

## Development of a Robot Balancing on a Ball

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**Abstract:** This paper proposes a robot balanced on a ball. In contrast to an inverted pendulum with two wheels, such as the Segway Human Transporter, an inverted pendulum using a ball can traverse in any direction without changing its orientation, thereby enabling stable motion. Such robots can be used in place of the two-wheeled robots. The robot proposed in this paper is equipped with three omnidirectional wheels with stepping motors that drive the ball and two sets of rate gyroscopes and accelerometers as attitude sensors. The robot has a simple design; it is controlled with a 16-bit microcontroller and runs on Ni-MH batteries. It can not only stand still but also traverse on floor and pivot around its vertical axis. Inverted pendulum control is applied in two axes for attitude control, and commanded motions are converted into velocity commands for the three wheels. The mechanism, control method, and experimental results are described in this paper.

**Keywords:** Inverted pendulum, Balance on ball, Omnidirectional wheel

### 1. INTRODUCTION

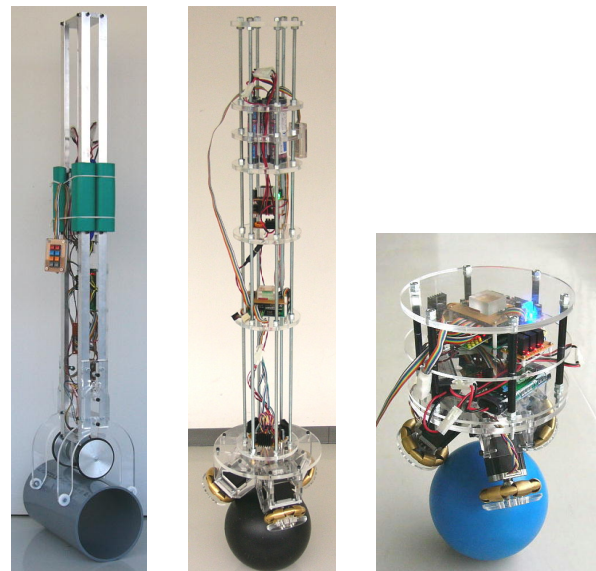
This paper describes a robot that rides and balances on a ball. Balancing on a ball is an acrobatic feat performed by human entertainers or animals in a circus. This study was motivated by the desire to create such a robot.

The robot is realized using inverted pendulum control and a system that provides an omnidirectional driving mechanism for the ball. We have developed several types of pendulums, including a wheel-driven pendulum that could ride on a pipe as shown in Fig. 1(a). This is a prototype robot, whose concept has been applied to the proposed robot. In addition, an omnidirectional mobile robot functions as a useful reference.

There have been several studies on such robots. The most famous of these is *Ballbot*[1] by Lauwers et al. that used an inverse mouse-ball driving mechanism for motion and LQR controller for feedback of sensory information. The aim of the study was to develop a dynamically stable mobile robot that was tall enough to interact with humans. Although the robot worked well, it could not rotate around yaw axis; this limitation of robot will be examined in the future.

*B.B.Rider*[2] was a type of wheel chair developed by Endo et al. by an approach similar to ours; it consisted of several omnidirectional wheels for turning. However, to the best of our knowledge, only the mechanical design and developed system were reported without practical results.

After developing a pipe-riding robot as a prototype three years ago, we began developing a ball-riding robot. In order to build a component to drive the ball, we focused our attention on mobile robots with omnidirectional wheels. These robots could move in any direction with rotation. Among several types of wheels, we decided to use the one developed by Asama et al.[3], [4]. This particular wheel was selected because it remains in contact at a single point during rotations while some of the other wheels have two trajectories. In addition, it was possible to fabricate this prototype ourselves.



(a) on pipe (b) on ball (1st) (c) on ball (2nd)

Fig. 1 Robots that balance on pipe and ball.

Finally, the robot was developed once we obtained the wheels.

This paper describes two robots that balance on a ball. The mechanism for driving the ball, the control method, and some experimental results are presented in this paper.

