

DS Problem Set No.1

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Contents

1	Part 1.1-1.2	2
2	Part 1.3	3
3	Part 1.4	5
4	Supplementary Exercises	8

1 Part 1.1-1.2

$$2^{9}$$

Problem 9: Part A)

$$(14)(12) = 168$$

Part B)

$$(14)(12)(6)(18) = 18144$$

Part C)

$$(8)(18)(6)(3)(14)(12)(14)(12) = 73156608$$

Problem 10:

Problem 15:

$$e\underline{x}e\underline{x}e\underline{x}e\underline{x}e \Rightarrow 4!$$

Problem 16: Part A)

$$40^{25}$$

Part B)

$$40 * 30 * 30 * \cdots * 30 * 30 * 40 = (40^2)(30^{23})$$

Problem 23: Part A)

$$\frac{12!}{3!2!2!2!}$$

Part B)

$$\frac{11!}{3!2!2!2!} + \frac{11!}{3!2!2!2!}$$

Part C)

$$\left(\frac{7!}{2!2!}\right)\left(\frac{6!}{2!3!}\right)$$

Problem 24:

$$\frac{6!}{2!} + \frac{6!}{2!2!} + \frac{6!}{2!2!} = \left(\frac{6!}{2!}\right) \left(1 + \frac{1}{2} + \frac{1}{2}\right) = 6!$$

Problem 25:

$$\frac{12!}{4!3!2!3!} = 277200$$

Problem 28:

$$\frac{14!}{7!7!} = \left(\begin{array}{c} 14\\7\end{array}\right)$$

Problem 31: Part A-B)

$$12*6*8 = 576$$

Problem 32: Part A) 5 Letter reverse word

$$26 * 26 * 26 * 1 * 1 = 26^3$$

6 Letter reverse word

$$26 * 26 * 26 * 1 * 1 * 1 = 26^3$$

Part B)

$$26 * 25 * 24 = 15600$$

Problem 38: Part A)

$$3! \begin{pmatrix} 8 \\ 2 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Part B)7200

Part C)5760

Problem 39: Part A)

$$10 + 128 + 128^2 = 16522$$
 Blocks, 8459264 Bytes

Part B)

$$10 + 128 + 128^2 + 128^3 = 2113674$$
 Blocks, 1082201088 Bytes

2 Part 1.3

Problem 4: Part A)

$$2^6 - 1 = 63$$

Part B)

$$\left(\begin{array}{c} 6\\3 \end{array}\right) = 20$$

Part C)

$$\left(\begin{array}{c} 6 \\ 2 \end{array}\right) + \left(\begin{array}{c} 6 \\ 4 \end{array}\right) + \left(\begin{array}{c} 6 \\ 6 \end{array}\right) = 31$$

Part D)

$$\left(\begin{array}{c}6\\4\end{array}\right)+\left(\begin{array}{c}6\\5\end{array}\right)+\left(\begin{array}{c}6\\6\end{array}\right)=22$$

Problem 6:

$$\left(\begin{array}{c} n \\ 2 \end{array}\right) + \left(\begin{array}{c} n-1 \\ 2 \end{array}\right) = \left(\frac{1}{2}\right)(n)(n-1) + \left(\frac{1}{2}\right)(n-1)(n-2)$$

$$= \left(\frac{1}{2}\right)(n-1)[n+(n-2)] = \left(\frac{1}{2}\right)(n-1)(2n-2) = (n-1)^2$$

Problem 7: Problem 13: Part A)

$$\left(\begin{array}{c}12\\3\end{array}\right)\left(\begin{array}{c}9\\3\end{array}\right)\left(\begin{array}{c}6\\3\end{array}\right)\left(\begin{array}{c}3\\3\end{array}\right)=\frac{12!}{(3!)^4}$$

$$\left(\begin{array}{c}12\\4\end{array}\right)\left(\begin{array}{c}8\\4\end{array}\right)\left(\begin{array}{c}4\\2\end{array}\right)\left(\begin{array}{c}2\\2\end{array}\right)=\frac{12!}{(4!)^2(2!)^2}$$

