Homework by Chenye Jin. Written in JupyterLab & $\LaTeX$.

给定一组数据点 $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$，拉格朗日插值的目标是找到一个 n 次多项式 $P(x)$，使得它经过所有这些数据点，即 $P(x_i) = y_i (i = 0, 1, \ldots, n)$。

Given a set of data points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$, the goal of Lagrange interpolation is to find an n th degree polynomial $P(x)$ such that it passes through all these data points, i.e., $P(x_i) = y_i (i = 0, 1, \ldots, n)$.

这个n次多项式$P(x)$就被表示为$W(x)$，其解析表达式为：

This n th degree polynomial $P(x)$ is represented as $W(x)$, whose analytical expression is:

$$W(x) = \Sigma(i = 0 -> n) y_i * L_i(x)$$

其中$L_i(x)$是拉格朗日基底多项式，定义为：

Where $L_i(x)$ is the Lagrange basis polynomial, defined as:

$$L_i(x) = \Pi(j = 0 -> n, j! = i)(x - x_j)/(x_i - x_j)$$

The following is my code implementation: （以下是我的代码实现：）

In [ ]:
```python
import numpy as np

def lagrange_interpolation(x, points):
    n = len(points) # n is the length of points tuples
    result = 0.0 # result will be the final W(x) result and we sum up on it

    # you can see W(x) is $Σ(i=0->n) y[i] * L[i](x)$ so the outside loop is calculating that summation
    # terms is the y[i] * L[i](x) inside the summation

    for i in range(n):
        term = points[i][1] # terms initialize with y[i] first.
        for j in range(n): # and inside the loop is *L[i](x) part
            if i != j: # also the contraints in formula.
                # you know anything/(points[i][0]-points[i==j][0]) makes divide 0. which is not allowed in both pro
```

```
                term *= (x - points[j][0]) / (points[i][0] - points[j][0])
                # every time upline executed, term just multiply L[j](x)
        # so when code comes here, term just = y[i] * L[1](x) * L[2](x) * ... * L[n](x)
        result += term
        # and then we sum up the term onto the final result
    return result

# format: (x,y) tuples in points list
example_points = [(1, 3), (2, 5), (3, 7)]

# so we define langrange interpolation upper here

# and we define a function in order to combine xy arrays easily
def combine_xy(x, y):
    if len(x) != len(y):
        raise ValueError("Input lists must have the same length.")

    points = []
    for i in range(len(x)):
        points.append((x[i], y[i]))

    return points
```

In [ ]:

```
import numpy as np
import pandas as pd

# homework functions
f1 = lambda x: np.abs(x)
f2 = lambda x: np.exp(np.abs(x))

# define interpolation nodes and function vals
X_a = [-5.0, 0.0, 5.0]
Yf1_a = [f1(x) for x in X_a]
Yf2_a = [f2(x) for x in X_a]

X_b = [-10.0, 0.0, 10.0]
Yf1_b = [f1(x) for x in X_b]
Yf2_b = [f2(x) for x in X_b]

X_c = [-5.0, -2.0, 0.0, 2.0, 5.0]
```

```
Yf1_c = [f1(x) for x in X_c]
Yf2_c = [f2(x) for x in X_c]

X_d = [-9.0, -5.0, 0.0, 5.0, 9.0]
Yf1_d = [f1(x) for x in X_d]
Yf2_d = [f2(x) for x in X_d]

X_e = [-10.0, -9.0, -5.0, -2.0, -1.0, 0.0, 1.0, 2.0, 5.0, 9.0, 10.0]
Yf1_e = [f1(x) for x in X_e]
Yf2_e = [f2(x) for x in X_e]

# create data frames
df_a = pd.DataFrame({"X": X_a, "f(x) = abs(x)": Yf1_a, "f(x) = exp(abs(x))": Yf2_a})
df_b = pd.DataFrame({"X": X_b, "f(x) = abs(x)": Yf1_b, "f(x) = exp(abs(x))": Yf2_b})
df_c = pd.DataFrame({"X": X_c, "f(x) = abs(x)": Yf1_c, "f(x) = exp(abs(x))": Yf2_c})
df_d = pd.DataFrame({"X": X_d, "f(x) = abs(x)": Yf1_d, "f(x) = exp(abs(x))": Yf2_d})
df_e = pd.DataFrame({"X": X_e, "f(x) = abs(x)": Yf1_e, "f(x) = exp(abs(x))": Yf2_e})

# display them all
print("Interpolation Nodes and Function Values:")
display(df_a)
display(df_b)
display(df_c)
display(df_d)
display(df_e)
```

```
Interpolation Nodes and Function Values:
```

