BeamDyn: A High-Fidelity Wind Turbine Blade Solver in the FAST Modular Framework

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BeamDyn, a Legendre-spectral-finite-element implementation of geometrically exact beam theory (GEBT), is developed to meet the design challenges associated with highly flexible composite wind turbine blades. In this paper, the governing equations of GEBT are reformulated into a nonlinear state-space form to support its coupling within the modular framework of the FAST wind turbine computer-aided-engineering (CAE) tool. Different time integration schemes (implicit and explicit) are implemented and examined for wind turbine analysis. Numerical examples and discussion on the efficiency and accuracy of time and space discretization will be presented in the final paper. A validation example of a realistic wind turbine blade will also be presented.

I.   Introduction

Wind power installations in the U.S. have exceeded 60 GW, and have become an increasingly important part of the overall energy portfolio. In recent years, the size of wind turbines has also increased in the quest for economies of scale. Larger wind turbine blades result in structures that are highly flexible. To ensure the performance and reliability of wind turbines, it is crucial to make use of computer-aided-engineering (CAE) tools that are capable of analyzing wind turbine blades in an accurate and efficient manner. Although modern computers enable three-dimensional (3D) analysis of a fully resolved blade, such analyses are too expensive for iterative design. More importantly, modern composite wind turbine blades are very well suited to nonlinear beam models, which can capture with high-fidelity the deformation response under realistic operating conditions, and in a small fraction of the time required by a fully resolved 3D simulation.

Beam models are widely used to analyze structures that have one of its dimensions being much larger than the other two. Many engineering structures are modeled as beams, e.g., bridges, joists, and helicopter rotor blades. Similarly, beam models are well suited to analyze, with high fidelity, wind turbine blades, towers, and shafts. Most wind turbine blades are constructed of composite materials, and analysis of composite beams is more complicated than isotropic beams due to the elastic coupling effects. The geometrically exact beam theory (GEBT), first proposed by Reissner[1], is a beam-deformation model capable of enabling efficient analysis of highly flexible composite structures. GEBT has demonstrated its efficacy in helicopter rotor analysis. Simo[2] and Simo and Vu-Quoc[3] extended Reissner's initial work to include 3D dynamic problems. Jelenić and Crisfield[4] derived a finite-element (FE) method that interpolates the rotation field thereby preserving the geometric exactness of this theory. It is noted that Ibrahimbegović and his colleagues implemented this theory for static[5] and dynamic[6] analysis. Readers are referred to Hodges[7], where comprehensive derivations and discussions on nonlinear composite-beam theories can be found. Recently, a mixed formulation of GEBT along with the numerical implementation was presented by Yu and Blair[8].

FAST is a CAE tool developed by the National Renewable Energy Laboratory (NREL) for the purposes of wind turbine analysis for both land-based and offshore wind turbines using realistic operating conditions. The current beam model in FAST is not capable of predictive analysis of highly flexible composite wind turbine blades. Recently, FAST has been reformulated under a new modularized framework that provides a rigorous means by which various mathematical systems are implemented in distinct modules and are interconnected to solve for the global, coupled, dynamic response of wind turbines and wind plants[9,10].

In this paper, a three-dimensional displacement-based implementation of the geometrically exact beam theory using Legendre spectral finite elements is presented. The theory is reformulated in a nonlinear state-space form for the purpose of integrating with the FAST framework, thereby introducing an optional high-fidelity beam model as an alternative to the current beam model. This work builds on previous efforts that showed the implementation GEBT and spatial discretization executed using Legendre spectral finite elements (LSFEs)[11–14] for analysis of composite wind

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turbine blades. The paper is organized as follows. First, the theoretical foundation of the geometrically exact beam theory along with the reformulation of the governing equations into a state-space form is introduced. Coupling to the FAST framework is then discussed. Finally, validation examples are provided to show the accuracy and efficiency of the present model for composite wind turbine blades.

