Nonlinear Legendre Spectral Finite Elements for Wind Turbine Blade Dynamics

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This paper presents a numerical implementation and examination of new wind turbine blade finite element model based on Geometrically Exact Beam Theory (GEBT) and a high-order spectral finite element method. The displacement-based GEBT is presented, which includes the coupling effects that exist in composite structures and geometric nonlinearity. Legendre spectral finite elements (LSFEs) are high-rder finite elements with nodes located at the Gauss-Legendre-Lobatto points. LSFEs can be an order of magnitude more efficient that low-order finite elements for a given accuracy level. The new LSFE code is implemented in the new FAST Modularization Framework for dynamic simulation of highly flexible composite-material wind turbine blades. The framework allows for fully interactive simulations of turbine blades in operating conditions. Numerical examples showing validation and LSFE performance are provided in the numerical examples section. It concludes that the implemented code can be used as a efficient high-fidelity beam tool in FAST.

I. Introduction

Wind power is becoming one of the most important renewable-energy sources in the United States. In recent years, the size of wind turbines has been increasing immensely to lower the cost, which, because of weight restrictions, also leads to highly flexible turbine blades. This huge electro-mechanical system poses a significant challenge for engineering design and analysis. Although possible with modern super computers, direct three-dimensional (3D) structural analysis is so computationally expensive that engineers are always seeking for efficient high-fidelity simplified models.

Beam models are widely used to represent and analyze engineering structures that have one of its dimensions much larger than the other two. Many engineering components can be idealized as beams: bridges in civil engineering, joists and lever arms in heavy-machine industries, and helicopter rotor blades. The blades, tower, and shaft in a wind turbine system can be considered as beams. In the weight-critical applications of beam structures, like high-aspect-ratio wings in aerospace and wind energy, composite materials are attractive due to their superior strength-to-weight and stiffness-to-weight ratios. However, analysis of composite-materials structures is more difficult than their isotropic counterparts due to elastic-coupling effects. The geometrically exact beam theory (GEBT) first first proposed by Reissner[2], is a method that has proven powerful for analysis of highly flexible composite beams in the helicopter engineering community. During the past several decades, much effort has been invested in this area. Simo[3] and Simo and Vu-Quoc[4] extended Reissner's work to deal with three-dimensional (3D) dynamic problems. Jelenić and Crisfield[5] implemented this theory using the finite-element method where a new approach for interpolating the rotation field was proposed that preserves the geometric exactness. Betsch and Steinmann[6] circumvented the interpolation of rotation by introducing a re-parameterization of the weak form corresponding to the equations of motion of GEBT. It is noted that Ibrahimbegović and his colleagues implemented this theory for static[7] and dynamic[8] analysis. In contrast to the displacement-based implementations, the geometric exact beam theory has also been formulated by mixed finite elements where both the primary and dual field are independently interpolated[9]. In the mixed formulation, all of the necessary ingredients, including Hamilton's principle and kinematic equations, are combined in a single variational formulation statement; Lagrange multipliers, motion variables, generalized strains, forces and moments, linear and angular momenta, and displacement and rotation variables are considered as independent quantities. Yu et al.[10,11] presented the implementation of GEBT in a mixed formulation; various rotation parameters were investigated and the code was validated against analytical and numerical solutions. Readers are referred to Hodges[12], where comprehensive derivations and discussions on nonlinear composite-beam theories can be found.

