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Invited Review Paper

Binary-Star Light-Curve Models

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ABSTRACT. The historical development of binary-star light-curve models is traced from the early 1900s to the present, with emphasis on recent progress. A major break with tradition occurred about 1970 when physical models, based on equipotentials and made possible by fast computers, replaced geometrical models. Physical models have been improving in accuracy, efficiency, generality, and user friendliness. Further improvements can be expected. Astrophysical advances due to the new models range from new ways to estimate mass ratios and rotation rates to confirmation of theoretical predictions about the structure of W Ursae Majoris stars and behavior of irradiated convective envelopes. The morphology of close binaries, including extensions to nonsynchronous rotation and orbital eccentricity, is interwoven with physical models and their applications. The origins and influence of the four morphological types—detached, semidetached, overcontact, and double contact—are inseparable from the development of light-curve models. Parameter adjustment is an active area, with contributions on Differential Corrections, the Marquardt algorithm, the Simplex algorithm, and other methods for reaching a least-squares minimum. Solutions with applied constraints and simultaneous solutions of two or more kinds of observations are coming into more frequent usage. Observables other than photometric brightness include radial velocity, polarization, photospheric spectral line profiles, and spectral distributions due to circumstellar flows. Some of the newer models extend into these areas and are leading to new kinds of observing programs.

1. LIGHT-CURVE MODELS IN PERSPECTIVE

It has been customary in reviews of the binary-star field to stress the fundamental importance of binaries in providing masses, luminosities, and radii of stars, and other measured properties as well. Indeed, the value of such knowledge is not easily overstated, but obtaining reliable numbers can be at least as satisfying as having them, and one breed of astronomer finds it more so. Three stages are involved—conceive a model, compute its consequences in terms of simulated observations, and compare with real observations. So there are modelers, computer programmers, and analyzers, and sometimes they may all be the same person. The review will give some idea of what these astronomers have been doing, so as to stay off the streets and out of trouble.

For a reasonably straightforward example, one now sits at a desktop computer, adjusting parameters, and arrives at a match between computed ("predicted") and real observations within a few hours. Inside the machine, computations are being made which could not have been produced by all the astronomers in history, even if they were somehow provided with ordinary electronic calculators. Most of the several hours will be spent in thinking about overall strategies and in consulting previous publications—the actual computing, massive as it is, typically requires only a minor part of the time. Such is the power of today's computers. What the machine is doing in all of this computation will be covered to some extent. However the review mainly will concern the intuitive connection between binary-star models and light curves, and the historical development of the field.

A program may be called a "light-curve" program although it computes radial velocities, polarization variables, and other observables in addition to brightness variations. It is a name that has a traditional meaning, and which serves to distinguish such programs from other binary-star programs, such as evolution programs. It may seem inappropriate that the commonly used name comes from only one of the program's main purposes. However the situation has arisen because, after one has created a program to compute light curves from a physical binary-star model, only minor programming is needed to generate a variety of other observable quantities. Thus the accepted name "light-curve program" may be interpreted within the wider meaning, "observables generating program."

1.1 Models versus Programs

We have come a long way from the spherical and "rectifiable" models which constituted the entire field for over half a century, but it has not been a gradual transition. The span of a few years around 1970 saw the appearance of "physical" models, which were not elaborations of spherical or ellipsoidal models, but a new beginning. By 1985 it was rare to see one of the old models used, and at this writing it is rare indeed. Today's physical models are based on a concept of long standing, known as the Roche model, or on extensions of the Roche Model. In a formal sense, the name only means that gravitation is idealized by that of point masses, but in common usage it also has come to imply synchronous rotation and circular orbits. Extended Roche Models may incorporate nonsynchronous rotation, eccentric orbits, and other generalizations. Numerous investigations have shown that the point-mass approximation represents the gravity field at the star surfaces quite adequately, largely because of the high degree of central condensation of real stars.

Among the new contributions, some were intended for general use and others for particular applications. A few were developed for specific projects, but then resurrected and further developed when the new wave of interest became apparent. While the number of original contributors is not large (of order ten), the situation is complicated by difficulties in distinguishing a new model or significant model attribute from merely a new program. Now a new program based on an existing model might be very useful—it might be notably accurate or flexible or user friendly. However here the main concern is with models and analytic methods rather than programs. Accordingly, a few words on the distinctions between computer models and computer programs seem appropriate.

The terms "computer model" and "computer program" are strongly intermeshed in recent scientific thinking. They are difficult to separate because the practical embodiment of a computer model must be in a computer program. This situation is particularly found in the binary-star field because, typically, one person or pair of persons has developed the model and the program. It could be that one person conceives the model and another does the programming, in which case the separation would be clear, but that has not been the usual situation.

Nevertheless it is useful to keep in mind the distinction between models and programs. Historically, there were models before there were programs (i.e., the Russell model, discussed later), and astrophysical interest naturally is in the models, rather than the programs. Some items are not so easily assigned to model versus program status. For example, eclipse effects may be handled in a wide variety of ways, perhaps with the eclipsing boundary represented by an approximation function or perhaps with it computed more or less directly. Is that a model or a program characteristic? Here the view will be, if it is a way of getting numbers, it is "program," and if it is astrophysical in any way, it is "model." For example, use of gravitational—centrifugal potentials or another means of locating the star surfaces is part of the model, as is the gravitational mass distribution (usually the point-mass approximation). Other physical attributes, such as treatment of "reflection," gravity brightening, and limb darkening also are model properties. On the other hand, ways of computing eclipse effects, and of computing system brightness in general from model characteristics, are program properties. A term that shall be avoided here is "computer code," which often is used elsewhere as a synonym for "computer program." To illustrate the reason, suppose that two computer programmers are given the same minor assignment, such as to generate Legendre functions. They will produce different "codes," perhaps with different efficiencies, compactness, etc., but little or no real programming is required for such a simple problem. Programming includes strategies for whole different ways of going about an intricate computation—not just how to apply mathematical techniques, but which ones to apply, as well as how to use constraints, how to avoid singularities and near singularities, and so on. Real programming is required for a binary-star light-curve program, which involves many interrelated strategies to bring the model into effective realization. The term "coding," on the other hand, describes simple translation of a computational recipe from one set of rules and symbols to another. All astrophysical binary-star light-curve programs are far beyond the level of coding.

Several questions arise at this point in regard to the availability and usefulness of binary-star computer models and programs. How many are there? Are there as many models as programs or do some of the programs correspond to practically the same model? How general are the various models and programs? What output quantities can they compute? Are they portable to a wide variety of computer systems? How easy are they to use? Which ones are being actively maintained? How does each solve the inverse problem of determining parameter values from observations? Let us look at how things have developed so far.

2. HISTORICAL VIEW

In a history of the binary-star field, the Russell Model (e.g., Russell 1912a, b, 1939, 1942, 1945, 1948; Russell and Merrill 1952; Russell and Shapley 1912) would command a very long chapter. For roughly half a century the Russell Model gained strength as it progressed from spherical stars to more intricate ellipsoidal configurations with "reflection effect," and it was the only model actually used very often to fit observations. During the 1940s and 50s, numerous important ideas concerning how to model and compute binary-star light curves were contributed by Kopal (e.g., 1941, 1942, 1943, 1946, 1954, 1959), and some of his central concepts, such as the specification of figures by equipotential surfaces, form the basis of today's models. A number of relations derived by Kopal were incorporated into the Russell model. His work in this field was summarized in *Close Binary Systems* (Kopal 1959). Most other light-curve publications of those times involved manipulation of equations for spherical stars. While much algebra and many mathematical symbols filled their pages, they have essentially no continuing value and did not even contribute much to the field in their day, in contrast to Russell's and Kopal's core of essential ideas. Both Russell and Kopal took logical steps for their time (i.e., a time without fast automatic computation) in bringing their models to practical realization. Central to the Russell Model was the process of "rectification," by which an observed light curve was corrected so as to correspond, as nearly as could be managed, with that of a hypothetical binary made of spherical, limb darkened, stars in circular orbit. One then analyzed the rectified light curve so as to determine the geometric and photometric properties of the spherical stars, which supposedly could be related back to those of the real stars by transformation formulas. Of course, the properties actually were related back to the ellipsoidal model stars, necessarily having the same axis ratios (shapes) and "similarly situated," which meant that the long axes had to be collinear and the other corresponding axes had to be mutually parallel. Several somewhat incorrect formulations of the local physics were adopted in order to keep the rectification process reasonably tractable, although it still was a great deal of work to rectify an observed light curve—far more than required to fit a light-curve with modern models and the aid of a computer. In essence, the rectification process separated the modeling problem into two main parts. In the first part, one used the observed brightness variation outside eclipse to infer the tidal deformations, surface-brightness effects of deformation, and amplitude of the reflection effect. The light curves were then corrected for those effects (rectification). The fitting process was carried out with the corrected curves. Solutions were almost entirely graphical. Computer programs based on rectifiable models were developed by Jurkevich (1964, 1970) and by Proctor and Linnell (1972), but these arrived at about the same time as the new physical models (discussed below) and were seldom used. The PL paper is a watershed in which the Russell model was fully automated and its results compared with one of the early physical model solutions (Wilson and Devinney 1971, hereafter WD). PL show thorough awareness of the limitations of rectifiable models, and include a fair appraisal of the strong and weak points on both sides. Of course, the Russell model had reached its acme then, while physical models were just beginning.

A tacit and unjustified assumption of rectifiable models in application was that eclipses provide the only worthwhile information about the binary configuration. Although in principle the between-eclipse variation could have been used to learn about the binary configuration, this seldom was done. Thus the tidal and mutual heating effects, which today provide valuable information for parameter estimates, were then treated as nuisances. Although some physics was embodied within rectification, much crucial physics also was omitted. Not only were solutions weakened by the absence of such physics, but it was common to find solutions that violated the dictates of gravitation and centrifugal force—for example, with the star sizes and figures inconsistent with their known or reasonably inferred masses.

Several procedures for determining parameters of spherical models binaries, and at least one for rectification, also were published by Kopal, but his enduring contributions are mainly to be found in *Close Binary Systems*, in which quantitative discussions of equipotentials and their gradients, contact configurations, and various related subjects are to be found. Kopal also developed formulas to generate light curves by correcting the light of a spherical model for Roche Model distortions, and these were used by Soderhjelm (1974, 1976a, 1978) to represent observations of several binaries. Soderhjelm also made comparisons between the Kopal corrected-sphere light curves and those from several of the frequently used physical and ellipsoidal model programs, although no obvious recommendations resulted from the experiments, except that the Kopal expressions seem acceptably accurate for $r/r_{\rm lobe}$ of the order of 0.8 or smaller. Of course, only the existence of the newer models allows us to know this with confidence. In the author's view, the lack of usage of the Kopal scheme has been due only partly to its inapplicability to overcontact binaries and loss of accuracy for lobe-filling and near-contact cases. Indeed, the scheme should be quite adequate for many published light curves, and its speed advantage (viz., Soderhjelm) would have been important until the improvements in computing machines of the last few years. The main reason for its neglect is that it cannot readily be generalized without going back to beginnings, while the directly physical models can accommodate nonsynchronous rotation, eccentric orbits, detailed reflection theory, spots, and various other improvements. Still, the Kopal light-curve model predated the earliest of the directly physical models by a decade and could have filled an ecological niche during the 1960s, 70s, and 80s, had it been more actively used.

In overview, Russell's work was primarily directed toward creation of a procedure by which astrophysical quantities could be estimated from real light curves, while Kopal's was more in fundamental ideas. Both kinds of contributions have been historically important. Perhaps fated to be lost on future generations is the insight, rigorous thinking, and cleverness of the Russell model. It is easy to forget how things were before one could do more computing in a second by pressing a button than in a lifetime by hand. Now we look back and see that the Russell model produced distorted results, but results would have been more distorted without the Russell model. The attraction of learning about real star dimensions and properties was there in those times, and what is one to do in the absence of adequate computers? In retrospect, Russell and Kopal jointly accomplished about all that could be done without fast computers.

