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## 1. Data Structures

### 1.1 Segment Tree

```
template<typename T, T (*merge)(T, T), T e>
struct Seg {
    int n;
    vector<T> seg;
    void init(int _n) {
        n = _n;
        seg.resize(2 * n);
    }
    void upt(int i, T p) {
        for (seg[i += n] += p; i >>= 1; )
            seg[i] = merge(seg[i << 1], seg[i << 1 | 1]);
    }
    T get(int i) { return seg[i + n]; }
    T query(int l, int r) {
        T nl = e, nr = e;
        for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
            if (l & 1) nl = merge(nl, seg[l++]);
            if (r & 1) nr = merge(seg[--r], nr);
        }
        return merge(nl, nr);
    }
};
```

### 1.2 Merge Sort Tree

```
struct MergeSortTree {
    int n;
    vector<int> arr;
    vector<vector<int>> seg;
    void Init(vector<int>& vec) {
        n = vec.size();
        arr.assign(vec.begin(), vec.end());
        seg.resize(4 * n);
        SetTree(1, 0, n - 1);
    }
    void SetTree(int num, int s, int e) {
        if (s == e) {
            seg[num].push_back(arr[s]);
            return;
        }
        int mid = s + e >> 1;
        SetTree(2 * num, s, mid);
```

```
        SetTree(2 * num + 1, mid + 1, e);
        vector<int>& now = seg[num];
        vector<int>& l = seg[2 * num], r = seg[2 * num + 1];
        int pl = 0, pr = 0;
        for (int i = 0; i < l.size() + r.size(); i++) {
            if (pl < l.size() && pr < r.size()) {
                if (l[pl] < r[pr]) now.push_back(l[pl]), pl++;
                else now.push_back(r[pr]), pr++;
            }
            else if (pl < l.size())
                now.push_back(l[pl]), pl++;
            else if (pr < r.size())
                now.push_back(r[pr]), pr++;
        }
    }
    int query(int num, int s, int e, int l, int r, int k) {
        if (r < s || e < l) return 0;
        if (l <= s && e <= r) {
            int idx = upper_bound(seg[num].begin(), seg[num].end(), k) -
                seg[num].begin();
            return seg[num].size() - idx;
        }
        int mid = s + e >> 1;
        return query(2 * num, s, mid, l, r, k) + query(2 * num + 1, mid + 1, e,
            l, r, k);
    }
    int query(int l, int r, int k) { return query(1, 0, n - 1, l, r, k); }
}tree;
```

### 1.3 Persistent Segment Tree

```
typedef long long ll;
const int MAX = 300001;
//[l..r] find k-th minimum number in O(logN)
struct PST {
    struct Node {
        ll cnt;
        Node* l, * r;
    };
    int n;
    Node* root[MAX];
    void func(int _n) {
        n = _n;
        root[0] = new Node();
```

```

    init(root[0], 0, n - 1);
}
void init(Node* node, int s, int e) {
    if (s == e) return;
    int mid = s + e >> 1;
    node->l = new Node();
    node->r = new Node();
    init(node->l, s, mid); init(node->r, mid + 1, e);
}
void update(Node* prv, Node* now, int s, int e, int idx, ll a) {
    if (s == e) {
        now->cnt += a;
        return;
    }
    int mid = s + e >> 1;
    if (idx <= mid) {
        now->l = new Node(); now->r = prv->r;
        now->l->cnt += prv->l->cnt;
        update(prv->l, now->l, s, mid, idx, a);
    }
    else {
        now->l = prv->l; now->r = new Node();
        now->r->cnt += prv->r->cnt;
        update(prv->r, now->r, mid + 1, e, idx, a);
    }
    now->cnt = now->l->cnt + now->r->cnt;
}
void update(int num, int idx, ll a) {
    root[num] = new Node();
    update(root[num - 1], root[num], 0, n - 1, idx, a);
}

ll query(Node* nl, Node* nr, int s, int e, int k) {
    if (s == e) return s;
    int mid = s + e >> 1;
    int cnt = nr->l->cnt - nl->l->cnt;
    if (k <= cnt)
        return query(nl->l, nr->l, s, mid, k);
    else
        return query(nl->r, nr->r, mid + 1, e, k - cnt);
}
ll query(int il, int ir, int k) { return query(root[il - 1], root[ir], 0,
n - 1, k); }
};

```

## 1.4 Lichao Tree

```

//Minimum
typedef long long ll;
const ll inf = 1e18;
struct Line {
    ll a, b;
    ll f(ll x) { return a * x + b; }
};
struct Lichao {
    struct Node {
        ll l, r;
        Line line;
    };
    ll n, psum, ns, ne;
    vector<Node> seg;
    vector<Line> lines;
    void init(int s, int e) {
        ns = s, ne = e;
        seg.push_back({ -1, -1, {0, inf} });
    }
    int size() { return lines.size(); }
    void insert(int num, int s, int e, Line l) {
        Line lo = seg[num].line, hi = l;
        if (lo.f(s) > hi.f(s)) swap(lo, hi);
        if (lo.f(e) <= hi.f(e)) {
            seg[num].line = lo;
            return;
        }
        int mid = s + e >> 1;
        if (lo.f(mid) < hi.f(mid)) {
            seg[num].line = lo;
            if (seg[num].r == -1) {
                seg[num].r = seg.size();
                seg.push_back({ -1, -1, {0, inf} });
            }
            insert(seg[num].r, mid + 1, e, hi);
        }
        else {
            seg[num].line = hi;
            if (seg[num].l == -1) {
                seg[num].l = seg.size();
                seg.push_back({ -1, -1, {0, inf} });
            }
            insert(seg[num].l, s, mid, lo);
        }
    }
};

