In [ ]:

```
# INTERSECTIONS AND UNION RULE
```

```
P[A ∪ B] = P[A] + P[B] - P[A ∩ B]     # Always True
```

In [ ]:

In [ ]:

In [ ]:

```
# CONDITIONAL PROBABILITY
```

```
P[A|B] = P[A ∩ B]/P[B]           # Always True
```

In [ ]:

In [ ]:

```
# A and B are independent events if
```

```
P[A|B] = P[A]
P[B|A] = P[B]
```

```
P[A ∩ B] = P[A] * P[B] if A and B are independent
P[A ∩ B ∩ C] = P[A] * P[B] * P[C]    if  A , B , C are indepenedent events
```

In [ ]:

In [ ]:

In [ ]:

```
# BAYES THEOREM
```

```
if E1 E2 E3  are mutually exhaustive events     E1 ∪ E2 ∪ E3 = S
E1 ∩ E2 = {}
E2 ∩ E3 = {}
E3 ∩ E1 = {}   And Mututally Exclusive events

P[E1|A] = p[A|E1] * p[E1] /                                # Always True
            P[A]

P[A] = P[A|E1]*P[E1] + P[A|E2]*P[E2] + P[A|E3]*P[E3]

P[E1|A] = (P[A|E1] * P[E1]) / (P[A|E1]*P[E1] + P[A|E2]*P[E2] + P[A|E3]*P[E3])
             # True only for Mutually Exclusive and Exhaustive evenets
```

In [ ]:

In [ ]:

```
∩
```

```
∪
```

In [ ]:

In [ ]:

In [ ]:

In [ ]:

Random Variable: A random variable is a variable that takes numerical values as a result of a random experiment or measurement ; associates a numerical value with each possible outcome.

