

there are n event

$E_i \cap E_j = \{\}$ \rightarrow all i, j $i \neq j$
"mutually exclusive"

$E_1 \cup E_2 \dots \cup E_n = S$ "exhaustive"

$$P[F] = P[F \cap E_1] + P[F \cap E_2] + \dots + P[F \cap E_n]$$

$$= P(F|E_1)P(E_1) + \dots + P(F|E_n)P(E_n)$$

$P(A|B) = P(A \cap B) / P(B)$

In []:

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A family has 2 children. Given that at least one child is a girl, what is the probability that both are girls

Click on an option to submit your answer

- A

1/2
- B

2/3
- C

1/4
- D

1/3

1
2
3
4
5
6
7
8
9

sample space : gg,by,gb,bb
already given that one child is a girl
so bb is not an element

so , sample space is : gg,bg,gb

one is girl that is given ,
the probability of both are girl is :
1/ 3

Family has 2 children. Atleast one is a girl. What is the probability that both are girls? \rightarrow only 28% got this right

$S = \{bb, bg, gb, gg\}$ $\xrightarrow{\text{numerator}}$

$P[\{gg\} | \{bg, gb, gg\}] = \frac{P[\{gg\}]}{P[\{bg, gb, gg\}]} = \frac{1/4}{3/4} = \frac{1}{3}$

$P[A|B] = \frac{P[A \cap B]}{P[B]}$

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In MCQ with 4 options, let 0.8 be the probability that the student knows the answer, and 0.2 the probability of guessing. What is the conditional probability that the student knew the answer to a question given that it was answered correctly?

- Click on an option to submit your answer
- A

10/17
- B

12/17
- C

14/17
- D

16/17

In [2]:

1

D

1

P[student knows the answer] = 0.8

2

P[~ student knows the ans] = 0.2 (guessing)

3

4

P[correct ans]

5

P[ans is wrong]

P[K] = 0.8

P[G] = 0.2

```

6
7
8 P[knows ans| ans correctly] = ? (Question )
9
10 P[K|C] = P[C|K]*P[K] / P[C|K]*P[K] + P[C|K']*P[K']
11          = P[C|K]*P[K] / P[C|K]*P[K] + P[C|G]*P[G]
12
13 P[C|K] probability of correct ans given he knows the ans = 1
14 P[k] is 0.8
15 P[C|G] probability of correct ans guessing out of 4 questions = 1/4
16 P[G] = 0.2
17
18          = (1*0.8) / ((1*(0.8)) + ((1/4)*0.2))
19
20
21
22

```

In [3]: 1 (1*0.8) / ((1*(0.8)) + ((1/4)*0.2))

Out[3]: 0.9411764705882353

In [4]: 1 16/17

Out[4]: 0.9411764705882353

In []: 1

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In []: 1 # A ∩ B
2 # A ∪ B
3 # A ⊂ B

P(E) = 0.6

What can we say about P(E|F) when E and F are mutually exclusive

Click on an option to submit your answer

A P(E|F) = 0.6

B P(E|F) >= 0.6

C P(E|F) <= 0.6

D P(E|F) = 0

```

1 P[E|F] = P[E ∩ F]/P[F]
2
3 here, E and F are mutually exclusive . that means :

```

```

4 E ∩ F = {}
5 P[E ∩ F] = 0
6
7 P[E|F] = 0/P[F]
8         = 0
9

```

In []:

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In []:

1

Let $P(E) = 0.6$

What can we say about $P(E|F)$ when E is a subset of F

Click on an option to submit your answer

A $P(E|F) = 0.6$

B $P(E|F) \geq 0.6$

C $P(E|F) \leq 0.6$

D $P(E|F) = 0$

```

1 E is a subset of F :
2
3 P[E ∩ F] = P[E]
4
5
6
7
8 P[E|F] = P[E ∩ F]/P[F]
9         = P[E]/P[F]
10        = 0.6/P[F]
11
12      P[E]/ P[E]  >= P[E]
13 ans is option B
14
15
16
17 P[E] = 0.6
18
19 P[E|F] = P[E]/P[F]
20
21 left say : P[E|F]== 0.5
22            0.5= 0.6 / P[F]

```

```

23         so , P[F] becomes >1 , which is not possible
24
25         thats why P[E|F] should be >= 0.6
26
27
28

```

In []:

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Let $P(E) = 0.6$

What can we say about $P(E|F)$ when F is a subset of E

Click on an option to submit your answer

- A $P(E|F) = 1$
- B $P(E|F) \geq 0.6$
- C $P(E|F) \leq 0.6$
- D $P(E|F) = 0$

In []:

```

1  F is a subset of E :
2      means  $F \cap E = F$ 
3
4   $P[E|F] = P[E \cap F] / P[F]$ 
5            $= P[F] / P[F]$ 
6            $= 1$ 
7
8  option A is the ans
9
10

