Mining and Application of Frequent Patterns with Counting Quantifiers

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Abstract. Frequent Pattern Mining (FPM) is a classical problem in graph theory. It is typically defined as discovering all the graph patterns Q whose occurrence frequencies are above a user-specified threshold τ, in a single large graph G. In recent years, we have witnessed wide applications of FPM, such as social network analysis, fraud detection, etc. However, emerging applications keep calling for more expressive graph patterns to capture more complex structures in a large graph. In light of this, we first incorporate counting quantifiers in graph patterns and introduce quantified graph patterns (QGPs) which are able to express richer semantics. We then develop an algorithm, denoted as QGPM, to mine QGPs in a single large graph, and provide effective strategies to optimize space and computational costs. As an application of QGPs, we introduce quantified graph pattern association rules (QGPARs) to identify links between a pair of entities in a large graph. Finally, using real graphs and synthetic graphs, we verified that (1) our QGPM algorithm is very efficient to discover QGPs and our optimization strategies are pretty effective in reducing space and computational costs; (2) compared with existing link prediction methods, QGPARs mined achieve even higher prediction accuracy.

1 Introduction

Graphs simulate complex connections between objects in a variety of applications such as chemistry, bioinformatics, social networks, and text retrieval, where entities are represented as nodes, and interactions between entities are represented as edges. Frequent pattern mining (FPM), the problem is to find subgraphs from a collection of small graphs or a single large graph whose support is not lower than a user-specified threshold. For a long time, FPM has been a core task in the field of graph mining, and related research has also made significant progress. Driven by application scenarios, existing work considers this problem in two different settings: transaction-based and single-graph-based. With the widespread popularity of applications such as social networks and network analysis, frequent pattern mining based on a single large graph has received more attention [13].

However, as application scenarios become more complex, more expressive patterns are required, especially those with counting quantifiers (CQs), which were not taken into account by previous mining methods. CQs are represented as ordinary mathematical symbols in first-order logic. The introduction of CQs can expand the expressive ability of semantics and help solve more complex problems [14].

Example 1. Graph patterns with CQs reflect the regularity of connections between entities in social networks. Patterns Q_1 and Q_2 of Fig. 1 show respectively:

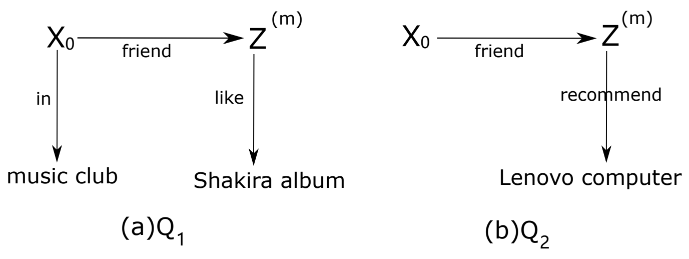


Fig. 1. Quantified graph patterns

◦ *If* (i) *person X_0 is in a music club, and* (ii) *among the people whom X_0 follows, at least m of them like Shakira's album, then X_0 may also like the album.*

◦ *If* (i) *m people Z whom X_0 follows,* (ii) *Z recommends Lenovo computer, then X_0 may buy a Lenovo computer.*

Intuitively, the above graph patterns contain the CQ m, and achieve quantitative aggregation through m. Example 1 shows that, compared with the traditional graph pattern, the graph pattern with CQs has richer semantics and can express more meaningful special structures. From the perspective of application, formulating a marketing strategy based on the above patterns is expected to outperform traditional marketing. Indeed, empirical studies suggest that "90% of customers believe in peer recommendations, while only 14% of customers trust advertisements [1]", and "the peer influence from one's friends leads to more than 50% increase in the probability of buying a product [6]".

To the best of our knowledge, no existing mining model can solve the mining problem of patterns with CQ. In this paper, firstly the significance of patterns with CQ is analyzed, and the difference between the patterns with CQ and traditional patterns is explored. Then, referring to the previous traditional FPM algorithm, a series of researches on this problem are carried out.

Contributions. Our contributions in this paper can be summarized as follows.

(1) We extend the traditional graph pattern and propose QGPs (Section 2). Using simple CQs, QGPs are more suitable for some complex situations.

(2) We propose QGPM, a framework for mining frequent QGPs in large single graphs. Based on QGPM, corresponding optimization strategies are proposed to improve the performance of the algorithm.

(3) As an application of QGP, we introduce Quantified Graph Pattern Association Rules (QGPARs; Section 4). QGPARs help us predict entity relationships in social networks.

(4) Using real-life and synthetic graphs, we experimentally verify the performance of our algorithms (Section 5). We find the following. (a) QGPs mining is feasible on large graphs. (b) QGPARs can predict missing relationships with an average accuracy of 84.3% on real-life graphs, and it can achieve higher prediction accuracy compared to existing link prediction methods.

Related Work. We categorize related work as follows.

Graph pattern mining. Frequent pattern mining is a core problem in many graph analysis applications [27, 21]. In recent years, many frequent pattern mining algorithms for large-scale data sets have emerged. [12] presented GraMi, a novel framework for frequent subgraph mining in a single large graph. GraMi is based on a novel approach, which is not storing all the appearances of a subgraph in a large graph, but only stores the subgraph's templates. [18] proposed FSSG, an algorithm that uses graph invariant properties and symmetries present in a graph to generate candidate subgraphs. FSSG reduces the generation of a large number of candidate subgraphs, thereby reducing the complexity of candidate generation and frequency counting. [20] introduced the SOCMI algorithm, its core idea is to store the appearance of patterns with pathgraph, which makes it easier for SOCMI to extend patterns and calculate frequency during mining. In addition, some mining methods such as weighted graph and dynamic graph have also been widely studied. [5, 19] proposed to mine weighted frequent subgraphs in a weighted single large graph. StreamFSM [25] and IncGM+ [3] are mining algorithms based on dynamic graphs.

