Proposal for Electro-Optically Tunable Kerr-Lens in BBO Crystal by Cascaded Second-Order Process

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Abstract: In this report, a linear electro-optically tunable non-phase matched second-order nonlinear process is transformed into a cascaded second-order nonlinear process in a bulk BBO crystal to generate the effect of electro-optically tunable Kerr-type nonlinearity. A phase mismatch condition is created by applying an electric field to the x-z plane of the crystal which introduces a nonlinear phase shift between the launched and reconverted fundamental waves from the generated second harmonic wave. The sign of this nonlinear phase shift is easily changed by changing the polarity of applied electric field. The nonuniform radial intensity distribution of incident Gaussian beam introduces a curvature to the fundamental wavefront which focuses or defocuses the incident beam while propagating through the medium. Hence, a positive or negative lens is established within the crystal. The focal length of that lens is also tuned by tuning the applied electric field. The maximum calculated effective nonlinear refractive index due to nonlinear phase shift is $\pm 6.6508\times10^{-14}$ cm^2/W.

Keywords: Electro-optical effect; Cascaded second-order nonlinear process; Kerr nonlinearity; Nonlinear phase shift; Nonlinear refractive index.

1. INTRODUCTION:

Materials having large nonresonant third-order nonlinear coefficients are very rare. Therefore materials having large second-order nonlinear effects has been developed rapidly as cascaded second-order nonlinearities can generate a large equivalent third-order nonlinearity ($\chi^{(3)}$) [1]. Cascaded second-order nonlinearity is growing interest in various fields of applications such as optical switching [2], mode-locking [3], pulse compression [4], transistor action [5], self-focusing and defocusing [6], spatial solitons [7], high-speed optical shutters [8], electro-optical detection [9], electro-optic switching [10] and many more. This phenomenon is developed by cascading of two nonlinear second-order processes ($\chi^{(2)} : \chi^{(2)}$) which can effectively enhance $\chi^{(3)}$ phenomenon of the crystal, obtained due to Kerr electro-optic effect [6].

Beta barium borate (BBO) can be broadly used to develop high indirect $\chi^{(2)}$ into cascaded nonlinearity like other materials such as KTP and $KNbO_3$ for its higher damage threshold [11]. For high power intracavity pulsed operation, $LiNbO_3$, DKDP, KTP Pockels cells have some limitations whereas BBO can support high average powers within a compact laser resonator due to its higher damage threshold than laser gain medium [12]. Also, in BBO Pockel cell the piezoelectric ringing effects are found to be negligible at repetition rates up to 6 kHz [13].

Earlier, few articles were reported on cascaded second-order nonlinearity which has been investigated by angle and temperature tuning methods [6, 13]. These methods of cascaded second-order nonlinearity require accurate calibration due to the extra setups like an oven in temperature and rotation stage in angle tuning. As a result, an undesired loss is introduced into the cavity [6, 13]. However, electric field application to nonlinear crystal has also been investigated along with other methods [14- 15]. Though, direct application of an electric field to BBO for cascaded second-order process has not been reported till now.

In this report, we propose for the first time to best of our knowledge, the cascaded second-order nonlinearity in a bulk BBO crystal by electro-optic tuning method where a DC electric field is applied to a crystal plane to create a phase mismatch into the fundamental waves (FW) propagating through the crystal. Hence, a nonlinear phase shift is achieved within the launched FW and reconverted FW from generated second harmonic wave (SHW). Due to the higher electro-optic coefficients, a minimum voltage is required to create the desired phase mismatch for bulk BBO compared to other crystals which transform the non-phase matched second-order nonlinear process into a cascaded second-order nonlinear process. This method creates the effect of electro-optically tunable Kerr-type nonlinearity into the crystal.

