Forecasting locally stationary time series

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Joint work with
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What do I mean by Nonstationary Time Series?

- I mean *NOT* second-order stationary.
- So, unconditional variance *changes* with time.
- Autocovariance, spectrum, etc. *change* with time.
- Typically assume $\mathbb{E}X_t = 0$ (assume mean removed).

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More interested in Variability and CI

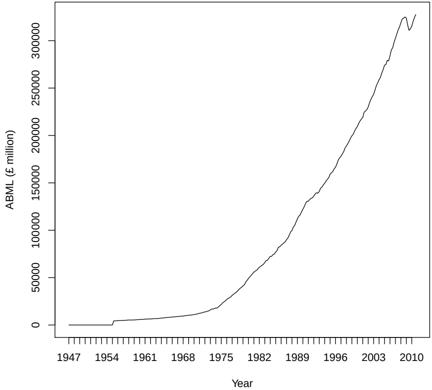
Structure of Presentation

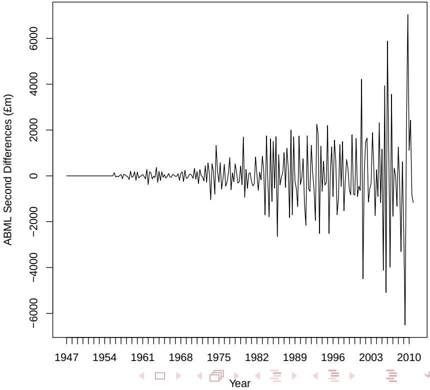
- Motivation
- Nonstationary forecasting
- The local partial autocorrelation function
- Forecasting using the lpacf

Motivation

Motivation - ABML

- ABML consists of gross value added amounts
- Component in the estimate of GDP
- 223 observations from Q1 1955 to Q3 2010
- We use second differences to remove trend
- Tests of stationarity reject H0.



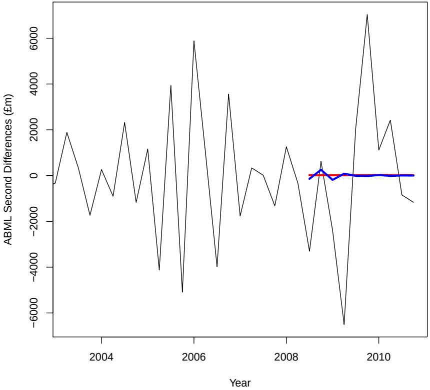


Motivation - ABML

- ONS currently use ARIMA models to forecast this data
- What is the danger in doing this?

- Red - Full series forecast
- Blue - Last 30 obs forecast

- Full fits ARMA(1,1) non-zero mean
- Last 30 obs fits AR(2)



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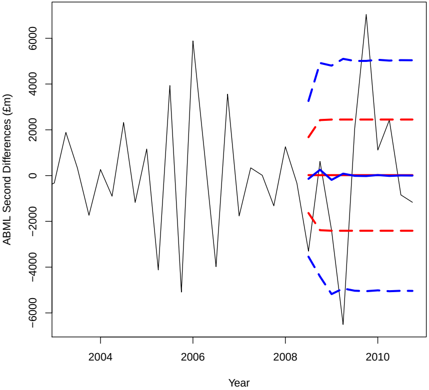
- Overconfident in forecast?



Recall: Forecasting Stationary TS— Notation

Suppose have data $x_1, \ldots, x_T$ from *stationary* series.

Want to make forecast $\hat{x}_T(h)$ made at time T for horizon h.

Want forecast using linear combination of past:

$$\begin{aligned}\hat{x}_T(1) &= \sum_{i=0}^{T-1} \varphi_i x_{T-i} \\ &= \varphi_0 x_T + \varphi_1 x_{T-1} + \varphi_2 x_{T-2} + \cdots\end{aligned}$$

Note: φ sequence DOES NOT depend on time (stationary x_t)

Theory can tell us optimal least-squares forecast (Box-Jenkins).

Extension to Nonstationary Time Series

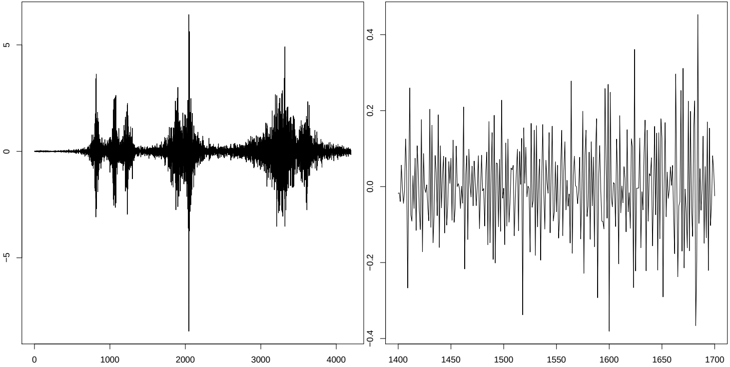
Modelling nonstationary time series

Modelling in the face of non-stationarity is no easy task!

