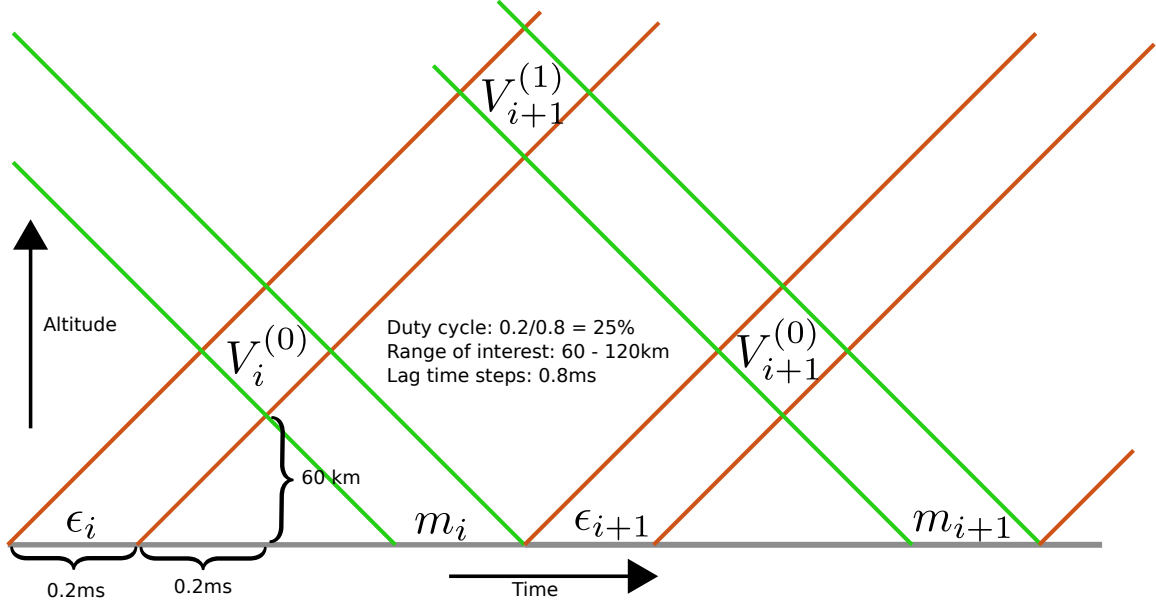


# A Simple Radar Code for the Low Altitude Ionosphere

Frank Hermann, 24.10.2021



We assume that the radar code  $\epsilon$  is periodic  $\epsilon_i = \epsilon_{i+L}$  and has already been transmitted for some time and that the radar continues transmitting the code indefinitely. According to the sketch above, we see that

$$\sum_{k=0}^{N-1} \epsilon_{i-k} V_i^{(k)} = m_i$$

where  $m_i$  is the  $i$ -th measured voltage and  $N$  is the total number of range gates considered along the radar beam. We calculate

$$\begin{aligned} \langle \epsilon_{j+\Delta} m_{j+\Delta+h}^* \epsilon_j^* m_{j+h} \rangle &= \epsilon_{j+\Delta} \epsilon_j^* \sum_{k=0}^{N-1} \epsilon_{j+\Delta+h-k}^* \epsilon_{j+h-k} \langle (V_{j+\Delta}^{(k)})^* V_j^{(k)} \rangle \\ &= \langle (V_{j+\Delta}^{(h)})^* V_j^{(h)} \rangle + \epsilon_{j+\Delta} \epsilon_j^* \sum_{k=0, k \neq h}^{N-1} \epsilon_{j+\Delta+h-k}^* \epsilon_{j+h-k} \langle (V_{j+\Delta}^{(k)})^* V_j^{(k)} \rangle \end{aligned}$$

where we have used the fact that signals from range gates of different altitudes do not correlate and assumed that  $\epsilon_i^* \epsilon_i = 1$ . Furthermore, we have introduced  $\Delta \in [1, M]$ , where  $M$  is the total number of measurement points in the auto correlation function. Those will have the (desired) spacing of 0.8ms. The auto correlation function is defined as  $\langle (V_{j+\Delta}^{(k)})^* V_j^{(k)} \rangle = R_{\Delta}^{(k)}$  assuming a wide sense stationary process for at least the time it takes to send the entire code once, which is  $T = L \cdot 0.8$  ms. We find for the auto correlation function at height  $h \in [0, N-1]$

$$R_{\Delta}^{(h)} = \langle \epsilon_{j+\Delta} m_{j+\Delta+h}^* \epsilon_j^* m_{j+h} \rangle - \sum_{k=0, k \neq h}^{N-1} \epsilon_{j+\Delta} \epsilon_j^* \epsilon_{j+\Delta+h-k}^* \epsilon_{j+h-k} R_{\Delta}^{(k)}.$$

In order to make the last term disappear we must average over the entire code to find the unbiased estimator

$$\hat{R}_{\Delta}^{(h)} = \frac{1}{L} \sum_{j=1}^L \langle \epsilon_{j+\Delta} m_{j+\Delta+h}^* \epsilon_j^* m_{j+h} \rangle - \sum_{k=0, k \neq h}^{N-1} R_{\Delta}^{(k)} \left[ \frac{1}{L} \sum_{j=0}^L \epsilon_{j+\Delta} \epsilon_j^* \epsilon_{j+\Delta+h-k}^* \epsilon_{j+h-k} \right]$$

and demand that (with  $w = k - h \neq 0$  so that  $|w| < N$ )

$$\frac{1}{L} \sum_{j=1}^L \epsilon_{j+\Delta} \epsilon_j^* \epsilon_{j+\Delta-w}^* \epsilon_{j-w} = 0.$$

Due to technical limitations of the radar we have  $\epsilon_i \in [-1, 1]$  and a naive brute force algorithm finds the following (shortest) codes for a given  $N$  and  $M$

$L$	$N$	$M$	Code
24	5	4	- + - + + + + + + - + + - + - + + - + + -
32	6	5	- + + - + + + + + + - - + + - - + - - - + + - + - -
40	7	4	- + - - + + + + + + + - + + + - - + - + + + - - - + - + + + - - + + + -

Note that we have used that every code which is valid for  $w$  is also valid for  $-w$ , because the value of the sum does not change if the code is shifted by  $w$ , due to the codes periodicity.