Machines now allow us to do far more. Our physical models are based on the hydrostatics of level surfaces, and require much more computation than do ellipsoids, but not so much as to overburden the machines. The fundamental ideas are that surfaces of constant density coincide with surfaces on which the potential energy per unit mass is constant (equipotential surfaces) and that local gravity and surface orientation are given by the potential gradient. In geometrical models there is some vagueness about the meaning of the radius of a nonspherical star—in particular about the connection between a mean radius derived from observations and the real physical mean radius. This problem disappears when the surface is specified by a single number related to its potential energy. Although many details need to be filled in along the way, these points provide a basis for computing light curves. A discussion of the main intuitive ideas can be found in Wilson (1974a).

In the late 1960s, Lucy became interested in W Ursae Majoris stars. These intriguing objects are overcontact main-sequence binaries with convective outer envelopes—i.e., the two stars actually touch one another and the overall configu-

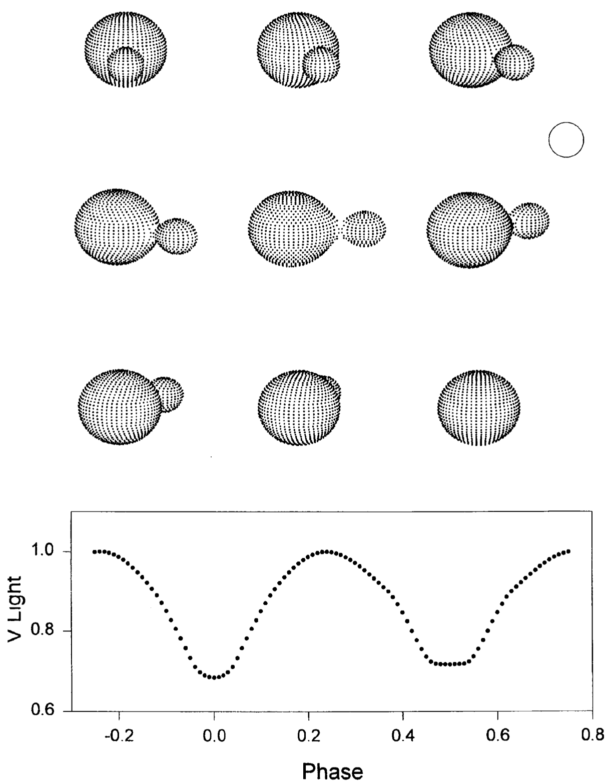

FIG. 1—A typical overcontact configuration is shown by these computer generated images of the W UMa-type binary RR Centauri and a theoretical light curve. The stars both overfill their Roche lobes, and have a common envelope whose surface follows a single equipotential. The absolute scale can be inferred from the circle at the upper right, which represents the Sun.

ration must somehow adjust to the thermodynamic interaction between their envelopes. Such a binary is said to have a common convective envelope. The matter of why they are interesting will be covered in Sec. 4, but relevant here is Lucy's need to compare observed W UMa light curves with computed ones without serious systematic error. It would not do to start with spheres and apply corrections for nonsphericity, since the real configurations are not even roughly similar to a pair of spheres (see Fig. 1). This problem led to a paper (Lucy 1968b) on a straightforward way to compute light curves of overcontact binaries, with a graphical example and a discussion of comparisons with observations. Lucy's method required a very finely spaced coordinate grid of surface elements to keep numerical noise reasonably small, and it only worked for overcontact cases, but it was a "first." Light curves of distorted stars were being computed directly, and real science was replacing the inverse science of corrected observations (rectification). Lucy's essential scheme still is used in one of the active programs (Rucinski 1973, 1974; Hill and Rucinski 1993). Several independent efforts to compute light curves began only shortly after Lucy's and some came into major usage, because of one or more advantageous characteristics. Among these were computer models by Hill (Hill and Hutchings 1970; Hill 1979; Hill and Rucinski 1993); Wilson and Devinney (WD; Wilson 1979; 1990); Wood (1971); Nelson, Davis, and Etzel [the NDE model] (Nelson and Davis 1972; Etzel 1975, 1981, 1993); Mochnacki and Doughty (1972a, b), and Budding (1977). Some others appeared at roughly the same time, including Cochrane (1970); Peraiah (1970); Mauder (1972); Berthier (1973, 1975); Nagy (1974; 1977) and Binnendijk (1977); and Eaton (1975) but have not been extensively used except by their originators, perhaps because they have not been made generally available, although they may have their own advantages. Others followed, including Yamasaki (1981, 1982); Linnell (1984, 1986, 1987; 1989); Antokhina (1988); Djurasevic (1992a, b, c); Hendry and Mochnacki (1992) and it may be too soon to comment on how widely they will be used. Models based on equipotentials are commonly called "physical models," and it is important to note that not all new and useful models are physical. The NDE and Budding models are based on spheres, which may seem primitive in today's environment, but they have computational advantages in terms of speed and numerical precision. The NDE model now includes first order corrections for asphericity and gravity brightening (Etzel 1993). It is intended for eccentric orbit binaries in which the stars really are nearly spherical. The Budding model is for spotted stars, some of which are not so nearly spherical, but the light curves are rectified before they are analyzed for star spots. Testimony to the usefulness of these specialized models is provided by the substantial number of papers in which they have been used. What is important in the long run is not the matter of which computer models reach popularity, but what can be learned from them to produce improved models. Notable in this regard is that each modeler–programmer has had strong convictions about what is important and where significant advances can be made. Some originators focus on numerical accuracy, some on astrophysical generality, some on special cases, and some on parameter adjustment. Their independent thinking is largely responsible for the vitality of the field.

3. INNOVATIONS FOR ACCURACY, EFFICIENCY, AND GENERALITY

Although physical models are not the only useful ones, they are the core of the field. The following is a brief discussion of ideas for accuracy, efficiency, and general capability, most of which concerns physical models. Given a formulation of the potential field, it is easy to compute the binary-star surface configuration, including the orientation of surface normals and surface-brightness distributions, to far more accuracy than necessary. The physical light-curve references explain this, although some coordinate systems require special procedures. A real difficulty is to achieve accurate quadrature of the system brightness while dealing with horizons, eclipses, and certain special problems. Accuracy is of great importance and improvements are always welcome. Because numerical derivatives may need to be formed, and because of possible sensitivity to other numerical problems, the problem is not finished when errors considerably smaller than those of good observations have been reached. One effective method was adopted by D. B. Wood (1971) for his ellipsoidal binary model. Although not based on a physical model, Wood's program has had a major influence because of its fast and accurate brightness quadrature. It applies a Gaussian quadrature scheme, which is well suited to an ellipsoid model, where integration limits can be computed by a simple explicit formula. Although application to a physical model is more difficult and involves some loss of efficiency, the Gaussian quadrature also is used in Hill's (1979) physical program LIGHT2. There are many special problems connected with integration of the light of overcontact binaries. Linnell (1984, 1989) has gone to extraordinary lengths to compute accurate light curves by attending to a variety of horizon and eclipse problems, including overcontact self-eclipses. A thorough discussion of accuracy is accompanied by informative tables in the 1984 paper. Some understanding of Linnell's methods can be obtained by inspecting Figs. 5 and 6 of his 1989 paper, which show pictures of model binaries. Such problems may have been dealt with in other models, but usually without published explanations. A computational economy used only in Linnell's programs so far is to compute only at regularly spaced phases and interpolate to the observed phases for comparison with observations. This procedure can require large amounts of memory when numerical derivatives need to be formed, but ever more memory is becoming available as machines evolve. Computation and memory can be saved by a variety of tricks, such as the use of both coarse and fine computing grids (the fine grid being used only where necessary), mirror imaging of memory locations to make use of symmetries, a user option to turn off time consuming detailed reflection computations in favor of simple reflection in simple situations, program switches to turn off redundant operations, reversible loop nesting, and other items (Wilson 1993a). The accuracy of numerical derivatives can be improved if they are computed with symmetrical excursions about the operating point, but at some cost in computation time (Wilson and Biermann 1976). Linnell (1989) computes the derivatives of a few parameters by analytic differentiation under the integral sign, followed by numerical integration (Leibniz's rule), which gives some accuracy advantage. However this method mainly applies to derivatives that can be computed rather accurately by simple differencing.

A quadrature scheme by Hendry and Mochnacki (1992) is quite different from any previously used and involves operations with simplexes (in this case, triangles) distributed over the stars. Hendry and Mochnacki have demonstrated impressive achievements in accuracy for given computing time. At present the method seems to have been applied only to overcontact systems, but should be generally applicable to a wide variety of binaries. It is not clear whether the simplex quadrature is more or less accurate than Linnell's for a given number of integration elements, but it probably is more accurate than any present competitor in terms of accuracy for given machine time, which is the main concern.

Generality is important in models and programs. If a binary has one characteristic that is modeled in program A but not in program B, and another that is in B but not in A, then neither program can properly handle the situation. The more general physical models now include attributes such as nonsynchronous rotation and orbital eccentricity, as well as most of the features of various specialty models, although the approach to full generality never ends. (Star spot modeling is included along with spotted stars in Sec. 4. Non-Planckian radiation, nonlinear limb darkening, and multiple reflection are discussed as recent improvements in Sec. 6.) Extension of a physical model to nonsynchronous rotation (Wilson 1979; Linnell 1984) requires multiplying the centrifugal term of the potential equation by F^2, where F is the ratio of angular rotation rate to the mean orbital revolution rate (Plavec 1958; Limber 1963). There are separate potential systems for the two stars for nonsynchronous rotation. If one neglects some minor effects, such as Coriolis forces and slight rotation-induced changes in the mass distribution, this formulation is all that is needed to treat stars in arbitrary states of uniform rotation. A nonsynchronous theory by Peraiah (1969, 1970) seems not to have been applied to real observations or built into a general light curve program, although it even includes nonuniform rotation. With rotation now included, one can estimate F from spectral line broadening and a preliminary light-curve solution and come to a consistent solution. The feasibility of rotation determinations from light curves alone is discussed in Sec. 4.

The eccentric orbit generalization is more troublesome because the star figures become time dependent. Several major difficulties arise in this case. One is that the extremely useful potential field no longer exists because spatial integrals of a time-varying force field are path dependent. In practical work, it has been assumed that an equipotential treatment is adequate over short arcs of the orbit, so long as a star can adjust to the changing force field on a short time scale (a small fraction of the period). The second difficulty is that of computing the instantaneous star figures, given that the full problem is not only dynamical, but also structural. If the real star cannot follow the forced oscillations faithfully, its figure will differ significantly from that of the model star. There is evidence that GP Velorum=HD 77581 shows this kind of behavior (Wilson and Terrell 1994). So structural complications can cause errors in both the size and shape of the model star, and this is without even considering the possibility of being near a resonance. The full problem needs to include modeling of nonradial oscillations, which has not yet been done, although Hadrava (1986) has investigated the structural response to a changing tidal field. One further difficulty is the need to be particularly careful in programming the orbital problem. Both eclipses of a real eccentric binary go through timewise excursions which are mutually 180° out of phase. Some light-curve programs have one of the eclipses pinned at zero phase and simply follow the motion of the other eclipse relative to the first one. While it is some trouble to mimic reality in this regard, it is important to do so for proper comparison with observations. Despite this collection of problems, programs for treating eccentric binaries have been written and have mainly been successful in applications. The Wood (1971) and NDE programs have been used very often for eccentric systems. Although they are not physical models (ellipsoids for Wood, corrected spheres for NDE), many persons have found them adequate for eccentric binaries, most of whose stars are small compared to their orbits and thus can be represented acceptably by simple geometry. This circumstance is no accident, as binaries that are not especially close can avoid having their orbits tidally circularized on a short time scale. Physical eccentric models have an advantage for binaries that are close enough for tides to be important, due to better treatment of variable figures and surface brightnesses. A good example is CO Lacertae (Wilson and Woodward 1983), which was the first eccentric binary fitted with a physical model. Physical models also can properly combine nonsynchronous and eccentric behavior. For example, a quite general definition of a limiting lobe becomes natural (Wilson 1979). It is the equipotential for which the effective gravity is zero on the line of centers at periastron. Physical models by Hill and Rucinski (Hill 1979; Hill and Rucinski 1993) and by Wilson (1979) include eccentric orbits on the assumption of instantaneously valid equipotentials (viz., Avni 1976). Hill and Rucinski assume fixed polar radius and Wilson assumes fixed volume. The fixed volume assumption involves an iteration at each phase to determine the potential corresponding to a specified volume. The present overall situation with regard to eccentric orbit models is that they are based on a variety of approximations, seem to be essentially adequate for parameter determinations in simple situations, but are not ready to cope with subtleties. They certainly are not ready for major structural complications, such as are seen in some supergiant X-ray binaries, but may be helpful in sounding out those problems.

