Conversion of Blade Beam Properties

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Overview

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- Stiffness Matrix
  - Extension and Bending terms
  - Shear and Torsion terms
- Mass Matrix
- Formulae for Neutral Axis, Shear Center, Structural and Mass twists
- Discussion points

Motivation

- Loads simulations are to be performed using both FAST v8 and Bladed at various phases of blade design. Fast v8 requires a generic 6x6 matrix format for beam properties. Whereas Bladed requires conventional simple stiffness terms. To be consistent in providing the same input to both these tools and be accurate in design iterations, algorithms are required that can seamlessly and accurately convert the blade beam properties from one format to the other.

Objectives

- An algorithm must be developed to convert the beam properties either ways i.e. in both FAST v8 to Bladed format and vice versa.
- Also this algorithm must be verified and validated for no lost of information (i.e. no errors) in conversion from one format to the other

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Understanding Blade Stiffness

Generic Stiffness

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} x & x & x & x & x & x \\ & x & x & x & x & x \\ & & x & x & x & x \\ & & & x & x & x \\ & sym & & & x & x \\ & & & & & x \end{bmatrix} \begin{Bmatrix} \gamma_x \\ \gamma_y \\ \varepsilon_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix} \begin{matrix} shear\_flap \\ shear\_edge \\ extension \\ bending\_edge \\ bending\_flap \\ torsion \end{matrix}$$

Generic Stiffness Limited to Orthotropic Layups i.e. NO Off-axis plies

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} \color{red}{x} & \color{red}{x} & 0 & 0 & 0 & \color{red}{x} \\ & \color{red}{x} & 0 & 0 & 0 & \color{red}{x} \\ & & \color{blue}{x} & \color{blue}{x} & \color{blue}{x} & 0 \\ & & & \color{blue}{x} & \color{blue}{x} & 0 \\ & sym & & & \color{blue}{x} & 0 \\ & & & & & \color{red}{x} \end{bmatrix} \begin{Bmatrix} \gamma_x \\ \gamma_y \\ \varepsilon_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix} \begin{matrix} \color{red}{shear\_flap} \\ \color{red}{shear\_edge} \\ \color{blue}{extension} \\ \color{blue}{bending\_edge} \\ \color{blue}{bending\_flap} \\ \color{red}{torsion} \end{matrix}$$

About a generic reference point and coordinate system

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Conversion of Extension & Bending Terms

Stiffness about Neutral Axis and in Principal Bending Directions

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} x & x & 0 & 0 & 0 & x \\ & x & 0 & 0 & 0 & x \\ & & EA & 0 & 0 & 0 \\ & & & EIxx & 0 & 0 \\ & sym & & & EIyy & 0 \\ & & & & & x \end{bmatrix} \begin{Bmatrix} \gamma_x \\ \gamma_y \\ \varepsilon_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix} \begin{matrix} shear\_flap \\ shear\_edge \\ extension \\ bending\_edge \\ bending\_flap \\ torsion \end{matrix}$$

Conventional to 6x6

1. Fill diagonal terms with EA, EIxx, EIyy as shown above (this represents stiffness about neutral axis and in Principal bending directions)
2. Rotate back to global frame
3. Translate from neutral axis to reference point

6x6 to Conventional

1. Reverse the steps above

Conversion of Shear & Torsion Terms

Stiffness about Shear Center and in Principal Shear Directions

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} k_x GA & 0 & 0 & 0 & 0 & 0 \\ & k_y GA & 0 & 0 & 0 & 0 \\ & & x & x & x & 0 \\ & & & x & x & 0 \\ & sym & & & x & 0 \\ & & & & & GJ \end{bmatrix} \begin{Bmatrix} \gamma_x \\ \gamma_y \\ \varepsilon_z \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix} \begin{array}{l} shear\_flap \\ shear\_edge \\ extension \\ bending\_edge \\ bending\_flap \\ torsion \end{array}$$

Conventional to 6x6

1. Fill diagonal terms with kxGA, kyGA and GJ as shown above (this represents stiffness about shear center and in Principal shear directions)
2. Rotate back to global frame***
3. Translate from shear center to reference point

6x6 to Conventional

1. Reverse the steps above

*** Most often this rotation is ignored as an approximation which in other words means ignoring coupling between two shear terms

