In The Name Of God

DS Problem Set No.2

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1 Part 8.1

Problem 2: Part A)

$$n = 4, k = 19 \begin{pmatrix} 4+19-1 \\ 19 \end{pmatrix} = \begin{pmatrix} 21 \\ 19 \end{pmatrix}$$

Part C)

$$y_3 = x_3 - 3 \to 0 \leq y_3 \leq 4$$

$$y_4 = x_4 - 3 \to 0 \leq y_4 \leq 5$$

$$c_1 : x_1 > 5$$

$$c_2 : x_2 > 6$$

$$c_3 : y_3 > 4$$

$$c_4 : y_4 > 5$$

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4})$$

$$= |S| - (N(c_1) + N(c_2) + N(c_3) + N(c_4)) + (N(c_1c_2) + N(c_2c_3) + N(c_3c_4) + N(c_1c_3) + N(c_2c_4) + N(c_1c_4))$$

$$-(N(c_1c_2c_3) + N(c_1c_3c_4) + N(c_2c_3c_4) + (c_1c_2c_4)) + (N(c_1c_2c_3c_4))$$

$$= \begin{pmatrix} 21 \\ 4 \end{pmatrix} - \left(\begin{pmatrix} 16 \\ 4 \end{pmatrix} + \begin{pmatrix} 15 \\ 4 \end{pmatrix} + \begin{pmatrix} 17 \\ 4 \end{pmatrix} + \begin{pmatrix} 16 \\ 4 \end{pmatrix} \right)$$

$$+ \left(\begin{pmatrix} 10 \\ 4 \end{pmatrix} + \begin{pmatrix} 12 \\ 4 \end{pmatrix} + \begin{pmatrix} 11 \\ 4 \end{pmatrix} + \begin{pmatrix} 11 \\ 4 \end{pmatrix} + \begin{pmatrix} 10 \\ 4 \end{pmatrix} + \begin{pmatrix} 12 \\ 4 \end{pmatrix} \right)$$

$$- \left(\begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} \right) + 0$$

Problem 3:

$$N = S_0 = \frac{11!}{(2!)^3}$$

$$N(c_1) = \frac{9!}{(2!)^2}, \begin{pmatrix} 6 \\ 1 \end{pmatrix} \left[\frac{9!}{(2!)^2} \right]$$

$$N(c_1c_2) = N(c_1c_3) = N(c_1c_6) = N(c_2c_4) = N(c_2c_5) = N(c_3c_4) =$$

$$N(c_3c_4) = N(c_3c_5) = N(c_4c_6) = N(c_5c_6) = 0$$

$$N(c_1c_4) = \frac{7!}{2!}, S_2 = 6 * \frac{7!}{2!}, S_3 = S_4 = S_5 = S_6 = 0$$

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}\bar{c_5}\bar{c_6}) = S_0 - S_1 + S_2 = \frac{11!}{(2!)^3} - \begin{pmatrix} 6 \\ 1 \end{pmatrix} \left[\frac{9!}{(2!)^2} \right] + 6 * \frac{7!}{2!}$$

Problem 5:

$$(x_1x_2x_3x_4x_5x_6x_7)_10 \to x_1 + x_2 + \cdots + x_7 = 31, 0 \leq x_i \leq 9 (1 \leq i \leq 7)$$

$$\rightarrow \binom{37}{31} - \binom{7}{1}\binom{27}{21} + \binom{7}{2}\binom{17}{11} - \binom{7}{3}\binom{7}{1}$$

Problem 8:

$$x_1 + x_2 + x_3 + x_4 = 9, 1 \leq i \leq 4 \rightarrow \binom{12}{9} - \binom{4}{1}\binom{8}{5} + \binom{4}{2}\binom{4}{1}$$

Problem 11:

$$\frac{[6^8 - \binom{6}{1}5^8 + \binom{6}{2}4^8 - \binom{6}{3}3^8 + \binom{12}{9}2^8 - \binom{12}{9}]}{6^8}$$

Problem 12:

$$10^9 - \binom{3}{1}(9^9) + \binom{3}{2}(8^9) - \binom{3}{3}(7^9)$$

Problem 15:

$$c_i \rightarrow x_i > 6$$

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}\bar{c_5}) = S_0 - S_1 + S_2 = \binom{19}{15} - \binom{5}{1}\binom{13}{9} + \binom{5}{2}\binom{7}{3}$$

$$= 3876 - 5 * 715 + 10 * 35 = 3876 - 3575 + 350 = 651$$

and the answer is :

$$\frac{651}{7776} = 0.08372$$

2 Part 8.2

Problem 2: Part A) 1.

$$E_2 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 = 544320 - (3)(80640) + (6)(5040) = 332640$$

2.

$$L_2 = S_2 - \binom{2}{1}S_3 + \binom{3}{1}S_4 = 398160$$

Part B) 1.

$$E_3 = S_3 - \binom{4}{1}S_4 = 60480$$

2.