Legendre spectral finite elements[13,14] (LSFEs) are p-type finite elements whose shape functions are Lagrangian interpolants with node locations at the Gauss-Lobatto-Legendre (GLL) points. LSFEs combine the accuracy of global

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spectral methods with geometric flexibility of *h*-type FEs. The spectral FEs have seen successful use in the simulation of fluid dynamics[13–15], two-dimensional elastic wave propagation in solid media in geophysics[16], elastodynamics[17], and acoustic wave propagation[18]. However, it has seen limited application to dynamic analysis of beam[19–22] and plate elements[23–25]. COMMENT: we need to add references on ”quadrature elements”

In this paper, we present a displacement-based implementation of geometrically exact beam theory using LSFEs. This work builds on a previous effort which showed the implementation of three-dimensional rotation parameters[11] and a demonstration example of two-dimensional nonlinear spectral beam elements[26] for static deformation. The code implemented in this work is in accordance to FAST Modularization Framework[27], which allows simulation of a whole turbine under realistic operating conditions. COMMENT: EXPAND ON FAST MODULARIZATION

The paper is organized as follows. The theoretical foundation of the geometrically exact beam theory is introduced first. Then the GEBT discretization by LSFEs is discussed. Finally, verification examples are provided to show the accuracy and efficiency of the GEBT LSFEs for isotropic and composite beams.

II. Geometrically Exact Beam Theory

For completeness, this section reviews the geometrically exact beam theory and linearization process of the governing equations. The content of this section can be found in many other papers and textbooks. Figure 1 shows a beam in its initial undeformed and deformed states. A reference frame $\mathbf{b}_i$ is introduced along the beam axis for the undeformed state; a frame $\mathbf{B}_i$ is introduced along each point of the deformed beam axis. Curvilinear coordinate x_1 defines the intrinsic parameterization of the reference line. In this paper, we use matrix notation to denote vectorial or

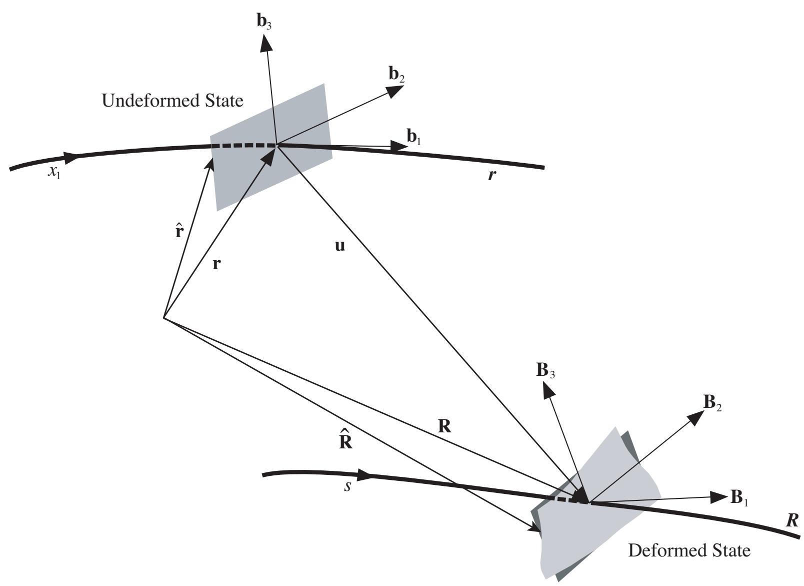


Figure 1: Schematic of beam deformation

vectorial-like quantities. For example, we use a underline to denote a vector $\underline{u}$, a bar to denote unit vector $\bar{n}$, and double underline to denote a tensor $\underline{\underline{\Delta}}$. Note that sometimes the underlines only denote the dimension of the corresponding matrix. The governing equations of motion for geometric exact beam theory can be written as[28]

$$\dot{\underline{h}} - \underline{F}' = \underline{f} \tag{1}$$

$$\dot{\underline{g}} + \dot{\tilde{u}}\underline{h} - \underline{M}' - (\tilde{x}_0' + \tilde{u}')\underline{F} = \underline{m} \tag{2}$$