Understanding Mass Matrix

Generic Mass Matrix

$$\mathbf{M}_s = \begin{bmatrix} m & 0 & 0 & 0 & 0 & -my_m \\ 0 & m & 0 & 0 & 0 & mx_m \\ 0 & 0 & m & my_m & -mx_m & 0 \\ 0 & 0 & my_m & I_{xx} & -I_{xy} & 0 \\ 0 & 0 & -mx_m & -I_{xy} & I_{yy} & 0 \\ -my_m & mx_m & 0 & 0 & 0 & I_{xx}+I_{yy} \end{bmatrix}$$

- m = mass per unit length
- xm, ym = center of mass
- Ixx,Iyy,Ixy = mass moments of inertia

Stiffness about Center of Mass and in Principal Inertia Directions

$$M_s = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ & m & 0 & 0 & 0 & 0 \\ & & m & 0 & 0 & 0 \\ & & & Ixx & 0 & 0 \\ & sym & & & Iyy & 0 \\ & & & & & Ip \end{bmatrix}$$

Conventional to 6x6

1. Fill diagonal terms with m, Ixx, Iyy and Ip as shown above (this represents mass matrix about center of mass and in Principal inertia directions)
2. Rotate back to global frame***
3. Translate from shear center to reference point

6x6 to Conventional

1. Reverse the steps above

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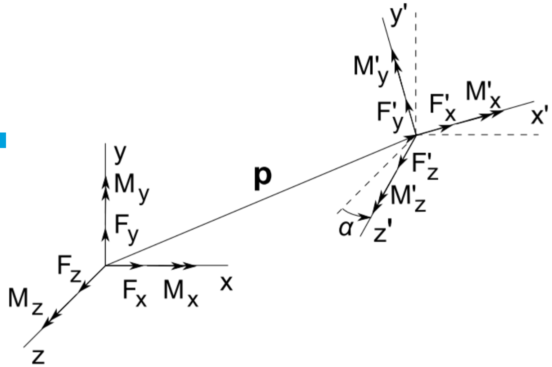
Transformations



- Translation from one reference point to other at P = [px,py,pz] from the first

$$\mathbf{T}_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -p_y & 1 & 0 & 0 \\ 0 & 0 & p_x & 0 & 1 & 0 \\ p_y & -p_x & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{M}' = \mathbf{T}_t \mathbf{M} \mathbf{T}_t^T$$

- Rotation from one reference frame to the other at α angle from the first

$$\mathbf{T}_r = \begin{bmatrix} cos(\alpha) & sin(\alpha) & 0 & 0 & 0 & 0 \\ -sin(\alpha) & cos(\alpha) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cos(\alpha) & sin(\alpha) & 0 \\ 0 & 0 & 0 & -sin(\alpha) & cos(\alpha) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{M}' = \mathbf{T}_r \mathbf{M} \mathbf{T}_r^T$$

 ***Note**: Above formulae are applicable for both Stiffness and Mass matrices*

Neutral Axis, Shear Center, Structure Twist, Mass Twist

If [K] is the stiffness matrix and [M] is the mass matrix (not applicable for Off-axis layups):

Neutral Axis: $x_{na} = \frac{-K_{35}}{K_{33}}$ $\quad$ $y_{na} = \frac{K_{34}}{K_{33}}$

Shear Center: $x_{sc} = \frac{K_{11}K_{26} - K_{12}K_{16}}{K_{11}K_{22} - {K_{12}}^2}$ $\quad$ $y_{sc} = \frac{K_{12}K_{26} - K_{16}K_{22}}{K_{11}K_{22} - {K_{12}}^2}$

Structure Twist: $\theta_p = atan\left(\frac{-\alpha \pm \sqrt{\alpha^2 + 4}}{2}\right)$ where $\alpha = \frac{K_{44} - K_{55}}{K_{45}}$

Mass Twist: $\theta_m = atan\left(\frac{-\alpha \pm \sqrt{\alpha^2 + 4}}{2}\right)$ where $\alpha = \frac{M_{44} - M_{55}}{M_{45}}$

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Discussion points

- Passing geometry information to compute Bladed parameters x',y'
- Element length parameter in shear and torsion terms???
- Recommended validation:
  - Use FASTv8 and Bladed to perform comparisons for the following
    - Blade mass
    - Deflection
      - Static – single point or multi-point pulls
      - Dynamic – constant rpm w/o wind?
    - Modal (natural frequencies) – static and steady state