Mirror Symmetry Detection in Digital Images

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Abstract: This article proposes an approach to the recognition of symmetrical objects in digital images, based on a quantitative asymmetry measure construction of such objects. The object asymmetry measure is determined through the Fourier descriptor of a discrete object boundary points sequence. A method has been developed for calculating the asymmetry measure and determining the most likely symmetry axis based on minimizing the asymmetry measure. The proposed solution using the Fourier descriptor has a quadratic complexity in the number of the object boundary points. A practical assessment of the efficiency and effectiveness of the algorithm is obtained by computational experiments with silhouettes of aircraft in remote sensing images.

1 INTRODUCTION

Symmetry is an important classification feature in solving various problems of analysis and recognition of digital images and video. The mirror symmetry property can be used when segmenting and classifying objects. The orientation of symmetrical objects can be determined on the image by the symmetry axes found. For example, the symmetrical silhouettes of aircraft and their orientation can be determined among the many objects obtained by segmenting images in remote images (Fig. 1).

Methods for determining symmetric objects in images solve the problem in various settings, for example, they look for objects with axial or central, global or local symmetry (Liu et al., 2010), (Lee and Liu, 2012), (Widynski et al., 2014). Another important aspect of the method is the use of preliminary segmentation of objects in the image or working directly with the image without preliminary processing. We consider the problem of determining global axial symmetry for segmented images. It is assumed that the segmentation of objects is carried out, but the quality of this segmentation is not very high. An example is shown in Fig. 1. Such a problem arises in the analysis of images obtained by remote sensing of the Earth. The image sizes are very large, the search for symmetrical objects without preliminary segmentation requires a lot of computational time. The source image is segmented based on thresholds or using trained neural networks. The result is a binary image in which it is necessary to recognize objects of a particular class by their shape. In this case, objects in the binary image are distorted by noise. Objects of the desired class are symmetric, but their binary images are not symmetrical in the strict mathematical sense. Thus, the task is reduced to determining the degree of symmetry of binary image objects in order to further classify their shape. Moreover, algorithms for calculating the degree of symmetry should be computationally efficient for use in processing a stream of large images.

Symmetry of human or animal figures can be used for pose definition on an object silhouette. Based on the symmetry evaluation of the object in the frame, the correct shooting angle can be selected when the camera is automatically positioned.

In order to determine whether an object is mirror symmetric, it is enough to find its symmetry axis explicitly, or to establish that there is no symmetry axis. This is an easy task for the human eye. Known algorithms allow to find symmetry axes for perfectly symmetric objects. Such objects can be found in high-quality images, where the symmetry shows up very well and is easily evaluated (Zahn and Roskies, 1972), (Yip et al., 1994). However, when automatic recognition of real digital images, there are problems associated with the inevitable image segmentation errors due to insufficient resolution, low contrast, poor illumination, etc. Such errors are clearly visible in the example in the figure 1. For these reasons, in practical work with real images and videos, the shape of symmetrical objects is far from being perfect. Therefore, the development of a reliable effective algorithm for estimating the object symmetry in images is rele-

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Figure 1: Symmetrical and asymmetrical objects in the airport image. Above is the original photo, below is the binary image obtained as a result of segmentation.

vant and in demand in solving practical problems of machine vision.

The disadvantage of the existing methods for solving the problems of determining the symmetry axis and calculating the symmetry measure is the high computational complexity. A naive search of boundary point pairs with a comparison to some similarity measure of the figure two halves to find the symmetry axis has a cubic complexity in the number of boundary points. Among the more effective existing algorithms for solving the described problems, two classes of algorithms can be distinguished: methods, the main part of which is the two halves comparison of the contour or skeletal object representation (Van Otterloo, 1988), (Sheynin et al., 1999), (Yang et al., 2008), (Kushnir et al., 2016), and methods for analyzing halftone images, for example, using integral transformations (Karkishchenko and Mnukhin, 2012), (Lepskiy, 2013). The article (Fedotova et al., 2016) presents an algorithm for refining the symmetry axis for binary discrete images.