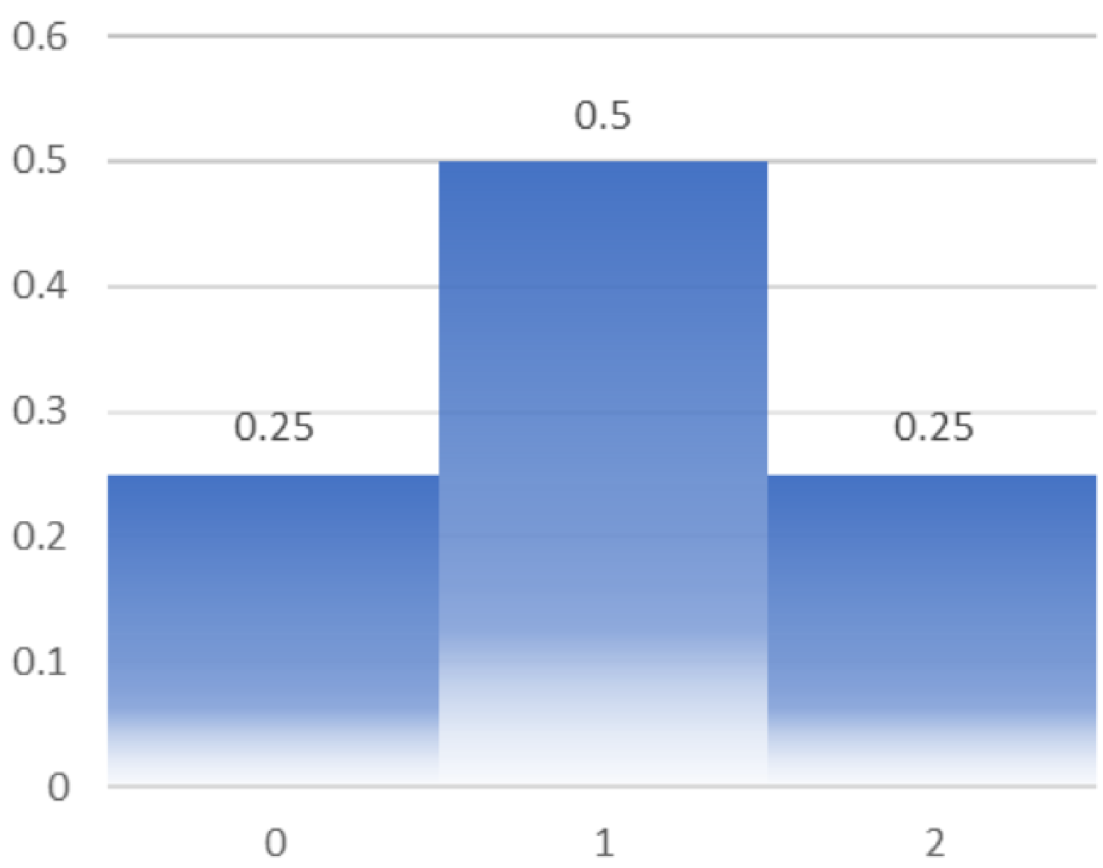
RVs must have numerical values

In [ ]:

```

```

```
Coin toss twice:

    Sample Space : S = {HH,HT,TH,TT}

X : no of heads in 2 tosses:

        HH 2
        HT 1
        TH 1
        TT 0

0 happened  1 times   probability P[TT] = 1/4
1 happened  2 times   probability P[HT,TH] = 2/4
2 happened  1 times   probability P[HH] = 1/4

```

```
#### Probability Mass Function
A probability mass function is a function that gives the probability that a discrete random variable is exactly equal to some value.
```




```
discrete RV
1. Constituting a seperate thing.
2. cosisting of unconnected distinct parts
3. Mathematics defined for a finite or countable set of values, not continuous .

```

```
that means , Random Variables must be Mutually Exclusive and Exhaustive
they cannot be overlaped

```

Bernoulli Random Variable :

```
The Bernoulli distribution, is the discrete probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability q=(1-p).
```

```
X = 0,1
P[1]  = p
P[0]  = 1-p

example :
in dice:

S = {1,2,3,4,5,6}  # all possible outcomes

if we define burnoulli RV

X = {
     0 , (odd events)       P[0] 1/2
     1 , (even events)      P[1] 1/2
```

```
    }

if
Y = {
    0, {1,2}         2/6    p
    1, {3,4,5,6}     4/6    1-p
    }


```

Let X be a Bernoulli random variable with parameter "p". What is the expectation E(X)

69 users have participated

A	1	9%
B ✓	p	30%
C	1-p	16%
D	p(1-p)	45%

In [ ]:

```

```

In [4]:

```
import math
```

Basic Counting Principle

```
## Basic Counting Principle

4 boxes are thre , and 10 balls !
How many ways we can put different balls into those 4 boxes . one in each box :



    _  _  _  _

    for first box, we have 10 choices of balls
        2nd box             9
        3rd bax             8
        4th box             7

    total ways : 10 * 9 * 8 *7    ("Permutations ")

      =  10 * 9 * 8 * 7 * ( 6 * 5 * 4 * 3 * 2 * 1)
        /   ( 6 * 5 * 4 * 3 * 2 * 1)

      = 10! / 6!
      = 10! / (10 - 4)!

 Permutation : General formula :

  nPr =     n!/
          (n-r)!


```

In [2]:

```
10 * 9 * 8 *7
```

Out[2]: 5040

In [5]:

```
math.perm(10,4)
```

Out[5]: 5040

In [ ]:

```

```

In [ ]:

```

```

In [ ]:

```

```

In [ ]:

In [ ]:

```
if we are tossing one coin 10 times :

 {HHHH...H , HHH...T , HHHHH...TH, .....}

    SUCH,
    total number of outcomes : 2**10 = 1024

  How many of 2^10 outcomes have  4 heads :

    we are intereseted in 4 Heads out of 1024 outcomes :
        choose 4 locations to place head

        10 * 9 * 8 * 7 (permutations)

        now lets say , there's one outcome 1,5,6,7 .
                                but there also will be 7,6,5,1.
                                (permutations)
             that is why:

    to get the combinations we have to divide the choices which are repeated in different order

        = 10 * 9 * 8 * 7 /
            4!

        = 10*9*8*7*6*5*4*3*2*1 /
               4! * (6*5*4*3*2*1)

        = 10! /
          4!*6!

        = 10!/
          4!(10-4!)


Combinations : nCr =    n!
                     / r!(n-r)!
```