3.1 Parameter Adjustment

A prevailing idea throughout the 1960s was that direct computer models (as opposed to rectifiable models) would have very limited value because there would be no very effective means to estimate parameters. However, subsequent work has exploited several good algorithms for solving this problem iteratively. While the algorithms have their proponents and detractors, all are much to be preferred over the old methods of fitting rectifiable models, which mainly consisted of graphical trial and error. Indeed, parameter adjustment has been an active area, and while most of the ideas and methods are from the mathematics literature, the nonlinearity and correlation of light-curve solutions have provided a good arena for their intercomparison. Some of the earlier direct models were fitted by trial and error, but WD, Lucy (1973), and later several others, applied the method of Differential Corrections (DC). In this method, one expresses the total error in an observation, represented by an (observed−computed) residual, as a numerical analog of the total differential. For light-curve solutions, each term is thus the product of a derivative (of observable flux with respect to a parameter) and the correction to the parameter needed to eliminate the systematic part of the residual. The residuals and derivatives are computed from the best approximate solution available, and one solves iteratively for the corrections by the ordinary methods of linear least squares. The DC method had been used much earlier (e.g., Wyse 1939; Wyse and Kron 1939; Irwin 1947) in spherical model solutions, always with analytically computed derivatives. Prior to 1971, however, no one in the binary-star field seemed to have noticed the elementary yet crucial point that numerical derivatives often can serve adequately when analytic formulas do not exist (although DC had been used with numerical derivatives in several other fields, such as celestial mechanics). The arrival together of direct physical models and the means to estimate their parameters by least squares was a double breakthrough that stimulated new interest in light curves and in close binaries.

Solutions are done today by DC, by the Marquardt (1963) algorithm, by the Simplex algorithm, occasionally by Iterative Minimization, by Steepest Descent, and by Controlled Random Search. All seek to minimize the weighted sum of squares of residuals, hereafter denoted by SS (least-squares criterion). Iterative Minimization (e.g., Horak 1970) seeks a least-squares minimum simply by cyclical adjustment of one parameter at a time, and thus avoids possible divergence due to correlations among parameters. Since it does not deal with correlations, it lacks capability to compute standard error estimates. Because it cycles through the entire parameter set a very large number of times, it also is slow. Steepest Descent (e.g., Berthier 1975) computes and follows the local negative gradient of SS, which is the vector whose components are the partial derivatives of SS with respect to the various parameters. This procedure also does not involve any consideration of correlations, and thus cannot compute standard errors. It also suffers from slow convergence in many practical settings. Controlled Random Search (Price 1976; Barone et al. 1988) essentially blankets all of parameter space (within specified limits) with a randomly chosen sprinkling of trial points so as not to miss any local minima. Since it looks everywhere, including places ruled out by intuition, it is very slow. DC currently is used in several programs (e.g., Linnell 1989; Eaton 1991) in addition to those mentioned above. It does take correlations into account, which is at once its strength and weakness. Because it incorporates correlations, it usually converges much faster than Steepest Descent, and also can produce standard error estimates, provided that the problem at hand is reasonably linear in the parameters. However, in a highly nonlinear situation, DC can produce false corrections because its least-squares equation contains only linear terms in most applications. Virtual tradeoffs among the correlated higher order (missing) terms introduce differences between proper corrections and those actually computed (Wilson 1983). For the same reason, the standard errors lose accuracy, although they still are preferable to a complete lack of such estimates. To circumvent this problem, Wilson and Biermann (1976) tried breaking the overall parameter set into subsets that are solved alternately, or sequentially if there are more than two subsets. According to experience, a few large correlations cause much less trouble than do many fairly large correlations. Applications have shown major improvements in DC convergence when this Method of Multiple Subsets (MMS) is used to reduce the complexity of correlation matrices. Standard deviations or probable errors are to be computed from a final run involving the full parameter set. The MMS now is being used in a substantial fraction of published DC solutions, perhaps more than half. In the extreme of only one parameter per subset, the MMS effectively is equivalent to Iterative Minimization, so that in the general case it provides options intermediate between DC and Iterative Minimization. A combination of DC, with second derivatives included, and Simplex (see below) has been developed by Plewa (1988). Despite the inevitable numerical noise associated with numerical second derivatives, Plewa reports improvement on ordinary DC solutions.

The Marquardt algorithm essentially strikes a compromise between the corrections provided by DC and those of Steepest Descent. It does not compute the two kinds of corrections separately, but incorporates a parameter, λ, which will lead to the DC results if set to zero and to the Steepest Descent results if set to very large values. At each iteration, λ is set according to rules designed to avoid the possible divergence of DC for small λ and the possible slow convergence of Steepest Descent at large λ. Fast and reliable convergence with the Marquardt algorithm has been reported by several persons and groups (e.g., Djurasevic 1992c; Hill and Rucinski 1993). The Marquardt algorithm is beginning to have wide application in astrophysical parameter adjustment problems.

The Simplex algorithm (viz. Kallrath and Linnell 1987; Linnell and Kallrath 1987; Kallrath 1993 and contained references; Plewa 1988) does not compute any derivatives, but instead explores parameter space by means of a geometrical figure that samples SS at each of its vertices. The figure is a simplex, which will be a triangle on a two-dimensional surface, a tetrahedron in a three-dimensional volume, etc., and in general will have one more vertex than the dimension of the space in which it lies. Here that dimension will be the number of parameters. A set of operating rules governs contractions, expansions, and certain other changes in the figure as it moves through parameter space. In the end, the simplex should contract down to very small size surrounding the least-squares minimum. According to which papers one reads, the Simplex method may be described as fast or slow compared to DC, so the realistic speed relative to DC may depend on circumstances. The main advantages are that it cannot diverge and that it is unaffected, or at least not seriously affected by correlations. Since no numerical derivatives are required, it should be bothered less by numerical inaccuracies than are methods that do need derivatives. The obvious disadvantages are that it cannot provide standard errors and that a large number of iterations usually will be needed. Because it has $n+1$ operating points in n parameter dimensions, rather than the single operating point of other methods, it is not well suited to addition or deletion of parameters during the course of a solution (cf. Interactive Branching later in this section).

3.2 Simultaneous Solutions

Most light-curve solutions involve at least two photometric bandpasses, such as B and V or u, v, b, and y. Wilson and Devinney (1972) gave results of simultaneous multibandpass solutions in an addendum to their original 1971 paper. They considered the method sufficiently obvious as not to require explanation, so some time passed before it was explicitly specified in print. When it did appear (Wilson 1979), the explanation was combined with one for the simultaneous solution of light and radial-velocity curves, so that one can now solve n multibandpass light curves and one or two radial-velocity curves simultaneously. Proper assignment of weights is important for simultaneous solutions (Wilson 1979). At each iteration with n bandpasses, the procedure gives one correction for each bandpass-independent parameter (e.g., mass ratio) and n corrections for each bandpass-dependent parameter (e.g., limb darkening). One advantage of such simultaneous solutions lies in a reduction of the number of free parameters. Of course, only one mass ratio, one eccentricity, etc., is needed, not the four values of each that would be needed in separate solutions of, say, three light curves and a velocity curve. Another advantage is one of internal consistency, since solutions with different mass ratios would be mutually inconsistent. Several authors have assumed that it is all right to strike an average of separate solutions in two or more bandpasses, but this is erroneous. Such an average is not a valid solution for any of the bandpasses (see discussion in Van Hamme and Wilson 1984). Simultaneous solutions have "caught on" in one sense, since most users of the WD program make simultaneous light-curve solutions and some make simultaneous light and velocity solutions. In another sense they have not caught on, as simultaneous solution capability is not the norm in other programs, almost a quarter of a century after its appearance. Note that one can find papers on "simultaneous" trial and error solutions, although the discussion here concerns only simultaneous solutions based on an objective criterion.

The future of simultaneous solutions eventually will include types of observations other than light and radial-velocity curves (Wilson 1993b). Only fairly minor additions to a physical light-curve model are needed in order to compute various other observable quantities, such as polarization curves (Wilson and Liou 1993), spectral line profiles (Mukherjee et al. 1994, hereafter MPW), pulse arrival times for X-ray binaries (Wilson and Terrell 1994), and narrow band index curves. Simultaneous solutions can be extended to include any or all such observables.

3.3 Constrained Solutions and Modes of Program Operation

The WD program can be operated in any one of eight modes, with each mode corresponding to a particular set of constraints on the physical configuration (Wilson 1993a). Constrained solutions have a prominent place in adjustment theory, and were included within WD almost from the beginning, but so far have been incorporated into few other light-curve programs. The relevance of constraints to light-curve problems can be illustrated as follows. Imagine that you know or are convinced of something about the parameters of a binary system, and your "fact" is not in the form of a definite value for a parameter, but rather is a functional relationship among parameters. You do not claim to know the value of parameter c, but if someone tells you the values of parameters a and b, you then can calculate c. The most common example has one of the stars accurately filling a limiting lobe (semidetached case). The physical situation is explained in the next section, but for now we need only recognize a connection between relative star size and system mass ratio. There are good reasons to believe that certain binaries (e.g., Algols) have this configuration, although it would be difficult to summarize the reasons in a few words. Assuming a circular orbit and synchronous rotation for simplicity, the mass ratio determines the relative size (R/a) of the lobe-filling star and, since the model expresses star size through the surface potential, only the mass ratio or the potential, but not both, should be adjusted. If we have lobe filling with nonsynchronous rotation, there is a functional relation among three parameters (now another one to express the rotation rate), and if the orbit is eccentric, we add another. In general, each constraint reduces the number of parameters by one. Since every parameter enlarges and complicates the parameter correlation matrix and thus weakens the solution, we should take advantage of every opportunity to eliminate free parameters. Essentially, we thereby introduce information from sources external to the light curves, and tell the least-squares algorithm "here is something about the binary which you cannot tell from the light curves, but we are letting you know so as to eliminate a whole dimension of incorrect solutions." Each constraint or set of constraints is associated with a particular mode of the WD light curve and differential corrections programs. Parameters set by constraints are no longer free. The program ignores their input values and computes them according to the constraint relation.

Not all constraints concern limiting lobes. Consider a compact X-ray source, such as a neutron star, paired with a normal star. There may be an X-ray eclipse but no detectable optical eclipse. If the X-ray eclipse has a well-defined duration, it constrains the size of the normal star. Chanan et al. (1976) show how to compute such eclipse durations for the synchronous–circular orbit case, and Wilson (1979) for the more general nonsynchronous–eccentric case. It happens that some X-ray binaries are nonsynchronous and eccentric, and their solutions can be improved by imposing the X-ray eclipse duration as a constraint. GP Vel/HD 77581/Vela X-1 even has pulsed X rays, so this one object incorporates a seldom used observable (pulse arrival times), simultaneous solutions including that observable, and a seldom used constraint (X-ray eclipse duration), along with an eccentric orbit and nonsynchronous rotation. Upon applying such an analysis, Wilson and Terrell (1994) infer interesting effects concerning forced oscillations with possible implications in regard to common envelope evolution.

