# Some Thoughts and Solutions

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# Solutions

## P53T30 Hunter with his dogs

### Problem

A hunter has two hunting dogs. One day, on the trail of some animal,the hunter comes to a place where the road diverges into two paths. He knows that each dog, independently of the other, will choose the correct path with probability p. The hunter decides to let each dog choose a path, and if they agree, take that one, and if they disagree, to randomly pick a path. Is his strategy better than just letting one of the two dogs decide on a path?

### Solution

Let the two hunting dogs be d1 and d2, event A = {d1 choose the correct path}, event B = {d2 choose the correct path}, event C = {The hunter choose the same path}.

As A and B are independent, $P(A \cap B) = P(A)P(B)$. Also, $P(A \cap B^c) = P(A)P(B^c),\ P(A^c \cap B) = P(A^c)P(B),\ P(A^c \cap B^c) = P(A^c)P(B^c)$

The situation can be divided into 4 parts:

1. The two dogs choose the same correct path.

$$P(C|A \cap B) = 1,\ P(A \cap B) = p^2$$

2. d1 choose the correct path but d2 does not.

$$P(C|A \cap B^c) = 1/2,\ P(A \cap B^c) = p(1-p)$$

3. d2 choose the correct path but d1 does not.

$$P(C|A^c \cap B) = 1/2,\ P(A^c \cap B) = p(1-p)$$

4. The two dogs both choose the wrong path.

$$P(C|A^c \cap B^c) = 0,\ P(A^c \cap B) = (1-p)(1-p)$$

Therefore, $P(C) = P(C|A\cap B)P(A\cap B) + P(C|A\cap B^c)P(A\cap B^c) + P(C|A^c\cap B)P(A^c\cap B) + P(C|A^c\cap B^c)P(A^c\cap B^c) = p$. This strategy is the same as just letting one of the two dogs decide on a path.

## P53T31 Communication through a noisy channel

### Problem

A binary (0 or 1) symbol transmitted through a noisy communication channel is received incorrectly with probability $\epsilon_0$ and $\epsilon_1$, respectively (see Fig 1). Errors in different symbol transmissions are independent.

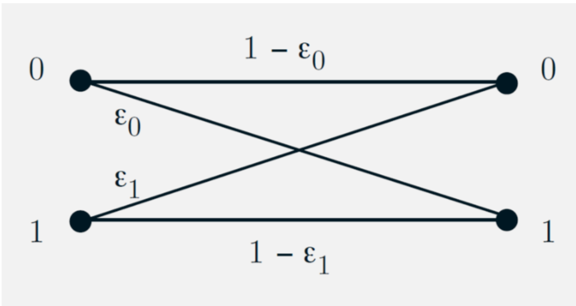


图 1: Error probabilities in a binary communication channel

(a) Suppose that the channel source transmits a 0 with probability p and transmits a 1 with probability 1−p. What is the probability that a randomly chosen symbol is received correctly?

(b) Suppose that the string of symbols 1011 is transmitted. What is the probability that all the symbols in the string are received correctly?

(c) In an effort to improve reliability, each symbol is transmitted three times and the received symbol is decoded by majority rule. In other words, a 0 (or 1) is transmitted as 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) if and only if the received three-symbol string contains at least two 0s (or 1s, respectively). What is the probability that a transmitted 0 is correctly decoded?

(d) For what values of $\epsilon_0$ is there an improvement in the probability of correct decoding of a 0 when the scheme part (c) is used?

(e) Suppose that the channel source transmits a 0 with probability p and transmits a 1 with probability 1 − p, and that the scheme of part (c) is used. What is the probability that a 0 was transmitted given that the received string is 101?

**Solution**

Let events $T_s = \{\text{string s is transmitted}\}$, $R_s = \{\text{string s is received}\}$, $C = \{\text{The symbol is received correctly}\}$

(a) The probability $P(C)$ that a randomly chosen symbol is received correctly:

$$\begin{aligned} P(C) &= P(C|T_0)P(T_0) + P(C|T_1)P(T_1) \\ &= (1-\epsilon_0)p + (1-\epsilon_1)(1-p) \\ &= 1-\epsilon_1 + (\epsilon_1 - \epsilon_0)p \end{aligned}$$

(b) As the correctness is independent with each other, the probability is: $(1-\epsilon_1)^3(1-\epsilon_0)$

(c) The symbols received should be among $S = \{000, 001, 010, 100\}$, the probability $P(S) = (1-\epsilon_0)^3 + \binom{3}{1}(1-\epsilon_0)^2\epsilon_0 = (1-\epsilon_0)^2(1+2\epsilon_0)$

(d) The probability is improved when:

$$(1-\epsilon_0)^2(1+2\epsilon_0) > 1-\epsilon_0 \Leftrightarrow 0 < \epsilon_0 < 1/2$$

(e) $P(R_{101}|T_0) = \epsilon_0^2(1-\epsilon_0),\ P(T_0) = p,\ P(R_{101}|T_1) = \epsilon_1(1-\epsilon_1)^2,\ P(T_1) = 1-p$. According to the Bayes Theorem, the probability would be:

$$\begin{aligned} P(T_0|R_{101}) &= \frac{P(R_{101}|T_0)P(T_0)}{P(R_{101}|T_1)P(T_1) + P(R_{101}|T_0)P(T_0)} \\ &= \frac{\epsilon_0^2(1-\epsilon_0)p}{\epsilon_1(1-\epsilon_1)^2(1-p) + \epsilon_0^2(1-\epsilon_0)p} \end{aligned}$$