Lecture 10

Intro to Machine Learning

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AEM 7130

Roadmap

1. Intro to machine learning
2. Supervised methods, validation, and cross-validation
3. Unsupervised methods

What is machine learning?

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Unlike standard econometrics, we aren't as interested in unbiased estimators or causality

We just care about getting the prediction right and it working rather than having formal statistical properties

We want a good prediction of y, not good estimates of coefficients

Econometricians are finding ways to do both (double selection, double ML, trees for heterogeneous causal effects, etc)

Terminology

You'll run into terms that have similar meaning to what we use in economics

1. **Features**: regressors, your xs
2. **Supervised learning**: settings where we have an outcome y and a set of features x, this will be called regression if y is continuous, or classification if y is categorical
3. **Unsupervised learning**: settings where we only have features x, there's no outcomes!
4. **Training sample**: the partition of the dataset used to estimate the model
5. **Validation sample**: the partition of the dataset used to validate the out-of-sample fit within a fold
6. **Test sample**: The partition of the dataset used to test out-of-sample fit of the final model

The key pieces

Out-of-sample validation: we will validate our methods by checking their fit and properties out-of-sample

The fact that we're trying to solve prediction problems is why we can do this: we see the actual realizations of y, so that we can test the quality of the fit

For causal inference problems we never observe the true β so we can't validate our solutions

We use our training sample to estimate our model and then test it on our test sample

The key pieces

Regularization: impose a penalty for overfitting the model

You can get great (perfect) in-sample prediction by having $N = K$

The problem is that this will lead to an over-fit model that will do very poorly out-of-sample

How much regularization do we want?

We typically use cross-validation methods to help us choose

The key pieces

Scalability: can handle a lot of data N or K

Could have thousands of features, billions of observations

Having parallelizable algorithms is important

The key pieces

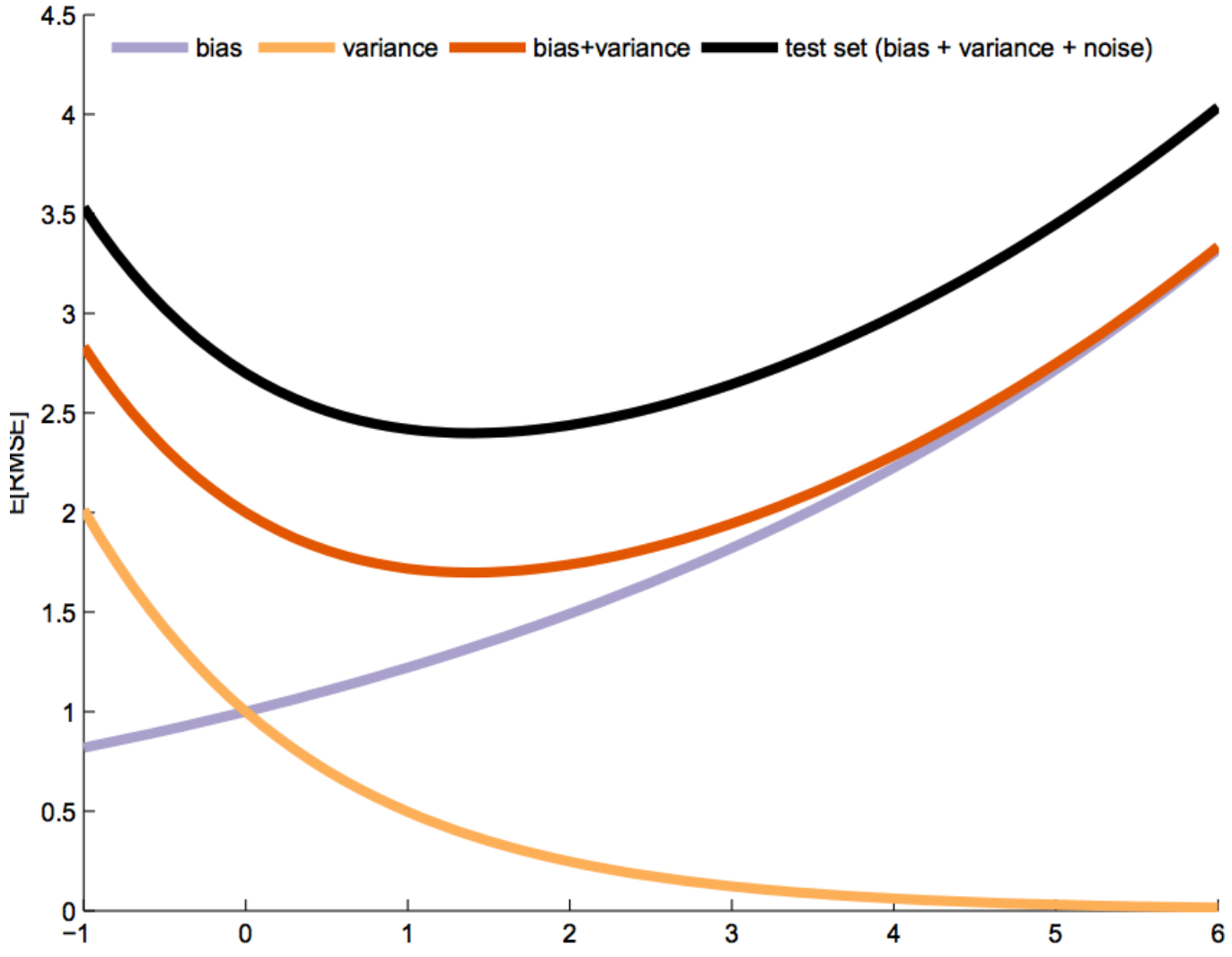
Bias-variance trade-off: expected mean squared error (MSE) of a prediction is a combo of bias and variance