where $\underline{h}$ and $\underline{g}$ are the linear and angular momenta resolved in the inertial coordinate system, respectively; $\underline{F}$ and $\underline{M}$ are the beam's sectional forces and moments, respectively; $\underline{u}$ is the 1D displacement of the reference line; $\underline{x}_0$ is the initial position vector of a point along the beam's reference line; $\underline{f}$ and $\underline{m}$ are the distributed force and moment applied to the beam structure. Notation $(\bullet)'$ indicates a derivative with respect to the beam axis x_1 and $\dot{(\bullet)}$ indicates a derivative with respect to time. The tilde operator $\widetilde{(\bullet)}$ defines a second-order, skew-symmetric tensor corresponding to the given vector. In the literature, it is also termed as ”cross-product matrix”. For example,

$$\widetilde{n} = \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix}$$

The constitutive equations relate the velocities to the momenta and the one-dimensional strain measures to the sectional resultants as

$$\begin{Bmatrix} \underline{h} \\ \underline{g} \end{Bmatrix} = \underline{\underline{\mathcal{M}}} \begin{Bmatrix} \underline{\dot{u}} \\ \underline{\omega} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \underline{F} \\ \underline{M} \end{Bmatrix} = \underline{\underline{\mathcal{C}}} \begin{Bmatrix} \underline{\epsilon} \\ \underline{\kappa} \end{Bmatrix} \tag{4}$$

where $\underline{\underline{\mathcal{M}}}$ and $\underline{\underline{\mathcal{C}}}$ are the 6×6 sectional mass and stiffness matrices,respectively, note that they are not really tensors; $\underline{\epsilon}$ and $\underline{\kappa}$ are the 1D strains and curvatures, respectively. $\underline{\omega}$ is the angular velocity vector that is defined by the rotation tensor $\underline{\underline{R}}$ as $\underline{\omega} = \mathrm{axial}(\underline{\underline{\dot{R}}}\,\underline{\underline{R}})$.

For a displacement-based finite element implementation, there are six degree-of-freedoms(DoFs) at each node: 3 displacement components and 3 rotation components. Here we use $\underline{q}$ to denote the elemental displacement array as $\underline{q} = \begin{bmatrix} \underline{u}^T & \underline{p}^T \end{bmatrix}$ where $\underline{u}$ is the 1D displacement and $\underline{p}$ is the rotation parameter vector. The acceleration array can thus be defined as $\underline{a} = \begin{bmatrix} \underline{\ddot{u}}^T & \underline{\dot{\omega}}^T \end{bmatrix}$. For nonlinear finite element analysis, the discretized and incremental forms of displacement, velocity, and acceleration array are written as

$$\underline{q}(x_1) = \underline{\underline{N}}\,\underline{\hat{q}} \quad \Delta \underline{q}^T = \begin{bmatrix} \Delta \underline{u}^T & \Delta \underline{p}^T \end{bmatrix} \tag{5}$$

$$\underline{v}(x_1) = \underline{\underline{N}}\,\underline{\hat{v}} \quad \Delta \underline{v}^T = \begin{bmatrix} \Delta \underline{\dot{u}}^T & \Delta \underline{\omega}^T \end{bmatrix} \tag{6}$$

$$\underline{a}(x_1) = \underline{\underline{N}}\,\underline{\hat{a}} \quad \Delta \underline{a}^T = \begin{bmatrix} \Delta \underline{\ddot{u}}^T & \Delta \underline{\dot{\omega}}^T \end{bmatrix} \tag{7}$$

where $\underline{\underline{N}}$ is the shape function matrix and $(\hat{\bullet})$ denotes a column matrix of nodal values. The governing equations for beams are highly nonlinear so that a linearization process is needed. According to Ref[28], the linearized governing equations in Eq. (1) and (2) are in the form of

$$\underline{\underline{\hat{M}}}\Delta\underline{\hat{a}} + \underline{\underline{\hat{G}}}\Delta\underline{\hat{v}} + \underline{\underline{\hat{K}}}\Delta\underline{\hat{q}} = \underline{\hat{F}}^{ext} - \underline{\hat{F}} \tag{8}$$