### 2. BASIC CONCEPT AND MECHANISM

The robot could be realized using two mechanisms: one was inverted pendulum control in two directions (back and forth and right and left) and the other was an omnidirectional mechanism for driving the ball. For the control, two independent control equations for the two directions using two attitude sensing systems placed orthogonally were applied to the robot to form a two-dimensional inverted pendulum. The outputs, that is the acceleration commands for ball motion, were combined and converted into velocity commands for the three

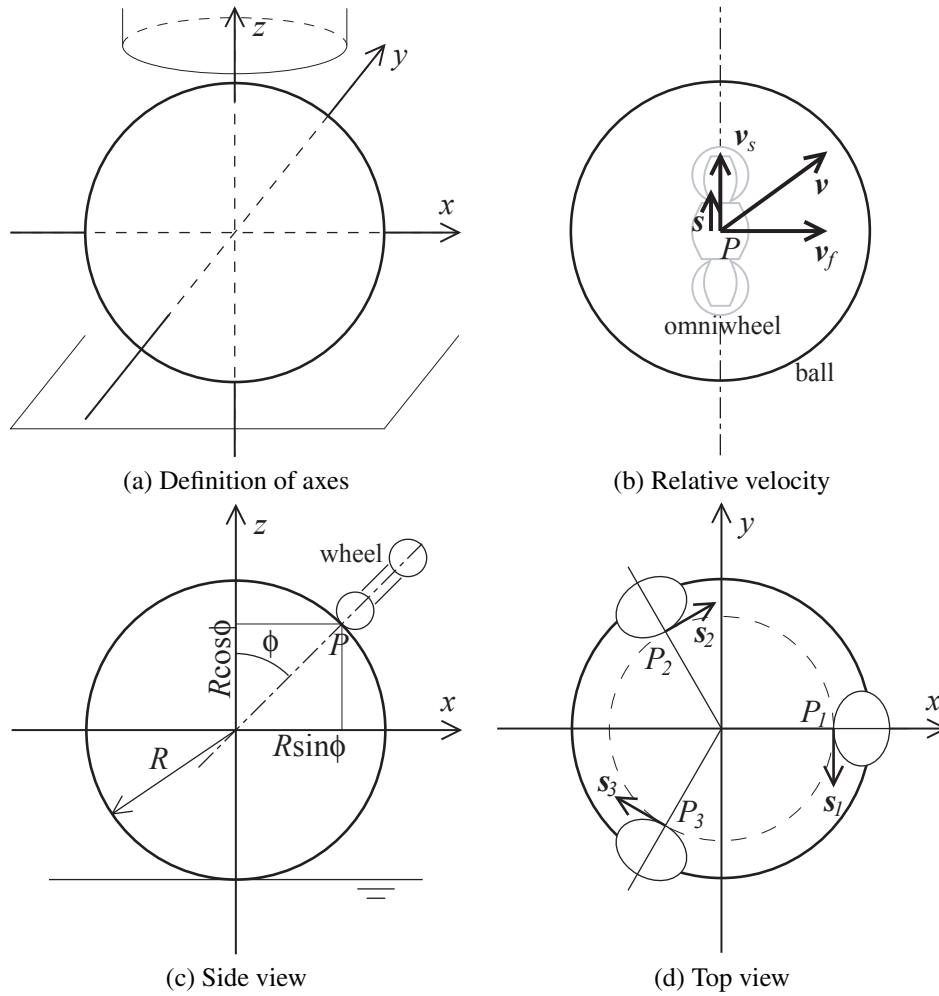


Fig. 3 Definition of axes and relationship between ball and wheels.

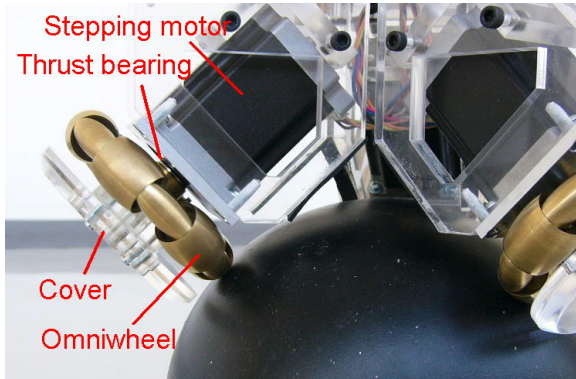


Fig. 2 Driving mechanism of the ball.

wheels that are described in the latter sections of this paper.