3.4 Insights—The Role of Experience

Ideas for least-squares solutions are not limited to formal algorithms, but also involve personal interactions in otherwise impersonal operations. The most common such interaction is the recognition of regions of parameter space that need not be explored. Perhaps this too will be done automatically some day, but for now it is not so easy to formalize what the mind does when it eliminates whole areas of parameter space. The situation is akin to that of computer chess programs, which perform less well than a human Grand Master, despite an enormous advantage in the number of moves evaluated. Personal intervention occurs also when one notices something that is just not right, after extensive trials. Sometimes the realization comes from spectroscopy, or another kind of observation, as with Linnell's (1991c, 1992) insights into SV Centauri. Least-squares methods adjust parameters but do not explicitly point out model deficiencies, or offer suggestions on how to improve a model. Also personal at present are decisions on which parameters are not to be adjusted, but adopted from theory, calibrated relationships, or other kinds of observations of the binary. Connected with all of these points is the issue of whether or not to iterate solutions automatically. At one extreme is the statement by one author that "an ideal method...should be fully automatic." At the other is the WD program, which intentionally is written to do only one iteration at each job submission, and thus requires personal involvement. Although there is only one iteration of WD per run, many subsets of the base parameter set can be solved, which permits Interactive Branching (Wilson 1988). This provision allows for changes in the parameter set, according to experience gained in the progress of the overall solution. In still another area, several authors have emphasized the importance of solving synthetic data that includes simulated observational scatter (e.g., Linnell 1970, 1973, 1984, 1986, 1989, 1991d; Hill 1979; Popper 1984). The idea is to verify that parameters are recovered within the accuracy indicated by their standard errors, and to uncover any problem areas in the overall procedure. It seems likely that most programs have been tested in this way, whether or not the outcome has seen publication. Linnell has discussed the practice especially thoroughly, including recovery of star spot parameters.

4. ASTROPHYSICS WITH THE NEW MODELS

Light-curve models provide insights and numbers for real binaries that can be used to improve theories of stellar structure and evolution. Unquestionably the insights and numbers now are strikingly better than before the new models appeared. The most important improvement is in our confidence in regard to both kinds of advance. Solutions of rectified light curves commonly turned up dubious or even unphysical characteristics. For example, suspiciously many solutions required an extra source of constant flux, or third light, with no confirmation (e.g., spectroscopic) of its reality. Algol systems often showed an unphysical negative luminosity for the secondary star.[1] W UMa binaries often came out detached, although physical arguments require thermal and mechanical contact.[2] Curiously, there was no tradition of derectifying the solution curves (applying rectification in reverse) so as to compare a computation with the direct observations, and in the few examples where this was done, agree-

[1]Algol-type binaries are understood to have undergone large scale mass exchange following evolutionary expansion of the originally more massive star, so as to make that star now the one of lower mass. An Algol thus consists of an evolved low-mass subgiant and a more massive main-sequence star, with a slow trickle of mass transfer still proceeding.

[2]W UMa-type binaries are main-sequence overcontact objects of sufficiently low temperature to have convective outer envelopes (spectral type F and cooler). They are discussed in the next section.

ment usually was poor, even to eye inspection. Such problems are greatly reduced with the new models, although lesser ones do persist. Continued refinements and additional features are needed to represent some of the very interesting binaries.

4.1 Background—Morphological Types

A facet of the light-curve modeling problem related to the properties of equipotentials is the morphology of close binaries, which concerns the relations between stars and limiting lobes (Kuiper 1941; Kopal 1954, 1959). One must have a good understanding of this subject to appreciate certain astrophysical advances made possible by physical models. A limiting lobe is the volume enclosed by a special equipotential surface—the one that marks the largest a binary-star member can be without having its surface material spill away via the "lobe overflow" mechanism. The physical concept is quite simple—somewhere on the line between the two stars the effective gravity (including the gravities of the two stars and centrifugal force) must be zero, since the main two forces (star gravities) oppose one another. Because net gravity is zero at this point, nothing counters the pressure gradient force of the gas, which will drive an outward flow. The equipotential configuration in the vicinity of the gravity null point creates a nozzle through which rapid mass ejection occurs, provided that the star is as large as the lobe. The most straightforward destiny for the gas is to fall onto the other star and be directly accreted, while another possibility is for the gas to form into a disk and spiral into the other star. The matter-ejecting star is said to fill, or be in contact with, its limiting lobe. A simple case is that of circular orbits, with both stars rotating at the same angular rate as the orbital motion (synchronous rotation), so that the whole system turns together. This synchronous, circular orbit case of a limiting lobe is traditionally called a Roche lobe (some persons use that name for any limiting lobe based on the point-mass approximation). A binary with one star in contact with a limiting lobe and the other lying inside its lobe is said to be semidetached, although semicontact would seem more directly descriptive. If neither star fills its limiting lobe, as with most binaries, the system is called detached. Nature shows that synchronously rotating systems can exist in which both stars overfill their limiting lobes. Each star would "like to" dump matter on the other, but a standoff has been reached in an essentially static situation, with no major flows either way. Such a binary has a common envelope whose surface follows a single equipotential higher than the Roche lobes, and is thus overcontact. The names of the three types seem to have been used first as a full set by Kopal, with the first really self-contained explanation in Kopal (1954), but Kuiper (1941) obviously understood the relevant principles and used the term overcontact. The three cases—detached, semidetached, and overcontact—form a complete set of physically meaningful morphological types for the case of synchronous rotation and circular orbits. However, we shall see in the next section that one more type is needed if rotation is nonsynchronous.

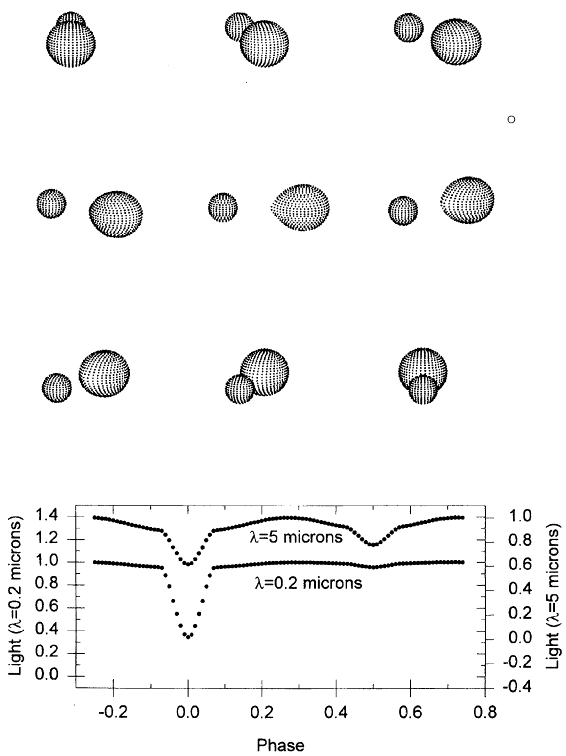

FIG. 2—The semidetached morphological type is illustrated by pictures of the Algol-type binary GT Cephei. Theoretical light curves in the infrared (5 μm) and ultraviolet (0.2 μm) show the strong wavelength dependence of eclipse depth for Algols. The orbital period of GT Cephei is 4.91 days.

4.2 Photometric Mass Ratios

In addition to the obvious structural and evolutionary consequences of the semidetached condition, there is an important connection with light curves. Eclipses can tell us the sizes of the stars relative to their separation, as shown by a few obvious thought experiments, which will be left to the reader (or see Wilson 1994, and think about eclipse durations and depths). If a star fills its Roche lobe (see Fig. 2), the star size tells the lobe size, again relative to the binary separation. Now the relative lobe size certainly must be related to the binary system mass ratio, since the point of zero effective gravity will be closer to the lower-mass star, at least in the synchronous case, so a light curve should be able to tell us the mass ratio. The logical chain is

$$\text{light curve} \rightarrow \text{star size} \rightarrow \text{lobe size} \rightarrow \text{mass ratio}.$$

However the case of one star filling its lobe is not the only one that can provide a mass ratio. In an overcontact binary, both stars are larger than their limiting lobes and share a common outer envelope whose surface follows a single equipotential. Here we do not have active lobe overflow in the sense of a fast moving stream, because each star faces onto the other, which provides a back pressure, rather than onto a vacuum, which does not. Obviously the geometry of this dumbbell shaped configuration also can tell the mass ratio, since the big end of the dumbbell goes with the more massive star (each star is only a little larger than its lobe). In the favorable case of total-annular eclipses, an overcontact photometric mass ratio is very accurate and reliable. Usually it is much more accurate than a spectroscopically determined mass ratio, in that situation. Notice that, contrary to a common misconception, it is primarily star size rather than star shape that tells the mass ratio. Experiments with synthetic data (Terrell, unpublished) show excellent reliability in recovering known mass ratios in suitably constrained light-curve solutions, but not in unconstrained ones. For a detached binary or one with partial eclipses, a light curve ordinarily carries insufficient information to fix the mass ratio reliably. Double-lined radial-velocity observations then offer the only way.

Kopal (1959, pp. 490–496) introduced photometric mass ratios in a discussion of mass estimates for semidetached systems. Much earlier however, F. B. Wood (1946) invoked the lobe-filling condition as a limiting case in finding a limit on the mass ratio of R Canis Majoris an Algol-type binary. R CMa is now considered semidetached, although that was not known in 1946. Thus Wood's limiting mass ratio becomes, in retrospect, the first photometric mass ratio. Kopal's (1954) paper on close binary morphology contains a discussion of point-contact configurations that essentially provides the idea of photometric mass ratios for overcontact binaries, but stops just short of completing the idea. The first physical model solutions of W UMa stars with complete eclipses naturally produced the first overcontact photometric mass ratios (Mochnacki and Doughty 1972a, 1972b; Wilson and Devinney 1973; Lucy 1973; Berthier 1975).

Photometric mass ratios provide a good example of physical thinking stimulated by physical models. Examine old papers that made use of a rectifiable model. Although there is no limitation by which a mainly geometrical model allows only geometrical thinking, in practice that was almost the invariable rule. Even with a rectifiable model one can make a fairly good mass ratio estimate simply from the mean relative radius (R/a) of a lobe-filling star, but this seldom was done. With a physical model the mass ratio is incorporated naturally into a solution, and it becomes unavoidable to think physically.