2 APPROACH TO ESTIMATING THE SYMMETRY OF SHAPES

The proposed solution of the problem is based on the symmetry estimator construction of figures in digital images, which allows to rank on the basis of symmetry any figures, both symmetrical, but deformed as a result of noise, and asymmetric. Our approach includes two elements:

- a finite set of possible straight lines-candidates for the symmetry axis of the object iteration;
- the optimal symmetry axis selection based on calculation of the measure of coincidence of the original figure with the figure obtained by mirroring relative to the candidate axis.

Formally the problem is formulated in the following way. Denote:

- S is a figure, a closed one-connected region on the Euclidean plane $\mathbb{R}^2$,
- G is a set of straight lines that have a non-empty intersection with the figure S, each line $g \in G$ is given by a pair $g = (s, \alpha)$, where $s \in S$ is the point of the figure through which the line g passes, α is the angle between the line g and the x-axis,
- $S(g)$ is a figure obtained by symmetrically reflecting S with respect to the straight line $g \in G$,
- $\mu(S, S(g))$ is a measure of the difference between the figures S and $S(g)$. If the figure coincides with its symmetrical image, this value is 0.

The shape asymmetry coefficient is defined as

$$\phi(S) = min_{g \in G} \mu(S, S(g)). \quad (1)$$

Further the figure classification on the basis of symmetry is performed by comparing this coefficient with the selected threshold M. If $\phi(S) \leq M$, then the figure is considered symmetric, if $\phi(S) > M$, then asymmetric.

Applied to a digital image, the figure S is a discrete set of pixels. The set G consists of lines passing through the centers of pixels, points with integer coordinates. The solution of the problem (1) is reduced to a combinatorial search of points from a finite set S and finding the best value of the angle α, minimizing the measure $\mu(S, S(g))$.

Our proposed solution is based on the following principles:

1. To calculate $\phi(S)$, only the set of lines passing through the figure S boundary points is used. The boundary ∂S of the figure S is described as a contour, as a closed chain of pixels connected in an 8-adjacent neighborhood structure.

2. The calculation of the difference measure between a figure and its symmetric reflection with respect to a straight line is determined based on the Fourier descriptor of the figure boundary contour.

The problem (1) solution is carried out on the basis of decomposition into two subtasks of minimization:

$$\phi(S) = min_{s\in\partial S}[min_{0\leq\alpha<\pi}F(\partial S, s, \alpha)], \quad (2)$$

where $F(\partial S, s, \alpha)$ is a symmetry measure of the boundary contour ∂S with respect to the line $g = (s, \alpha)$. The external minimization problem solution is based on iterating over the boundary contour ∂S points for the time $O(n)$, where n is the number of pixels in the boundary contour. To solve the inner subtask, we propose a method based on the use of the boundary contour Fourier descriptor, which allows us to find the best value of α at a given point s also in time $O(n)$. Thus, a joint problem solution has complexity $O(n^2)$. This theoretical complexity estimate is implemented in practice with a very small coefficient value at n^2 by using two simple heuristic rules: one for iterating over contour points in the outer subtask, and the other for minimizing the functional in the inner subtask.

The threshold M for classifying the figure according to the asymmetry coefficient is determined experimentally based on machine learning.

3 ESTIMATION OF OBJECTS MIRROR SYMMETRY BASED ON FOURIER DESCRIPTOR

Let the original object be a simply connected domain. The set $P = \{(x_l, y_l)\}_{l=0}^{N-1}$ consists of the coordinates of all boundary pixels of a silhouette in a 4-adjacent neighborhood structure, and describes a closed curve connected in an 8-adjacent neighborhood structure. The contour construction methods are described in (Mestetskiy, 2009). Convert P to a vector of complex numbers: $U = \{u_l = x_l + iy_l\}_{l=0}^{N-1}$. Then perform a discrete Fourier transform for U, taking the distance between neighboring points of the contour constant (by construction, it can take the values $\{1, \sqrt{2}\}$).

$$f_l = u_0 + \sum_{k=1}^{N-1} u_k \cdot exp(-i \cdot \frac{2\pi}{N} \cdot l \cdot k), l = \overline{0, N-1}.$$

The vector $\{f_l\}_{l=0}^{N-1}$ will be called Fourier descriptor of the contour U. We denote the complex conjugation of x as x^*.