2. THEORY AND PROPOSED SCHEME:

When FWs propagate through a noncentrosymmetric crystal along $x-$axis in type I phase matching direction, the coupled amplitude equations for nondegenerate two-wave interaction in the approximation of slowly varying amplitude can be derived from Maxwell's equations as

$$\frac{dE_1}{dx} = \frac{i\omega\chi^{(2)}(\omega;2\omega:-\omega)\,\mathrm{E}_1^{*}\,E_2 e^{-i\Delta kx}}{2n_1 c} \tag{1}$$

$$\frac{dE_2}{dx} = \frac{i\omega\chi^{(2)}(\omega;2\omega:\omega)E_1 E_1 e^{-i\Delta kx}}{2n_2 c} \tag{2}$$

Here E_1 is the electric field of incident FW, and E_2 is the electric field of the generated SHW. n_1 is the refractive indices of incident FW and n_2 is the refractive index of the generated SHW. ω is the fundamental frequency; c is the speed of light. $\chi^{(2)}$ is second-order nonlinear susceptibility [6]. Δk is the phase mismatch vector. The propagating distance (x) is assumed to be L. Hence, the term ΔkL can be defined by the relation of refractive indices of the crystal as

$$\Delta kL = L(2k_1 - k_2) = (4\pi L/\lambda)(n_1 - n_2) \tag{3}$$

Here k_1 and k_2 are the wave vectors of the FW and generated SHW respectively. From equation (3), it is clearly understood that Δk depends on the refractive indices of the crystal which can be varied by applied electric field to the crystal.

Fig. 1 is showing the proposed scheme where FW is incident on $y-z$ plane of a bulk BBO crystal having $\chi^{(2)}$ nonlinearity in type I phase matching direction. During back conversion, a part of the generated SHW is reconverted into FW followed by the same propagation path in phase matching direction. The electric field is applied on $x-z$ plane of the crystal which changes the refractive indices of the crystal along the principal axes as shown in fig. 1.

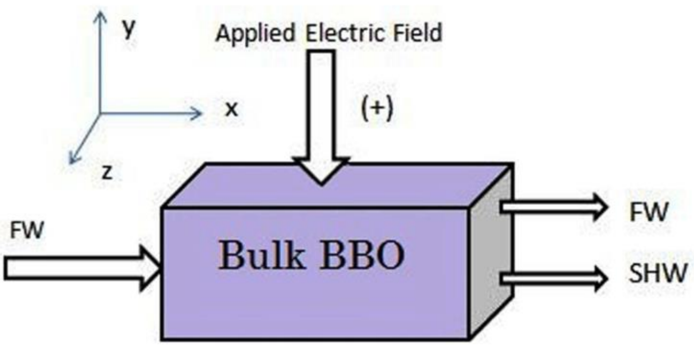


Fig. 1 Diagram of proposed scheme.

Due to the external DC electric field along the $y-$direction of the BBO, the refractive indices of the crystal changes as [16]

$$\mathrm{n}_{\mathrm{x}}^{'} = \mathrm{n}_{o}\left(1+\frac{1}{2}\mathrm{n}_{o}^{2}\mathrm{r}_{22}E_{y}\right);\ \mathrm{n}_{y}^{'} = \mathrm{n}_{0}\left(1-\frac{1}{2}\mathrm{n}_{o}^{2}\mathrm{r}_{22}E_{y}\right);\ \mathrm{n}_{\mathrm{z}}^{'} = \mathrm{n}_{e} \qquad (4)$$

And due to Pockel effect, the index ellipsoid of the crystal is modified [16] as

$$\left[\frac{1}{n_x^{'2}} - r_{22}E_y\right]x^2 + \left[\frac{1}{n_y^{'2}} + r_{22}E_y\right]y^2 + \left[\frac{1}{n_z^{'2}}\right]z^2 = 1 \qquad (5)$$