Pattern semantic extension. In practice, traditional graph patterns are not suitable for complex situations, and the necessity of semantic expansion of graph patterns is gradually revealed. SPARQLog extends SPARQL language with first-order logic (FO) rules including existential and universal quantification of nodes in the graph [10]. For social networks, SocialScope [4] and SNQL [26] are algebraic languages with numerical aggregations on node and edge sets. Furthermore, social recommendation rules are studied in [22], and support counts are introduced as constraints.

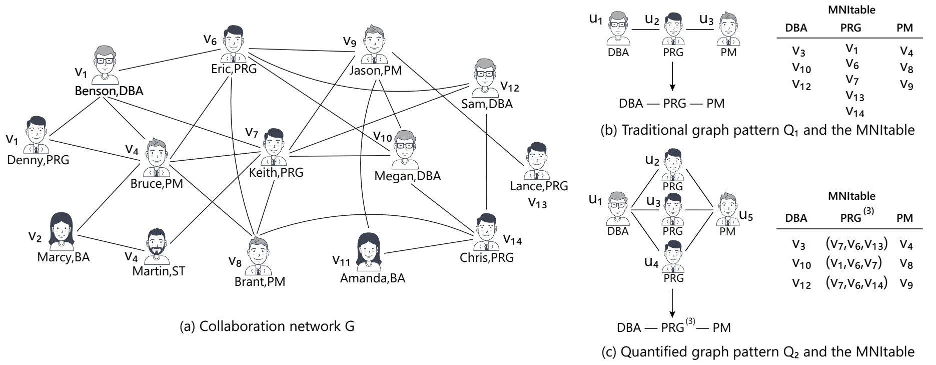
Previous studies have shown that existing extensions remain limited in emerging applications requiring patterns with complex features, notably counting quantifiers (CQs), predicates and negation [23]. Among these features, only CQs have received some attention [8, 15, 13, 23]. The QGraph [8] allows annotating edges of a graph pattern with CQs of the form [min, max] that can express different semantics (*e.g.,* negated ([0, 0]) and optional ([0, 1]) edges). From this, users can formulate query conditions consisting of nodes and edges. In [15], a quantified graph pattern has been proposed where CQs can express numeric and ratio aggregates of the forms: "=p(%)", "≥p(%)", and "=0" for negation. [15] also introduced a quantified matching model that can be used to identify potential customers in social media marketing. In [13], CQs are expressed in the following forms: "[min, max]", "≥min" and "≤max", which are used to annotate edges to extend the traditional graph pattern. [23] proposed the Conditional Graph Pattern (CGP), which extended the traditional graph pattern with CQs.

2 Preliminaries

In this section, we first review traditional graph patterns; we then introduce related concepts of quantified graph patterns(QGPs).

2.1 Conventional Graph Pattern Mining

Graph. A *graph* is defined as $G = (V, E, L)$, where (1) V is a set of nodes; (2) $E \subseteq V \times V$ is a set of edges; and (3) each node v in V carries a tuple $L(v) = (A_1 = a_1, \cdots, A_n = a_n)$, where $A_i = a_i (i \in [1, n])$ represents that the node v has a value a_i for the attribute A_i, and is denoted as $v.A_i = a_i$, *e.g.,* v.name ="*Sam*", v.job_title ="*DBA*".



MNItable

DBA	PRG	PM
v_3	v_1	v_4
v_{10}	v_6	v_8
v_{12}	v_7	v_9
	v_{13}	
	v_{14}	

MNItable

DBA	PRG$^{(3)}$	PM
v_3	(v_7,v_6,v_{13})	v_4
v_{10}	(v_1,v_6,v_7)	v_8
v_{12}	(v_7,v_6,v_{14})	v_9

Fig. 2. Graph & Traditional graph pattern & Quantified graph pattern(QGP)

A graph $G' = (V', E', L')$ is a *subgraph* of $G = (V, E, L)$, denoted by $G' \subseteq G$, if $V' \subseteq V$, $E' \subseteq E$, and moreover, for each $v \in V'$, $L'(v) = L(v)$.

Pattern. A *pattern* Q is defined as a graph (V_p, E_p, L_v), where V_p and E_p are the set of nodes and edges, respectively; for each u in V_p, it is associated with a predicate $L_v(u)$ defined as a conjunction of atomic formulas of the form of '$A = a$' such that A denotes an attribute of the node u and a is a value of A. Intuitively, $L_v(u)$ specifies search conditions imposed by u.

Graph pattern matching. Consider graph $G = (V, E, L)$ and pattern $Q = (V_p, E_p, L_v)$, a node v in G satisfies the search conditions of a pattern node u in Q, denoted as $v \sim u$, if for each atomic formula '$A = a$' in $L_v(u)$, there exists an attribute A in $L(v)$ such that $v.A = a$.

Further, a match of pattern Q in graph G is a *bijective function* f from the nodes of Q to the nodes of a subgraph G, such that (1) for each node $u \in V_p$, $f(u) \sim u$, and (2) (u, u') is an edge in Q if and only if $(f(u), f(u'))$ is an edge in G. We denote by $Q(G)$ the set of matches of Q in G.

Example 2. Consider Q_1 (Fig. 2(b)) and G (Fig. 2(a)), $Q_1(G) = \{\{v_3, v_1, v_4\}, \{v_3, v_7, v_4\}, \{v_3, v_7, v_8\}, \{v_3, v_7, v_9\}, \{v_3, v_6, v_9\}, \{v_3, v_6, v_8\}, \{v_3, v_6, v_4\}, \{v_{10}, v_7, v_9\}, \{v_{10}, v_7, v_8\}, \{v_{10}, v_7, v_4\}, \{v_{10}, v_6, v_9\}, \{v_{10}, v_6, v_8\}, \{v_{10}, v_6, v_4\}, \{v_{10}, v_{13}, v_9\}, \{v_{10}, v_{14}, v_8\}, \{v_{12}, v_6, v_9\}, \{v_{12}, v_6, v_8\}, \{v_{12}, v_6, v_4\}, \{v_{12}, v_7, v_4\}, \{v_{12}, v_7, v_8\}, \{v_{12}, v_7, v_9\}, \{v_{12}, v_{14}, v_8\}\}$.