	X	f(x) = abs(x)	f(x) = exp(abs(x))
0	-5.0	5.0	148.413159
1	0.0	0.0	1.000000
2	5.0	5.0	148.413159

	X	f(x) = abs(x)	f(x) = exp(abs(x))
0	-10.0	10.0	22026.465795
1	0.0	0.0	1.000000
2	10.0	10.0	22026.465795

	X	f(x) = abs(x)	f(x) = exp(abs(x))
0	-5.0	5.0	148.413159
1	-2.0	2.0	7.389056
2	0.0	0.0	1.000000
3	2.0	2.0	7.389056
4	5.0	5.0	148.413159

	X	f(x) = abs(x)	f(x) = exp(abs(x))
0	-9.0	9.0	8103.083928
1	-5.0	5.0	148.413159
2	0.0	0.0	1.000000
3	5.0	5.0	148.413159
4	9.0	9.0	8103.083928

	X	f(x) = abs(x)	f(x) = exp(abs(x))
0	-10.0	10.0	22026.465795
1	-9.0	9.0	8103.083928
2	-5.0	5.0	148.413159
3	-2.0	2.0	7.389056
4	-1.0	1.0	2.718282
5	0.0	0.0	1.000000
6	1.0	1.0	2.718282
7	2.0	2.0	7.389056
8	5.0	5.0	148.413159
9	9.0	9.0	8103.083928
10	10.0	10.0	22026.465795

In [ ]:
```
eval_x_values = np.arange(-10, 11)
# define the x values as the homework told [-10, -9 ... 9, 10]
print(f"Xvalues:\n{eval_x_values}")
```

```
Xvalues:
[-10  -9  -8  -7  -6  -5  -4  -3  -2  -1   0   1   2   3   4   5   6   7
   8   9  10]
```

我使用每对 $(X_{a,b,c,d,e}, Y_{f1,f2})$ 值得到了10个插值函数，并使用这些函数来计算 $X = [-10, -9, \ldots, 9, 10]$ 的 $W(X)$ 值。以下是我的代码实现：

I get interpolation functions with each $(X_{a,b,c,d,e}, Y_{f1,f2})$ values. and use those 10 functions to obtain the $X = [-10, -9 \ldots 9, 10]$'s $W(X)$ values. My code implementation is here:

In [ ]:
```
# now calculate W(x) the assignment requested
```

```
# Evaluation points
x = np.arange(-10, 11)

# Combine interpolation nodes and function values into tuples
points_f1_a = combine_xy(X_a, Yf1_a)
points_f2_a = combine_xy(X_a, Yf2_a)

points_f1_b = combine_xy(X_b, Yf1_b)
points_f2_b = combine_xy(X_b, Yf2_b)

points_f1_c = combine_xy(X_c, Yf1_c)
points_f2_c = combine_xy(X_c, Yf2_c)

points_f1_d = combine_xy(X_d, Yf1_d)
points_f2_d = combine_xy(X_d, Yf2_d)

points_f1_e = combine_xy(X_e, Yf1_e)
points_f2_e = combine_xy(X_e, Yf2_e)

# Compute the interpolation polynomials for each set of data
W_f1_a = [lagrange_interpolation(x_val, points_f1_a) for x_val in x]
W_f2_a = [lagrange_interpolation(x_val, points_f2_a) for x_val in x]

W_f1_b = [lagrange_interpolation(x_val, points_f1_b) for x_val in x]
W_f2_b = [lagrange_interpolation(x_val, points_f2_b) for x_val in x]

W_f1_c = [lagrange_interpolation(x_val, points_f1_c) for x_val in x]
W_f2_c = [lagrange_interpolation(x_val, points_f2_c) for x_val in x]

W_f1_d = [lagrange_interpolation(x_val, points_f1_d) for x_val in x]
W_f2_d = [lagrange_interpolation(x_val, points_f2_d) for x_val in x]

W_f1_e = [lagrange_interpolation(x_val, points_f1_e) for x_val in x]
W_f2_e = [lagrange_interpolation(x_val, points_f2_e) for x_val in x]

# Create a DataFrame to display the results
data1 = {
'x': x,
'W(x): f(x) = abs(x), X_a': W_f1_a,
'W(x): f(x) = abs(x), X_b': W_f1_b,
```

```
'W(x): f(x) = abs(x), X_c': W_f1_c,
'W(x): f(x) = abs(x), X_d': W_f1_d,
'W(x): f(x) = abs(x), X_e': W_f1_e
}
data2 = {
'x': x,
'W(x): f(x) = exp(abs(x)), X_a': W_f2_a,
'W(x): f(x) = exp(abs(x)), X_b': W_f2_b,
'W(x): f(x) = exp(abs(x)), X_c': W_f2_c,
'W(x): f(x) = exp(abs(x)), X_d': W_f2_d,
'W(x): f(x) = exp(abs(x)), X_e': W_f2_e
}

df1 = pd.DataFrame(data1)
df2 = pd.DataFrame(data2)
display(df1)
display(df2)
```

	x	W(x): f(x) = abs(x), X_a	W(x): f(x) = abs(x), X_b	W(x): f(x) = abs(x), X_c	W(x): f(x) = abs(x), X_d	W(x): f(x) = abs(x), X_e
0	-10	20.0	10.0	-87.142857	8.095238	10.000000
1	-9	16.2	8.1	-48.600000	9.000000	9.000000
2	-8	12.8	6.4	-22.857143	8.838095	75.964894
3	-7	9.8	4.9	-7.000000	7.933333	70.461435
4	-6	7.2	3.6	1.542857	6.571429	32.448120
5	-5	5.0	2.5	5.000000	5.000000	5.000000
6	-4	3.2	1.6	5.257143	3.428571	-2.421873
7	-3	1.8	0.9	3.857143	2.028571	0.048202
8	-2	0.8	0.4	2.000000	0.933333	2.000000
9	-1	0.2	0.1	0.542857	0.238095	1.000000
10	0	0.0	0.0	0.000000	0.000000	0.000000
11	1	0.2	0.1	0.542857	0.238095	1.000000
12	2	0.8	0.4	2.000000	0.933333	2.000000
13	3	1.8	0.9	3.857143	2.028571	0.048202
14	4	3.2	1.6	5.257143	3.428571	-2.421873
15	5	5.0	2.5	5.000000	5.000000	5.000000
16	6	7.2	3.6	1.542857	6.571429	32.448120
17	7	9.8	4.9	-7.000000	7.933333	70.461435
18	8	12.8	6.4	-22.857143	8.838095	75.964894
19	9	16.2	8.1	-48.600000	9.000000	9.000000

	x	W(x): f(x) = abs(x), X_a	W(x): f(x) = abs(x), X_b	W(x): f(x) = abs(x), X_c	W(x): f(x) = abs(x), X_d	W(x): f(x) = abs(x), X_e
20	10	20.0	10.0	-87.142857	8.095238	10.000000

