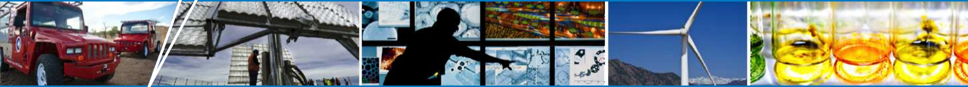
Nonlinear Legendre Spectral Finite Elements for Wind Turbine Blade Dynamics



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Motivation

- Beam model currently used in FAST
  - Euler-Bernoulli beam model with shortening effect
  - Two degree-of-freedoms
  - Assumed-mode method
- Beam models used in other wind turbine tools
  - Multibody-formulation
  - Linear beam models
  - Constraints introduced between linear beams to describe large deflections and rotations
  - Finite element method

Objective

- Objective: create efficient high-fidelity beam models for wind turbine blade analysis that can
  - Capture geometrical nonlinearity systematically
  - Capture anisotropic and heterogeneous behavior of composite materials rigorously
  - Modeling moving beams (translation and rotation)
  - Achieve the speed of computational design without significant loss of accuracy comparing to the ultimate accuracy obtained by 3D nonlinear FEA
  - Compatible with the FAST modularization framework

NWTC static and dynamic blade tests showing typical large, elastic deflections.

Approach

- Implementation
  - Geometrically Exact Beam Theory (GEBT)

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    - Mixed implementation (Yu and Blair, 2012)
  - Legendre Spectral Finite Element (LSFE) (**?**)

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    - A *p*-version high-order finite element
    - Successfully applied to simulation of fluid dynamics, geophysics, elastodynamics
    - Limited usage in structural dynamics
  - FAST Modularization Framework (Jonkman, 2013)
    - State-space formulation for tight-coupling scheme
    - Time integrator for first-order PDEs
- Result: BeamDyn, which can be used as a structural module of FAST

- Governing Equation

$$\dot{\underline{h}} - \underline{F}' = \underline{f}$$

$$\dot{\underline{g}} + \dot{\tilde{u}}\underline{h} - \underline{M}' - (\tilde{x}_0' + \tilde{u}')\underline{F} = \underline{m}$$

- Constitutive Equation

$$\begin{Bmatrix} \underline{h} \\ \underline{g} \end{Bmatrix} = \underline{\underline{\mathcal{M}}} \begin{Bmatrix} \dot{\underline{u}} \\ \underline{\omega} \end{Bmatrix}$$

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- Timoshenko-like beam model

- Strain Measures

$$\begin{Bmatrix} \underline{\epsilon} \\ \underline{\kappa} \end{Bmatrix} = \begin{Bmatrix} \underline{x}_0' + \underline{u}' - (\underline{\underline{R}}\,\underline{\underline{R}}_0)\bar{\imath}_1 \\ \underline{k} \end{Bmatrix}$$

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- Geometrically exact: deformed beam geometry is represented exactly

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- Geometrically exact: deformed beam geometry is represented exactly
- Small strains

State-space Formulation

- Governing Equation

$$\underline{\underline{\mathfrak{M}}}\,\underline{a} + f(\underline{q},\underline{v},t) = 0$$

$$\underline{q}^T = \begin{bmatrix} \underline{u}^T & \underline{p}^T \end{bmatrix} \qquad \underline{v}^T = \begin{bmatrix} \underline{\dot{u}}^T & \underline{\omega}^T \end{bmatrix} \qquad \underline{a}^T = \begin{bmatrix} \underline{\ddot{u}}^T & \underline{\dot{\omega}}^T \end{bmatrix}$$

- State-Space Form

$$\underline{\underline{A}}\,\dot{\underline{\hat{x}}}(t) = \mathfrak{f}(\underline{\hat{x}}(t), t)$$

$$\underline{x}(t) \equiv \begin{Bmatrix} \underline{q}(t) \\ \underline{v}(t) \end{Bmatrix} \qquad \underline{\underline{A}}(\underline{\hat{x}}(t)) = \begin{bmatrix} \underline{\underline{D}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{M}} \end{bmatrix}$$

$$\underline{\underline{D}}(\underline{\hat{x}}(t)) = \int_0^l \underline{\underline{N}}^T \begin{bmatrix} \underline{\underline{I}}_3 & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{H}} \end{bmatrix} \underline{\underline{N}}\, dx_1 \qquad \mathfrak{f}(\underline{\hat{x}}(t), t) = \begin{Bmatrix} \int_0^l \underline{\underline{N}}^T \underline{v}\, dx_1 \\ \underline{F}(\underline{\hat{x}}(t), t) \end{Bmatrix}$$