```

In []:

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```
In [ ]: 1
In [ ]: 1
In [ ]: 1
```

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and when he flips it, it shows heads. What is the probability that it is the fair coin?

Click on an option to submit your answer

- A

1/2
- B

1/3
- C

1/5
- D

1

```
1 Fair coin      Biased coin
2 (HT)          (HH)
3
4 P[F] = 1/2     P[B] = 1/2
5
6 P[F|H] = ? (Question )
7
8
9
10 P[F|H]  = P[H|F]*P[F] / P[H|F]*P[F] + P[H|B]*P[B]
11          = 1/2 * 1/2   /   1/2 * 1/2 +   1 * 1/2
12
13      P[H|F] probability of heads given the coin is fair is 50%
14      P[F] probability of fair coin is 1/2
15      P[H|B] : probability of heads given the biased coin :
16                as biased coin has both sides heads , probability of heads is 1
17      P[B] = P[F] due to randomness P[B] is 1/2 and P[F] is 1/2
```

```
In [5]: 1 ((1/2) * (1/2)) / ( ((1/2) * (1/2)) + ( 1 * (1/2) ) )
```

Out[5]: 0.3333333333333333

```
In [6]: 1 1/3 # ans is option B
```

Out[6]: 0.3333333333333333

```
In [ ]: 1
In [ ]: 1
In [ ]: 1
In [ ]: 1
In [ ]: 1
In [ ]: 1
In [ ]: 1
```

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and he flips it twice. It shows heads both the times. What is the probability that it is the fair coin?

Click on an option to submit your answer

A

1/2

B

1/3

C

1/5

D

1

```
1 Fair coin      Biased coin
2
3
4 P[F] = 1/2      P[B] = 1/2
5
6 P[F|HH] = ? (Question )
7
8
9
10 P[F|HH]  = P[HH|F]*P[F] / P[HH|F]*P[F] + P[HH|B]*P[B]
11           = 1/4 * 1/2    /    1/4 * 1/2 +    1 * 1/2
12
13 P[HH|F]  probability of two heads(two flips) given the coin is fair  is 25% (1/2 * 1/2)
14 P[F]     probability of fair coin is 1/2
15 P[HH|B] : probability of both heads given the biased coin :
16           as biased coin has both sides heads , probability of two heads is 1
17 P[B] = P[F] due to randomness P[B] is 1/2 and P[F] is 1/2
```

```
In [7]: 1 ((1/4) * (1/2)) / ( ((1/4) * (1/2)) + ( 1 * (1/2) ) )
```

Out[7]: 0.2

```
In [8]: 1 1/5 # ans is Option C
```

Out[8]: 0.2

```
In [ ]: 1
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In [ ]: 1
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In [ ]: 1
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In [ ]: 1
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In [ ]: 1
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```
In [ ]: 1
```

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and he flips it three times. He gets {HHT}. What is the probability that it is the fair coin?

Click on an option to submit your answer

A

1/2

B

1/3

C

1/5

D

1

```
1 Fair coin      Biased coin
2
3
4 P[F] = 1/2      P[B] = 1/2
5
6 P[F|HHT] = ? (Question )
```

```
7
8
9
10 P[F|HHT] = P[HHT|F]*P[F] / P[HHT|F]*P[F] + P[HHT|B]*P[B]
11           = 1/8 * 1/2 / 1/8 * 1/2 + 0 * 1/2
12           = 1/16 / 1/16
13           = 1
14
15
16 probability of tails in a two headed coin P[HHT|B] = 0
17
18
19 Option D = 1 is the ans
```

In []: 1

In []: 1

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In []: 1

Type *Markdown* and LaTeX: α^2

In []: 1

1

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In []: 1

In []: 1

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Q3. Color blind

Unresolved



Stuck somewhere?

Ask for help from a TA and get it resolved.

[Get help from TA.](#)

Suppose 5 percent of men and 0.25 percent of the women are color-blind. A random color-blind person is chosen. What is the probability of this person being male? Assume there are equal number of men and women overall.