Problem 21: Part A)

three 1, seven 0 two 1, one 2, seven 0 one 3, nine 0
$$\underbrace{\begin{pmatrix} 10 \\ 3 \end{pmatrix}} + \underbrace{\begin{pmatrix} 10 \\ 1 \end{pmatrix} \begin{pmatrix} 9 \\ 2 \end{pmatrix}} + \underbrace{\begin{pmatrix} 10 \\ 1 \end{pmatrix}} = 220$$

Part B)

$$\left(\begin{array}{c}10\\4\end{array}\right)+\left(\begin{array}{c}10\\2\end{array}\right)+\left(\begin{array}{c}10\\1\end{array}\right)\left(\begin{array}{c}9\\2\end{array}\right)+\left(\begin{array}{c}10\\1\end{array}\right)\left(\begin{array}{c}9\\1\end{array}\right)=705$$

Part C)

$$2^{10} * \sum_{i=0}^{5} \begin{pmatrix} 10 \\ 2i \end{pmatrix}$$

Problem 30:Part A) 2^3

Part B) 2¹⁰

Part C) 3¹⁰

Part D) 4^5

Part E) 4¹⁰

Problem 31:

$$\begin{pmatrix} 2n \\ n \end{pmatrix} + \begin{pmatrix} 2n \\ n-1 \end{pmatrix} = \frac{(2n)!}{n!n!} + \frac{(2n)!}{(n-1)!(n+1)!} = \frac{(2n)! + (n+1)^2 + (2n)!(n)(n+1)}{(n+1)!(n+1)!}$$

$$= \frac{1}{2} * \frac{(2n)!(2n+2)(n+1) + (2n)!(n)(2n+2)}{(n+1)!(n+1)!}$$

$$= \frac{1}{2} * \frac{(2n)!(2n+2)(2n+1)}{(n+1)!(n+1)!}$$

$$= \frac{1}{2} * \frac{(2n+2)!}{(n+1)!(n+1)!}$$

$$= \frac{1}{2} \begin{pmatrix} 2n+2 \\ n+1 \end{pmatrix}$$
Problem 34:

$$(1+2)^n = 3^n$$

3 Part 1.4

Problem 1: Part A)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10 \rightarrow \begin{pmatrix} 5 + 10 - 1 \\ 10 \end{pmatrix} = \begin{pmatrix} 14 \\ 10 \end{pmatrix}$$

Part B)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 5 \rightarrow \begin{pmatrix} 5+5-1 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

Part C)

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10, x_5 \ge 2 \to \begin{pmatrix} 5+8-1 \\ 8 \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \end{pmatrix}$$

Problem 6:

$$\left(\begin{array}{c} 22\\12 \end{array}\right) \left(\begin{array}{c} 12\\4,4,4 \end{array}\right)$$

Problem 7: Part A)

$$\left(\begin{array}{c}4+32-1\\32\end{array}\right) = \left(\begin{array}{c}35\\32\end{array}\right)$$

Part B)

$$\left(\begin{array}{c} 4+28-1\\28 \end{array}\right) = \left(\begin{array}{c} 31\\28 \end{array}\right)$$

Part C)

$$\left(\begin{array}{c} 4+8-1\\ 8 \end{array}\right) = \left(\begin{array}{c} 11\\ 8 \end{array}\right)$$

Part D)

1

Part E)

$$\left(\begin{array}{c} 4+40-1\\ 40 \end{array}\right) = \left(\begin{array}{c} 43\\ 40 \end{array}\right)$$

Part F)

$$\left(\begin{array}{c}4+28-1\\28\end{array}\right)-\left(\begin{array}{c}4+3-1\\3\end{array}\right)=\left(\begin{array}{c}31\\28\end{array}\right)-\left(\begin{array}{c}6\\3\end{array}\right)$$

Problem 9:

$$230230 = \begin{pmatrix} n+20-1\\20 \end{pmatrix} = \begin{pmatrix} n+19\\20 \end{pmatrix} \Rightarrow n=7$$