```

```

    }
}
void insert(Line l) {
    l.b -= psum;
    lines.push_back(l);
    insert(0, ns, ne, l);
}
void apply() {
    for (auto& l : lines) l.b += psum;
    for (auto& l : seg) l.line.b += psum;
    psum = 0;
}
ll query(int num, int s, int e, ll x) {
    if (num == -1) return inf;
    int mid = s + e >> 1;
    ll d = seg[num].line.f(x) + psum;
    if (x <= mid) return min(d, query(seg[num].l, s, mid, x));
    else return min(d, query(seg[num].r, mid + 1, e, x));
}
ll query(ll x) { return query(0, ns, ne, x); }
};

```

## 2. Graph

### 2.1 Bellman-Ford

```

const ll inf = 1e18;
struct Line { ll u, v, c; };
vector<ll> bellman(int n, vector<Line>& e) {
    vector<ll> dst(n, inf);
    dst[1] = 0;
    for (int i = 0; i < n; i++) for (auto& l : e) {
        if (dst[l.u] != inf && dst[l.v] > dst[l.u] + l.c) {
            dst[l.v] = dst[l.u] + l.c;
            if (i == n - 1) return vector<ll>(n, -1);
        }
    }
    return dst;
}

```

### 2.2 Lowest Common Ancestor

```

const int MAX = 30001;
const int LV = 17;
int dep[MAX], dp[LV + 1][MAX];
vector<int> V[MAX];
void dfs(int pos, int d = 0, int p = 0) {

```

```

    dep[pos] = d;
    dp[0][pos] = p;
    for (int i = 1; i <= LV; i++)
        dp[i][pos] = dp[i - 1][dp[i - 1][pos]];

    for (int w : V[pos]) {
        if (w == p) continue;
        dfs(w, d + 1, pos);
    }
}
int lca(int a, int b) {
    if (dep[a] < dep[b]) swap(a, b);
    int d = dep[a] - dep[b];
    for (int i = 0; d; i++, d >>= 1)
        if (d & 1) a = dp[i][a];
    if (a == b) return a;
    for (int i = LV; ~i; i--)
        if (dp[i][a] != dp[i][b]) a = dp[i][a], b = dp[i][b];
    return dp[0][a];
}

```

### 2.3 Strongly Connected Component

```

struct SCC {
    vector<int> visited, scc_id;
    int scc_cnt, n;
    vector<vector<int>> adj;
    vector<vector<int>> scc;
    stack<int> st;
    void init(int _n) {
        n = _n;
        scc_cnt = 0;
        adj.clear(); adj.resize(n);
        visited.clear(); visited.resize(n, -1);
        scc_id.clear(); scc_id.resize(n, -1);
        scc.clear();
    }
    int dfs(int cur) {
        int ret = visited[cur] = scc_cnt++;
        st.push(cur);

        for (auto nxt : adj[cur]) {
            if (visited[nxt] == -1) ret = min(ret, dfs(nxt));
            else if (scc_id[nxt] == -1) ret = min(ret, visited[nxt]);
        }
    }
}

```

```

    if (ret == visited[cur]) {
        vector<int> v;
        while (true) {
            int t = st.top(); st.pop();
            scc_id[t] = scc.size() + 1;
            v.push_back(t);
            if (t == cur) break;
        }
        scc.push_back(v);
        scc_cnt++;
    }
    return ret;
}

void get_scc() {
    for (int i = 1; i <= n; i++) {
        if (visited[i] == -1) dfs(i);
    }
}
};

```

## 2.4 2-Satisfiability

```

struct TwoSat{
    int n;
    SCC scc;
    void init(int _n) {
        n = _n;
        scc.init(2*n);
    }
    int inv(int i) { return i + n; }
    void add_clause(int a, int b, bool arev = false, bool brev = false) {
        int u1 = (arev) ? inv(a) : a, v1 = (brev) ? b : inv(b);
        int u2 = (brev) ? inv(b) : b, v2 = (arev) ? a : inv(a);

        scc.adj[u1].push_back(v1);
        scc.adj[u2].push_back(v2);
    }

    bool correct() {
        for (int i = 0; i < n; i++)
            if (scc.scc_id[i] == scc.scc_id[inv(i)]) return false;
        return true;
    }
}

```

## 2.5 Heavy Light Decomposition

```

const int MAX = 300001;
vector<int> V[MAX], g[MAX];
int sz[MAX], dep[MAX], top[MAX], par[MAX], in[MAX], out[MAX];

bitset<MAX> vit;
void dfs(int pos) {
    vit[pos] = true;
    for (int& w : V[pos]) {
        if (vit[w]) continue;
        g[pos].push_back(w);
        dfs(w);
    }
}
void dfs1(int pos) {
    sz[pos] = 1;
    for (int& w : g[pos]) {
        dep[w] = dep[pos] + 1, par[w] = pos;
        dfs1(w);
        sz[pos] += sz[w];
        if (sz[w] > sz[g[pos][0]]) swap(w, g[pos][0]);
    }
}
int pv;
void dfs2(int pos) {
    in[pos] = ++pv;
    for (int& w : g[pos]) {
        top[w] = w == g[pos][0] ? top[pos] : w;
        dfs2(w);
    }
    out[pos] = pv;
}

void update(int a, int b, int diff) {
    int ans = 0;
    while (top[a] != top[b]) {
        if (dep[top[a]] < dep[top[b]]) swap(a, b);
        int x = top[a];
        tree.update(in[x], in[a], diff);
        a = par[x];
    }
    if (dep[a] > dep[b]) swap(a, b);
    tree.update(in[a], in[b], diff);
}

