coarse sintered-glass disk, heating the face of the disk a short time in the flame, and "sticking" the two together. The coarse sintered-glass disk must fit the cation exchange tube closely. The juncture between the disk and capillary tube need not be tight and only strong enough to withstand the operations of insertion and removal. Gum rubber tubing (2 mm.) and a capillary tubing tip attached to the column outlet gave an adjustable head. If the head was maintained at 35 cm. the liquid level stopped automatically at the top of the resin, thus requiring less operator attention. Fourteen milliliters of resin in this column quantitatively removed sodium from nine samples containing 1 millimole each of sulfuric acid and sodium chloride. The removal of metal ions is more efficient from less acid solutions. According to Samuelson (*4*) the removal of sodium ions is more difficult than all other common ions except lithium.

REAGENTS AND SOLUTIONS

Alizarin Red S. Prepare a 0.2% aqueous solution.

Ammonium Sulfate. Prepare a 0.1000*M* solution by dissolving 13.214 grams of ammonium sulfate in water and diluting to exactly 1 liter.

Barium Chloride. Prepare a stock solution which is approximately 0.1*M*. Adjust the pH to 3 to 3.5. Standardize against a standard solution of sulfuric acid using the procedure described below.

Barium Perchlorate. Prepare a 0.1*M* stock solution and adjust the pH to about 3. Standardize against a standard solution of sulfuric acid using the procedure described below.

Cation Exchange Resin. Dowex 50, cross-linking 16, 50- to 100-mesh resin.

Hydrochloric Acid. Dilute reagent grade hydrochloric acid to 3.5*N*.

Magnesium Acetate, approximately 0.25*M* solution.

Methanol, A.C.S. grade.

Sulfuric Acid. Prepare an approximately 0.01*M* stock solution and standardize by titration with a standard sodium hydroxide solution.

o-(2-Hydroxy-3,6-disulfo-1-naphthylazo)-benzenearsonic Acid Disodium Salt (Thorin). Prepare a 0.025% solution of the sodium salt in water.

PROCEDURE

General Procedure. For macrotitration dissolve a sample containing 2 to 4 millimoles of sulfate in 45 ml. of water, add 40 ml. of methanol, and adjust the pH to 3.0 to 3.5 with dilute magnesium acetate or perchloric acid. Rapidly add about 90% of the required barium chloride or perchlorate, then add 5 drops of Alizarin Red S and titrate to the first permanent pink. Allow a time lapse of 3 to 5 seconds between addition of the last few increments of titrant.

For titrations on a semimicro scale use a 0.2- to 0.8-millimole sample, 10 ml. of water, 10 ml. of methanol, and 1 drop of indicator.

Ion Exchange Procedure. Agitate the resin (H form) in a beaker and decant the finer particles. Repeat this several times. With the column open pour in 14 ml. of resin, measured wet. Backwash with a slow stream of distilled water for a few minutes, then place the sintered-glass disk, with capillary tube attached, on top of the resin column and run 50 ml. of 3.5*N* hydrochloric acid through from the bottom at a flow rate of about 4 ml. per minute, washing it through with distilled water. Continue the washing until only a faint chloride test is obtained. This will require about 150 ml. and 30 minutes.

Introduce the 5-ml. sample containing 1 millimole of sulfate by pipet directly onto the resin column and wash through with 20 ml. of distilled water in small portions, allowing the liquid level to come to rest at the top of the resin column each time before a new rinse is added. Titrate the sulfate in the eluate by the standard macroprocedure.

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Tables for Evaluating Bateman Equation Coefficients for Radioactivity Calculations

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Tables of decay constants and functions thereof are presented to simplify the problem of calculating the constants involved in the Bateman equation. These tables make it possible to calculate any constant involved in any of the four radioactive series by a maximum of three mathematical operations, either by three divisions or by two multiplications and a division. They are useful and time-saving where a large number of such calculations are involved.

RADIOACTIVE equilibrium in the thorium, uranium, or neptunium series can be disturbed by chemical treatment of materials containing some or all of the members of these series. As a result of such a break in the radioactive chain, it is important to know the variation with time of the quantity or activity of a particular decay product.

Calculations involving radioactive equilibrium were simplified by Bateman (*1*) and his method is standard procedure (Rutherford *et al.*, *5*). The solution of the general case of n products is given by

$$N = N' \left(C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} \ldots C_n e^{-\lambda_n t}\right)$$

where N atoms of a given species are formed from N' atoms of a parent species in time t, λ is the decay constant, e is the Naperian base, and C_n is a constant having the form

$$C_2 = \frac{\lambda_1 \lambda_2 \ldots\ldots \lambda_{n-1}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)\ldots\ldots(\lambda_n - \lambda_2)}$$

These constants, although simple in themselves, are tedious to calculate, especially where there are a large number of calculations, and where many terms are involved. To facilitate the evaluation of these Bateman constants a number of tables have been computed. The half-life data are taken from the publications of Way *et al.* (*6*), Fleming (*2*), Ginnings *et al.* (*3*), and Hollander *et al.* (*4*).