4.3 Understanding the W UMa Stars

What have we learned astrophysically from the new light curve models beyond some accurate mass ratios? Of course there are the absolute masses obtained for single-lined binaries having photometric mass ratios, and there are more accurate star dimensions, but there is much more. Whole new areas of investigation have developed, some with a substantial literature already and others rapidly building toward one. The first major area was the structure and evolution of overcontact binaries and, in particular, the W UMa stars. With the new light-curve models, we suddenly had a tool for extracting reasonably unbiased estimates of dimensions, mass ratios, and other properties of overcontact binaries, and this made it possible to draw new evolutionary conclusions and test evolutionary models. W UMa's are main-sequence binaries. Unlike the moderately advanced stages of stellar evolution, in which numerous basic problems remain unsolved, the structural theory of main-sequence stars is a mature field in which newly recognized problems do not appear often. With the W UMa binaries, Kuiper (1941) had pointed to a serious paradox—although they surely are main-sequence objects, and thus should follow the main-sequence mass–luminosity law, they obviously do not, with the secondaries grossly overluminous for their low masses. The paradox can be viewed in terms of surface temperature, which increases strongly with mass on the main sequence, whereas components of W UMa binary have unequal masses but almost equal surface temperatures. What is going on? Kuiper (1948) suggested that thermal energy is being exchanged by flows, with net energy transfer from the higher to lower-mass star. Kuiper had uncovered the kernal of an important problem and provided an idea through which it might be solved. Regrettably, Kuiper's suggestion did not lead to a solution, apparently because his one page note drew no significant attention. The next two decades saw important observational activity, with many accurate W UMa light curves published (mostly by Binnendijk), but Kuiper's paradox remained unresolved. In fact, an entire school of thought arose, according to which the W UMa's were not overcontact at all. This notion seems to have been spawned partly by the frequent finding of detached configurations by the light-curve models of the time. There may also have been some wishful thinking by advocates of those models, which clearly did not apply to overcontact binaries, so it would be convenient if there were no overcontact binaries. Although it was well established in the literature that W UMa secondaries are enormously overluminous for their masses, with about the same temperatures as the more massive primaries, the "detached" school steadfastly held its ground. But how are the overluminous secondaries to be explained if the stars are separated and cannot exchange thermal energy? This uneasy standoff persisted until work by Lucy (1967, 1968a, 1968b) made it clear that, if the adiabatic parts of the two convective envelopes touch one another, the thermal contact needed to transfer luminosity will be provided. So W UMa-type binaries have common convective envelopes and must be overcontact. Lucy's idea stimulated an avalanche of papers on the structure, evolution, and observational situation of W UMa stars, such as Moss and Whelan (1970), Moss (1971); Whelan (1972a; 1972b); Hazlehurst and Meyer-Hofmeister (1973); Rucinski (1973; 1974); Biermann and Thomas (1973); Flannery (1976); Lucy (1976); Shu et al. (1976); Lubow and Shu (1977); Robertson and Eggleton (1977); Webbink (1977); Wilson (1978); Lucy and Wilson (1979), and many others. While this spate of contributions left several major issues unresolved, it convinced essentially everyone that W UMa's are indeed overcontact, and thus that their light curves need to be based on the mathematics of level surfaces rather than that of corrected spheres or ellipsoids. Indeed, the new light-curve models (e.g., Mochnacki and Doughty 1972a, 1972b; Wilson and Devinney 1973; Lucy 1973) did find overcontact in nearly every case, with the few exceptions attributable to accidental or slight systematic error. With the overcontact geometry handled correctly the various parameters can be regarded as meaningful (viz., e.g., Rafert and Twigg 1980 on

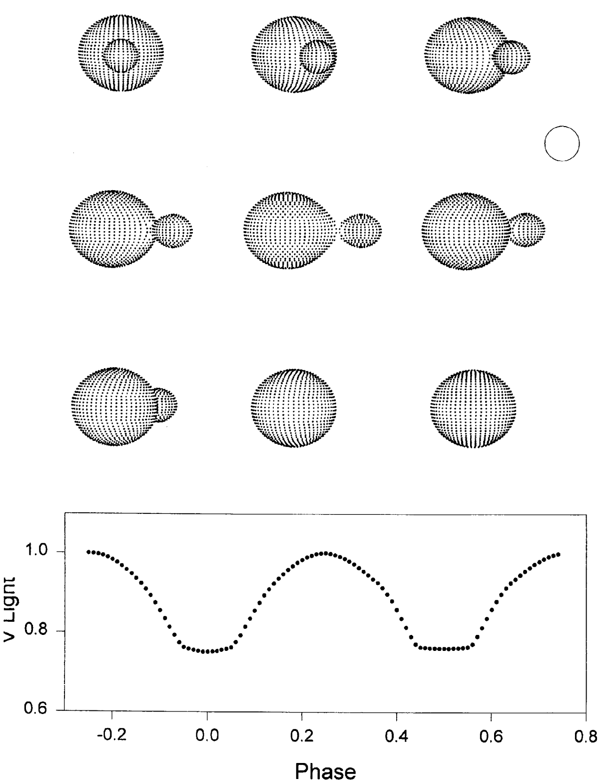

FIG. 3—FG Hydrae is a binary of the W UMa-type that belongs to the A subclass. Together with the light curve, the pictures of FG Hya illustrate the main basis for photometric mass ratios for completely eclipsing overcontact systems. Note that the eclipses, although total annular, are not very deep, which implies rather unequal masses. The mass ratio of FG Hya is about 0.15, and the period is 0.328 days.

the statistics of gravity brightening), whereas with rectifiable model solutions they were nearly useless. In retrospect, it should have been realized that the almost equal effective temperatures imply thermal contact, and thus mechanical overcontact. However, few did recognize the implication, so the serious need for new light-curve models went mainly unrecognized until the work of the late 1960s and early 1970s. The new models also provide a basis for understanding the two observationally defined subclasses of W UMa's, the W-types and the A-types (defined observationally by Binnendijk 1965, 1970), in structural and evolutionary terms (Wilson, 1978; Lucy and Wilson 1979). It was found that the W-types are only slightly overcontact, are close to the zero age main sequence (ZAMS), and have moderate mass ratios such as 0.4 to 0.6, while the A-types are well overcontact, are evolved somewhat beyond the ZAMS, and have mass ratios even as low as 0.07 for the remarkable system of AW UMa.

Such spectacularly extreme mass ratios are themselves one of the more astonishing discoveries made possible by the new models. Actually, one can demonstrate that binaries such as AW UMa, FG Hydrae, and RR Centauri have mass ratios very far from unity merely by inspection of their light curves. Figure 3 shows computer-generated pictures and observational light curves of FG Hya. Note that the eclipses are obviously total annular (nearly flat bottomed), yet are not very deep. What does that tell? We already know that the component surface brightnesses are nearly equal in W UMa binaries so, in the first approximation, only relative sizes are involved in determining eclipse depths. Just by thinking about the overlap of projected areas, we assign the largest depths to equal-size stars, smaller depths to unequal sizes, and realize that, for the total-annular case, eclipse depth must depend monotonically on size ratio. Finally, recall that the overcontact equipotential configuration relates unequal sizes to unequal masses. Thus the small light-curve amplitudes unequivocally translate into very unequal masses, which could have been noticed well before the new light-curve models provided accurate mass ratio estimates, yet it was not noticed. It seems that having a physical model really does spur physical thinking.

4.4 The Reflection Effect

The history of the reflection effect provides an interesting example of the value of direct comparison of a proper model with unaltered observations, as opposed to comparison of an overly simplified model with rectified observations. To appreciate this point, it must be realized that illuminated stellar atmospheres were considered an important theoretical problem as least as early as work by Milne (1926) and that checks against light curves of photoelectric accuracy have been available since the 1920s. Now it happened that despite this background, a major discrepancy between theory and observation, a factor of 2 in a readily observed effect, went unnoticed until 1957. It seems not to have aroused much attention until the late 1960s and early 1970s, by which time hundreds of papers had been published with solutions of rectified light curves with virtually no notice of the problem. At issue here is the amplitude of the reflection effect from stars whose outer envelopes transfer thermal energy mainly by convection (briefly called convective envelopes). Now suppose one takes a properly constructed light-curve model, either one of today or of 20 years ago, and assumes conservation of energy in the local heating and reradiation of the reflection effect, which at first sight seems reasonable. Next, compare the output with observed light curves of Algols. The computed curves will not fit the observed ones because the reflection variation simply will be too large in the theory. This discrepancy will be seen with most of the Algols one might try, and in some non-Algols where the main reflection effect arises from a star with a convective envelope. For Algol itself, the discrepancy is very obvious. One cannot miss this when plotting direct theoretical light curves of appropriate binaries together with directly observed ones. How was it missed until 1957? It was missed because of preoccupation with the phase law of reflection (the form of the variation with phase) rather than the overall amplitude. The key assumption was that local energy conservation in reradiation could be trusted, so whatever amplitude of reflection was indicated by the light curve must be all right and need not be checked. It was Hosokawa (1957, 1958, 1959) who did notice. Sobieski (1965) confirmed Hosokawa's finding by comparing predicted and observed reflection for seven binaries, most of which are Algols. Understanding of the phenomenon came with Rucinski's (1969, 1973) work on the structure of irradiated convective envelopes, and it is now standard practice to recognize a parameter called the bolometric albedo, which is the local ratio of reradiated to incident energy over all wavelengths. The bolometric albedo should be unity for radiative envelopes and about 0.5 or so for convective envelopes, as originally estimated by Rucinski. Put simply, this means that a star with a convective envelope reradiates locally only about half of the external heating energy, while the rest comes out globally over the entire surface. Rucinski's idea soon was tested in five Algols by Napier (1971) with a spherical star reflection model, and for Algol itself by Wilson et al. (1972) with the physical WD model. As in Hosokawa's original statistics, the observed effect agreed with the Rucinski theory. In a way, this discovery did not need the new light-curve models because it is obvious even with, say, the Russell model. The discovery was long delayed because no one was making direct comparisons between light curves based on proper radiative theory and observations. Instead, the observed light curves were altered (rectification), and the rectification process adopted by fiat whatever reflection amplitude appeared in the observations. In another way, therefore, the discovery did need the new light-curve models, with their direct confrontation between theory and observation. Had those models somehow magically existed in the 1930's, convective global redistribution of irradiance energy would have been found 20 years sooner.

Comparisons of observed and theoretical light curves uncovered another very interesting aspect of the reflection effect in the dwarf X-ray binary Hercules X-1/HZ Herculis, and perhaps it is not a coincidence that this happened soon after the first physical light-curve models appeared. Briefly, HZ Her has a far different phase law than theory predicts (Wilson 1973a), which is not a subtle point, as the theoretical law needs to be shifted half a cycle and turned upside down to agree with observation. This phase law is what one expects if the real irradiated cap extends more than half way around the star, rather than less than half way, as predicted by simple line-of-sight irradiation (Wilson 1973a, b). Since superficial surface currents have insufficient heat content to carry energy for long distances, the envelope must be affected to deep layers. Rucinski's (1969) mechanism may be at work to carry thermal energy to longitudes that are out of sight of the X-ray emitting neutron star. Some years later, Kippenhahn and Thomas (1979) rejected Wilson's starting point, that superficial currents cannot do the job, and developed a circulation model which might be able to carry sufficient flux. The circulation extends well within the envelope (KT mention 20% of the radius), so they actually are in agreement with Wilson, although they claim to be opposed. In any case, further work is needed on the transport of irradiation energy in stellar envelopes.