```

```

int query(int a, int b) {
    int ans = 0;
    while (top[a] != top[b]) {
        if (dep[top[a]] < dep[top[b]]) swap(a, b);
        int x = top[a];
        ans += tree.query(in[x], in[a]);
        a = par[x];
    }
    if (dep[a] > dep[b]) swap(a, b);
    ans += tree.query(in[a], in[b]);
    return ans;
}

```

## 2.6 Dominator Tree

```

namespace dtree { // by cki86201
    const int MAXN = 300001;
    vector<int> E[MAXN], RE[MAXN], rdom[MAXN];

    int S[MAXN], RS[MAXN], cs;
    int par[MAXN], val[MAXN], sdom[MAXN], rp[MAXN], dom[MAXN];

    void clear(int n) {
        cs = 0;
        for (int i = 0; i <= n; i++) {
            par[i] = val[i] = sdom[i] = rp[i] = dom[i] = S[i] = RS[i] = 0;
            E[i].clear(); RE[i].clear(); rdom[i].clear();
        }
    }

    void add_edge(int x, int y) { E[x].push_back(y); }
    void Union(int x, int y) { par[x] = y; }
    int Find(int x, int c = 0) {
        if (par[x] == x) return c ? -1 : x;
        int p = Find(par[x], 1);
        if (p == -1) return c ? par[x] : val[x];
        if (sdom[val[x]] > sdom[val[par[x]]]) val[x] = val[par[x]];
        par[x] = p;
        return c ? p : val[x];
    }

    void dfs(int x) {
        RS[S[x] = ++cs] = x;
        par[cs] = sdom[cs] = val[cs] = cs;
        for (int e : E[x]) {
            if (S[e] == 0) dfs(e), rp[S[e]] = S[x];
            RE[S[e]].push_back(S[x]);
        }
    }
}

```

```

}
}
int solve(int s, int* up) { // Calculate idoms
    dfs(s);
    for (int i = cs; i; i--) {
        for (int e : RE[i]) sdom[i] = min(sdom[i], sdom[Find(e)]);
        if (i > 1) rdom[sdom[i]].push_back(i);
        for (int e : rdom[i]) {
            int p = Find(e);
            if (sdom[p] == i) dom[e] = i;
            else dom[e] = p;
        }
        if (i > 1) Union(i, rp[i]);
    }
    for (int i = 2; i <= cs; i++) if (sdom[i] != dom[i]) dom[i] =
dom[dom[i]];
    for (int i = 2; i <= cs; i++) up[RS[i]] = RS[dom[i]];
    return cs;
}
}

```

## 3. Flow

### 3.1 Bitpartite Matching

```

const int MAX = 501;
vector<int> V[MAX];
int ma[MAX], mb[MAX];
bool vit[MAX];
bool dfs(int pos) {
    vit[pos] = true;
    for (int w : V[pos]) {
        if (mb[w] == -1 || !vit[mb[w]] && dfs(mb[w])) {
            ma[pos] = w;
            mb[w] = pos;
            return true;
        }
    }
    return false;
}

int match(int n) {
    memset(ma, -1, sizeof(ma));
    memset(mb, -1, sizeof(mb));
    int ans = 0;
    for (int i = 0; i < n; i++) {
        if (ma[i] == -1) {

```

```

        memset(vit, false, sizeof(vit));
        ans += dfs(i);
    }
}
return ans;
}

```

### 3.2 Hopcroft-Karp

```

struct HopcroftKarp {
    int n;
    vector<vector<int>> V;
    vector<int> ma, mb, lv;
    vector<bool> vit;
    void init(int _n) {
        n = _n;
        ma.resize(n); mb.resize(n);
        lv.resize(n); V.resize(n);
        vit.resize(n);
    }
    void add_edge(int u, int v) { V[u].push_back(v); }
    bool bfs() {
        queue<int> q;
        fill(lv.begin(), lv.end(), 0);
        for (int i = 0; i < n; i++)
            if (ma[i] == -1 && !lv[i])
                q.push(i), lv[i] = 1;
        bool ok = false;
        while (q.size()) {
            int top = q.front(); q.pop();
            for (int w : V[top]) {
                if (mb[w] == -1) ok = true;
                else if (!lv[mb[w]]) {
                    lv[mb[w]] = lv[top] + 1;
                    q.push(mb[w]);
                }
            }
        }
        return ok;
    }
    bool dfs(int a) {
        if (vit[a]) return false;
        vit[a] = true;
        for (int w : V[a]) {
            if (mb[w] == -1 || (!vit[mb[w]] && lv[mb[w]] == lv[a] + 1 &&

```

```

dfs(mb[w]))) {
            ma[a] = w;
            mb[w] = a;
            return true;
        }
    }
    return false;
}
int match() {
    fill(ma.begin(), ma.end(), -1);
    fill(mb.begin(), mb.end(), -1);
    int ans = 0;
    while (bfs()) {
        fill(vit.begin(), vit.end(), false);
        for (int i = 0; i < n; i++)
            if (ma[i] == -1 && dfs(i)) ans++;
    }
    return ans;
}
};

```

### 3.3 MCMF

```

template<typename T>
struct MCMF {
    struct Edge {
        int to;
        T cap, f, cost;
        int dual;
        T spare() { return cap - f; }
    };
    int n;
    T ans, cot;
    vector<vector<Edge>> E;
    void init(int _n) {
        n = _n;
        E.clear(); E.resize(n);
    }
    void add_edge(int u, int v, T cap, T cost) {
        E[u].push_back({ v, cap, 0, cost });
        E[v].push_back({ u, 0, 0, -cost });
        E[u].back().dual = E[v].size() - 1;
        E[v].back().dual = E[u].size() - 1;
    }
    bool spfa(int s, int t, bool apply = true) {
        vector<T> dst(n, 1e9);