Table I shows the half lives and decay constants of the various

Table I. Half Lives and Decay Constants

Nuclide	Half Life	λ, Sec.$^{-1}$	Nuclide	Half Life	λ, Sec.$^{-1}$
Tl 207	4.79 m	2.411785596 (−3)	Rn 222	3.825 d	2.097395244 (−6)
208	3.1 m	3.726597744 (−3)	Fr 221	4.8 m	2.406761043 (−3)
209	2.2 m	5.251115003 (−3)	223	21 m	5.501168099 (−4)
210	1.32 m	8.751858339 (−3)	Ra 223	11.2 d	7.162979295 (−7)
Pb 206	Stable		224	3.64 d	2.203993629 (−6)
207	Stable		225	14.8 d	5.420632980 (−7)
208	Stable		226	1,622 y	1.354190985 (−11)
209	3.22 h	5.979530542 (−5)	228	6.7 y	3.278354894 (−9)
210	22 y	9.984080813 (−10)	Ac 225	10.0 d	8.022536811 (−7)
211	36.1 m	3.200125487 (−4)	227	22.0 y	9.984080813 (−10)
212	10.6 h	1.816423428 (−5)	228	6.13 h	3.140960578 (−5)
214	26.8 m	4.310616794 (−4)	Th 227	18.6 d	4.313191834 (−7)
Bi 209	Stable		228	1.90 y	1.156051462 (−8)
210	5.02 d	1.598114902 (−6)	229	7,340 y	2.992503786 (−12)
211	2.16 m	5.348357874 (−3)	230	8.0 (4) y	2.745622224 (−13)
212	60.5 m	1.909496365 (−4)	231	25.64 h	7.509394831 (−6)
213	47 m	2.457968725 (−4)	232	1.39 (10) y	1.580214229 (−18)
214	19.7 m	5.864189344 (−4)	234	24.10 d	3.328853448 (−7)
Po 210	138.39 d	5.797049505 (−8)	Pa 231	34,300 y	6.403783615 (−13)
211	0.52 s	1.332975347	233	27.4 d	2.927933142 (−7)
212	3.04 (−7) s	2.280089409 (6)	234	1.175 m	9.831874900 (−3)
213	4.2 (−6) s	1.650350429 (5)	U 233	1.62 (5) y	1.355862826 (−13)
214	1.637 (−4)s	4.234252782 (3)	234	2.475 (5) y	8.874738501 (−14)
215	1.83 (−3) s	3.787689510 (2)	235	7.13 (8) y	3.080642045 (−17)
216	0.158 s	4.387007471	238	4.498 (9) y	4.883276521 (−18)
218	3.05 m	3.787689510 (−3)	Np 237	2.20 (6) y	9.984080813 (−15)
At 217	0.018 s	3.850817669 (1)	Pu 241	14 y	1.568926985 (−9)
Rn 219	3.92 s	1.768232603 (−1)	Am 241	470 y	4.673399529 (−11)
220	54.5 s	1.271829688 (−2)			

Table II. Products of Differences of Decay Constants of Uranium-238 Series

(In sec.$^{-1}$)

	y →	λU^{238}	λTh^{234}	λPa^{234}	λU^{234}	λTh^{230}	λRa^{226}	λRn^{222}
1	$(\lambda U^{238} - y)$		−3.328853448 (−7)	−9.831874900 (−3)	−8.874250173 (−14)	−2.745573391 (−13)	−1.354190497 (−11)	−2.097395244 (−6)
2	$1 \times (\lambda Th^{234} - y)$	3.328853448 (−7)		9.666249117 (−5)	−2.954107042 (−20)	−9.139603910 (−20)	−4.507718322 (−18)	3.700874670 (−12)
3	$2 \times (\lambda Pa^{234} - y)$	3.272887066 (−9)	−3.272776254 (−9)		−2.904441088 (−22)	−8.985944228 (−22)	−4.431932256 (−20)	3.637877458 (−14)
4	$3 \times (\lambda U^{234} - y)$	2.904441861 (−22)	1.089458961 (−15)	−9.503735207 (−7)		1.669721766 (−34)	5.962350470 (−31)	−7.630066555 (−20)
5	$4 \times (\lambda Th^{230} - y)$	7.974358289 (−35)	−3.626646227 (−22)	9.343953564 (−9)	−5.396882485 (−35)		−7.910457638 (−42)	1.600326320 (−25)
6	$5 \times (\lambda Ra^{226} - y)$	1.079880021 (−45)	1.207208268 (−28)	−9.186858238 (−11)	−7.260513690 (−46)	2.215277911 (−45)		−3.356495141 (−31)
7	$6 \times (Rn^{222} - y)$	2.264935220 (−51)	2.130130939 (−34)	9.030477245 (−13)	−1.522816624 (−51)	4.646312745 (−51)	−1.659124910 (−47)	
8	$7 \times (\lambda Po^{218} - y)$	8.578871374 (−54)	8.067565524 (−37)	−5.458187863 (−15)	−5.767956552 (−54)	1.759879004 (−53)	−6.284249994 (−50)	−1.270632154 (−33)
9	$8 \times (\lambda Pb^{214} - y)$	3.698022702 (−57)	3.474932769 (−40)	5.131140462 (−17)	−2.486345038 (−57)	7.586163985 (−57)	−2.708899271 (−53)	−5.450608950 (−37)
10	$9 \times (\lambda Bi^{214} - y)$	2.168590532 (−60)	2.036609617 (−43)	−4.743973320 (−19)	−1.548039808 (−60)	4.448670208 (−60)	−1.588549787 (−56)	−3.184908211 (−40)
	Branch via Po^{214} (99.96%)							
11	$10 \times (\lambda Po^{214} - y)$	9.182360493 (−57)	8.623519937 (−40)	−2.008713558 (−15)	−6.173709113 (−57)	1.883679420 (−56)	−6.726321355 (−53)	−1.348570645 (−36)
12	$11 \times (\lambda Pb^{210} - y)$	9.167742877 (−66)	−2.862033615 (−46)	1.974941841 (−17)	−6.163333169 (−66)	1.880163568 (−65)	−6.624526361 (−62)	2.827139233 (−42)
13	$12 \times (\lambda Bi^{210} - y)$	1.465110651 (−71)	−3.621129523 (−52)	−1.941422493 (−19)	−9.849714035 (−72)	3.004716899 (−71)	−1.058666459 (−67)	1.411535043 (−48)
14	$13 \times (\lambda Po^{210} - y)$	8.493318974 (−79)	9.955022789 (−59)	1.908771053 (−21)	−5.709919245 (−79)	1.741841011 (−78)	−6.135708236 (−75)	−2.878719501 (−54)
15	$14 \times (\lambda Pb^{206} - y)$	4.147522513 (−96)	−3.313881194 (−65)	−1.876679820 (−23)	5.067404016 (−92)	−4.782437390 (−91)	8.308920780 (−86)	6.037812590 (−60)
	Branch via Tl^{210} (0.04%)							
16	$10 \times (\lambda Tl^{210} - y)$	1.897919713 (−62)	1.782344090 (−45)	5.123569750 (−22)	−1.276055785 (−62)	3.893413146 (−62)	−1.390276268 (−58)	−2.786718547 (−42)
17	$16 \times (\lambda Pb^{210} - y)$	1.894898370 (−71)	−5.915367202 (−52)	−5.037429171 (−24)	−1.273911161 (−71)	3.886146165 (−71)	−1.369236065 (−67)	5.842067945 (−48)
18	$17 \times (\lambda Bi^{210} - y)$	3.028265323 (−77)	−7.484297424 (−58)	4.951932304 (−26)	−2.035856297 (−77)	6.210507029 (−77)	−2.188178018 (−73)	2.916829682 (−54)
19	$18 \times (\lambda Po^{210} - y)$	1.755500399 (−84)	2.057544502 (−64)	−4.868649186 (−28)	−1.180194167 (−84)	3.600244618 (−84)	−1.268201308 (−80)	−5.948654642 (−60)
20	$19 \times (\lambda Pb^{206} - y)$	−8.572593881 (−102)	−6.849264110 (−71)	4.786794973 (−30)	1.047391461 (−97)	−9.884911635 (−97)	1.717386778 (−91)	1.247667995 (−65)