	x	W(x): f(x) = exp(abs(x)), X_a	W(x): f(x) = exp(abs(x)), X_b	W(x): f(x) = exp(abs(x)), X_c	W(x): f(x) = exp(abs(x)), X_d	W(x): f(x) = exp(abs(x)), X_e
0	-10	590.652636	22026.465795	2126.103472	13197.242074	22026.465795
1	-9	478.618635	17841.627294	1407.259301	8103.083928	8103.083928
2	-8	378.377687	14097.298109	889.375725	4573.850652	2934.593021
3	-7	289.929792	10793.478239	530.688483	2266.643016	1081.544334
4	-6	213.274949	7930.167686	294.346753	878.902871	406.136598
5	-5	148.413159	5507.366449	148.413159	148.413159	148.413159
6	-4	95.344422	3525.074527	65.863766	-146.702095	50.995349
7	-3	54.068737	1983.291922	24.588081	-187.977780	17.988978
8	-2	24.586105	882.018632	7.389056	-116.607696	7.389056
9	-1	6.896526	221.254658	1.983084	-33.444560	2.718282
10	0	1.000000	1.000000	1.000000	1.000000	1.000000
11	1	6.896526	221.254658	1.983084	-33.444560	2.718282
12	2	24.586105	882.018632	7.389056	-116.607696	7.389056
13	3	54.068737	1983.291922	24.588081	-187.977780	17.988978
14	4	95.344422	3525.074527	65.863766	-146.702095	50.995349
15	5	148.413159	5507.366449	148.413159	148.413159	148.413159
16	6	213.274949	7930.167686	294.346753	878.902871	406.136598
17	7	289.929792	10793.478239	530.688483	2266.643016	1081.544334
18	8	378.377687	14097.298109	889.375725	4573.850652	2934.593021

	x	W(x): f(x) = exp(abs(x)), X_a	W(x): f(x) = exp(abs(x)), X_b	W(x): f(x) = exp(abs(x)), X_c	W(x): f(x) = exp(abs(x)), X_d	W(x): f(x) = exp(abs(x)), X_e
19	9	478.618635	17841.627294	1407.259301	8103.083928	8103.083928
20	10	590.652636	22026.465795	2126.103472	13197.242074	22026.465795

In [ ]:
```
# try with scipy one
# results seems like same with mine algorithm
import numpy as np
import pandas as pd
from scipy.interpolate import lagrange

x = np.arange(-10, 11)

points_f1_a = np.column_stack((X_a, Yf1_a))
points_f2_a = np.column_stack((X_a, Yf2_a))

points_f1_b = np.column_stack((X_b, Yf1_b))
points_f2_b = np.column_stack((X_b, Yf2_b))

points_f1_c = np.column_stack((X_c, Yf1_c))
points_f2_c = np.column_stack((X_c, Yf2_c))

points_f1_d = np.column_stack((X_d, Yf1_d))
points_f2_d = np.column_stack((X_d, Yf2_d))

points_f1_e = np.column_stack((X_e, Yf1_e))
points_f2_e = np.column_stack((X_e, Yf2_e))

W_f1_a = [lagrange(points_f1_a[:, 0], points_f1_a[:, 1])(xi) for xi in x]
W_f2_a = [lagrange(points_f2_a[:, 0], points_f2_a[:, 1])(xi) for xi in x]

W_f1_b = [lagrange(points_f1_b[:, 0], points_f1_b[:, 1])(xi) for xi in x]
W_f2_b = [lagrange(points_f2_b[:, 0], points_f2_b[:, 1])(xi) for xi in x]

W_f1_c = [lagrange(points_f1_c[:, 0], points_f1_c[:, 1])(xi) for xi in x]
W_f2_c = [lagrange(points_f2_c[:, 0], points_f2_c[:, 1])(xi) for xi in x]
```

```
W_f1_d = [lagrange(points_f1_d[:, 0], points_f1_d[:, 1])(xi) for xi in x]
W_f2_d = [lagrange(points_f2_d[:, 0], points_f2_d[:, 1])(xi) for xi in x]

W_f1_e = [lagrange(points_f1_e[:, 0], points_f1_e[:, 1])(xi) for xi in x]
W_f2_e = [lagrange(points_f2_e[:, 0], points_f2_e[:, 1])(xi) for xi in x]

data1 = {
    'x': x,
    'W(x): f(x) = abs(x), X_a': W_f1_a,
    'W(x): f(x) = abs(x), X_b': W_f1_b,
    'W(x): f(x) = abs(x), X_c': W_f1_c,
    'W(x): f(x) = abs(x), X_d': W_f1_d,
    'W(x): f(x) = abs(x), X_e': W_f1_e
}
data2 = {
    'x': x,
    'W(x): f(x) = exp(abs(x)), X_a': W_f2_a,
    'W(x): f(x) = exp(abs(x)), X_b': W_f2_b,
    'W(x): f(x) = exp(abs(x)), X_c': W_f2_c,
    'W(x): f(x) = exp(abs(x)), X_d': W_f2_d,
    'W(x): f(x) = exp(abs(x)), X_e': W_f2_e
}

df1 = pd.DataFrame(data1)
df2 = pd.DataFrame(data2)
display(df1)
display(df2)
```