```

```

vector<int> prv(n, -1);
vector<Edge*> sel(n);
vector<bool> chk(n);
dst[s] = 0;
queue<int> q;
q.push(s); chk[s] = true;
while (q.size()) {
    int top = q.front(); q.pop();
    chk[top] = false;
    for (auto& l : E[top]) {
        if (l.spare() > 0 && dst[top] + l.cost < dst[l.to]) {
            dst[l.to] = dst[top] + l.cost;
            prv[l.to] = top;
            sel[l.to] = &l;
            if (!chk[l.to]) {
                q.push(l.to);
                chk[l.to] = true;
            }
        }
    }
}
if (prv[t] == -1) return false;
if (apply) {
    T flow = 1e9;
    for (int i = t; i != s; i = prv[i]) flow = min(flow, sel[i]->spare());
    for (int i = t; i != s; i = prv[i]) {
        sel[i]->f += flow;
        E[sel[i]->to][sel[i]->dual].f -= flow;
        cot += flow * sel[i]->cost;
    }
    ans += flow;
}
return true;
}
pair<T, T> flow(int s, int t) {
    ans = 0; cot = 0;
    while (spfa(s, t));
    return { ans, cot };
}
};

```

### 3.4 Dinic

```

template<typename T>
struct Dinic {
    struct Edge {
        int to;
        T cap, f;
        int dual;
        T spare() { return cap - f; }
    };
    int n;
    T ans;
    vector<vector<Edge*>> E;
    vector<int> lv, work;
    void init(int _n) {
        n = _n;
        E.clear(); E.resize(n);
        lv.resize(n); work.resize(n);
    }
    void add_edge(int u, int v, T cap) {
        E[u].push_back({ v, cap, 0 });
        E[v].push_back({ u, 0, 0 });
        E[u].back().dual = E[v].size() - 1;
        E[v].back().dual = E[u].size() - 1;
    }
    bool bfs(int s, int t) {
        fill(lv.begin(), lv.end(), -1);
        lv[s] = 0;
        queue<int> q; q.push(s);
        while (q.size()) {
            int top = q.front(); q.pop();
            for (auto& l : E[top]) {
                if (lv[l.to] == -1 && l.spare()) {
                    lv[l.to] = lv[top] + 1;
                    q.push(l.to);
                }
            }
        }
        return lv[t] != -1;
    }
    T dfs(int pos, int t, T flow) {
        if (pos == t) return flow;
        for (int& i = work[pos]; i < E[pos].size(); i++) {
            auto& l = E[pos][i];
            if (lv[l.to] == lv[pos] + 1 && l.spare()) {

```



```

    T df = dfs(l.to, t, min(flow, l.spare()));
    if (df) {
        l.f += df;
        E[l.to][l.dual].f -= df;
        return df;
    }
}
}
return 0;
}
T flow(int s, int t) {
    ans = 0;
    while (bfs(s, t)) {
        fill(work.begin(), work.end(), 0);
        while (1) {
            T flow = dfs(s, t, 1e9);
            if (!flow) break;
            ans += flow;
        }
    }
    return ans;
}
};

```

### 3.5 Circulation

```

template<typename T>
struct LRFlow { //by sgc109
    Dinic<T> dinic;
    int n, src, sink, fsrc, fsink;
    vector<T> inSum, outSum;
    void init(int _n, int _src, int _sink) {
        n = _n, src = _src, sink = _sink;
        fsrc = n, fsink = n + 1;
        inSum = vector<T>(n);
        outSum = vector<T>(n);
    }
    void add_edge(int u, int v, int l, int r) {
        dinic.add_edge(u, v, r);
        inSum[v] += l;
        outSum[u] += l;
    }
    int flow() {
        for (int i = 0; i < n; i++)
            if (inSum[i]) dinic.add_edge(fsrc, i, inSum[i]);
        for (int i = 0; i < n; i++)

```

```

            if (outSum[i]) dinic.add_edge(i, fsink, outSum[i]);
            dinic.add_edge(sink, src, 1e9);
            return dinic.flow();
        }
    }
};

```

## 4. Strings

### 4.1 KMP

```

vector<int> KMP(string from, string to) {
    int n = from.size(), m = to.size();
    vector<int> fail(m + 1);
    for (int i = 1, j = 0; i < m; i++) {
        while (j && to[i] != to[j]) j = fail[j];
        if (to[i] == to[j]) j++;
        fail[i + 1] = j;
    }
    vector<int> ans;
    for (int i = 0, j = 0; i < n; i++) {
        while (j && from[i] != to[j]) j = fail[j];
        if (from[i] == to[j]) j++;
        if (j == m) ans.push_back(i - m + 1), j = fail[j];
    }
    return ans;
}

```

### 4.2 Trie

```

struct Trie {
    map<char, Trie*> to;
    Trie* fail;
    bool end;
    void insert(int idx, string& vec) {
        if (idx == vec.size()) {
            end = true;
            return;
        }
        if (to.find(vec[idx]) == to.end())
            to[vec[idx]] = new Trie();
        to[vec[idx]]->insert(idx + 1, vec);
    }
};

