

# Class10

2018年4月17日 星期二 10:00

\* 从简单的数学问题开始:

$$A = A_1 \times \dots \times A_n$$

$$= \bigcup_{i=1}^n B_i^1 \times \dots \times B_m^i$$

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

$$1_A = \sum_{i=1}^m 1_{B_i} \pmod{2}$$

$$A_1 \begin{array}{|c|c|} \hline B_1 & B_2 \\ \hline B_3 & B_4 \\ \hline \end{array} A_2$$

类似于盒子划分

若  $B_j^i \neq A_j \forall i, j$ , 则  $n \geq 2^m$

pf: Subbox  $C = C_1 \times \dots \times C_m, C_j \subseteq A_j$

Odd:  $|C| = \text{odd} \Leftrightarrow \forall j, |C_j| = \text{odd}$

令  $\Theta(A) = \{\text{odd subbox of } A\}$

$P(\text{每个维度中取盒子} \in \Theta(A)) = \frac{1}{2} \leftarrow 2^m \text{中奇偶子集各半}$

$$\Theta_i(A) = \{C \in \Theta(A) : C \cap B_i^1 \in \Theta(A)\}$$

$$\Rightarrow \Theta(A) = \bigcup \Theta_i(A)$$

$$\frac{|\Theta_i(A)|}{|\Theta(A)|} = \frac{1}{2^m} \Rightarrow n \geq 2^m$$

$$\because |C_k|, |C_k \cap \{x_k\}| \equiv \{0, 1\} \pmod{2} \text{ (-奇-偶)}$$

$$|C_k \cap B_k^i|, |(C_k \cap \{x_k\}) \cap B_k^i| \equiv \{0, 1\} \pmod{2}$$

(B 在各个方向投影均未占满, 选取与其投影不交  $x_1, \dots, x_m$ )

\*  $|\Omega| < \infty, MC \text{ on } \Omega$

$$f: \Omega \rightarrow \Omega$$

$$(\pi_0, \pi_1, \dots)$$

$$(f(\pi_0), f(\pi_1), \dots)$$

higher-order MC (依赖于前有限集)

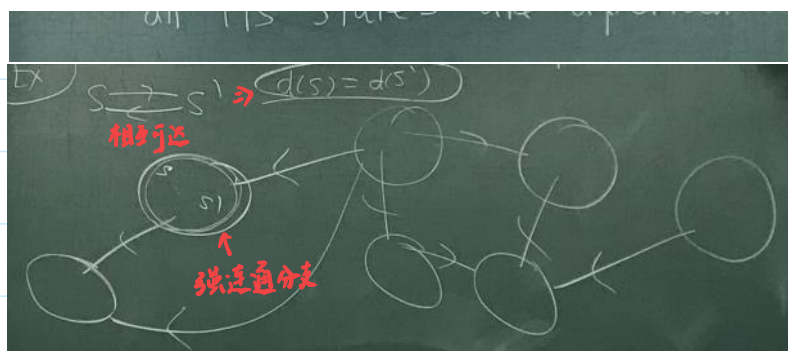
\* MC transition matrix  $P$ , state set  $= \{s_1, \dots, s_k\}$

period of state  $s_i$ :  $d(s_i) \triangleq \gcd \{n \geq 1 : (P^n)_{ii} > 0\}$

if  $d(s_i) = 1$ , we say  $s_i$  is aperiodic 非周期

A MC is aperiodic provided all its states are aperiodic.

Ex  $s \rightarrow s' \Rightarrow d(s) = d(s')$



HW

\* Thm:

Thm: A MC with transition matrix  $P_{(k \times k)}$  is aperiodic iff there exists  $N < \infty$  such that  $(P^n)_{ii} > 0$  for all  $n > N$  and for all  $i \in \{1, \dots, k\}$ .