## 2.1 Mechanism of robot

We developed two types of robots: the one built first is shown in Fig. 1(b). Its total height was approximately 1,300 [mm], and its weight without the ball was 12 [kg]; the ball weighed 3.6 [kg]. The second robot, developed after the first one, is shown in Fig. 1(c). The intended purpose of the new robot was transportation of load, and it was designed to be shorter than the old one. It was

approximately 500 [mm] in height and 11 [kg] in weight. The ball used in each robot was a rubber-coated bowling ball (coated using liquid rubber spray); its diameter was approximately 200 [mm].

The body of the robot consisted of an electronic apparatus including sensors and a ball driving mechanism that used three stepping motors and omnidirectional wheels. The driving mechanism of the ball is shown in Fig. 2. Each wheel was directly attached to the shaft of the motors with no reduction gears, which reduced the mechanism backlash. The use of the stepping motors reduces the cost of the mechanism, driving circuits, and control software because they have open loop characteristics and their torque is larger than that of DC servomotor. The direct-drive mechanism provided smooth and low oscillatory motion to the robot. The shaft of the motor was reinforced with thrust bearings between the face of the motor and the wheel.

Three wheels were fixed symmetrically at intervals of 120 [deg] as shown in Fig. 3(d). Each wheel was fixed through a shaft so that it was perpendicular to the tangent plane of the ball, as shown in Fig. 3(c). The zenith angle  $\phi$  was 45 [deg] for our first prototype and 40 [deg] for the second one. The omnidirectional wheel used for the robot was that proposed by Asama et al.[3], [4]. This

type of wheel has only one contact line, which makes it easy to drive the robot on an undulated surface. We manufactured these wheels in our workshop. The diameter of wheel was 100 [mm].

The control system consisted of MEMS attitude sensors and a 16-bit microcontroller (Renesas H8/3052) that can run on three 7.2 [V] Ni-MH batteries (also used to run the motors). We employed two sets of an angular velocity sensor (rate gyro) ADXRS401 (Analog devices) and an accelerometer ADXL203 (Analog devices). For fast movements, i.e., high-frequency components, we used the rate gyros. However, they had to be integrated in order to obtain the inclination of the robot; this induced a deflection in the inclination value when only the rate gyros were used. The accelerometer covered inclinations near the DC component, i.e., low-frequency components. However, it cannot be used over the entire band of signals because it detects not only gravity acceleration but also the motion of the robot. Therefore, a combination of these two types of sensors was necessary. We used a first-order digital filter to combine the signals from the two sensors for inclination. The angular velocity detected by the rate gyros was directly used in the control equation because the drift of the sensor signal was small.

We also used a micro-step controller TA8435 (Toshiba semiconductor) for the stepping motors. It divided each step of the motors, 1.8 [deg/step], into 0.225 [deg/step]; this made the rotation of the wheel much smoother. The motor used was KH56QM2-913 (Japan servo), whose nominal maximum torque is 1.3 [Nm].

## 2.2 Deciding optimum wheel speed for driving the ball

The calculation of the speed of each wheel is discussed in this section.

As shown in Fig. 3(a), we define a coordinate system with its origin at the center of the ball. The axes are fixed on the body of the robot: the  $z$  axis is along the vertical line passing through the ball center and the center of mass of the robot; the  $x$  and  $y$  axes are perpendicular to  $z$ .

The speed of each wheel to acquire angular velocity of the ball  $\omega$  is derived as follows. Let the point of contact between the wheel  $i$  and the ball be  $P_i$ , whose position vector is  $\mathbf{p}_i$ . The circumferential speed  $\mathbf{v}_i$  at  $P_i$  corresponding to angular velocity of the ball  $\omega$  is obtained by

$$\mathbf{v}_i = \omega \times \mathbf{p}_i \quad (1)$$

Next, we derive the relation between the circumferential speed of the ball  $\mathbf{v}_i$  and that of the wheel  $\mathbf{v}_s$ . As shown in Fig. 3(b), the speed  $\mathbf{v}$  can be resolved into two components:  $\mathbf{v}_f$ , which is parallel to the axis of the wheel and  $\mathbf{v}_s$ , which is perpendicular to it. The former is generated by the free rotation of the small rollers of the wheel. The latter,  $\mathbf{v}_s$ , is the circumferential speed of the wheel. The relation between  $\mathbf{v}$  and  $\mathbf{v}_s$  is obtained as follows:

$$\begin{aligned} \mathbf{v} \cdot \mathbf{s} &= \mathbf{v}_s \cdot \mathbf{s} + \mathbf{v}_f \cdot \mathbf{s} \\ &= |\mathbf{v}_s| |\mathbf{s}| + 0 = |\mathbf{v}_s| \end{aligned} \quad (2)$$

where  $\mathbf{s}$  is the unit vector in the driving direction of the wheel.

