Deep Recurrent Factor Models

ACM40960 Projects in Mathematical Modelling

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Agenda

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- Objectives
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Introduction

Stock Returns: Definition and Computation

- Stock returns measure the change in value of an investment over a period of time
- The mathematical definition of returns is

$$Return(t) = \frac{Price(t) - Price(t-1)}{Price(t-1)},$$

where *Price* refers to the '*Closing Price*' of a particular time period.

- 'Classical' research in *Quantitative Finance* is mainly interested in modelling **monthly returns**
- Monthly returns are often used by banks, hedge funds or other industry participants to inform their decision making process

Stock Return Prediction in Quantitative Finance

- Research works in Quantitative Finance try to model the statistical behavior of returns
- Accurate predictions of returns of stocks or stock-market indices are vital
- Industry professionals, like portfolio managers, rely on statistical models.
- A popular approach to forecasting returns are **Factor Models**

Factor Models

- are quantitative frameworks used to explain and predict the returns of financial assets.
- are based on the idea that a small number of underlying factors drive the variation in asset returns.
- assume a **linear** relationship between factors and stock returns

Classic Example: Fama-French Three-Factor Model

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,M} \cdot (R_{M,t} - R_{f,t}) + \beta_{i,SMB} \cdot SMB_t + \beta_{i,HML} \cdot HML_t + \varepsilon_{i,t}$$

1. **Market Risk Factor (RM-RF):** Represents the excess return of the market portfolio over the risk-free rate.
2. **Size Factor (SMB - Small Minus Big):** Reflects the historical outperformance of small-cap stocks compared to large-cap stocks.
3. **Value Factor (HML - High Minus Low):** Represents the historical tendency of value stocks (stocks with low price-to-book ratios) to outperform growth stocks (stocks with high price-to-book ratios).

Disadvantages of classical Factor Model

While classical factor models have been widely used in quantitative finance, they come with certain limitations. To name 2 major ones:

1. **Linearity Assumption:** Classical models assume linear relationships between factors and asset returns, which may not capture complex non-linear interactions or effects.
2. **Lack of Time-Dependent Effects:** Classical models often overlook time-varying relationships and fail to account for changing market dynamics over time, even though recent research shows that time dependence is important - see Neuhierl et al. (2023)

New Idea: Deep Recurrent Factor Models

- **Deep Recurrent Factor Model** is a term coined by Nakagawa et al. (2019).
- The authors challenge the idea of linear factor models to predict stock returns
- The authors use **Long-Short-Term Memory networks (LSTM)** in conjunction with **layer-wise-relevance propagation (LRP)** to construct a time-varying factor model that outperforms equivalent linear models, whilst providing insights into the relevance of particular factors in the prediction.

Why should Finance care?

- Linear models are often employed due to their simplicity, but not necessarily due to their predictive performance
- Neural networks are often considered **black box** algorithms in the context of stock return predictions, which has led to their limited adoption
- Interpretability of neural networks is a significant concern in finance, where understanding the factors and variables driving predictions is crucial for
  - decision-making,
  - risk assessment and
  - regulatory compliance

...LRP makes neural networks more explainable and could foster adoptation in quantitative investing, portfolio management and asset pricing, whilst providing superior predictive performance

Objectives

Objectives

The objectives of the project can be separated into:

- Research Objectives
- Technical Objectives

Project: Objectives

Research Objectives

- Replicate the approach used in Nakagawa et al. (2019) to predict stock returns of the SP500 stock-market index.
- Analyse whether Deep Recurrent Factor Models provide a better predictive performance than linear models.
- Analyse the importance of factors through LRP.

Project: Objectives

Technical Objectives

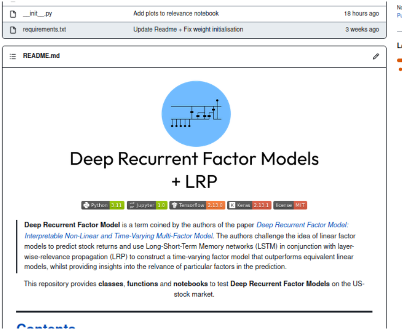
- Build classes and methods on top of `Keras` and `Tensorflow` to render LRP possible for deep LSTM networks.
- Do so by not interfering with typical `Keras` workflow
- Make classes and methods **openly available** and for others to re-use

Results and Outcomes

Results: LRP for Deep LSTM Networks

- We implemented LRP rules and algorithms according to
  - Arras et al. (2017) and
  - Arjona-Medina et al. (2019)

- We made the classes and methods available on Github for easy download and use:



Results: Replication of Nakagawa et al. (2019)

- We fitted a **linear model** and a **deep recurrent factor model** to returns of SP500 stock market index
- We compared the models predictive performance over the time frame 1991-2021
- We tried multiple network architectures, which almost always were superior to the linear model
- Numerical Results[1]:

Model	**MSE**	**RMSE**
Linear Model	1.3338	1.1549
Deep Recurrent Factor Model	0.9865	0.9932

[1] For the full analysis and a complete list of the results for different architectures (incl. regularisation + dropout) we refer to the GitHub repository.

