# DS Problem Set No.2

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#### 1 Part 8.1

Problem 2: Part A)

$$n = 4, k = 19 \begin{pmatrix} 4+19-1\\19 \end{pmatrix} = \begin{pmatrix} 21\\19 \end{pmatrix}$$

Part C)

$$y_3 = x_3 - 3 \to 0 \le y_3 \le 4$$

$$y_4 = x_4 - 3 \to 0 \le y_4 \le 5$$

$$c_1 : x_1 > 5$$

$$c_2 : x_2 > 6$$

$$c_3 : y_3 > 4$$

$$c_4 : y_4 > 5$$

$$N(\bar{c}_1 \bar{c}_2 \bar{c}_3 \bar{c}_4)$$

$$= |S| - (N(c_1) + N(c_2) + N(c_3) + N(c_4)) + (N(c_1c_2) + N(c_2c_3) + N(c_3c_4) + N(c_1c_3) + N(c_2c_4) + N(c_1c_4))$$

$$- (N(c_1c_2c_3) + N(c_1c_3c_4) + N(c_2c_3c_4) + (c_1c_2c_4)) + (N(c_1c_2c_3c_4))$$

$$= \begin{pmatrix} 21 \\ 4 \end{pmatrix} - \begin{pmatrix} 16 \\ 4 \end{pmatrix} + \begin{pmatrix} 15 \\ 4 \end{pmatrix} + \begin{pmatrix} 17 \\ 4 \end{pmatrix} + \begin{pmatrix} 16 \\ 4 \end{pmatrix}$$

$$+ \begin{pmatrix} \begin{pmatrix} 10 \\ 4 \end{pmatrix} + \begin{pmatrix} 12 \\ 4 \end{pmatrix} + \begin{pmatrix} 11 \\ 4 \end{pmatrix} + \begin{pmatrix} 11 \\ 4 \end{pmatrix} + \begin{pmatrix} 11 \\ 4 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \end{pmatrix} + \begin{pmatrix} 12 \\ 4 \end{pmatrix}$$

Problem 3:

$$N = S_0 = \frac{11!}{(2!)^3}$$

$$N(c_1) = \frac{9!}{(2!)^2}, \begin{pmatrix} 6 \\ 1 \end{pmatrix} \left[ \frac{9!}{(2!)^2} \right]$$

$$N(c_1c_2) = N(c_1c_3) = N(c_1c_6) = N(c_2c_4) = N(c_2c_5) = N(c_3c_4) =$$

$$N(c_3c_4) = N(c_3c_5) = N(c_4c_6) = N(c_5c_6) = 0$$

$$N(c_1c_4) = \frac{7!}{2!}, S_2 = 6 * \frac{7!}{2!}, S_3 = S_4 = S_5 = S_6 = 0$$

$$N(\bar{c}_1\bar{c}_2\bar{c}_3\bar{c}_4\bar{c}_5\bar{c}_6) = S_0 - S_1 + S_2 = \frac{11!}{(2!)^3} - \begin{pmatrix} 6 \\ 1 \end{pmatrix} \left[ \frac{9!}{(2!)^2} \right] + 6 * \frac{7!}{2!}$$

 $-\left(\left(\begin{array}{c}6\\4\end{array}\right)+\left(\begin{array}{c}5\\4\end{array}\right)+\left(\begin{array}{c}6\\4\end{array}\right)+\left(\begin{array}{c}5\\4\end{array}\right)\right)+0$ 

Problem 5:

$$(x_1x_2x_3x_4x_5x_6x_7)_10 \rightarrow x_1 + x_2 + \dots + x_7 = 31, 0 \le x_i \le 9(1 \le i \le 7)$$

$$\rightarrow \left(\begin{array}{c} 37\\ 31 \end{array}\right) - \left(\begin{array}{c} 7\\ 1 \end{array}\right) \left(\begin{array}{c} 27\\ 21 \end{array}\right) + \left(\begin{array}{c} 7\\ 2 \end{array}\right) \left(\begin{array}{c} 17\\ 11 \end{array}\right) - \left(\begin{array}{c} 7\\ 3 \end{array}\right) \left(\begin{array}{c} 7\\ 1 \end{array}\right)$$