```

### 4.3 Rabin-Karp Fingerprint

```
typedef long long ll;
template<ll key = 29, ll mod = 1'000'000'007>
struct RabinKarp {
    int n;
    vector<ll> p;
    void init(int n) {
        this->n = n;
        p.resize(n);
        p[0] = 1;
        for (int i = 1; i < n; i++) p[i] = (p[i - 1] * key) % mod;
    }
    vector<Q> hashing(string& arr, int gap) {
        assert(arr.size() <= n);
        vector<Q> ans;
        ll now = 0, idx = 0;
        for (int i = 0; i < arr.size(); i++) {
            if (i >= gap) {
                ans.push_back({ now, idx++ });
                now = (now - p[gap - 1] * arr[i - gap] % mod + mod) % mod;
            }
            now = (now * key) % mod;
            now = (now + arr[i]) % mod;
        }
        ans.push_back({ now, idx });
        return ans;
    }
};
```

### 4.4 Manacher

```
int manacher(string str) {
    string arr;
    for (char c : str) {
        arr.push_back('#');
        arr.push_back(c);
    }
    arr.push_back('#');
    swap(str, arr);
    int n = str.size();
    vector<int> vec(n);
    int r = 0, p = 0;
    for (int i = 0; i < n; i++) {
        if (i > r) vec[i] = 0;
        else vec[i] = min(r - i, vec[2 * p - i]);
```

```
        while (i - vec[i] - 1 >= 0 && i + vec[i] + 1 < n && str[i - vec[i] - 1]
== str[vec[i] + i + 1])
            vec[i]++;

        if (r < i + vec[i]) {
            r = i + vec[i];
            p = i;
        }
    }
    return *max_element(vec.begin(), vec.end());
}
```

### 4.5 Aho-Corasick

```
Trie* CreateTrie(vector<string> str) {
    Trie* trie = new Trie();
    for (auto& s : str) trie->insert(0, s);
    queue<Trie*> q;
    q.push(trie->fail = trie);
    while (q.size()) {
        Trie* top = q.front(); q.pop();
        for (auto& p : top->to) {
            char c = p.first; Trie* nxt = p.second;
            if (top == trie) nxt->fail = trie;
            else {
                Trie* f = top->fail;
                while (f != trie && f->to.find(c) == f->to.end())
                    f = f->fail;
                if (f->to.find(c) != f->to.end())
                    f = f->to[c];
                nxt->fail = f;
            }
            if (nxt->fail->end) top->end = true;
            q.push(nxt);
        }
    }
    return trie;
}

bool AhoCorasick(string str, Trie* root) {
    bool ans = false;
    Trie* curr = root;
    for (char c : str) {
        while (curr != root && curr->to.find(c) == curr->to.end())
            curr = curr->fail;
        if (curr->to[c])
```

```

    curr = curr->to[c];
    if (curr->end) {
        ans = true;
        break;
    }
}
return ans;
}

```

## 4.6 Suffix Array and LCP

```

vector<int> buildSA(string& str) {
    int n = str.size();
    vector<int> sa(n), r(n + 1), nr(n + 1);
    for (int i = 0; i < n; i++) sa[i] = i, r[i] = str[i];
    for (int d = 1; d < n; d <= 1) {
        auto cmp = [&](int i, int j) -> bool {
            return r[i] < r[j] || (r[i] == r[j] && r[i + d] < r[j + d]);
        };
        sort(sa.begin(), sa.end(), cmp);

        nr[sa[0]] = 1;
        for (int i = 1; i < n; i++)
            nr[sa[i]] = nr[sa[i - 1]] + cmp(sa[i - 1], sa[i]);
        r = nr;
    }
    return sa;
}

vector<int> buildLCP(string& str, vector<int>& sa) {
    int n = str.size();
    vector<int> lcp(n + 1), isa(n + 1);
    for (int i = 0; i < n; i++) isa[sa[i]] = i;
    for (int i = 0, k = 0; i < n; i++) {
        if (isa[i]) {
            for (int j = sa[isa[i] - 1]; str[i + k] == str[j + k]; k++);
            lcp[isa[i]] = (k ? k-- : 0);
        }
    }
    return lcp;
}

```

## 5. Geometry

### 5.1 Line-Segment Intersection

```

//BOJ 12555, by shwldus067
typedef pair<int, int> pi;

```

```

#define x first
#define y second
struct Line { pi s, e; };
int ccw(pi a, pi b, pi c) {
    int ret = (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x);
    if (ret > 0) return 1;
    else if (ret == 0) return 0;
    return -1;
}
int cross(pi a, pi b, Line l) {
    int p = ccw(a, b, l.s), q = ccw(a, b, l.e);
    int r = ccw(l.s, l.e, a), s = ccw(l.s, l.e, b);
    if (p == 0 && q == 0 && r == 0 && s == 0) {
        if (a > b) swap(a, b);
        if (l.s > l.e) swap(l.s, l.e);
        if (l.s == b || a == l.e) return 1;
        return (a < l.e && l.s < b) << 3;
    }
    return p * q <= 0 && r * s <= 0;
}
int rectCross(pi s, pi e, Line l) {
    vector<pi> sq = { {s.x, s.y}, {s.x, e.y}, {e.x, e.y}, {e.x, s.y}, {s.x, s.y} };
    int res = 0;
    for (int i = 0; i < 4; i++) {
        res += cross(sq[i], sq[i + 1], l);
        if (cross(sq[i], sq[i], l)) res--;
    }
    return min(res, 4);
}

```

### 5.2 Convex Hull

```

typedef long long ll;
typedef pair<ll, ll> pi;
#define x first
#define y second
ll ccw(pi a, pi b, pi c, bool area = false) {
    ll _ = (b.x - a.x) * (c.y - a.y) - (c.x - a.x) * (b.y - a.y);
    if (area) return _;

    if (_ < 0) return -1;
    else if (_ > 0) return 1;
    return 0;
}
ll pw(ll x) { return x * x; }

