Approximate Static Condensation: Numerical Comparison of ASC(0) and ASC(1)

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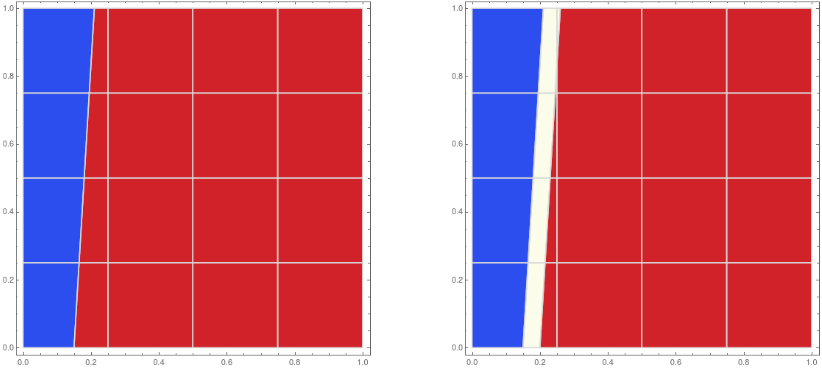
Overview

1. ASC(0) $\rightarrow$ ASC(1): Motivation
2. Robustness Test
3. Convergence Test
4. TODO List

Exact Recovery of Linear Solutions

Figure: ASC(0) $\mathbb{L}^2$ Temperature Error



(a) $||p - p_h||_{\mathbb{L}^2(\Omega)} = 1.7 \times 10^{-2}$

(b) $||p - p_h||_{\mathbb{L}^2(\Omega)} = 1.8 \times 10^{-2}$

Here $\mathbf{K}_i = \mathbf{K}_j$ and the exact soln is linear. ASC(0) produces errors due to const trace approximation, and ACS(1) recovers the exact soln

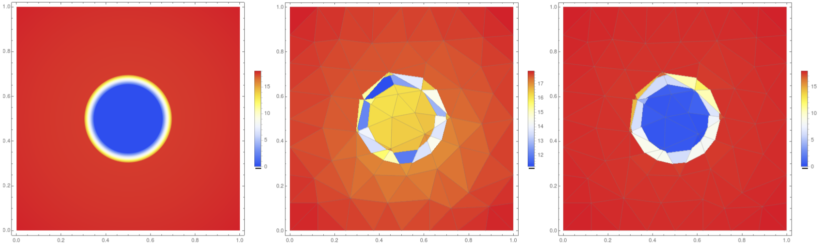
Highly Heterogeneous Benchmark 1 / 2

We choose $\mathbf{K} = k\,\mathbf{I}$, $k = .001$ inside the ring and 1 outside. Exact solution is pw quadratic. We recover measure of the inclusion via MOF

ring_base.png	ring_mmcs.png	ring_mini.png
(a) Base Mesh	(b) MMCs	(c) Interface Rec

Highly Heterogeneous Benchmark 2 / 2

Figure: ASC(0) $\mathbb{L}^2$ Temperature Error

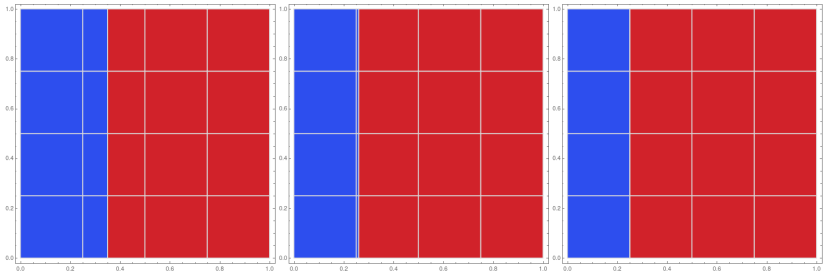


(a) Reference, p (b) ASC(0), p_h (c) ASC(1), p_h

$||p - p_h||_{\mathbb{L}^2(\Omega)} = 2.7 \times 10^{-1}$ for ASC(0) and 1.5×10^{-2} for ASC(1). ASC(1) produces ~ 18 times smaller error on the same mesh for this example

Robustness Test: Geometry

Figure: $w :=$ width of the left minimesh cells



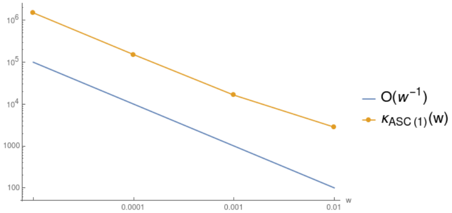
(a) $w = .1$ (b) $w = .01$ (c) $w = .001$