In [6]:

```
math.comb(10,4)
```

Out[6]: 210

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

Binomial Random Variable :

```
Two pparameters  :
    > n : process consists of sequence of n trials
    > only two exclusive outcomes are : success and failure :

                P[sucess] = p
                P[failure] = 1-p

   all trials are independent, outcome of previous trials do not influence further trials.
```

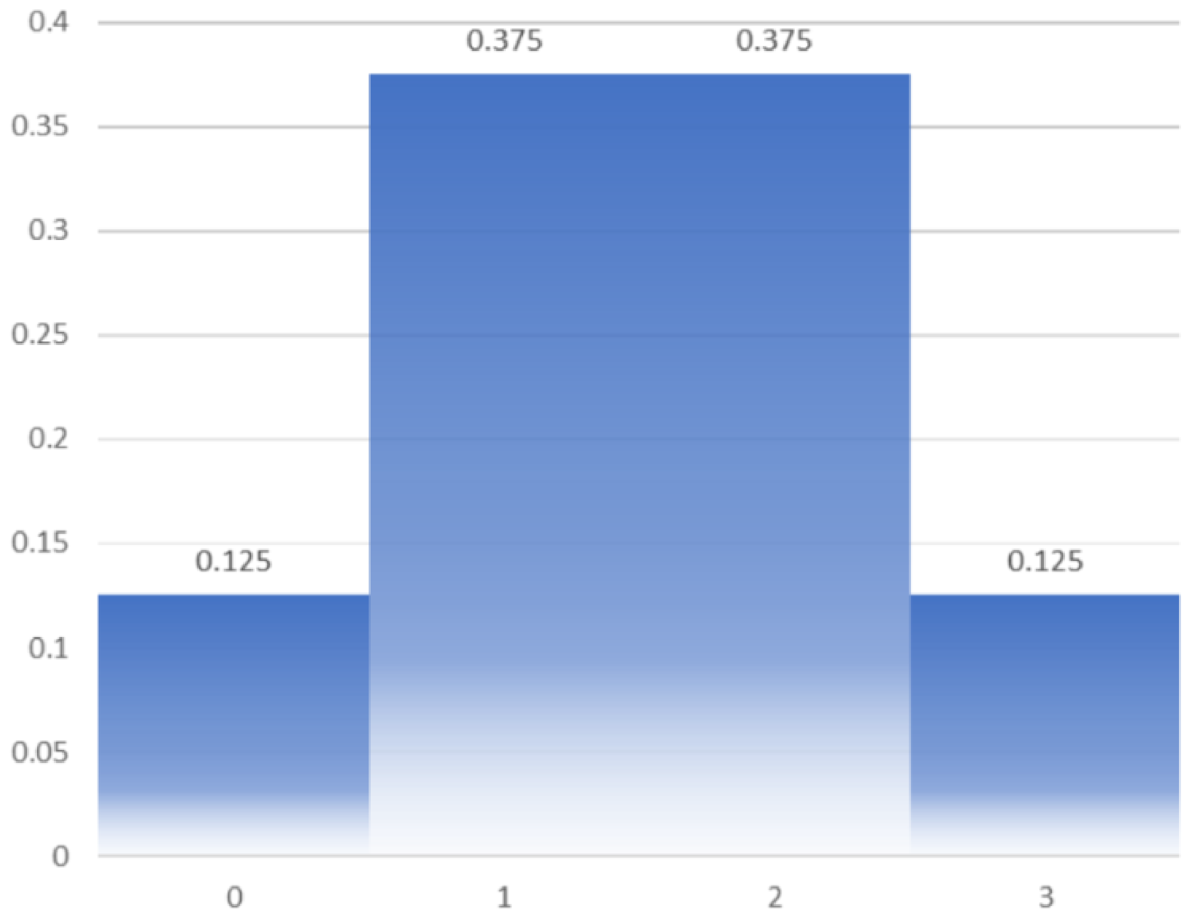
BINOMIAL EXPERIMENT

A binomial experiment has the following characteristics:

1. The process consists of a sequence of n trials.
2. Only two exclusive outcomes are possible in each trial. One outcome is called a “success” and the other a “failure.”*
3. The probability of a success denoted by p, does not change from trial to trial. The probability of failure is $1 - p$ and is also fixed from trial to trial.
4. The trials are independent; the outcome of previous trials to not influence future trials.

In [ ]:

```
for 3 trial coin toss :

n = 3

Probability of heads is p

S = {    # of heads     P[x]
    HHH : 3
    HHT : 2
    HTH : 2
    HTT : 1                                 P[TTT] = (1-p)^3
    THH : 2                                 P[HHH] = (p)^3
    THT : 1                                 P[HHT] = p^2 * (1-p)^1
    TTH : 1                                 P[TTH] = (1-p)^2 * p
    TTT : 0
    }


  so , the random variable will take values: X = {0,1,2,3}

  x  P[x]
  0  1/8     0  : 1 times  P[x = 0]  = 1 * (1-p)^3
  1  3/8     1  : 3 times  P[x = 1]  = 3 * p * ((1-p)^2)
  2  3/8     2  : 3 times  P[x = 2]  = 3 * (p^2) * (1-p)
  3  1/8     3  : 1 times  P[x = 3]  = 1 * p^3



   P[x = 1]   {HTT,THT,TTH}
               A    B   C     A or B or C

   P[A U B U C] = P[A] + P[B] + P[C]
                = p(1-p)^2 + p(1-p)^2 + p(1-p)^2
                = 3 * p(1-p)^2


    P[x = 2]   {HHT,HTH,THH}
                 A    B   C     A or B or C

   P[A U B U C] = P[A] + P[B] + P[C]
                = p^2 * (1-p)^1 + p^2 * (1-p)^1 + p^2 * (1-p)^1
                = 3 * p^2 * (1-p)^1

    P[x = 3]    {HHH}  (H and H and H)

             P[H] = p

             P[HHH] = p^3

    P[x = 0]
                TTT      (T and T and T)
                (1-p)^3
```




```
  x  P[x]
  0  1/8
  1  3/8
  2  3/8
  3  1/8
```

In [ ]:
```

```

In [ ]:
```

```

```
x : no of heads in n trials(n tosses)
p : probability of heads

P[x = k]            = nCk          * (p^k) * (1-p)^n-k
probability of      total number
number of heads     of outcomes
in n trials         with k heads
```