where the $\underline{\underline{\hat{M}}}$, $\underline{\underline{\hat{G}}}$, and $\underline{\underline{\hat{K}}}$ are the elemental mass, gyroscopic, and stiffness matrices, respectively; $\underline{\hat{F}}$ and $\underline{\hat{F}}^{ext}$ are the elemental forces and externally applied loads, respectively. They are defined as follows

$$\underline{\underline{\hat{M}}} = \int_0^l \underline{\underline{N}}^T \underline{\underline{\mathcal{M}}}\,\underline{\underline{N}} dx_1 \tag{9}$$

$$\underline{\underline{\hat{G}}} = \int_0^l \underline{\underline{N}}^T \underline{\underline{\mathcal{G}}}^I\,\underline{\underline{N}} dx_1 \tag{10}$$

$$\underline{\underline{\hat{K}}} = \int_0^l \left[\underline{\underline{N}}^T (\underline{\underline{\mathcal{K}}}^I + \underline{\underline{\mathcal{Q}}})\,\underline{\underline{N}} + \underline{\underline{N}}^T \underline{\underline{\mathcal{P}}}\,\underline{\underline{N}}' + \underline{\underline{N}}'^T \underline{\underline{\mathcal{C}}}\,\underline{\underline{N}}' + \underline{\underline{N}}'^T \underline{\underline{\mathcal{O}}}\,\underline{\underline{N}} \right] dx_1 \tag{11}$$

$$\underline{\hat{F}} = \int_0^l (\underline{\underline{N}}^T \underline{\mathcal{F}}^I + \underline{\underline{N}}^T \underline{\mathcal{F}}^D + \underline{\underline{N}}'^T \underline{\mathcal{F}}^C) dx_1 \tag{12}$$

$$\underline{\hat{F}}^{ext} = \int_0^l \underline{\underline{N}}^T \underline{\mathcal{F}}^{ext} dx_1 \tag{13}$$

The new matrix notations in Eq. (9) to (13) are briefly introduced here. $\underline{\underline{\mathcal{M}}}$ is the sectional mass matrix resolved in inertial system; $\underline{\mathcal{F}}^C$ and $\underline{\mathcal{F}}^D$ are elastic forces obtained from Eq. (1) and (2) as

$$\underline{\mathcal{F}}^C = \begin{Bmatrix} \underline{F} \\ \underline{M} \end{Bmatrix} = \underline{\underline{\mathcal{C}}} \begin{Bmatrix} \underline{\epsilon} \\ \underline{\kappa} \end{Bmatrix} \tag{14}$$

$$\underline{\mathcal{F}}^D = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{0}} \\ (\tilde{x}_0' + \tilde{u}')^T & \underline{\underline{0}} \end{bmatrix} \underline{\mathcal{F}}^C \equiv \underline{\underline{\Upsilon}}\,\underline{\mathcal{F}}^C \tag{15}$$

where $\underline{\underline{0}}$ denotes a 3×3 null matrix. The $\underline{\underline{\mathcal{G}}}^I$, $\underline{\underline{\mathcal{K}}}^I$, $\underline{\underline{\mathcal{O}}}$, $\underline{\underline{\mathcal{P}}}$, $\underline{\underline{\mathcal{Q}}}$, and $\underline{\mathcal{F}}^I$ in Eq. (10), Eq. (11), and Eq. (12) are defined as

$$\underline{\underline{\mathcal{G}}}^I = \begin{bmatrix} \underline{\underline{0}} & (\tilde{\omega} m \underline{\eta})^T + \tilde{\omega} m \tilde{\eta}^T \\ \underline{\underline{0}} & \tilde{\omega} \underline{\underline{\varrho}} - \widetilde{\underline{\underline{\varrho}} \omega} \end{bmatrix} \tag{16}$$