```

```

ll dst(pi a, pi b) { return pw(a.x - b.x) + pw(a.y - b.y); }
vector<pi> hull(vector<pi>& vec) {
    swap(vec[0], *min_element(vec.begin(), vec.end()));
    sort(vec.begin() + 1, vec.end(), [&](auto& a, auto& b) -> bool {
        ll cw = ccw(vec[0], a, b);
        if (cw != 0) return cw > 0;
        return dst(vec[0], a) < dst(vec[0], b);
    });

    vector<pi> ans;
    for (auto& p : vec) {
        while (ans.size() > 1 && ccw(ans[ans.size() - 2], ans.back(), p) <= 0)
            ans.pop_back();
        ans.push_back(p);
    }
    return ans;
}

```

### 5.3 Smallest Enclosing Circle

```

#include <random>
namespace cover_2d {
    //https://www.secmem.org/blog/2019/04/08/Smallest-Enclosing-Circle/
    double eps = 1e-9;
    using Point = complex<double>;
    struct Circle { Point p; double r; };
    double dist(Point p, Point q) { return abs(p - q); }
    double area2(Point p, Point q) { return conj(p) * q).imag(); }
    bool in(const Circle& c, Point p) { return dist(c.p, p) < c.r + eps; }
    Circle INVALID = Circle{ Point(0, 0), -1 };
    Circle mCC(Point a, Point b, Point c) {
        b -= a; c -= a;
        double d = 2 * (conj(b) * c).imag(); if (abs(d) < eps) return INVALID;
        Point ans = (c * norm(b) - b * norm(c)) * Point(0, -1) / d;
        return Circle{ a + ans, abs(ans) };
    }
    Circle solve(vector<Point> p) {
        mt19937 gen(0x94949); shuffle(p.begin(), p.end(), gen);
        Circle c = INVALID;
        for (int i = 0; i < p.size(); ++i) if (c.r < 0 || !in(c, p[i])) {
            c = Circle{ p[i], 0 };
            for (int j = 0; j <= i; ++j) if (!in(c, p[j])) {
                Circle ans{ (p[i] + p[j]) * 0.5, dist(p[i], p[j]) * 0.5 };
                if (c.r == 0) { c = ans; continue; }
                Circle l, r; l = r = INVALID;
                Point pq = p[j] - p[i];

```

```

                for (int k = 0; k <= j; ++k) if (!in(ans, p[k])) {
                    double a2 = area2(pq, p[k] - p[i]);
                    Circle c = mCC(p[i], p[j], p[k]);
                    if (c.r < 0) continue;
                    else if (a2 > 0 && (l.r < 0 || area2(pq, c.p - p[i]) >
area2(pq, l.p - p[i]))) l = c;
                    else if (a2 < 0 && (r.r < 0 || area2(pq, c.p - p[i]) <
area2(pq, r.p - p[i]))) r = c;
                }
                if (l.r < 0 && r.r < 0) c = ans;
                else if (l.r < 0) c = r;
                else if (r.r < 0) c = l;
                else c = l.r <= r.r ? l : r;
            }
        }
        return c;
    }
};

```

### 5.4 Point In Convex Polygon Check

```

typedef long long ll;
typedef pair<ll, ll> pi;
#define x first
#define y second
bool f(vector<pi>& cv, pi p) {
    int n = cv.size();
    if (ccw(cv[0], cv[1], p) < 0 ||
        ccw(cv[0], cv.back(), p) > 0) return false;
    int lo = 1, hi = n - 1, ans = 1;
    while (lo <= hi) {
        int mid = lo + hi >> 1;
        if (ccw(cv[0], cv[mid], p) > 0)
            lo = mid + 1, ans = mid;
        else hi = mid - 1;
    }
    return ccw(cv[ans], cv[(ans + 1) % n], p) >= 0;
}

```

### 5.5 Point In Non-Convex Polygon Check

```

typedef long long ll;
typedef pair<ll, ll> pi;
#define x first
#define y second
bool pointInRect(pi& p, vector<pi>& pos) {
    int cnt = 0;

```

```

for (int i = 0; i < pos.size(); i++) {
    int nxt = (i + 1) % pos.size();
    double sx = pos[i].x, sy = pos[i].y;
    double ex = pos[nxt].x, ey = pos[nxt].y;

    if ((sy > p.y) != (ey > p.y)) {
        double x = (ex - sx) * (p.y - sy) / (ey - sy) + sx;
        if (p.x < x) cnt++;
    }
}

return cnt % 2;
}

```

## 5.6 Rotating Calipers

```

pi operator-(pi a, pi b) {
    return { a.x - b.x, a.y - b.y };
}

ll get(vector<pi>& arr) {
    vector<pi> cv = hull(arr);
    int l = 0, r = 0;
    for (int i = 0; i < cv.size(); i++) {
        if (cv[l].x > cv[i].x) l = i;
        if (cv[r].x < cv[i].x) r = i;
    }

    pi line = { 0, 1 };

    ll ans = dst(cv[l], cv[r]);
    int sz = cv.size();
    for (int i = 0; i < sz; i++) {
        if (ccw(cv[(l + 1) % sz] - cv[l], cv[r] - cv[(r + 1) % sz]) > 0)
            l = (l + 1) % sz;
        else
            r = (r + 1) % sz;

        ans = max(ans, dst(cv[l], cv[r]));
    }

    return ans;
}