Problem 8:

$$x_1 + x_2 + x_3 + x_4 = 9, 1 \le i \le 4 \rightarrow \begin{pmatrix} 12 \\ 9 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 8 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Problem 11:

$$\frac{\left[6^{8} - \left(\begin{array}{c}6\\1\end{array}\right)5^{8} + \left(\begin{array}{c}6\\2\end{array}\right)4^{8} - \left(\begin{array}{c}6\\3\end{array}\right)3^{8} + \left(\begin{array}{c}12\\9\end{array}\right)2^{8} - \left(\begin{array}{c}12\\9\end{array}\right)\right]}{6^{8}}$$

Problem 12:

$$10^9 - \left(\begin{array}{c} 3\\1 \end{array}\right) (9^9) + \left(\begin{array}{c} 3\\2 \end{array}\right) (8^9) - \left(\begin{array}{c} 3\\3 \end{array}\right) (7^9)$$

Problem 15:

$$c_i \to x_i > 6$$

$$N(\bar{c}_1\bar{c}_2\bar{c}_3\bar{c}_4\bar{c}_5) = S_0 - S_1 + S_2 = \begin{pmatrix} 19\\15 \end{pmatrix} - \begin{pmatrix} 5\\1 \end{pmatrix} \begin{pmatrix} 13\\9 \end{pmatrix} + \begin{pmatrix} 5\\2 \end{pmatrix} \begin{pmatrix} 7\\3 \end{pmatrix}$$
$$= 3876 - 5*715 + 10*35 = 3876 - 3575 + 350 = 651$$

and the answer is:

$$\frac{651}{7776} = 0.08372$$

#### 2 Part 8.2

Problem 2: Part A) 1.

$$E_2 = S_2 - \begin{pmatrix} 3 \\ 1 \end{pmatrix} S_3 + \begin{pmatrix} 4 \\ 2 \end{pmatrix} S_4 = 544320 - (3)(80640) + (6)(5040) = 332640$$

2.

$$L_2 = S_2 - \begin{pmatrix} 2 \\ 1 \end{pmatrix} S_3 + \begin{pmatrix} 3 \\ 1 \end{pmatrix} S_4 = 398160$$

Part B) 1.

$$E_3 = S_3 - \begin{pmatrix} 4 \\ 1 \end{pmatrix} S_4 = 60480$$

2.

$$L_3 = S_3 - \begin{pmatrix} 3 \\ 2 \end{pmatrix} S_4 = 65520$$

Problem 4:

$$E_3 = S_3 - \left(\begin{array}{c}4\\1\end{array}\right) S_4 + \left(\begin{array}{c}5\\2\end{array}\right) S_5 - \left(\begin{array}{c}6\\3\end{array}\right) S_6 + \left(\begin{array}{c}7\\4\end{array}\right) S_7 = \left(\begin{array}{c}7\\3\end{array}\right) 4^{10} - \left(\begin{array}{c}4\\1\end{array}\right) \left(\begin{array}{c}7\\4\end{array}\right) 3^{10}$$

$$+ \begin{pmatrix} 5 \\ 2 \end{pmatrix} \begin{pmatrix} 7 \\ 5 \end{pmatrix} 2^{10} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} \begin{pmatrix} 7 \\ 6 \end{pmatrix} 1^{10} + \begin{pmatrix} 7 \\ 4 \end{pmatrix} \begin{pmatrix} 7 \\ 7 \end{pmatrix} 0^{10} = 28648200$$

$$L_3 = S_3 - \begin{pmatrix} 3 \\ 2 \end{pmatrix} S_4 + \begin{pmatrix} 4 \\ 2 \end{pmatrix} S_5 - \begin{pmatrix} 3 \\ 2 \end{pmatrix} S_6 + \begin{pmatrix} 6 \\ 2 \end{pmatrix} S_7$$

$$= \begin{pmatrix} 7 \\ 3 \end{pmatrix} 4^{10} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{pmatrix} 7 \\ 4 \end{pmatrix} 3^{10} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 7 \\ 5 \end{pmatrix} 2^{10} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} \begin{pmatrix} 7 \\ 6 \end{pmatrix} 1^{10}$$

