Linear Algebra for Electrical Engineers

Homework V

Due 10/20 23:59

Please checkout the eTL homework announcement for the submission format. Skeleton code is available at this link: https://github.com/3dvision-snu/linear-algebra-2020-fall

1. You are given a $n \times m$ matrix A. Write a program that takes matrix A as input and calculates the matrix Q, R, which are QR-decomposition outputs of A. The pseudocode is given as following:

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Algorithm 1: QR decomposition

Input: n \times m matrix A

Output: Q, R, where q_i the ith column of Q and r_{ij} is the (i, j) element of R

Q \leftarrow A

R \leftarrow \text{zero matrix}

for 0 \le i < n do

\begin{vmatrix} r_{ii} = ||q_i|| \\ q_i = q_i/r_{ii} \\ \text{for } i < j < n \text{ do} \end{vmatrix}

\begin{vmatrix} r_{ij} = q_i^T q_j \\ q_j = q_j - r_{ij} q_i \\ \text{end} \end{vmatrix}
```

Verify the algorithm by running the $check_qr(Q, R, A)$ function, where it checks 1) whether QR = A, 2) Q is orthogonal, and 3) R is upper triangular. With the correct algorithm, the program should output correct!!!. In this homework, it is important that you should not touch anything besides the $qr_decompose(A)$ function (0.7 points).

2. Briefly explain why the algorithm guarantees QR = A (0.3 points).