```

## 5.7 Half Plane Intersection

[//https://www.secmem.org/blog/2019/09/17/Half-Plane-Intersection/](https://www.secmem.org/blog/2019/09/17/Half-Plane-Intersection/)

```

#define sz(x) ((int)x.size())
typedef long long ll;
typedef long double ld;
struct point {
    ld x, y;
    point() {}
    point(ld x, ld y) : x(x), y(y) {}
};
struct line {
    point s, t;
    line() {}
    line(point s, point t) : s(s), t(t) {}
};
inline bool equals(ld a, ld b) { return abs(a - b) < 1e-9; }
bool line_intersect(point& s1, point& e1, point& s2, point& e2, point& v) {
    ld vx1 = e1.x - s1.x, vy1 = e1.y - s1.y;
    ld vx2 = e2.x - s2.x, vy2 = e2.y - s2.y;
    ld det = vx1 * (-vy2) - (-vx2) * vy1;
    if (equals(det, 0)) return 0;
    ld s = (ld)((s2.x - s1.x) * (-vy2) + (s2.y - s1.y) * vx2) / det;
    v.x = s1.x + vx1 * s;
    v.y = s1.y + vy1 * s;
    return 1;
}
bool bad(line& a, line& b, line& c) {
    point v;
    if (!line_intersect(a.s, a.t, b.s, b.t, v)) return 0;
    ld crs = (c.t.x - c.s.x) * (v.y - c.s.y) - (c.t.y - c.s.y) * (v.x - c.s.x);
    return crs < 0 || equals(crs, 0);
}
vector<point> HPI(vector<line>& ln) {
    auto lsgn = [&](const line& a) {
        if (a.s.y == a.t.y) return a.s.x > a.t.x;
        return a.s.y > a.t.y;
    };
    sort(ln.begin(), ln.end(), [&](const line& a, const line& b) {
        if (lsgn(a) != lsgn(b)) return lsgn(a) < lsgn(b);
        return (a.t.x - a.s.x) * (b.t.y - b.s.y) - (a.t.y - a.s.y) * (b.t.x - b.s.x) > 0;
    });
    deque<line> dq;
    for (int i = 0; i < sz(ln); i++) {
        while (dq.size() >= 2 && bad(dq[dq.size() - 2], dq.back(), ln[i]))
            dq.pop_back();
    }
}

```

```

while (dq.size() >= 2 && bad(dq[0], dq[1], ln[i]))
    dq.pop_front();
if (dq.size() < 2 || !bad(dq.back(), ln[i], dq[0]))
    dq.push_back(ln[i]);
}
vector<point> res;
if (dq.size() >= 3) for (int i = 0; i < sz(dq); i++) {
    int j = (i + 1) % sz(dq);
    point v;
    if (!line_intersect(dq[i].s, dq[i].t, dq[j].s, dq[j].t, v)) continue;
    res.push_back(v);
}
return res;
}

```

## 6. Math

### 6.1 FFT, XOR-FFT

```

namespace FFT {
using ll = long long;
using cpx = complex<double>;
const double PI = acos(-1);
void FFT(vector<cpx>& v, bool inv) {
    ll S = v.size();
    for (ll i = 1, j = 0; i < S; i++) {
        ll bit = S / 2;
        while (j >= bit) {
            j -= bit;
            bit /= 2;
        }
        j += bit;
        if (i < j) swap(v[i], v[j]);
    }
    for (ll k = 1; k < S; k *= 2) {
        double angle = (inv ? PI / k : -PI / k);
        cpx w(cos(angle), sin(angle));
        for (ll i = 0; i < S; i += k * 2) {
            cpx z(1, 0);
            for (ll j = 0; j < k; j++) {
                cpx even = v[i + j];
                cpx odd = v[i + j + k];
                v[i + j] = even + z * odd;
                v[i + j + k] = even - z * odd;
                z *= w;
            }
        }
    }
}

```

```

    }
}
if (inv)
    for (ll i = 0; i < S; i++) v[i] /= S;
}

vector<ll> multiply(vector<ll>& v, vector<ll>& u) {
    vector<cpx> vc(v.begin(), v.end());
    vector<cpx> uc(u.begin(), u.end());
    ll S = 2;
    while (S < v.size() + u.size()) S *= 2;
    vc.resize(S); FFT(vc, false);
    uc.resize(S); FFT(uc, false);
    for (ll i = 0; i < S; i++) vc[i] *= uc[i];
    FFT(vc, true);
    vector<ll> w(S);
    for (ll i = 0; i < S; i++) w[i] = round(vc[i].real());
    return w;
}

namespace XORFFT {
using ll = long long;
using cpx = complex<double>;
const double PI = acos(-1);
void XORFFT(vector<ll>& v, bool inv) {
    ll S = v.size();
    for (ll i = 1, j = 0; i < S; i++) {
        ll bit = S / 2;
        while (j >= bit) {
            j -= bit;
            bit /= 2;
        }
        j += bit;
        if (i < j) swap(v[i], v[j]);
    }
    for (ll k = 1; k < S; k *= 2) {
        for (ll i = 0; i < S; i += k * 2) {
            for (ll j = 0; j < k; j++) {
                ll even = v[i + j];
                ll odd = v[i + j + k];
                v[i + j] = even + odd;
                v[i + j + k] = even - odd;
            }
        }
    }
}

```

```

    }
    if (inv)
        for (ll i = 0; i < S; i++) v[i] /= S;
}
vector<ll> XORmultiply(std::vector<ll>& v, std::vector<ll>& u) {
    vector<ll> vc(v.begin(), v.end());
    vector<ll> uc(u.begin(), u.end());
    ll S = 2;
    while (S < v.size() + u.size()) S *= 2;
    vc.resize(S); XORFFT(vc, false);
    uc.resize(S); XORFFT(uc, false);
    for (ll i = 0; i < S; i++) vc[i] *= uc[i];
    XORFFT(vc, true);
    vector<ll> w(S);
    for (ll i = 0; i < S; i++) w[i] = vc[i];
    return w;
}
}