Problem 5:

Problem 7: Part A)

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}) = \begin{pmatrix} 52\\13 \end{pmatrix} - \begin{pmatrix} 4\\1 \end{pmatrix} \begin{pmatrix} 39\\13 \end{pmatrix} + \begin{pmatrix} 4\\2 \end{pmatrix} \begin{pmatrix} 26\\13 \end{pmatrix} - \begin{pmatrix} 4\\3 \end{pmatrix} \begin{pmatrix} 13\\13 \end{pmatrix}$$

and the answer is:

$$\frac{N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4})}{\begin{pmatrix} 52\\13 \end{pmatrix}}$$

Part B)

$$E_1 = S_1 + \begin{pmatrix} 2 \\ 1 \end{pmatrix} S_2 + \begin{pmatrix} 3 \\ 2 \end{pmatrix} S_3 - \begin{pmatrix} 4 \\ 3 \end{pmatrix} S_4$$
$$= \begin{pmatrix} 39 \\ 13 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 26 \\ 13 \end{pmatrix} + 3 \begin{pmatrix} 4 \\ 3 \end{pmatrix} \begin{pmatrix} 13 \\ 13 \end{pmatrix} - 0$$

and the answer is:

$$E_1/\left(\begin{array}{c}52\\13\end{array}\right)$$

Part C)

$$E_2 = S_2 - \begin{pmatrix} 3 \\ 1 \end{pmatrix} S_3 = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 26 \\ 13 \end{pmatrix} - 3 \begin{pmatrix} 13 \\ 13 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

and the answer is:

$$E_2/\left(\begin{array}{c}52\\13\end{array}\right)$$

Problem 8: Part B)

$$L_{t-1} = L_t + E_{t-1}, E_{t-1} = S_{t-1} - tS_t$$

Part C)

$$L_{t-1} = L_t + E_{t-1} = S_t + S_{t-1} - tS_t = S_{t-1} - (t-1)S_t = S_{t-1} - \begin{pmatrix} t-1 \\ t-2 \end{pmatrix} S_t$$

Part D)

$$L_m = L_{m+1} + E_m$$

$$L_{t-1} = S_{t-1} - \begin{pmatrix} t - 1 \\ t - 2 \end{pmatrix} S_t, L_t = S_t$$

$$L_{k+1} = S_{k+1} - \begin{pmatrix} k+1 \\ k \end{pmatrix} S_{k+2} + \begin{pmatrix} k+2 \\ k \end{pmatrix} S_{k+3} - \dots + (-1)^{t-k-1} \begin{pmatrix} t - 1 \\ k \end{pmatrix} S_t$$

$$L_k = L_{k+1} + E_k = \left[ S_{k+1} - \begin{pmatrix} k+1 \\ k \end{pmatrix} S_{k+2} + \begin{pmatrix} k+2 \\ k \end{pmatrix} S_{k+3} - \dots + (-1)^{t-k-1} \begin{pmatrix} t - 1 \\ k \end{pmatrix} S_t \right] + \left[ S_k - \begin{pmatrix} k+1 \\ 1 \end{pmatrix} S_{k+1} + \begin{pmatrix} k+2 \\ 2 \end{pmatrix} S_{k+2} - \dots + (-1)^{t-k} \begin{pmatrix} t \\ t-k \end{pmatrix} S_t \right]$$

and then we have:

$$L_k = S_k - \binom{k}{k-1} S_{k+1} + \binom{k+1}{k-1} S_{k+2} - \dots + (-1)^{t-k} \binom{t-1}{k-1} S_t$$