Problem 7: Part A)

$$\left(\begin{array}{c} 10+5-1\\ 5 \end{array}\right) = \left(\begin{array}{c} 14\\ 5 \end{array}\right)$$

Part B)

$$\begin{pmatrix} 7+5-1 \\ 5 \end{pmatrix} + 3\begin{pmatrix} 7+4-1 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 7+3-1 \\ 3 \end{pmatrix} + \begin{pmatrix} 7+2-1 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \end{pmatrix} + 3\begin{pmatrix} 10 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 9 \\ 3 \end{pmatrix} + \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

Problem 12:

Problem 15:

$$x_1 + x_2 + x_3 + x_4 = 24, 1 \le x_i \to \begin{pmatrix} 4 + 20 - 1 \\ 20 \end{pmatrix} = \begin{pmatrix} 23 \\ 20 \end{pmatrix}$$

and answer is:

$$(24!)\left(\begin{array}{c}23\\20\end{array}\right)$$

Problem 20: Part A)

$$x_1 + x_2 + x_3 = 6 \rightarrow \begin{pmatrix} 3+6-1 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

 $x_4 + x_5 + x_6 + x_7 = 31 \rightarrow \begin{pmatrix} 4+31-1 \\ 31 \end{pmatrix} \begin{pmatrix} 34 \\ 31 \end{pmatrix}$

and answer is:

$$\left(\begin{array}{c} 8\\6 \end{array}\right) \left(\begin{array}{c} 34\\31 \end{array}\right)$$

Part B)

$$\begin{pmatrix} 5 \\ 3 \end{pmatrix} \begin{pmatrix} 34 \\ 31 \end{pmatrix}$$

Problem 24:

$$\begin{pmatrix} n+2 \\ 3 \end{pmatrix} = \sum_{i=1}^{i+1} \begin{pmatrix} i+1 \\ 2 \end{pmatrix} \Rightarrow \frac{(n+2)(n+1)n}{6} = \frac{1}{2} \sum_{i=1}^{n} (i+1)i \Rightarrow$$

$$\frac{(n+2)(n+1)n}{6} = \frac{1}{2} \sum_{i=1}^{n} i^2 + \frac{1}{2} \sum_{i=1}^{n} i \Rightarrow \frac{1}{2} \sum_{i=1}^{n} i^2 = \frac{(n+2)(n+1)n}{6} - \frac{(n+1)n}{4} \Rightarrow$$

$$\sum_{i=1}^{n} i^2 = n(n+1) \left[\frac{n+2}{3} - \frac{1}{2} \right] = n(n+1) \left[\frac{2n+4-3}{6} \right] = \frac{n(n+1)(2n+1)}{6}$$

Problem 25: Part A) take above C language program : sum = 0 inti, j, k, m;

$$for(i = 1; i <= n; i + +)$$

$$for(j = 1; j <= i; j + +)$$

$$for(k = 1; k <= j; k + +)$$

$$for(m = 1; m <= k; m + +)$$

$$sum + +;$$

after this command rum value of sum equal to

$$\left(\begin{array}{c} n+3\\4 \end{array}\right) = \sum_{i=1}^{n} \left(\begin{array}{c} i+2\\3 \end{array}\right)$$

Part B)

$$(1) \binom{n+3}{4} = \sum_{i=1}^{n} \binom{i+2}{3}$$

$$(2) \binom{1}{4!} (n+3)(n+2)(n+1)(n) = \sum_{i=1}^{n} \binom{1}{6} (i+2)(i+1)(i)$$

$$(3) \binom{1}{4} (n+3)(n+2)(n+1)(n) = \sum_{i=1}^{n} (i^3+3i^2+2i)$$

$$(4) \binom{1}{4} (n+3)(n+2)(n+1)(n) = \sum_{i=1}^{n} i^3 + (3) \binom{1}{6} (n)(n+1)(2n+1) + (2) \binom{1}{2} (n+1)(n)$$

