Class10

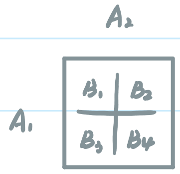
2018年4月17日 星期二 10:00

* 从简单的数学问题开始：

$$A = A_1 \times \cdots \times A_n = \bigcup_{i=1}^{n} B_1^i \times \cdots \times B_m^i$$

$A \times B = \{(a,b) \mid a \in A, b \in B\}$

$$1_A = \sum_{i=1}^{m} 1_{B^i} \pmod 2$$



类似于盒子划分

若 $B_j^i \neq A_j \ \forall i,j$，则 $n \geq 2^m$

pf: subbox $C = C_1 \times \cdots \times C_m$, $C_j \subseteq A_j$

Odd: $|C| = \text{odd} \iff \forall j, |C_j| = \text{odd}$

令 $\theta(A) = \{\text{odd subbox of } A\}$　　P(每个维度中取奇子集 $\in \theta(A)$) $= \frac{1}{2} \leftarrow 2^{n}$中奇偶子集各半

$\theta_i(A) = \{C \in \theta(A) : C \cap B^i \in \theta(A)\}$

$\Rightarrow \theta(A) = \bigcup \theta_i(A)$　　$\dfrac{|\theta_i(A)|}{|\theta(A)|} = \dfrac{1}{2^m} \Rightarrow n \geq 2^m$

$\because |C_k|, |C_k \triangle \{x_k\}| \equiv \{0,1\} \bmod 2$ (一奇一偶)

$|C_k \cap B_k^i|, |(C_k \triangle \{x_k\}) \cap B_k^i| \equiv \{0,1\} \bmod 2$

(B在各个方向投影均未占满，选取与其投影不交 $x_1, \cdots, x_m$)

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* $|\Omega| < \infty$, MC on Ω

$f: \Omega \to \Omega$　　$(x_0, x_1, \cdots)$
　　　　　　$(f(x_0), f(x_1), \cdots)$

higher-order MC (依赖于前有限步)

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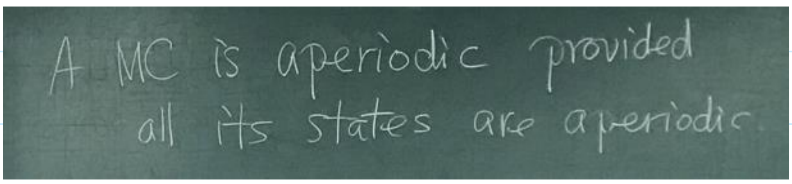
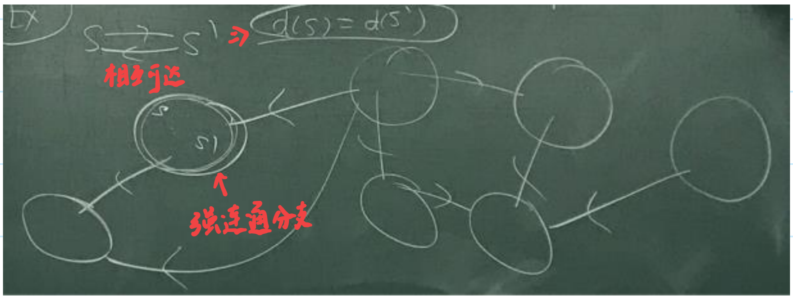
* MC transition matrix P, state set $= \{s_1, \cdots, s_k\}$

period of state s_i: $d(s_i) \triangleq \gcd\{n \geq 1 : (P^n)_{ii} > 0\}$

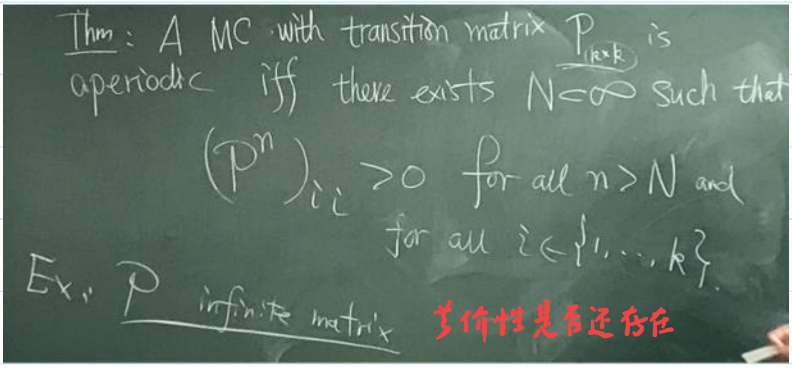
if $d(s_i) = 1$, we say s_i is aperiodic 非周期



