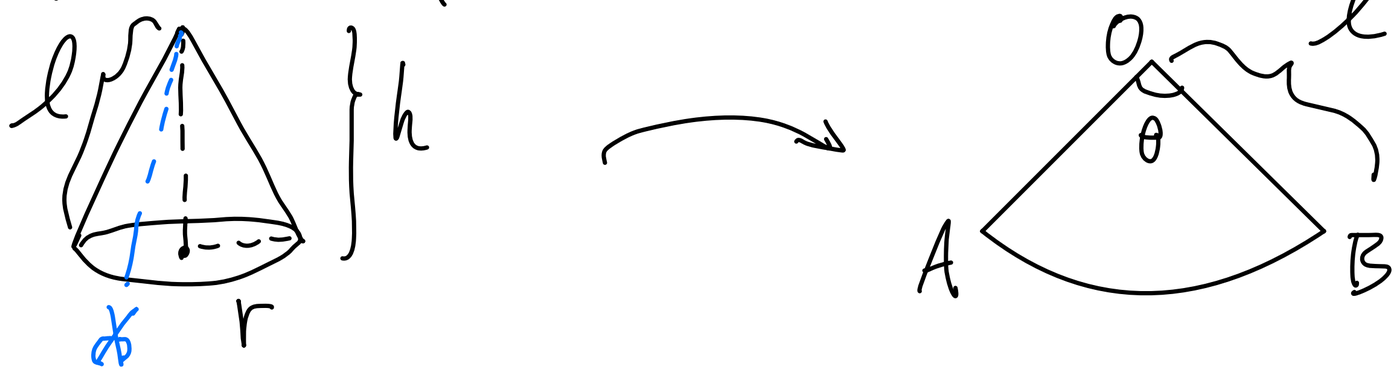
Surface area of a cone (w/o base)

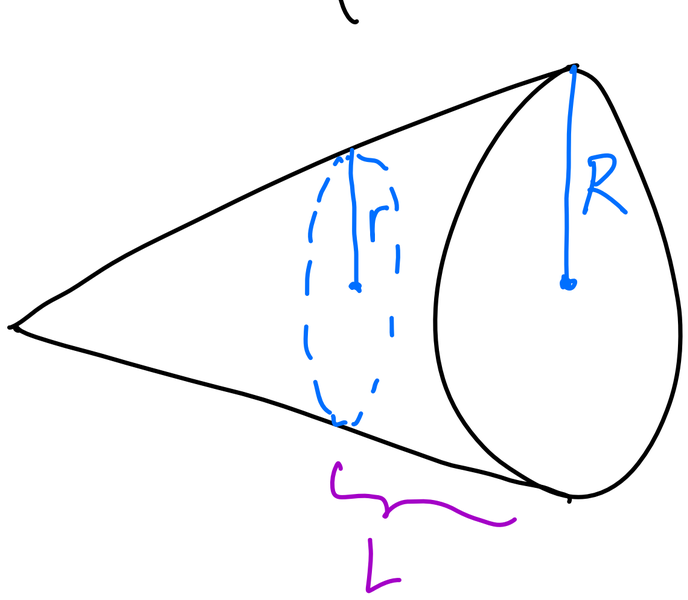


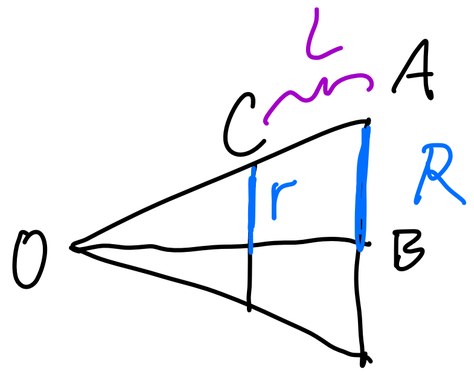
① Area $= \frac{\theta}{2\pi}\pi l^2$; want to express with no θ.

② $\frac{\theta}{2\pi} = \frac{\overset{\frown}{AB}}{2\pi l} = \frac{\text{Circumference of cone base}}{2\pi l}$ $\frac{2\pi r}{2\pi l} = \frac{r}{l}$

①, ② $\Rightarrow$ Area $= \frac{r}{l}\pi l^2 = \pi r l$.

Conical frustum





Area of frustum

= Area of Big cone
− Area of Small cone

$= \pi|OA|R - \pi|OC|r$

$= \pi(|OC|+L)R - \pi|OC|r$ ③

Want to "kill" $|OC|$

$\frac{|OC|}{|OC|+L} = \frac{r}{R} \Rightarrow |OC|R = |OC|r + Lr$

$\Rightarrow |OC| = \frac{Lr}{R-r}$ ④

$\Rightarrow |OC| + L = \frac{Lr + LR - Lr}{R-r} = \frac{LR}{R-r}$ ⑤

Sub ④ & ⑤ in ③ :

$$\text{Area} = \pi(|OC|+L)R - \pi|OC|r$$

$$= \pi\frac{LR}{R-r}R - \pi\frac{Lr}{R-r}r = \pi\frac{L}{R-r}(R^2-r^2)$$

$$= \pi(R+r)L.$$