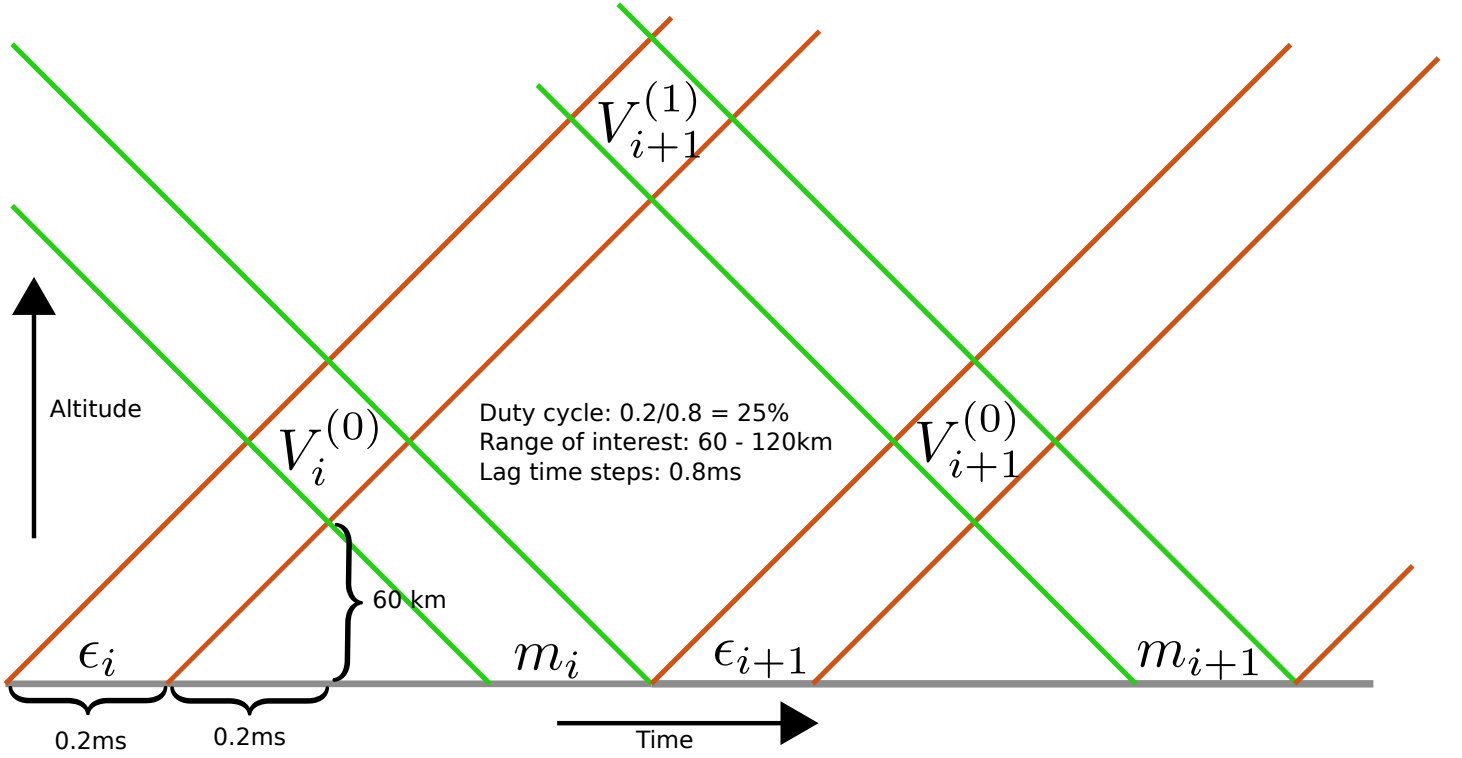


A Simple Radar Code for the Low Altitude Ionosphere

Frank Hermann, 24.10.2021



We assume that the radar code ϵ is periodic $\epsilon_i = \epsilon_{i+L}$ and has already been transmitted for some time and that the radar continues transmitting the code indefinitely. According to the sketch above, we see that

$$\sum_{k=0}^{N-1} \epsilon_{i-k} V_i^{(k)} = m_i$$

where m_i is the i -th measured voltage and N is the total number of range gates considered along the radar beam. We calculate

$$\langle \epsilon_{j+\Delta} m_{j+\Delta}^* \epsilon_j^* m_j \rangle = \epsilon_{j+\Delta} \epsilon_j^* \sum_{k=0}^{N-1} \epsilon_{j+\Delta-k}^* \epsilon_{j-k} \langle (V_{j+\Delta}^k)^* V_j^{(k)} \rangle = \langle (V_{j+\Delta}^{(0)})^* V_j^{(0)} \rangle + \epsilon_{j+\Delta} \epsilon_j^* \sum_{k=1}^{N-1} \epsilon_{j+\Delta-k}^* \epsilon_{j-k} \langle (V_{j+\Delta}^k)^* V_j^{(k)} \rangle$$

where we have used the fact that signals from range gates of different altitudes do not correlate and assumed that $\epsilon_i^* \epsilon_i = 1$. Furthermore, we have introduced $\Delta \in [1, M]$, where M is the total number of measurement points in the auto correlation function. Those will have the (desired) spacing of 0.8ms. The auto correlation function is defined as $\langle (V_{j+\Delta}^{(k)})^* V_j^{(k)} \rangle = R_{\Delta}^{(k)}$ assuming a wide sense stationary process for at least the time it takes to send the entire code once, which is $T = L \cdot 0.8$ ms. We find for the auto correlation function of interest

$$R_{\Delta}^{(0)} = \langle \epsilon_{j+\Delta} m_{j+\Delta}^* \epsilon_j^* m_j \rangle - \sum_{k=1}^{N-1} \epsilon_{j+\Delta} \epsilon_j^* \epsilon_{j+\Delta-k}^* \epsilon_{j-k} R_{\Delta}^{(k)}.$$

In order to make the last term disappear we must average over the entire code to find the unbiased estimator

$$\hat{R}_{\Delta}^{(0)} = \frac{1}{L} \sum_{j=1}^L \langle \epsilon_{j+\Delta} m_{j+\Delta}^* \epsilon_j^* m_j \rangle - \sum_{k=1}^{N-1} R_{\Delta}^{(k)} \left[\frac{1}{L} \sum_{j=1}^L \epsilon_{j+\Delta} \epsilon_j^* \epsilon_{j+\Delta-k}^* \epsilon_{j-k} \right]$$

and demand that

$$\frac{1}{L} \sum_{j=1}^L \epsilon_{j+\Delta} \epsilon_j^* \epsilon_{j+\Delta-k}^* \epsilon_{j-k} = 0.$$

Due to technical limitations of the radar we have $\epsilon_i \in [-1, 1]$ and a naive brute force algorithm finds the following (shortest) codes for a given N and M

L	N	M	Code
24	5	4	- + - + + + + + + - + + - - + - + + - + + -
32	6	5	- + + - + + + + + + + - - - + + - - - + - - - + + - + - - -
40	7	4	- + - - + + + + + + + + - + + + - - - + - + + - - - - + - + + + - - + + + + -

However, a code with $N \cdot 120 \text{ km} > H$ where H is the total height of the ionosphere, is required.