Here $\mathrm{n}_{\mathrm{x}}^{'}, \mathrm{n}_{y}^{'}$, $\mathrm{n}_{\mathrm{z}}^{'}$ are the modified refractive indices along principal direction after applying DC electric field (E_y) to the crystal. n_o is the ordinary refractive index and n_e is the extra-ordinary refractive index. r_{22} is the electro optic coefficient of the crystal. n_{x}, n_{y}, n_{z} are the refractive indices along x, y, z principal axes before applying any electric field to the crystal respectively. However, due to the modified refractive indices, the reconverted FW will have a different phase from the launched FW. Thus, a phase mismatch condition is occurred between FWs which introduces a nonlinear phase shift ($\Delta\phi^{NL}$) between the propagating FWs [17]. This $\Delta\phi^{NL}$ is achieved with an intensity dependent refractive index coefficient (n_2^{eff}), proportional to $\chi^{(3)}$ nonlinearity similar to Kerr medium. This $\chi^{(3)}$ can be positive or negative depending on ΔkL which leads to self-focusing and defocusing of propagating FWs within the medium. Now the amplitude equation of the propagating wave can be solved from the coupled mode equation (1) as

$$\frac{d^2E_1}{dx^2} + i\Delta k\frac{dE_1}{dx} + \psi\frac{\Gamma^2}{E_0} = 0 \qquad (6)$$

Here the constant parameters are, $\psi = (I + I_1)/I_0$ and $\Gamma = \frac{\omega d_{eff}E_0}{c\sqrt{n_1^2 n_2}}\left(\frac{2I_0}{c\varepsilon_0}\right)^{1/2}$. E_0 is the field of incident FW. I is the total peak intensity ($I = I_1$) where I_1 is the peak intensity of incident

FW for type I phase matching. I_0 is the normalized peak intensity which is equal to 1 GW/cm^2. Γ is a nonlinear coupling coefficient, ε_0 is the free space permittivity and d_{eff} is the effective nonlinear coefficient [6].

The $\Delta\phi^{NL}$ acquired by the FWs at the exit surface of the crystal (L) is obtained from the relation between the $\Delta\phi^{NL}$ of the FW and phase mismatch condition by assuming negligible depletion of FWs [6] as

$$\Delta\phi^{NL} = \frac{\Delta kL}{2}\{1-[1+4\Gamma^2\xi/(\Delta k)^2]^{1/2}\} \tag{7}$$

For a large phase-mismatch or low intensity of light $|\Delta k| >> |\Gamma\xi|$. Hence $\Delta\phi^{NL}$ varies linearly with the intensity of incident wave similar to optical Kerr effect [6] which can be derived by solving equation (7) at unit intensity,

$$\Delta\phi^{NL} \cong \frac{\Gamma^2 L^2}{\Delta kL} \tag{8}$$

However, the change in refractive index is also proportional to the intensity of the light traveling through the medium in cascaded second-order process as similar to optical Kerr effect. Hence an intensity dependent refractive index is achieved by the relation of $n(I) = n_0 + n_2^{eff} I$ where n_2^{eff} is the effective nonlinear refractive index, n_0 is the linear refractive index. The n_2^{eff} is related to $\Delta\phi^{NL}$ of the FW as

$$\Delta\phi^{NL} = (2\pi L n_2^{eff} I)/\lambda \tag{9}$$

Here λ is the wavelength of the propagating wave [6] and n_2^{eff} can be defined as

$$n_2^{eff} = -\frac{4\pi}{c\varepsilon_0} \times \frac{L}{\lambda} \times \frac{d_{eff}^2}{\left(n'_{\omega 1}\right)^2 n'_{\omega 2}} \times \frac{1}{\Delta kL} \tag{10}$$

From Equation (10), it is clearly seen that n_2^{eff} is proportional to the $\left[d_{eff}^2 / \left(n'_{\omega 1}\right)^2 n'_{\omega 2}\right]$ term which can be comparable to $\chi^{(2)}$ material as

$$n_2^{eff} \cong d_{eff}^2 / n^3 \tag{11}$$

Since the incident beam is a Gaussian beam, so the radial intensity distribution of the propagating beam is not uniform. Hence a different $\Delta\phi^{NL}$ will be created within the propagating FWs which introduce a curvature to the fundamental wavefront while propagating through the medium. Therefore, cascaded second-order process will act as a focusing and defocusing lens likewise in optical Kerr media that depends on the intensity of the incident beam. The focal length of this lens can be tuned by tuning the applied electric field. For an incident beam having lower intensity and diffraction length more than the crystal length, the focal length of the cascaded Kerr-lens can be approximated by the formula

