Solving Alphametics

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1 Problem Definition

We want to find a set of operations to solve an alphametic puzzle. An example of

- Find $E, M, N, O, R, Y \in \mathbb{N}_0$ such that

$$\begin{array}{cccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline = & M & O & N & E & Y \end{array}$$

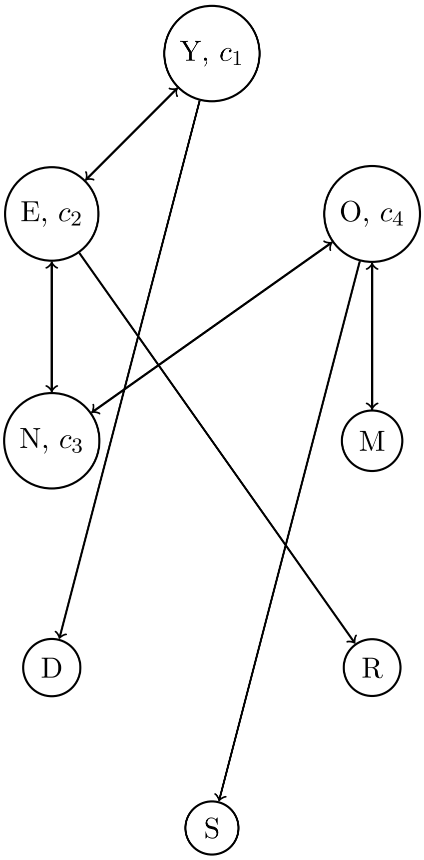
- where the rules of standard addition apply, this can be written as:

$$\begin{aligned} Y &= (D + E) && \pmod{10} \\ E &= \left(N + R + \lfloor\frac{D+E}{10}\rfloor\right) && \pmod{10} \\ N &= \left(E + O + \lfloor\frac{N + R + \lfloor\frac{D+E}{10}\rfloor}{10}\rfloor\right) && \pmod{10} \\ &\vdots \end{aligned}$$

- A term c_n can be introduced for the overflow where $c \in \{0, 1\}$.

$$
\begin{aligned}
Y &= D + E && \pmod{10} \\
E &= N + R + c_1 && \pmod{10} \\
N &= E + O + c_2 && \pmod{10} \\
O &= S + M + c_3 && \pmod{10} \\
M &= c_4 && \pmod{10}
\end{aligned}
$$

- this can be generalized the maximum for the overflow term is linked to the number of addends $c_{max} = \max\{n_{addends} - 1, 9\}$



2 A Simpler Example

Let's experiment by solving a simple example

$$
\begin{array}{cccc}
 & & \mathrm{T} & \mathrm{O} \\
+ & & \mathrm{G} & \mathrm{O} \\
\hline
= & \mathrm{O} & \mathrm{U} & \mathrm{T}
\end{array}
$$

gives us the following equations

$$
\begin{align}
T &= O + O \pmod{10} \tag{1}\\
U &= T + G + c_1 \pmod{10} \tag{2}\\
O &= c_2 \pmod{10} \tag{3}
\end{align}
$$