II.   Geometrically Exact Beam Theory

This section reviews the geometric exact beam theory for completeness of this paper. The content of this section can be found in many other papers and textbooks. Figure 1 shows a beam in its initial undeformed and deformed states. A reference frame $\mathbf{b}_i$ is introduced along the beam axis for the undeformed state; a frame $\mathbf{B}_i$ is introduced along each point of the deformed beam axis. Curvilinear coordinate x_1 defines the intrinsic parameterization of the reference line. In this paper, we use matrix notation to denote vectorial or vectorial-like quantities. For example, we
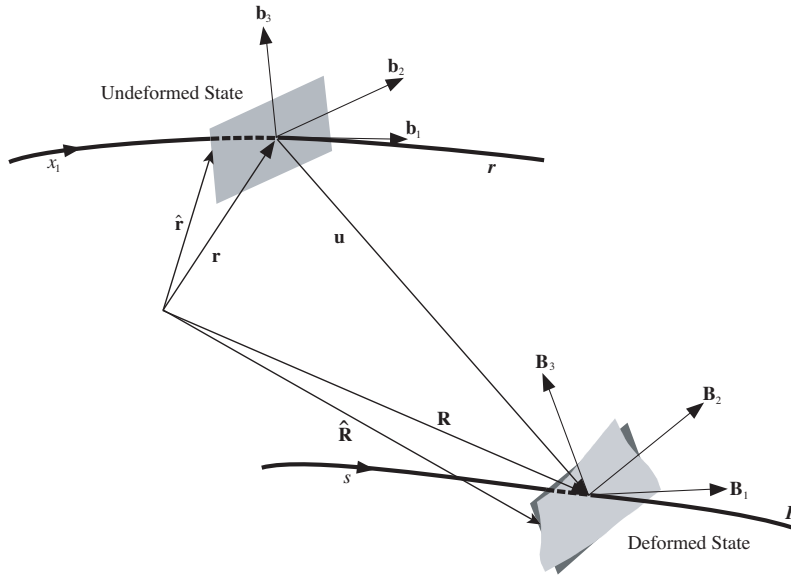

Figure 1: Schematic of beam deformation

use a underline to denote a vector $\underline{u}$, a bar to denote unit vector $\bar{n}$, and double underline to denote a tensor $\underline{\underline{\Delta}}$. Note that sometimes the underlines only denote the dimension of the corresponding matrix. The governing equations of motion for geometric exact beam theory can be written as [15]

$$\dot{\underline{h}} - \underline{F}' = \underline{f} \tag{1}$$

$$\dot{\underline{g}} + \dot{\tilde{u}}\underline{h} - \underline{M}' + (\tilde{x}_0' + \tilde{u}')^T \underline{F} = \underline{m} \tag{2}$$

where $\underline{h}$ and $\underline{g}$ are the linear and angular momenta resolved in the inertial coordinate system, respectively; $\underline{F}$ and $\underline{M}$ are the beam's sectional forces and moments, respectively; $\underline{u}$ is the 1D displacement of the reference line; $\underline{x}_0$ is the position vector of a point along the beam's reference line; $\underline{f}$ and $\underline{m}$ are the distributed force and moment applied to the beam structure. Notation $(\bullet)'$ indicates a derivative with respect to the beam axis x_1 and $(\dot{\bullet})$ indicates a derivative with respect to time. The tilde operator $(\tilde{\bullet})$ defines a second-order, skew-symmetric tensor corresponding to the given vector. In the literature, it is also termed as "cross-product matrix". For example,

$$\tilde{n} = \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix}$$

The constitutive equations relate the velocities to the momenta and the one-dimensional strain measures to the sectional resultants as

$$\left\{ \frac{\underline{h}}{\underline{g}} \right\} = \underline{\underline{\mathcal{M}}} \left\{ \frac{\dot{\underline{u}}}{\underline{\omega}} \right\} \tag{3}$$

$$\left\{ \frac{\underline{F}}{\underline{M}} \right\} = \underline{\underline{\mathcal{C}}} \left\{ \frac{\epsilon}{\kappa} \right\} \tag{4}$$

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where $\underline{\underline{M}}$ and $\underline{\underline{C}}$ are the 6×6 sectional mass and stiffness matrices, respectively (note that they are not really tensors); $\underline{\epsilon}$ and $\underline{\kappa}$ are the 1D strains and curvatures, respectively. $\underline{\omega}$ is the angular velocity vector that is defined by the rotation tensor $\underline{\underline{R}}$ as $\underline{\omega} = axial(\underline{\dot{R}}\,\underline{R})$. The 1D strain measures are defined as

$$\left\{ \begin{matrix} \underline{\epsilon} \\ \underline{\kappa} \end{matrix} \right\} = \left\{ \begin{matrix} \underline{x}_0' + \underline{u}' - (\underline{\underline{R}}'\,\underline{\underline{R}}_0)\bar{\imath}_1 \\ \underline{k} + \underline{\underline{R}}\,\underline{k}_i \end{matrix} \right\} \tag{5}$$

where $\underline{k} = axial(\underline{\underline{R}}'\,\underline{\underline{R}}^T)$ is the sectional curvature vector resolved in the inertial basis, $\underline{k}_i$ is the corresponding initial curvature vector, and $\bar{\imath}_1$ is the unit vector along x_1 direction in the inertial basis. It is noted that the three sets of equations, including equations of motion Eq. (1) and (2), constitutive equations Eq. (3) and (4), and kinematical equations Eq. (5), provided a full mathematical description of elasticity problems.