members of the uranium-238, uranium-235, thorium-232, and neptunium-237 series arranged in the order of increasing atomic number and mass.

Tables II through V are products of differences of decay constants for each of the four series. Each entry has been carried to at least nine digits to reduce round-off error. The powers of 10 for each figure are indicated below the figure in parentheses—for example, (-25) indicates $\times\ 10^{-25}$. The last column of each table in Tables II through V represents the expression in the numerator of the constant, as $(\lambda_1 - \lambda_n)(\lambda_2 - \lambda_n)\ldots$ is equal to $\lambda_1 \lambda_2 \ldots$ when λ_n represents the end or stable member of the series and is numerically equal to zero.

The product of difference tables must be read from top to bottom. Where the desired product is not shown in the tables, it is easily calculated by dividing out the undesired part.

For example, in the table for the thorium series

$$(\lambda_{Ra^{224}} - \lambda_{Th^{228}})(\lambda_{Rn^{220}} - \lambda_{Th^{228}})(\lambda_{Po^{216}} - \lambda_{Th^{228}})$$

can be obtained by dividing the entry in the seventh row of column 4 by the third entry in the same column. Thus, the numerical evaluation of the desired product above is obtained as follows:

$$\frac{(\lambda_{Th^{232}} - \lambda_{Th^{228}})(\lambda_{Ra^{228}} - \lambda_{Th^{228}})\ldots\ldots(\lambda_{Po^{216}} - \lambda_{Th^{228}})}{(\lambda_{Th^{232}} - \lambda_{Th^{228}})(\lambda_{Ra^{228}} - \lambda_{Th^{228}})(\lambda_{Ac^{228}} - \lambda_{Th^{228}})} =$$

$$\frac{3.677449098 \times 10^{-28}}{3.006238142 \times 10^{-21}} = 1.223272716 \times 10^{-7}$$

As a further example, consider the complete Bateman coefficient

$$C = \frac{\lambda_{Rn^{222}} \lambda_{Po^{218}} \lambda_{Pb^{214}} \lambda_{Bi^{214}} \lambda_{Po^{214}}}{(\lambda_{Po^{218}} - \lambda_{Rn^{222}})(\lambda_{Pb^{214}} - \lambda_{Rn^{222}})(\lambda_{Bi^{214}} - \lambda_{Rn^{222}}) \times (\lambda_{Pb^{210}} - \lambda_{Rn^{222}})}$$

The numerator can be evaluated by dividing the eleventh entry under Pb^{206} by the sixth entry in the same column. The denominator can be evaluated by dividing the twelfth entry under Rn^{222} by the sixth entry in the same column.

Thus

$$C = \frac{4.484328657 \times 10^{-74}/5.273738631 \times 10^{-63}}{2.827139233 \times 10^{-42}/-3.356495141 \times 10^{-31}} =$$

$$-\frac{0.8503139266 \times 10^{-11}}{0.8411890885 \times 10^{-11}} =$$

$$-1.009526346$$

CONCLUSIONS

These tables enable one to calculate any constant involved in any of the four radioactive series by a maximum of three mathematical operations, either by three divisions or by two multiplications and a division. The tables have been found to be extremely useful and time-saving where a large number of calculations of this nature are involved.

