Some Thoughts

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1 Problem For a sequence of random variables $X_1, X_2, \cdots$ with Markov property and a function f on X. It is obvious that $f(X_1), f(X_2), \cdots$ are also random variables, however not necessarily holding Markov property.

Solution In discrete cases, a function on X is a process to merge some states just as what dynamic programming does, which requires the states with similar state transition rules.

When f is injective, Markov property must hold. Situation is not consistent on other cases. I only present a simple counterexample.

Consider 2 random variables X_1, X_2 and the state universe is $X = \{1, 2, 3, 4\}$. The transition matrix P and the function f is following. Let $\mu = \{p_1, p_2, p_3, p_4\}$ be the distribution of X_1, then the distribution λ of X_2 is $\{0, 0, p_1 + p_3, p_2 + p_4\}$. Acted on f, $f(\lambda) = \{0, p_1 + p_3, p_2 + p_4\}$, which cannot be expressed as $f(\mu)P' = \{p_1 + p_2, p_3, p_4\}P'$. So $f(X_1), f(X_2)$ do not hold Markov property any more.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad f(x) = \begin{cases} 1 & x = 1, 2 \\ 2 & x = 3 \\ 3 & x = 4 \end{cases}$$