姓名 学号 年级 专业、班 学院

密 封 线

# 重庆大学《算法分析与设计》课程试卷

A卷 B卷

**2014 — 2015 学年 第 1 学期**

开课学院：计算机学院 课程号：18016435 考试日期：2013.12.26

考试方式：开卷 闭卷 其他 考试时间：120 分钟

| 题 号 | 一 | 二 | 三 | 四 | 五 | 六 | 七 | 八 | 九 | 十 | 总 分 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 得 分 | | | | | | | | | | | |

## 考试提示

1.严禁随身携带通讯工具等电子设备参加考试；

2.考试作弊，留校察看，毕业当年不授学位；请人代考、替他人考试、两次及以上作弊等，属严重作弊，开除学籍。

## 一、（15 分）算法复杂度渐进分析

(1) Sort the follwing items ascendingly (升序) by their asymptoticy (5 分)

$(\log(n))^{0.5}$ 100000 $(2.00001)^n$ $1.001^{n*(n+1)}$ n+1/n+log(n)

(2) Work out the following function by expansion method (展开法) (10 分)

T(n) = 2T(n/2)+n, T(1) = Θ(1).

## 二、 （20 分）快速排序

(1) What is the best time(最好时间) complexity to sort an array with quicksort? Describe such a situation briefly(简短叙述). (3 分)

(2) What is the worst time(最坏时间) complexity to sort the above array with quicksort? Describe such a situation briefly. (3 分)

(3) During each iteration, if we assume that the initial array is split(划分) according to the ration 1:9, then

A) write the recurrence(递推函数) of this situation; (3 分)

B) draw out the corresponding recursion tree; (3 分)

C) prove the tight bound (Θ) of your recurrence with substitution(替代法). (8 分)

## 三、 （15 分）动态规划（钢管切割问题）

For rod-cutting problem, we have following prices list:

| length $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| price $p_i$ | 1 | 5 | 8 | 9 | 10 | 17 | 17 | 20 | 24 | 30 |

Let r(n) be the maximum revenue(最大收入) by cutting up a rod of length n and selling the pieces.

(1) Give the recursive formula of r(n). (5 分)

(2) Write the algorithm into a program in pseudo code or other programming languages. (5 分)

(3) Work out r(1), r(2),…, r(8). (5 分)

## 四、 （10 分）贪心算法（Huffman Encoding）

Given the frequencies of 6 letters to be encoded as a:40%, b:13%; c:8%; d:%16; e:%9; f:14%,

(1) give the corresponding Huffman tree; (5 分)

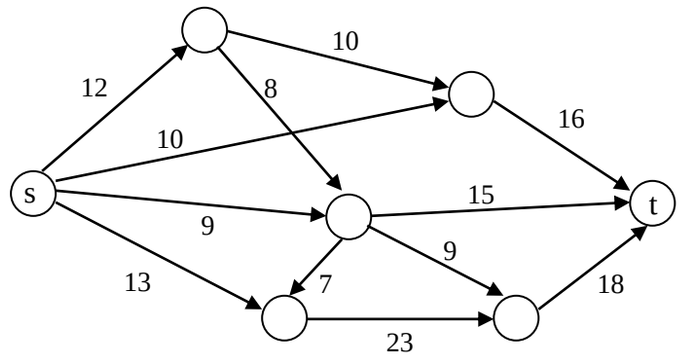
(2) write the corresponding binary codes(2 进制码) for each letter, and compute the averaged coding length(编码长度的平均值). (5 分)

## 五、 （20 分）动态规划（文件配置问题）

Suppose we want to replicate(复制) a file over a collection(集合) of n servers(服务器), labeled $S_1$, $S_2$, … , $S_n$. To place a copy of the file at server $S_i$ results in a placement cost(配置代价) of $\mathbf{c_i}$ ($c_i$ is a positive integer). Now if a user requests the file from server $S_i$, and no copy of the file is present at $S_i$, then servers $S_{i-1}$,$S_{i-2}$,$S_{i-3}$ … are searched in order until a copy of the file is finally found, say at

命题人：陈波 组题人：夏云霓 审题人：李佳 命题时间：2015.1.6 教务处制

server $S_j$, where j < i. This results in an access cost(搜索代价) of **i - j**. The access cost is 0 if $S_i$ holds a copy of file. We will require that a copy of the file be placed at server $S_1$, so that all such searches will terminate, at the latest at $S_1$.
We'd like to place copies of the files at servers so as to minimize(最小化) the sum of placement and access costs(配置代价与搜索代价之和). Formally, we say that a configuration is a choice, for each server $S_i$ with i = 2, 3, …n, of whether to place a copy of the file at $S_i$ or not. (Recall that a copy is always placed at $S_1$.) The total cost of a configuration is the sum of all placement costs for servers with a copy of the file, plus the sum of all access costs associated with all n servers.
For simplicity, let W(i) denote the minimal sum of all placement costs and access costs for servers $S_1,S_2,\ldots,S_i$ ($1 \leqslant i \leqslant n$). So W(n) is the optimal solution of the entire problem.
(1) If in the optimal solution for servers $S_1,S_2,\ldots,S_i$ (i>1), $S_i$ is placed with a copy of the file. In this case, W(i) = _____ . (2分)
(2) Assume servers $S_j, S_{j+1},\ldots,S_{i-1},S_i$ (j<i) and only $S_j$ has a placement of a copy of the file. Work out the total access costs of servers $S_{j+1},\ldots,S_i$ to $S_j$. (3分)
(3) Design a dynamic programming algorithm to solve W(n), write down your brief idea and the complete recursive formula of W(i). (10分)
(4) Analyze the computational complexity of your algorithm. (5分)




## 六、 （20 分）最大流

Let G=(V, E) be a flow network and | *f* | be the value of a flow *f* on G, i.e., | *f* | = *f*(s,V) with s being source of G.

(1) Prove | *f* | = *f*(V, t) with t being sink of G. (5 分)
(2) Work out the maximum flow of the following flow network, where the positive integers denote the capacities of each edge respectively. During each iteration, you should draw the residue network(剩余流量图) and find out an augmenting path (增广路径) (if exists). (15 分)