	x	W(x): f(x) = abs(x), X_a	W(x): f(x) = abs(x), X_b	W(x): f(x) = abs(x), X_c	W(x): f(x) = abs(x), X_d	W(x): f(x) = abs(x), X_e
0	-10	20.0	10.0	-87.142857	8.095238	10.000000
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20	10	20.0	10.0	-87.142857	8.095238	10.000000

	x	W(x): f(x) = exp(abs(x)), X_a	W(x): f(x) = exp(abs(x)), X_b	W(x): f(x) = exp(abs(x)), X_c	W(x): f(x) = exp(abs(x)), X_d	W(x): f(x) = exp(abs(x)), X_e
0	-10	590.652636	22026.465795	2126.103472	13197.242074	22026.465795
1	-9	478.618635	17841.627294	1407.259301	8103.083928	8103.083928
2	-8	378.377687	14097.298109	889.375725	4573.850652	2934.593021
3	-7	289.929792	10793.478239	530.688483	2266.643016	1081.544334
4	-6	213.274949	7930.167686	294.346753	878.902871	406.136598
5	-5	148.413159	5507.366449	148.413159	148.413159	148.413159
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15	5	148.413159	5507.366449	148.413159	148.413159	148.413159
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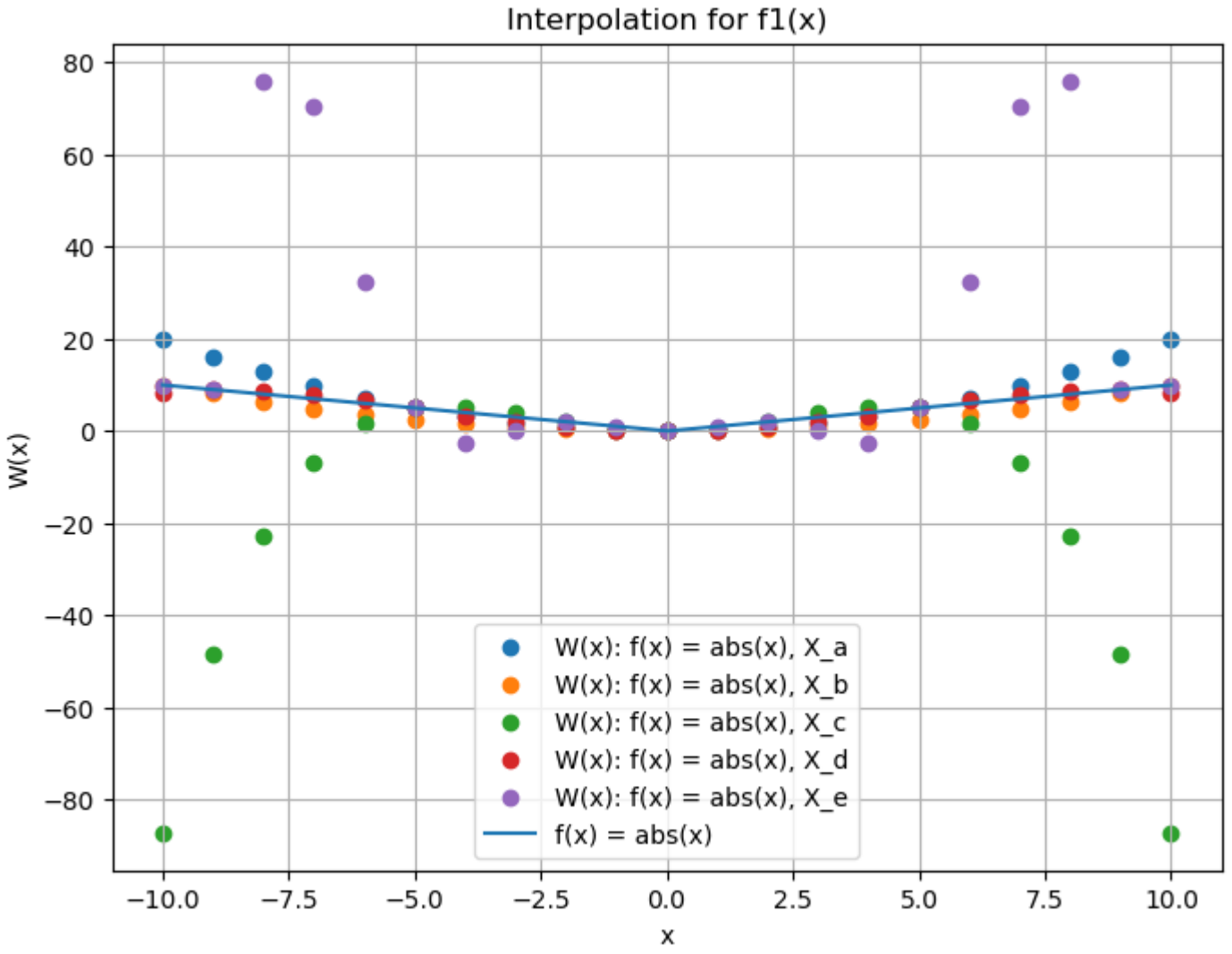
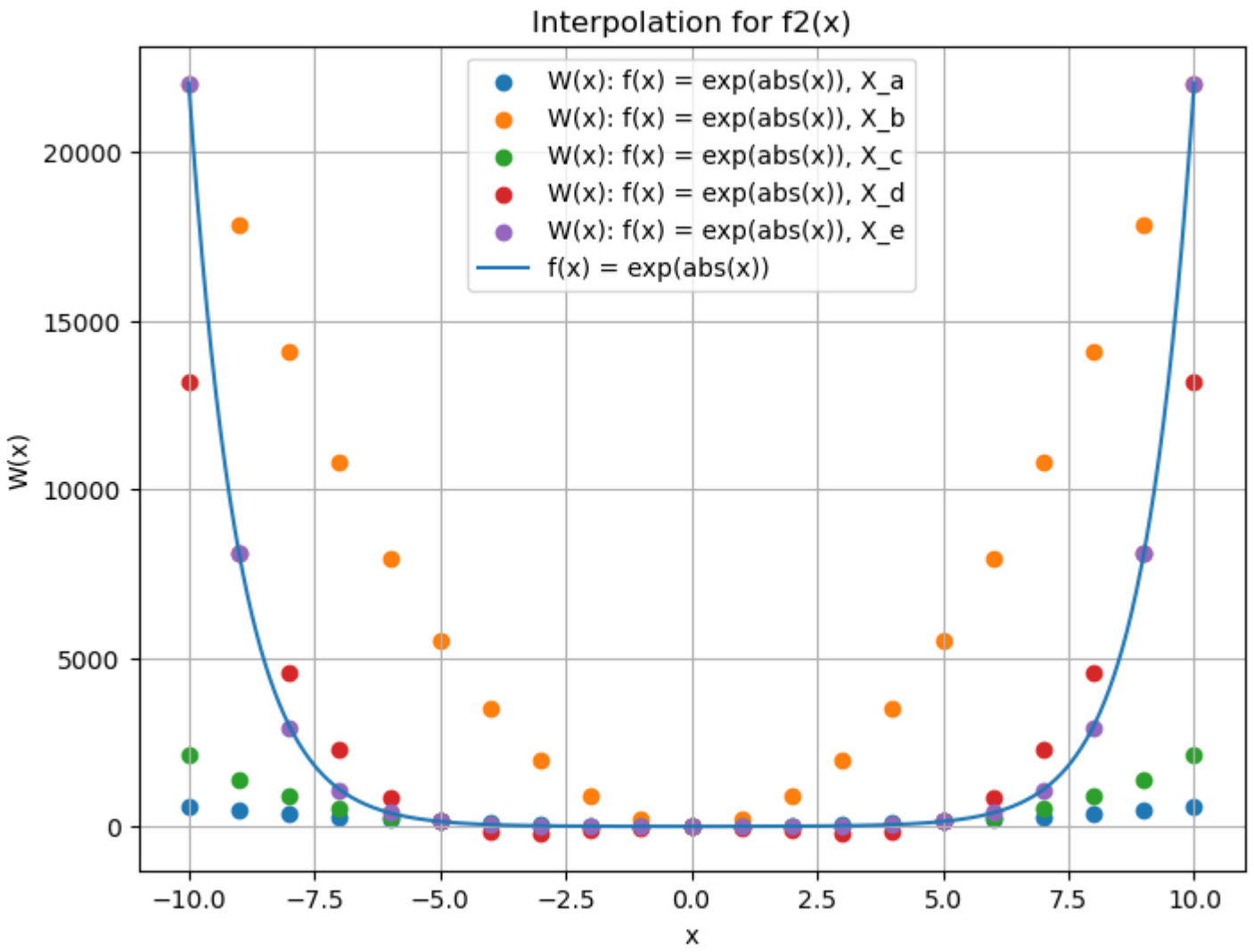
19	9	478.618635	17841.627294	1407.259301	8103.083928	8103.083928
20	10	590.652636	22026.465795	2126.103472	13197.242074	22026.465795