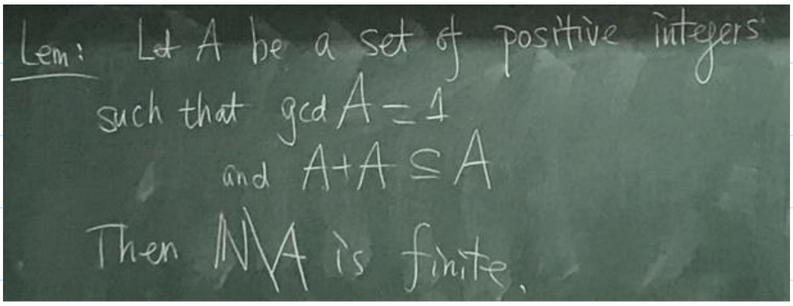
HW



Thm:



Lem:



pf:

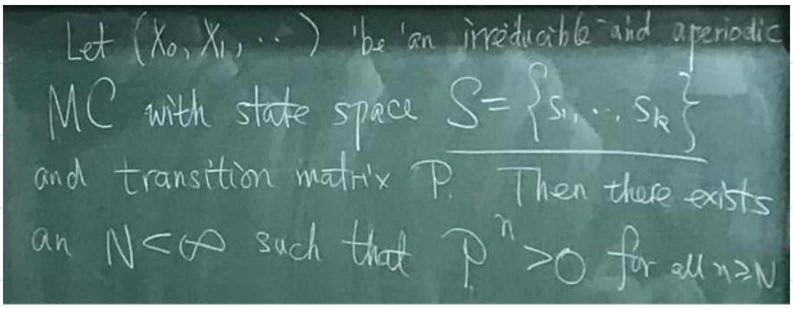
$1=\sum x_j a_j$, $a_j\in A_r$ $x_j\neq 0$

设 $c=\sum|x_j|a_j$, $N=c^2$, $\forall n\ge N$

$n=qc+r$ ($q\ge c$, $0\le r<c$)

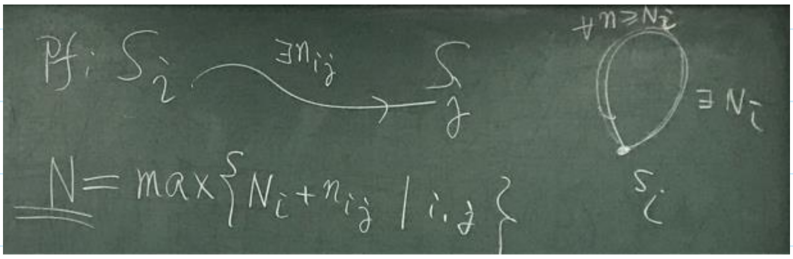
$=\sum(q|x_j|+rx_j)a_j$

由Lem，定义走回步数为A，N/A为有限 $i=1,\dots,k$ 有限×有限 ⇒ $\exists N<\infty$



前Thm说明非周期导出对角线>0

加上非周期则有，所有元素>0

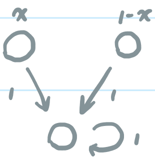


当 i 可走至 j（n_{ij} 步）

则先在 s_i 走 N_i 自环

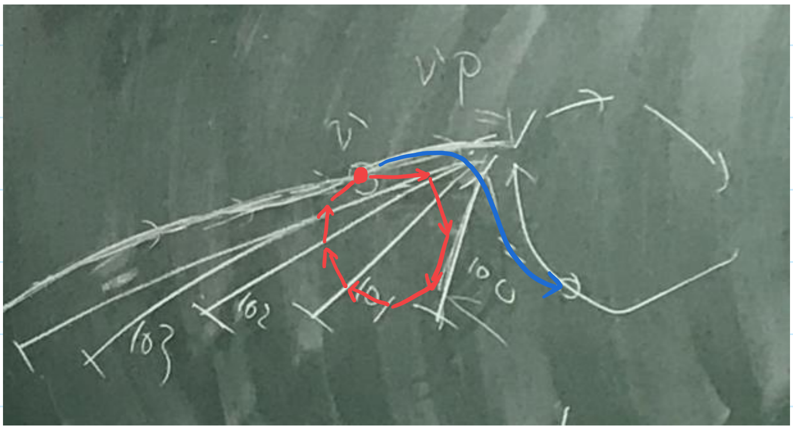
* Markov Chain must have a beginning : $\#\{\beta : \min i .\ \beta P^i = \alpha\}$

$\mathrm{lcm}(d_i) = d$ HW: $\lim_{n\to\infty} P^{nd}_{ij} = \pi_{ij}$ 非周期极限存在



pf: $v^{(n)} P^{nd} = v \longrightarrow \begin{cases} v^{(n_i)} \to u \\ P^{nd} \to \pi \end{cases}$ 紧集中收敛子序列

$\Rightarrow v = u\pi = u\pi P^d = vP^d$



若有无限长链，走d步后一定会回到自己
故不存在，但无法否定有无限条有限长链存在