

Class10

2018年4月17日 星期二 10:00

* 从简单的数学问题开始:

$$A = A_1 \times \dots \times A_n$$

$$= \bigcup_{i=1}^n B_i^1 \times \dots \times B_m^i$$

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

$$1_A = \sum_{i=1}^m 1_{B_i} \pmod{2}$$

$$A_1 \begin{array}{|c|c|} \hline B_1 & B_2 \\ \hline B_3 & B_4 \\ \hline \end{array} \quad \text{类似于盒子划分}$$

若 $B_j^i \neq A_j \quad \forall i, j$, 则 $n \geq 2^m$

pf: Subbox $C = C_1 \times \dots \times C_m, C_j \subseteq A_j$

Odd: $|C| = \text{odd} \Leftrightarrow \forall j, |C_j| = \text{odd}$

令 $\Theta(A) = \{\text{odd subbox of } A\}$

$P(\text{每个维度中取盒子} \in \Theta(A)) = \frac{1}{2} \leftarrow 2^m \text{中奇偶子集各半}$

$$\Theta_i(A) = \{C \in \Theta(A) : C \cap B_i^1 \in \Theta(A)\}$$

$$\Rightarrow \Theta(A) = \bigcup \Theta_i(A)$$

$$\frac{|\Theta_i(A)|}{|\Theta(A)|} = \frac{1}{2^m} \Rightarrow n \geq 2^m$$

$$\because |C_k|, |C_k \cap \{x_k\}| \equiv \{0, 1\} \pmod{2} \quad (-\text{奇}-\text{偶})$$

$$|C_k \cap B_k^i|, |(C_k \cap \{x_k\}) \cap B_k^i| \equiv \{0, 1\} \pmod{2}$$

(B 在各个方向投影均未占满, 选取与其投影不交 x_1, \dots, x_m)

* $|\Omega| < \infty, MC \text{ on } \Omega$

$$f: \Omega \rightarrow \Omega$$

$$(\pi_0, \pi_1, \dots)$$

$$(f(\pi_0), f(\pi_1), \dots)$$

higher-order MC (依赖于前有限集)

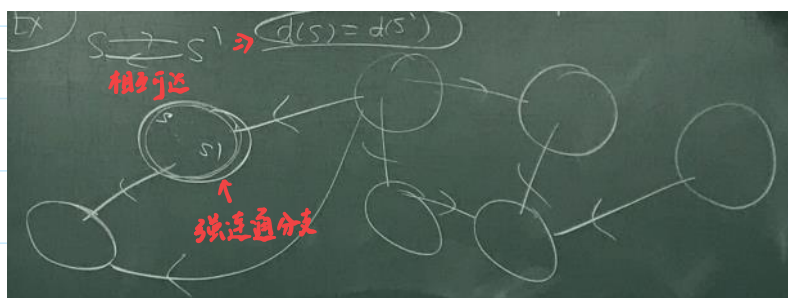
* MC transition matrix P , state set $= \{s_1, \dots, s_k\}$

period of state s_i : $d(s_i) \triangleq \gcd \{n \geq 1 : (P^n)_{ii} > 0\}$

if $d(s_i) = 1$, we say s_i is aperiodic 非周期

A MC is aperiodic provided all its states are aperiodic.

\Rightarrow



HW

* Thm:

Thm: A MC with transition matrix $P_{(k \times k)}$ is aperiodic iff there exists $N < \infty$ such that $(P^n)_{ii} > 0$ for all $n > N$ and for all $i \in \{1, \dots, k\}$.

Ex: P infinite matrix 周期性是否还存在

* Lem:

Lem: Let A be a set of positive integers such that $\gcd A = 1$ and $A + A \subseteq A$. Then N/A is finite.

pf:

$$1 = \sum x_j a_j, \quad a_j \in A, \quad x_j \geq 0$$

$$\text{设 } c = \sum |x_j| a_j, \quad N = c^2, \quad \forall n \geq N$$

$$n = qc + r \quad (q \geq c, 0 \leq r < c)$$

$$= \sum (q|x_j| + r x_j) a_j$$

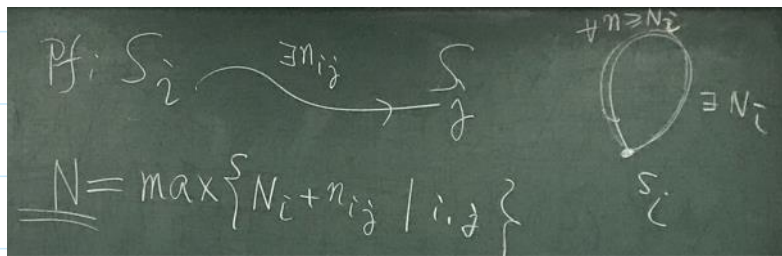
由 Lem, 定义走回步数为 A , N/A 为有限 $i=1, \dots, k$ 有限 \times 有限 $\Rightarrow \exists N < \infty$

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Let (X_0, X_1, \dots) be an irreducible and aperiodic MC with state space $S = \{s_1, \dots, s_k\}$ and transition matrix P . Then there exists an $N < \infty$ such that $P^n > 0$ for all $n \geq N$.

前 Thm 说明非周期导出, 对角线 > 0

加上非周期则有, 所有元素 > 0

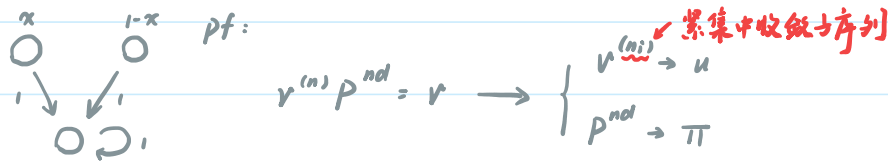


当 i 可走至 j (n_{ij} 步)

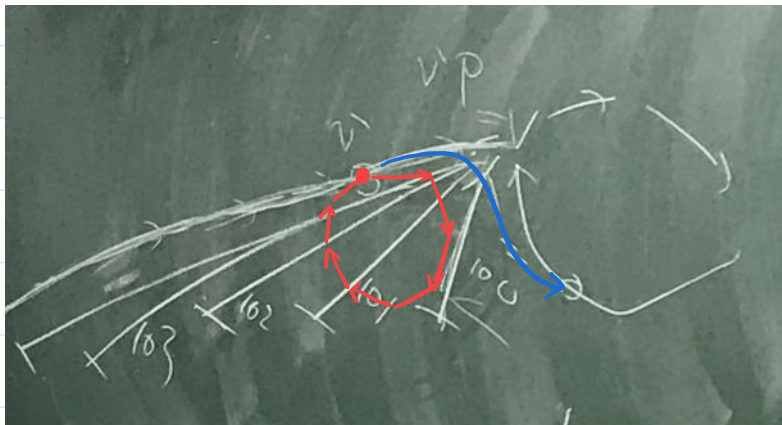
则先在 S_i 走 $\geq N_i$ 自环

* Marker Chain must have a beginning: $\# \{ p : \min i, p p^i = a \}$

$\text{lcm}(d_i) = d$ HW: $\lim_{n \rightarrow \infty} P_{ij}^{nd} = \pi_{ij}$ 非周期极限存在



$\Rightarrow v = u\pi = u\pi p^d = v p^d$



若有限长链，走 d 步后一定会回到自己
故不存在，但无法否定有无限条有限长链存在