

In The Name of God

DS Problem Set No.1

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1 Part 1.1-1.2

Problem 4: Problem 7:

$$2^9$$

Problem 9: Part A)

$$(14)(12) = 168$$

Part B)

$$(14)(12)(6)(18) = 18144$$

Part C)

$$(8)(18)(6)(3)(14)(12)(14)(12) = 73156608$$

Problem 10:

$$(14)(15!)$$

Problem 15:

$$\underline{e}\underline{x}\underline{e}\underline{x}\underline{e}\underline{x}\underline{e}\underline{x}\underline{e} \Rightarrow 4!$$

Problem 16: Part A)

$$40^{25}$$

Part B)

$$40 * 30 * 30 * \dots * 30 * 30 * 40 = (40^2)(30^{23})$$

Problem 23: Part A)

$$\frac{12!}{3!2!2!2!}$$

Part B)

$$\frac{11!}{3!2!2!2!} + \frac{11!}{3!2!2!2!}$$

Part C)

$$\left(\frac{7!}{2!2!}\right)\left(\frac{6!}{2!3!}\right)$$

Problem 24:

$$\frac{6!}{2!} + \frac{6!}{2!2!} + \frac{6!}{2!2!} = \left(\frac{6!}{2!}\right)\left(1 + \frac{1}{2} + \frac{1}{2}\right) = 6!$$

Problem 25:

$$\frac{12!}{4!3!2!3!} = 277200$$

Problem 28:

$$\frac{14!}{7!7!} = \binom{14}{7}$$

Problem 31: Part A-B)

$$12 * 6 * 8 = 576$$

Problem 32: Part A) 5 Letter reverse word

$$26 * 26 * 26 * 1 * 1 = 26^3$$

6 Letter reverse word

$$26 * 26 * 26 * 1 * 1 * 1 = 26^3$$

Part B)

$$26 * 25 * 24 = 15600$$

Problem 38: Part A)

$$3! \binom{8}{2} \binom{6}{2} \binom{4}{2} \binom{2}{2}$$

Part B)7200

Part C)5760

Problem 39: Part A)

$$10 + 128 + 128^2 = 16522 \text{ Blocks, } 8459264 \text{ Bytes}$$

Part B)

$$10 + 128 + 128^2 + 128^3 = 2113674 \text{ Blocks, } 1082201088 \text{ Bytes}$$

2 Part 1.3

Problem 4: Part A)

$$2^6 - 1 = 63$$

Part B)

$$\binom{6}{3} = 20$$

Part C)

$$\binom{6}{2} + \binom{6}{4} + \binom{6}{6} = 31$$

Part D)

$$\binom{6}{4} + \binom{6}{5} + \binom{6}{6} = 22$$

Problem 6:

$$\begin{aligned} \binom{n}{2} + \binom{n-1}{2} &= \left(\frac{1}{2}\right)(n)(n-1) + \left(\frac{1}{2}\right)(n-1)(n-2) \\ &= \left(\frac{1}{2}\right)(n-1)[n + (n-2)] = \left(\frac{1}{2}\right)(n-1)(2n-2) = (n-1)^2 \end{aligned}$$

Problem 7: Problem 13: Part A)

$$\binom{12}{3} \binom{9}{3} \binom{6}{3} \binom{3}{3} = \frac{12!}{(3!)^4}$$

Part B)

$$\binom{12}{4} \binom{8}{4} \binom{4}{2} \binom{2}{2} = \frac{12!}{(4!)^2(2!)^2}$$

Problem 21: Part A)

$$\overbrace{\binom{10}{3}}^{\text{three 1,seven 0}} + \overbrace{\binom{10}{1} \binom{9}{2}}^{\text{two 1,one 2,seven 0}} + \overbrace{\binom{10}{1}}^{\text{one 3,nine 0}} = 220$$

Part B)

$$\binom{10}{4} + \binom{10}{2} + \binom{10}{1} \binom{9}{2} + \binom{10}{1} \binom{9}{1} = 705$$