(*Continued on page 1600*)

Table II. Products of Differences of Decay Constants of Uranium-238 Series (*Continued*)

(In sec.$^{-1}$)

$\lambda_{Po^{218}}$	$\lambda_{Pb^{214}}$	$\lambda_{Bi^{214}}$	$\lambda_{Po^{214}}$	$\lambda_{Tl^{210}}$	$\lambda_{Pb^{210}}$	$\lambda_{Bi^{210}}$	$\lambda_{Po^{210}}$	$\lambda_{Pb^{206}}$
−3.787689510 (−3)	−4.310616794 (−4)	−5.864189344 (−4)	−4.234252782 (3)	−8.751858339 (−3)	−9.984080764 (−10)	−1.598114902 (−6)	−5.797049505 (−8)	4.883276521 (−18)
1.434533096 (−5)	1.856706774 (−7)	3.436919564 (−7)	1.792889662 (7)	7.659211102 (−5)	−3.313585974 (−16)	2.021982209 (−12)	−1.593694994 (−14)	1.625571188 (−24)
8.670583980 (−8)	1.745455359 (−9)	3.177588849 (−9)	−7.591530411 (10)	8.272074834 (−8)	−3.257875946 (−18)	1.987664477 (−14)	−1.566891742 (−16)	1.598241256 (−26)
−3.284147999 (−10)	−7.523989184 (−13)	−1.863398267 (−12)	3.214445876 (14)	−7.239602712 (−10)	3.252400544 (−27)	−3.176516044 (−20)	9.083335091 (−24)	1.418397321 (−39)
1.243933292 (−12)	3.243303411 (−16)	1.092732026 (−15)	−1.361077639 (18)	6.335997737 (−12)	−3.246330000 (−36)	5.076436753 (−26)	−5.265629380 (−31)	3.894383207 (−52)
−4.711633064 (−15)	−1.398063771 (−19)	−6.407987355 (−19)	5.763146799 (21)	−5.545175454 (−14)	3.197200598 (−45)	−8.112660479 (−32)	3.051798352 (−38)	5.273738631 (−63)
1.783632098 (−17)	5.997194248 (−23)	3.744325034 (−22)	−2.440262027 (25)	4.851895962 (−16)	6.702601218 (−51)	4.050491898 (−38)	6.223913088 (−44)	1.106111432 (−68)
	2.013034912 (−25)	1.198659756 (−24)	1.033267703 (29)	−2.408563070 (−18)	2.538736563 (−53)	1.533553252 (−40)	2.357388951 (−46)	4.189606668 (−71)
−5.986989140 (−20)		−1.862204894 (−28)	−4.375116201 (32)	2.004116355 (−20)	1.096634375 (−56)	6.586052460 (−44)	1.016043381 (−49)	1.805978886 (−74)
1.916597217 (−22)	3.127395781 (−29)		1.852534538 (36)	−1.636449066 (−22)	6.430860667 (−60)	3.851660597 (−47)	5.957681762 (−53)	1.059060214 (−77)
Branch via Po^{214} (99.96%)								
8.115349838 (−19)	1.324218294 (−25)	−7.885045162 (−25)			2.722988967 (−56)	1.630890459 (−43)	2.522633058 (−49)	4.484328657 (−74)
−3.073841735 (−21)	−5.708184396 (−29)	4.623931909 (−28)	−7.844099521 (39)			−2.604722052 (−49)	−1.437196700 (−56)	4.477189970 (−83)
1.163784574 (−23)	2.451457218 (−32)	−2.704171648 (−31)	3.321390020 (43)		4.348930592 (−62)		−2.213490459 (−62)	7.155064010 (−89)
−4.407987158 (−26)	−1.056587153 (−35)	1.585620694 (−34)	−1.406360493 (47)		2.477676519 (−69)	4.011648100 (−55)		4.147826028 (−96)
1.669608672 (−28)	4.554542326 (−39)	−9.298379977 (−38)	5.954885830 (50)		−2.473732259 (−78)	−6.411074610 (−61)	1.283171377 (−69)	
Branch via Tl^{210} (0.04%)								
9.514312162 (−25)	2.602242437 (−31)	−1.520572122 (−30)			5.628197514 (−62)	3.370303252 (−49)	5.214044144 (−55)	9.268744965 (−80)
−3.603725088 (−27)	−1.121724397 (−34)	8.916907653 (−34)		1.432196877 (−24)		−5.382766914 (−55)	−2.970549764 (−62)	9.253989876 (−89)
1.364403254 (−29)	4.817397597 (−38)	−5.214793241 (−37)		−1.253209536 (−26)	8.988887081 (−68)		−4.575075605 (−68)	1.478893912 (−94)
−5.167856798 (−32)	−2.076316224 (−41)	3.057751191 (−40)		1.096783968 (−28)	5.121156565 (−75)	8.290238357 (−61)		8.573221220 (−102)
1.957423698 (−34)	8.950203585 (−45)	−1.793123195 (−43)		−9.598897916 (−31)	−5.113004100 (−84)	−1.324875346 (−66)	2.652193977 (−75)	

Table III. Products of Differences of Decay Constants of Uranium-235 Series

(In sec.$^{-1}$)