#### 3 Part 8.3

Problem 3:

$$\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 5! \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 5*4*3 - 5*4 + 5 - 1 = 60 - 20 + 5 - 1 = 44$$

Problem 4:

$$7!\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!}\right) = 1854$$

and the answer is:

$$5040 - 1854 = 3186$$

Problem 5: Part A)

$$(d_7 = (7!)e^{-1})7! - d_7$$

Part B)

$$d_{26} = (26!)e^{-1}$$

Problem 8: Part A)

$$(4!)d_4 = (4!)^2 e^{-1}$$

Part B,C)

$$\frac{2*3^2*6}{(4!)^2e^{-1}}$$

Problem 10: Part A) 1.

$$\frac{d_n}{n!}$$

2.

$$\frac{n(d_{n-1})}{n!}$$

$$1-\frac{d_n}{n!}$$

4.

$$\frac{\binom{n}{r}d_{n-r}}{n!}$$

Part B) 1.

$$e^{-1}$$

2.

$$e^{-1}$$

3.

$$1-e^{-1}$$

4.

$$\frac{1}{r!} * e^{-1}$$

Problem 11: Part A)

$$(d_10)^2$$

Part B)

$$N(\bar{c_1}\bar{c_2}\cdots \bar{c_{10}}) = (10!)^2 - \begin{pmatrix} 10\\1 \end{pmatrix} (9!)^2 + \begin{pmatrix} 10\\2 \end{pmatrix} (8!)^2 - \cdots + (-1)^{10} \begin{pmatrix} 10\\0 \end{pmatrix} (0!)^2$$

Problem 13:

$$n! = \begin{pmatrix} n \\ 0 \end{pmatrix} d_0 + \begin{pmatrix} n \\ 1 \end{pmatrix} d_1 + \begin{pmatrix} n \\ 2 \end{pmatrix} d_2 + \dots + \begin{pmatrix} n \\ n \end{pmatrix} d_n = \sum_{k=0}^n \begin{pmatrix} n \\ k \end{pmatrix} d_k$$

Problem 14: Part A)

$$N(c_i) = (n-1)!$$

$$N(c_i c_j) = (n-2)!$$

$$N(c_i c_j c_k) = (n-3)!$$

$$N(c_1 c_2 \cdots c_{n-1}) = (n-(n-1))!$$

$$N(\bar{c}_1 \bar{c}_2 \cdots \bar{c}_{n-1}) =$$

$$n! - \binom{n-1}{1}(n-1)! + \dots + (-1)^k \binom{n-1}{k}(n-k)! + \dots + (-1)^{n-1} \binom{n-1}{n-1}(n-(n-1))!$$

Part B)

$$\begin{bmatrix} n! - \binom{n}{1} (n-1)! + \binom{n}{2} (n-2)! - \dots + (-1)^n \binom{n}{n} (n-n)! \end{bmatrix}$$

$$+ \left[ (n-1)! - \binom{n-1}{1} (n-2)! + \binom{n-1}{2} (n-3)! - \dots + (-1)^{n-1} \binom{n-1}{n-1} ((n-1) - (n-1))! \right]$$

and then the coefficient of (n-k)! in  $d_n + d_{n-1}$ :

$$(-1)^k \binom{n}{k} + (-1)^{k-1} \binom{n-1}{k-1} =$$

$$(-1)^{k-1} [[(n-1)!/[(k-1)!(n-k)!]] - [n!/[k!(n-k)!]]$$

$$= (-1)^{k-1} [[k(n-1)!-n!]/[k!(n-k)!]]$$

$$= (-1)^{k-1} (n-1)![(k-n)/[k!(n-k)!]]$$

$$(-1)^k (n-1)!/[k!(n-k-1)!] = (-1)^k \binom{n-1}{k}$$

Problem 15:

$$\left(\begin{array}{c} n \\ 0 \end{array}\right)(n-1)! - \left(\begin{array}{c} n \\ 1 \end{array}\right)(n-2)! + \left(\begin{array}{c} n \\ 2 \end{array}\right)(n-3)! - \dots + (-1)^{n-1} \left(\begin{array}{c} n \\ n-1 \end{array}\right)(0!) + (-1)^n \left(\begin{array}{c} n \\ n \end{array}\right)$$