$$\underline{\underline{\mathcal{K}}}^I = \begin{bmatrix} \underline{\underline{0}} & \dot{\tilde{\omega}} m \tilde{\eta}^T + \tilde{\omega}\tilde{\omega} m \tilde{\eta}^T \\ \underline{\underline{0}} & \ddot{\tilde{u}} m \tilde{\eta} + \underline{\underline{\varrho}}\dot{\tilde{\omega}} - \widetilde{\underline{\underline{\varrho}}\dot{\omega}} + \tilde{\omega}\underline{\underline{\varrho}}\tilde{\omega} - \tilde{\omega}\widetilde{\underline{\underline{\varrho}}\omega} \end{bmatrix} \tag{17}$$

$$\underline{\underline{\mathcal{O}}} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{C}}_{11}\tilde{E}_1 - \tilde{F} \\ \underline{\underline{0}} & \underline{\underline{C}}_{21}\tilde{E}_1 - \tilde{M} \end{bmatrix} \tag{18}$$

$$\underline{\underline{\mathcal{P}}} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{0}} \\ \tilde{F} + (\underline{\underline{C}}_{11}\tilde{E}_1)^T & (\underline{\underline{C}}_{21}\tilde{E}_1)^T \end{bmatrix} \tag{19}$$

$$\underline{\underline{\mathcal{Q}}} = \underline{\underline{\Upsilon}}\, \underline{\underline{\mathcal{O}}} \tag{20}$$

$$\underline{\mathcal{F}}^I = \begin{Bmatrix} m\ddot{\underline{u}} + (\dot{\tilde{\omega}} + \tilde{\omega}\tilde{\omega}) m \underline{\eta} \\ m\tilde{\eta}\ddot{\underline{u}} + \underline{\underline{\varrho}}\dot{\underline{\omega}} + \tilde{\omega}\underline{\underline{\varrho}}\underline{\omega} \end{Bmatrix} \tag{21}$$

The following notations were introduced to simply the writing of the above expressions

$$\underline{E}_1 = \underline{x}_0' + \underline{u}' \tag{22}$$

$$\underline{\underline{\mathcal{C}}} = \begin{bmatrix} \underline{\underline{C}}_{11} & \underline{\underline{C}}_{12} \\ \underline{\underline{C}}_{21} & \underline{\underline{C}}_{22} \end{bmatrix} \tag{23}$$

The derivation and linearization of governing equations of geometrically exact beam theory can be found in Ref[28].

III. Legendre Spectral Finite Elements

The displacement fields in a element are interpolated as

$$\underline{u}(s) = h^k(s)\hat{\underline{u}}^k \tag{24}$$

$$\underline{u}'(s) = h^{k\prime}(s)\hat{\underline{u}}^k \tag{25}$$

where $h^k(s)$ is the Lagrangian-interpolan shape function of node k, $k = 1, 2, ..., n + 1$, where n is the polynomial order of of the basis functions, and $\hat{\underline{u}}^k$ is the k^{th} nodal value. However, as discussed in Ref[29], the three-dimensional rotation field cannot be simply interpolated as the displacement field in the form of

$$\underline{c}(s) = h^k(s)\hat{\underline{c}}^k \tag{26}$$

$$\underline{c}'(s) = h^{k\prime}(s)\hat{\underline{c}}^k \tag{27}$$

where $\underline{c}$ is the rotation field in a element and $\hat{\underline{c}}^k$ is the nodal value at the k^{th} node, for three reasons: 1) rotations do not form a linear space so that they must be "composed" instead of added; 2) a rescaling operation is needed to eliminate the singularity existing in the vectorial rotation parameters; 3) the rotation field lacks objectivity, which, as defined by Crisfield and Jelenić[5], refers to the invariance of strain measures computed through interpolation to the addition of a rigid-body motion. Therefore, we adopt the more robust interpolation approach proposed by Crisfield and Jelenić[5] to deal with the finite rotations. Our approach is described as follows

Step 1: Compute the nodal relative rotations, $\hat{\underline{r}}^k$ by removing the rigid body rotation, $\hat{\underline{c}}^1$, from the finite rotation at each node, $\hat{\underline{r}}^k = \hat{\underline{c}}^{1-} \oplus \hat{\underline{c}}^k$.

