Linear Algebra for Electrical Engineers

Homework VIII

Due 11/19 23:59

Please checkout the eTL homework announcement for the submission format. Skeleton code is available at this link: `https://github.com/3dvision-snu/linear-algebra-2020-fall`

In this homework, you are asked to implement an image deblurring algorithm using least square methods. Let H and W be the height and width of the image. We start formalizing the problem of image deblurring by setting $\mathbf{x} \in \mathbb{R}^{HW}$ be the original image, and $\mathbf{y} \in \mathbb{R}^{HW}$ to be the blurred noisy image. Given a blurry image $\mathbf{y}$, our objective is to find deblurred image $\hat{\mathbf{x}}$, an estimate of original image by minimizing the cost function

$$\|\mathbf{x} - \mathbf{y}\| + \lambda(\|D_h \mathbf{x}\|^2 + \|D_v \mathbf{x}\|^2), \tag{1}$$

where D_h and D_v represent the vertical and horizontal differencing operations. Specifically, assuming the vector $\mathbf{x}$ contains the pixel values of the 2D image in a column-wise format,

$$D_h = \begin{bmatrix} -I & I & 0 & \cdots & 0 & 0 & 0 \\ 0 & -I & I & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -I & I & 0 \\ 0 & 0 & 0 & \cdots & 0 & -I & I \end{bmatrix}, \tag{2}$$

where all blocks have size $H \times H$, and D_v be the $(H-1)W \times HW$ matrix

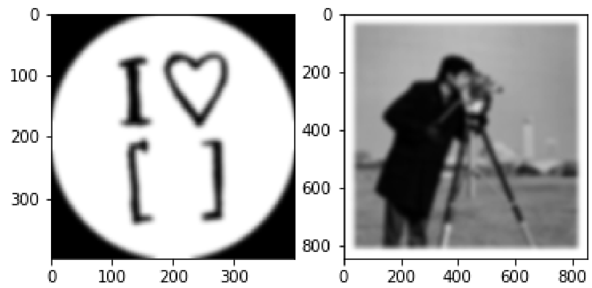
$$D_h = \begin{bmatrix} D & 0 & \cdots & 0 \\ 0 & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & D & \cdots & D \end{bmatrix}, \tag{3}$$

with $(H-1) \times H$ matrix D

$$D = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \tag{4}$$

The first term in Eq. 1 is to ensure that the estimate is close to given image $\mathbf{y}$, while the second term penalizes the non-smoothenss in the reconstructed image. See textbook page 321 for further details.

1. Write the Eq. 1 in least squares form, i.e. $\|A\mathbf{x} - \mathbf{y}\|^2$ (0.2 points).
2. Implement the least-squares deblurring algorithm and deblur the given images. Find



the hyperparameter λ that best pleases our eyes for each image and write it in the report with images attached. You may use the numpy's least square solver, but **do not use any open source deblurring algorithm** (0.8 points).