### 4 Supplementary Exercises

Problem 2:

$$n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 = 37, 0 \le n_i \le 9 (1 \le i \le 6), 0 \le n_7 \le 37 \rightarrow \begin{pmatrix} 43 \\ 37 \end{pmatrix} - \begin{pmatrix} 6 \\ 1 \end{pmatrix} \begin{pmatrix} 33 \\ 27 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

Problem 6: Part A)

$$N(\bar{c}_1\bar{c}_2\bar{c}_3\bar{c}_4\bar{c}_5) = k^5 - \left(\begin{array}{c} 5 \\ 1 \end{array}\right)k^4 + \left(\begin{array}{c} 5 \\ 2 \end{array}\right)k^3 - \left(\begin{array}{c} 5 \\ 3 \end{array}\right)k^2 + \left(\begin{array}{c} 5 \\ 4 \end{array}\right)k - \left(\begin{array}{c} 5 \\ 5 \end{array}\right)k$$

Part B)

$$k = 1, 2 \to 0, k = 3 \to 30$$

Part C)

$$k^6 - \left(\begin{array}{c} 6 \\ 1 \end{array}\right) k^5 + \left(\begin{array}{c} 6 \\ 2 \end{array}\right) k^4 - \left(\begin{array}{c} 6 \\ 3 \end{array}\right) k^3 + \left(\begin{array}{c} 6 \\ 4 \end{array}\right) k^2 - \left(\begin{array}{c} 6 \\ 5 \end{array}\right) k + \left(\begin{array}{c} 6 \\ 6 \end{array}\right) k$$

Problem 8:

Problem 9: Part A)

$$T = \frac{13!}{(2!)^5}$$

$$S_5 = \begin{pmatrix} 5 \\ 5 \end{pmatrix} (8!), S_4 = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \left(\frac{9!}{2!}\right), S_3 = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \left[\frac{10!}{(2!)^2}\right]$$

$$E_3 = \frac{\left[S_3 + \begin{pmatrix} 4 \\ 1 \end{pmatrix} S_4 + \begin{pmatrix} 5 \\ 2 \end{pmatrix} S_5\right]}{T}$$

$$E_5 = S_5, E_4 = [S_4 - \begin{pmatrix} 5 \\ 1 \end{pmatrix} S_5]$$

and the answer is:

$$\frac{[T - (E_4 + E_5)]}{T}$$

Problem 10:

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}) = \frac{16!}{(4!)^2} - \left(\begin{array}{c} 4 \\ 1 \end{array}\right) \left[\frac{13!}{(4!)^3}\right] + \left(\begin{array}{c} 4 \\ 2 \end{array}\right) \left[\frac{10!}{(4!)^2}\right] - \left(\begin{array}{c} 4 \\ 3 \end{array}\right) \left(\frac{7!}{4!}\right) + \left(\begin{array}{c} 4 \\ 4 \end{array}\right) (4!)$$

Problem 11: Part A)

$$\left(\begin{array}{c} n-m\\r-m \end{array}\right) = \left(\begin{array}{c} n-m\\n-r \end{array}\right)$$

Part B)

$$\begin{pmatrix} n-m \\ n-r \end{pmatrix} = N(\bar{c_1}\bar{c_2}\cdots\bar{c_m}) = \sum_{i=0}^m (-1)^i \begin{pmatrix} m \\ i \end{pmatrix} \begin{pmatrix} n-i \\ r \end{pmatrix}$$