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	7	fail*
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathbb{N}$	7	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	ε	ε	ascend

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$\mathrm{X}_0[+\ \mathrm{X}]$	$\mathbb{N}$	$5 + 3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}]$	ε	$+ 3 + 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 3 + 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	X	7	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	7	fail*
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathbb{N}$	7	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	ε	ε	ascend
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	ε	ascend

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$\mathrm{X}_0[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 3 + 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	X	7	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	7	fail*
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathbb{N}$	7	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	ε	ε	ascend
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	ε	ascend
$\mathrm{X}_0[+\ \mathrm{X}]$	ε	ε	ascend

Parsing $5 + 3 + 7$ with $\mathrm{X} ::= \mathrm{X} + \mathrm{X} \mid \mathbb{N}$

Stack	Grammar	Input	Action
ε	X	$5 + 3 + 7$	descend X
X_0	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$5 + 3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$\mathbb{N}$	$5 + 3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}]$	ε	$+ 3 + 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 3 + 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}]$	X	$3 + 7$	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	$3 + 7$	loop X, [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$\mathbb{N}$	$3 + 7$	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	$+ 7$	continue [+ X]
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	$+\ \mathrm{X}$	$+ 7$	parse +
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	X	7	descend X
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathrm{X} + \mathrm{X} \mid \mathbb{N}$	7	fail*
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	$\mathbb{N}$	7	parse $\mathbb{N}$
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}];\mathrm{X}_4$	ε	ε	ascend
$\mathrm{X}_0[+\ \mathrm{X}];\mathrm{X}_2[+\ \mathrm{X}]$	ε	ε	ascend
$\mathrm{X}_0[+\ \mathrm{X}]$	ε	ε	ascend
ε	ε	ε	done

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    ::= (\x y → 10 * x + y) <$> number <*> digit
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Combined with Data Types à la Carte (6)

Conclusion

- We can combine
  - lightweight
  - embedded
  - generalized
  - monadic
- Parser combinators

Future work

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- Disambiguation (Layout, Precedence, Fixity)

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- Disambiguation (Layout, Precedence, Fixity)
- Memoization
- Higher-order combinators
- Actually start writing parsers

References

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