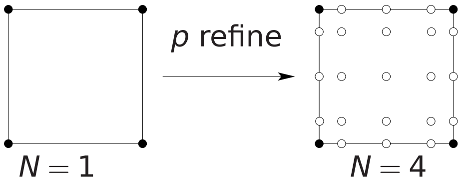
- Second-order Adams-Moulton (AM2)

$$\underline{\underline{A}}_{k+1}(\underline{\hat{x}}_{k+1} - \underline{\hat{x}}_k - \frac{\Delta t}{2}\dot{\underline{\hat{x}}}_k) = \frac{\Delta t}{2}\mathfrak{f}(\underline{\hat{x}}_{k+1}, t_{k+1})$$

Legendre Spectral Finite Elements

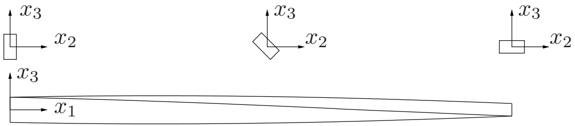
LSFE methods combine the geometric flexibility of the FE method with the accuracy of global spectral methods.

- Solution improved through increased basis polynomial order (p-refinement)
- LSFEs employ Lagrangian interpolant shape functions with nodes at Gauss-Lobatto-Legendre (GLL) points
- *Exponential* convergence rates for sufficiently smooth solutions



Example 1: Initially Twisted Beam

- Sketch of Initially Twisted Beam



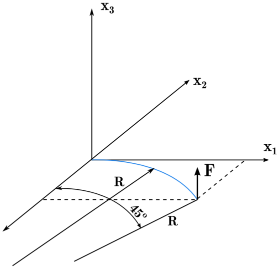
- Result

Table: Comparison of tip displacements of an initially twisted beam

	u_1 (m)	u_2 (m)	u_3 (m)
BeamDyn	-1.132727	-1.715123	-3.578671
ANSYS	-1.134192	-1.714467	-3.584232
Percent Error	0.129%	0.038%	0.155%

Example 1: Initially Curved Beam

- Sketch of Initially Curved Beam



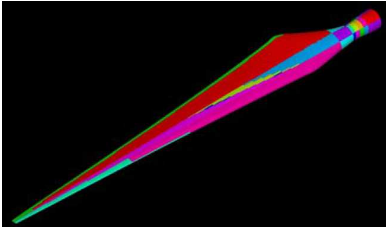
- Result

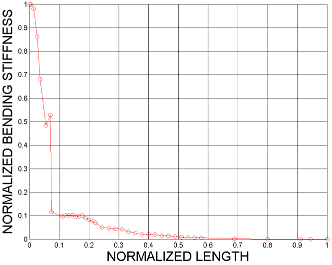
Table: Comparison of tip displacements of an initially curved beam

	u_1 (inches)	u_2 (inches)	u_3 (inches)
BeamDyn (one LSFE)	-23.7	13.5	53.4
Bathe-Bolourchi **?**	-23.5	13.4	53.4

Example 2: CX-100

- Sketch of CX-100



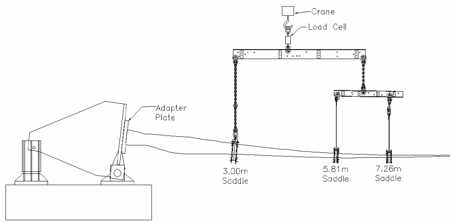


- Sectional Properties at 2.2 m

$$C = 10^3 \times \begin{bmatrix} 193,000 & -75.4 & 12.2 & -75.2 & -1970 & -3500 \\ -75.4 & 19,500 & 4,760 & 62.6 & 67.3 & 11.3 \\ 12.2 & 4,760 & 7,210 & -450 & 17.0 & 2.68 \\ -75.2 & 62.6 & -450 & 518 & 1.66 & -1.11 \\ -1,970 & 67.3 & 17.0 & 1.66 & 2,280 & -879 \\ -3,500 & 11.6 & 2.68 & -1.11 & -875 & 4,240 \end{bmatrix}$$