$$(5) \binom{1}{4} (n+1)(n)[(n+3)(n+2) - 2(2n+1) - 4] = \sum_{i=1}^{n} i^3$$

$$(6) \binom{1}{4} (n+1)(n)[n^2 + 5n + 6 - 4n - 6] = \sum_{i=1}^{n} i^3$$

$$(7) \binom{1}{4} (n+1)^2(n)^2 = \sum_{i=1}^{n} i^3$$

Problem 26:

$$(r)(r+1)(r+2)\cdots(r+n-1) = P(r+n-1,r-1)$$

Problem 27: Part A) put 1 object in every containers

$$\left(\begin{array}{c} n+(m-n)-1\\ m-n \end{array}\right) = \left(\begin{array}{c} m-1\\ m-n \end{array}\right) = \left(\begin{array}{c} m-1\\ n-1 \end{array}\right)$$

Part B) put r objects in every containers

$$\begin{pmatrix} n+(m-rn)-1 \\ m-rn \end{pmatrix} = \begin{pmatrix} m-1+(1-r)n \\ m-rn \end{pmatrix} = \begin{pmatrix} m-1+(1-r)n \\ n-1 \end{pmatrix}$$

4 Supplementary Exercises

Problem 4: Part A)

$$\underbrace{\begin{array}{c} \text{Book 1} \\ 25 \\ 2 \end{array} \cdots \begin{array}{c} \text{Book 3} \\ 25 \\ 2 \end{array}}_{} = 300 * 300 * 300 * 300$$

Part B)

$$3\left(\begin{array}{c}25\\1\end{array}\right)^2\left(\begin{array}{c}25\\4\end{array}\right)+6\left(\begin{array}{c}25\\1\end{array}\right)\left(\begin{array}{c}25\\2\end{array}\right)\left(\begin{array}{c}25\\3\end{array}\right)+\left(\begin{array}{c}25\\2\end{array}\right)^3$$

Problem 6:

$$x_1 + x_2 + x_3 + \dots + x_{15} + x_{16} = 45$$
 $x_1 \ge 0, x_{16} \ge 0$ $(2 \le i \le 15), x_i > 0$
 $y_1 + y_2 + y_3 + \dots + y_{15} + y_{16} = 31$ $(1 \le i \le 16), y_i \ge 0$
 $\begin{pmatrix} 16 + 31 - 1 \\ 31 \end{pmatrix} = \begin{pmatrix} 46 \\ 15 \end{pmatrix}$

Problem 11: Part A)

$$2+4+1+5=12$$

Part B)

$$2+4+5+8+10+20=49$$

Problem 15: Part A) 1.

$$\left(\begin{array}{c}5\\4\end{array}\right)+\left(\begin{array}{c}5\\2\end{array}\right)\left(\begin{array}{c}4\\2\end{array}\right)+\left(\begin{array}{c}4\\4\end{array}\right)$$

2.

$$\left(\begin{array}{c}5+4-1\\4\end{array}\right)+\left(\begin{array}{c}5+2-1\\2\end{array}\right)\left(\begin{array}{c}4+2-1\\2\end{array}\right)+\left(\begin{array}{c}4+4-1\\4\end{array}\right)=\left(\begin{array}{c}8\\4\end{array}\right)+\left(\begin{array}{c}6\\2\end{array}\right)\left(\begin{array}{c}5\\2\end{array}\right)+\left(\begin{array}{c}7\\4\end{array}\right)$$

3.

$$\left(\begin{array}{c} 8 \\ 4 \end{array}\right) + \left(\begin{array}{c} 6 \\ 2 \end{array}\right) \left(\begin{array}{c} 5 \\ 2 \end{array}\right) + \left(\begin{array}{c} 7 \\ 4 \end{array}\right) - 9$$

Part B) 1.

$$\left(\begin{array}{c}5\\1\end{array}\right)\left(\begin{array}{c}4\\3\end{array}\right)+\left(\begin{array}{c}5\\3\end{array}\right)\left(\begin{array}{c}4\\1\end{array}\right)$$

2-3.