Results: Replication of Nakagawa et al. (2019)

Linear vs. Deep Recurrent Factor Model

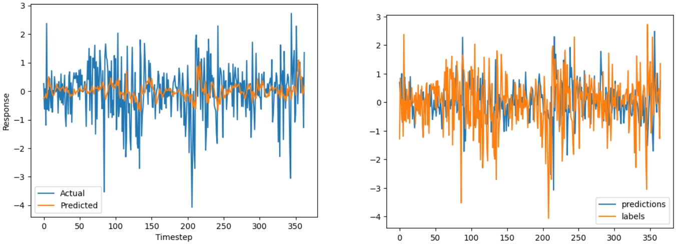


Figure 1: Deep Recurrent Factor Model (Left), Linear Model (Right)

Results: Replication of Nakagawa et al. (2019)

Factor Relevance across time

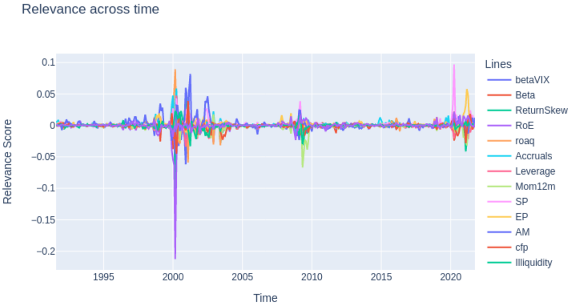


Figure 2: Relevance Scores of features over time

Results: Replication of Nakagawa et al. (2019)

... after grouping the features according to the factor categories *Risk*, *Quality*, *Momentum*, *Value* and *Size*

we get:

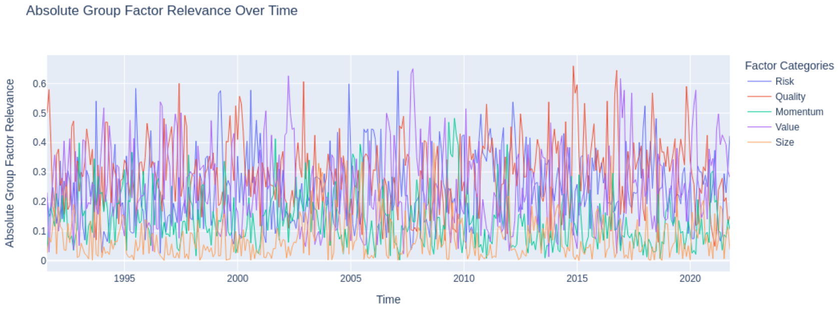


Figure 3: Relevance Scores of factors over time

Results: Replication of Nakagawa et al. (2019)

... we smoothed the curves by computing simple moving averages of the 5 time series:

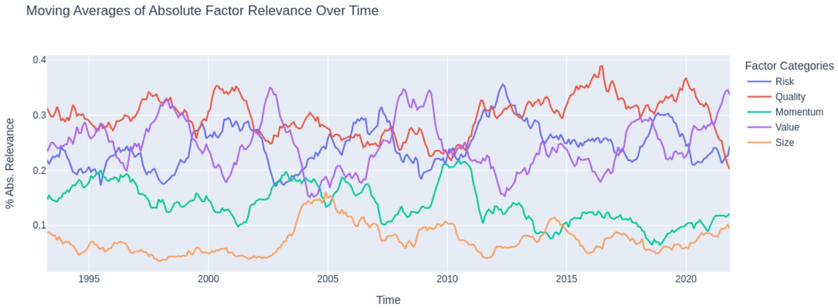
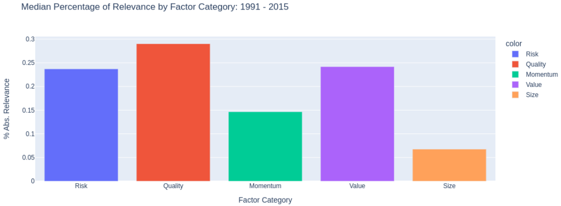
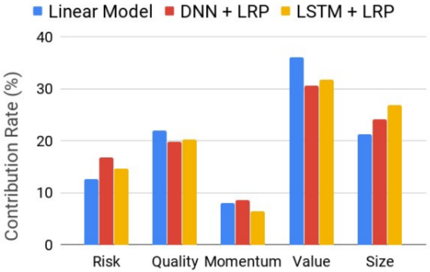


Figure 4: Relevance Scores of factors over time (MA)

Results: Replication of Nakagawa et al. (2019)

... we computed the median absolute relevance for each factor group:





Results: Replication of Nakagawa et al. (2019)

Areas for Further Research

- New LRP rules
- Try out different subsets of factors
- Try out different network architectures
- Analyse relevance across time

References

Arjona-Medina, Jose A, Michael Gillhofer, Michael Widrich, Thomas Unterthiner, Johannes Brandstetter, and Sepp Hochreiter. 2019. "Rudder: Return Decomposition for Delayed Rewards." *Advances in Neural Information Processing Systems* 32.

Arras, Leila, Grégoire Montavon, Klaus-Robert Müller, and Wojciech Samek. 2017. "Explaining Recurrent Neural Network Predictions in Sentiment Analysis." *arXiv Preprint arXiv:1706.07206*.

Nakagawa, Kei, Tomoki Ito, Masaya Abe, and Kiyoshi Izumi. 2019. "Deep Recurrent Factor Model: Interpretable Non-Linear and Time-Varying Multi-Factor Model." *arXiv Preprint arXiv:1901.11493*.

Neuhierl, Andreas, Otto Randl, Christoph Reschenhofer, and Josef Zechner. 2023. "Timing the Factor Zoo." *Available at SSRN*.