Example 2: CX-100 (Continued)

- Static Test Configuration



- Deflection

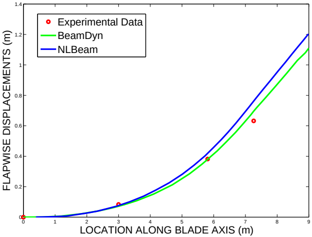
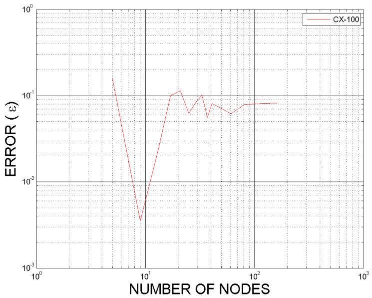


Table: Experimental and BeamDyn simulation results for the CX-100 static test

	u_3 at saddle #1 (m)	u_3 at saddle #2 (m)	u_3 at saddle #3 (m)
Experimental	0.083530	0.381996	0.632460
BeamDyn	0.072056	0.381074	0.698850
Percent Error	13.74%	0.24%	10.5%

Example 2: Convergence Study

- Validation



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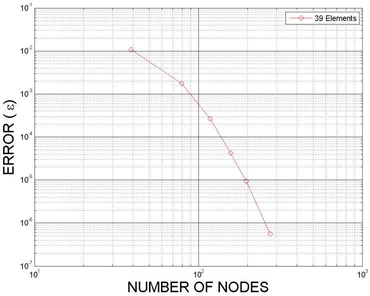

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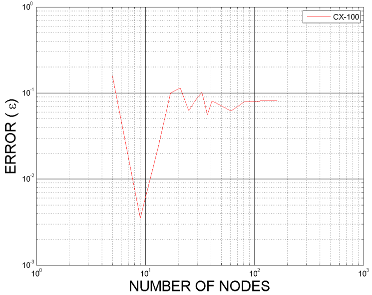

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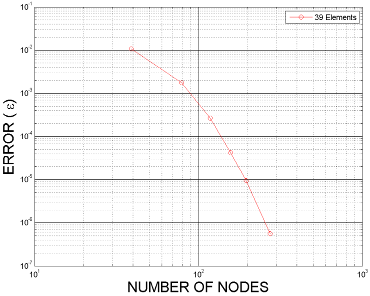
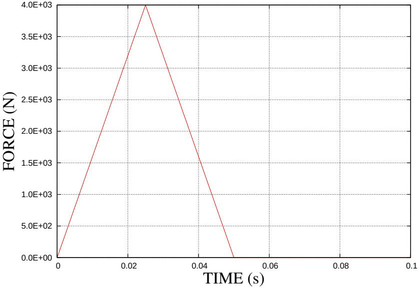


- Sharp gradients in sectional properties
- Erratic data

Example 3: Damping Effect

- Cantilever Beam Under Impulsive Load



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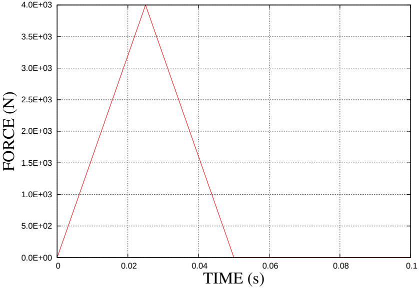


- Viscous Damping

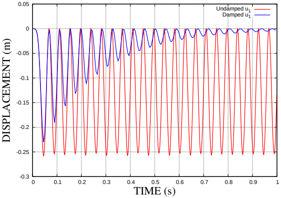
$$\underline{f}_d = \underline{\underline{\mu}}\ \underline{\underline{\mathcal{C}}} \begin{Bmatrix} \dot{\epsilon} \\ \dot{\kappa} \end{Bmatrix}$$