4.5 Fast Rotation and Photometric Rotation Rates

With the experiences of the overcontact W UMa binaries and convective albedos in hand, it is natural to ask if we have been missing some other major physical effect. Indeed, there is one that is sure to be important in some binaries, namely, fast rotation. Although it has been known for many decades that some of the more interesting Algols have fast rotating primaries, no actively used light-curve models incorporated nonsynchronous rotation until about 15 years ago, and most still do not. We are not dealing here with a subtle or even slightly esoteric phenomenon. The figure and surface-brightness distribution of a star are strongly affected by its rotation as it becomes oblate and, for extreme examples, even lenticular. Figure 4 shows RZ Scuti, while other examples can be found in the *Pictorial Atlas* (Terrell et al. 1992). Of course, we cannot expect to obtain reliable results if we ignore fast rotation where it occurs. Rotation rates derived from light curves have just begun to come in over the past few years. A number of Algol primaries rotate very fast, yet their rotations were paid little attention until new light-curve solutions made it clear just how fast they do rotate—up to the centrifugal limit in a few examples. One usually thinks of spectral line broadening as the carrier of information on rotation, but in favorable cases a light curve does so also. To appreciate why this is so, think about the physical effects of rotation on a star and then about the consequences of those effects for the light curve. A rotating star develops both oblateness and a pole to equator variation in surface brightness, with the latter arising from the phenom-

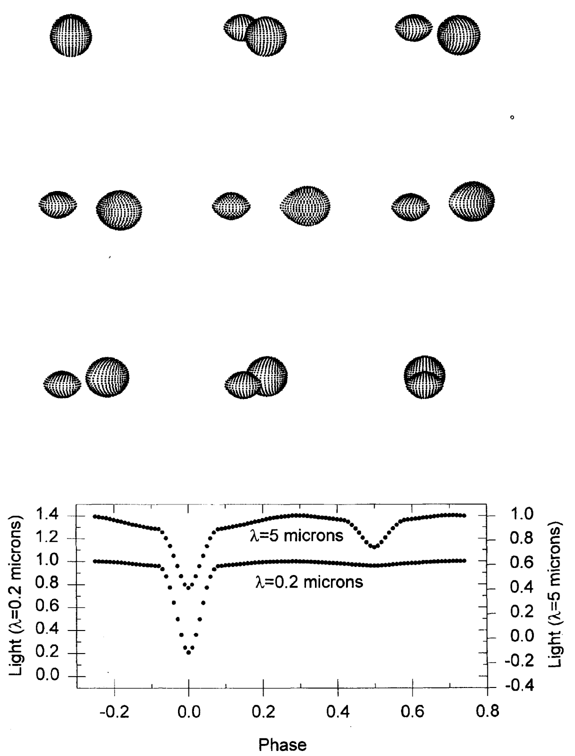

FIG. 4—RZ Scuti is a very unusual semidetached binary with a primary star, classified B2 II, that rotates at or near to its centrifugal limit and an A0 III Roche lobe-filling secondary. Notice the strong oblateness of the B2 star, caused by its rotation. This binary may be a double contact system. The orbital period is 15.9 days. A sense of the large scale of RZ Sct is provided by the small Sun circle at the upper right.

enon of gravity brightening (Von Zeipel 1924a, b, c), which also is called gravity darkening. Gravity brightening is strong in stars hot enough to have radiative envelopes, such as Algol primaries, where the bolometric flux is directly proportional to local gravity. For modest rotations, such stars can be approximated by ellipsoids, but a star with extremely fast rotation actually becomes lenticular and its equatorial surface brightness becomes very low, theoretically approaching zero in the centrifugal limit. In a binary member, effective gravity will not actually go to zero everywhere on the equator in the centrifugal limit, but only at the substellar point. However it will be very low around the rest of the equator. So we have a star with a major geometric distortion and highly nonuniform surface brightness. Surely these effects will influence the light curve. A little intuition shows that the depths and shapes of eclipses will be affected, as will the variation between eclipses. There seems to be no competing parameter whose variation would mimic that of rotation, so it should be possible to estimate rotation from light curves, at least in some favorable cases. In fact, this has been done for some time now. The modeling concepts (Wilson 1979) were published six years before the first practical example (Wilson et al. 1985, hereafter WVP), but now photometric rotation determinations are becoming fairly common (e.g., Wilson and Plavec 1988; Wilson and Mukherjee 1988; Etzel and Olson 1993; Olson 1989; Van Hamme and Wilson 1990, 1993; others). Experience and intuition agree, however, that photometric rotation estimates are feasible only for quite rapid rotation, say at least half the centrifugal limit. Insensitivity to rotation occurs because the physical effects grow nonlinearly with increasing rotation (Wilson 1989), only becoming really large as the centrifugal limit is approached. It must be said that, although light curve and line broadening rotation measures often are in reasonably good agreement, there are counterexamples. It is not clear whether the discrepancies are due to timewise variation of rotation (with the epoch of the spectra much different from that of the light curves), ordinary observational uncertainty, differential rotation, or some other problem. However we are only at the beginning. Refinements surely are needed and refinements surely will come.

4.6 Slow Rotation

Subsynchronous rotation is a timely subject now because of great interest in the phenomenon of common envelope evolution, which is understood to produce a variety of very small bizarre binaries. Examples are the tiny cataclysmic variables (CV's), which contain white dwarf stars and explode as classical novae, recurrent novae, and dwarf novae. Common envelope evolution (of the type under discussion here) proceeds via the evolutionary expansion of one member of a binary so that its outer envelope begins to engulf the companion (viz., e.g., Sparks and Stecher 1974; Paczynzki 1976). The subject now has a very large literature, with recent reviews by Webbink (1992), Taam and Bodenheimer (1992), and Iben and Livio (1994). The orbit decays in a tight spiral so that the orbital motion becomes faster than the rotation, which cannot keep up through the usual tidal locking mechanism. The circumstance can occur in an X-ray binary, where the companion is a neutron star, of which HD 77581/Vela X1 (Van Paradijs et al. 1977; Wickramasinghe et al. 1974; Zuiderwijk et al. 1974; Wilson and Terrell 1994; many others) is an example. Naturally, a light-curve model for such a star needs to incorporate nonsynchronous rotation, especially since the supergiant may be very close to filling its limiting lobe. Particularly interesting for HD 77581 is that the supergiant could not produce eclipses of the observed width if it rotated as fast as synchronously—it would be limited by its Roche lobe to be smaller than it actually is.

4.7 Will the Real Undersized Subgiants Please Stand Up?

The case of the "undersized subgiants" provides another example of progress helped by physical light-curve models. Undersized subgiants were advanced (e.g., Kopal and Shapley 1956; Kopal 1959; Hall 1974) as a new class of evolved binaries, and indeed most examples have turned out to be among the extremely interesting RS Canum Venaticorum stars, which in fact contain "undersized" (i.e., detached) subgiants. However some of the originally proposed examples (e.g., S Cancri, TT Hya) have light curves typical of Algol (i.e., semidetached) systems, which are understood to be in a quite different evolutionary state than are RS CVn's. Physical light-curve models have been very useful in distinguishing actual "undersized" subgiants from ordinary Algols. Developments with S Cnc have been particularly unusual—one might even say bizarre. As shown by Weis (1976), Batten (1976), and Popper and Tomkin (1984), the radial velocities, upon which its supposedly detached nature was based, are not useful for measuring orbital motion. Not only do they have excessive scatter and wrong amplitude, but even have wrong phase behavior. The old velocities show the primary star approaching before its eclipse and receding afterward, which obviously disqualifies them for estimates of system parameters. Subsequently, Popper and Tomkin (1984) produced much better velocity curves for both components, and these have been used together with light curve analysis (Van Hamme and Wilson 1993; Olson and Etzel 1993) to investigate S Cnc's dimensions and masses. However the story is not over. While VW assumed a semidetached configuration and arrived at an acceptable (to VW) overall solution, OE's light curve fitting experiments led them to the opinion that the low-mass subgiant is detached! This seemingly small distinction between filling versus only near filling has major consequences in regard to the intermittency of lobe filling, time scales for changes in radius, and what is meant by an Algol-type system. Is the OE detached solution for S Cnc essentially correct? For now, we await a new round of observations. Do the Algol-type secondary stars in general fill their limiting lobes virtually exactly, as the bulk of present evidence is ordinarily interpreted, or are some photospheres sensibly below the lobe surfaces? While this might seem a fine point, it could influence our estimates and validity of photometric mass ratios. We may have a new active problem.

4.8 RS CVn's, W UMa's and Spot Modeling

A discussion of the RS CVn stars, including the observational and interpretive contributions of Struve, Hiltner, Gratton, Grygar, Popper, Oliver, and Hall, among others, is contained in Hall (1976). A history of spotted stars of several kinds is that by Hall (1994). Key theoretical papers are by Popper and Ulrich (1977) and Morgan and Eggleton (1979). Consider whether RS CVn's, being detached binaries, can adequately be handled by ellipsoidal or spherical models. In a typical configuration the hotter star is quite small compared to its Roche lobe and the cooler star larger, but also well detached. It sounds as if a simple model will do. However, one of the main problems with light curves of RS CVn's concerns reliable extraction of information on the dark magnetic star spots that cover large parts of their surfaces, mostly on the cooler components. The light-curve distortion waves produced by spots are mixed in with the ordinary proximity effects due to tides and external heating and, while the proximity effects are small in some RS CVn's, they are large enough to cause problems in some others. When this point is coupled with the well-known difficulties of estimating spot parameters even for uncomplicated cases, it is not clear that good results can be had from simple models in general, although a simple model may often be satisfactory. The implications of spots for models of W UMa systems also are quite important (viz., e.g., Linnell 1991a, b, d). With W UMa's proximity effects are large and must be modeled accurately.

Accordingly, there are many general approaches. Budding (1977) and his collaborators rectify for spot effects (i.e., remove them from the observations), then solve the spot-rectified light curves for ordinary binary parameters. This procedure is iterative, so the first step is to fit the originally observed curves without considering spots, and the final step is to fit a distortion wave (obtained by differencing observed curves and unspotted model curves) for spot parameters. This procedure has been tested via solutions of simulated observations (Rhodes et al. 1990) and has produced apparently good results (e.g., Budding and Zeilik 1987 and contained references) for numerous RS CVn's. The underlying model is of spherical stars. Budding's spot generation algorithm has been adopted in Linnell's (1989) model. Most modelers (Eaton and Hall 1979; Yamasaki 1982; Poe and Eaton 1985; Strassmeier 1988; Kang and Wilson 1989; Linnell 1991a, b, d; Wilson 1992) include spots in the overall model and do not rectify. According to this latter approach, spot effects need not be separated methodwise in order to be separated parameterwise. Furthermore, with a unified method, one may be able to learn about correlations between spot parameters and other parameters. These are equipotential star models, except for Poe–Eaton (spherical) and Strassmeier (ellipsoidal). The Eaton–Hall and Strassmeier spot models consist of irregular shapes and bands of undulating brightness, while the others are simple geometrical figures, mainly circles. The WD model now allows spots to move with the physical surface of a nonsynchronously rotating star (Wilson 1992), which could be useful for following spot migrations over several months. Observationally, both short term and long term time coverage is crucial for proper interpretation, including migrations. Most spot solutions to date are by trial and error. Exceptions are Kang and Wilson (1989) on three RS CVn's and Samec et al. (1993) on CE Leonis, which are by least squares (although the present version of the WD program also adjusts spots by least squares, the programming is independent of that in Kang and Wilson 1989). So the combinations are many—rectification or no rectification—spheres, ellipsoids, or equipotentials for the stars—circles or more complicated shapes for the spots—moving or stationary spots—trial and error fitting or least squares. Add to these combinations that some applications are to RS CVn's and others to W UMa's. In the long run the more sophisticated and general models and procedures should win out, but there are practical hardware and software problems along the way and the simpler methods have a useful place, for now. It could be advantageous to combine light-curve spot modeling with the successful work of Vogt and collaborators (e.g., Vogt 1981; Vogt and Penrod 1983) on spot reconstruction from absorption line profiles. Although spot information from light curves is comparatively rough, light curves reach fainter limiting magnitudes, are more readily obtained, and exist over longer time spans than suitable line profile observations. Spot models obviously are in their early stages of development, and exploration of their full possibilities has only begun.

5. LIGHT-CURVE MODELS AND MORPHOLOGY

The concept of close binary morphology outlined above is threaded through nearly all research on interacting binaries, whether on classes or individual systems, whether on structure, evolution, statistics, or other general areas. The three types defined by Kuiper and Kopal continue to guide us through the labyrinth of the binary-star zoo, while testimony to the soundness of the underlying ideas is shown by their increased usefulness over the years. Close binary morphology enables us to recognize configurations into which real systems have been driven by physical processes and conditions, and thus to understand those processes and conditions. Examples are evolutionary expansion, gravitational radiation, mass loss and exchange, and magnetic braking. For example, we see the overcontact configuration in the W UMa binaries, where it shows that a range of overlap in star dimensions can be tolerated without serious disturbance of hydrostatic equilibrium. Attempts to understand how the W UMa's enter and maintain this uneasy state have led to large numbers of contributions on the physics of stellar interactions. We recognize the semidetached condition in Algols, cataclysmic variables, some X-ray binaries, and other types, as a situation produced by evolutionary expansion or orbit shrinkage. The wealth of opportunities for testing astrophysical theory here could not be summarized within a reasonable length, but all such theory needs to be consistent with close binary morphology. Even the detached type can help in our astrophysical understanding, as with the RS CVn stars, which are evolved yet detached binaries. The detective work which produced our present view of RS CVn's (Popper and Ulrich 1977; Morgan and Eggleton 1979) included morphological arguments. The overall lesson is that special equipotential configurations (morphological types) play major roles in binary-star astrophysics because natural processes drive binaries into those configurations and maintain them there for significant intervals.