Ex:  $P$  infinite matrix 周期性是否还存在

\* Lem:

Lem: Let  $A$  be a set of positive integers such that  $\gcd A = 1$  and  $A + A \subseteq A$ . Then  $N/A$  is finite.

pf:

$$1 = \sum x_j a_j, a_j \in A, x_j \geq 0$$

$$\text{设 } c = \sum |x_j| a_j, N = c^2, \forall n \geq N$$

$$n = qc + r \quad (q \geq c, 0 \leq r < c)$$

$$= \sum (q|x_j| + r x_j) a_j$$

由 Lem, 定义走回步数为  $A$ ,  $N/A$  为有限  $i=1, \dots, k$  有限 \* 有限  $\Rightarrow \exists N < \infty$

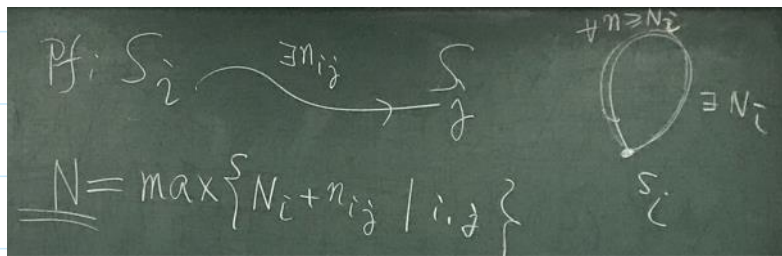
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Let  $(X_0, X_1, \dots)$  be an irreducible and aperiodic MC with state space  $S = \{s_1, \dots, s_k\}$  and transition matrix  $P$ . Then there exists an  $N < \infty$  such that  $P^n > 0$  for all  $n \geq N$ .

前 Thm 说明非周期导出, 对角线  $> 0$

加上非周期则有, 所有元素  $> 0$

pf:

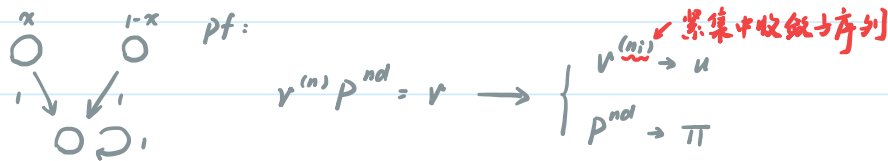


当  $i$  可走到  $j$  ( $n_{ij}$  步)

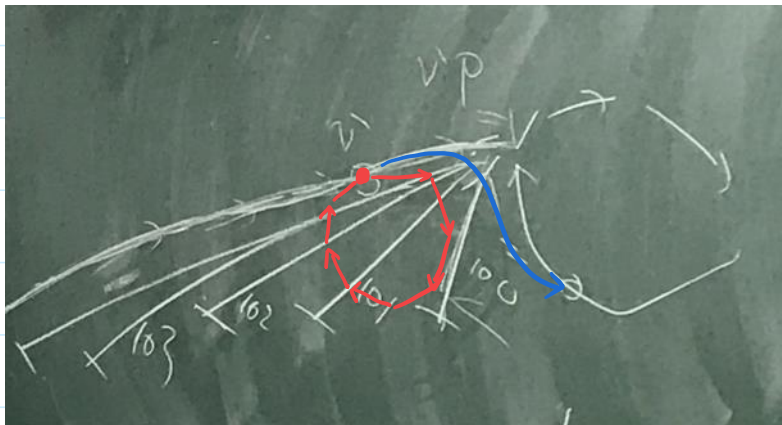
则先在  $S_i$  走  $\geq N_i$  自环

\* Marker Chain must have a beginning:  $\# \{ p : \min i, p p^i = a \}$

$\text{lcm}(d_i) = d$  HW:  $\lim_{n \rightarrow \infty} P_{ij}^{nd} = \pi_{ij}$  非周期极限存在



$\Rightarrow v = u\pi = u\pi p^d = v p^d$



若有限长链，走  $d$  步后一定会回到自己  
故不存在，但无法否定有无限条有限长链存在