Typically as economists we want low (zero) bias estimators because we care about the sign and interpretation of coefficients

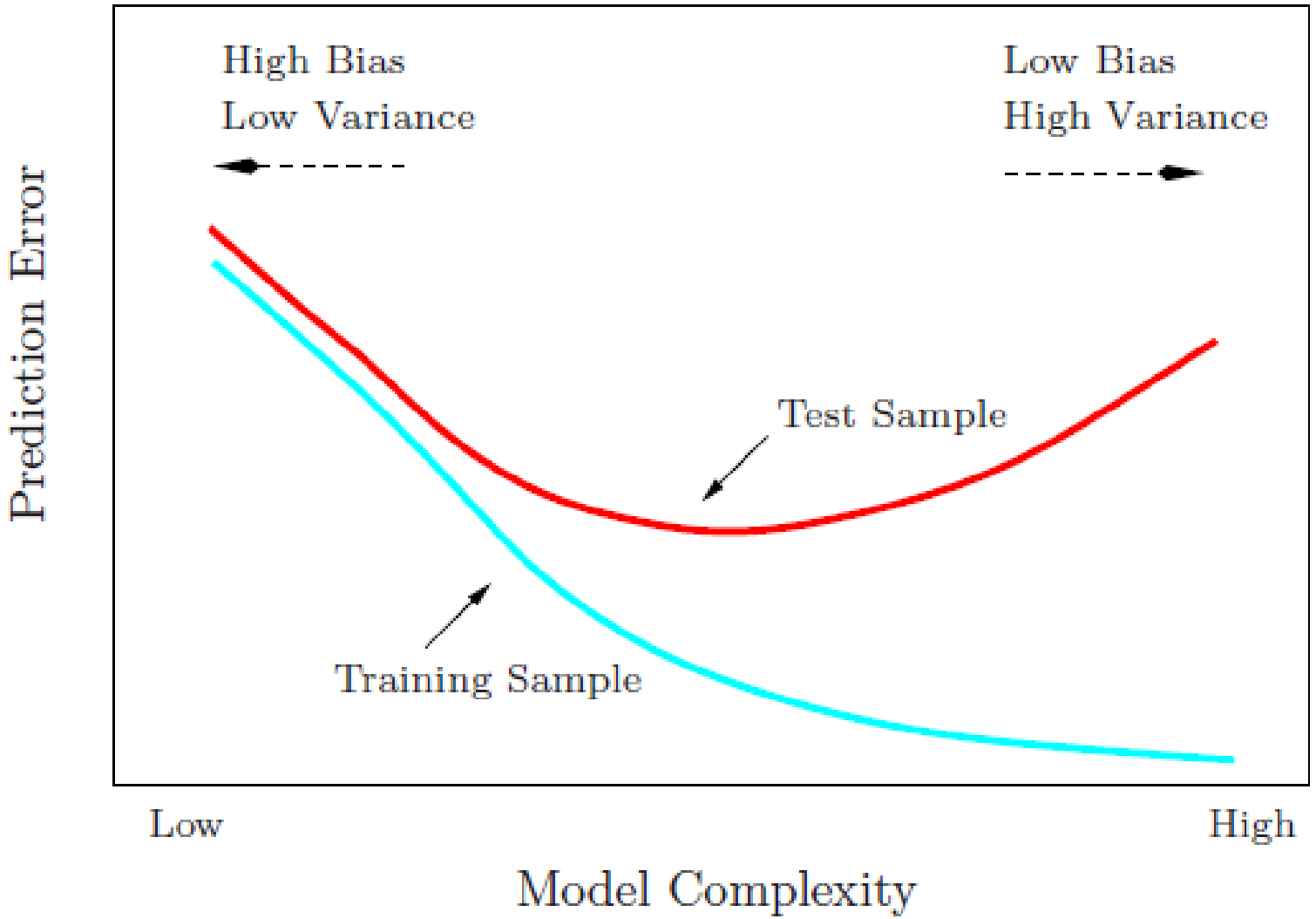
If we want a good prediction of y, we may be willing to allow more bias to reduce variance and decrease MSE