$$L_3 = S_3 - \binom{3}{2}S_4 = 65520$$

Problem 4:

$$E_3 = S_3 - \binom{4}{1}S_4 + \binom{5}{2}S_5 - \binom{6}{3}S_6 + \binom{7}{4}S_7 = \binom{7}{3}4^{10} - \binom{4}{1}\binom{7}{4}3^{10}$$

$$+\binom{5}{2}\binom{7}{5}2^{10}-\binom{6}{3}\binom{7}{6}1^{10}+\binom{7}{4}\binom{7}{7}0^{10}=28648200$$

$$L_3=S_3-\binom{3}{2}S_4+\binom{4}{2}S_5-\binom{3}{2}S_6+\binom{6}{2}S_7$$

$$=\binom{7}{3}4^{10}-\binom{3}{2}\binom{7}{4}3^{10}+\binom{4}{2}\binom{7}{5}2^{10}-\binom{5}{2}\binom{7}{6}1^{10}$$

Problem 5:
Problem 7: Part A)

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4})=\binom{52}{13}-\binom{4}{1}\binom{39}{13}+\binom{4}{2}\binom{26}{13}-\binom{4}{3}\binom{13}{13}$$

and the answer is:

$$\frac{N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4})}{\binom{52}{13}}$$

Part B)

$$E_1=S_1+\binom{2}{1}S_2+\binom{3}{2}S_3-\binom{4}{3}S_4$$

$$=\binom{39}{13}\binom{4}{1}-2\binom{4}{2}\binom{26}{13}+3\binom{4}{3}\binom{13}{13}-0$$

and the answer is :

$$E_1/\binom{52}{13}$$

Part C)

$$E_2=S_2-\binom{3}{1}S_3=\binom{4}{2}\binom{26}{13}-3\binom{13}{13}\binom{4}{3}$$

and the answer is:

$$E_2/\binom{52}{13}$$

Problem 8: Part B)

$$L_{t-1}=L_t+E_{t-1},E_{t-1}=S_{t-1}-tS_t$$

Part C)

$$L_{t-1}=L_t+E_{t-1}=S_t+S_{t-1}-tS_t=S_{t-1}-(t-1)S_t=S_{t-1}-\binom{t-1}{t-2}S_t$$

Part D)

$$L_m=L_{m+1}+E_m$$

Part E)

$$L_{t-1} = S_{t-1} - \binom{t-1}{t-2} S_t, L_t = S_t$$

$L_{k+1} = S_{k+1} - \binom{k+1}{k} S_{k+2} + \binom{k+2}{k} S_{k+3} - \cdots + (-1)^{t-k-1} \binom{t-1}{k} S_t$

$L_k = L_{k+1} + E_k = \left[S_{k+1} - \binom{k+1}{k} S_{k+2} + \binom{k+2}{k} S_{k+3} - \cdots + (-1)^{t-k-1} \binom{t-1}{k} S_t\right] +$
$\left[S_k - \binom{k+1}{1} S_{k+1} + \binom{k+2}{2} S_{k+2} - \cdots + (-1)^{t-k} \binom{t}{t-k} S_t\right]$

and then we have :

$$L_k = S_k - \binom{k}{k-1} S_{k+1} + \binom{k+1}{k-1} S_{k+2} - \cdots + (-1)^{t-k} \binom{t-1}{k-1} S_t$$

3 Part 8.3

Problem 3:

$\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 5! \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 5*4*3 - 5*4 + 5 - 1 = 60 - 20 + 5 - 1 = 44$

Problem 4:

$$7! \left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!}\right) = 1854$$

and the answer is :

$$5040 - 1854 = 3186$$

Problem 5: Part A)

$$(d_7 = (7!)e^{-1})7! - d_7$$

Part B)

$$d_{26} = (26!)e^{-1}$$

Problem 8: Part A)

$$(4!)d_4 = (4!)^2 e^{-1}$$

Part B,C)

$$\frac{2 * 3^2 * 6}{(4!)^2 e^{-1}}$$

Problem 10: Part A) 1.

$$\frac{d_n}{n!}$$

2.

$$\frac{n(d_{n-1})}{n!}$$

3.

$$1 - \frac{d_n}{n!}$$

4.

$$\frac{\left(\begin{array}{c} n \\ r \end{array} \right) d_{n-r}}{n!}$$

Part B) 1.

$$e^{-1}$$

2.

$$e^{-1}$$

3.

$$1 - e^{-1}$$

4.

$$\frac{1}{r!} * e^{-1}$$

Problem 11: Part A)

$$(d_1 0)^2$$

Part B)

$$N(\bar{c_1}\bar{c_2}\cdots\bar{c_{10}}) = (10!)^2 - \left(\begin{array}{c} 10 \\ 1 \end{array} \right) (9!)^2 + \left(\begin{array}{c} 10 \\ 2 \end{array} \right) (8!)^2 - \cdots + (-1)^{10} \left(\begin{array}{c} 10 \\ 0 \end{array} \right) (0!)^2$$

Problem 13:

$$n! = \left(\begin{array}{c} n \\ 0 \end{array} \right) d_0 + \left(\begin{array}{c} n \\ 1 \end{array} \right) d_1 + \left(\begin{array}{c} n \\ 2 \end{array} \right) d_2 + \cdots + \left(\begin{array}{c} n \\ n \end{array} \right) d_n = \sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) d_k$$