Step 2: Interpolate the relative rotation field: $\underline{r}(s) = h^k(s)\hat{\underline{r}}^k$ and $\underline{r}'(s) = h^{k\prime}(s)\hat{\underline{r}}^k$. Find the curvature field $\underline{\kappa}(s) = \underline{\underline{R}}(\hat{\underline{c}}^1)\underline{\underline{H}}(\underline{r})\underline{r}'$.

Step 3: Restore the rigid body rotation removed in Step 1: $\underline{c}(s) = \hat{\underline{c}}^1 \oplus \underline{r}(s)$.

where H is the tangent tensor that relates the curvature vector k and rotation vector p as

$$\underline{k} = \underline{\underline{H}}\, \underline{p}' \tag{28}$$

In the LSFE approach, shape functions (e.g., those composing $\underline{\underline{N}}$) are n^{th}-order Lagrangian interpolants, where nodes are located at the $n + 1$ GLL-quadrature points in the $[-1, 1]$ element natural-coordinate domain. Need more work here: a figure shows some LS elements (non-evenly placed internal nodes) and a short discussion of its advantages. The implementation of GEBT into the FE code, BeamDyn, carries out the integration in the space domain by using reduced Gauss quadrature. Time integration is performed using generalized-α scheme, which is a unconditionally stable, second-order accurate algorithm. More details regarding on the generalized-α method can be found in Ref[28,30]

IV. Numerical Examples

A. Example 1: Static bending of a cantilever beam

The first example is a benchmark problem for geometrically nonlinear analysis of beams[3,31]. We calculate the static deflection of a cantilever beam that is subjected at its free end to a constant moment M. The length of the beam L is $10\,in$ and the cross-sectional stiffness matrix is given below:

$$C^* = 10^3 \times \begin{bmatrix} 1770 & 0 & 0 & 0 & 0 & 0 \\ & 1770 & 0 & 0 & 0 & 0 \\ & & 1770 & 0 & 0 & 0 \\ & & & 8.16 & 0 & 0 \\ & & & & 86.9 & 0 \\ & & & & & 215 \end{bmatrix} \tag{29}$$

It is pointed out that the term with a asterisk denotes it is resolved in the material coordinate system. A sketch of this case can be found in Figure 2. The load applied at the tip is given by the following equation:

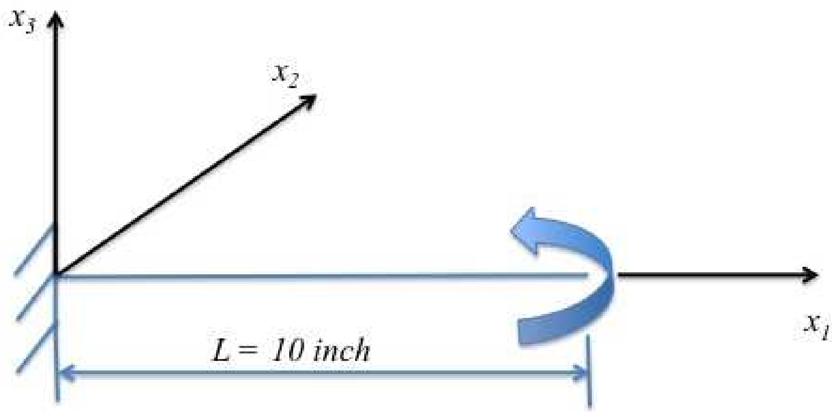


Figure 2: Sketch of a cantilever beam.

$$M_2 = \lambda \bar{M}_2 \tag{30}$$

where $\bar{M}_2 = \pi \frac{EI_2}{L}$. The parameter λ will vary between 0 and 2. In this case, the beam is discretized with two 5^{th} order elements. The deformations of the beam are shown in Figure 3. The calculated results are compared with the

