Flexible AC Transmission Systems (FACTS)

(EE1422)

Department Elective I

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Module 1, II part
Transmission Line and compensation

Analysis of uncompensated line: transmission line equations, performance of a line connected to unity power factor load, performance of a symmetrical line, passive reactive power compensation: distributed and discrete power compensation, compensation by a series capacitor connected at the mid-point of the line, shunt capacitor compensation connected at the midpoint of the line, comparison between series and shunt capacitor compensation

AC Transmission Line and Reactive Power Compensation

In this Module, the reactive power control in AC power transmission lines is examined. The requirements are to

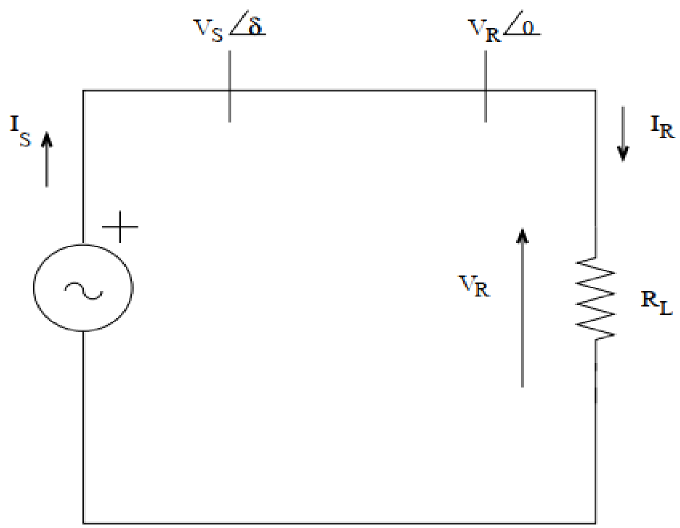
(a) transmit as much power as feasible on a line of specified voltage and

(b) to control the voltage along the line within limits.

Analysis of Uncompensated AC Line

1.1 General

A transmission line has distributed circuit parameters. We will be assuming the line to be symmetric in three phases and operating with balanced (positive sequence) voltages and currents.



In Fig 1 it is assumed that the sending end is connected to a generator and the receiving end is connected to a (unity power factor) load. The line has series resistance r and inductance l, shunt conductance g and capacitance c (all parameters expressed per unit length).

Fig 1: A transmission line supplying a unity power factor load.

1.2. Transmission Line Equations

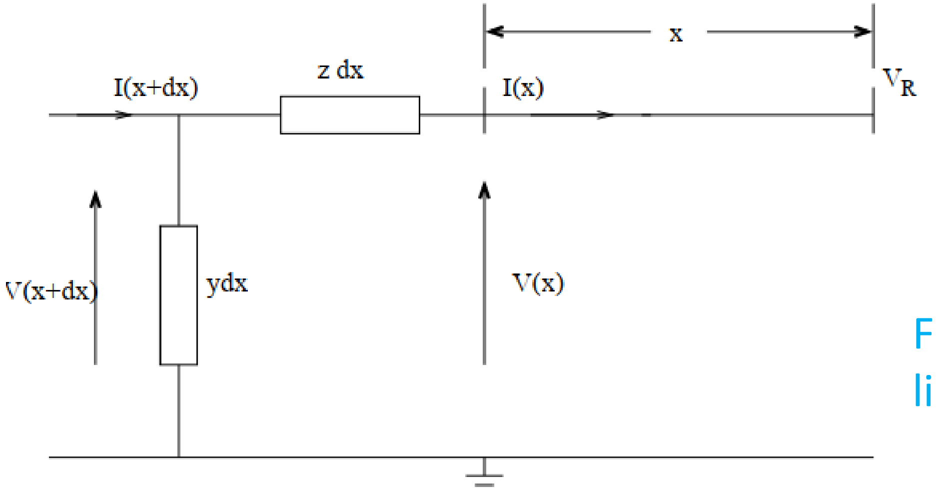


Fig 2: Voltage and current variation in a long line

It is assumed that in steady state all the voltages and currents in the line are sinusoidal of frequency (!rad/sec) and expressed in phasors.