We solve the diffusion problem w/ $\mathbf{K} = k\,\mathbf{I}$, $k = 1$ on the left part and .1 on the right. Exact solution is pw linear

Robustness Test: Spectrum

Figure: Condition Numbers of ASC(0) / ASC(1) Matrices

w	$\kappa_{\mathrm{ASC}(0)}$	$\kappa_{\mathrm{ASC}(1)}$
10^{-1}	41	1 730
10^{-2}	45	2 817
10^{-3}	48	16 391
10^{-4}	49	152 325
10^{-5}	49	1.5×10^6



$\kappa_{\mathrm{ASC}(0)}$ does not depend on w, and $\kappa_{\mathrm{ASC}(1)}$ is proportional to w^{-1}. However, if we remove 3 smallest eig values (corresponding to 3 int MM faces), **we will have $\kappa_{\mathrm{ASC}(1)} = \kappa_{\mathrm{ASC}(0)}$**. Starting from some iteration CG behaves like extreme eig values are not present; that is, several small eig values is not a problem

Problem Setup

We solve the diffusion problem w/ $\mathbf{K} = k\,\mathbf{I}$, $k = 1$ on the left part and .01 on the right. Exact solution is pw linear. We compare convergence of ASC(0) and ASC(1)

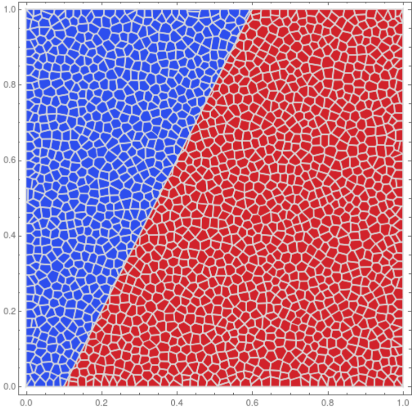
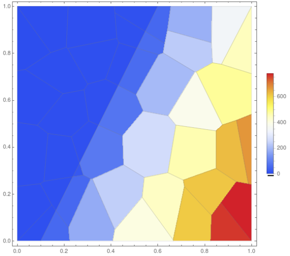
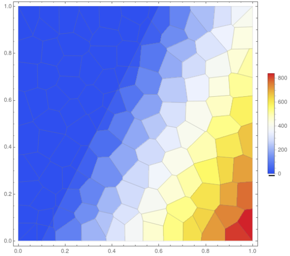


Figure: $h = 3.2 \times 10^{-2}$

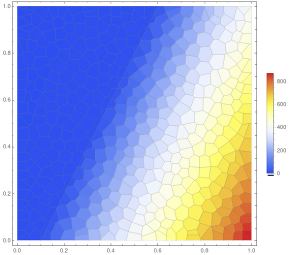
ASC(1) Solutions: Mesh Refinement



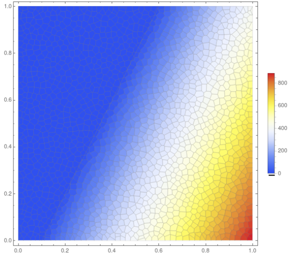
(a) $h = 2.4 \times 10^{-1}$



(b) $h = 1.1 \times 10^{-1}$



(c) $h = 6.4 \times 10^{-2}$

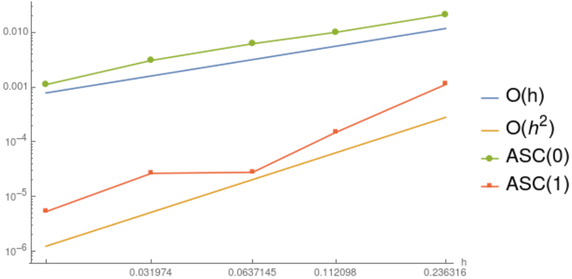


(d) $h = 3.2 \times 10^{-2}$

Temperature Errors

Figure: $e := ||p - p_h||_{\mathbb{L}^2(\Omega)}$ for ASC(0) and ASC(1)

h	$e_{ASC(0)}$	$e_{ASC(1)}$
2.4×10^{-1}	2.1×10^{-2}	1.1×10^{-3}
1.1×10^{-1}	1.0×10^{-2}	1.5×10^{-4}
6.4×10^{-2}	6.2×10^{-3}	2.8×10^{-5}
3.2×10^{-2}	3.1×10^{-3}	2.6×10^{-5}
1.6×10^{-2}	1.1×10^{-3}	5.2×10^{-6}



TODO List

- Implement homogenization approach and compare to ASC(0) / ASC(1)
- Compare convergence of ASC(0) and ASC(1) when we have base faces w/ 3 materials on all refinement levels