Solving Alphametics

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Contents

1 Problem Definition

We want to find a set of operations to solve an alphametic puzzle. An example of such a puzzle is this:

$$\begin{array}{cccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline = & M & O & N & E & Y \end{array}$$

Where

- $E, N, O, R, Y \in \mathbb{N}_0$ and $M, S \in \mathbb{N}$
- $E \neq N \neq O \neq R \neq Y \neq M \neq S \neq E$
- $$\begin{aligned} Y &= D + E && \pmod{10} \\ E &= N + R + \lfloor \frac{D+E}{10} \rfloor && \pmod{10} \\ N &= E + O + \lfloor \frac{N + R + \lfloor \frac{D+E}{10} \rfloor}{10} \rfloor && \pmod{10} \\ &\vdots \end{aligned}$$

We can introduce a term c_n for the overflow where $c \in \{0, 1\}$ in this case.

$$\begin{aligned} Y &= D + E && \pmod{10} \\ E &= N + R + c_1 && \pmod{10} \\ N &= E + O + c_2 && \pmod{10} \\ O &= S + M + c_3 && \pmod{10} \\ M &= c_4 && \pmod{10} \end{aligned}$$

c_{max} can be generalized as it is linked to the number of addends where $c_{max} = \max\{n_{addends} - 1, 9\}$

2 A Simpler Example

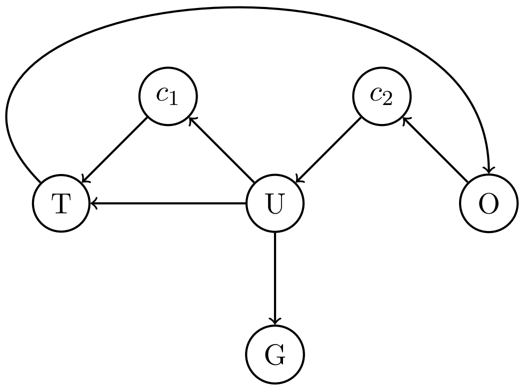
Let's experiment by solving a simple example

$$\begin{array}{cccc} & & \mathrm{T} & \mathrm{O} \\ + & & \mathrm{G} & \mathrm{O} \\ \hline = & \mathrm{O} & \mathrm{U} & \mathrm{T} \end{array}$$

This alphametic results in the following equations

$$T = O + O \pmod{10} \tag{1}$$
$$U = T + G + c_1 \pmod{10} \tag{2}$$
$$O = c_2 \pmod{10} \tag{3}$$

We can construct a graph like structure to see what variables depend on each other:



3 Solving the more complicated case

the alphametic from the introduction above gives us the following graph