- RMS Error

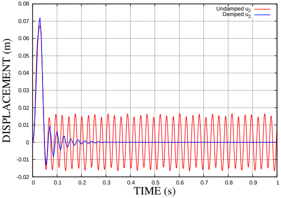
$$\varepsilon_{RMS} = \sqrt{\frac{\sum_{k=0}^{n_{max}} [u_3^k - u_b(t^k)]^2}{\sum_{k=0}^{n_{max}} [u_b(t^k)]^2}}$$

Example 3: Root forces and moments

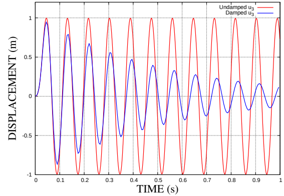
- u_1



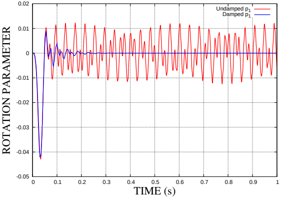
- u_2



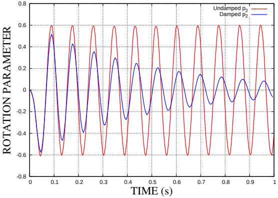
- u_3



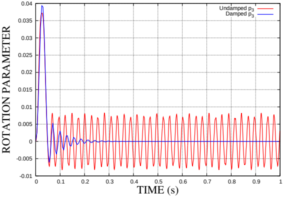
- p_1



- p_2



- p_3



Example 4: NREL 5-MW Blade

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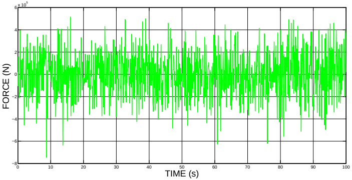
- Time History of Applied Force



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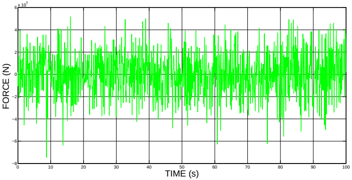
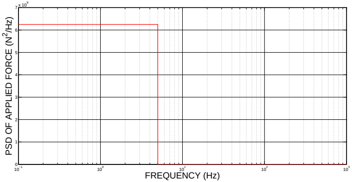


- PSD of Applied Force



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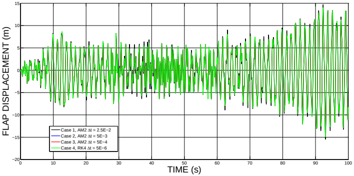
- Flapwise Response



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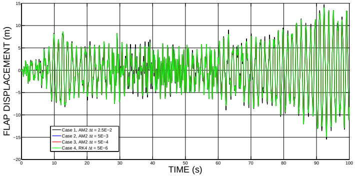
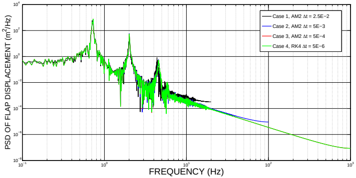


- PSD of Flapwise Response



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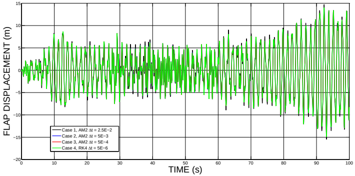

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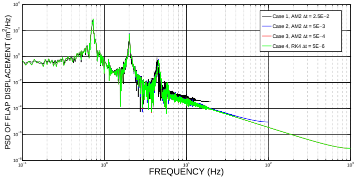

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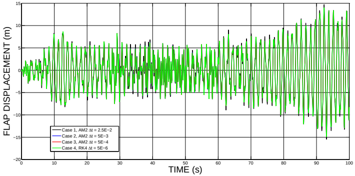

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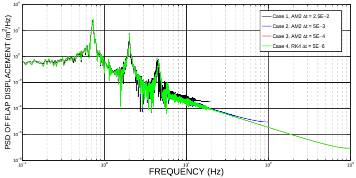

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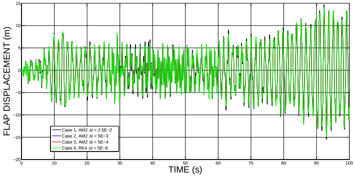