Definition 1. *The contour $U = \{u_l\}_{l=0}^{N-1}$ is called perfectly symmetric if the following conditions are satisfied:*

$$Im(u_0) = 0, u_l = u_{N-l}^*, l = \overline{1, N-1}.$$

Statement 1. *Let $U = \{u_l\}_{l=0}^{N-1}$ be a perfectly symmetric contour. Then for the Fourier descriptor $F = \{f_l\}_{l=0}^{N-1}$ of the contour U the following equalities are fulfilled:*

$$Im(f_l) = 0, l = \overline{0, N-1}.$$

Proof. For convenience, we introduce $u_N \equiv u_0$ and a new contour $\hat{U} = \{u_{N-l}^*\}_{l=0}^{N-1}, l = \overline{0, N-1}$.

For the Fourier descriptor $\hat{F} = \{\hat{f}_l\}_{l=0}^{N-1}$ of the contour $\hat{U}$, the following equalities are satisfied:

$$\begin{aligned}
\hat{f}_l &= \sum_{k=0}^{N-1} u_{N-k}^* \cdot exp(-i \cdot \frac{2\pi}{N} \cdot l \cdot k) = \\
&= u_0^* + \sum_{m=1}^{N-1} u_m^* \cdot exp(-i \cdot \frac{2\pi}{N} \cdot l \cdot (N-m)) = \\
&= u_0^* + \sum_{m=1}^{N-1} u_m^* \cdot 1 \cdot exp(i \cdot \frac{2\pi}{N} \cdot l \cdot m) = \\
&= u_0^* + \sum_{m=1}^{N-1} (u_m exp(-i \cdot \frac{2\pi}{N} \cdot l \cdot m))^* = \\
&= u_0^* + (\sum_{m=1}^{N-1} u_m exp(-i \cdot \frac{2\pi}{N} \cdot l \cdot m))^* = \\
&= u_0^* + (f_l - u_0)^* = f_l^*, l = \overline{0, N-1}.
\end{aligned}$$

Since U is perfectly symmetric, $U = \hat{U}$ is executed. Then $f_l = \hat{f}_l$, $f_l = \hat{f}_l^*$, $l = \overline{0, N-1}$. So $Im(f_l) = Im(\hat{f}_l) = 0$ for all $l \in \overline{0, N-1}$.

□

The Fourier descriptor has the following properties (Theodoridis and Koutroumbas, 2003):

Lemma 1. *Shifting the contour U to a fixed vector Δu changes only one coefficient f_0 of the Fourier descriptor.*

Lemma 2. *After rotating U by a given angle α around the origin, all Fourier descriptor coefficients f_l are multiplied by the constant $exp(i\alpha)$.*

Lemma 3. *Let $U^p = \{u_{(l+p) mod N}\}_{l=0}^{N-1}$ be the contour obtained by cyclic shift of contour points $U = \{u_l\}_{l=0}^{N-1}$ by p positions, $p \in \overline{0, N-1}$, $F = \{f_l\}_{l=0}^{N-1}$ is the Fourier descriptor of the contour U, $F^p = \{f_l^p\}_{l=0}^{N-1}$ is the Fourier descriptor of the contour U^p. Then*

$$f_l^p = f_l \cdot exp(i \cdot \frac{2\pi}{N} \cdot l \cdot p), l = \overline{0, N-1}.$$

Using the described facts about the behavior of the Fourier descriptor under contour elementary transformations, we can generalize the statement 1 to an arbitrary symmetric contour:

Statement 2. *Let the following conditions be satisfied for the contour $U = \{u_l\}_{l=0}^{N-1}$:*

1. *The contour is symmetrical;*
2. *There is a contour point u_p lying on the symmetry axis;*
3. *The symmetry axis has an angle α to the abscissa axis.*

Let $F = \{f_l\}_{l=0}^{N-1}$ be the Fourier descriptor of the contour U. Then the following equality holds for all $l = 1..N-1$:

$$Im[f_l \cdot exp(i \cdot \frac{2\pi}{N} \cdot l \cdot (N-p)) \cdot exp(-i\alpha)] = 0.$$

Proof. Let's perform a sequential shift of the contour by $\Delta u = -u_p$ and a rotation by the angle $-\alpha$. Next, we perform a cyclic shift of the obtained contour points by $N-p$ positions (now the initial point of the contour is u_p, lying on the axis of symmetry which is coincides with the x-axis). The resulting contour $\widetilde{U}$ satisfies the conditions of the statement 1. By performing inverse transformations of the Fourier descriptor according to the rules of the lemmas 1, 2, 3, we obtain the necessary equalities. □

In practice, in noisy images, the equalities from statement 2 are only approximate.