$$
\begin{aligned}
E(y - \hat{f}\,(x)^2) &= E[y^2] + E[\hat{f}^{\,2}] - 2[Ey\hat{f}\,] \\
&= var(y) + E[y^2] + var(\hat{f}\,) + E[\hat{f}^{\,2}] - 2fE[\hat{f}\,] \\
&= var(y) + var(\hat{f}\,) + (f - E[\hat{f}\,])^2 \\
&= \sigma^2 + variance + bias^2
\end{aligned}
$$

Bias-variance trade-off



Bias-variance trade-off



Shrinkage/regularization methods

One way to reduce variance is to shrink the βs, or even set some to zero ($var(0) = 0$)

A common way we implement this is by penalizing deviation in βs different than zero:

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda \times Penalty(\beta \neq 0)$$

If we set estimates to zero we will end up with **sparse** representations

Bet on the **sparsity principle**: use a procedure that does well in sparse problems, since no procedure does well in dense problems (Hastie, Tibshirani and Wainwright 2015)

Shrinkage/regularization methods

There are three common specifications for this approach depending on how we specify the penalty function

- **Ridge regression**: Penalty = $\sum_l \beta_l^2$
- **Least Absolute Shrinkage and Selection Operator (LASSO)**: Penalty = $\sum_l |\beta_l|$
- **Elastic Net**: Penalty = $(1-\alpha)\sum_l \beta_l^2 + \alpha \sum_l |\beta_l|$

Ridge regression

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda \sum_{l} \beta_l^2$$

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda(||\beta||_2)^2$$

Ridge regression penalizes coefficients based on their L_2 norm, this tends to **shrink** coefficients toward zero

It rarely sets coefficients exactly equal to zero since the penalty is smooth

It does a good job with fixing ill-conditioning problems and in cases where $K > N$

It also has a closed form solution: $\hat{\beta} = (X'X + \lambda I)^{-1} X'Y$

Ridge regression

Ridge has a nice Bayesian interpretation

If

- The prior distribution of β is $\mathcal{N}(0, \tau^2 \times I)$
- The error term is distributed $\mathcal{N}(0, \sigma^2)$
- $\lambda = \sigma^2/\tau^2$

Then

$\hat{\beta}_{ridge}$ is the posterior mean, median, and mode

Sidebar: normalization

When regularizing we generally want to normalize our features and outcome

Why?