In [ ]:

```
import matplotlib.pyplot as plt

plt.figure(figsize=(8, 6))
plt.title('Interpolation for f1(x)')
plt.xlabel('x')
plt.ylabel('W(x)')

for key, value in data1.items():
    if key != 'x':
        label = "W(x): "+ key.split(': ')[1]
        plt.scatter(data1['x'], value, label=label)

x_for_orignal = np.arange(-10,10.01,0.01)

plt.plot(x_for_orignal, [f1(x) for x in x_for_orignal], label = "f(x) = abs(x)")

plt.legend()
plt.grid(True)
plt.show()

plt.figure(figsize=(8, 6))
plt.title('Interpolation for f2(x)')
plt.xlabel('x')
plt.ylabel('W(x)')

for key, value in data2.items():
    if key != 'x':
        label = "W(x): "+ key.split(': ')[1]
        plt.scatter(data2['x'], value, label=label)

plt.plot(x_for_orignal, [f2(x) for x in x_for_orignal], label = "f(x) = exp(abs(x))")
```

```
plt.legend()
plt.grid(True)
plt.show()
```





In [ ]:
```python
# batchly get the f1, f2 values with numpy

f1_a = np.abs(x)
f2_a = np.exp(np.abs(x))

f1_b = np.abs(x)
f2_b = np.exp(np.abs(x))
```

```
f1_c = np.abs(x)
f2_c = np.exp(np.abs(x))

f1_d = np.abs(x)
f2_d = np.exp(np.abs(x))

f1_e = np.abs(x)
f2_e = np.exp(np.abs(x))

# calculate relative error and absolute errors

relative_error_f1_a = np.abs((f1_a - W_f1_a) / f1_a)
absolute_error_f1_a = np.abs(f1_a - W_f1_a)

relative_error_f2_a = np.abs((f2_a - W_f2_a) / f2_a)
absolute_error_f2_a = np.abs(f2_a - W_f2_a)

relative_error_f1_b = np.abs((f1_b - W_f1_b) / f1_b)
absolute_error_f1_b = np.abs(f1_b - W_f1_b)

relative_error_f2_b = np.abs((f2_b - W_f2_b) / f2_b)
absolute_error_f2_b = np.abs(f2_b - W_f2_b)

relative_error_f1_c = np.abs((f1_c - W_f1_c) / f1_c)
absolute_error_f1_c = np.abs(f1_c - W_f1_c)

relative_error_f2_c = np.abs((f2_c - W_f2_c) / f2_c)
absolute_error_f2_c = np.abs(f2_c - W_f2_c)

relative_error_f1_d = np.abs((f1_d - W_f1_d) / f1_d)
absolute_error_f1_d = np.abs(f1_d - W_f1_d)

relative_error_f2_d = np.abs((f2_d - W_f2_d) / f2_d)
absolute_error_f2_d = np.abs(f2_d - W_f2_d)

relative_error_f1_e = np.abs((f1_e - W_f1_e) / f1_e)
absolute_error_f1_e = np.abs(f1_e - W_f1_e)

relative_error_f2_e = np.abs((f2_e - W_f2_e) / f2_e)
absolute_error_f2_e = np.abs(f2_e - W_f2_e)
```

```
/tmp/ipykernel_14039/528344189.py:20: RuntimeWarning: invalid value encountered in divide
  relative_error_f1_a = np.abs((f1_a - W_f1_a) / f1_a)
