Class10

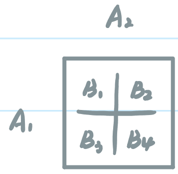
2018年4月17日 星期二 10:00

* 从简单的数学问题开始：

$A = A_1 \times \cdots \times A_n$ $\quad A \times B = \{(a,b) \mid a \in A, b \in B\}$

$= \bigcup_{i=1}^{n} B_1^i \times \cdots \times B_m^i$ (with $B_j^i \subseteq A_j$)

$1_A = \sum_{i=1}^{m} 1_{B^i} \pmod 2$



类似于盒子划分

若 $B_j^i \neq A_j \ \forall i,j$，则 $n \geq 2^m$

pf: subbox $C = C_1 \times \cdots \times C_m$, $C_j \subseteq A_j$

Odd: $|C| = odd \iff \forall j, |C_j| = odd$

令 $\theta(A) = \{odd \text{ subbox of } A\}$ $\quad P(\text{每个维度中取奇子集} \in \theta(A)) = \frac{1}{2} \leftarrow 2^k$中奇偶子集各半

$\theta_i(A) = \{C \in \theta(A) : C \cap B^i \in \theta(A)\}$ $\quad \downarrow$

$\Rightarrow \theta(A) = \bigcup \theta_i(A)$ $\quad \frac{|\theta_i(A)|}{|\theta(A)|} = \frac{1}{2^m} \Rightarrow n \geq 2^m$

$\because |C_k|, |C_k \triangle \{x_k\}| \equiv \{0,1\} \bmod 2$ (一奇一偶)

$|C_k \cap B_k^i|, |(C_k \triangle \{x_k\}) \cap B_k^i| \equiv \{0,1\} \bmod 2$

(B在各个方向投影均未占满，选取与其投影不交 $x_1, \cdots, x_m$)

* $|\Omega| < \infty$, MC on Ω

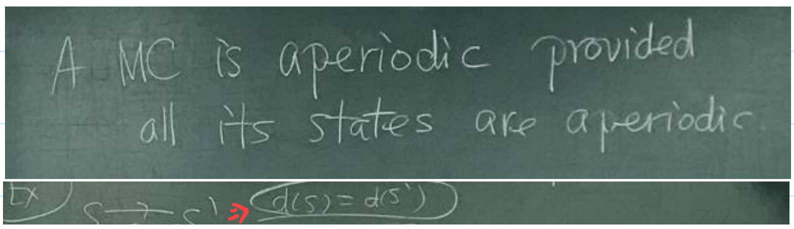
$f: \Omega \to \Omega$ $\quad (x_0, x_1, \cdots)$

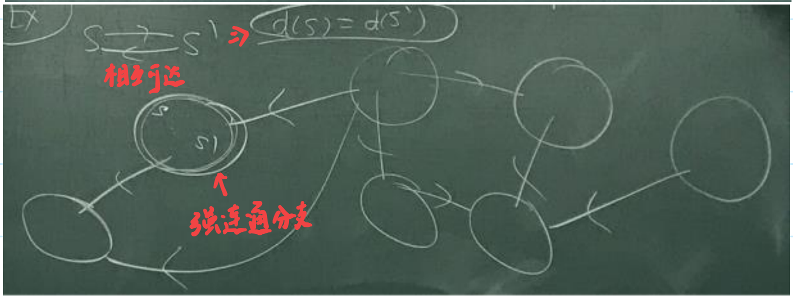
$\quad (f(x_0), f(x_1), \cdots)$ $\quad$ higher-order MC (依赖于前有限个)

* MC transition matrix P, state set $= \{s_1, \cdots, s_k\}$

period of state s_i: $d(s_i) \triangleq \gcd\{n \geq 1 : (P^n)_{ii} > 0\}$

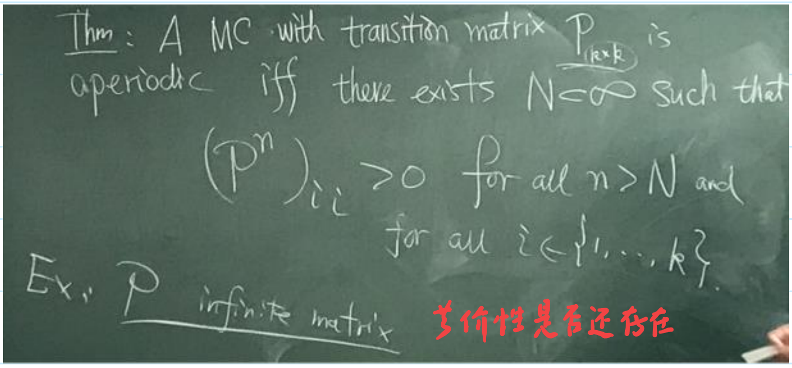
if $d(s_i) = 1$, we say s_i is aperiodic 非周期