In [ ]:
```

```

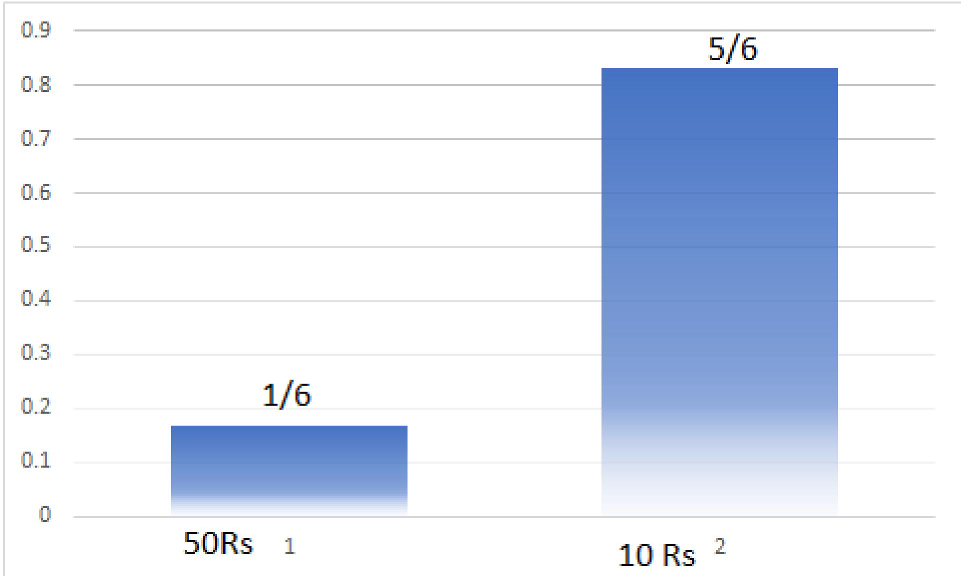
In [ ]:
```

```

Expected value :

weighted average

Example :

In [ ]:
```
# roll a dice: {1,2,3,4,5,6}
# everytime 6 comes up , u get 50rs
# else : 10 rs

RV :
    10 : {1,2,3,4,5}      5/6
    50 : {6}              1/6


```



In [ ]:

```

# We toss 1000 times :

# How much money do we expect to get :



```

In [8]:

```
1000*((5/6*10)+(1/6*50))
```

Out[8]: 16666.666666666664

In [ ]:

```

```

```
for example : if we get 320 times {6} out of 1000
                         and 680 times {1,2,3,4,5,}

     ( (320 * 50Rs) + (680 * 10Rs))/1000
        k1              k2

    total expected amount(average) to get per toss : K =     (k1 + k2)
                                                        ((k1*50) + (k2*10))/k
                          ((k1*50) + (k2*10))/ k1 + k2

                          (1/6 * 50)    + (5/6 * 10)       this is per toss
                         ((1/6 * 50)    + (5/6 * 10))   * 1000 toss



```

```
Expected value (Mean of a random vvariable )

E(X) = Σ(x*P(x))
     = 50*(1/6)  + 10*(5/6)
     = 16.666 Rs

for 1000 times  16666.67 Rs

```

In [11]:

```
(50*(1/6)  + 10*(5/6))*1000
```

Out[11]: 16666.666666666664

PROJECTED PROFITS $E(X)$

Below is the probability distribution for Terrific Taco's projected profits (in $million).

$$x = -1, 0, .5, 1, 1.5, 2$$

x	$P(x)$
-1	.08
0	.22
.5	.24
1	.31
1.5	.10
2	.05
Σ	1

What is the $E(x)$ or μ profit ($million) for Terrific Taco Company?

x	$P(x)$	$xP(x)$
-1	.08	$-1 \times .08 = -.08$
0	.22	$0 \times .22 = 0$
.5	.24	$.5 \times .24 = .12$
1	.31	$1 \times .31 = .31$
1.5	.10	$1.5 \times .15 = .225$
2	.05	$2 \times .05 = .10$
Σ	1	$.675$ or $\$675{,}000$

The expected value is simply the mean of a random variable; the average expected outcome. ***It does not have to be a value the discrete random variable can assume.***

$$E(X) = \mu = \Sigma x P(x)$$

- $E(X)$ is the expected value or mean of the outcomes x
- μ is the mean
- $\Sigma x P(x)$ is the sum of each random variable value x multiplied by its own probability $P(x)$

A WEIGHTED AVERAGE

Type *Markdown* and LaTeX: α^2

Let X be a RV taking values {1, 2, 3, 4, 5, 6} for a dice thrown. What is the expectation E(X)?

60 users have participated

A	3	18%
B	3.5	63%
C	4	15%
D	4.5	3%

In [12]:
```
(1*(1/6))+(2*(1/6))+(3*(1/6))+(4*(1/6))+(5*(1/6))+(6*(1/6))
# E(X) = Σ(x*P(x))
```
Out[12]: 3.5

Let "X" denote random variable which is the number of heads in two coin tosses for a fair coin. Find the expectation: E(X)

58 users have participated

A	1/2	36%
B	1/4	29%
C	1	26%

```
two coin toss :

    X : no of heads
        {
            HH   2
            HT   1
            TH   1
            TT   0
        }

    RV

    x      P(x)
    0 : 1  1/4
    1 : 2  2/4
    2 : 1  1/4

     E(X) = Σ(x*P(x))
```