On the other hand, if we arrange the wheels as shown in Fig. 3(c)(d), where the contact points are located at the zenith angle,  $\phi$ , and are equally spaced in the horizontal plane; point  $\mathbf{p}_i$  can be expressed as

$$\begin{aligned} \mathbf{p}_1 &= (R \sin \phi, 0, R \cos \phi) \\ \mathbf{p}_{2,3} &= \left(-\frac{1}{2} R \sin \phi, \pm \frac{\sqrt{3}}{2} R \sin \phi, R \cos \phi\right) \end{aligned} \quad (3)$$

where  $R$  is the radius of the ball and  $\mathbf{p}_2$  and  $\mathbf{p}_3$  are combined because of symmetry. The driving direction  $\mathbf{s}$  has only a horizontal component because we did not skew the wheel in the mechanical design. Thus, we obtain the following values:

$$\begin{aligned} \mathbf{s}_1 &= (0, -1, 0) \\ \mathbf{s}_{2,3} &= (\pm(\sqrt{3}/2), (1/2), 0) \end{aligned} \quad (4)$$

From the above settings, we derive the wheel speed  $|\mathbf{v}_{si}|$  when the ball rotates around the  $x$  axis with an angular velocity  $\omega$ , when  $\omega = (\omega, 0, 0)$ :

$$\begin{aligned} \mathbf{v}_i &= (\omega, 0, 0) \times (p_{ix}, p_{iy}, p_{iz}) \\ &= (0, -p_{iz}\omega, p_{iy}\omega) \\ |\mathbf{v}_{s1}| &= \mathbf{v}_1 \cdot \mathbf{s}_1 \\ &= (0, -p_{1z}\omega, p_{1y}\omega) \cdot (0, -1, 0) \\ &= p_{1z}\omega = R \cos \phi \omega \\ |\mathbf{v}_{s2,3}| &= (0, -p_{iz}\omega, p_{iy}\omega) \cdot (\pm(\sqrt{3}/2), (1/2), 0) \\ &= -(1/2)p_{iz}\omega = -(1/2)R \cos \phi \omega. \end{aligned} \quad (5)$$

Similarly, when  $\omega = (0, \omega, 0)$ ,  $|\mathbf{v}_{si}|$  can be calculated as

$$\begin{aligned} \mathbf{v}_i &= (p_{iz}\omega, 0, -p_{ix}\omega) \\ |\mathbf{v}_{s1}| &= 0 \\ |\mathbf{v}_{s2,3}| &= \pm(\sqrt{3}/2)p_{iz}\omega = \pm(\sqrt{3}/2)R \cos \phi \omega. \end{aligned} \quad (6)$$

Since these two components are independent, we can add them to get the actual speed of the wheel for the inverted pendulum control.

In addition, we can consider the case of rotation around the vertical axis by setting  $\omega = (0, 0, \omega)$ .

$$\begin{aligned} \mathbf{v}_i &= (-p_{iy}\omega, p_{ix}\omega, 0) \\ |\mathbf{v}_{s1}| &= -p_{1x}\omega = -R \sin \phi \omega \\ |\mathbf{v}_{s2,3}| &= (-p_{iy}\omega, p_{ix}\omega, 0) \cdot (\pm(\sqrt{3}/2), (1/2), 0) \\ &= -R \sin \phi \omega \end{aligned} \quad (7)$$

Thus, all wheels rotate at the same speed of  $(-R \sin \phi \omega)$ .