<u>Support</u>. The support of a pattern Q in a graph G, denoted by $sup(G, Q)$, indicates the occurrence frequency of Q in G.

Having an anti-monotone support metric is of crucial importance since it allows the development of methods to effectively prune the search space; several anti-monotone support metrics exist such as minimum image based (MNI) [9], harmful overlap (HO) [16], and maximum independent sets (MIS) [17]. In this paper, we adopt the MNI metric because it can be calculated more efficiently.

Let f be the set of isomorphic mappings of pattern $Q = (V_p, E_p, L_v)$ in G. The length of the set f is m, while $F(u) = \{f_1(u), f_2(u), \cdots, f_m(u)\}$ is the mapping function for each u on the set f after de-duplication, where $u \in V_s$. The minimum image based support (MNI) of Q in G, denoted by $sup(G, Q)$, is defined as $sup(G, Q) = \min\{t | t = |F(u)|, u \in V_p\}$.

Intuitively, MNI computes the support of a pattern Q as the minimum number of distinct graph vertices matching each $u_i \in V_p$. We use MNItable to represent the MNI field of pattern Q. An MNItable consists of a set of MNIcol; the MNI metric returns the length of the smallest MNIcol. An MNIcol(u_i) contains a list of distinct valid nodes, *i.e.,* nodes corresponding to pattern node u_i in the match list for Q in G.

Example 3. Recall Example 2, it can be easily verified that MNIcol(u_1) = $\{v_3, v_{10}, v_{12}\}$, MNIcol(u_2) = $\{v_1, v_6, v_7, v_{13}, v_{14}\}$, MNIcol($u_3$) = $\{v_4, v_8, v_9\}$, thus $sup(G, Q_1) = 3$. Fig. 2(b) shows the resulting MNItable.

2.2 Quantified Graph Patterns

Quantified Graph Patterns (QGPs). A QGP $Q^{(m)}_{(u_0)}$ is defined as (V_p, E_p, L_u, f_u), where (1) V_p and E_p are the set of pattern nodes and edges, respectively; (2) L_u is a function that assigns a label to a node u ($u \subseteq V_p$); and (3) f_u assigns a CQ m (a natural number) to the specified quantifier-constrained node u_0 such that $f_u(u_0) = m$.

Intuitively, the quantified graph pattern extends the traditional graph pattern by incorporating CQ. It supports counting on a single node, *i.e.,* for two or more nodes with the same label, if they all connect to the same set of nodes, they can be shrunk into a single node with a CQ. *e.g.,* in Fig. 2(c), Q_2 is a quantified graph pattern and can be represented as $DBA - PRG^{(3)} - PM$, *i.e., PRG* is associated with a superscript "(3)" as a quantifier.

In particular, we only consider the case where a single node in the pattern has a CQ. For a QGP, when the quantifier constraints are not considered, the corresponding pattern is called a quantifier-free pattern, and is denoted as Q^w. *e.g.,* Q_1 in Fig. 2(b) is the quantifier-free pattern of Q_2 in Fig. 2(c).

Support of QGP. Inspired by MNI support. Since QGP carries CQs constraints, when calculating MNItable, we regard the nodes with CQ as a column, and then calculate their MNI support. *e.g.,* in Fig. 2(c), for Q_2, denote MNIcol($PRG^{(3)}$)=$\{(v_7, v_6, v_{13}), (v_1, v_6, v_7), (v_7, v_6, v_{14})\}$, MNIcol($PM$)=$\{v_4, v_8, v_9\}$, MNIcol($DBA$)=$\{v_3, v_{10}, v_{12}\}$, hence $sup(G, Q_2) = 3$.

Theorem 1. *Given a QGP* ${Q_1}^{(m)}_{(v)}$*, if there is another QGP* ${Q_2}^{(m')}_{(v)}$*, they have the same* Q^w*, but* $m > m'$*, then* ${Q_1}^{(m)}_{(v)}$ *implies* ${Q_2}^{(m')}_{(v)}$*, denoted as* ${Q_1}^{(m)}_{(v)} \Rightarrow {Q_2}^{(m')}_{(v)}$*. For this case, only need to consider the pattern with the largest CQ.*

Example 4. Consider two QGPs, $Q^{(2)}_{(B)} : A - B^{(2)} - C$ and $Q^{(3)}_{(B)} : A - B^{(3)} - C$, we will only focus on the latter, because (1) $Q^{(3)}_{(B)} \Rightarrow Q^{(2)}_{(B)}$. (2) According to the "anti-monotone" property, if $Q^{(3)}_{(B)}$ is frequent, $Q^{(2)}_{(B)}$ must also be frequent.

In view of this, in this paper, for frequent patterns with the same Q^w, we only discuss the case with the largest CQ.

1-itemset. *1-itemset* Refers to the structure of $X - Y^{(m)} (m \geq 1)$ in the graph, where X and Y are quantifier-free node and quantifier-constrained node, respectively. It is frequent when the support of a *1-itemset* meets the threshold.

Forward & Backward expansion. The forward extension on a pattern Q essentially introduces a new edge from one node in Q; while the backward extension includes a new edge from two existing nodes. Interested readers can refer to [29].

symbols	**notations**
CQ	counting quantifier
$Q_{(v)}^{(m)}$	QGP, v is the quantifier-constrained node and m is the value of CQ.
Q^w	an quantifier-free pattern corresponding to a QGP
$Q1_{(v)}^{(m)} \Rightarrow Q2_{(v)}^{(m')}$	$Q1_{(v)}^{(m)}$ implies $Q2_{(v)}^{(m')}$
QnodeLabel	quantifier-constrained node label
Qnum	value of CQ
SimpleNode	quantifier-free node
InstanceSet.size	number of pattern matches
MNItable.SimplenodeCol(u_i)	column corresponding to quantifier-free node u_i in MNItable
MNItable.QuantifiednodeCol	column of quantifier-constrained node in MNItable

Table 1. Notations used in the paper

The notations of this paper are summarized in Table 1.