	$y \rightarrow$	λU^{235}	λTh^{231}	λPa^{231}	λAc^{227}	λTh^{227}	λFr^{223}
1	$(\lambda U^{235} - y)$		−7.509394831 (−6)	−6.403475551 (−13)	−9.984080505 (−10)	−4.313191834 (−7)	−5.501168099 (−4)
2	$1 \times (\lambda Th^{231} - y)$	7.509394831 (−6)		−4.808622210 (−18)	−7.496443435 (−15)	−3.052909808 (−12)	2.984974602 (−7)
3	$2 \times (\lambda Pa^{231} - y)$	4.808622620 (−18)	5.639100592 (−11)		7.479709146 (−24)	1.316776610 (−18)	−1.642084704 (−10)
4	$3 \times (\lambda Ac^{227} - y)$	4.800967536 (−27)	−4.234060271 (−16)	−4.797887936 (−27)		−5.666363317 (−25)	9.033367595 (−14)
				Branch via Th^{227} (99%)			
5	$4 \times (\lambda Th^{227} - y)$	2.070749397 (−33)	2.996899890 (−21)	−2.069418034 (−33)	3.218674239 (−30)		
6	$5 \times (\lambda Ra^{223} - y)$	1.483273506 (−39)	−2.035823136 (−26)	−1.482318528 (−39)	2.302316143 (−36)	−1.614793113 (−31)	
7	$6 \times (\lambda Rn^{219} - y)$	2.622772572 (−40)	−3.599655965 (−27)	−2.621083949 (−40)	4.071030443 (−37)	−2.855322865 (−32)	
8	$7 \times (\lambda Po^{215} - y)$	9.934248158 (−38)	−1.363437887 (−24)	−9.927852178 (−38)	1.541979930 (−34)	−1.081507645 (−29)	
9	$8 \times (\lambda Pb^{211} - y)$	3.179084072 (−41)	−4.260786398 (−28)	−3.177037273 (−41)	4.934513879 (−38)	−3.456295429 (−33)	
10	$9 \times (\lambda Bi^{211} - y)$	1.700287933 (−43)	−2.275621455 (−30)	−1.699193232 (−43)	2.639154123 (−40)	−1.848401411 (−35)	
				Branch via Po^{211} (99.68%)			
11	$10 \times (\lambda Po^{211} - y)$	2.266441895 (−43)	−3.033330211 (−30)	−2.264982688 (−43)	3.517927383 (−40)	−2.463872716 (−35)	
12	$11 \times (\lambda Pb^{207} - y)$	−6.982096194 (−60)	2.277847421 (−35)	1.450445902 (−55)	−3.512327129 (−48)	1.062715568 (−41)	
				Branch via Tl^{207} (0.32%)			
13	$10 \times (\lambda Tl^{207} - y)$	4.100729941 (−46)	−5.471222507 (−33)	−4.098089762 (−46)	6.365071266 (−43)	−4.457150648 (−38)	
14	$13 \times (\lambda Pb^{207} - y)$	−1.263288107 (−62)	4.108557001 (−38)	2.624328007 (−58)	−6.354938590 (−51)	1.922454578 (−44)	
				Branch via Fr^{223} (1%)			
15	$4 \times (\lambda Fr^{223} - y)$	2.641092945 (−30)	−2.297432499 (−19)	−2.639398803 (−30)	4.114706267 (−27)		
16	$15 \times (\lambda Ra^{223} - y)$	1.891809408 (−36)	1.560668159 (−24)	−1.890594207 (−36)	2.943247423 (−33)		−4.962936782 (−17)
17	$16 \times (\lambda Rn^{219} - y)$	3.345159074 (−37)	2.759507124 (−25)	−3.343010316 (−37)	5.204346023 (−34)		−8.748324676 (−18)
18	$17 \times (\lambda Po^{215} - y)$	1.267042393 (−34)	1.045215598 (−22)	−1.266228510 (−34)	1.971244684 (−31)		−3.313588948 (−15)
19	$18 \times (\lambda Pb^{211} - y)$	4.054694655 (−38)	3.266331709 (−26)	−4.052090120 (−38)	6.308210673 (−35)		7.624709368 (−19)
20	$19 \times (\lambda Bi^{211} - y)$	2.168595808 (−40)	1.744498274 (−28)	−2.167202810 (−40)	3.373856193 (−37)		3.658519359 (−21)
				Branch via Po^{211} (99.68%)			
21	$20 \times (\lambda Po^{211} - y)$	2.890684750 (−40)	2.325360093 (−28)	−2.888827918 (−40)	4.497267130 (−37)		4.874703498 (−21)
22	$21 \times (\lambda Pb^{207} - y)$	−8.905164980 (−57)	−1.746204706 (−33)	1.849042889 (−52)	−4.490107846 (−45)		−2.681656338 (−24)
				Branch via Tl^{207} (0.32%)			
23	$20 \times (\lambda Tl^{207} - y)$	5.230188133 (−43)	4.194255683 (−31)	−5.226828521 (−43)	8.137014402 (−40)		6.810951294 (−24)
24	$23 \times (\lambda Pb^{207} - y)$	−1.611233746 (−59)	−3.149632194 (−36)	3.347147884 (−55)	−8.124060937 (−48)		−3.746818798 (−27)

Table IV. Products of Differences of Decay Constants of Thorium-232 Series

(In sec.$^{-1}$)