If features vary dramatically in magnitude or have different scales (dollars of GDP vs percent GDP), variables that are numerically large will get penalized more just because of their units

If we set all variables to mean zero, variance one they are on a common playing field for regularization

Regularizing the outcome will get rid of the intercept term as well

For ridge, normalizing results in coefficients being shrunk by a factor of $1/(1+\lambda)$

LASSO

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda \sum_{l} |\beta_l|$$

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda ||\beta||_1$$

LASSO penalizes coefficients based on their L_1 norm, this tends to **select** a subset of ceofficients, i.e. it sets a number of them equal precisely to zero and generates a sparse solution

LASSO is generally used for variable or model selection

LASSO has no analytic solution, need to use convex optimization routines

LASSO

LASSO also has a nice Bayesian interpretation

If

- The prior distribution of β is Laplacian
- The error term is distributed $\mathcal{N}(0, \sigma^2)$

Then

$\hat{\beta}_{LASSO}$ is the posterior mode

L_p regularization

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda(||\beta||_p)^{1/p}$$

Ridge and LASSO are special cases of a general L_p regularizer

Another special case is subset selection is we use the L_0 norm

Subset selection induces the estimates to be the OLS estimates but it is computationally tough to solve so it is not often used

Ridge vs LASSO

One way to reframe ridge and LASSO are as their dual, constrained problems:

Ridge:

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 \text{ subject to } \sum_{l} \beta_l^2 \leq s$$

LASSO:

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 \text{ subject to } \sum_{l} |\beta_l| \leq s$$

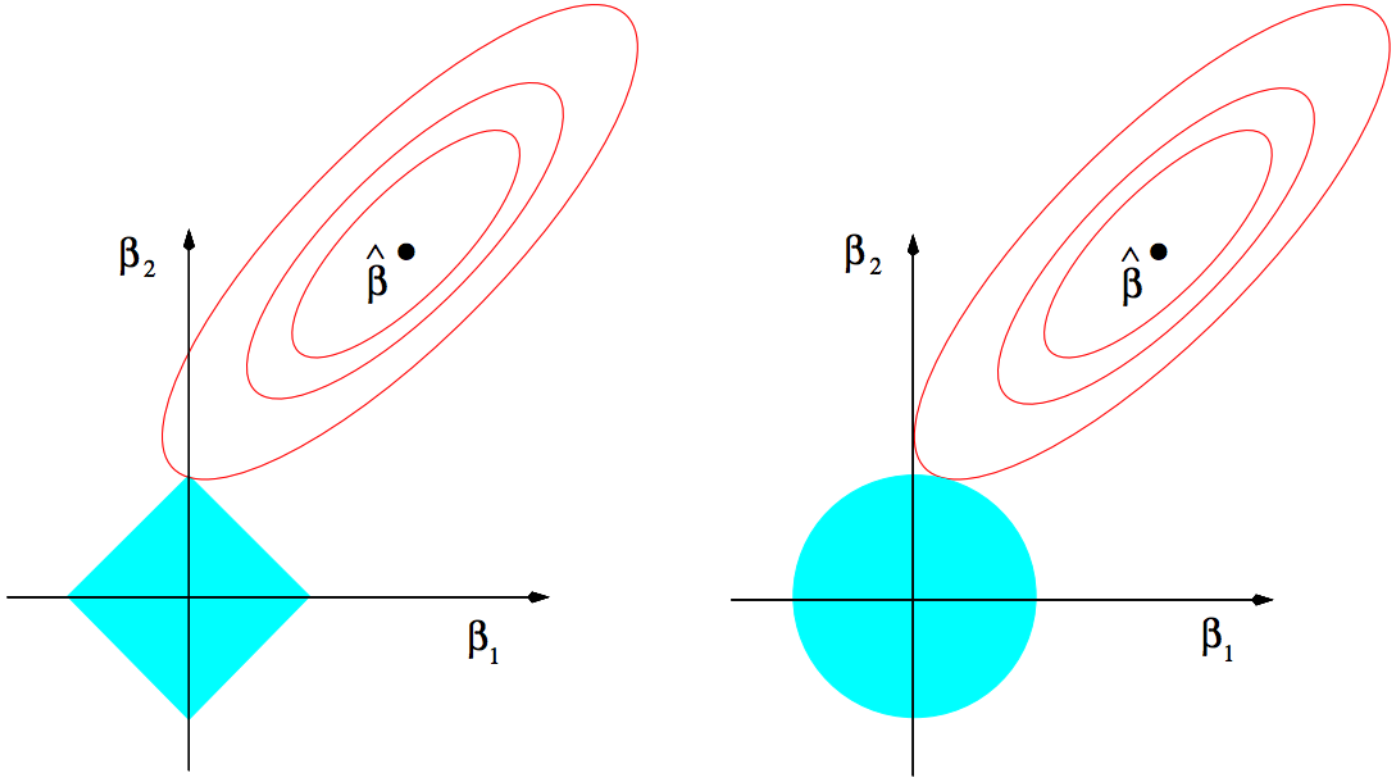
We can then plot the constraints and the contours of the unconstrained problem to see how they differ

Ridge vs LASSO

LASSO induces a constraint set with kinks at $x_1 = 0; x_2 = 0, \ldots$

$\rightarrow$ solutions will generally be at the kinks and we get lots of zero coefficients

Ridge induces a spherical constraint set, it tends to shrink coefficients toward zero without setting them exactly to zero



Elastic Net

$$\min_{\beta} \sum_{i=1}^{N} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda[(1-\alpha)(||\beta||_2)^2 + \alpha||\beta||_1]$$

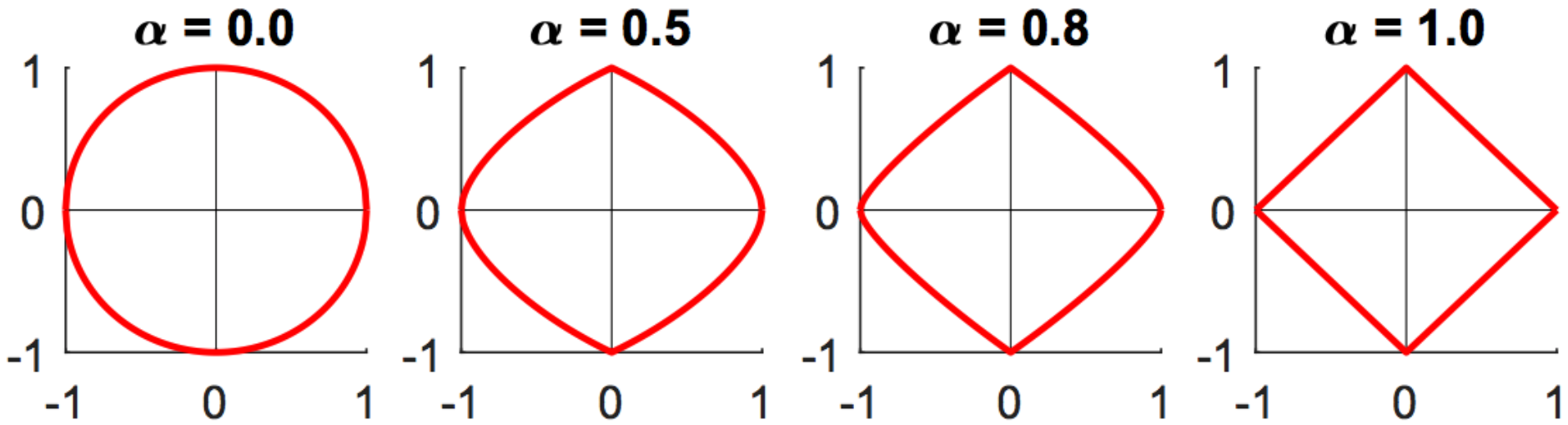
Elastic net tries to get the best of both ridge and LASSO by using a convex combination of their penalties