3 Frequent QGPs mining

In this section, we first investigate the frequent QGP mining problem (Section 3.1); we then develop an algorithm for the problem (Section 3.2) and provide an optimization strategy to improve performance (Section 3.3).

3.1 Problem Statement

The *frequent QGPs mining* (QGPMining) problem can be stated as follows.

- Input: A graph G, support threshold τ.
- Output: A set $\mathbb{S}$ of frequent QGPs Q of G such that $sup(G, Q) \geq \tau$ for any Q in $\mathbb{S}$.

According to Theorem 1, here $\mathbb{S}$ only contains all frequent QGPs with the largest CQ in the same Q^w structure.

Proposition 2: *The decision problem of QGPMining is NP-hard.*

To see Proposition 2, observe that the subgraph isomorphism (ISO) problem is embedded in QGPMining problem, thus QGPMining problem must be at least as hard as ISO problem [11]. Since ISO is an NP-complete problem, thus QGPMining problem must be NP-hard.

Algorithm 1: *QGPM*

```
Input: graph $G$, support threshold $\tau$.
Output: All QGPs $Q$ of $G$ such that sup$(G, Q) \geq \tau$.
1  $result \leftarrow \emptyset$;
2  Let $fEdges$ be the set of frequent single edge patterns in $G$;
3  Let $fItemSet$ be the set of frequent 1-itemset in $G$;
4  for each $item \in fItemSet$ do
5      $result \leftarrow result \cup$ QuanExt($item, G, \tau, fItemSet$);
6      Remove $item$ from $fItemSet$;
7  for each $fwitem \in result$ do
8      $result \leftarrow result \cup$ FWExt($fwitem, G, \tau, fEdges$);
9  for each $bwitem \in result$ do
10     $result \leftarrow result \cup$ BWExt($bwitem, G, \tau, fEdges$);
11 return $result$;
```

3.2 frequent QGP mining algorithm

Theorem 3. *QGP is constrained by counting quantifiers, compared to the traditional pattern, under the same support threshold, the number of QGP will be much less.*

According to Theorem 1 and Theorem 3, we consider storing all matches corresponding to QGP.

Proposition 4: *The expansion process adopts the method of quantifiers from high to low, which can ensure that the first frequent pattern found by the algorithm must be the pattern with the largest CQ in the same Q^w.*

Star-QGP. Star-QGP has the characteristics of star topology. The pattern structure takes a quantifier-constrained node as the center, and other quantifier-free nodes are connected with the center node.

Proposition 5: *The generation of Star-QGP starts with a 1-itemset, and then gradually adds adjacent 1-itemsets.*

Specifically, the Star-QGP is generated by merging nodes with the same quantifier-constrained node labels in both patterns, and then the CQ value of the new pattern takes the smaller CQ of the two patterns.

Remark. Unless otherwise stated, the patterns we mention below are QGP (quantifier-constrained node and CQ are not given).

We next provide an algorithm, denoted by QGPM. Below, we describe the details of the algorithm.

Initialization. QGPM first initializes a set of parameters: set *result* for keeping track of frequent QGPs (line 1), sets *fEdge* and *fItemSet* are used to maintain frequent single-edge patterns and frequent *1-itemsets*, respectively(lines 2-3). It should be noted that *1-itemset* here is the largest case of quantifiers in the same Q^w.

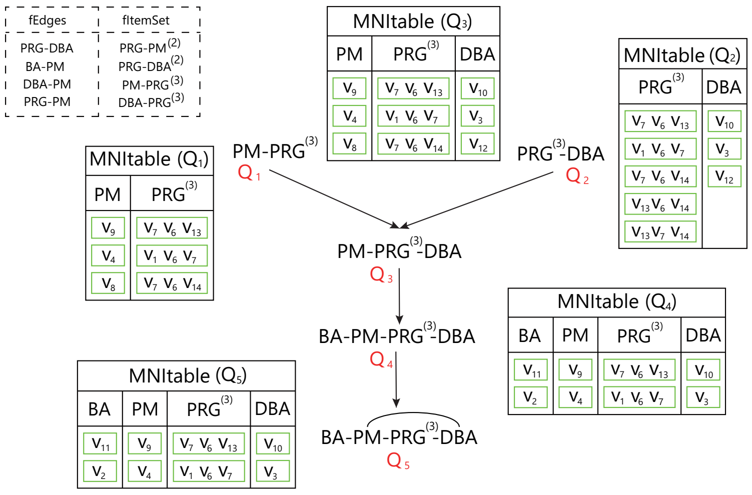


Fig. 3. Example of pattern extension process

Frequent QGPs mining. Based on *fEdge* and *fItemSet*, QGPM is divided into different stages to obtain results. In the first stage, for each *1-itemset* (from *fItemSet*), QGPM applies the procedure *QuanExt* to generate a set of frequent Star-QGPs (lines 4-6); the second is forward expansion, QGPM calls the procedure *FWExt* to extend the Star-QGP produced in the first stage by adding new edges (from *fEdges*) (lines 7-8); finally, QGPM employs the procedure *BWExt* for backward extension (lines 9-10). In addition, to exclude the already generated extensions, we adopt the VF2-based graph isomorphism detection algorithm. Next, we describe the details of each stage.

