

HW

* Thm:

Thm: A MC with transition matrix $P_{(k \times k)}$ is aperiodic iff there exists $N < \infty$ such that $(P^n)_{ii} > 0$ for all $n > N$ and for all $i \in \{1, \dots, k\}$.

Ex: P infinite matrix 周期性是否还存在

* Lem:

Lem: Let A be a set of positive integers such that $\gcd A = 1$ and $A + A \subseteq A$. Then N/A is finite.

pf:

$$1 = \sum x_j a_j, \quad a_j \in A, \quad x_j \geq 0$$

$$\text{设 } c = \sum |x_j| a_j, \quad N = c^2, \quad \forall n \geq N$$

$$n = qc + r \quad (q \geq c, \quad 0 \leq r < c)$$

$$= \sum (q|x_j| + r x_j) a_j$$

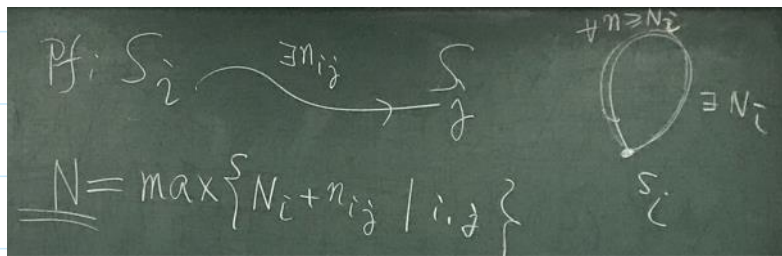
由 Lem, 定义走回步数为 A , N/A 为有限 $i=1, \dots, k$ 有限 \times 有限 $\Rightarrow \exists N < \infty$

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Let (X_0, X_1, \dots) be an irreducible and aperiodic MC with state space $S = \{s_1, \dots, s_k\}$ and transition matrix P . Then there exists an $N < \infty$ such that $P^n > 0$ for all $n \geq N$.

前 Thm 说明非周期导出, 对角线 > 0

加上非周期则有, 所有元素 > 0

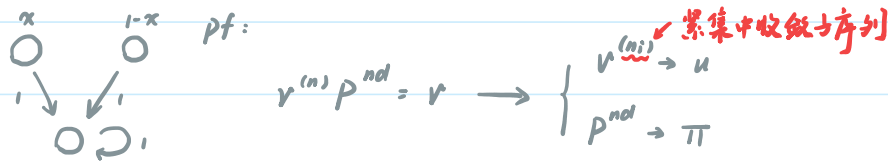


当 i 可走到 j (n_{ij} 步)

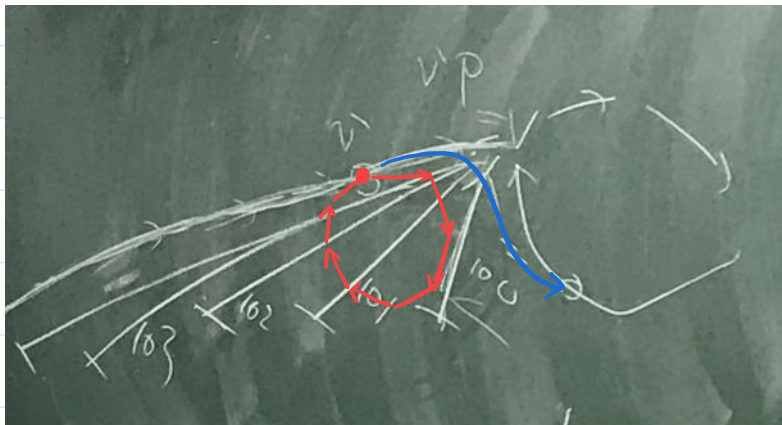
则先在 S_i 走 $\geq N_i$ 自环

* Marker Chain must have a beginning: $\# \{ p : \min i, p p^i = a \}$

$\text{lcm}(d_i) = d$ HW: $\lim_{n \rightarrow \infty} P_{ij}^{nd} = \pi_{ij}$ 非周期极限存在



$\Rightarrow v = u\pi = u\pi p^d = v p^d$



若无限长链，走 d 步后一定会回到自己
故不存在，但无法否定有无限条有限长链存在