Part C)

$$2^{10} * \sum_{i=0}^5 \binom{10}{2i}$$

Problem 30:Part A) 2^3

Part B) 2^{10}

Part C) 3^{10}

Part D) 4^5

Part E) 4^{10}

Problem 31:

$$\begin{aligned} \binom{2n}{n} + \binom{2n}{n-1} &= \frac{(2n)!}{n!n!} + \frac{(2n)!}{(n-1)!(n+1)!} = \frac{(2n)! + (n+1)^2 + (2n)!(n)(n+1)}{(n+1)!(n+1)!} \\ &= \frac{1}{2} * \frac{(2n)!(2n+2)(n+1) + (2n)!(n)(2n+2)}{(n+1)!(n+1)!} \\ &= \frac{1}{2} * \frac{(2n)!(2n+2)(2n+1)}{(n+1)!(n+1)!} \\ &= \frac{1}{2} * \frac{(2n+2)!}{(n+1)!(n+1)!} \\ &= \frac{1}{2} \binom{2n+2}{n+1} \end{aligned}$$

Problem 34:

$$(1+2)^n = 3^n$$

3 Part 1.4

Problem 1: Part A)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10 \rightarrow \binom{5+10-1}{10} = \binom{14}{10}$$

Part B)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 5 \rightarrow \binom{5+5-1}{5} = \binom{9}{5}$$

Part C)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10, x_5 \geq 2 \rightarrow \binom{5+8-1}{8} = \binom{12}{8}$$

Problem 6:

$$\binom{22}{12} \binom{12}{4,4,4}$$

Problem 7: Part A)

$$\binom{4+32-1}{32} = \binom{35}{32}$$

Part B)

$$\binom{4+28-1}{28} = \binom{31}{28}$$

Part C)

$$\binom{4+8-1}{8} = \binom{11}{8}$$

Part D)

$$1$$

Part E)

$$\binom{4+40-1}{40} = \binom{43}{40}$$

Part F)

$$\binom{4+28-1}{28} - \binom{4+3-1}{3} = \binom{31}{28} - \binom{6}{3}$$

Problem 9:

$$230230 = \binom{n+20-1}{20} = \binom{n+19}{20} \Rightarrow n = 7$$

Problem 7: Part A)

$$\binom{10+5-1}{5} = \binom{14}{5}$$

Part B)

$$\binom{7+5-1}{5} + 3 \binom{7+4-1}{4} + 3 \binom{7+3-1}{3} + \binom{7+2-1}{2} = \\ \binom{11}{5} + 3 \binom{10}{4} + 3 \binom{9}{3} + \binom{8}{2}$$

Problem 12:

$$x_1 + x_2 + \cdots + x_5 < 40 \Rightarrow x_1 + x_2 + \cdots + x_5 \leq 39 \Rightarrow x_1 + x_2 + \cdots + x_6 = 39 \\ \rightarrow \binom{6+39-1}{39} = \binom{44}{39}$$

Problem 15:

$$x_1 + x_2 + x_3 + x_4 = 24, 1 \leq x_i \rightarrow \binom{4+20-1}{20} = \binom{23}{20}$$

and answer is :

$$(24!) \binom{23}{20}$$

Problem 20: Part A)

$$x_1 + x_2 + x_3 = 6 \rightarrow \binom{3+6-1}{6} = \binom{8}{6}$$

$$x_4 + x_5 + x_6 + x_7 = 31 \rightarrow \binom{4+31-1}{31} \binom{34}{31}$$

and answer is :

$$\binom{8}{6} \binom{34}{31}$$

Part B)

$$\binom{5}{3} \binom{34}{31}$$

Problem 24:

$$\binom{n+2}{3} = \sum_{i=1}^{i+1} \binom{i+1}{2} \Rightarrow \frac{(n+2)(n+1)n}{6} = \frac{1}{2} \sum_{i=1}^n (i+1)i \Rightarrow \\ \frac{(n+2)(n+1)n}{6} = \frac{1}{2} \sum_{i=1}^n i^2 + \frac{1}{2} \sum_{i=1}^n i \Rightarrow \frac{1}{2} \sum_{i=1}^n i^2 = \frac{(n+2)(n+1)n}{6} - \frac{(n+1)n}{4} \Rightarrow \\ \sum_{i=1}^n i^2 = n(n+1) \left[\frac{n+2}{3} - \frac{1}{2} \right] = n(n+1) \left[\frac{2n+4-3}{6} \right] = \frac{n(n+1)(2n+1)}{6}$$

Problem 25: Part A) take above C language program : $sum = 0$

$inti, j, k, m;$

$for(i = 1; i \leq n; i++)$

$for(j = 1; j \leq i; j++)$

$for(k = 1; k \leq j; k++)$

$for(m = 1; m \leq k; m++)$

$sum++;$

after this command sum value of sum equal to

$$\binom{n+3}{4} = \sum_{i=1}^n \binom{i+2}{3}$$

Part B)

$$(1) \binom{n+3}{4} = \sum_{i=1}^n \binom{i+2}{3}$$

$$(2) \left(\frac{1}{4!}\right) (n+3)(n+2)(n+1)(n) = \sum_{i=1}^n \left(\frac{1}{6}\right) (i+2)(i+1)(i)$$

$$(3) \left(\frac{1}{4}\right) (n+3)(n+2)(n+1)(n) = \sum_{i=1}^n (i^3 + 3i^2 + 2i)$$

$$(4) \left(\frac{1}{4}\right) (n+3)(n+2)(n+1)(n) = \sum_{i=1}^n i^3 + (3) \left(\frac{1}{6}\right) (n)(n+1)(2n+1) + (2) \left(\frac{1}{2}\right) (n+1)(n)$$

$$(5) \left(\frac{1}{4}\right) (n+1)(n)[(n+3)(n+2) - 2(2n+1) - 4] = \sum_{i=1}^n i^3$$

$$(6) \left(\frac{1}{4}\right) (n+1)(n)[n^2 + 5n + 6 - 4n - 6] = \sum_{i=1}^n i^3$$

$$(7) \left(\frac{1}{4}\right) (n+1)^2(n)^2 = \sum_{i=1}^n i^3$$

Problem 26:

$$(r)(r+1)(r+2) \cdots (r+n-1) = P(r+n-1, r-1)$$

Problem 27: Part A) put 1 object in every containers

$$\binom{n+(m-n)-1}{m-n} = \binom{m-1}{m-n} = \binom{m-1}{n-1}$$

Part B) put r objects in every containers

$$\binom{n+(m-rn)-1}{m-rn} = \binom{m-1+(1-r)n}{m-rn} = \binom{m-1+(1-r)n}{n-1}$$

4 Supplementary Exercises

Problem 4: Part A)

$$\overbrace{\binom{25}{2}}^{\text{Book 1}} \cdots \overbrace{\binom{25}{2}}^{\text{Book 3}} = 300 * 300 * 300$$

Part B)

$$3 \binom{25}{1}^2 \binom{25}{4} + 6 \binom{25}{1} \binom{25}{2} \binom{25}{3} + \binom{25}{2}^3$$

Problem 6:

$$x_1 + x_2 + x_3 + \cdots + x_{15} + x_{16} = 45 \quad x_1 \geq 0, x_{16} \geq 0 \quad (2 \leq i \leq 15), x_i > 0$$

$$y_1 + y_2 + y_3 + \cdots + y_{15} + y_{16} = 31 \quad (1 \leq i \leq 16), y_i \geq 0$$

$$\binom{16+31-1}{31} = \binom{46}{15}$$

Problem 11: Part A)

$$2 + 4 + 1 + 5 = 12$$

Part B)

$$2 + 4 + 5 + 8 + 10 + 20 = 49$$

Problem 15: Part A) 1.