In [13]:
```
(0*(1/4))+(1*(2/4))+(2*(1/4))
```
Out[13]: 1.0

Let "X" denote random variable which is the number of heads in two coin tosses for coin whose probability of heads is 3/4. Find the expectation: E(X)

53 users have participated

A	1/2	21%
B	1	9%
C	2	8%
D	3/2	62%

In [ ]:
```

```

```
    x      P(x)
    0 : 1  1/4  * 1/4
    1 : 2  1/4 * 3/4 * 2
    2 : 1  3/4 * 3/4
```

```
E(X) = Σ(x*P(x))
E(X) = (0 * P[x=0]) + (1 * P[x=1]) + (2 * P[x=2])

   (0 * ((1/4)^2)) + (1 * (2*(1/4)*(3/4))) + (2 * ((3/4)^2))
```

In [21]:
```
(0 * ((1/4)**2)) + (1 * 2*(1/4)*(3/4)) + (2 * ((3/4)**2))
```
Out[21]: 1.5

In [22]:
```
3/2
```
Out[22]: 1.5

Let X be a Bernoulli random variable with parameter "p". What is the expectation E(X)

69 users have participated

A	1	9%
B ✓	p	30%
C	1-p	16%
D	p(1-p)	45%

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

Q6. Exactly 3 baskets Solved

Stuck somewhere?

Ask for help from a TA and get it resolved.

Get help from TA.

A basketball player takes 5 independent free throws with a probability of 0.6 of getting a basket on each shot. Find the probability that he gets exactly 3 baskets.

Choose the correct answer from below:

- ○ 0.536
- ◉ 0.3456

In [24]:
```
math.comb(5,3)
```
Out[24]: 10

In [25]:
```
0.6**3
```
Out[25]: 0.21599999999999997

In [26]:
```
(1-0.6)**2
```
Out[26]: 0.16000000000000003

In [27]:
```
10*0.21599999999999997*0.16000000000000003

```
Out[27]: 0.3456

In [ ]:
```

```

Q5. Find npq ✓ Solved

**Stuck somewhere?**
Ask for help from a TA and get it resolved.
Get help from TA.

For a binomial distribution, the mean is 3 and the standard deviation is 3/2. The values of n(number of trials), p(probability of success), and q(probability of failure) are:

Choose the correct answer from below:

- ○ n=12, p=3/4, q=1/4
- ● n=12, p=1/4, q=3/4

```
mean = 3 = np
std = 3/2

std = sq(npq)
3/2 = sq(3 * q)
q = 3/4
p = 1-q = 1/4
n = mean/p = 3 / (1/4) = 12
```

In [ ]:
```

```

In [ ]:
```

```

Q7. Defective Bulbs ✓ Solved

Stuck somewhere?
Ask for help from a TA and get it resolved.
Get help from TA.

In a factory, the probability of producing a defective bulb is 0.25. A sample of 40 bulbs is collected. What is the probability that exactly 10 bulbs are defective?

Choose the correct answer from below:

- ○ 0.10
- ○ 0.12
- ○ 0.11
- ● 0.14

In [98]:
```
(math.comb(40,10)) * (0.25**10) * ((1-0.25)**(40-10))
```
Out[98]: 0.14436434635625678

In [ ]:
```

```

Q7. Mean of the tosses

Solved

Stuck somewhere?

Ask for help from a TA and get it resolved.

Get help from TA.

If you toss a coin 10 times, which let's say represents a binomial distribution here. What's the mean and variance value of the number of heads?

Choose the correct answer from below:

- ( ) 10, 5
- ( ) 5, 2
- (•) 5, 2.5
- ( ) 2.5,5

```
n * p = (10 * (1/2))
n*p*q = variance = 10*(1/2)*(1/2)
```

In [101]:
```
(10 * (1/2)),(10*(1/2)*(1/2))
```

Out[101]: (5.0, 2.5)

In [ ]:
```

```

Q8. Archer

Solved

Stuck somewhere?

