Class 15

2018年5月15日 星期二 10:02

generating function X r.v. $g_X(z) = E(z^X)$

Laplace transform

moment generating function

characteristic function (Fourier transform)

* 母函数

互相唯一决定

$$(a_n)_{n=0}^{\infty} \to \sum_{n=0}^{\infty} a_n \cdot z^n = g(z)$$

$$a_n = P(x=n) \quad \begin{cases} E(x) = \sum n \cdot a_n = g'(1) \to Ex^2 - Ex = g''(1) \\ Var(x) = Ex^2 - (Ex)^2 = g''(1) - g'(1)^2 + g'(1) \end{cases} \Rightarrow a_j = \frac{1}{j!} g^{(j)}(0)$$

* 唯一性定理:

$$z=0 \qquad \sum_{j=1}^{\infty} a_j z^j = \sum_{j=1}^{\infty} b_j z^j$$

不同概率分布对应不同母函数

$z=0 \to a_0 = b_0 \to /z \to z=0 \to a_1 = b_1 \cdots$

* 卷积:

$$(a_j) * (b_k) \Rightarrow (c_l)$$

$$c_l = \sum_{j=0}^{l} a_j b_{l-j}$$

* 当 a,b 独立且 $c = a+b$, $g_a(z) \cdot g_b(z) = g_c(z)$

* eg 1:

$$P(T=j) = P_j = q^{j-1} p, \quad j \geq 1$$

$$g(z) = \sum_{j=1}^{\infty} P_j \cdot z^j = \frac{p}{q} \sum_{j=1}^{\infty} (qz)^j = \frac{pz}{1-qz}$$

抛硬币第一次看到n次向上:

$$S_n = T_1 + T_2 + \cdots + T_n$$

$$g_{S_n}(z) = (g_T(z))^n = \left(\frac{pz}{1-qz}\right)^n$$

$$= \sum_{k=n}^{\infty} \binom{k-1}{n-1} p^n q^{k-n} z^k$$

$$P(S_n = n+j) = \binom{n+j-1}{j} p^n q^j = \binom{-n}{j} p^n (-q)^j$$

* Laplace 变换：

X: r.v. $\to E(e^{-\lambda X})$, $\lambda \in [0,\infty)$

$\int_{-\infty}^{\infty} e^{-\lambda u} f(u)\,du$, $\sum_{j=0}^{\infty} a_j e^{-j\lambda}$

