

## Question

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If  $X_1, X_2, X_3, \dots$  are independent random variables,

(1)  $Y_i = X_i X_{i+1} X_{i+2}$ , are the random variables, are they independent?

(2)  $S_1, S_2$  are any subsets of  $\mathbf{N}^*$ ,  $Y_S = \prod_{i \in S} X_i$ . When are  $S_1$  and  $S_2$  independent?

## Solution

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(1) No.

As  $X_1, X_2, X_3, \dots$  are independent random variables,

$$E[Y_1] = E[X_1 X_2 X_3] = E[X_1]E[X_2]E[X_3]$$

Similarly, we have

$$E[Y_2] = E[X_2 X_3 X_4] = E[X_2]E[X_3]E[X_4]$$

So

$$E[Y_1]E[Y_2] = E[X_1]E[X_2]^2 E[X_3]^2 E[X_4]$$

But

$$E[Y_1 Y_2] = E[X_1 X_2^2 X_3^2 X_4] = E[X_1]E[X_2^2]E[X_3^2]E[X_4]$$

Since we could not derive  $E[X_2^2] = E[X_2]^2$ , so

$$E[Y_1 Y_2] \neq E[Y_1]E[Y_2]$$

So they are not independent.

(2) With the condition

$$E[X_i^2] = E[X_i]^2, \quad \forall i \in \mathbf{N}^*$$

We can show that  $S_1$  and  $S_2$  are independent. It is also easy to show that the condition is also necessary.