Class 14

2018年5月10日 星期四 10:00

Gamma distribution: $X \sim \gamma(\alpha, \beta)$

$$f(x) = \begin{cases} \dfrac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$\Gamma(\alpha)$ ← 归一化参数

$\Gamma(\alpha) = (\alpha-1)!$ $\alpha = 1, 2, \cdots$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

引题：

$$\left.\begin{array}{l} X \sim \text{gamma}(n, \lambda) \\ Y \sim \text{exponential}(\lambda) \end{array}\right\} \Rightarrow X + Y \sim \text{gamma}(n+1, \lambda)$$

引题：

Let α be an integer and suppose X has distribution $\text{Gamma}(\alpha, \beta)$. Then $P(X \leq x) = P(Y \geq \alpha)$ where $Y \sim \text{Poisson}(x\beta)$

* Cauchy distribution: $\text{Cauchy}(\alpha, \beta)$ 人工反例分布，γ与泊松更为自然

$$f(x) = \frac{1}{\beta\pi} \cdot \frac{1}{1 + (x-\alpha)^2/\beta^2}$$

$N(\mu, \sigma)$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

* 特征函数：

此放缩未必好算好用，加入f(x)

$$P(X \geq a) = E(1_{X \geq a}) \leq E\left(\frac{X}{a}\right) = \frac{EX}{a}$$

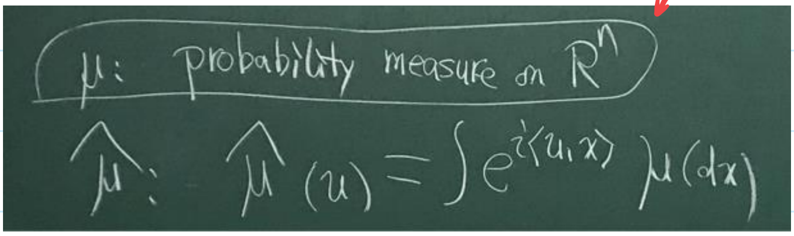
$f(x) \geq b$

$\frac{f(x)}{b}$ 寻找f(x)更好算

定义：$X \in R^n$, $\varphi_X(u) = E\left(e^{i\langle u, X\rangle}\right)$ ← E 即为积分

$= \int e^{i\langle u,x\rangle} P(dx)$

任意R^n上函数，如密度函数



* 母函数:

$(b_0, b_1, \dots,) \to \sum_{i=0}^{\infty} b_i t^i$

错排问题:

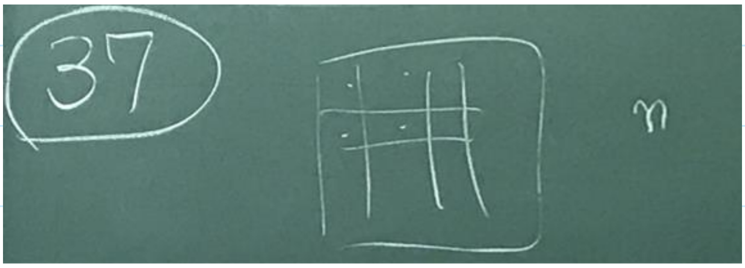
$b_0 = 1$, $b_n = P(X_n = 0)$ $n \geq 1$ 其中 $X_n = \mathrm{Fix}\,\sigma$, $\sigma \in S_n$

$B(t) = \sum_{i=0}^{\infty} b_i \cdot t^i$ $\frac{1}{1-t} = B(t)e^t$

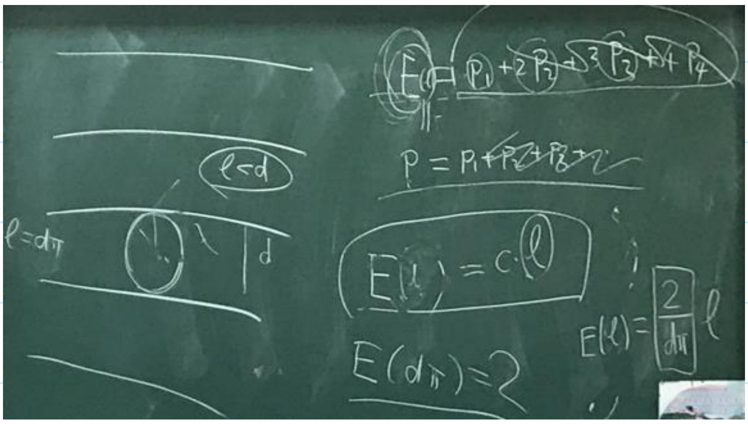
$(1+t+t^2+\cdots) = (\sum_{i=0}^{\infty} b_i \cdot t^i)(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots)$ } pf: $1 = \sum_k \frac{b_{n-k}}{k!}$ 全概率公式

$1 = \sum_k \frac{b_{n-k}}{k}$ $\iff$ $1 = \sum_k P\{\sigma : |\mathrm{Fix}\,\sigma| = k\}$