/tmp/ipykernel_14039/528344189.py:26: RuntimeWarning: invalid value encountered in divide
  relative_error_f1_b = np.abs((f1_b - W_f1_b) / f1_b)
/tmp/ipykernel_14039/528344189.py:32: RuntimeWarning: invalid value encountered in divide
  relative_error_f1_c = np.abs((f1_c - W_f1_c) / f1_c)
/tmp/ipykernel_14039/528344189.py:38: RuntimeWarning: invalid value encountered in divide
  relative_error_f1_d = np.abs((f1_d - W_f1_d) / f1_d)
/tmp/ipykernel_14039/528344189.py:44: RuntimeWarning: invalid value encountered in divide
  relative_error_f1_e = np.abs((f1_e - W_f1_e) / f1_e)
```

In [ ]:

```python
import pandas as pd

function_names = ['f1_Xa', 'f2_Xa', 'f1_Xb', 'f2_Xb', 'f1_Xc', 'f2_Xc', 'f1_Xd', 'f2_Xd', 'f1_Xe', 'f2_Xe']
relative_errors = [relative_error_f1_a, relative_error_f2_a, relative_error_f1_b, relative_error_f2_b,
                   relative_error_f1_c, relative_error_f2_c, relative_error_f1_d, relative_error_f2_d,
                   relative_error_f1_e, relative_error_f2_e]
relative_errors = [np.nanmean(error) for error in relative_errors]
absolute_errors = [absolute_error_f1_a, absolute_error_f2_a, absolute_error_f1_b, absolute_error_f2_b,
                   absolute_error_f1_c, absolute_error_f2_c, absolute_error_f1_d, absolute_error_f2_d,
                   absolute_error_f1_e, absolute_error_f2_e]
absolute_errors = [np.nanmean(error) for error in absolute_errors]

# mean errors calculated here

data = {'Function': function_names, 'Mean Relative Error': relative_errors, 'Mean Absolute Error': absolute_errors}

# create a dataframe
df = pd.DataFrame(data)

# display the dataframe
display(df)
```

	Function	Mean Relative Error	Mean Absolute Error
0	f1_Xa	0.500000	2.857143
1	f2_Xa	0.980650	3119.601974
2	f1_Xb	0.450000	1.571429
3	f2_Xb	40.819824	4758.501854
4	f1_Xc	2.377143	19.678912
5	f2_Xc	0.373227	2798.028503
6	f1_Xd	0.228571	0.725624
7	f2_Xd	4.505829	1203.535569
8	f1_Xe	2.455895	15.928392
9	f2_Xe	0.019657	6.653424

Clearly, the best fit for the function $f1(x) = |x|$ is obtained by interpolating with Lagrange interpolation on the data set X_d.

The best fit for the function $f2(x) = \exp(|x|)$ is obtained by interpolating with Lagrange interpolation on the data set X_e.

结论：

显然， 由 X_d 拉格朗日插值的 $f1(x) = abs(x)$ 的插值函数拟合得最好。

由 X_e 拉格朗日插值的 $f2(x) = exp(abs(x))$ 的插值函数拟合得最好。