Algorithm 2: *QuanExt*

```
Input: QGP $S$, graph $G$, support threshold $\tau$, frequent itemset
       $fItemSet$.
Output: frequent Star-QGPs.
1  $result \leftarrow S$, $candidateSet \leftarrow \emptyset$;
2  for each $item \in fItemSet$ do
3      if $item.QnodeLabel$ equals $S.QnodeLabel$ then
4          Let $G'_s$ be the extension of $S$ with $item$;
5          if $G'_s$ was not generated then
6              $candidateSet \leftarrow candidateSet \cup \{G'_s\}$;
7  for each $c \in candidateSet$ do
8      if ISFrequent$(c, G, \tau)$ then
9          $result \leftarrow result \cup$ QuanExt$(c, G, \tau, fItemSet)$;
10     else
11         $candidateSet \leftarrow candidateSet \cup$ CQdec$(c, G, \tau)$;
12 return $result$;
```

(I) Star-QGP generation. QGPM employs procedure *QuanExt* to generate Star-QGPs. More specifically, *QuanExt* takes a QGP S as input and tries to extend it with the *1-itemset* of *fItemSet* (lines 2-5). That is, when S and the current *1-itemset* have the same quantifier-constrained node label, it means that they can be merged to generate a new candidate pattern. All applicable extensions that have not been previously considered are stored in *candidateSet* (line 6). Then, *QuanExt*(lines 7-11) evaluates all candidate patterns generated. If the candidate is frequent, the program continues to extend and evaluates it recursively. Otherwise, decrement the CQ for the current pattern, and check whether the pattern after the decrement is frequent(line 11, details of *CQdec* will be given shortly).

Example 5. Given Fig. 3 with τ=2 and G (Fig. 2(a)). Fig. 3 shows the expansion process of this pattern and the corresponding MNItable. Specifically, QGPM first initializes the sets *fEdge* and *fItemSet* (shown in the upper left corner of Fig. 3), as their support is greater than 2, and then QGPM runs in stages. For example, considering Q_1 and Q_2(Fig. 3), QGPM applies procedure *QuanExt* to extend them, and generates candidate Star-QGP Q_3 (sup$(G, Q_3) = 3$).

(II) Forward expansion. QGPM applies *FWExt* for forward expansion, which is aimed at the quantifier-free nodes in QGPs. *FWExt* takes QGP S as input, and first obtains the quantifier-free nodes in S (line 2), then extends S by adding new edges (from *fEdges*) to generate candidate patterns (lines 3-6). After that, *FWExt* evaluates the support of candidate patterns and eliminates the members of the candidate set that do not satisfy the support threshold τ. Finally, *FWExt* is recursively executed to further expand the frequent subgraphs (lines 9-10).For the contents of line 12, please refer to *QuanExt*.

Algorithm 3: *FWExt*

```
Input: QGP $S$, graph $G$, support threshold $\tau$, frequent edgeset
       $fEdges$.
Output: frequent QGPs after forward expansion.
1  $result \leftarrow S$, $candidateSet \leftarrow \emptyset$;
2  $SimpleNodeSet \leftarrow SimpleNode$ of $S$;
3  for each $e \in fEdges$ and $u \in SimpleNodeSet$ do
4      if $e$ can be used to extend $u$ then
5          Let $G'_s$ be the extension of $S$ with $e$;
6          if $G'_s$ was not generated then
7              $candidateSet \leftarrow candidateSet \cup \{G'_s\}$;
8  for each $c \in candidateSet$ do
9      if ISFrequent$(c, G, \tau)$ then
10         $result \leftarrow result \cup$ FWExt$(c, G, \tau, fEdges)$;
11     else
12         $candidateSet \leftarrow candidateSet \cup$ CQdec$(c, G, \tau)$;
13 return $result$;
```

```
Algorithm 4: BWExt
  Input: QGP $S$, graph $G$, support threshold $\tau$, frequent edgeset
         fEdges.
  Output: frequent QGPs after backward expansion.
1 $result \leftarrow S$, $candidateSet \leftarrow \emptyset$;
2 $SimpleNodeSet \leftarrow SimpleNode$ of $S$;
3 for each $u \in SimpleNodeSet$ do
4     for each $v \in SimpleNodeSet$ do
5         if $S$ does not contain $Edge(u, v)$ and fEdges contains
          $Edge(u, v)$ then
6             Let $G'_s$ be the extension of $S$ with $Edge(u, v)$;
7             if $G'_s$ is not already generated then
8                 $candidateSet \leftarrow candidateSet \cup \{G'_s\}$;
9 for each $c \in candidateSet$ do
10    if ISFrequent$(c, G, \tau)$ then
11        $result \leftarrow result \cup$ BWExt$(c, G, \tau, fEdges)$;
12    else
13        $candidateSet \leftarrow candidateSet \cup$ CQdec$(c, G, \tau)$;
14 return result;
```

(III) Backward expansion. QGPM applies *BWExt* for backward expansion. Similar to *FWExt*, *BWExt* takes QGP S as input and then extends S by adding new edges (from *fEdges*), but the difference is that *BWExt* does not introduce new nodes (i.e. two nodes of the extended edge already exist in pattern S). Algorithm 4 shows the process of backward expansion.

Example 6. Continuing with Example 5, *FWExt* generates candidate pattern Q_4 by enlarging Q_3 with frequent single-edge pattern $PM - BA$. Then Q_5 is generated by extending Q_4 through *BWExt*.

```
Algorithm 5: CQdec
  Input: QGP $c$, graph $G$, support threshold $\tau$.
  Output: prePattern: the input pattern after the quantifier is
          decremented
1 $Qnum := c.Qnum$;
2 $prePattern \leftarrow c$;
3 while $Qnum > 2$ do
4     $prePattern.Qnum := --Qnum$;
5     if ISFrequent$(c, G, \tau)$ then
6         return prePattern;
7         break;
8 return;
```

Also, during the different extension phases of the pattern, QGPM uses the procedure *CQdec* to reduce the CQ value of the pattern. Specifically, if a QGP is infrequent but its CQ $m > 2$, we reduce m and then continue to determine if the pattern is frequent.