Considering a small element of the line of length (dx) at a distance x from the receiving end, (see Fig. 2) the following equations apply,

$I(x + dx) = I(x) + (ydx)V(x + dx)$ ....(1)

$V(x + dx) = V(x) + (zdx)I(x)$..(2)

where $y = g + jb$; $z = r + jx$; $b = \omega c$; $x = \omega l$....................(3)

It is to be noted that both V and I are phasors that are functions of x. From the above equations, we get the following difierential equations for V and I.

$dV/dx = zI$..(4)

$dI/dx = yV$...(5)

Solving equation 1 to 5

$V(x) = VR\cosh(\gamma x) + I_R Zc\sinh(\gamma x)$ ……………………(6)

$I(x) = VR/Z_c \sinh(\gamma x) + I_R\cosh(\gamma x)$……………………(7)

γ is termed as the propagation constant. α is called the attenuation constant and β is called the phase constant.

Expressions for a Lossless line

Neglecting r and g, the propagation constant γ is purely imaginary with

$\gamma = j\beta = j\omega\sqrt{lc}$………………………………………….(8)

and the characteristics impedance Z_c is purely resistive with

$Z_c = \sqrt{\frac{l}{c}} = Zn$ ……………………………………………………(9)

In this case the Z_c is termed as surge impedance or natural impedance (Z_n)

When x = λ, such that

$\beta\lambda = 2\pi$………………………………………(10)

λ, is defined as the wavelength, which depends on the frequency f. It can be shown that

$\lambda = uT = u/f = \frac{u2\pi}{\omega}$ ………………………………………………(11)

where u is the velocity of propagation of the (voltage or current) wave given by $u=1/\sqrt{(lc)}$

Typically, the value of u for overhead high voltage transmission lines is slightly less than the velocity of light ($u = 3 \times 10^8$ m/sec).

Substituting x = d

$V_S = V_R \cos\theta + jI_R Z_n \sin\theta$……………………………(12)
$I_S = j V_R / Z_n \sin\theta + I_R \cos\theta$…………………………..(13)

$\theta = \beta d = \omega\sqrt{(lcd)} = \frac{2\pi}{\lambda} d$ ……………………………………(14)

is termed as the electrical length of the line expressed in radians.

1.3 Performance of a Line Connected to Unity Power Factor Load

Assuming that the sending end voltage of the line is held constant at V_S, the receiving end voltage V_R varies with the load. It will be assumed that the line is lossless.
It is convenient to represent the line by Thevenin equivalent at the receiving end. The Thevenin voltage is the open circuit voltage at the receiving end given by,
$V_{Th} = V_S / \cos\theta$ ……………………………………………….(15)
and Thevenin impedance is obtained as

$Z_{Th} = -\frac{V_R}{I_R}\Big|_{V_S=0} = jZ_n \tan\theta$……………………….(16)

$V_R = V_S \cos\delta / \cos\theta$ ………………………………………..(17)

and the power flow in the line, $P = P_R$ is given by

$$P = V_{Th} V_R \sin\delta / Z_n \tan\theta = \frac{V_S^2 \sin 2\delta}{Z_n \sin 2\theta} \quad \text{.....................} \quad (18)$$