$$f = r^2 / (4In_2^{eff} L) \tag{12}$$

Here r is the beam radius [18]. The focusing or defocusing of the FW depend on the sign of Δk as the FW focuses due to the positive values of phase mismatch i.e., $\Delta k > 0$ and defocuses for the negative values of phase mismatch, i.e., $\Delta k < 0$ [19]. The focal length of the lens is related to the nonlinear phase which can be derived by solving the equations (9) and (12)

$$f = (r^2\pi) / (2\Delta\phi^{NL}\lambda) \tag{13}$$

Equation (13) can be used to find the focal length of the cascaded lens. Thus it is clearly seen from equation (13), the focal length of the cascaded lens is dependent on $\Delta\phi^{NL}$ of FW having low intensity.

From the above analytical analysis, we can say that the sign of n_2^{eff} is responsible for the change in phase and focusing or defocusing of the fundamental beam in the optical Kerr medium. While in cascaded nonlinearity, the presence of phase mismatch between SHW and FW is also responsible for the nonlinear phase change of the FW and focusing or defocusing of the beam.

3. RESULTS & DISCUSSION:

In this work, FW is propagating along the $x-$direction of BBO where one component of the propagating FW is polarized in the $y-$direction having a refractive index of n_y; another is polarized in $z-$direction having refractive index value of n_z. The FW of 1064 nm is assumed to be propagating along the type I phase matching direction of the crystal making an angle of θ=22.3902286° to the principle axis. The dimension of the BBO crystal is assumed to be 10×5×4 mm^3.The refractive indices of the crystal along the principal axes are n_{o1} = 1.6551, n_{e1} =1.5425 [20] for FW as superscript 1 defines the FW. For SHW (superscript 2), n_{o2} = 1.6749, n_{e2} = 1.555 [20]. The electro-optic co-efficient of BBO is considered to be r_{22}= 2.1×$10^{(-12)}$ pm/V [21]. A calculated electric field of ∓ 6 KV/mm is applied along the $y-$axis that modifies the index ellipsoid and the principal refractive indices of the crystal. By the equation, $V = E_y \times d$, the DC applied electric field can be converted into a voltage where d is the thickness of the crystal and V is the applied voltage. The change in the principal refractive indices of the crystal changes the refractive indices of the propagating medium for the FW and SHW. Hence a tunable phase mismatch condition is created within the propagating FWs in type I phase matching direction by the application of a tunable DC electric field to the crystal as shown in fig. 2.