Ask for help from a TA and get it resolved.

Get help from TA.

An Archer can shoot an arrow into the bull's eye with a probability of 0.72. What is the probability that the archer misses shooting the bull's eye and also calculate its variance?

Choose the correct answer from below:

- ( ) 0.72,0.30
- ( ) 0.72, 0.20
- (•) 0.28 ,0.20

In [31]:
```
(1-0.72),(0.72*(1-0.72))
```

Out[31]: (0.28, 0.2016)

In [ ]:
```

```

Q10. Final exam-avg score

Solved

Stuck somewhere?

Ask for help from a TA and get it resolved.

Get help from TA.

A teacher is teaching two Statistics classes. On the final exam, the 25 students in the first class averaged 90 while the 15 students in the second class averaged only 87. If the teacher combines the classes, what will the average final exam score be?

Choose the correct answer from below:

- ( ) 87
- ( ) 87.5
- ( ) 88
- (•) **88.8**

In [ ]:
```
students    grades
25           90       25*90 = 2250
15           87       15*87 = 1305
total Students : 40       total : 3555

    expected final average :  3555/40 = 88.8


```

In [52]:
```
3555/40
```

Out[52]:
```
88.875
```

In [ ]:
```

```

Q11. Right measure

Solved

Stuck somewhere?

Ask for help from a TA and get it resolved.

Get help from TA.

Given a sample of values [25, 25, 40, 45, 30, 41, 50, 30, 30, 1000] which measure of central tendency would you choose to represent the sample more correctly?

Choose the right combination of measure of central tendency and the value.

Choose the correct answer from below:

- (•) **Median, 35**
- ( ) Mean, 131
- ( ) Mode, 30

In [54]:
```
x =  np.array([25, 25, 40, 45, 30, 41, 50, 30, 30, 1000])
```

```
<IPython.core.display.Javascript object>
```

In [57]:
```
np.median(x)
```

```
<IPython.core.display.Javascript object>
```

Out[57]:
```
35.0
```

In [ ]:
```

```

In [ ]:
```

```

In [ ]:

```

```

Q13. New average ⊞ Solved

Stuck somewhere?
Ask for help from a TA and get it resolved.
Get help from TA.

The average of a set of 15 observations is recorded, but later it is found that for one observation, the digit in the tens place was wrongly recorded as 8 instead of 3. After correcting the observation, the average is

Choose the correct answer from below:

- ○ reduced by 1/3
- ○ increased by 10/3
- ● reduced by 10/3

In [66]:

```
np.mean(np.array([100,180,20,100,100,100,100,100,100,100,100,100,100,100,100]))
```

```
<IPython.core.display.Javascript object>
<IPython.core.display.Javascript object>
```

Out[66]:

```
100.0
```

In [ ]:

```
100
```

In [67]:

```
np.mean(np.array([100,130,20,100,100,100,100,100,100,100,100,100,100,100,100]))
```

```
<IPython.core.display.Javascript object>
<IPython.core.display.Javascript object>
```

Out[67]:

```
96.66666666666667
```

In [68]:

```
96.66/100
```

Out[68]:

```
0.9666
```

In [70]:

```
(96/2)/2
```

Out[70]:

```
24.0
```

In [71]:

```
10/3
```

Out[71]:

```
3.3333333333333335
```

In [74]:

```
96.666+3.333
```

Out[74]:

```
99.999
```

In [ ]:

```

```

In [ ]:

```

```

Q15. Mean-Median impact ⊞ Solved

Stuck somewhere?
Ask for help from a TA and get it resolved.
Get help from TA.

For the given data below, If a data point beyond the Q3+1.5 IQR is removed, then what can you say about the mean and median.

data=
[10,23,24,24,28,29,20,32,33,25,38,29,25,41,50,25,31,60,70]

Choose the correct answer from below:

- ○ Mean and median both will have equal impact.
- ● Mean will have significant impact compared to median.
- ○ Median will have significant impact compared to mean.