Now for a question: are any further types needed, beyond the original three? The possibilities for each star are that it can be detached from its lobe (symbol $=d$), in accurate contact with its lobe (symbol $=c$), or overfill its lobe (symbol $=o$), so the possibilities for two stars are dd, cc, oo, dc, do, and co. We recognize dd, dc, and oo, respectively, as the original types of detached, semidetached, and overcontact binaries. The do and co configurations are not expected to achieve hydrostatic equilibrium because their level surfaces do not close. Therefore they should not persist for longer than the binary system's dynamical time scale, which in most cases of interest is not longer than a few days. What about cc? One can imagine two stars placed such that each just fills its Roche lobe. If each already had the size dictated by the astrophysics of stellar structure, there would be no violation of hydrostatic equilibrium, and the system could remain essentially in that state for a significant time. However, there would seem to be no known or readily imaginable natural sequence of events to cause a binary to attain such a configuration, nor would there be any reason for the configuration to persist if perturbed. Any size change due to evolution would render the binary semidetached or overcontact. So it would seem natural to conclude that the cc type should play no role in astrophysics.

5.1 A New Morphological Type

At least that is the situation under one of the rules by which the morphology game has been played—synchronous rotation. Remember now the fundamental requirement for recognizing a morphological situation as meaningful—a physical process that drives a system into the situation and keeps it there. Now, as mentioned above, some Algol primary stars rotate much faster than synchronously, with a few even close to or approximately at the centrifugal limit. The only reasonable mechanism to account for such fast rotation is spin-up by the accretion process. That is, gas transferred from the contact component arrives with considerable angular momentum and acts as an agent for converting orbital to rotational angular momentum. With supersynchronous rotation there will still be a limiting lobe, but it will be smaller than for synchronous rotation because centrifugal force (outward) is now larger and will balance gravitation (inward) closer to the star. Of course, a small star can rotate faster than a large one before its surface becomes centrifugally unbound. Here we have a new idea for having a star in contact with its limiting lobe. Instead of the star expanding to reach the lobe, we have the lobe contract (via spin-up) to meet the star. Since the synchronously rotating secondary star already fills its lobe, both stars now accurately fill their limiting lobes and we have a fourth morphological type, a double contact binary (Wilson 1979). Solutions of light and velocity curves (cf. Sec. 4) for fast rotating Algols have turned up a few candidates for double contact such as in WVP, Etzel and Olson (1993), and MPW. While such objects are rare, they may be missing links in binary-star evolution.

6. RECENT IMPROVEMENTS

Light-curve modeling is an active field, with many advances now in progress between conception and full development. Among those that are new and essentially completed, or have achieved new levels of accuracy or sophistication are

(A) improved radial-velocity measurement by means of a light-curve model (Hill et al. 1989; Hill 1993; Hill and Rucinski 1993),
(B) improved computation of predicted radial-velocity curves via a light-curve model (Van Hamme and Wilson 1994),
(C) accurate inclusion of stellar atmosphere radiative behavior (Linnell 1991c; Milone et al. 1992),
(D) thorough discussion and tabulation of limb darkening (Van Hamme 1993),
(E) accurate and efficient treatment of reflection (Wilson 1990),
(F) use of a light-curve program to estimate rotation from spectral line broadening (Rucinski 1992, 1993a, b; Mukherjee 1994; MPW),
(G) direct computation of polarimetric curves for combined photospheric and circumstellar polarization (Wilson and Liou 1993),
(H) interactive screen graphics programs (Bradstreet 1993a, b; Terrell 1992).

Some of the above and certain other improvements were discussed by about 40 authors and participants during General Assembly XXI of the International Astronomical Union, at a meeting organized by Milone. A number of contributions from that meeting, "Light-Curve Modeling of Eclipsing Binary Stars," are among the references.

6.1 Radial Velocities

Items (A) and (B) represent two basically different ways to attack the problem of velocity curve distortions due to surface effects. Briefly, radial-velocity measurements are not just those expected for a point source, but are affected by a star's nonsphericity, surface brightness, and line strength variation over the surface, aspect dependence of spectral line strength, and eclipses. In binaries with strong tidal distortions or reflection heating, this problem can produce significant errors in mass estimates and other quantities derived from velocities. In approach (A), one heads off the problem in the measurement process by computing line profiles, including wavelength shifts, and using them as templates for cross correlation. Hill (1993) and Hill and Rucinski (1993) discuss progress in method (A). In (B), one applies corrections to the computed velocities. While each way is a major improvement on the simple point source assumption, notice that once a theoretical profile of incorrect shape has been fitted to an observed profile, especially an asymmetrical one, it is no longer rigorously clear what the estimated wavelength (or velocity) means. Therefore the Hill, Fisher, and Holmgren method (see Van Hamme and Wilson 1985, for original suggestion) is to be preferred, where available. On the other hand, approach (B) needs only an ordinary computer and proper software, and can be applied to already published data, whereas approach (A) requires measurement apparatus that now exists only in a few places. Item (B) actually has a long history going back at least in concept to Sterne (1941) and in quantitatively useful form to Hutchings (1973); Crampton and Hutchings (1974); and Wilson and Sofia (1976). Van Hamme and Wilson (1994) now include weighting of velocity integrals by line strength, which is influenced by aspect, local temperature, and gravity.

6.2 Local Radiation

The Linnell (1991c) and the Milone et al. (1992) stellar atmosphere capabilities are important because, contrary to some elementary text books, most stars are approximated only very roughly by black bodies. Some light-curve programs have included stellar atmosphere corrections (Wilson 1992) or empirical corrections (Hill and Rucinski 1993) for years, but mainly for main-sequence stars and based on atmospheres less accurate than those now available. Inadequate radiative physics can be quite a significant problem when the binary components have very different temperatures, as in Algols, for example. Typically a computed eclipse will be too deep in one bandpass and not deep enough in another, with no possibility for reconciliation because there is only one radiative parameter, the effective temperature. Van Hamme and Wilson (1986b, 1990) discuss practical ways to cope, but the problem should go away if the model stars can be made to radiate like real stars. Milone et al. (1992) report much improved consistency in fitting multibandpass light curves with the help of their new radiative software, which is based on Kurucz's stellar atmosphere program, ATLAS.

Contributions concerning the form of limb darkening laws and numerical values for their coefficients (e.g., Grygar 1965; Kiperman and Shul'berg 1969; Klinglesmith and Sobieski 1970; Manduca et al. 1977; Al-Naimiy 1978; Wade and Rucinski 1985; Claret and Gimenez 1990; Diaz-Cordoves and Gimenez 1992) are summarized in Van Hamme (1993). An often expressed opinion has been that the linear cosine law, $I/I_0 = 1 - x + x\cos(\gamma)$, is essentially adequate for binary-star light curves because the coefficient x typically is rather highly correlated with other parameters, and thus uncertain. Why apply an elaborate law when the coefficient of the ordinary simple law cannot realistically be determined? However, determination of limb darkening is not the only issue—one should use the best model available so as not to bias results. Furthermore, no significant computing time is needed to apply a more accurate law, and sometimes limb darkening can be separated from other parameters. Probably the most telling remark in this regard is that the linear law usually is quite a poor approximation to real limb darkening, and one extra term can give major improvements. With these thoughts in mind, Van Hamme investigated very thoroughly the accuracies of the main contending nonlinear laws by comparison with the ATLAS stellar atmosphere program by Kurucz (1979, 1991). He also generated bolometric and bandpass-specific coefficients (16 commonly used bandpasses) for the Klinglesmith and Sobieski logarithmic law and for the Diaz-Cordoves square root law, by fitting those laws to ATLAS intensities. In some regimes of temperature and log g, the logarithmic law agrees better with Kurucz model stellar atmospheres than does the square root law, while in other regimes the situation is reversed. An early advocate of the importance of nonlinear laws was Linnell (1984, 1985, 1989). Some form of nonlinear limb darkening now is included in most light-curve models.

6.3 Computing the Reflection Effect

In addition to the structural reflection problem of Sec. 4 for convective envelopes, there have been improvements in the computation of the reflection effect, as the stellar surfaces interact radiatively. The history of this subject is extensive but a reasonably complete summary is contained in Vaz (1985), and further historical comments are in Wilson (1990). In modern computer models the essential problem is not one of reflection theory, which is understood well enough for most purposes, but in applying the theory efficiently. The central difficulty is that a rigorous treatment involves radiative interactions among surface grid elements of two stars, so that computing time scales with the square of the number of grid points per star, assuming equal numbers of points on the two stars. For fine surface grids, reflection can amount to most of the whole light-curve computation. There also is the problem of multiple reflection which increases complexity and computation time. A flexible way to provide for this in a program is to allow the user to choose the number of reflections. The accuracy, efficiency, and multiple reflection problems are developed together in Wilson (1990). Some aspects are to be found in earlier publications, such as Hutchings (1968); Hill and Hutchings (1970); Mochnacki and Doughty (1972a); Binnendijk (1977); Linnell (1984); Kitamura and Yamasaki (1984); and others referenced by Vaz and by Wilson. Comparisons with approximate treatments show excellent agreement in some cases and poor agreement in others. The cases of excellent agreement are those in which the stars have mainly tidal distortions but little rotational distortion, are not overcontact, and for which multiple reflection is a negligible effect (perhaps due to very unequal temperatures). Those can be handled approximately, via an inverse square law in distance from the center of the irradiating star plus penumbral and ellipsoidal corrections, without significant loss of accuracy. However rigor is required for fast rotating stars, overcontact systems, and binaries with multiple reflection, and it now can be obtained within acceptable computing time—usually with not much load on the overall computation. To accomplish this, a strategy for formation and use of lumped computed quantities needs to be properly thought through and applied. That is, the local geometric, bolometric, and wavelength-specific quantities should be grouped for storage according to how often they need to be computed (Wilson 1990). It is helpful if a program provides the option of either a relatively simple and fast or a rigorous reflection computation, so that time is not wasted where rigor is not needed. Numerical and graphical examples related to the importance of various effects, and the issue of when a relatively simple treatment is adequate, are given in many papers on reflection, such as Napier (1968); Kitamura and Yamasaki (1984); and Wilson (1990). Recent contributions on the wavelength-dependence of reradiation and on bolometric albedos are those by Nordlund and Vaz (1990); and Claret and Gimenez (1992).

6.4 Rotation from Spectral Line Broadening

The most commonly used means of estimating rotation is by comparison of photospheric linewidths with those of rotational standard stars (e.g., Slettebak et al. 1975). Rucinski (1992, 1993a, b) now has a process that generates broadening functions from entire spectral windows for overcontact binaries. A slowly rotating standard star is used to remove sources of broadening other than rotation. However, Rucinski's method does not need fast rotating standards, as does the traditional method. Not only is a physical model brought in when fitting the broadening function, but the broadening function is produced by a deconvolution that emphasizes only the really useful sections of the window. Mukherjee (1994) and MPW discuss and apply another new method that directly compares observed and computed profiles, without the intermediary of standard stars. Theoretical profiles are generated by joining line broadening theory with a binary-star light-curve program. Advantages are that the theory can be upgraded without involving any standard stars, that whole profiles are used rather than only half widths, that departures from axisymmetry are included naturally, that observations in eclipse can be utilized, that the amount of observing is reduced by elimination of standard star observations, and that the fitting process is by least squares and thus impersonal, rather than subjective. It can be applied not only to binaries, but also to single stars. Combined with CCD observations, this procedure should yield major increases in the quality and quantity of rotation statistics.