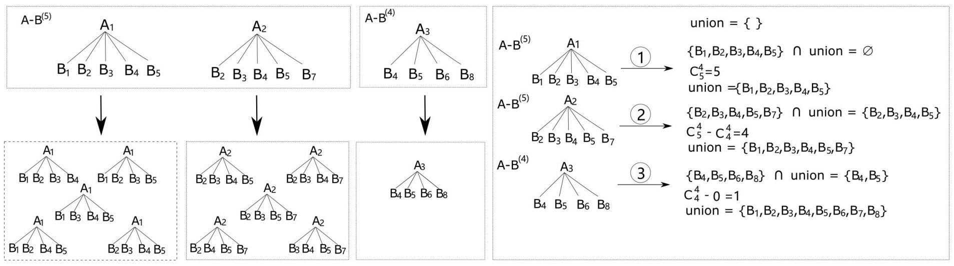
Algorithm 6: *ISFrequent*

```
   Input: QGP $S$, graph $G$, support threshold $\tau$.
   Output: true if $S$ is a frequent QGP of $G$, false otherwise
1  $InstanceSet \leftarrow$ all instance of $S$;
2  if $InstanceSet.size < \tau$ then
3      return false;
4  for each $instance \in InstanceSet$ do
5      for each $v \in instance.SimpleNode$ do
6          $MNItable.SimplenodeCol(u_i).add(v)$;
7      $MNItable.QuantifiednodeCol.add(instance.QuantifiedNode)$;
8      $minlength = MinLength(MNItable)$;
9      if $minlength \geq \tau$ then
10         return true;
11 return false;
```

Support evaluation. According to Section 2, a QGP S is frequent in G (*i.e.,* $sup(G, Q) \geq \tau$) if there exist at least τ nodes in each domain (MNIcol(v_1), $\cdots$, MNIcol(v_n)) that are valid variable assignments (*i.e.,* are part of a match) for the corresponding variables ($v_1, \cdots, v_n$). To evaluate support, a procedure *ISFrequent* is employed by QGPM. It returns *true* iff S is a frequent QGP in G or *false* otherwise. In a nutshell, if QGP has fewer matches (*i.e.,* instances) than τ, the procedure directly returns *false* (Line 3). Following, *ISFrequent* considers each match and places nodes assigned to variables in the corresponding domains (lines 5-7). If all domains have at least τ nodes, then S is frequent in G. Otherwise, *ISFrequent* continues to consider other matches.

3.3 Optimization

To further improve the performance of QGPM, we propose the following optimization methods called storage reduction and computation optimization from the perspective of intermediate result storage and support computation.
Storage reduction. To see Theorem 1, if $Q1_{(v)}^{(m)} \Rightarrow Q2_{(v)}^{(m')}$, it is not difficult to conclude that each match of $Q1_{(v)}^{(m)}$ must contain a certain number of matches of $Q2_{(v)}^{(m')}$. As shown in Fig. 4(a), denote $A - B^{(5)}$ as $Q_{(B)}^{(5)}$, and denote $A - B^{(4)}$ as $Q_{(B)}^{(4)}$. It is observed that two matches of $Q_{(B)}^{(5)}$ each contain five matches of $Q_{(B)}^{(4)}$(listed in the figure). In light of this, when m is large, the matching of $Q1_{(v)}^{(m)}$ must contain a huge number of matching of $Q2_{(v)}^{(m')}$. If these split matches are stored, the space overhead is undoubtedly huge. To this end, we propose a non-split storage method, that is, the matching of patterns with large quantifiers is directly assigned to patterns with small quantifiers without spliting. This alleviates the memory overhead to some extent.


A-B$^{(5)}$
A$_1$
B$_1$ B$_2$ B$_3$ B$_4$ B$_5$
A$_2$
B$_2$ B$_3$ B$_4$ B$_5$ B$_7$
A-B$^{(4)}$
A$_3$
B$_4$ B$_5$ B$_6$ B$_8$
union = { }
{B$_1$,B$_2$,B$_3$,B$_4$,B$_5$} ∩ union = ∅
C_5^4=5
union ={B$_1$,B$_2$,B$_3$,B$_4$,B$_5$}
{B$_2$,B$_3$,B$_4$,B$_5$,B$_7$} ∩ union = {B$_2$,B$_3$,B$_4$,B$_5$}
C_5^4 - C_4^4=4
union = {B$_1$,B$_2$,B$_3$,B$_4$,B$_5$,B$_7$}
{B$_4$,B$_5$,B$_6$,B$_8$} ∩ union = {B$_4$,B$_5$}
C_4^4 - 0 =1
union = {B$_1$,B$_2$,B$_3$,B$_4$,B$_5$,B$_6$,B$_7$,B$_8$}

(a) Storage reduction (b) Computation optimization

Fig. 4. Optimization

Computation optimization. Since the verification of the support does not need to obtain the exact result, it only needs to know whether the number of valid matches corresponding to all the nodes of the current candidate QGP meets the threshold τ. Therefore, according to the above storage method, for quantifier nodes, we use estimation to predict their matches without enumerating them.

Fig. 4(b) shows the specific process, an empty set *union*(recording the matches that have already appeared) is initialized first, and then the existing matches are traversed in turn. For the first match, there are 5 matches of $Q^{(4)}_{(B)}$, and then update the *union*. For the second match, the intersection with the union is $\{B_2, B_3, B_4, B_5\}$, indicating that a match formed by the intersection has been generated before, and if such a match occurs again, it must be subtracted, so the number of matches $Q^{(4)}_{(B)}$ formed by the second match is 4, and the *union* is then updated as well. For the third match, its intersection with *union* is $\{B_4, B_5\}$, and the cardinality of the intersection is less than the number of quantifiers, indicating that the previous match will not be generated, so there is 1 match. The final match count is the sum of the matches obtained each time, *i.e.,* the matches of $Q^{(4)}_{(B)}$ is 10.

4 QGPARs generation

As an application of QGPs, we introduce Quantified Graph Pattern Association Rules(QGPARs), to identify regularity between entities in graphs in general, and potential customers in social graphs in particular.

Graph Pattern Association Rule(GPAR) [29]. A GPAR R is defined as $Q_l \to Q_r$, where Q_l and Q_r (1) are different connected graph patterns, (2) share nodes but no common edges. We refer to Q_l and Q_r are the antecedent and consequent of R, respectively.