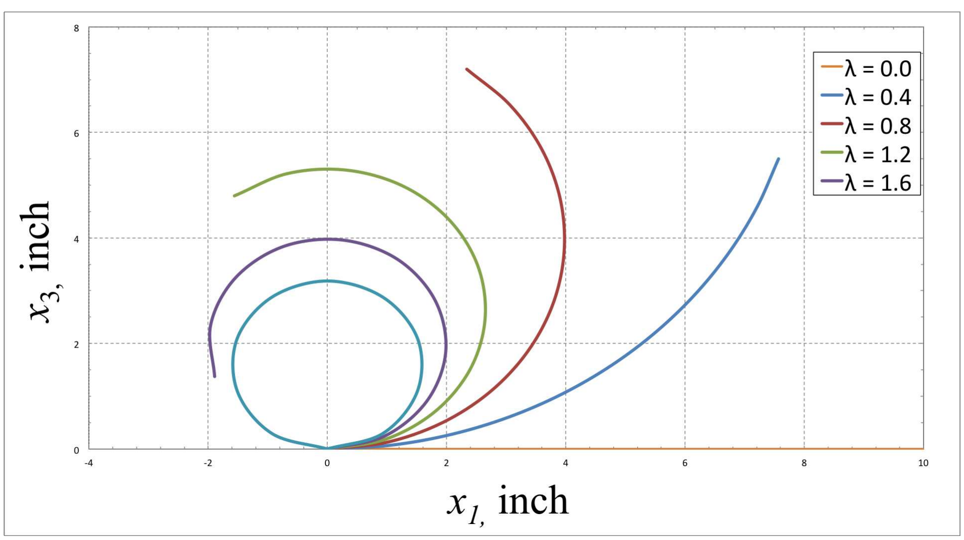


Figure 3: Deformations of a cantilever beam under several constant bending moments.

Table 1: Axial displacement u_1 of a cantilever beam subject to a constant moment (in inches).

λ	Analytical	BeamDyn	% Error
0.4	-2.4317	-2.4317	0.00
0.8	-7.6613	-7.6613	0.00
1.2	-11.5591	-11.5591	0.00
1.6	-11.8921	-11.8921	0.00
2.0	-10.0000	-10.0000	0.00

Table 2: Vertical displacement u_3 of a cantilever beam subject to a constant moment (in inches).

λ	Analytical	BeamDyn	% Error
0.4	5.4987	5.4987	0.00
0.8	7.1978	7.1979	0.0013
1.2	4.7986	4.7986	0.00
1.6	1.3747	1.3747	0.00
2.0	0.0000	0.0000	0.00

analytical solution, which can be found in Ref.[32] as

$$u_1 = \rho sin\left(\frac{x_1}{\rho}\right) - x_1 \qquad u_3 = \rho\left(1 - cos\left(\frac{x_1}{\rho}\right)\right) \tag{31}$$

The results can be found in Table 1 and 2, respectively. Good agreement can be observed between these two sets of results.

The rotation parameters at each node along beam axis x_1 obtained from BeamDyn are plotted in Figure 4 for $\lambda = 0.8$ and $\lambda = 2.0$, respectively. It is noted that the three-dimensional rotations are represented by Wiener-Milenković parameter defined in the following equation:

$$\underline{p} = 4tan\frac{\phi}{4}\bar{n} \tag{32}$$

where ϕ is the rotation angle and $\bar{n}$ is the unit vector of rotation axis. The singularity exists in the above definition can be removed by a rescaling operation, which can be observed in Figure 4. Figure 5 shows the normalized error