In [75]:
```
x = np.array([10,23,24,24,28,29,20,32,33,25,38,29,25,41,50,25,31,60,70])
```
```
<IPython.core.display.Javascript object>
```

In [76]:
```
x
```
Out[76]:
```
array([10, 23, 24, 24, 28, 29, 20, 32, 33, 25, 38, 29, 25, 41, 50, 25, 31,
       60, 70])
```

In [85]:
```
np.quantile(x,0.75),np.quantile(x,0.25)
```
```
<IPython.core.display.Javascript object>

<IPython.core.display.Javascript object>
```
Out[85]:
```
(35.5, 24.5)
```

In [82]:
```
np.sort(x)
```
```
<IPython.core.display.Javascript object>
```
Out[82]:
```
array([10, 20, 23, 24, 24, 25, 25, 25, 28, 29, 29, 31, 32, 33, 38, 41, 50,
       60, 70])
```

In [83]:
```
len(x)
```
Out[83]:
```
19
```

In [86]:
```
35.5-24.5
```
Out[86]:
```
11.0
```

In [87]:
```
35.5+(1.5*11)
```
Out[87]:
```
52.0
```

In [89]:
```
y = np.array([10, 20, 23, 24, 24, 25, 25, 25, 28, 29, 29, 31, 32, 33, 38, 41,])
```
```
<IPython.core.display.Javascript object>
```

In [90]:
```
np.mean(y),np.mean(x)
```
```
<IPython.core.display.Javascript object>

<IPython.core.display.Javascript object>
```
Out[90]:
```
(27.3125, 32.473684210526315)
```

In [91]:
```
np.median(y),np.median(x)
```
```
<IPython.core.display.Javascript object>

<IPython.core.display.Javascript object>
```
Out[91]:
```
(26.5, 29.0)
```

In [ ]:
```

```

In [ ]:
```

```

Q16. Weighted mean

Solved

Stuck somewhere?

Ask for help from a TA and get it resolved.

Get help from TA.

Suppose a firm conducts a survey of 1000 households to determine the average number of children living in each household. The data showed a large number of households have two or three children and a smaller number with one or four children. Every household in the sample has at least one child and no household with more than 4 children. Find the average number of children living per household.

No.of children per household	Number of households
1	70
2	385
3	523
4	22

Choose the correct answer from below:

- (•) 2.49
- ( ) 2.63
- ( ) 3.50
- ( ) 4.23

In [97]:
```
70+385+523+22
```
Out[97]: 1000

In [ ]:

In [95]:
```
(1*(70/1000))+(2*(385/1000))+(3*(523/1000))+(4*(22/1000))
```
Out[95]: 2.497

In [ ]:

In [ ]:
```
If a normal distribution with μ = 200 have P(X > 225) = 0.1587, then P(X < 175) equal to:
```

In [ ]:

Q5. PnC 05

Solved

Stuck somewhere?

Ask for help from a TA and get it resolved.

Get help from TA.

In how many ways can we arrange the word **FUZZTONE** so that all the vowels come together?

Choose the correct answer from below:

- ( ) 1440
- ( ) 6
- (•) 2160
- ( ) 4320

```

                 FUZZTONE
                 FZZTN(UOE)
(n-r)!
n!

There are 3 vowels (U,E,O) which can be arranged in 3! ways.
```

```
Let the vowels be in one group.
Now, we have (8−3=)5 characters + 1 group = 6
This can be arranged in 6! ways.
But the alphabet Z is twice so we need to divide by 2!.
This give us

6!/2!

Total ways to arrange the letters = 3!× 6!/2!
                                     =2160
 Hence, the value of FUZZTONE after applying permutation is 2160.
```

In [ ]:

In [ ]:

In [102]:

```python
0.25*0.1
```

Out[102]: 0.025

In [103]:

```python
0.75*0.9
```

Out[103]: 0.675

In [104]:

```python
0.025/(0.025+0.675)
```

Out[104]: 0.03571428571428571

In [ ]: