```
> restart:
  with(LinearAlgebra):

  # REQUIRES THE Kronecker.mpl and Hyperplane.mpl FILES

  read("Kronecker.mpl"):
  read("Hyperplane.mpl"):
```

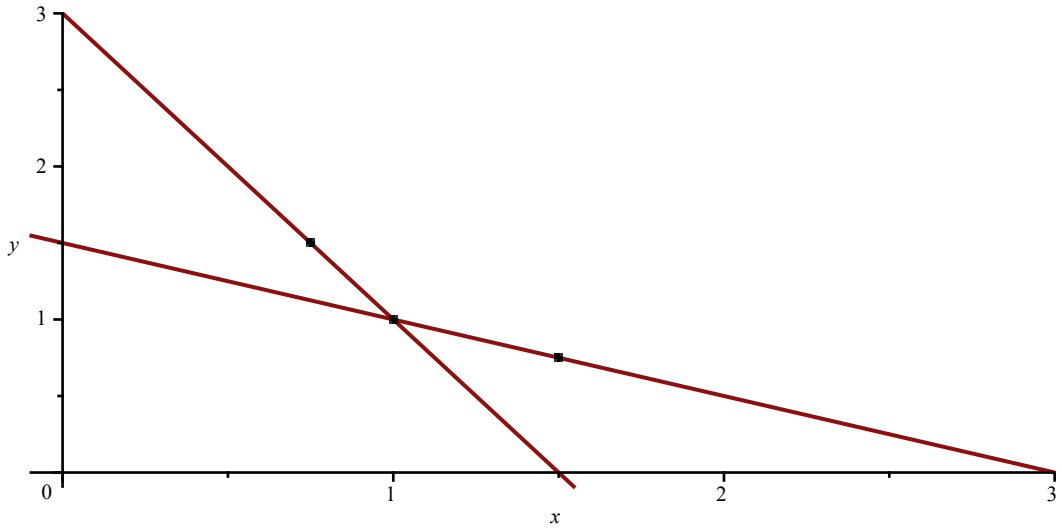
```
> ##########################
  # Example 1 From Paper
  ##########################
```

```
> l1 := (1-2/3*x-1/3*y):
  l2 := (1-1/3*x-2/3*y):
  H := l1*l2;
  G := 1:
  r := [1,1]:
```

$$H := \left(1 - \frac{x}{3} - \frac{2\,y}{3}\right)\left(1 - \frac{2\,x}{3} - \frac{y}{3}\right) \tag{1}$$

```
> # Find only contributing points
  contrib:= contribSing(1,H,[1,1],[x,y],u);
  cps := map(a->a[2],contrib):
  P1 := plots[implicitplot]([l1,l2],x=-0.1..3,y=-0.1..3):
  P2 := plots[pointplot](cps,symbol=solidcircle):
  plots[display](P1,P2);
```

$$contrib := \{[[u+4, [1,1,1,1], -4.], [1., 1.], \{1,2\}], [[4u-7, [3,6,8], 1.75000], [0.7500000000, 1.500000000], \{2\}], [[4u+13, [6,3,8], -3.25000], [1.500000000, 0.7500000000], \{1\}]\}$$




```
> # List of contributing singularities, the flats they're on, and
  asymptotic contributions
  for k in contrib do
  aa,rr := ASMContrib(G,H,r,[x,y],k,u):
  print(k[2..3], aa);
```

```
od:
```

$$[[1., 1.], \{1, 2\}], 3$$

$$[[0.7500000000, 1.500000000], \{2\}], -\frac{4\left(\frac{8}{9}\right)^n}{\sqrt{n}\sqrt{\pi}}$$

$$[[1.500000000, 0.7500000000], \{1\}], -\frac{4\left(\frac{8}{9}\right)^n}{\sqrt{n}\sqrt{\pi}} \quad \textbf{(2)}$$

```
> # Detect non-generic direction
  contribSing(1,H,[2,1],[x,y],u);
Error, (in contribSing) Not generic direction |Hyperplane.mpl:66|
> #############
  # Example 2
  #############
  l1 := 1-2*x-y:
  l2 := 1-x-2*y:
  l3 := 1-4*x-3*y/2:
  l4 := 1-2*x/3-2*y/3:
  H := l1*l2*l3*l4;

  G := 1;
  r := [1,1];
```

$$H := (1-2x-y)\,(1-x-2y)\left(1-4x-\frac{3y}{2}\right)\left(1-\frac{2x}{3}-\frac{2y}{3}\right)$$

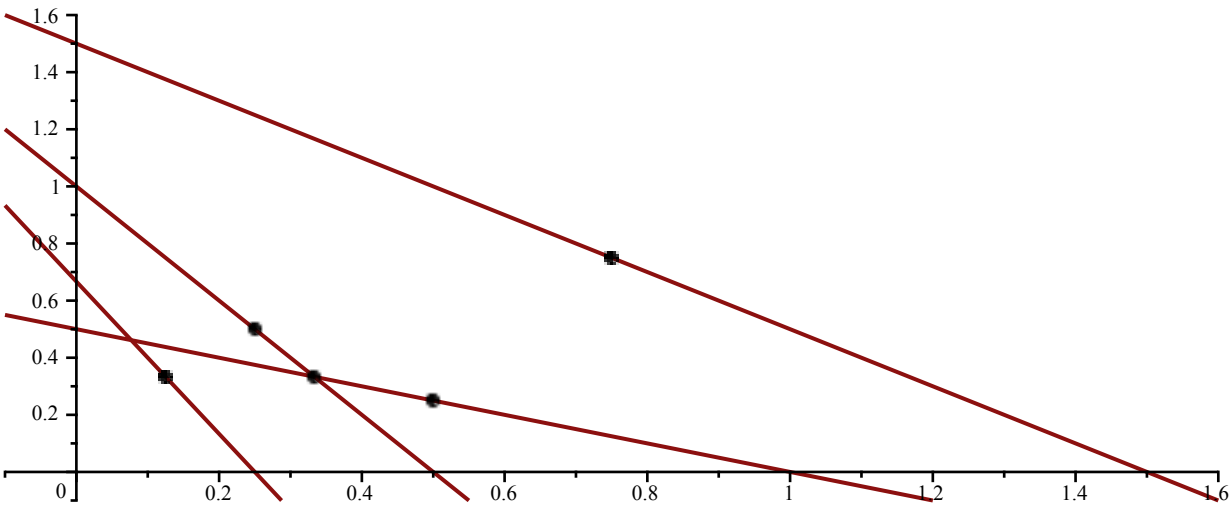
$$G := 1$$

$$r := [1, 1] \quad \textbf{(3)}$$

```
> contrib:= contribSing(1,H,[1,1],[x,y],u):
  cps := map(a->a[2],contrib);
```

$$cps := \{[0.1250000000, 0.3333333333], [0.2500000000, 0.5000000000], [0.3333333333, 0.3333333333], [0.5000000000, 0.2500000000], [0.7500000000, 0.7500000000]\} \quad \textbf{(4)}$$

```
> # Plots of flats and contributing singularities
  P1 := plots[implicitplot]([l1,l2,l3,l4],x=-0.1..1.6,y=-0.1..1.6):
  P2 := plots[pointplot](cps, symbol=solidcircle):
  plots[display](P1,P2,labels=["",""]);
```




```
> # List of contributing singularities, the flats they're on, and
  asymptotic contributions
  for k in contrib do
  aa,rr := ASMContrib(G,H,r,[x,y],k,u):
  print(k[2..3], aa);
  od:
```

$$[[0.2500000000, 0.5000000000], \{1\}], \frac{32\ 8^n}{3\sqrt{n}\sqrt{\pi}}$$

$$[[0.3333333333, 0.3333333333], \{1, 2\}], -\frac{162\ 9^n}{25}$$

$$[[0.5000000000, 0.2500000000], \{2\}], \frac{64\ 8^n}{11\sqrt{n}\sqrt{\pi}}$$

$$[[0.7500000000, 0.7500000000], \{4\}], -\frac{128\left(\frac{16}{9}\right)^n}{625\sqrt{n}\sqrt{\pi}}$$

$$[[0.1250000000, 0.3333333333], \{3\}], \frac{10368\ 24^n}{625\sqrt{n}\sqrt{\pi}} \quad \textbf{(5)}$$

```
> ##################
  # Example 3
  ##################

  l1 := 1+2*x+y+z+w:
  l2 := 1-x-3*y-z-w:
  l3 := 1-x+y-4*z+w:
  l4 := 1-x-y+z-5*w:
  G := 1;
  H := l1*l2*l3*l4;
  r := [1,2,1,2];
```

$$G := 1$$

$$H := (1+2x+y+z+w)\,(1-x-3y-z-w)\,(1-x+y-4z+w)\,(1-x-y+z-5w)$$

$$r := [1,2,1,2] \qquad \textbf{(6)}$$

```
> cont := contribSing(G,H,r,[x,y,z,w],u):
```

```
> for k in cont do
    aa,rr := ASMContrib(G,H,r,[x,y,z,w],k,u):
    print(k[2],evalf(subs(u=rr,aa)));
  od:
```

$$[0.1666666667, 0.1111111111, 0.1666666667, 0.3333333333],\ -\frac{0.08414040090\ 26244.^{n}}{n^{3/2}}$$

$$[-0.08333333333, -0.3333333333, -0.1666666667, -0.3333333333],\ \frac{0.009526787421\ 5832.^{n}}{n^{3/2}}$$

$$[0.1666666667, 0.3333333333, -0.1666666667, 0.06666666667],\ \frac{0.3918653665\ (-72900.)^{n}}{n^{3/2}}$$

$$[0.1666666667, -0.3333333333, 0.04166666667, -0.3333333333],\ \frac{0.009827079381\ 11664.^{n}}{n^{3/2}}$$

$$[-0.2380952381, -0.3333333333, 0.1428571429, -0.3333333333],\ -\frac{0.02350113643\ (-2381.400000)^{n}}{n}$$

$$[-1.351351351, 0.3243243243, 0.8108108108, 0.5675675676],\ -0.06700132275\ (-26.93506350)^{n}$$

$$[0.1666666667, 0.1263111081, 0.2608296366, 0.1935703724],\ \frac{1.192577620\ 38479.85727^{n}}{n}$$

$$[0.1666666667, 0.3141650824, -0.1774963033, 0.06833438953],\ -\frac{0.1095419419\ (-73344.48638)^{n}}{n}$$

$$[-0.5701451904, -0.7303039929, 0.3420871143, 0.5285072595],\ \frac{0.02542055024\ (-34.41664680)^{n}}{\sqrt{n}}$$

$$[6.496071116, -10.26969601, -3.897642670, 0.1751964442],\ \frac{0.00003047625289\ (-0.01220060365)^{n}}{\sqrt{n}}$$

$$[-1.562687101, 0.2186564497, 0.6355725838, 1.271145168],$$

$$\frac{0.007329368640\ (-13.03310612)^n}{n}$$

$$[-0.1706462327,\ 0.9146768836,\ -0.5244614727,\ -1.048922945],$$

$$-\frac{0.003902611800\ 12.13855879^n}{n}$$

$$[-1.175841744,\ 1.395643924,\ -0.1666666667,\ 0.1227062307],$$

$$-\frac{0.005575382315\ 173.9882368^n}{n}$$

$$[-0.1574915894,\ -0.8956439237,\ -0.1666666667,\ 0.3772937693],$$

$$-\frac{0.01960643172\ 333.6290433^n}{n}$$

$$[0.1666666667,\ 0.6890163923,\ 0.05772677642,\ -1.291442620],$$

$$-\frac{0.01386753458\ 131.2697398^n}{n}$$

$$[-0.4399271006,\ 0.7800364497,\ 0.2639562604,\ -1.164138509],$$

$$\frac{0.01381666878\ (-10.44355148)^n}{\sqrt{n}}$$

$$[0.1666666667,\ -0.9208229684,\ 0.06930042770,\ 0.3646913459],$$

$$-\frac{0.03549494560\ 767.7364017^n}{n}$$

$$[0.1666666667,\ 0.7541563018,\ 0.4219276425,\ 0.1002209348],$$

$$-\frac{0.01219831420\ 2489.279394^n}{n}$$

$$[-0.7686616281,\ 0.6156691860,\ -0.2574536819,\ 0.1791077520],$$

$$\frac{0.04698446837\ 415.5680192^n}{\sqrt{n}}$$

$$[-1.526447255,\ 0.2367763725,\ 1.131819967,\ 0.6842981700], \quad \textbf{(7)}$$

$$\frac{0.009852538640\ (-22.04821499)^n}{\sqrt{n}}$$

```
> # Highest exponential growth is
  max([seq(abs(1/mul(k[2][j]^r[j],j=1..4)),k=cont)]);
```

$$73344.48637 \quad \textbf{(8)}$$

```
> # This comes from the 8th term - THIS CAN CHANGE ON DIFFERENT RUNS
  OF WORKSHEET - VERIFY INDEX IS CORRECT FOR YOU!
  aa,rr := ASMContrib(G,H,r,[x,y,z,w],cont[8],u);
  subs(u=evalf(rr),aa);
```

$aa, rr :=$

$$\frac{1}{2\,n\,(756\,u-16551)\,\pi}\left(\left(-\frac{\frac{41183843148\,u}{3125}-\frac{926340202314}{3125}}{756\,u-16551}\right)^{n}\right.$$

$$\left.\sqrt{\frac{756\,u-16551}{\frac{11252849832\,u}{125}-\frac{207123447924}{125}}}\left(\frac{197400039363\,u}{265000}-\frac{109062076041549}{3710000}\right)\right),$$

$$\mathit{RootOf}\left(378\,\_Z^2-16551\,\_Z+108785,\,35.731433246869640915\right)$$

$$-\frac{0.1095419419\,(-73344.48637)^{n}}{n} \tag{9}$$

```
> # A few other simple tests
```

```
> cont := contribSing(1,1-x-y,[1,1],[x,y],u):
  aa,rr := ASMContrib(1,1-x-y,[1,1],[x,y],cont[1],u):
  aa, simplify(convert(series(factorial(2*n)/factorial(n)^2,n=
  infinity,2),polynom)) assuming n>0;
```

$$\frac{4^{n}}{\sqrt{n}\,\sqrt{\pi}},\ \frac{4^{n}}{\sqrt{n}\,\sqrt{\pi}} \tag{10}$$

```
> cont := contribSing(1,1-x-y-z,[1,1,1],[x,y,z],u):
  aa,rr := ASMContrib(1,1-x-y-z,[1,1,1],[x,y,z],cont[1],u):
  aa, simplify(convert(series(factorial(3*n)/factorial(n)^3,n=
  infinity,2),polynom));
```

$$\frac{27^{n}\sqrt{3}}{2\,n\,\pi},\ \frac{27^{n}\sqrt{3}}{2\,n\,\pi} \tag{11}$$

```
> ########################
  # Non-simple example
  ########################
  l1 := 1-2*x-2*y+2*z;
  l2 := 1-2*x;
  l3 := 1-2*y;
  l4 := 1-2*z;
  l5 := 1-x-y;
```

$$l1 := 1-2\,x-2\,y+2\,z$$

$$l2 := 1-2\,x$$

$$l3 := 1-2\,y$$

$$l4 := 1-2\,z$$

$$l5 := 1-x-y \tag{12}$$

```
> H := l1*l2*l3*l4*l5^2:
  H1 := (4*(-1+2*y))*(-1+2*z)*(-1+x+y)^4;
  H2 := (4*(-1+2*x+2*y-2*z))*(-1+2*y)*(-1+x+y)^4:
```

```
H3 := (4*(-1+2*x))*(-1+2*z)*(-1+x+y)^4:
H4 := (4*(-1+2*x+2*y-2*z))*(-1+2*x)*(-1+x+y)^4:
1/H = 1/H1+1/H2+1/H3+1/H4;
```

$$H1 := 4\,(-1+2y)\,(-1+2z)\,(-1+x+y)^4$$

$$\frac{1}{(1-2x-2y+2z)\,(1-2x)\,(1-2y)\,(1-2z)\,(1-x-y)^2} \tag{13}$$

$$= \frac{1}{4\,(-1+2y)\,(-1+2z)\,(-1+x+y)^4} + \frac{1}{4\,(-1+2x+2y-2z)\,(-1+2y)\,(-1+x+y)^4} + \frac{1}{4\,(-1+2x)\,(-1+2z)\,(-1+x+y)^4} + \frac{1}{4\,(-1+2x+2y-2z)\,(-1+2x)\,(-1+x+y)^4}$$

```
> normal(1/H-1/H1-1/H2-1/H3-1/H4);
```

$$0 \tag{14}$$

```
> for HH in [H1,H2,H3,H4] do
  print('H' = HH);
  cont := contribSing(1,HH,[1,2,3],[x,y,z],u):
  for k in cont do
    aa,rr := ASMContrib(1,HH,[1,2,3],[x,y,z],k,u):
    print(aa);
  print();
  od:od:
```

$$H = 4\,(-1+2y)\,(-1+2z)\,(-1+x+y)^4$$

$$\frac{2\,64^n\,n^3}{3}$$

$$-\frac{81\,54^n\,n^{5/2}\sqrt{21}\,\sqrt{2}}{224\,\sqrt{\pi}}$$

$$H = 4\,(-1+2x+2y-2z)\,(-1+2y)\,(-1+x+y)^4$$

$$-\frac{(-27648)^n}{4\,n\,\pi}$$

$$H = 4\,(-1+2x)\,(-1+2z)\,(-1+x+y)^4$$

$$\frac{81\,54^n\,n^{5/2}\sqrt{21}\,\sqrt{2}}{224\,\sqrt{\pi}}$$

$$H = 4\,(-1+2x+2y-2z)\,(-1+2x)\,(-1+x+y)^4 \\ -\frac{(-27648)^n}{5\,n\,\pi}$$

(15)