Problem 14: Part A)

$$N(c_i) = (n-1)!$$

$$N(c_i c_j) = (n-2)!$$

$$N(c_i c_j c_k) = (n-3)!$$

$$N(c_1 c_2 \cdots c_{n-1}) = (n-(n-1))!$$

$$N(\bar{c_1}\bar{c_2}\cdots\bar{c_{n-1}}) =$$

$$n! - \left(\begin{array}{c} n-1 \\ 1 \end{array} \right) (n-1)! + \cdots + (-1)^k \left(\begin{array}{c} n-1 \\ k \end{array} \right) (n-k)! + \cdots + (-1)^{n-1} \left(\begin{array}{c} n-1 \\ n-1 \end{array} \right) (n-(n-1))!$$

Part B)

$$d_n + d_{n-1} =$$

$$\left[n! - \left(\begin{array}{c} n \\ 1 \end{array} \right) (n-1)! + \left(\begin{array}{c} n \\ 2 \end{array} \right) (n-2)! - \cdots + (-1)^n \left(\begin{array}{c} n \\ n \end{array} \right) (n-n)! \right]$$

$$+\left[(n-1)!-\binom{n-1}{1}(n-2)!+\binom{n-1}{2}(n-3)!-\cdots+(-1)^{n-1}\binom{n-1}{n-1}((n-1)-(n-1))!\right]$$

and then the coefficient of $(n-k)!$ in $d_n + d_{n-1}$:

$$(-1)^k\binom{n}{k}+(-1)^{k-1}\binom{n-1}{k-1}=$$

$$(-1)^{k-1}[[(n-1)!/[(k-1)!(n-k)!]]-[n!/[k!(n-k)!]]$$
$$=(-1)^{k-1}[[k(n-1)!-n!]/[k!(n-k)!]]$$
$$=(-1)^{k-1}(n-1)![(k-n)/[k!(n-k)!]]$$
$$(-1)^k(n-1)!/[k!(n-k-1)!]=(-1)^k\binom{n-1}{k}$$

Problem 15:

$$\binom{n}{0}(n-1)!-\binom{n}{1}(n-2)!+\binom{n}{2}(n-3)!-\cdots+(-1)^{n-1}\binom{n}{n-1}(0!)+(-1)^n\binom{n}{n}$$

4 Supplementary Exercises

Problem 2:

$$n_1+n_2+n_3+n_4+n_5+n_6+n_7=37, 0\le n_i\le 9 (1\le i\le 6), 0\le n_7\le 37 \rightarrow \binom{43}{37}-\binom{6}{1}\binom{33}{27}+\binom{6}{2}$$

Problem 6: Part A)

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}\bar{c_5})=k^5-\binom{5}{1}k^4+\binom{5}{2}k^3-\binom{5}{3}k^2+\binom{5}{4}k-\binom{5}{5}k$$

Part B)

$$k=1,2\rightarrow 0, k=3\rightarrow 30$$

Part C)

$$k^6-\binom{6}{1}k^5+\binom{6}{2}k^4-\binom{6}{3}k^3+\binom{6}{4}k^2-\binom{6}{5}k+\binom{6}{6}k$$

Problem 8:

Problem 9: Part A)

$$T=\frac{13!}{(2!)^5}$$

$$S_5=\binom{5}{5}(8!), S_4=\binom{5}{4}\left(\frac{9!}{2!}\right), S_3=\binom{5}{3}\left[\frac{10!}{(2!)^2}\right]$$

$$E_3=\frac{\left[S_3+\binom{4}{1}S_4+\binom{5}{2}S_5\right]}{T}$$

Part B)

$$E_5 = S_5, E_4 = [S_4 - \left(\begin{array}{c} 5 \\ 1 \end{array} \right) S_5]$$

and the answer is:

$$\frac{[T - (E_4 + E_5)]}{T}$$

Problem 10:

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}) = \frac{16!}{(4!)^2} - \left(\begin{array}{c} 4 \\ 1 \end{array} \right) \left[\frac{13!}{(4!)^3} \right] + \left(\begin{array}{c} 4 \\ 2 \end{array} \right) \left[\frac{10!}{(4!)^2} \right] - \left(\begin{array}{c} 4 \\ 3 \end{array} \right) \left(\frac{7!}{4!} \right) + \left(\begin{array}{c} 4 \\ 4 \end{array} \right) (4!)$$

Problem 11: Part A)

$$\left(\begin{array}{c} n-m \\ r-m \end{array} \right) = \left(\begin{array}{c} n-m \\ n-r \end{array} \right)$$

Part B)

$$\left(\begin{array}{c} n-m \\ n-r \end{array} \right) = N(\bar{c_1}\bar{c_2}\cdots\bar{c_m}) = \sum_{i=0}^{m} (-1)^i \left(\begin{array}{c} m \\ i \end{array} \right) \left(\begin{array}{c} n-i \\ r \end{array} \right)$$