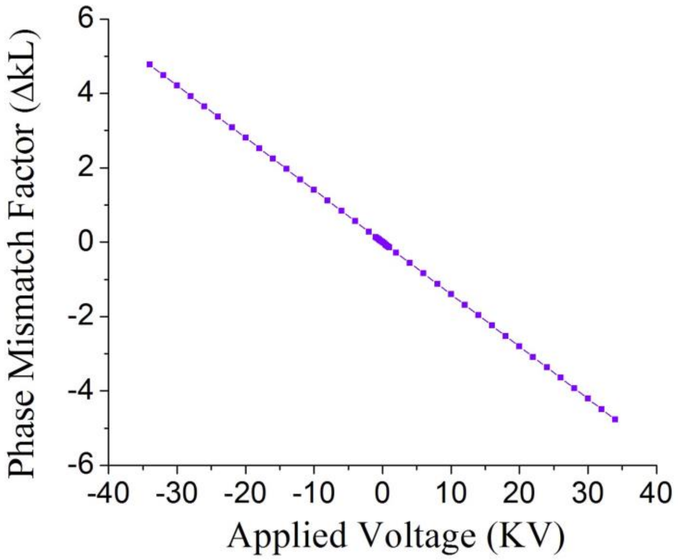
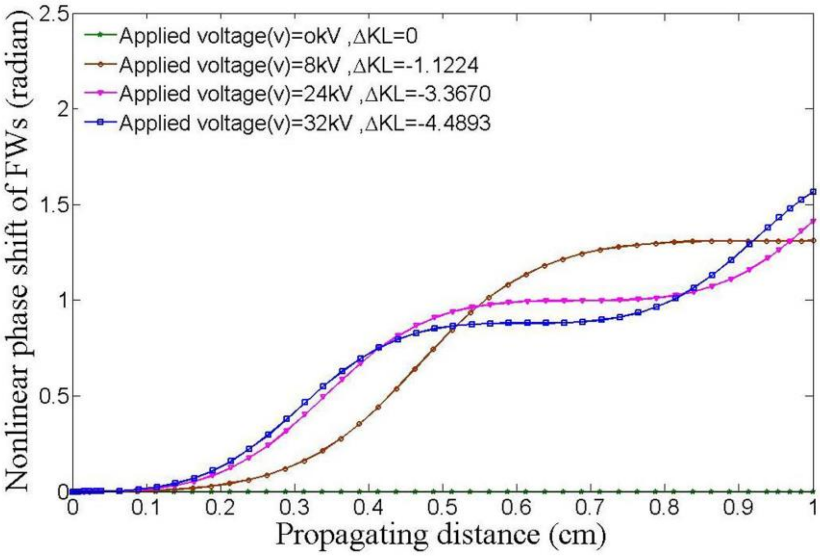
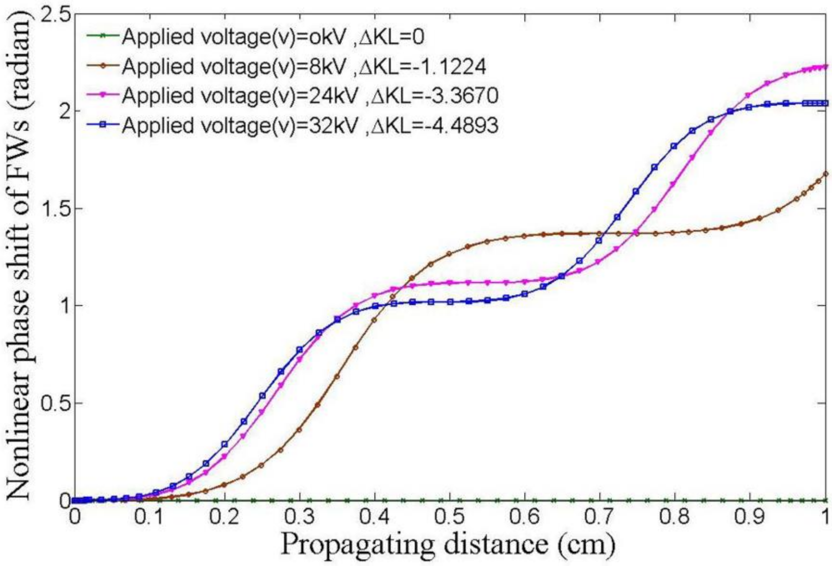


Fig. 2 Tuning of ΔkL w.r.t applied voltage (KV).

Fig. 2 indicates that ΔkL can easily be tuned by varying the magnitude and polarity of the applied voltage to the crystal. From fig. 2 it is clearly understood that for zero applied voltage, phase matching condition is sustained within the propagating FWs. As the applied voltage increases, phase mismatch condition is appeared within the propagating FWs. Hence tunable phase mismatch condition can be created by tuning the applied voltage to the crystal. The maximum calculated ΔkL is ± 4.7699 radian for an applied voltage of ∓ 34 KV. When FW incident on the crystal, initially the beam does not focus as the FW is not phase shifted. After consecutive up and down conversion of FWs along the propagating length, required $\Delta\phi^{NL}$ appears within the propagating FWs for focusing the propagating beam along the type I phase matching direction at that applied voltage. Hence, the beam focuses or defocuses within the crystal. This $\Delta\phi^{NL}$ can be calculated for different intensities of FWs as shown in Fig. 3.



(a)



(b)

Fig. 3 Change of nonlinear phase shift of the FWs along the propagation length at an intensity of (a) 0.4 GW/cm^2 and (b) 0.8 GW/cm^2.