```

## 6.2 Extended Euclidean

```

typedef long long ll;
struct Euclid {
    ll g, x, y;
};
//ax+by=1, get a, b
Euclid egcd(ll a, ll b) {
    if (b == 0) return { a, 1, 0 };
    Euclid ret = egcd(b, a % b);
    return { ret.g, ret.y, ret.x - (a / b) * ret.y };
}

```

## 6.3 Z2 Matrix

```

namespace Z2mat {
    const int MAX = 501;
    int n;
    bitset<2 * MAX> mat[MAX];
    void init() { for (int i = 0; i < MAX; i++) mat[i].reset(); }
    void input(vector<vector<bool>> arr) {
        assert(arr.size() == arr[0].size());
        init();
        n = arr.size();
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++) {
                mat[i][j] = arr[i][j];
                if (i == j) mat[i][j + n] = true;
            }
    }
}

```

```

    }
}
vector<vector<bool>> rev() {
    for (int i = 0; i < n; i++) {
        if (!mat[i][i])
            for (int j = i + 1; j < n; j++)
                if (mat[j][i]) swap(mat[i], mat[j]);
        assert(mat[i][i]);
        for (int j = 0; j < n; j++)
            if (i != j && mat[j][i]) mat[j] ^= mat[i];
    }
    vector ans(n, vector<bool>(n));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++) ans[i][j] = mat[i][j + n];
    return ans;
}
}

```

## 6.4 Miller-Rabin

```

typedef unsigned long long ull;
vector<int> test = { 2, 7, 61 };
ull mypow(ull x, ull cnt, ull mod) {
    x %= mod;
    ull ans = 1LL;
    for (; cnt; ans = (ans * ans) % mod, cnt >>= 1LL)
        if (cnt & 1) ans = (ans * x) % mod;
    return ans;
}
bool miller_rabin(ull n, int a) {
    if (a % n == 0) return true;
    ull d = n - 1;
    while (d) {
        ull k = mypow(a, d, n);
        if (k == n - 1) return true;
        if (d & 1) return k == n - 1 || k == 1;
        d >>= 1;
    }
}
bool prime(ull x) {
    for (auto& l : test)
        if (!miller_rabin(x, l)) return false;
    return true;
}

```

## 6.5 Euler phi Function

```
typedef long long ll;
const int MAX = 100001;
ll phi[MAX], low[MAX];
//phi(n) is equal to the number of integers from 1 to n that are prime to n.
ll f(ll n) {
    ll i; ll ret = n;
    for (i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            ret -= ret / i;
            while (n % i == 0) n /= i;
        }
    }
    if (n != 1) ret -= ret / n;
    return ret;
}
void fillPhi(int n) {
    phi[1] = 1;
    for (int i = 2; i <= n; i++) {
        for (int j = i; j <= n; j += i) {
            if (!low[j]) low[j] = i;
        }
        phi[i] = i;
        for (int j = i; j != 1; ) {
            int p = low[j];
            while (j % p == 0) {
                j /= p;
            }
            phi[i] = (1ll * phi[i] * (p - 1)) / p;
        }
    }
}
```

## 6.6 Mobius Function

```
typedef long long ll;
const ll MAX = 1000000;
ll u[MAX + 1];
//find the number of square-free numbers less than or equal to num
ll func(ll num) {
    ll cnt = 0;
    for (ll i = 1; i * i <= num; i++)
        cnt += u[i] * (num / (i * i));
    return cnt;
}
```

```
}
void init() {
    u[1] = 1;
    for (int i = 1; i <= MAX; i++)
        for (int j = 2 * i; j <= MAX; j += i)
            u[j] -= u[i];
}
```

## 6.7 Modular Integer

```
template <int MOD = 998'244'353>
struct Modular {
    int value;
    static const int MOD_value = MOD;

    Modular(long long v = 0) { value = v % MOD; if (value < 0) value += MOD; }
    Modular(long long a, long long b) : value(0) { *this += a; *this /= b; }

    Modular& operator+=(Modular const& b) { value += b.value; if (value >= MOD) value -= MOD; return *this; }
    Modular& operator-=(Modular const& b) { value -= b.value; if (value < 0) value += MOD; return *this; }
    Modular& operator*=(Modular const& b) { value = (long long)value * b.value % MOD; return *this; }

    friend Modular mexp(Modular a, long long e) {
        Modular res = 1; while (e) { if (e & 1) res *= a; a *= a; e >>= 1; }
        return res;
    }
    friend Modular inverse(Modular a) { return mexp(a, MOD - 2); }

    Modular& operator/=(Modular const& b) { return *this *= inverse(b); }
    friend Modular operator+(Modular a, Modular const b) { return a += b; }
    friend Modular operator-(Modular a, Modular const b) { return a -= b; }
    friend Modular operator-(Modular const a) { return 0 - a; }
    friend Modular operator*(Modular a, Modular const b) { return a *= b; }
    friend Modular operator/(Modular a, Modular const b) { return a /= b; }
    friend std::ostream& operator<<(std::ostream& os, Modular const& a)
    { return os << a.value; }
    friend bool operator==(Modular const& a, Modular const& b) { return a.value == b.value; }
    friend bool operator!=(Modular const& a, Modular const& b) { return a.value != b.value; }
};
```



## 7. Miscellaneous

### 7.1 CHT, DNC dp optimization

### 7.2 Lucas Theorem

### 7.3 Burnside's Lemma

### 7.4 Hall's Theorem