Quantified GPARs (QGPAR). Analogous to GPARs, a QGPAR R is also defined as $Q_l \to Q_r$, with exception that Q_l is a quantified pattern rather than a traditional one. Then, the rule R states that in any graph G, if there is an isomorphism mapping h_l from Q_l to the subgraph G_1, there likely exists another mapping h_r from Q_r to the subgraph G_2, such that for each node $u \in V_l \cap V_r$, if it is mapped to the node v in G_1 by h_l, then it is also mapped to the same node v in G_2 by h_r. For a QGPAR R, we model R as a QGP Q_R by extending Q_l with the edge set of Q_r.

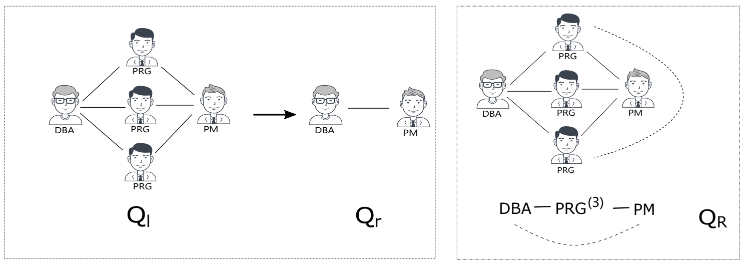


Fig. 5. QGPAR $R : Q_l \rightarrow Q_r$

Example 7. Fig. 5 shows a QGPAR R found in Fig. 2(a), expressed as $Q_l \rightarrow Q_r$, with Q_l and Q_r as its antecedent and consequent, respectively. The QGPAR R can be modeled as a graph pattern Q_R by extending Q_l with the edge set of Q_r.

Support & Confidence. By considering QGPAR R as pattern Q_R, the support of R in G is defined as:

$$Sup(G, R) = Sup(G, Q_R) \tag{1}$$

For the confidence metric, it is defined the same as that of GPARs, *i.e.,*

$$Conf(G, R) = \frac{Sup(G, Q_R)}{Sup(G, Q_l)} \tag{2}$$

QGPARs generation: Along the same line as traditional strategy applied by association rule mining. For any frequent QGP $Q^m_{(v)}$, we enumerate its non-empty subgraph Q_l (including quantifier-constrained node v), and $Q_r = Q^m_{(v)} \backslash Q_l$ (excluding v), then we can generate rules R: $Q_l \rightarrow Q_r$. All rules that meet the support and confidence are valid.

5 Experimental Study

Using real-life and synthetic data, we conducted two sets of experiments to evaluate (1) The efficiency of our algorithm QGPM and the effectiveness of optimization techniques for QGPM; (2) the effectiveness of for identifying correlated entities in large real-world graphs.

DataSets	$\lvert V \rvert$	$\lvert L \rvert$	$\lvert E \rvert$
Citeseer	3312	6	4519
Lasftm_asia	7624	12	27806
Mico	100000	29	1080298
DBLP	425957	25	1049866
Amazon	334863	80	925872
AstroPh	18772	50	396160

Table 2. Description of graphs ($|L|$ indicates no. of node labels).

Experimental setting. We used six graphs, (a) CiteSeer [12] represents a graph consisting of publications (nodes) and citations between them (edges). (b) lasftm_asia [2], a social network consisting of users (nodes) and following relationships (edges) between them. (c) Mico [12] models the Microsoft co-authorship information. (d) DBLP [2] represents a collaborative network consisting of authors (nodes) and partnerships (edges) between them. (e) Amazon [2], is a product co-purchasing network. (f) AstroPh [2] covers scientific collaboration between author papers submitted to the Astrophysics category. Table 2 shows the statistics of each graph.

Experimental results. We next report our findings.

Exp-1: Performance of QGPM & QGPM-opt. In this set of experiments, we evaluated the performance of algorithm QGPM compared with QGPM-opt.

The results presented in Figure 6(a)-6(f) show the results of the algorithm on CiteSeer, lasftm_asia, MiCo, DBLP, Amazon, and AstroPh, respectively, and demonstrate the following.(1) All algorithms take longer with a small τ because more candidate patterns and their matches need to be verified. (2) QGPM-opt performs better than QGPM, as unnecessary computations are avoided owing to the strategy QGPM-opt applied.

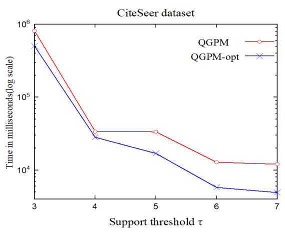
Frequency threshold. By observing the results, support threshold τ is a key evaluation metric because it determines whether a pattern is frequent. As the threshold value decreases, there is an exponential number of candidate pattern that leads to exponential runtime. For a given time budget, an efficient algorithm should be able to solve mining problems with low τ values. When τ is given, the efficiency is determined by the execution time.

Case study. We also manually examined QGPs mined from DBLP and AstroPh. *e.g.,* Fig. 6(g) shows three QGPs discovered from DBLP. Fig. 6(h) shows three QGPs discovered from AstroPh.

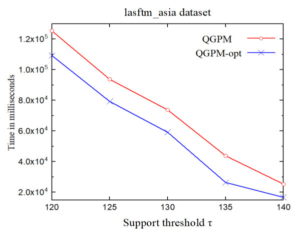
Exp-2: Prediction accuracy. In this set of tests, we evaluated the prediction accuracy of QGPARs vs. Jaccard [24] and SimRank [24]. The settings of the evaluation are as follows.

The prediction accuracy of QGPARs is tested via cross validation. That is, given a graph G, we partition it into two fragments F_1 and F_2, mine and select the top-k (k ranges from 10 to 50 in 10 increments) QGPARs from F_1, and evaluate their prediction accuracy, which is defined as $Acc(R) = \frac{Sup(F_2,Q_R)}{Sup(F_2,Q_l)}$, on F_2. We used metrics Gain [7] and Interest [28] to rank the QGPARs, where θ is set as 0.4 for Gain.