Fig. 3 (a) and (b) are showing the variation of $\Delta\phi^{NL}$ of the FWs along the propagation length at different intensities viz. 0.4 GW/cm^2 and 0.8 GW/cm^2 according to the applied voltages to the crystal. In both figures, it is clearly seen that when applied voltage is zero, phase matching condition will occur means $\Delta\phi^{NL}$ is also zero as denoted by the green line in fig. 3 (a) and (b). However, at different applied voltages viz. 8 KV, 24 KV and 32 KV, $\Delta\phi^{NL}$ of the FWs are changed significantly which are indicated by brown, pink, blue lines respectively in both figures. Although, from fig. 3 it is also understood that the change of $\Delta\phi^{NL}$ at 0.8 GW/cm^2 is higher than that of 0.4 GW/cm^2. So it can be said that $\Delta\phi^{NL}$ is intensity dependent parameter, as the intensity of incident beam increases $\Delta\phi^{NL}$ also increases. Here the beam waist of propagating beam is assumed to be 56 µm. Therefore the diffraction length of the beam should be greater than the crystal length of the beam waist. For the total intensity of 0.4 GW/cm^2, the phase acquired by the FW at the end of the crystal makes the focal length of the configurable lens less than the diffraction length of the beam. However, from theoretical analysis, we can say that by controlling the applied voltage to the crystal $\Delta\phi^{NL}$ and n_2^{eff} can be controlled which are the function of ΔkL. Fig. 4 shows the controlling of $\Delta\phi^{NL}$ and n_2^{eff} by the tuning of applied voltage to the crystal.

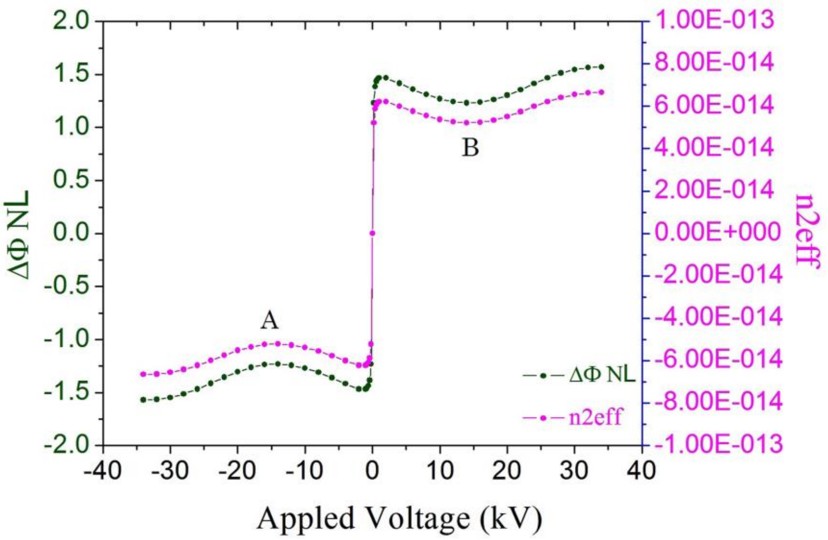


Fig. 4 Controlling of $\Delta\phi^{NL}$ and n_2^{eff} w.r.t applied voltage (KV).

From fig. 4 it is clearly observed that at zero applied voltage i.e. in phase matching condition $\Delta\phi^{NL}$ is zero within the crystal. Hence n_2^{eff} is also zero as it is a $\Delta\phi^{NL}$ dependent parameter. Therefore, by varying magnitude and polarity of the applied voltage a noticeable change in n_2^{eff} and as well as $\Delta\phi^{NL}$ are also observed. Moreover, beside n_2^{eff} a change in $\Delta\phi^{NL}$ is also noticed for the change of intensity of the incident beam. The change of $\Delta\phi^{NL}$ and n_2^{eff} by tuning applied voltage is denoted by region around A (0 to - 34 KV) and B (0 to + 34 KV) respectively as shown in fig. 4. By controlling the applied voltage to the crystal, maximum $\Delta\phi^{NL}$ and n_2^{eff} have been calculated as ± 1.5702 radian and ± 6.6508×10^{-14} respectively for applied voltage of ±34 KV. On the other hand, due to the regions around A and B, FWs focus and defocus due to the appearance of a curvature within the wave front of FWs while propagating through the crystal. Therefore, the crystal itself acts like a cascaded Kerr-lens that can be configured by controlling applied voltage. The focal length of this configurable lens can also be tuned by varying the applied voltage to the crystal as shown in Fig. 5.