- Various approaches have been explored, built on models that fit particular types of non-stationarity:
  - assume piecewise stationarity;
  - use parametric models with time-changing coefficients, e.g. Time Varying AR (tvAR).
- Processes with a slowly time-varying second order structure are known as locally stationary (LS).
  - Advanced LS models (ARCH) (Dahlhaus and Subba Rao, 2006).
  - Locally stationary fourier processes (Dahlhaus, 1997).
  - Locally stationary wavelet processes (Nason et al., 2000).

Locally stationary models

If you take a small enough region, it will appear stationary as the structure varies slowly over time.



Application areas include;

- medicine, finance
- environmental processes, e.g. wind speeds.

Locally stationary wavelet (LSW) processes

LSW processes (Nason *et al.*, 2000):

$$X_{t,T} = \sum_{j=-J(T)}^{-1} \sum_{k \in \mathbb{Z}} w_{j,k;T} \psi_{j,k}(t) \xi_{j,k}, \quad t = 1, \dots, T.$$

- $\{\psi_{j,k}\}$ is a collection of discrete non-decimated wavelets.
- $\{\xi_{j,k}\}_{j,k}$ is a sequence of zero-mean, orthonormal random variables.
- Smoothness of wavelet amplitudes $w_{j,k;T}$ as a function of k controls the degree of non-stationarity.
- LSW processes encapsulate other models and represent processes whose variance and autocorrelation function vary over time.
- This leads to a localised measure of autocovariance $c(t, \tau)$.

Forecasting in LSW framework

Given observations $x_0, \ldots, x_{t-1}$, we:

- Predict $\hat{x}_t = \sum_{s=t-p}^{t-1} b_{t-1-i,T} X_s$, where

- p is the number of latest observations used for prediction and
- $\mathbf{b}$ is the solution to localized Yule-Walker equations.

$$\begin{bmatrix} c(t,1) \\ c(t,2) \\ \vdots \\ c(t,p) \end{bmatrix} = \begin{bmatrix} c(t-1,0) & c(t-2,-1) & \cdots \\ c(t-1,1) & c(t-2,0) & \cdots \\ \vdots & \vdots & \ddots \\ c(t-1,p) & c(t-2,p-1) & \cdots \end{bmatrix} \begin{bmatrix} b_{t-1} \\ b_{t-2} \\ \vdots \\ b_{t-1-p} \end{bmatrix}$$

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These require knowledge of the covariance structure at time t which is the point we are trying to predict.

- The covariance at time t is similar to that at time $t-1$.
- We extrapolate at the smoothing step.

Previous nonstationary forecasting work

Fryzlewicz et al. (2003) propose a method for LSW forecasting:

- smooth and extrapolate the covariances directly by kernel smoothing;
- choose p (and bandwidth) by in sample optimization.

For a practitioner this leaves many questions.

- How much data do I train on?
- Do I update p and bandwidth simultaneously or another method?
- When updating, how many alternative options do I consider?
- What kernel smoother should I use?

Ultimately this p is hard to choose.

Our approach

Our work:

- introduces *localised partial* acf
- shows that local partial ACF is an interesting tool in its own right
- gives encouraging forecasting results

Thus we,

- mirror the stationary process and
- produce a data driven approach for practitioners.

The local partial autocorrelation function (lpacf)

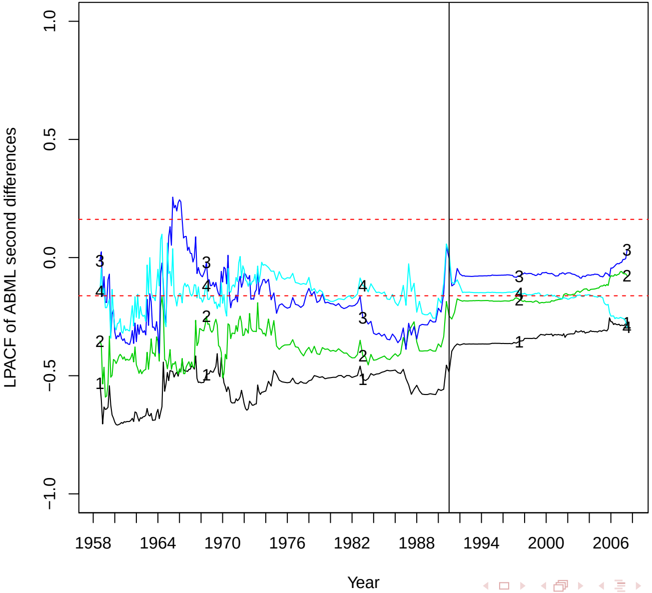
We define the lpacf at time t and lag τ as:

$$q(t,\tau) = \text{Corr}(X_t, X_{t-\tau} | \{X_{t-1} \ldots, X_{t-\tau+1}\})$$

$$q_T(t,\tau) = \begin{cases} \frac{c\left(\frac{t-1}{T},1\right)}{\sqrt{c\left(\frac{t}{T},0\right)c\left(\frac{t-1}{T},0\right)}}, & \text{for } \tau = 1 \\ \varphi_{t,\tau,\tau}\sqrt{\frac{\text{MSPE}(\hat{X}_t, X_t | X_{t-1} \ldots, X_{t-\tau+1})}{\text{MSPE}(\hat{X}_{t-\tau}, X_{t-\tau} | X_{t-1} \ldots, X_{t-\tau+1})}}, & \text{for } \tau \geq 2. \end{cases}$$

For stationary models the square root equals 1 and the $\varphi_{t,\tau,\tau}$ is the usual estimate of the pacf.

ABML lpacf



LPACF of ABML second differences
1.0
0.5
0.0
−0.5
−1.0
1958
1964
1970
1976
1982
1988
1994
2000
2006
Year

Forecasting Simulations

Comparisons with ARMA

Range of models considered:

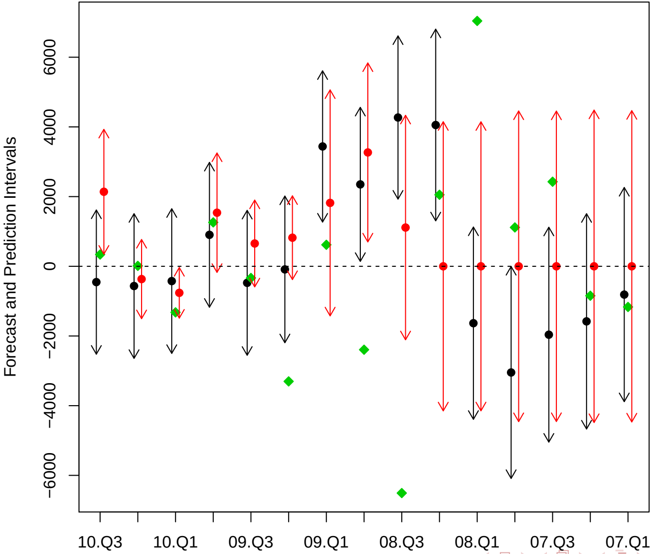
- TVAR(1)
- TVAR(2)
- TVAR(12)
- TVMA(1)
- TVMA(2)
- Uniformly modulated white noise
- LSW process

Summary

lpacf greatly improves on ARMA in 2/3 of cases
comparable results (ratio ± 0.05) in 1/3.

AMBML - lpacf forecasting

RMSE: lpacf=3430m, B-J=4290m, Fryzlewicz=8830m, 11/15, 8/15, 6/15



Summary

- Motivated why forecasting nonstationary time series is important.
- Proposed a new measure – the local partial autocorrelation function – and associated theoretical justification.
- Used the lpacf to choose p for the localised Yule-Walker equations.
- Showed increased forecasting performance when using the lpacf.

- We have used the lpacf as a tool for forecasting but it can be used in a variety of settings.

References I

R. Dahlhaus and S. Subba Rao.
Statistical inference for time-varying ARCH processes.
Annals of Statistics, 34(3):1075–1114, 2006.

R. Dahlhaus.
Fitting Time Series Models to Nonstationary Processes.
Annals of Statistics, 25(1):1–37, 1997.

G.P. Nason, R. von Sachs and G. Kroisandt.
Wavelet Processes and Adaptive Estimation of the Evolutionary Wavelet Spectrum.
JRSSB, 62(2):271–292, 2000.

P. Fryzlewicz, S. Van Bellegem and R. von Sachs.
Forecasting non-stationary time series by wavelet process modelling.
Ann. Inst. Statist. Math., 55(4):737–764, 2003.