For a displacement-based finite element implementation, there are six degree-of-freedoms at each node: three displacement components and three rotation components. Here, we use $\underline{q}$ to denote the elemental displacement array as $\underline{q}^T = \begin{bmatrix} \underline{u}^T & \underline{p}^T \end{bmatrix}$ where $\underline{u}$ is the displacement and $\underline{p}$ is the rotation-parameter vector. The acceleration array can thus be defined as $\underline{a}^T = \begin{bmatrix} \underline{\ddot{u}}^T & \underline{\dot{\omega}}^T \end{bmatrix}$. For nonlinear finite-element analysis, the discretized form of displacement, velocity, and acceleration are written as

$$\underline{q}(x_1) = \underline{\underline{N}}\,\hat{\underline{q}} \quad \underline{q}^T = \begin{bmatrix} \underline{u}^T & \underline{p}^T \end{bmatrix} \tag{6}$$

$$\underline{v}(x_1) = \underline{\underline{N}}\,\hat{\underline{v}} \quad \underline{v}^T = \begin{bmatrix} \underline{\dot{u}}^T & \underline{\omega}^T \end{bmatrix} \tag{7}$$

$$\underline{a}(x_1) = \underline{\underline{N}}\,\hat{\underline{a}} \quad \underline{a}^T = \begin{bmatrix} \underline{\ddot{u}}^T & \underline{\dot{\omega}}^T \end{bmatrix} \tag{8}$$

where $\underline{\underline{N}}$ is the shape function matrix and $(\hat{\cdot})$ denotes a column matrix of nodal values.

To accommodate the FAST modular framework, the governing equations (1) and (2) need to be reformulated into a state-space form. Firstly we recast these equations in compact form as

$$\underline{\mathcal{F}}^I - \underline{\mathcal{F}}^{C\prime} + \underline{\mathcal{F}}^D = \underline{\mathcal{F}}^{ext} \tag{9}$$

where $\underline{\mathcal{F}}^I, \underline{\mathcal{F}}^C$ and $\underline{\mathcal{F}}^D$, and $\underline{\mathcal{F}}^{ext}$ are the inertial forces, elastic forces, and externally applied forces, respectively; their definitions are

$$\underline{\mathcal{F}}^I = \left\{ \begin{matrix} \underline{\dot{h}} \\ \underline{\dot{g}} \end{matrix} \right\} + \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{0}} \\ \tilde{\underline{\dot{u}}} & \underline{\underline{0}} \end{bmatrix} \left\{ \begin{matrix} \underline{h} \\ \underline{g} \end{matrix} \right\} \tag{10}$$

$$\underline{\mathcal{F}}^C = \left\{ \begin{matrix} \underline{F} \\ \underline{M} \end{matrix} \right\} \tag{11}$$

$$\underline{\mathcal{F}}^D = \left\{ \begin{matrix} \underline{0} \\ (\tilde{x}_0' + \tilde{u}')^T \underline{F} \end{matrix} \right\} \tag{12}$$

$$\underline{\mathcal{F}}^{ext} = \left\{ \begin{matrix} \underline{f} \\ \underline{m} \end{matrix} \right\} \tag{13}$$

Along with the constitutive equations (3) and (4), the inertial force $\underline{\mathcal{F}}^I$ can be written explicitly as

$$\begin{aligned} \underline{\mathcal{F}}^I &= \left\{ \begin{matrix} m\underline{\ddot{u}} + (\tilde{\dot{\omega}} + \tilde{\omega}\tilde{\omega})m\eta \\ m\tilde{\eta}\underline{\ddot{u}} + \underline{\varrho}\underline{\dot{\omega}} + \tilde{\omega}\underline{\varrho}\underline{\omega} \end{matrix} \right\} \\ &= \begin{bmatrix} m\underline{\underline{I}} & m\tilde{\eta}^T \\ m\tilde{\eta} & \underline{\varrho} \end{bmatrix} \left\{ \begin{matrix} \underline{\ddot{u}} \\ \underline{\dot{\omega}} \end{matrix} \right\} + \begin{bmatrix} \underline{\underline{0}} & m\tilde{\omega}\tilde{\eta}^T \\ \underline{\underline{0}} & \tilde{\omega}\underline{\varrho} \end{bmatrix} \left\{ \begin{matrix} \underline{\dot{u}} \\ \underline{\omega} \end{matrix} \right\} \\ &\equiv \underline{\underline{\mathfrak{M}}}\underline{a} + \underline{\underline{\mathcal{G}}}\underline{v} \end{aligned} \tag{14}$$

where m is the mass density per unit span; $\underline{\eta}$ is the center of mass location; $\underline{\varrho}$ is the moment of inertia; $\underline{\underline{I}}$ is the identity matrix. The definitions of the acceleration vector $\underline{a}$ and velocity vector $\underline{v}$ can be found in Eq. (8) and (7), respectively. By the newly introduced matrices, the compact form of equations of motion can be rewritten as