```
1 p[CB|M] = 0.05
2 p[CB|F] = 0.0025
```

```
1 P[M|CB] = ?
2
3 M|CB = (CB|M * M) / ((CB|M * M) + (CB|F * F))
```

In [15]: `0.05 * 0.5 / ((0.05*0.5) + (0.0025*0.5))`

Out[15]: 0.9523809523809523

In []:

```
1
```

Q4. Three coins

Solved



Stuck somewhere?

Ask for help from a TA and get it resolved.

[Get help from TA.](#)

There are three coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

Choose the correct answer from below:

☐ 1/9

☐ 2/9

☐ 1/3

☒ 4/9

In []:

```
1
```

```
1 TOTAL 3 COINS :
2
3 p[2headed] = 1/3
4 P[fair] = 1/3
5 P[biased] = 1/3
6
```

```
7
8     P[H|biased] = 0.75
9     P[H|2headed] = 1
10    P[H|fiar] = 1/2
11
12    P[2headed|H] = P[H|2headed] * P[2headed] /
13                  (P[H|2headed] * P[2headed]) + (P[H|2headed'] * P[2headed'])
```

In [63]: 1 1 * (1/3)

Out[63]: 0.3333333333333333

In [66]: 1 ((0.75 + (1/2))/2) * (1-(1/3))

Out[66]: 0.41666666666666674

In [67]: 1 (1 * (1/3)) + ((0.75 + (1/2))/2) * (1-(1/3))

Out[67]: 0.75

In [69]: 1 (1 * (1/3)) / ((1 * (1/3)) + ((0.75 + (1/2))/2) * (1-(1/3)))

Out[69]: 0.44444444444444444

In [73]: 1 4/9

Out[73]: 0.44444444444444444

In []: 1

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Q5. Ten coins

  Solved

 **Stuck somewhere?**
Ask for help from a TA and get it resolved. [Get help from TA.](#)

There are ten coins numbered from 1 to 10, where the probability of the heads is $i/10$, $i=1,2,\dots,10$. One of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the 5th coin?

Choose the correct answer from below:

- ☐ 5/11
- ☒ 1/11
- ☐ 5/10
- ☐ 1/10

```
In [ ]: 1 P[C5|H] = ?
        2 P[]
        3
        4
        5
        6 P[C5|H] =
        7           P[H|C5] * P[C5] /
        8           P[H|C5] * P[C5] + P[H|C5'] * P[C5']
        9
```

```
In [ ]: 1
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```
In [ ]: 1
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In [ ]: 1
```

```
In [58]: 1 (0.1+0.2+0.3+0.4+0.6+0.7+0.8+0.9+1)/9
```

Out[58]: 0.5555555555555556

```
In [59]: 1 1-(1/10)
```

Out[59]: 0.9

```
In [60]: 1 (5/10)*(1/10)
```

Out[60]: 0.05

```
In [61]: 1 (5/10)*(1/10) + (0.555*0.9)
```

Out[61]: 0.5495000000000001

```
In [62]: 1 0.05/0.549500
```

Out[62]: 0.09099181073703368

```
In [74]: 1 1/11
```

Out[74]: 0.09090909090909091

```
In [ ]: 1
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In [ ]: 1
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In [ ]: 1
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In [ ]: 1
```

Q6. Two Urns

Solved





Stuck somewhere?

Ask for help from a TA and get it resolved.

[Get help from TA.](#)

Urn 1 has five white and seven black balls. Urn 2 has three white and twelve black balls. We flip a fair coin. If the outcome is heads, then a ball from urn 1 is selected, while if the outcome is tails, then a ball from urn 2 is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?

Choose the correct answer from below:

- ☒ 12/37
- ☐ 25/37
- ☐ 20/37
- ☐ 5/37

```
1 Urn1          Urn2
2 5 white balls  3 white balls
3 7 black balls  12 black balls
4
5 Heads          Tails
6
7
8 if heads : ball from Urn1 is selected
9 if tails : ball from Urn2 is selected
10
11 probability of tails given white ball is selected :
12 P[T|W] = ?
13
14 P[T|W] = P[W|T]* P[T]/
15         P[W|T]* P[T] + P[W|T']* P[T']
16
17         = P[W|T]* P[T]/
18         P[W|T]* P[T] + P[W|H]* P[H]
19
20 P[W|T]  white balls given tails : 3 / (3+12) = 3/15
21 P[W|H]  white ball given not tail : 5 / (5+7) = 5/ 12
22
23
24 P[T|W]=          P[W|T]* P[T]/
25                 P[W|T]* P[T] + P[W|H]* P[H]
26
27
28
```

```
In [79]: 1 (3/15)*(1/2)
```

Out[79]: 0.1

```
In [80]: 1 (3/15)*(1/2) + ((5/12)*0.5)
```

Out[80]: 0.30833333333333335

```
In [81]: 1 0.1/0.308333
```

Out[81]: 0.3243246749455945

```
In [78]: 1 12/37
```

Out[78]: 0.32432432432432434

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In [ ]: 1
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