The maximum power (theoretical limit) occurs at $\delta = 45^0$ and is given by

$$P_{max} = \frac{V_S^2}{Z_n \sin 2\theta} \quad \text{..}(19)$$

At this value of power, the receiving end voltage is

$$V_{Rm} = \frac{V_S}{\sqrt{2}\cos\theta} \quad \text{..}(20)$$

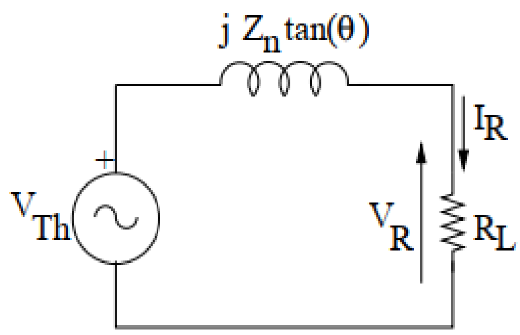


Fig 4: Equivalent circuit of the line connected to a unity p.f. load

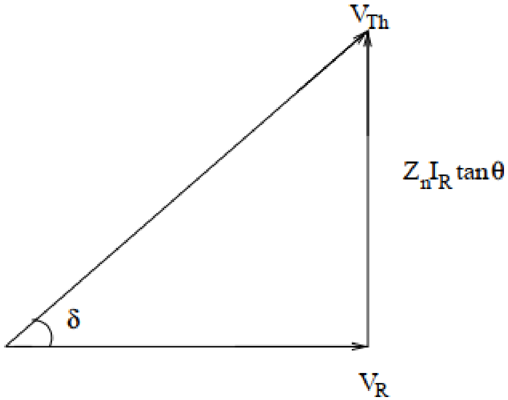


Fig 6: Phasor diagram of voltages

At no load ($P_R = 0$), the voltage at the receiving end is higher than the sending due to the line charging. This is termed as Ferranti Effect. The no load voltage at the receiving end is given by

$V_{R0} = V_S / \cos\theta$ ……………………………………..(21)

This can be excessive as θ increases. At line lengths approaching quarter wavelength, V_{R0} is very high. Note that V_{R0} is bounded in real lines as the resistance of the line cannot be ignored at high charging currents. At no load, the sending end current is the charging current of the line and is given by

$$\hat{I}_{S0} = j\frac{\sin\theta \hat{V}_S}{Z_n \cos\theta} = j\frac{\hat{V}_S}{Z_n}\tan\theta \quad \text{……………………………(22)}$$

This also increases as θ increases and can overload the generator. The no load reactive power Q_{S0} is obtained as

$$Q_{S0} = \mathrm{Im}[V_S I_{S0}^*] = -\frac{V_S^2 \tan\theta}{Z_n} \quad \text{………………………..(23)}$$

1.4 Performance of a Symmetrical Line

To control the receiving end voltage and increase the power transfer capability of the line it is necessary to have a generator or a controlled reactive power source (with fast control) such as a SVC at the receiving end (see Fig. 6). The reactive power injected is $-Q_R$. If line is symmetrical ($V_S = V_R = V$), then from symmetry, $Q_S = -Q_R$. Thus, the reactive power requirements of the line are shared equally at both ends of the line.

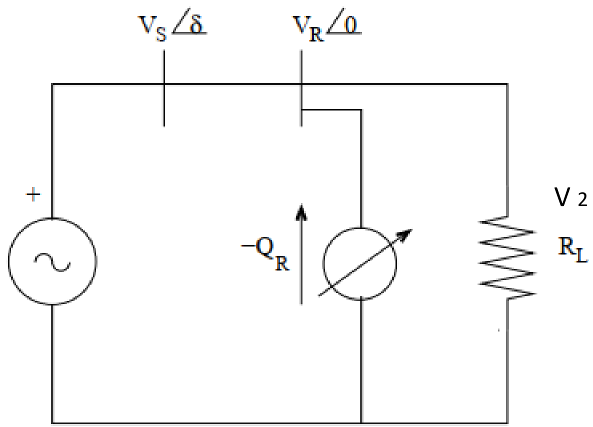


Fig 6: A transmission line with dynamic voltage support at the receiving end

From Fig. 6, it can be derived that if the voltage V_R is controlled using a reactive power source in parallel with R_L, then the power transfer on the line is given by

$$P = \frac{V^2}{Z_n \sin\theta} \sin\delta \quad \text{...(24)}$$

$$\hat{I}_R = \frac{\hat{V}_S - \hat{V}_R \cos\theta}{jZ_n \sin\theta} = \frac{V\angle{-\delta} - V^2 \cos\theta}{jZ_n \sin\theta} \quad \text{.......................... (25)}$$

The complex power (S_R) at the receiving end is defined by

$$S_R = P_R + jQ_R \quad = \hat{V}_R I_R^* = \frac{V^2\angle\text{-}\delta\text{-}V^2\cos\theta}{\text{-}jZ_n\sin\theta} \quad \text{..............................(26)}$$

$$P = \frac{V^2\sin\delta}{Z_n\sin\theta} = \frac{P_n\sin\delta}{\sin\theta} \quad \text{......................................(27)}$$

where Pn is termed as Surge Impedance Loading (SIL) defined by

$P_n = V^2/Z_n$

$Q_R = V^2/(Z_n \sin\theta)*(\cos\delta - \cos\theta) = -Q_S$ ....**(28)**

The voltage profile along the line varies as the loading varies. For $P = P_n$ (SIL) the voltage profile is flat. The voltage variation at the midpoint is maximum for the symmetrical line as the load varies from zero to the maximum value. (Fig 7) To compute the midpoint voltage (V_m) we can divide the line into two equal sections of half the length. For the line section connecting the sending end to the midpoint, we have

$$V_S = V\angle\delta = \hat{V}_m\cos\frac{\theta}{2} + j\hat{I}_m Z_n \sin\frac{\theta}{2} \quad \text{......................................(29)}$$

where I_m is the current flowing at the midpoint.