The zenith angle  $\phi$  is significant because it determines the ratio of the wheel rotation to the ball rotation, and it also affects the support of the robot body. When  $\phi$  is near  $\pi/2$ , i.e., the contact point is close to the  $xy$  plane, rotation around the  $z$  axis occurs easily because  $\sin \phi$  is large. However, it is obviously difficult to rotate the ball around the  $x$  axis and  $y$  axis; this is impossible in the case of  $\phi = 0$ . In such a case, however, the ball can rotate, but its motion is mainly restricted to the passive rollers and it cannot be driven actively. On the other hand, when  $\phi$  is small, the speed of the ball around vertical axis respect

to the wheels is increases because  $\sin \phi$  decreases, which makes the rotation of the ball sensitive to the motor speed. Moreover, this robot may fall down with such a narrow supporting triangle because this robot simply rides on the ball.

We used  $\phi = 45 [\text{deg}]$  for our first prototype, in accordance with the above considerations. The wheels can drive the ball even when  $\phi = \pi/2$ , if the wheels are skewed; however, this complicates the mechanical design. These discussion in this section can also be applied when we use more than three wheels by deciding  $P_i(p_i)$  and  $s_i$  though it is possible that some of wheels lose contact with the ball due to geometrical constraint.

### 2.3 Control method

The robot had two sets of accelerometers and rate gyros functioning as attitude sensors in two horizontal axes (around the  $x$  and  $y$  axes in Fig. 3 (a)), as mentioned before. The signals from both the sensors were combined to obtain the inclination and angular velocity around each axis.

The control equations are represented as follows:

$$\begin{aligned} a_x &= K_A \theta_x + K_{AV} \dot{\theta}_x + K_T(x - x_0) + K_V v_x \\ a_y &= K_A \theta_y + K_{AV} \dot{\theta}_y + K_T(y - y_0) + K_V v_y, \end{aligned} \quad (11)$$

where  $a$  is the command acceleration of a *virtual* wheel,  $\theta$  is the inclination,  $x$  and  $y$  are the traveling distances and  $v$  is the velocity calculated from the *virtual* wheel. The suffixes  $x$  and  $y$  denote state variables of the axes, and  $K$  is the constant gain tuned through experiments.  $v$  and  $x(y)$  were obtained by numerical integration of the command acceleration  $a$ . It should be noted that we used acceleration as the command, while conventional inverted pendulum control systems command torque (or force) of the actuator, because we used the stepping motor as the actuator. From some experimental results affected by disturbances, it appears that the acceleration command method is more robust than the torque command method; however this has not been analyzed theoretically.

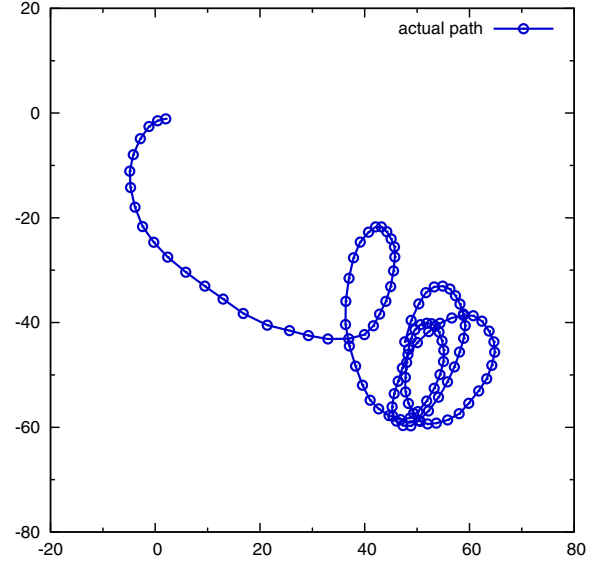
The command velocities of *virtual* wheels derived from acceleration were converted into the velocities of the three *real* wheels:

$$\begin{aligned} v_{s1} &= K_1 v_x + K_2 v_r \\ v_{s2} &= K_1 \{-(1/2)v_x + (\sqrt{3}/2)v_y\} + K_2 v_r \\ v_{s3} &= K_1 \{-(1/2)v_x - (\sqrt{3}/2)v_y\} + K_2 v_r \end{aligned} \quad (12)$$