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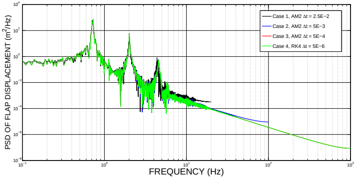

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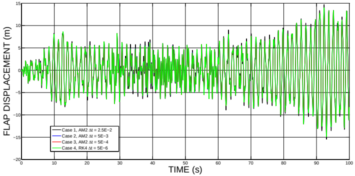
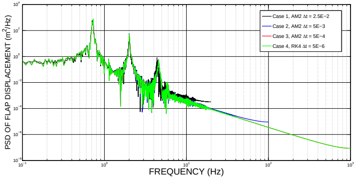
- For an AM2 time step of 0.025 s, the solution remains stable and tracks the other solutions, but error grows at higher frequencies
- The spikes at 0.7 Hz and 2 Hz correspond to the first and second blade flapwise natural frequencies

Example 4: NREL 5-MW Blade (Continued)

- Flapwise Response



- PSD of Flapwise Response



- For an implicit AM2 time step beyond 0.005 s, the solution is nearly identical to the fully resolved explicit RK4 solution
- For an AM2 time step of 0.025 s, the solution remains stable and tracks the other solutions, but error grows at higher frequencies
- The spikes at 0.7 Hz and 2 Hz correspond to the first and second blade flapwise natural frequencies
- The spike above 5 Hz–above the frequency range of excitation–is brought about by nonlinear effects

Example 4: NREL 5-MW Blade (Continued)

- Convergence Rate

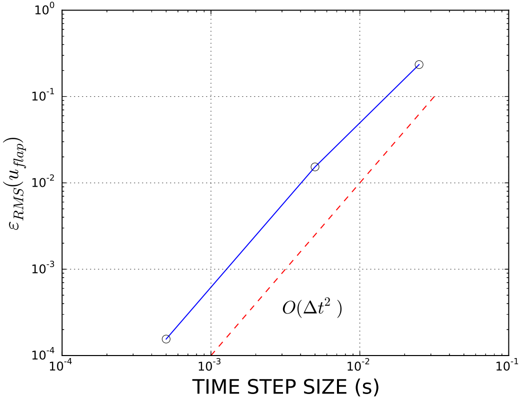


Figure: Normalized RMS error of flapwise displacement histories as a function of time step size for AM2 time integrator. The dashed line shows ideal second-order convergence.

Example 4: NREL 5-MW Blade (Continued)

- Solver Statistics

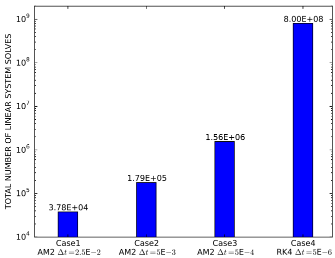


Figure: Total number of linear system solves.

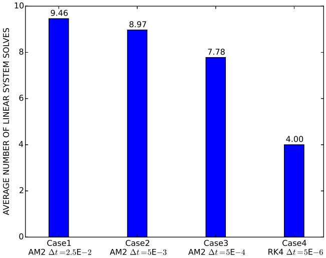


Figure: Average number of linear system solves per step.

Summary

- Conclusion
  - Based on geometrically exact beam theory, BeamDyn is capable of dealing with geometric nonlinear beam problems with arbitrary magnitude of displacements and rotations for both static and dynamic analyses
  - Along with a preprocessor like PreComp or VABS, BeamDyn takes full elastic coupling effects into account
  - The governing equations are reformulated into state-space form, thus, making it amendable into FAST for tight-coupling analysis
  - The space is discretized by spectral finite elements, which is a p-version finite element, so that exponential convergence rate can be expected for smooth solutions
  - Different time integrators have been implemented in BeamDyn; users will have options based on their needs
  - BeamDyn is implemented following the programming requirements (data structures and interfaces) of the FAST modularization framework

Summary (Continued)

- Future Work
  - Coupling BeamDyn to FAST
  - Full-Turbine validation
  - Enhancement of numerical performance

Questions?

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