	$y \rightarrow$	λTh^{232}	λRa^{228}	λAc^{228}	λTh^{228}	λRa^{224}
1	$(\lambda Th^{232} - y)$		−3.278354892 (−9)	−3.140960578 (−5)	−1.156051462 (−8)	−2.203993629 (−6)
2	$1 \times (\lambda Ra^{228} - y)$	3.278354892 (−9)		9.864603636 (−10)	9.574602864 (−17)	4.850362443 (−12)
3	$2 \times (\lambda Ac^{228} - y)$	1.029718348 (−13)	−1.029610872 (−13)		3.006238142 (−21)	1.416578043 (−16)
4	$3 \times (\lambda Th^{228} - y)$	1.190407402 (−21)	−8.527401702 (−22)	−3.097292715 (−14)		−3.105752610 (−22)
5	$4 \times (\lambda Ra^{224} - y)$	2.263650330 (−27)	−1.876638317 (−27)	9.045832975 (−19)	6.590976051 (−27)	
6	$5 \times (\lambda Rn^{220} - y)$	3.336836381 (−29)	−2.386763710 (−29)	1.147634632 (−20)	8.382591395 (−29)	−3.949303868 (−24)
7	$6 \times (\lambda Po^{216} - y)$	1.463872613 (−28)	−1.047075022 (−28)	5.034645657 (−20)	3.677449098 (−28)	−1.732561687 (−23)
8	$7 \times (\lambda Pb^{212} - y)$	2.659012510 (−33)	−1.901588333 (−33)	−6.668575210 (−25)	6.675553378 (−33)	−2.765210146 (−28)
9	$8 \times (\lambda Bi^{212} - y)$	5.077374722 (−37)	−3.631013668 (−37)	−1.063904694 (−28)	1.274617318 (−36)	−5.219213668 (−32)
				Branch via Po^{212} (66.3%)		
10	$9 \times (\lambda Po^{212} - y)$	1.157686833 (−30)	−8.279035808 (−31)	−2.425797825 (−22)	2.906241447 (−30)	−1.190027381 (−25)
11	$10 \times (\lambda Pb^{208} - y)$	−1.829393206 (−48)	2.714161756 (−39)	7.619335338 (−27)	−3.359764674 (−38)	2.622812766 (−31)
				Branch via Tl^{208} (33.7%)		
12	$9 \times (\lambda Tl^{208} - y)$	1.892133318 (−39)	−1.353131544 (−39)	−3.931328005 (−31)	4.749971286 (−39)	−1.943840676 (−34)
13	$12 \times (\lambda Pb^{208} - y)$	−2.989975992 (−57)	4.436045419 (−48)	1.234814628 (−35)	−5.491211250 (−47)	4.284212466 (−40)

Table III. Products of Differences of Decay Constants of Uranium-235 Series (*Continued*)

(In Sec.$^{-1}$)

λRa^{223}	λRn^{219}	λPo^{215}	λPb^{211}	λBi^{211}	λPo^{211}	λTl^{207}	λPb^{207}
−7.162979295 (−7)	−1.768232603 (−1)	−3.787689510 (2)	−3.200125487 (−4)	−5.348357874 (−3)	−1.332975347	−2.411785596 (−3)	3.080642045 (−17)
−4.865881246 (−12)	3.126513755 (−2)	1.434659154 (5)	1.000049308 (−7)	2.856476902 (−5)	1.776813266	5.798598710 (−6)	2.313375745 (−22)
3.485417546 (−18)	−5.528403555 (−3)	−5.434043428 (7)	−3.200283273 (−11)	−1.527746073 (−7)	−2.368448280	−1.398497684 (−8)	1.481435769 (−34)
−2.493117502 (−24)	9.775503353 (−4)	2.058246929 (10)	1.024127612 (−14)	8.170931214 (−10)	3.157083168	3.372875175 (−11)	1.479077444 (−43)
Branch via Th^{227} (99%)							
7.104854996 (−31)	−1.728532158 (−4)	−7.796000294 (12)	−3.272919614 (−18)	−4.369754002 (−12)	−4.208312671	−8.133196979 (−14)	6.379544753 (−50)
	3.056434536 (−5)	2.952882848 (15)	1.045030962 (−21)	2.336787818 (−14)	5.609574030	1.960970153 (−16)	4.569654698 (−56)
1.256298535 (−31)		−1.117938200 (18)	1.844513588 (−22)	4.007004630 (−15)	−6.485520722	3.420156962 (−17)	8.080212421 (−57)
4.758468774 (−29)	1.157142054 (−2)		6.986438866 (−20)	1.517707509 (−12)	−2.447868842 (3)	1.295441016 (−14)	3.060533582 (−54)
1.519361239 (−32)	−2.042393307 (−3)	4.234399216 (20)		−7.631557458 (−15)	3.262165469 (3)	−2.709768601 (−17)	9.794091520 (−58)
8.124999331 (−35)	3.502191930 (−4)	−1.603836302 (23)	3.513022721 (−22)		−4.330938919 (3)	7.957431353 (−20)	5.238230650 (−60)
Branch via Po^{211} (99.68%)							
1.083041798 (−34)	4.049066509 (−4)	6.053455195 (25)	4.681648468 (−22)	−1.013186165 (−14)			6.982432318 (−60)
−7.757805975 (−41)	−7.159691413 (−5)	−2.292860874 (28)	−1.498186258 (−25)	5.418882203 (−17)	5.773034808 (3)		
Branch via Tl^{207} (0.32%)							
1.958993643 (−37)	−6.108224592 (−5)	6.074795256 (25)	7.348446241 (−25)	2.241062007 (−17)			1.263348923 (−62)
−1.403223090 (−43)	1.080076187 (−5)	−2.300943827 (28)	−2.351595010 (−28)	−1.198600163 (−19)		−1.919161832 (−22)	
Branch via Fr^{223} (1%)							
−1.369720032 (−27)	−1.723158705 (−4)	−7.795988980 (12)	2.356561275 (−18)	−3.920609768 (−12)	−4.206577266	−6.279176432 (−14)	8.136653651 (−47)
	3.046933059 (−5)	2.952878562 (15)	−7.524411799 (−22)	2.096601580 (−14)	5.607260779	1.513952951 (−16)	5.828268163 (−53)
−2.421973806 (−28)		−1.117936578 (18)	−1.328083120 (−22)	3.595145513 (−15)	−6.482846252	2.640507668 (−17)	1.030573378 (−53)
−9.173684762 (−26)	1.153544870 (−2)		−5.030362252 (−20)	1.361710266 (−12)	−2.446859400 (3)	1.000135951 (−14)	3.903491973 (−51)
−2.929123150 (−29)	−2.036044160 (−3)	4.234393072 (20)		−6.847149450 (−15)	3.260820231 (3)	−2.092057426 (−17)	1.249166415 (−54)
−1.566390074 (−31)	3.491304736 (−4)	−1.603833975 (23)	−2.529439851 (−22)		−4.329152945 (3)	6.143477841 (−20)	6.680989032 (−57)
Branch via Po^{211} (99.68%)							
−2.087958231 (−31)	4.036479257 (−4)	6.053446412 (25)	−3.370871509 (−22)	−9.090460408 (−15)			8.905593673 (−57)
1.495600158 (−37)	−7.137434224 (−5)	−2.292857547 (28)	1.078721183 (−25)	4.861903550 (−17)	−5.770654149 (3)		
Branch via Tl^{207} (0.32%)							
−3.776675016 (−34)	−6.089236076 (−5)	6.074786442 (25)	−5.291014104 (−25)	2.010714926 (−17)			1.611311311 (−59)
2.705224494 (−40)	1.076718576 (−5)	−2.300940488 (28)	1.693190909 (−28)	−1.075402301 (−19)		−1.481675137 (−22)	