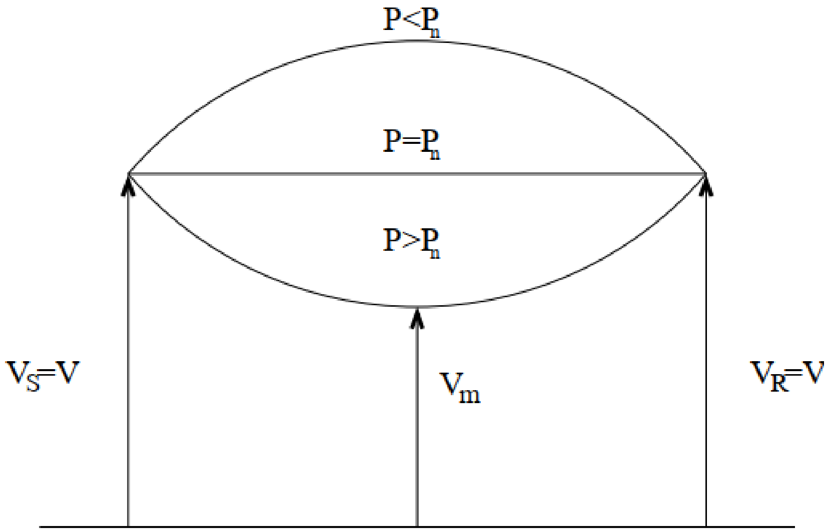


Fig 7: Voltage profile along the line

For the line section connecting midpoint to the receiving end, we have

$$V_R = V\angle 0 = \hat{V}_m \cos\frac{\theta}{2} - j\hat{I}_m Z_n \sin\frac{\theta}{2} \text{..............................(30)}$$

$$\hat{V}_m = \frac{V\cos\delta/2}{\cos\frac{\theta}{2}} \text{Ð}\delta/2 \text{..(31)}$$

$$\hat{I}_m = \frac{V\sin\delta/2}{Z_n\sin\frac{\theta}{2}} \angle\delta/2 \text{...(32)}$$

$$P = V_m I_m = \frac{V^2\sin\delta}{Z_n\sin\theta} \text{..(33)}$$

The no load voltge at the midpoint (V_{mo})
is given by

$$V_{mo} = \frac{V}{\cos\theta/2} \text{..(34)}$$

The charging reactive power at no load is given by

$$Q_{S0}=-Q_{R0}=\frac{V^2}{Z_n \sin\theta}(\cos\theta-1)=-\frac{V^2}{Z_n}\tan\frac{\theta}{2} \text{........(35)}$$

Fig 8: A transmission line with Thevenin equivalent at both ends

For simplifying the analysis we can consider $E_S = E_R = E$ and $x_S = x_R = x$. In this case, it can be shown that

$$V_m = \frac{E\cos\delta/2}{\cos(\theta/2)(1-\frac{x}{Z_n}\tan(\theta/2))}\angle\delta/2 \qquad \text{............(36)}$$

$$I_m = \frac{E\sin\delta/2}{Z_n\sin(\theta/2)(1+\frac{x}{Z_n}\cot(\theta/2))}\angle(\delta/2) \qquad \text{..........(37)}$$

The power flow (P) is obtained as $$P = V_m I_m = \frac{E^2\sin\delta}{Z_n\sin\theta\left[1+\frac{2x}{Z_n}\cot\theta-\frac{x^2}{Z_n^2}\right]} \qquad \text{..........(38)}$$

1.2.1 Passive Reactive Power Compensation

Let us consider distributed series compensation (capacitive) whose effect, in steady state, is to counteract the effect of the distributed series inductance of the line. Similarly, by providing distributed shunt (inductive) compensation, the effect of line capacitance is reduced

The phase constant (β') of a compensated line is given by

$$\begin{aligned}\beta' &= \sqrt{\omega l' \cdot \omega c'} = \sqrt{\omega l(1-k_{se})\omega c(1-k_{sh})} \\ &= \beta\sqrt{(1-k_{se})(1-k_{sh})}\end{aligned} \qquad \text{............(39)}$$

where β is the phase constant of the uncompensated line, k_{se} is the degree of series compensation and k_{sh} is the degree of shunt compensation. It is assumed that both k_{se} and k_{sh} are less than unity.

The surge impedance (Z'_n) of the compensated line is

$$Z'_n = \sqrt{\frac{\omega \ell'}{\omega c'}} = Z_n\sqrt{\frac{(1-k_{se})}{(1-k_{sh})}} \qquad \text{............(40)}$$

the electrical length (θ’) of the compensated line given by

$$\theta' = d\beta' = \theta\sqrt{(1-k_{se})(1-k_{sh})} \quad \text{......................(40)}$$

is reduced by both series and shunt compensation. On the other hand, Z_n is reduced by series compensation (capacitive) and increased by shunt compensation (inductive)

For a lossless symmetrical line, the power flow in a compensated line is given by

$$P' = \frac{V^2}{Z_n' \sin\theta'}\sin\delta \simeq \frac{V^2}{Z_n'\theta'}\sin\delta = \frac{V^2\sin\delta}{Z_n\theta(1-k_{se})} \quad \text{........................(41)}$$

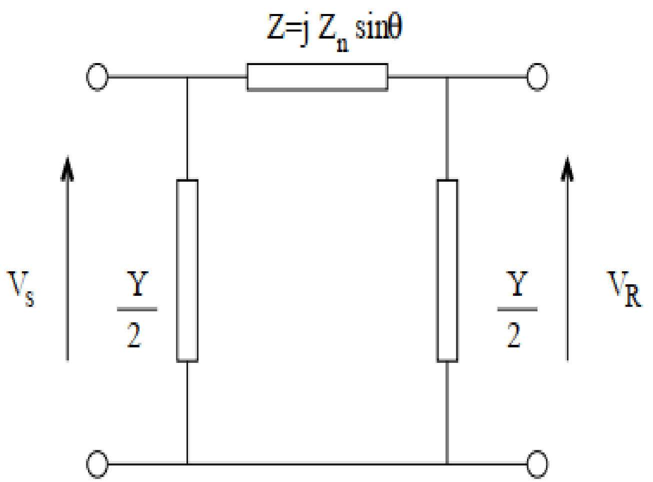
for small values θ’.

The no load voltage at the mid-point (V'_{m0}) is given by

$$Q'_{So} = -Q'_{Ro} = -\frac{V^2}{Z_n'}\tan\frac{\theta'}{2} \simeq \frac{V^2}{Z_n'}\frac{\theta'}{2} = \frac{V^2\theta}{2Z_n}(1-k_{sh}) \text{...................(42)}$$

1.2.2 Discrete Passive Compensation

It is not practical to provide distributed compensation. Here, we will consider discrete series and shunt compensation. Before taking this up, it is instructive to derive an equivalent circuit (in steady state) of the distributed parameter, uncompensated line. Figure 9 shows the equivalent π circuit of the line



$$Y/2 = \frac{j\tan(\theta/2)}{Z_n}$$

Fig 9: The exact π equivalent circuit of a lossless uncompensated line

This is obtained by comparing the A and B constants of the line and the equivalent circuit as given below

$$A = 1 + \frac{YZ}{2} = \cos\theta, \quad B = Z = jZ_n \sin\theta \quad \text{................(43)}$$

We can also express Z and Y as

$$Z = jX^n \frac{\sin\theta}{\theta}, \quad \frac{Y}{2} = j\frac{B^n}{2}\left(\frac{\tan\frac{\theta}{2}}{\frac{\theta}{2}}\right) \text{...............(44)}$$

where $X^n = \omega l d = Z_n \theta$ the total (nominal) reactance of the line and $B^n = \omega c d = \frac{\theta}{Z_n}$ is the total (nominal) susceptance of the line.

For small values of θ, $\frac{\sin\theta}{\theta} \simeq 1$ and $\tan\frac{\theta}{2} \simeq \frac{\theta}{2}$. Hence $Z \simeq jX^n$, $Y \simeq jB^n$. With this approximation, this equivalent circuit is called as the 'nominal' π circuit of the transmission line.

3. Compensation by a Series Capacitor Connected at the Midpoint of the Line

The two cases of the series compensation are considered.

Case 1: Series compensation accompanied by shunt compensation:

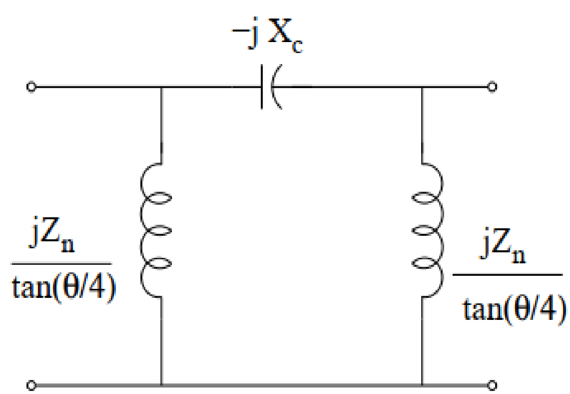
The equivalent circuit of the line with the series and (full) shunt compensation connected at the midpoint in addition to the two ends of the line (10(a)) is shown in Fig. 10(b).

The power flow in the compensated line is given by

$$P = \frac{V_S V_R}{\left(2Z_n \sin\frac{\theta}{2} - X_c\right)} \sin\delta \qquad \text{.......................(45)}$$

It is to be noted that when $V_S = V_R = V$, the no load voltage at the midpoint of the line is V (and at both terminals of the series capacitor as the current through the capacitor is zero at no load). The Eq. (45) can also be expressed as

$$P = \frac{V_S V_R \sin\delta}{Z_n \sin\theta \left[\frac{1}{\cos\frac{\theta}{2}} - \frac{X_c}{Z_n \sin\theta}\right]} = \frac{V_S V_R \sin\delta \cdot \cos\frac{\theta}{2}}{Z_n \sin\theta \left[1 - \frac{X_c}{2Z_n \sin\frac{\theta}{2}}\right]} \qquad \text{............(46)}$$



(a) Series capacitor and shunt reactors connected at the midpoint

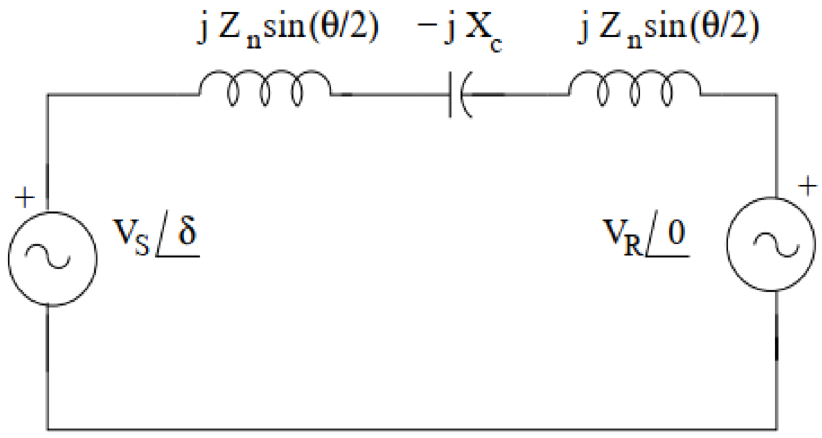


Fig 10: Representation of a compensated line

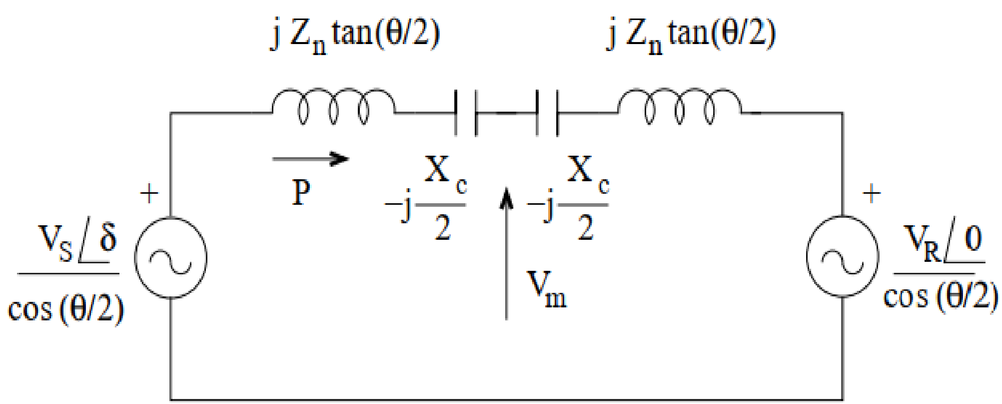


Fig 11: Equivalent circuit of a line with pure series compensation

Case 2: With pure series compensation at the midpoint:

If no shunt reactors are used at the terminals the series capacitor, the equivalent circuit of half the line can be obtained { as shown in Fig. 4 and the combined equivalent circuit of the series compensated line is shown in Fig. 11

The power flow (P) in the line is given by

$$P = \frac{V_S V_R \sin\delta}{\cos^2\frac{\theta}{2}\left[2Z_n \tan\frac{\theta}{2} - X_c\right]} = \frac{V_S V_R \sin\delta}{Z_n \sin\theta\left[1 - \frac{X_c}{2Z_n \tan\frac{\theta}{2}}\right]} \quad \text{..................(47)}$$

It is to be noted that when $V_S = V_R = V$, the midpoint voltage (V_m) is unaffected by the series capacitor while the midpoint current (I_m) is modified to

$$I_m^{se} = \frac{V \sin\frac{\delta}{2}}{Z_n \sin\frac{\theta}{2}(1 - k_{se})} \quad \text{..........................(48)}$$

$$k_{se} = \frac{X_c}{2Z_n \tan\frac{\theta}{2}} \quad \text{..............................(49)}$$

It can be shown that both $(\hat{V}_m)$ and $(\hat{I}_m^{se})$ are in phase and the power flow (P) is

$$P = V_m I_m^{se} \quad \text{..............................(50)}$$

4. Shunt Compensation Connected at the Midpoint of the Line

As mentioned earlier, the control of no load voltage requires a shunt reactor. On the other hand, increase in the power flow in a line requires shunt capacitor. Unlike in the case of the series capacitor, the location of the shunt capacitor is very crucial. The best location is at the midpoint of the line to maximize the power flow in the line. The equivalent circuit of the line with the shunt susceptance connected at the midpoint, is shown in Fig. 12

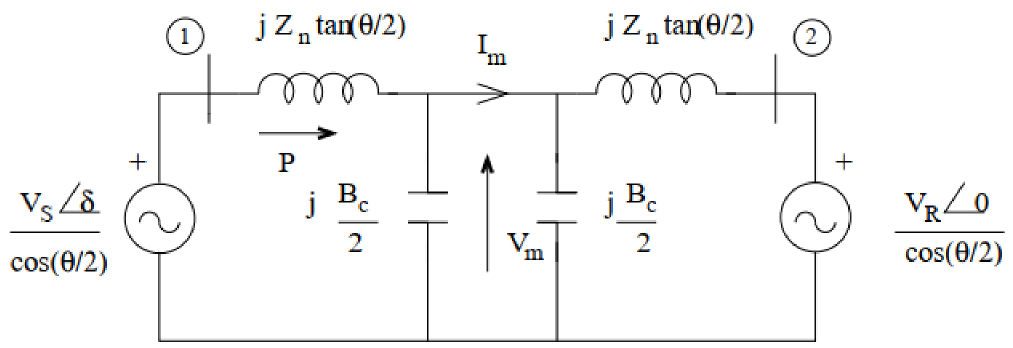


Fig 12: Equivalent circuit of a line with shunt capacitor at the midpoint

The transfer reactance (X_t) between the nodes (1) and (2) is given by

$$X_t = 2Z_n \tan\frac{\theta}{2} - B_c Z_n^2 \tan^2\frac{\theta}{2} \qquad \text{.............................(51)}$$

The power flow (P) in the line is given by

$$P = \frac{V_S V_R \sin\delta}{X_t \cos^2\frac{\theta}{2}} = \frac{V_S V_R \sin\delta}{Z_n \sin\theta\left[1 - B_c\frac{Z_n}{2}\tan\frac{\theta}{2}\right]} \quad \text{..............................(52)}$$

If $V_S = V_R = V$, the midpoint voltage (V_m) is given by

$$V_m^{sh} = \frac{V\cos\frac{\delta}{2}}{\cos\frac{\theta}{2}\left[1 - \frac{Z_n B_c}{2}\tan\frac{\theta}{2}\right]} \quad \text{............................(53)}$$

5. Comparison between Series and Shunt Capacitor

The maximum power flow in the line is given by substituting $\delta = \delta_{max}$ in the expression for the power flow (P). δ_{max} is chosen from considerations of the steady state margin that will not result in the power flow exceeding limits during a contingency.