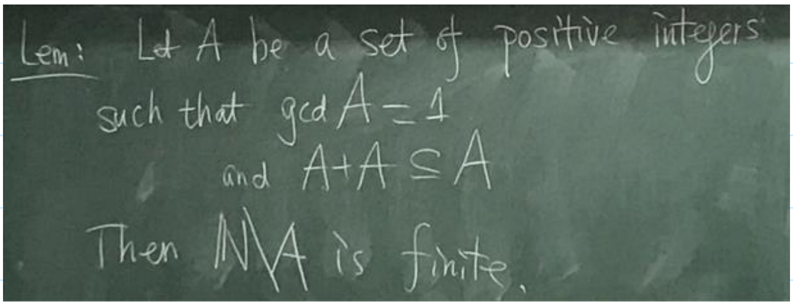


HW

※ **Thm:**



※ **Lem:**



pf:

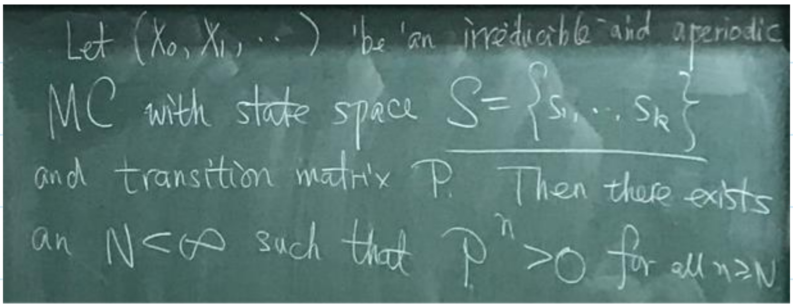
$1 = \sum x_j a_j$, $a_j \in A_r$ $x_j \neq 0$

设 $c = \sum |x_j| a_j$, $N = c^2$, $\forall n \geq N$

$n = qc + r$ $(q \geq c,\ 0 \leq r < c)$

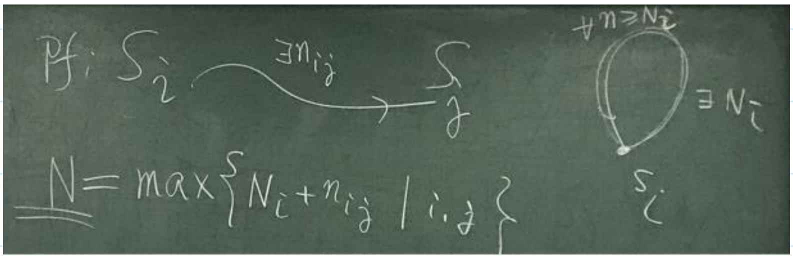
$= \sum (q|x_j| + r x_j) a_j$

由Lem，定义走回步数为A，N/A为有限 $i = 1, \dots, k$ 有限×有限 ⇒ $\exists N < \infty$



前Thm说明非周期导出对角线>0
加上非周期则有，所有元素>0

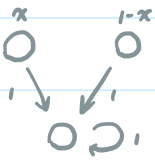
pf:



当i可走至j（n_{ij}步）
则先在 s_i 走 N_i 自环

Marker Chain must have a beginning : $\#\{p : \min i \,.\, p p^i = \alpha\}$

$lcm(d_i) = d$ **HW**: $\lim_{n\to\infty} P_{ij}^{nd} = \pi_{ij}$ 非周期极限存在

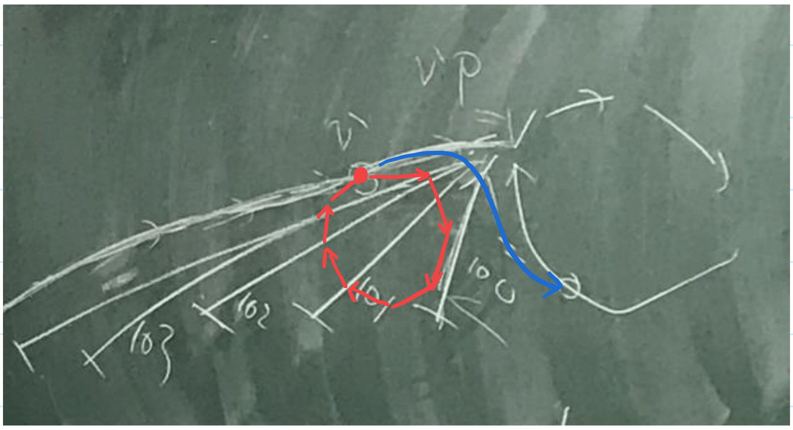


pf:

紧集中收敛子序列

$$v^{(n)} P^{nd} = v \longrightarrow \begin{cases} v^{(n_i)} \to u \\ P^{nd} \to \pi \end{cases}$$

$\Rightarrow v = u\pi = u\pi P^d = vP^d$



若有无限长链，走d步后一定会回到自己
故不存在，但无法否定有无限条有限长链存在