$$\underline{\underline{\mathfrak{M}}}\,\underline{a} + f(\underline{q}, \underline{v}, t) = 0 \tag{15}$$

where

$$f(\underline{q}, \underline{v}, t) = \underline{\mathcal{F}}^F - \underline{\mathcal{F}}^{C\prime} + \underline{\mathcal{F}}^D - \underline{\mathcal{F}}^{ext} \tag{16}$$

$$\begin{aligned} \underline{\mathcal{F}}^F &= \underline{\underline{\mathcal{G}}}\underline{v} \\ &= \begin{bmatrix} \underline{\underline{0}} & m\tilde{\omega}\tilde{\eta}^T \\ \underline{\underline{0}} & \tilde{\omega}\underline{\varrho} \end{bmatrix} \left\{ \begin{matrix} \underline{\dot{u}} \\ \underline{\omega} \end{matrix} \right\} \end{aligned} \tag{17}$$

A weighted residual formulation will be used to enforce the the dynamic equilibrium conditions in Eq. (15)

$$\int_0^l \underline{N}^T (\underline{\underline{\mathfrak{M}}} \underline{a} + \underline{\mathcal{F}}^F - \underline{\mathcal{F}}^{C\prime} + \underline{\mathcal{F}}^D - \underline{\mathcal{F}}^{ext}) dx_1 = 0 \tag{18}$$

The above equation can be recast as

$$\underline{\underline{M}} \hat{a} = F(\underline{q}, \underline{v}, t) \tag{19}$$

where

$$\underline{\underline{M}} = \int_0^l \underline{\underline{N}}^T \underline{\underline{\mathfrak{M}}} \, \underline{\underline{N}} \, dx_1 \tag{20}$$

$$\underline{F}(\underline{q}, \underline{v}, t) = \int_0^l \underline{\underline{N}}^T (-\underline{\mathcal{F}}^F + \underline{\mathcal{F}}^{C\prime} - \underline{\mathcal{F}}^D + \underline{\mathcal{F}}^{ext}) dx_1 \tag{21}$$

To derive the state-space form of the governing equations, $\underline{x}(t)$ is introduced as

$$\underline{x}(t) \equiv \left\{ \begin{matrix} \underline{q}(t) \\ \underline{v}(t) \end{matrix} \right\} \tag{22}$$

It is noted that the second component of $\underline{x}(t)$ is not $\dot{\underline{q}}$ but $\underline{v}$ in that the angular velocity $\underline{\omega}$ cannot be calculated as time derivative of the rotation parameter $\underline{p}$. Substituting the discretized quantities in Eqs. (6) to (8) into Eq. (22) and using the relation

$$\underline{a} = \dot{\underline{v}} = \left\{ \begin{matrix} \ddot{\underline{u}} \\ \dot{\underline{\omega}} \end{matrix} \right\} \tag{23}$$

The state-space form can be obtained as

$$\dot{\hat{\underline{x}}}(t) = \mathfrak{f}(\hat{\underline{x}}(t), t) \tag{24}$$

$$\hat{\underline{x}}(0) = \hat{\underline{x}}_0 \tag{25}$$

where

$$\mathfrak{f}(\hat{\underline{x}}(t), t) = \underline{\underline{A}}^{-1}(\hat{\underline{x}}(t)) \underline{b}(\hat{\underline{x}}(t), t) \tag{26}$$

$$\underline{\underline{A}}(\hat{\underline{x}}(t)) = \begin{bmatrix} \underline{I} & \underline{0} \\ \underline{0} & \underline{M} \end{bmatrix} \tag{27}$$

$$\underline{b}(\hat{\underline{x}}(t), t) = \left\{ \begin{matrix} \dot{\hat{\underline{q}}} \\ \underline{F}(\hat{\underline{x}}(t), t) \end{matrix} \right\} \tag{28}$$

$$\hat{\underline{x}}_0 = \left\{ \begin{matrix} \hat{\underline{q}}_0 \\ \hat{\underline{v}}_0 \end{matrix} \right\} \tag{29}$$

It is noted that the state-space form, in Eq. (24) and (25), can be solved with any number of first-order ordinary differential equations (ODE) integrators for first-order-in-time systems.

III.   Content of Full Paper

In the full paper, we will provide more details of the theoretical derivations along with various numerical examples. Different time integration schemes, including explicit and implicit methods, will be implemented and examined in the numerical studies. Moreover, we will briefly introduce the format of the current code, which is in accordance to the FAST modularization framework. Finally, BeamDyn will be validated against experimental data for a realistic wind turbine blade.

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