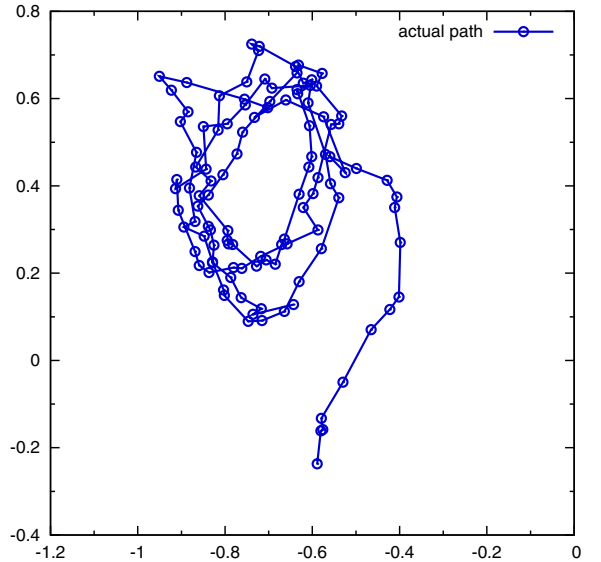
where  $v_s$  is wheel circumferential velocity of each wheel and  $K_1$  and  $K_2$  are the constant gains. The coefficients accompanying  $K_1$  were derived from the relation between the expected speed of the ball ( $v$ ) and the wheel speed ( $v_s$ ), as mentioned above. In addition to  $v_x$  and  $v_y$ , another term  $v_r$  was introduced for command rotation (pivoting) around the vertical axis.

## 3. EXPERIMENTS

We developed the robot and carried out experiments to tune its parameters. Before employing rubber-coated bowling ball, we used a basketball. The robot with the



(a) Fluctuation in robot position ([mm]).

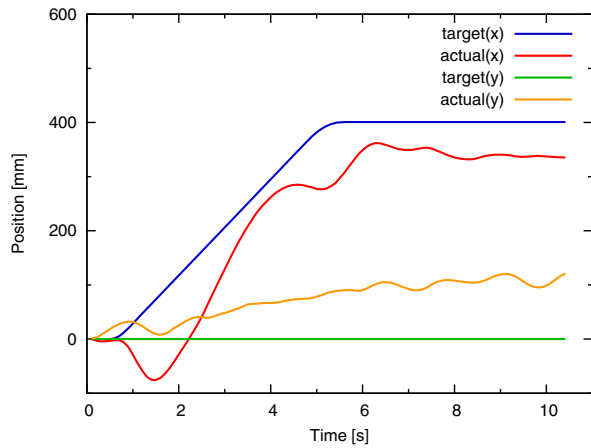


(b) Fluctuation in inclination ([deg]).

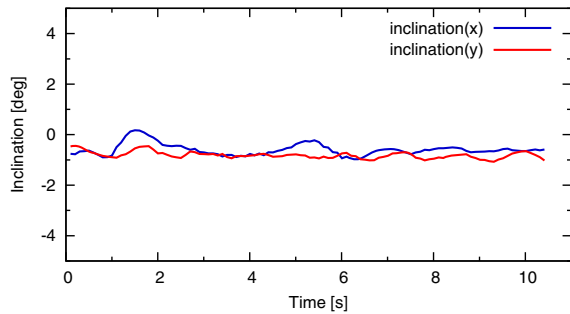
Fig. 4 Experimental results when robot was commanded to stand still. Acquisition interval (points on graphs) was 50 [ms].

basketball could maintain balance, though it was unsteady. This seemed to be due to the lack of stiffness of the ball as it was dented, and it started vibrating when the wheel accelerated. Thus, we replaced the ball with a more rigid one.

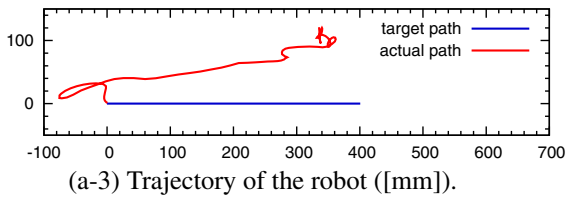
Both the robots developed in this study were controlled stably. Typical data obtained from the tall robot is shown in Fig. 4 and Fig. 5. The robot was commanded to stand still on a flat floor, as demonstrated in Fig. 4. The results indicate the deviation in the steady state. The robot stayed within an approximate 30 [mm] range (it was 200 [mm] when we used the basketball). Note that the position was calculated from the wheel rotation command, and it should be different from that in actual motion in details. The offsets in the inclination were induced by the calibration error of the sensing system because the in-



(a-1) Change in target point and actual position.