$E(z^{X_1+X_2}) = E(z^{X_1}) \cdot E(z^{X_2})$

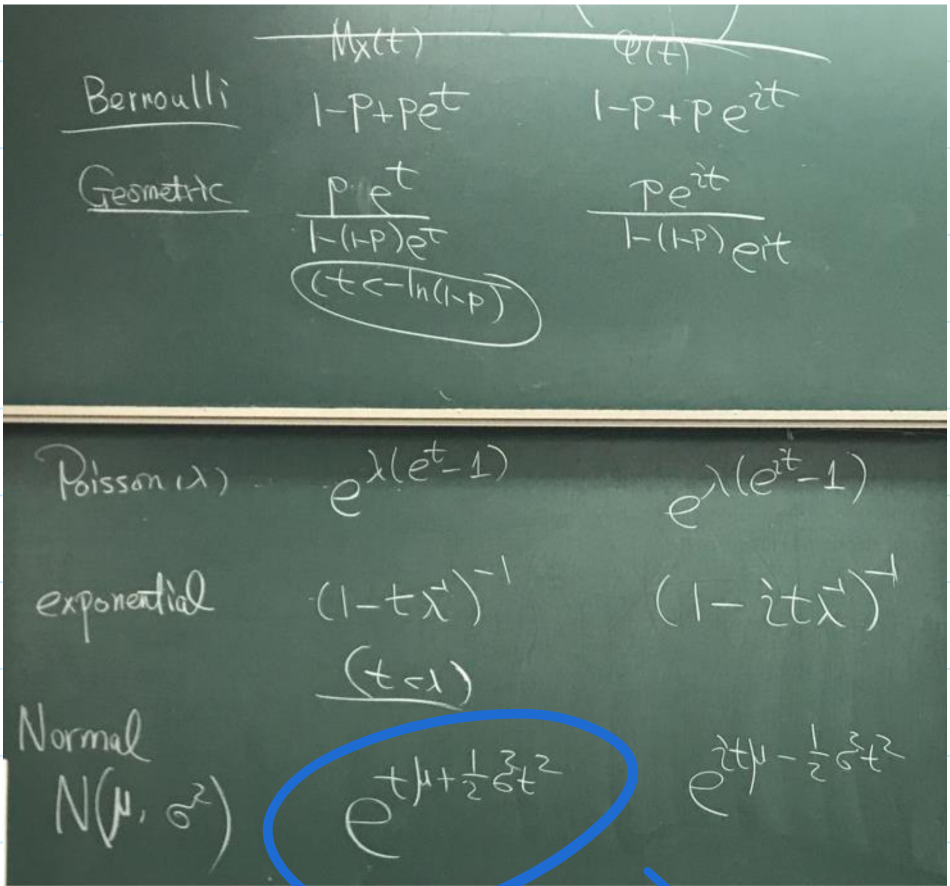
* 矩母函数：（见课本）

X: r.v. $M_X(t) = E(e^{tX}) \overset{\text{Taylor展开}}{=} E\left(1+tX+\frac{t^2X^2}{2!}+\cdots\right) \overset{\text{期望线性性}}{=} 1+tEX+\frac{t^2EX^2}{2!}\cdots$

{ 唯一性
独立性质

* 特征函数：

$\varphi_X(\theta) = E(e^{i\theta X})$



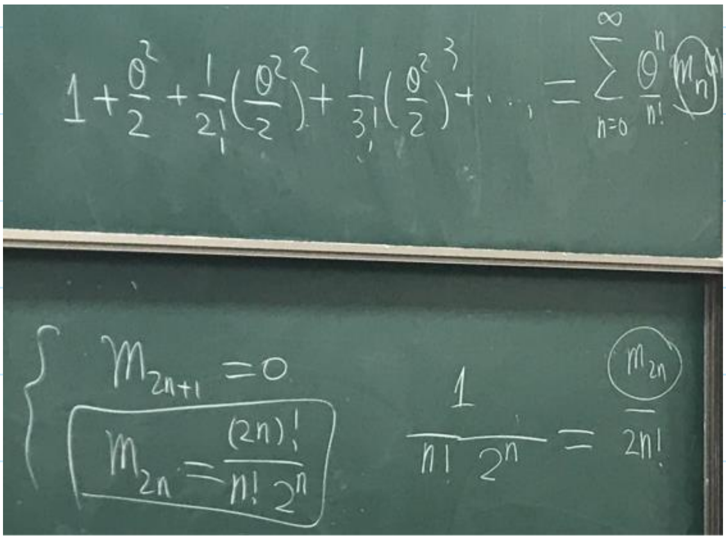
* P_{226} - 42 , P_{227} - 43 可能考！

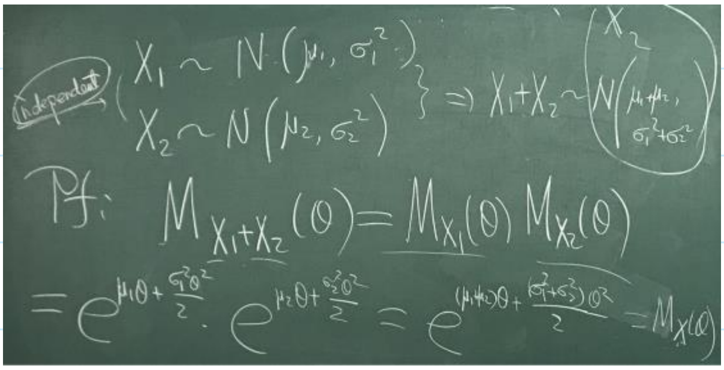
* $X \sim N(0,1)$

$$M_X(\theta) = \int_{-\infty}^{\infty} e^{\theta x} \varphi(x)\,dx \quad , \quad \varphi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(\theta x - \frac{x^2}{2}\right) dx$$