$$\binom{5}{4} + \binom{5}{2} \binom{4}{2} + \binom{4}{4}$$

2.

$$\binom{5+4-1}{4} + \binom{5+2-1}{2} \binom{4+2-1}{2} + \binom{4+4-1}{4} = \binom{8}{4} + \binom{6}{2} \binom{5}{2} + \binom{7}{4}$$

3.

$$\binom{8}{4} + \binom{6}{2} \binom{5}{2} + \binom{7}{4} - 9$$

Part B) 1.

$$\binom{5}{1} \binom{4}{3} + \binom{5}{3} \binom{4}{1}$$

2-3.

$$\binom{5}{1} \binom{4+3-1}{3} + \binom{5+3-1}{3} \binom{4}{1} = \binom{5}{1} \binom{6}{3} + \binom{7}{3} \binom{4}{1}$$

Problem 17: Part A)

$$2 \binom{9}{4} + \binom{9}{3}$$

Part B)

$$\binom{9+4-1}{4} = \binom{12}{4}$$

$$2 \binom{12}{4} - 9$$

Problem 19: Part A)

$$5 * (9!)$$

Part B)

$$3 * (8!)$$

Problem 20: Part A)

$$x_1 + x_2 + x_3 = 6 \rightarrow \binom{3+6-1}{6} = \binom{8}{6}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 15 \rightarrow \begin{cases} x_1 + x_2 + x_3 = 6 \\ x_4 + x_5 = 9 \end{cases}$$

$$x_4 + x_5 = 9 \rightarrow \binom{2+9-1}{9} = \binom{10}{9}$$

and answer is :

$$\binom{10}{9} \binom{8}{6}$$

Part B) define $0 \leq k \leq 6$

$$x_1 + x_2 + x_3 = k \rightarrow \binom{3+k-1}{k} = \binom{k+2}{k}$$

$$x_4 + x_5 \leq 15 - k \rightarrow \binom{3+15-k-1}{15-k} = \binom{17-k}{15-k}$$

and answer is :

$$\sum_{k=0}^6 \binom{k+2}{k} \binom{17-k}{15-k}$$

Problem 23:

$$\frac{2(n+r-k-1)}{(n-k)!(r-1)!} + \frac{(n+r-k-1)!}{(n-k)!(r-2)!}$$

Problem 33: Part A)

$$\frac{11!}{4!7!}$$

Part B)

$$\binom{11!}{7!4!} - \binom{4!}{2!2!} \binom{4!}{3!1!}$$

Part C) 1.

$$\binom{11!}{7!4!} + \binom{10!}{6!3!1!} + \binom{9!}{5!2!2!} + \binom{8!}{4!1!3!} + \binom{7!}{3!0!4!}$$

2.

$$\left[\binom{11!}{7!4!} + \binom{10!}{6!3!1!} + \binom{9!}{5!2!2!} + \binom{8!}{4!1!3!} + \binom{7!}{3!0!4!} \right] - \left\{ \left[\binom{4!}{2!2!} + \binom{3!}{1!1!1!} + \binom{2!}{2!} \right] * \left[\binom{4!}{3!1!} + \binom{4!}{3!1!} + \binom{3!}{2!1!} \right] \right\}$$

Problem 34: Part A)

$$\frac{7!}{4!3!} = 35$$

Part B)

$$\frac{7!}{6!1!} = 7$$

Part C)

$$35 - 7 = 28$$

Part D)

$$\binom{7}{4} - \binom{7}{5} = 35 - 21$$

Part E)

$(m, n) \rightarrow (m+1, n+1)$, if the $(n+1)$ -st ballot is for first student.
 $(m, n) \rightarrow (m+1, n-1)$, if the $(n+1)$ -st ballot is for second student.

$$\binom{13}{8} - \binom{13}{9}$$