We introduce a value characterizing the asymmetry of the contour. If α is the approximate angle of the symmetry axis passing through the point u_p, then this value is close to zero. To find the symmetry axis, we will minimize it by α for each value of p:

$$t(\alpha, p) = \sum_{l=1}^{N-1} [Im(f_l \cdot exp(i \cdot \frac{2\pi}{N} \cdot l \cdot (N-p) - i\alpha))]^2$$

This value is the norm square of the Fourier coefficients imaginary parts deviation from zero after rotating by the angle $-\alpha$ when choosing u_p as the origin and starting point of the contour.

Calculate the optimal value of α given p, f_l, $l = \overline{1, N-1}$.

$$t(\alpha, p) \to \min_{\alpha \in [0,\pi)} \tag{3}$$

Denote $f_l^p = f_l \cdot exp(i \cdot \frac{2\pi}{N} \cdot l \cdot (N-p))$, $l = \overline{0, N-1}$ be Fourier coefficients after the shift of the contour starting point. Let $exp(-i\alpha) = x + i \cdot y$, $f_l^p = a_l + i \cdot b_l$, $l = \overline{1, N-1}$. It is clear that in this case $x = \cos\alpha$, $y = -\sin\alpha$:

$$f_l^p \cdot exp(-i\alpha) = (a_l + i \cdot b_l) \cdot (x + i \cdot y) = \\ = (x \cdot a_l - y \cdot b_l) + i \cdot (x \cdot b_l + y \cdot a_l),$$

$$t(\alpha, p) = \sum_{l=1}^{N-1} (x \cdot b_l + y \cdot a_l)^2 = \\ = x^2 \cdot \sum_{l=1}^{N-1} b_l^2 + 2xy \cdot \sum_{l=1}^{N-1} (a_l \cdot b_l) + y^2 \cdot \sum_{l=1}^{N-1} a_l^2.$$

Denote $k_1 = \sum_{l=1}^{N-1} b_l^2$, $k_2 = 2 \cdot \sum_{l=1}^{N-1} (a_l \cdot b_l)$, $k_3 = \sum_{l=1}^{N-1} a_l^2$. Then (3) can be rewritten like this:

$$k_1 \cos^2\alpha - k_2 \cos\alpha \cdot \sin\alpha + k_3 \sin^2\alpha \to \min_{\alpha \in [0,\pi]}$$

The optimized function is continuously differentiable by α, it is easy to find the local minima of the given function on the segment $(0, \pi)$. The minimum can also be reached at the end of the segment. There can be three local minima points α, denote them as $\alpha_1, \alpha_2, \alpha_3$ and the global minimum as α^p.

$$\alpha_1 = \begin{cases} \frac{\pi}{4}, & k_1 = k_3 \\ \frac{1}{2}(\arctan(\frac{k_2}{k_1 - k_3}) \mod \pi), & k_1 \neq k_3 \end{cases},$$

$$\alpha_2 = \alpha_1 + \frac{\pi}{2},$$

$$\alpha_3 = 0.$$

The solution finally has the form:

$$k = \arg\min_{i \in \{1,2,3\}} t(\alpha_i, p),$$

$$\alpha^p = \alpha_k.$$

To find the symmetry axis, we perform the described procedure for finding the optimal angle for each contour point, and in addition, calculate the following value of the descriptor coefficients deviation from the found optimal axis for all possible p:

$$Q(p) = \frac{\sqrt{\sum_{l=1}^{N-1} Im(f_l^p \cdot exp(-i\alpha^p))^2}}{N-1}$$

The closer $Q(p)$ is to zero, the more the found line resembles the symmetry axis. Choose $P = \arg\min_{p \in \overline{0, N-1}} Q(p)$. The most suitable symmetry axis passes through the selected vertex, and the value $Q(P) = Q$ makes sense as a asymmetry measure of the figure. There is a classification criterion: we consider a figure symmetric if the value Q for this figure does not exceed the specified threshold M. The described algorithm for calculating the asymmetry measure and determining the symmetry axis is quadratic in terms of the contour points number. The M value is determined based on training.