6.5 Polarimetry

Polarimetry of binaries eventually will be done from space, thus providing long, essentially continuous polarization curves. Interrupted polarimetry from the ground has very limited usefulness because the main phenomena are nonperiodic, so one cannot get around the daily interruptions by folding the observations. Photospheric polarization should repeat with the orbital period, but is a minute effect that may have been detected in, at most, a few data sets, of which Kemp et al. (1983) is by far the best example. Scattering from circumstellar gas has been a much more successful area for detections, but to derive major astrophysical information we will have to develop hydrodynamic flow–polarization models and, of course, there will have to be extended observations from space on which to try the models. Terrell and Wilson (paper in preparation) have such a model, although the model's realism is difficult to evaluate in the absence of appropriate observations. The connection with light-curve models comes in mainly through the photospheric effects, in which spatial symmetry is broken by eclipses, gravity brightening, and perhaps reflection. Eclipse polarization signatures are sensitive to geometrical circumstances, so polarimetry offers a way to fine-tune light-curve solutions where those signatures can be observed with reasonably good signal-to-noise ratios. Also, the sense of projected orbital motion and the position angle of the projected normal to the orbit can be found in cases with both eclipse and circumstellar polarization, if the gas can be assumed to be symmetric about the orbit plane. Polarimetric output (photospheric and circumstellar) may be an option in readily available light-curve programs within a few years.

6.6 Interactive Graphics Packages

Interactive user-friendly programs are welcome new arrivals. These generate binary-star pictures on the screen of a personal computer, and display light and radial-velocity curves. One can fit light curves interactively, which makes the process go faster, or at least seem to, as "time goes fast when you are having fun." Some limitations have been imposed by the available memory and speed capabilities of personal computers, but that may be a problem of the past, or soon will be. Because of the use of special facilities (e.g., screen graphics), one needs a version specific to the type of personal computer on which the program will be run. Bradstreet's (1993a, b) Binary Maker is the best known interactive program. Binary Maker includes sample sets of real light and velocity curve observations so as to make start-up easy for beginners, and even has a zoom feature, by which the pictures can be expanded and contracted. The full amenities of Binary Maker can only be appreciated by working with it. Of course any of the various light-curve programs can be made interactive by means of an interface, or "front end." This has been done for the WD program by Terrell (1992), and is called the Wilson–Devinney User Interface. With an interface program, capabilities unique to a given source language program can be accessed interactively, with screen graphics and other advantages of personal computers.

7. WINDOWS OF OPPORTUNITY

The breadth of the field is illustrated by the variety of potential work that could not be included in a review of reasonable length. Proper coverage of relatively untapped areas will not be attempted here, but it may be useful to mention problems that could be more thoroughly explored. The field is more than "light-curve modeling"—a name that has the merit of brevity but does not convey the full scope of related activities. To be more inclusive, one can think of the field as encompassing observables other than flux and covering physical situations which need more thorough attention. Even for light curves, modeling contributions continue to be needed for extended atmospheres, physical eccentric orbit and nonsynchronism effects, and realistic effects of radiation pressure. The hot spots, disks, and other features of cataclysmic binaries need to be modeled more directly and more physically.

7.1 Rectification as a Signpost of Rewarding Work Areas

An indicator of a primitive model is the practice of correcting observations for intrinsically astrophysical effects. This practice can be included under the general heading of rectification. There may be a genuine need to correct for observer-dependent effects, such as due to the Earth's atmosphere and motion around the Sun, but corrections for phenomena inherent to the object of interest are appropriate only in preliminary work. This recognition makes possible quick identification of fertile regions for growth of new models–just scan the journals for rectification procedures (which may not be identified by that name). For example, rectification is now the rule in analysis of binary-star polarization curves. If the immediate interest is in circumstellar polarization, the first step may be to remove estimated photospheric polarization, due to tides and gravity brightening, and then analyze the corrected curves. Much better is to build all effects into the model.

7.2 Spectral Distributions of Circumstellar Gas as Observables

Spectral features of circumstellar gas flows are traditionally seen as unrelated to light-curve models, except as contaminants. The flows are fully dynamical, as opposed to the static or quasistatic physics of light-curve models. They appear most usefully in spectrometry rather than light curves, and are nonperiodic. How could they be more different, and why mention them in a light curve review? Well, think about this. Spectrometry of circumstellar flows contains an impressive wealth of information, certainly far more than typical of light curves, and we have no lack of spectrometric observations. Space observatories return much spectrometry, but little in the way of light curves. Yet binary-star insights from light curves have been at least comparable to those from spectrometry. How can those modest-looking light curves compete in terms of real productivity, as they obviously do? Perhaps because light curves exploit timewise variation better than does spectrometry? With observatories in space, spectrometry may reverse that. Perhaps because static problems are easier than hydrodynamic flow problems, and continuum radiation is easier to compute than line radiation? However those points are intimately connected with the greater information conveyed by flows and by line radiation. In short, spectrometry seems to have all the really fundamental advantages, so again, how do light curves compete so successfully? They compete because the models are direct and quantitative. Light-curve analysis involves computation of hard numbers, while spectrometric analysis typically involves sketches and order of magnitude estimates. However light curves may not retain this advantage indefinitely, because direct and quantitative synthesis of circumstellar flow spectrometry is on the way (Terrell and Wilson 1993). Light-curve programs are to be regarded as observables generating programs, and spectra of circumstellar gas are one kind of synthesized observable. Very general extensions of today's light-curve models will stimulate thoroughly new strategies for observing from space when more fully developed.

7.3 Are Double Contact Binaries Astrophysically Important?

One problem concerns the reality of double contact as a morphological type. If double contact is to be genuinely meaningful, it should not only exist in a formal sense, but be useful in explaining some class of binaries. The binaries it has been proposed to explain (Wilson 1979; WVP; Wilson 1989) are the W Serpentis stars (Plavec and Koch 1978; Plavec 1980). The logical connection between double contact and W Ser stars is that large scale mass transfer spins up accreting stars, probably in some cases to the centrifugal limit, and the idea is that the optically thick disks of W Ser stars represent transferred gas that cannot be accommodated by the centrifugally limited star. The obvious difficulty in trying to confirm this is that the accreting stars are hidden by their opaque disks, so that few properties are subject to observation and measurement. Faced with this problem, several persons and groups (e.g., WVP; Wilson and Plavec 1988; Van Hamme and Wilson 1990; Olson 1989; Etzel and Olson 1993; MPW) have been looking for binaries on the edge of the W Ser phenomenon. That is, they would be ending the rapid phase of mass transfer (Plavec 1968) and becoming normal Algols. The search is for Algols with centrifugally limited (or nearly so) primary stars and reasonably well behaved light and velocity curves that permit reliable estimates of rotation and dimensions. Of course, critically fast rotation and good behavior are almost incompatible characteristics, so only a small number of interesting candidates are known. Part of the evidence is provided by rotational statistics of accreting stars, which show continuity in rotation up to the centrifugal limit (Wilson 1988; 1989; Van Hamme and Wilson 1990; MPW). Some critical rotations seem to have been found, but nearly all are from light-curve analysis, which is still in the testing stage. Only for V356 Sagittarii (R. Polidan, private communication) and perhaps RZ Sct (MPW) is critical rotation indicated by the traditional means of spectral line broadening. The situation is similar to that of judging whether ordinary Algols are in a semidetached configuration, where the first step is to establish that Algol secondaries about match their Roche lobes in size. To do this, one must have an accurate estimate of the mass ratio. The conclusion that they accurately fill their lobes is based on that approximate match plus observations and physical arguments related to modes of mass transfer. With the rotational lobe filling of double contact the situation is trickier because not only is the mass ratio needed, but so is the rotation rate. Again, the overall judgment—accurate lobe filling or not—must be based on physical evidence in addition to a size match. Such evidence could be that the binary behaves in the way expected for centrifugally limited accretion, with backed-up circumstellar gas around the accreting star.

7.4 From Special Geometry to Physical Models—ϵ Aurigae and β Lyrae

A certain amount of exploratory work with purely geometrical models is needed when one deals with poorly comprehended light-curve phenomena, before physical models can take command. For normal eclipsing stars, we passed through this stage from about 1910 to 1930. We have not yet entirely exited that early level of understanding for the classical strange binaries ϵ Aurigae and β Lyrae, whose mysteries have been with us for more than a century. As always, the basic situation must be understood prior to development of a physical model. Recent work on the eclipses of ϵ Aurigae strongly supports the thin opaque ring model with semitransparent central opening around the (almost unobservable) secondary object, proposed by Wilson (1971). The ring, which is topologically similar to the Saturn ring system, produces unique and variable eclipses as it slides by the F supergiant every 27 years. Now Carroll et al. (1989); Ferluga (1990); and Carroll et al. (1991) have added details, in the form of further gaps, to the ring structure, that improve the fit and help in the interpretation of epoch to epoch changes. Observations of mid-eclipse brightening in the 1982–84 eclipse have ruled out the once favored Huang (1965) thick disk model, as emphasized by Ferluga and by Carroll et al. Similar brightening was present in the 1955–57 eclipse (Wilson 1971). The suggestion by Lissauer and Backman (1984) and Eggleton and Pringle (1985) that the secondary object may itself be a binary is likely to be correct, as this could explain a multitude of dynamical and evolutionary difficulties. In particular, Figs. 9 and 10 of the smoothed-particle hydrodynamic disks around binaries by Artymowicz and Lubow (1994) show cleared-out central regions which tie the Lissauer–Backman–Eggleton–Pringle central binary to the Wilson model for the eclipses. No structural model for this kind of disk has so far been attempted, except for exploratory computations (Van Hamme and Wilson 1986a), but a modest extension of the Artymowicz and Lubow models could be exactly what is needed. For example, ring gaps similar to those postulated by Ferluga and Carroll et al. should arise from resonances between the inner and outer binary periods. One could start from hydrodynamic disk calculations and compute eclipse light curves of ϵ Aurigae for comparison with observations.

For β Lyrae, the schematic disk model by Huang (1963) was quantified into a partly physical, partly geometrical light curve model (Wilson 1974b; Wilson and Lapasset 1981). Antokhina (1988) has developed a light curve program based on the same ideas. There are now two structural disk models for β Lyrae (Wilson 1981; 1982; Hubeny and Plavec 1991; Hubeny et al. 1994), and fully physical light-curve computations might help to distinguish between them.

7.5 Applications

This paper primarily concerns models rather than model applications, which could generate another review. Modeling innovations stimulate the imagination, but use of the capabilities that already exist in widely disseminated programs also can be rewarding. New contributions are needed on the statistics of parameters such as limb darkening and bolometric albedos, along the lines of that for gravity brightening by Rafert and Twigg (1980). Even third light has become more interesting, now that photometric indications of companions can be checked with the new generation of interferometers.

Excessive space would be needed to discuss particular findings, oddities, and perplexing items from papers on individual binaries, but anyone interested enough to have read this far will have his or her favorites. Many well accepted astrophysical phenomena were discovered from small things that did not fit the standard picture of the time, so one should not expect ordinary binaries to be ordinary, nor be too quick to sweep strange results under the rug. There has never been a lack of applications of models to real binaries since the first simple models appeared. Where there are models and observations, they will be in confrontation. Now that light curves are being gathered by automated photoelectric telescopes (APT's) in ever increasing numbers, there will be incentive for ever more refined and general models. They will continue to arrive.

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