A Simple Radar Code for the Low Altitude Ionosphere

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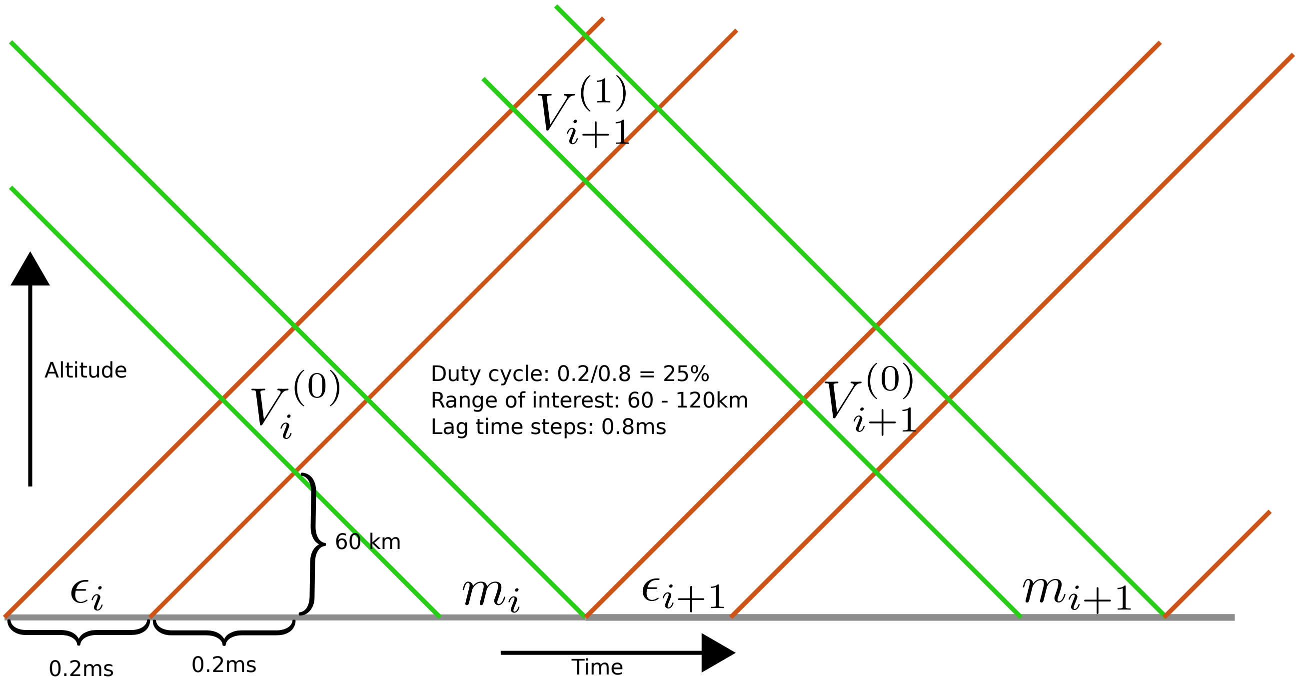

We assume that the radar code ϵ is periodic $\epsilon_i = \epsilon_{i+L}$ and has already been transmitted for some time and that the radar continues transmitting the code indefinitely. According to the sketch above, we see that

$$\sum_{k=0}^{N-1} \epsilon_{i-k} V_i^{(k)} = m_i$$

where m_i is the i-th measured voltage and N is the total number of range gates considered along the radar beam. We calculate

$$\begin{aligned}\langle \epsilon_{j+\Delta} m^*_{j+\Delta+h} \epsilon^*_j m_{j+h} \rangle &= \epsilon_{j+\Delta}\epsilon^*_j \sum_{k=0}^{N-1} \epsilon^*_{j+\Delta+h-k} \epsilon_{j+h-k} \langle (V_{j+\Delta}^{(k)})^* V_j^{(k)} \rangle \\ &= \langle (V_{j+\Delta}^{(h)})^* V_j^{(h)} \rangle + \epsilon_{j+\Delta}\epsilon^*_j \sum_{k=0, k\neq h}^{N-1} \epsilon^*_{j+\Delta+h-k} \epsilon_{j+h-k} \langle (V_{j+\Delta}^{(k)})^* V_j^{(k)} \rangle\end{aligned}$$

where we have used the fact that signals from range gates of different altitudes do not correlate and assumed that $\epsilon_i^* \epsilon_i = 1$. Furthermore, we have introduced $\Delta \in [1, M]$, where M is the total number of measurement points in the auto correlation function. Those will have the (desired) spacing of 0.8 ms. The auto correlation function is defined as $\langle (V_{j+\Delta}^{(k)})^* V_j^{(k)} \rangle = R_\Delta^{(k)}$ assuming a wide sense stationary process for at least the time it takes to send the entire code once, which is $T = L \cdot 0.8\,\mathrm{ms}$. We find for the auto correlation function at height h

$$R_\Delta^{(h)} = \langle \epsilon_{j+\Delta} m^*_{j+\Delta+h} \epsilon^*_j m_{j+h} \rangle - \sum_{k=0, k\neq h}^{N-1} \epsilon_{j+\Delta}\epsilon^*_j \epsilon^*_{j+\Delta+h-k} \epsilon_{j+h-k} R_\Delta^{(k)}.$$

In order to make the last term disappear we must average over the entire code to find the unbiased estimator

$$\hat{R}_\Delta^{(h)} = \frac{1}{L}\sum_{j=1}^{L} \langle \epsilon_{j+\Delta} m^*_{j+\Delta+h} \epsilon^*_j m_{j+h} \rangle - \sum_{k=0, k\neq h}^{N-1} R_\Delta^{(k)} \left[\frac{1}{L} \sum_{j=0}^{L} \epsilon_{j+\Delta}\epsilon^*_j \epsilon^*_{j+\Delta+h-k} \epsilon_{j+h-k} \right]$$

and demand that (with $W > w = k + H_1 - h + 1 > 0$ where $h \in [H_0, H_1]$ and $W = N + H_1 - H_0 + 1$)

$$\frac{1}{L}\sum_{j=1}^{L} \epsilon_{j+\Delta}\epsilon^*_j \epsilon^*_{j+\Delta-w} \epsilon_{j-w} = 0.$$

Due to technical limitations of the radar we have $\epsilon_i \in [-1, 1]$ and a naive brute force algorithm finds the following (shortest) codes for a given W and M

L	W	M	Code
24	5	4	− + − + + + + + + + − + + − − + − + + − + + + −
32	6	5	− + + − + + + + + + + + − − − + + − − − + − − − − + + − + − − −
40	7	4	− + − − + + + + + + + + + − + + + − − + − + + + − − − − + − + + + − − + + + + −

Note that for $H_1 = H_0 = 0$ we can use that $k \neq h$ and find $W = N$.