

Math I Diagnostic

Everaise Academy

2021

Below is a diagnostic to test whether your student is ready for Everaise Academy's Math I course. Everaise Academy is not collecting your scores on this test; it is only for your personal use. Instructions are as follows.

- Your student should attempt every problem below without randomly guessing, a time limit, or tools including outside help or a calculator unless otherwise specified.
- After your student has made their best possible attempt, check their answers against the answer key attached at the bottom of this PDF.
- Allow your student another attempt at all the problems they got incorrect or left blank.

If your student is able to answer at least 4 of the 12 problems below correctly on their second attempt, they are ready to take Math I!

Algebra

1. Suppose $1 < a < b < c$ are positive integers such that $1, a, b$ form a geometric progression, in that order, and a, b, c form an arithmetic progression, in that order. Find the smallest possible value of c .
2. Paul likes playing with polynomials. In particular, his favorite polynomial is

$$P(x) = x + 2x^2 + 3x^3 + \cdots + 20x^{20}.$$

What is the coefficient of x^{21} in $(P(x))^2$, once everything is expanded and all like terms are combined?

3. Define the product P as

$$P = \log_2 9 \cdot \log_3 16 \cdot \log_4 25 \cdots \log_{1023} 1024^2$$

Compute $\lfloor \log_2 P \rfloor$.

Geometry

1. $\triangle ABC$ has $AB = 3$, $AC = 5$ and $\angle A = 120^\circ$. Find the circumradius of $\triangle ABC$.
2. Consider quadrilateral $ABCD$ inscribed in circle ω such that $AB = 1$, $BC = 8$, $CD = 4$ and $DA = 7$. Compute the diameter of ω .
3. Let $ABCD$ be a square with side length 8, and denote E as the midpoint of AB . If the circle through A, D, E intersects EC at $F \neq E$, then the area of quadrilateral $DEBF$ can be expressed as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m + n$.

Combinatorics

1. How many ways are there to seat 6 people around a circular table, where seatings differing by a rotation or reflection are considered the same.
2. The Lakers and Suns are playing in a playoff series to win the best of 5 games. (Note: once either team wins 3 games, the series ends and no more games are played) Suppose the Suns are favored at winning any game with probability $2/3$ and the Lakers win with probability $1/3$. Determine the probability the Lakers win the series.
3. There were 10 students in the class before lunch. When the bell rang, the students all rushed to the cafeteria and organized in a single-file line to the lunch line. Interestingly, exactly one student had a shorter person immediately in front of him. If every student has a distinct height, how many possible lines could have been formed?

Number Theory

1. Determine the smallest integer n that has a remainder of 0 when divided by 2, a remainder of 1 when divided by 3, and a remainder of 5 when divided by 7.
2. The integer n can be expressed in base 9 as $\underline{1}\underline{a}\underline{0}$ and in base 4 as $\underline{2}\underline{b}\underline{c}\underline{1}$ for integer digits a, b, c . Find n .
3. Let a and b be positive integers such that

$$\gcd(a^2, b^2) \cdot \text{lcm}(a, b) - 5ab = 6\text{lcm}(a, b)$$

Find the value of $\gcd(a^4, b^4)$.

Answer Key

A1. 6

A2. 1540

A3. 1025

G1. $\frac{7\sqrt{3}}{3}$

G2. $\sqrt{65}$

G3. 149

C1. 60

C2. 17/81

C3. 1013

N1. 40

N2. 153

N3. 1296