$\Rightarrow$ $B(t) = \frac{e^{-t}}{1-t}$ $\lim_{n\to\infty} b_n = e^{-1} = 1 - 1 + \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} \cdots = 0.37$



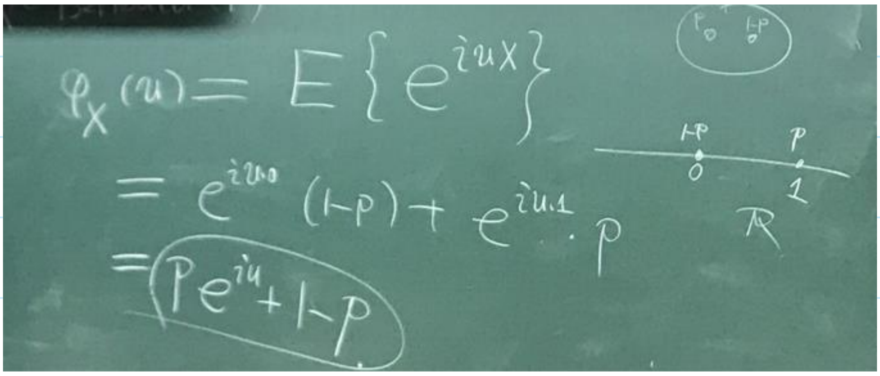
扔下n枚炸弹，格子不被炸到概率为0.37



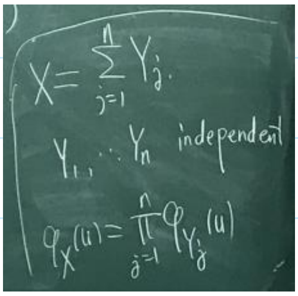
布丰投针

期望线性性

* $X \sim \mathrm{Bernoulli}(P)$:



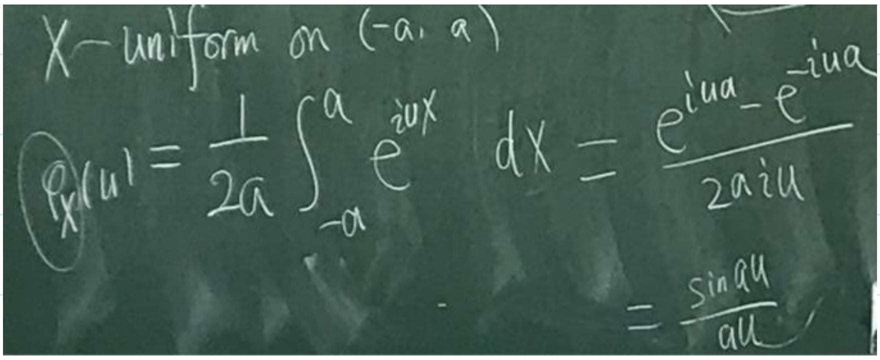
Binomial (p, n) , $\varphi_X(u) = (pe^{iu} + 1-p)^n$



习题

X ~ Poisson (λ) :

$\varphi_X(u) = e^{\lambda(e^{iu}-1)}$ $\quad \frac{\lambda^k}{k!} e^{-\lambda}$

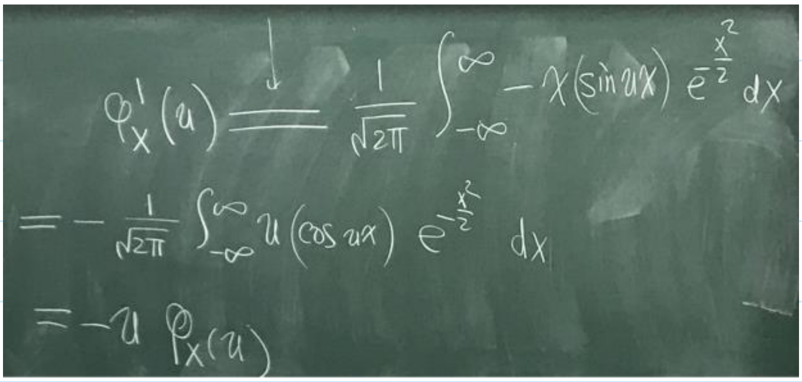


X ~ N(0,1) :

$\varphi_X(u) = \int e^{iux} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx$

$= \int_{-\infty}^{\infty} \frac{\cos ux}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + i \cdot \int_{-\infty}^{\infty} \frac{\sin ux}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \cos ux\, e^{-\frac{x^2}{2}} dx$



$$\frac{\varphi_x'}{\varphi_x} = -u \quad \to \quad \ln|\varphi_x(u)| = -\frac{u^2}{2} + C$$

$$\varphi_x(0) = 1, \quad \varphi_x(u) = e^{-\frac{u^2}{2}}$$

* $X \sim$ exponential (λ):

$$\varphi_x(u) = \frac{\lambda}{\lambda - iu}$$

⇝ double exponential - Laplace distribution $(\alpha = 0, \beta = 1)$

$$f_x(x) = \frac{1}{2} e^{-|x|}, \quad \varphi_x(u) = \frac{1}{1+u^2}$$

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