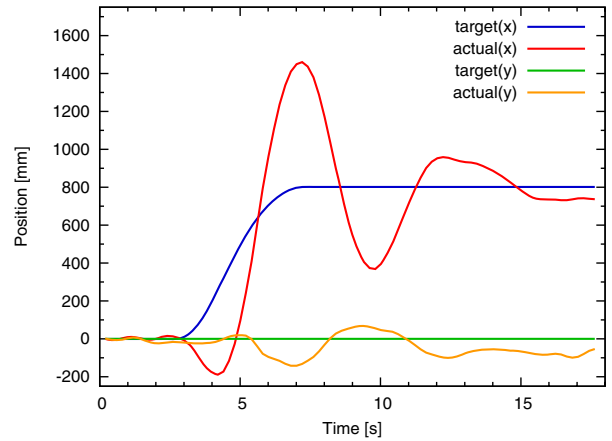


(a-2) Change in inclination of the robot.

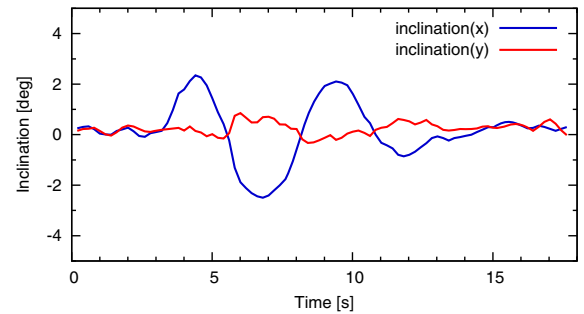


(a-3) Trajectory of the robot ([mm]).

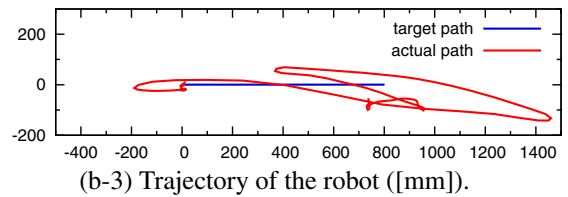
Response for commanded velocity of 90 [mm/s].



(b-1) Change in target point and actual position.



(b-2) Change in inclination of the robot.



(b-3) Trajectory of the robot ([mm]).

Response for commanded velocity of 300 [mm/s].

Fig. 5 Experimental results when robot was commanded to move straight.

verted pendulum must be perfectly vertical; that is, the center of mass must be just above the supporting point, in the steady state.

The results obtained when the robot was commanded to move straight are shown in Fig. 5. The commanded speeds were approximately 90 [mm/s] and 300 [mm/s]. In the result obtained at the lower speed, lesser fluctuations in the inclination and fewer position errors were recorded than those at the faster speed. In both the cases, undershoots were observed when the robot started moving; this occurred due to the nature of the inverted pendulum. The robot needed to incline itself toward the direction of motion in order to accelerate itself. To do so, it kicked the ball back. However, the kick-back motion initiated a fluctuation in the robot, which caused a large overshoot at the higher speed (the maximum actual speed was almost 1000 [mm/s]). The optimization of undershoot and overshoot (acceleration and deceleration) is needed for further applications. The direction error seen in Fig. 5 (a-3) seems to be caused by the coupling of motion and control in the two axes.

In other experiments, the robot could rotate through

60 [deg/s]. In addition, the second robot could carry several books with a total weight of 11 [kg] while moving straight and turning without changing its control gains.

## 4. CONCLUSION

In this paper, we report two new robots that we developed to ride and balance on a ball. The robots could not only traverse but also pivot around their vertical axis. This method provides a solution for a mobile robot with a two-wheel inverted pendulum.

The second robot showed the ability to transport loads in any direction. We could also move it without applying a force of restitution by setting the gain  $K_T$  to zero; which can be used in transport assistance machines in its present form. In the future, we aim at improving the usability of the robot as a carrier.

## Acknowledgments

The use of the omnidirectional wheel in this study was permitted by RIKEN, the patent administrator and the inventors of the wheel. We are grateful for their permission. Most of the mechanical parts were manufactured by engi-

neers in the workshop of Tohoku Gakuin University. We express our thanks for their help.

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