4 IMPROVING THE COMPUTATIONAL EFFICIENCY OF THE ALGORITHM

Efficiency improvement is achieved by reducing the search within the quadratic algorithm.

First, the symmetry axis of the figure must be the symmetry axis of its convex hull, which can be constructed in subquadratic time, and due to this fact it is enough to check only a few points from the contour instead of a complete search $p = 0, \ldots, N-1$. Namely, on the symmetry axis can lie either a point-vertex of the convex hull, or a point that lies on a line passing through the center of mass (the arithmetic mean of the contour points) and the middle of the convex hull edge. Since all values are determined numerically, we also check the small neighborhood of these points (example in Fig. 2).

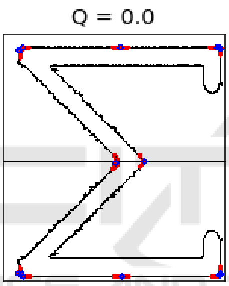

Figure 2: Silhouette of the Σ symbol. Blue dots are vertices to iterate based on convex hull. The red dots are their nearest neighbors. Q is the asymmetry measure.

Secondly, due to the distribution of the Fourier descriptor coefficients modules, we can see that an important contribution to the form description is made by only a small number of Fourier coefficients, namely having a large enough module. To speed up the method, we calculate α^p and $Q(p)$ by a truncated set of Fourier coefficients. That is, we sum only by indices l where the Fourier coefficient module $|f_l|$ is above a given threshold. Usually the first and last harmonics are the most useful. There is an example in Fig. 3. The absolute values of the Fourier coefficients, except for f_0, are invariant to transformations such as shift, rotation and change of the contour starting point, so this operation is correct. In fact, this procedure is equivalent to conducting a contour frequency filtering for approximate localization of the symmetry axis, while losing some information about the original circuit. In the small neighborhood of the found vertex P, we calculate the full set of coefficients for the exact determination of the figure symmetry measure.

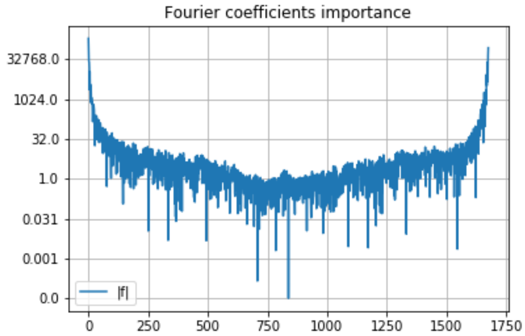

Figure 3: The graph shows the coefficients importance for the silhouette of the symbol Σ (logarithmic scale on the y-axis).

The fast Fourier transform is performed in subquadratic time, thus the final asymptotic of the algorithm is $O(NlogN + mk)$, where m is the number of contour points to iterate, k is the number of Fourier coefficients to compute truncated sums. The described heuristics work at $m \approx 0.05N$, $k \approx 0.3N$.

5 EXPERIMENTS

In the first experiment, a symmetric object silhouette was chosen as the initial image, the symmetry axis is parallel to the x-axis. There is a symbol Σ. The contour is constructed as a boundary pixels sequence, it consists of 1676 points. Next, the algorithm found the value of the asymmetry measure equal to 0 (Fig. 2).

In the case of a symmetric object, the graph $Q(p)$ has two close local minima, one of which is global (Fig. 4). These are values at two opposite contour points lying on the symmetry axis.

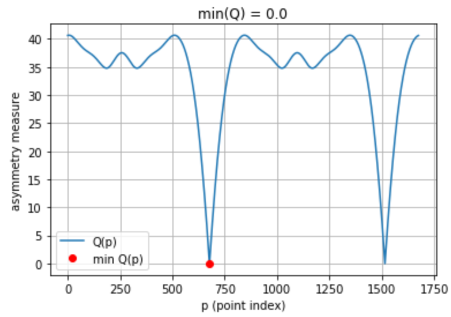

Figure 4: The symbol Σ asymmetry measure with respect to the symmetry axes passing through the various contour points.