Table IV. Products of Differences of Decay Constants of Thorium-232 Series (*Continued*)

(In Sec.$^{-1}$)

λRn^{220}	λPo^{216}	λPb^{212}	λBi^{212}	λPo^{212}	λTl^{208}	λPb^{208}
−1.271829688 (−2)	−4.387007471	−1.816423428 (−5)	−1.909496365 (−4)	−2.280089409 (6)	−3.726597744 (−3)	1.580214229 (−18)
1.617550338 (−4)	1.924583454 (1)	3.298798583 (−10)	3.646113767 (−8)	5.198807713 (12)	1.388751853 (−5)	5.180503051 (−27)
−2.052167879 (−6)	−8.443101540 (1)	4.369381274 (−15)	−5.817011023 (−12)	−1.185374640 (19)	−5.131699374 (−8)	1.627175586 (−31)
2.610005661 (−8)	3.703994943 (2)	−7.931595284 (−20)	1.110688893 (−15)	2.702760162 (25)	1.912371998 (−10)	1.881098715 (−39)
−3.318907443 (−10)	−1.624944532 (3)	1.265901695 (−24)	−2.096376892 (−19)	−6.162534820 (31)	−7.122426317 (−13)	4.145929583 (−45)
	7.107977275 (3)	1.607711945 (−26)	2.626204127 (−21)	1.405113030 (38)	−6.404271459 (−15)	5.272916328 (−47)
−1.451786090 (−9)		7.053015111 (−26)	−1.152067565 (−20)	−3.203777174 (44)	−2.807172059 (−14)	2.313232332 (−46)
1.843787592 (−11)	−3.118262030 (4)		1.990604576 (−24)	7.304898403 (50)	1.041021093 (−16)	4.201809402 (−51)
−2.309776740 (−13)	1.367924339 (5)	1.218658053 (−29)		−1.665582148 (57)	−3.680684258 (−19)	8.023339780 (−55)
Branch via Po^{212} (66.3%)						
−5.266497452 (−7)	3.118983796 (11)	2.778649320 (−23)	4.538756411 (−18)			1.829393206 (−48)
6.698087811 (−9)	−1.368300521 (−12)	−5.047203723 (−28)	−8.666738868 (−22)	3.797676215 (63)		
Branch via Tl^{208} (33.7%)						
2.076881753 (−15)	−5.995996591 (5)	4.519312361 (−32)	7.038077303 (−27)			2.989975992 (−57)
−2.641439872 (−17)	2.630448184 (6)	−8.208984851 (−37)	−1.343918303 (−30)		1.371642965 (−21)	

Table V. Products of Differences of Decay Constants of Neptunium-237 Series

(In sec.$^{-1}$)