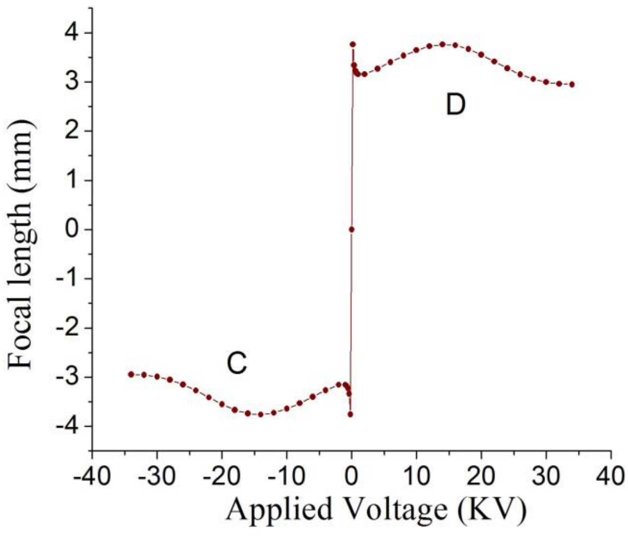


Fig. 5 Variation of the focal length of the cascaded Kerr-lens w.r.t applied voltage (KV).

Fig. 5 shows that when applied voltage to the crystal is zero, the focal length of the cascaded Kerr-lens is also zero. As the applied voltage changes with the magnitude and polarity, the focal length of the cascaded Kerr-lens also varies in both negative and positive values as shown by region around C (0 to -34 KV) and D (0 to + 34 KV) respectively in fig. 5. However, the region around C and D indicate the focusing and defocusing conditions of the incident beam for positive and negative lens respectively. By tuning the applied voltage from 0 to ± 34 KV, the focal length of the cascaded Kerr-lens can also be tuned from 0 to ± 3.7603 mm.

4. CONCLUSION:

In this work, we report analytically an electro-optically tunable cascaded Kerr-lens in a bulk BBO crystal. The direction of propagation of FWs are in type I phase matching direction, making θ = 22.3902286° along the $x-$direction. By applying maximum voltage of $\pm$ 34 KV along the crystal's thickness of 4 mm, a phase mismatch factor of $\mp$ 4.7699 radian has been calculated to achieve $\Delta\phi^{NL}$ within the propagating FWs which is dependent on n_2^{eff}. The maximum calculated n_2^{eff} is ± 6.6508×10^{-14} cm^2/W. However, the minimum calculated focal length of the cascaded Kerr-lens is ± 2.947 mm for an applied voltage of ± 34 KV for focusing and defocusing the incident Gaussian beam for a peak beam intensity of 0.4 GW/cm^2 having a beam waist 56 μm.

In electro optically tunable cascaded Kerr-lens method, the bulk BBO itself behaves like a configurable lens, hence the controlling of the focal length is easier with higher precision and accuracy unlike other method [22] of tunable cascaded Kerr-lens. Therefore direct electro-optical controlling of cascaded Kerr-lens mode-locking with higher precision is possible. Moreover, for creating the required phase mismatch condition, this method doesn't require any extra setup like others [6, 14]. However, in this method n_2^{eff} can be precisely changed by tuning the applied voltage to the crystal without changing its physical position whereas in angle tuning the amount of the phase mismatch is tuned by changing the incident angle which is crucial to adjust [5]. Hence cavity losses can be minimized. The proposed electro-optically tuned cascaded Kerr-lens can also be used in a laser cavity for adaptive optical requirement to compensate the effect of thermal lensing of the gain medium at high pump power level.

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