$$\left(\begin{array}{c}5\\1\end{array}\right)\left(\begin{array}{c}4+3-1\\3\end{array}\right)+\left(\begin{array}{c}5+3-1\\3\end{array}\right)\left(\begin{array}{c}4\\1\end{array}\right)=\left(\begin{array}{c}5\\1\end{array}\right)\left(\begin{array}{c}6\\3\end{array}\right)+\left(\begin{array}{c}7\\3\end{array}\right)\left(\begin{array}{c}4\\1\end{array}\right)$$

$$2\begin{pmatrix} 9\\4 \end{pmatrix} + \begin{pmatrix} 9\\3 \end{pmatrix}$$

Part B)

$$\begin{pmatrix} 9+4-1 \\ 4 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$$
$$2 \begin{pmatrix} 12 \\ 4 \end{pmatrix} - 9$$

Problem 19: Part A)

$$5*(9!)$$

Part B)

$$3*(8!)$$

Problem 20: Part A)

$$x_1 + x_2 + x_3 = 6 \to \begin{pmatrix} 3+6-1 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$
$$x_1 + x_2 + x_3 + x_4 + x_5 = 15 \to \begin{cases} x_1 + x_2 + x_3 = 6 \\ x_4 + x_5 = 9 \end{cases}$$
$$x_4 + x_5 = 9 \to \begin{pmatrix} 2+9-1 \\ 9 \end{pmatrix} = \begin{pmatrix} 10 \\ 9 \end{pmatrix}$$

and answer is:

$$\left(\begin{array}{c} 10\\9 \end{array}\right) \left(\begin{array}{c} 8\\6 \end{array}\right)$$

Part B) define $0 \le k \le 6$

$$x_1 + x_2 + x_3 = k \to \begin{pmatrix} 3+k-1 \\ k \end{pmatrix} = \begin{pmatrix} k+2 \\ k \end{pmatrix}$$

 $x_4 + x_5 \le 15 - k \to \begin{pmatrix} 3+15-k-1 \\ 15-k \end{pmatrix} = \begin{pmatrix} 17-k \\ 15-k \end{pmatrix}$

and answer is:

$$\sum_{k=0}^{6} \binom{k+2}{k} \binom{17-k}{15-k}$$

Problem 23:

$$\frac{2(n+r-k-1)}{(n-k)!(r-1)!} + \frac{(n+r-k-1)!}{(n-k)!(r-2)!}$$

Problem 33: Part A)

$$\frac{11!}{4!7!}$$

$$\left(\frac{11!}{7!4!}\right) - \left(\frac{4!}{2!2!}\right) \left(\frac{4!}{3!1!}\right)$$

Part C) 1.

$$\left(\frac{11!}{7!4!}\right) + \left(\frac{10!}{6!3!1!}\right) + \left(\frac{9!}{5!2!2!}\right) + \left(\frac{8!}{4!1!3!}\right) + \left(\frac{7!}{3!0!4!}\right)$$

2.

$$\left[\left(\frac{11!}{7!4!} \right) + \left(\frac{10!}{6!3!1!} \right) + \left(\frac{9!}{5!2!2!} \right) + \left(\frac{8!}{4!1!3!} \right) + \left(\frac{7!}{3!0!4!} \right) \right] - \left\{ \left[\left(\frac{4!}{2!2!} \right) + \left(\frac{3!}{1!1!1!} \right) + \left(\frac{2!}{2!} \right) \right] * \left[\left(\frac{4!}{3!1!} \right) + \left(\frac{4!}{3!1!} \right) + \left(\frac{3!}{2!1!} \right) \right] \right\}$$

Problem 34: Part A)

$$\frac{7!}{4!3!} = 35$$

Part B)

$$\frac{7!}{6!1!} = 7$$

Part C)

$$35 - 7 = 28$$

Part D)

$$\left(\begin{array}{c} 7 \\ 4 \end{array}\right) - \left(\begin{array}{c} 7 \\ 5 \end{array}\right) = 35 - 21$$

Part E)

 $(m,n) \to (m+1,n+1)$,if the (n+1)-st ballot is for first student. $(m.n) \to (m+1,n-1)$,if the (n+1)-st ballot is for second student.

$$\left(\begin{array}{c}13\\8\end{array}\right)-\left(\begin{array}{c}13\\9\end{array}\right)$$