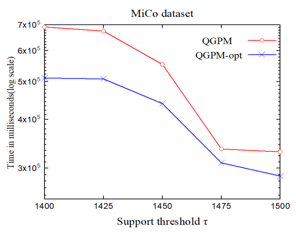
We compared our rule-based method vs. Jaccard and SimRank, on three real-life graphs. The tests of Jaccard and SimRank are along the same line as above but with 4-fold cross-validation. Specifically, (1) we extracted a small subgraph G_F of size (10000,30729), (5000,88822), and (10000,5678) from Amazon, AstroPh, and DBLP respectively, since SimRank is too costly to run on the entire graph; (2) we run Jaccard and SimRank on the training set, and obtain a list of node pairs; (3) we then sort node pairs according to their similarity and next select top-L node pairs as the predicted edges. Here, the prediction accuracy is defined as $Acc = \frac{L_r}{L}$, where L ranges from 50 to 250 in 50 increments and L_r is the number of edges that are



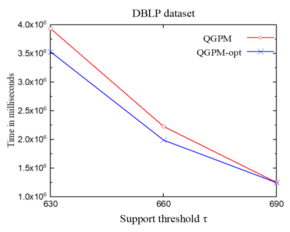
(a) Varying τ (*Citeseer*)



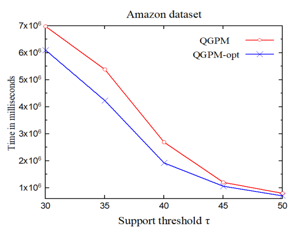
(b) Varying τ (*lasftm_asia*)



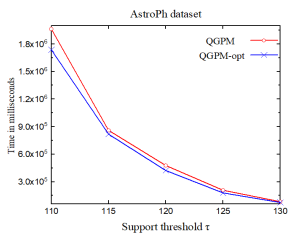
(c) Varying τ (*Mico*)



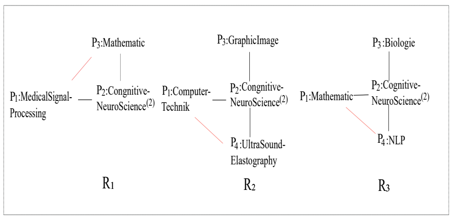
(d) Varying τ (*DBLP*)

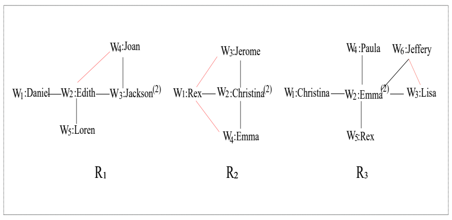


(e) Varying τ (*Amazon*)



(f) Varying τ (*AstroPh*)


P3:Mathematic
P1:MedicalSignal-Processing
P2:Congnitive-NeuroScience(2)
R1
P3:GraphicImage
P1:Computer-Technik
P2:Congnitive-NeuroScience(2)
P4:UltraSound-Elastography
R2
P3:Biologie
P1:Mathematic
P2:Cognitive-NeuroScience(2)
P4:NLP
R3
(g) Real-life QGPs: DBLP



(h) Real-life QGPs: AstroPh

Fig. 6. Performance of QGPM & QGPM-opt on real-life graphs

		Amazon					AstroPh					DBLP				
		top-k(Acc(R),%)					top-k(Acc(R),%)					top-k(Acc(R),%)				
		10	20	30	40	50	10	20	30	40	50	10	20	30	40	50
QGPARs	Gain	83.4	81.1	78.6	75.9	73.7	92.9	93.3	93.4	92.2	83.0	90.9	90.1	88.9	86.9	83.8
	Interest	79.1	76.9	76.8	73.8	71.7	92.7	91.9	90.8	90.0	89.7	91.5	86.1	86.1	85.1	82.7
		top-L(Acc,%)					top-L(Acc,%)					top-L(Acc,%)				
		50	100	150	200	250	50	100	150	200	250	50	100	150	200	250
Jaccard		43.5	42.5	43.6	43.4	42.0	56.5	43.0	43.0	33.5	27.0	47.0	49.7	48.7	48.4	48.1
Simrank		18.0	20.2	22.1	25.2	22.4	13.0	14.0	13.8	17.7	21.1	73.5	44.2	31.0	23.6	19.6

Table 3. Prediction accuracy of QGPARs vs. Jaccard and SimRank

correctly predicted. Note that the parameter C used by SimRank is set as 0.8, and the number of iterations is fixed as 5.

Table 3 shows the prediction accuracy of QGPARs, compared with Jaccard and SimRank. We find that 1) using the top-50 QGPARs can predict missing relationships with an average accuracy of up to 84.3% on three real graphs, and the higher k is, the lower $Acc(R)$ is, as expected; 2) our rule-based method is superior to Jaccard and SimRank in terms of prediction accuracy. Taking Amazon as an example, when top-50 interest QGPARs are used, the prediction accuracy (*i.e.,* 71.7%) is 170.71% and 320.0% of that of Jaccard and SimRank (L=250) respectively; 3) two different metrics *i.e.,* Gain and Interest leads to different while close accuracy.

6 Conclusion

Incorporating counting quantifiers into traditional graph patterns can capture more complex structural relationships in graph data. Accordingly, we have studied the problem of mining Quantified Graph Pattern (QGP) in a single large graph and propose QGPM to identify QGPs in the graph that satisfy threshold conditions. As an application of QGP, we also proposed Quantified Graph Pattern Association Rules (QGPAR), which can model more complex associations between social entities. Our experimental study validates the efficiency and effectiveness of the algorithm. Therefore, we think our method is promising in social network analysis.

The study of QGP is still in its infancy. One topic for future work is to extend quantifiers to multiple nodes to make graph patterns more diverse. Another topic involves the mining and application of quantified graph patterns in distributed environment.

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