

# 重庆大学《算法分析与设计》课程试卷

☒ A卷  
☐ B卷

2016—2017 学年 第 1 学期

开课学院：计算机学院 课程号：18016435 考试日期：\_\_\_\_\_

考试方式： ☐ 开卷 ☒ 闭卷 ☐ 其他 考试时间：120 分钟

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											

## 一、(15 分) 算法分析基础

(1) Prove the following two equations (10 分)

$$O(f(n)) + O(g(n)) = O(\min\{f(n) + g(n)\})$$

$$\Theta(f(n)) + \Theta(g(n)) = \Theta(f(n) + g(n))$$

(2) Sort the following items ascendingly (升序) by their asymptoticity (5 分)

$$2^{(n/\log n)} \quad n^2 \cdot n^{1.9} \quad n^{1.9} \quad \log(\log n + n^{1.9}) \quad n^{(n/\log n)}$$

## 二、(20 分) 算法分析应用

(1) Draw out the recursion tree (递归树) of equation  $T(n) = 3T(n/4) + n^2$  (7 分), and prove it by substitution(替代法) (10 分)

(2) What is the time complexity of the following code? (3 分)

```
int find_max( int *array, int n ) {
    max = array[0];
```

```
for ( int i = n-10; i < n; ++i ) {
    if ( array[i] > max ) {
        max = array[i];
    }
}
return max;
}
```

## 三、(15 分) 动态规划 (矩阵列相乘问题)

Given a sub-sequence of matrices  $A_i A_{i+1} \dots A_j$ , and dimensions  $p_{i-1} p_i \dots p_j$  where  $A_x$  is of dimension  $p_{x-1} \times p_x$ , We try to determine the best multiplication order for minimum multiply operations.

(1) Let  $m[i, j]$  denote the minimum number of multiplications needed to compute the said sequence. Please fill out the following recursive formula for  $m[i, j]$ . (5 分)

$$m[i, j] = \begin{cases} & i == j \\ & i < j \end{cases}$$

(2) We have following given sequence of matrices

matrix	A1	A2	A3	A4	A5
dimension	30 x 35	35 x 10	10 x 5	5 x 15	15 x 20

Fill out the table and write down the best parenthesization (最佳括号打法) according to your computation, like  $(A1A2)((A3A4)A5)$ .

命题人：陈波、张敏

组题人：夏云霓

审题人：李佳

命题时间：2016.12.30

教务处制

j	1	2	3	4	5	
	0	10500	7000(1)	9625(3)	11500(3)	1
		0	1750	4375(3)	6000(3)	2
			0	750	2500(4)	3
				0	1500	4
					0	5
						i

#### 四、 (10 分) 贪心算法 (0-1 背包问题)

There are  $n$  objects and a knapsack with capacity  $W$  kilograms. Prove whether the following locally optimal choice satisfies greedy property (局部最优选择满足贪心原则).

- (1) If each object has the same weight as its value(重量与价值一样), select the object with maximal weight. (2 分)
- (2) If each object has the same weight as its value, select the object with minimal weight. (2 分)
- (3) If all objects have the same value, select the object with minimal weight. (6 分)

#### 五、 (20 分) 动态规划 (应用题)

(Longest palindrome subsequence) A *palindrome* is a nonempty string over some alphabet that reads the same forward and backward (顺读和倒读都一样), such as *c*, *cc*, *civic*, *racecar*, *aibohphobia*. Give an efficient algorithm to find the longest palindrome that is a subsequence(子序列) of a given input string (sequence). For example, given the input character, the optimal solution is *carac* with length 5.

1. For a given sequence  $A[1..n]$ , let  $L[i,j]$  be the length of the longest palindromic subsequence of the sequence from character  $i$  to character  $j$ . Fill out the following recursive formula (8分)

$$L[i,j] = \begin{cases} 1 & i = j \\ i + 1 = j & A[i] = A[j] \\ A[i] <> A[j] \end{cases}$$

2. Write a program in pseudo code or other popular programming language based on above algorithm. (7 分)
3. Analyze the computational complexity of your algorithm. (5 分)

#### 六、 (20 分) 最大流

Let  $G=(V, E)$  be a flow network and  $|f|$  be the value of a flow  $f$  on  $G$ , i.e.,  $|f|=f(s,V)$  with  $s$  being source of  $G$ .

- (1) Prove  $f(V, t) = |f|$  where  $t$  is sink of  $G$ . (5 分)
- (2) Work out the maximum flow of the following flow network by Edmond-Karp algorithm, where the positive integers denote the capacities (容量) of each edge respectively. During each iteration, you should draw the residue network(画出剩余流量图) and find out an shortest augmenting path (找出最短增广路径) (if exists) using Breadth-First Search (广度优先遍历). (15 分)