	$y \rightarrow$	λPu^{241}	λAm^{241}	λNp^{237}	λPa^{233}	λU^{233}	λTh^{229}	λRa^{225}
1	$(\lambda Pu^{241} - y)$		1.522192990 (−9)	1.568917001 (−9)	−2.912243872 (−7)	1.568791399 (−9)	1.565934481 (−9)	−5.404943710 (−7)
2	$1 \times (\lambda Am^{241} - y)$	−1.522192990 (−9)		7.330609554 (−20)	8.525494168 (−14)	7.310318326 (−20)	6.849630979 (−20)	2.929568986 (−13)
3	$2 \times (\lambda Np^{237} - y)$	2.388194461 (−18)	−7.112296233 (−20)		−2.496207607 (−20)	−9.181920776 (−33)	−2.042915937 (−31)	−1.588011797 (−19)
4	$3 \times (\lambda Pa^{233} - y)$	6.955004684 (−25)	−2.082100357 (−26)	2.146353393 (−26)		−2.688403770 (−39)	−5.981460144 (−38)	3.958436749 (−26)
5	$4 \times (\lambda U^{233} - y)$	−1.091095153 (−33)	9.702256404 (−37)	2.695867120 (−39)	7.308725597 (−27)		1.708853818 (−49)	−2.145722742 (−32)
6	$5 \times (\lambda Th^{229} - y)$	1.708583522 (−42)	−4.243911660 (−47)	8.040476807 (−51)	−2.139924119 (−33)	−7.680547786 (−51)		1.163111125 (−38)
7	$6 \times (\lambda Ra^{225} - y)$	9.234797760 (−49)	−2.300270390 (−53)	4.358447295 (−57)	−5.334188505 (−40)	−4.163342022 (−57)	9.263018226 (−56)	
8	$7 \times (\lambda Ac^{225} - y)$	7.394161774 (−55)	−1.845292873 (−59)	3.496580343 (−63)	−2.717557633 (−46)	−3.340055898 (−63)	7.431262750 (−62)	3.026303292 (−45)
9	$8 \times (\lambda Fr^{221} - y)$	1.779596890 (−57)	−4.441178911 (−62)	8.415433353 (−66)	−6.539716161 (−49)	−8.038716417 (−66)	1.788527366 (−64)	7.281948420 (−48)
10	$9 \times (\lambda At^{217} - y)$	6.852903148 (−56)	−1.710217022 (−60)	3.240629945 (−64)	−2.518325435 (−47)	−3.095563121 (−64)	6.887292782 (−63)	2.804145525 (−46)
11	$10 \times (\lambda Bi^{213} - y)$	1.684411410 (−59)	−4.203659154 (−64)	7.965367054 (−68)	−6.182591670 (−51)	−7.608797334 (−68)	1.692875006 (−66)	6.877301757 (−50)
	Branch via Po^{213} (98%)							
12	$11 \times (\lambda Po^{213} - y)$	2.779869093 (−54)	−6.937510688 (−59)	1.314564693 (−62)	−1.020344281 (−45)	−1.255718194 (−62)	2.793836992 (−61)	1.134995790 (−44)
13	$12 \times (\lambda Pb^{209} - y)$	1.662187600 (−58)	−4.148302462 (−63)	7.860479731 (−67)	−6.071304794 (−50)	−7.508605276 (−67)	1.670583279 (−65)	6.725218035 (−49)
14	$13 \times (\lambda Bi^{209} - y)$	−2.607850980 (−67)	1.938667477 (−73)	−7.847966486 (−81)	1.777637452 (−56)	1.018063877 (−79)	−4.999226787 (−77)	−3.645493868 (−55)
	Branch via Tl^{209} (2%)							
15	$11 \times (\lambda Tl^{209} - y)$	8.845035383 (−62)	−2.207389745 (−66)	4.182705844 (−70)	−3.246368966 (−53)	−3.995466984 (−70)	8.889481337 (−69)	3.610977450 (−52)
16	$15 \times (\lambda Pb^{209} - y)$	5.288777149 (−66)	−1.319914408 (−70)	2.501061734 (−74)	−1.931671087 (−57)	−2.389101680 (−74)	5.315492250 (−73)	2.139621211 (−56)
17	$16 \times (\lambda Bi^{209} - y)$	−8.297705187 (−75)	6.168487373 (−81)	−2.497080247 (−88)	5.655803795 (−64)	3.239294155 (−87)	−1.590663068 (−84)	−1.159810130 (−62)

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Determination of Surface Zinc Oxide on Zinc Sulfide Phosphors

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In an acidimetric method for the determination of surface zinc oxide on zinc sulfide phosphors the zinc oxide is determined directly by titration with weak hydrochloric acid, under which condition the zinc sulfide is not dissolved. The limit of detection of zinc oxide is approximately 0.02%.

LUMINESCENT zinc sulfide phosphor particles, and particularly electroluminescent materials, usually have a surface layer of zinc oxide, produced during synthesis. During investigations concerning the effects of oxygen on luminescence, it became necessary to determine the amount of zinc oxide surface layer.

Two-step digestion methods have been reported for zinc oxide–zinc sulfide mixtures, wherein the zinc oxide is digested, and zinc is then determined by a standard method (*2*). Shakhov (*4*) used a polarographic method to determine zinc oxide in zinc sulfide, and Rooksby (*3*) has reported detecting as little as 0.2% zinc oxide in zinc sulfide by x-ray diffraction methods. The method evolved in the present investigation takes advantage of the fact that zinc sulfide, particularly fired (crystallized) zinc sulfide, is not attacked by weak hydrochloric acid. Zinc sulfide will not dissolve above a pH of about 2.4 (*5*). Any reaction is still further slowed because the zinc sulfide is in a fired state. No hydrogen sulfide was detectable during the titration.

The zinc oxide is determined acidimetrically by direct titration with 0.1N hydrochloric acid. A weighed sample is slurried with water, and is titrated to the methyl orange end point. Per cent by weight of zinc oxide is calculated from:

$$\% \text{ ZnO} = 4.07nN/S$$

where n is milliliters of acid, N is the normality of the acid, and S is the weight of sample taken, in grams.

PROCEDURE

To 10 ml. of distilled water, add 1 drop of methyl orange indicator (0.1%) and enough acid to develop fully the pink acid color of the indicator (about 1 drop of 0.1N hydrochloric acid). Add the weighed sample, 0.5 to 1.0 gram, and stir to break up the aggregates. If there is no change in the indicator color, zinc oxide is absent.