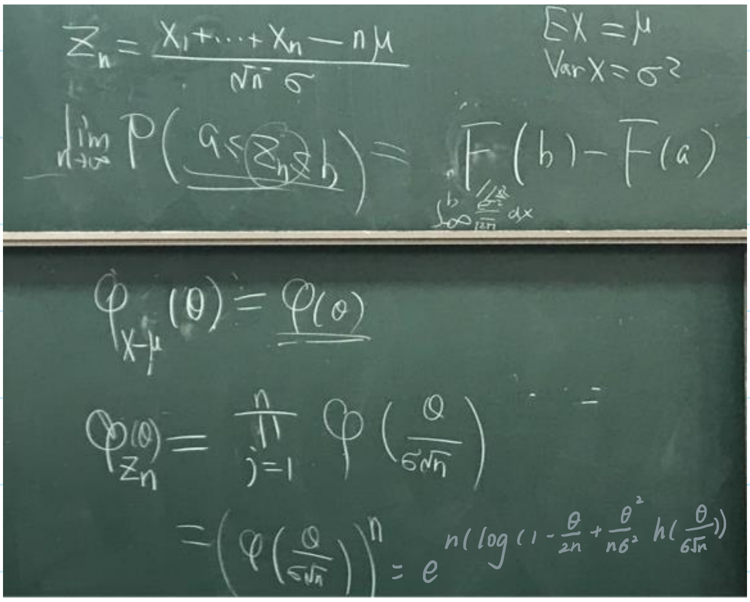
$$= e^{\frac{\theta^2}{2}} \int_{-\infty}^{\infty} \varphi(x-\theta)\,dx = e^{\frac{\theta^2}{2}}$$

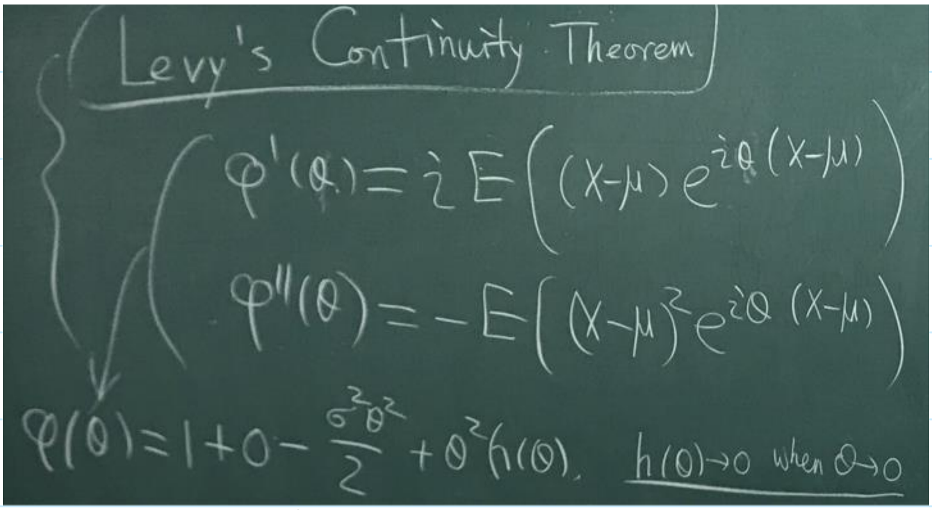




* Central Limit Theorem: P237

$x_1, x_2, \ldots$ i.i.d 独立同分布 $x_i \sim X$ $\begin{cases} EX = \mu \\ \mathrm{Var}\,X = \sigma^2 \end{cases}$





$\lim_{n\to\infty} \varphi_{Z_n}(\theta) = e^{-\frac{\theta^2}{2}}$