For the same amount of maximum power transfer, we obtain the following relation

$$\frac{B_c}{2} Z_n \tan\frac{\theta}{2} = \frac{X_c}{2Z_n\tan\frac{\theta}{2}} \quad \text{.....................................(54)}$$

While transferring maximum power, the reactive power (Q_{se}) supplied by the series capacitor (for a symmetric line with $V_S = V_R = V$) is given by

$$Q_{se} = I_m^2 X_c = \frac{V^2 \sin^2 \frac{\delta_{\max}}{2} X_c}{Z_n^2 \sin^2 \frac{\theta}{2}(1-k_{se})^2} \quad \text{..........................(55)}$$

The reactive power (Q_{sh}) supplied by the shunt capacitor (B_c) at $P = P_{max}$ is obtained as

$$Q_{sh} = V_m^2 B_c = \frac{V^2 \cos^2 \frac{\delta_{\max}}{2} B_c}{\cos^2 \frac{\theta}{2}(1-k_{sh})^2} \quad \text{..........................(56)}$$

$$k_{sh} = \frac{B_c Z_n}{2} \tan \frac{\theta}{2} \quad \text{.....................................(57)}$$

Since $k_{se} = k_{sh}$

$$\frac{Q_{se}}{Q_{sh}} = \frac{\tan^2 \frac{\delta_{\max}}{2}}{Z_n^2 \tan^2 \frac{\theta}{2}} \left(\frac{X_c}{B_c}\right) = \tan^2 \frac{\delta_{\max}}{2} \quad \text{...............................(58)}$$

The above relation shows that the series capacitor is much more effective than the shunt capacitor in increasing power transfer

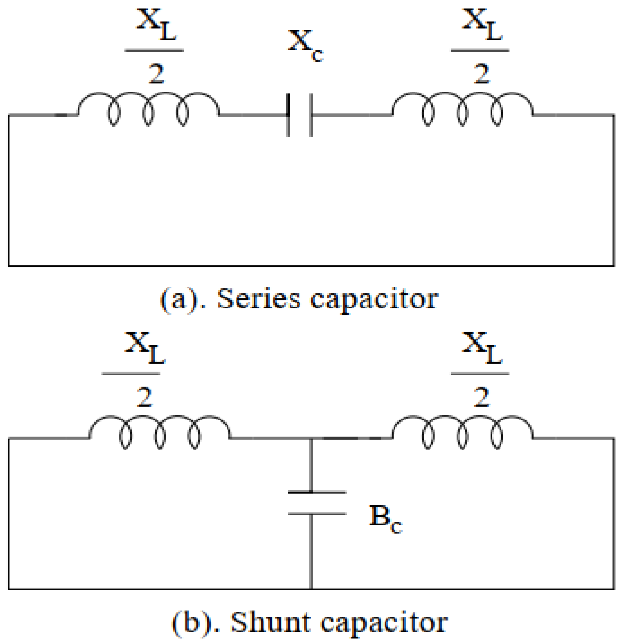


Fig 13: Equivalent circuits for determining resonance frequencies

Another factor in the comparison of the series and shunt (capacitor) compensation is the electrical resonance frequency.

The electrical resonance frequency (f^{se}_{er}) for the series capacitor compensation

$$f^{se}_{er} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = f_0\sqrt{\frac{X_c}{X_L}} = f_0\sqrt{(1-k_{se})} \quad \text{..................(59)}$$

The resonance frequency for the shunt capacitor compensation is obtained as

$$f_{er}^{sh} = f_0\sqrt{\frac{4}{B_c X_L}} = f_0\sqrt{\frac{1}{(1-k_{sh})}}$$

where f_0 is the operating system frequency (50 or 60 Hz).