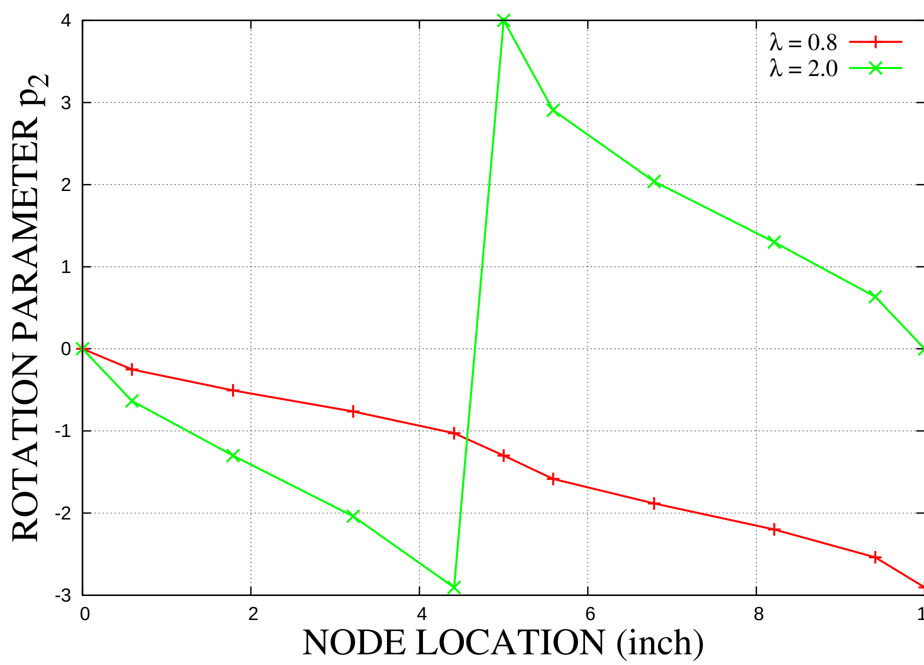


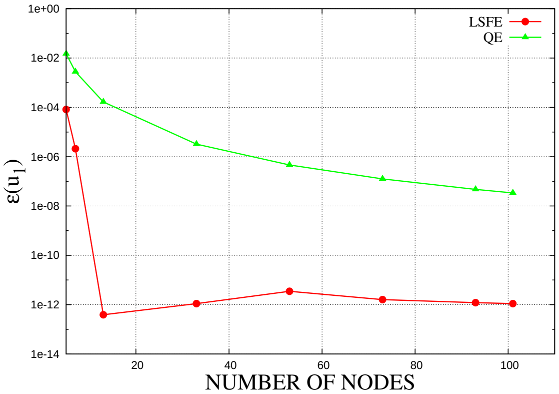
Figure 4: Wiener-Milenković rotation parameters along beam axis x_1 .

$\epsilon(u)$, where u is the tip displacement (at $x = L$), as a function of the number of model nodes for the calculation with

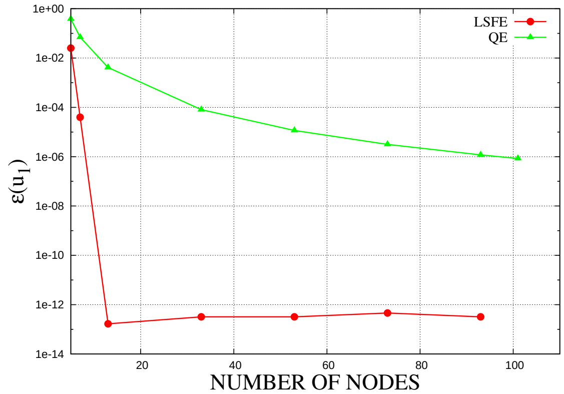
Dymore quadratic elements (QE) and a single Legendre spectral element (LSE), where

$$\epsilon(u) = \left| \frac{u - u^a}{u^a} \right| \tag{33}$$

and u is the test solution and u^a is the analytical solution. The parameter λ is set to 1.0 for this case. The Legendre spectral elements (with p-refinement) exhibit highly desirable exponential convergence to machine precision error.



(a) u_1



(b) u_3

Figure 5: Normalized error of the (a) u_1 and (b) u_3 displacements as a function of the total number of nodes

Acknowledgments

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