We will solve the following classification problem. Silhouettes of butterflies are given. It is required to classify from what position the picture is made: from above or from the side. Silhouettes of the first type are symmetrical, but allow noise as a result of sampling and segmentation. Silhouettes of the second type are not symmetrical. So, to solve the problem it is enough to determine the symmetry measure of each silhouette, and choose the optimal threshold to maximize classifier accuracy. For the butterfly sample shown in Fig. 5, the threshold value was found to be 2.

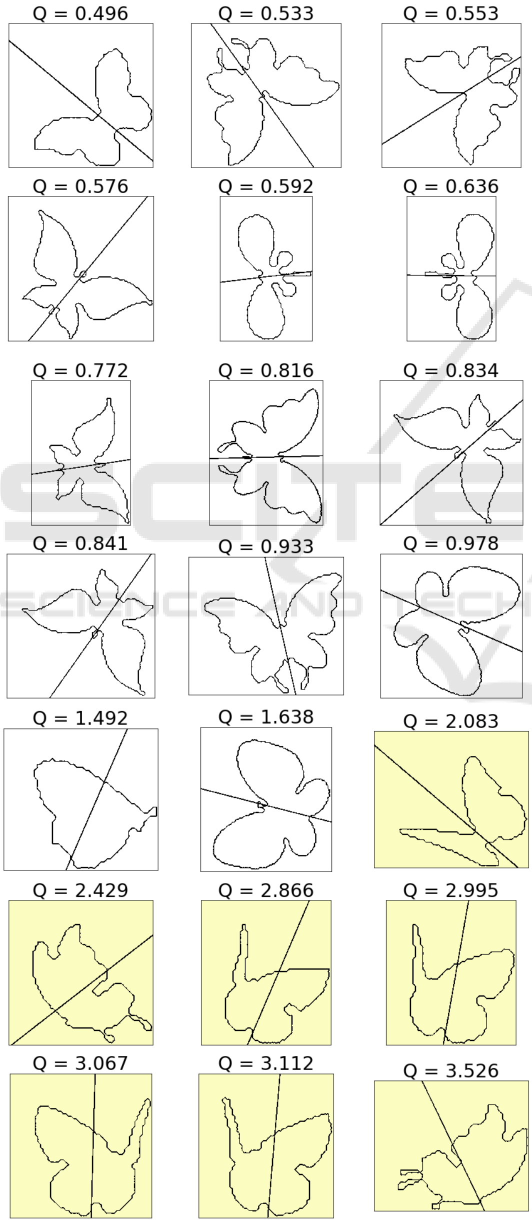

Figure 5: Q is an asymmetry measure. Silhouettes are ordered in Q ascending order. Yellow background indicates silhouettes classified as asymmetrical.

Also earlier computational experiments were carried out on real data: the aircraft silhouettes selected from remote sensing of the Earth images are given, it is required to determine the lines that specify these objects orientation (Mestetskiy and Zhuravskaya, 2019). 2208 binary images with a size of about 120×120 pixels were studied, the average image processing time was 0.1 seconds, the orientation was determined correctly in 98% of cases based on visual analysis, the errors are associated with sampling error (low resolution of images). A few examples are given in Fig. 6.

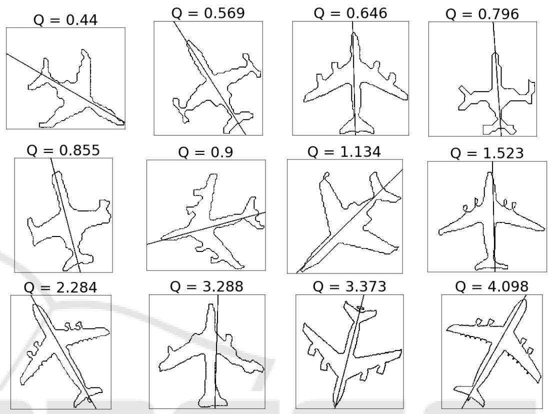

Figure 6: Q is an asymmetry measure. Silhouettes are ordered in Q ascending order.

6 CONCLUSION

A method for solving the classification problem of symmetric and asymmetric binary silhouettes is developed. An efficient algorithm implementing the created method is constructed. The reliability of the solution is established by computational experiments. A new measure of object symmetry is proposed. A new axial symmetry criterion is formulated for a connected object in a digital image. The applicability of this criterion as a feature for solving the classification problem is studied.

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