LASSO has one big problem:

Selection with Collinearity: if features are highly correlated LASSO tends to select one and ignore the others

The ridge penalty helps get around these issues by allowing us to select multiple of the correlated variables

Elastic Net



Validation and cross-validation

One thing we haven't discussed yet is how we select λ, our penalty parameter

Big lambdas tend to result in a lot of shrinkage and sparsity,
as $\lambda \to 0$ our solution approaches the OLS solution

There are two general ways to select λ

1. Select model with lowest AIC/BIC/other plug-in criterion
    - This uses no out-of-sample information for selection but is fast
2. Cross-validate by testing on our hold-out test sample
    - Variants of cross-validation are most commonly used

Cross-validation

When we perform cross-validation we split our sample into three different pieces: a training sample, a validation sample, and a test sample

First you randomly allocate some fraction of your data to the test sample

Next you perform cross-validation on the remaining data

A common way to do this is called **k-fold cross-validation**

k-fold cross-validation

In k-fold cross-validation we do the following:

- Create a grid of λs
- For each λ:
  - Split data into k mutually-exclusive folds of about equal size, usually choose $k = 5, 10$
  - For $j = 1, \ldots, k$
    - fit the model using all folds but fold j
    - Predict out-of-sample on fold j
  - Compute average mean squared prediction error across the k folds:
    $\bar{Q}(\lambda) = \frac{1}{k} \sum_{j=1}^{k} \sum_{i \in \text{fold j}} \left(y_i - (\alpha_0 + x_i'\beta)\right)^2 + \lambda ||\beta||_1$
- Choose $\hat{\lambda}_{min} = argmin_{\lambda} \bar{Q}(\lambda)$ or to avoid modest overfitting choose the largest λ such that $\bar{Q}(\lambda) \leq \hat{\lambda}_{min} + \sigma_{\hat{\lambda}_{min}}$ (1 standard deviation rule)

Supervised learning examples: Preliminaries

We need `tidyverse` to work with the data, `glmnet` to do the ML, , `caret` to do some higher-level tuning, and `tidymodels` to use a similar grammar and structure to `tidyverse`

We will be working with the mtcars dataset

```
## Loading required package: pacman
```

```
if (!require("pacman")) install.packages("pacman")
require(devtools)
devtools::install_github("tidymodels/tidymodels")
pacman::p_load(tidymodels, tidyverse, glmnet, caret)
set.seed(123)
```

Supervised learning examples: Preliminaries

```
mtcars ← mtcars %>% as_tibble()
mtcars
```

```
## # A tibble: 32 x 11
##      mpg   cyl  disp    hp  drat    wt  qsec    vs    am  gear  carb
##    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
##  1  21       6  160    110  3.9   2.62  16.5     0     1     4     4
##  2  21       6  160    110  3.9   2.88  17.0     0     1     4     4
##  3  22.8     4  108     93  3.85  2.32  18.6     1     1     4     1
##  4  21.4     6  258    110  3.08  3.22  19.4     1     0     3     1
##  5  18.7     8  360    175  3.15  3.44  17.0     0     0     3     2
##  6  18.1     6  225    105  2.76  3.46  20.2     1     0     3     1
##  7  14.3     8  360    245  3.21  3.57  15.8     0     0     3     4
##  8  24.4     4  147.    62  3.69  3.19  20       1     0     4     2
##  9  22.8     4  141.    95  3.92  3.15  22.9     1     0     4     2
## 10  19.2     6  168.   123  3.92  3.44  18.3     1     0     4     4
## # … with 22 more rows
```

Supervised learning example: pre-processing

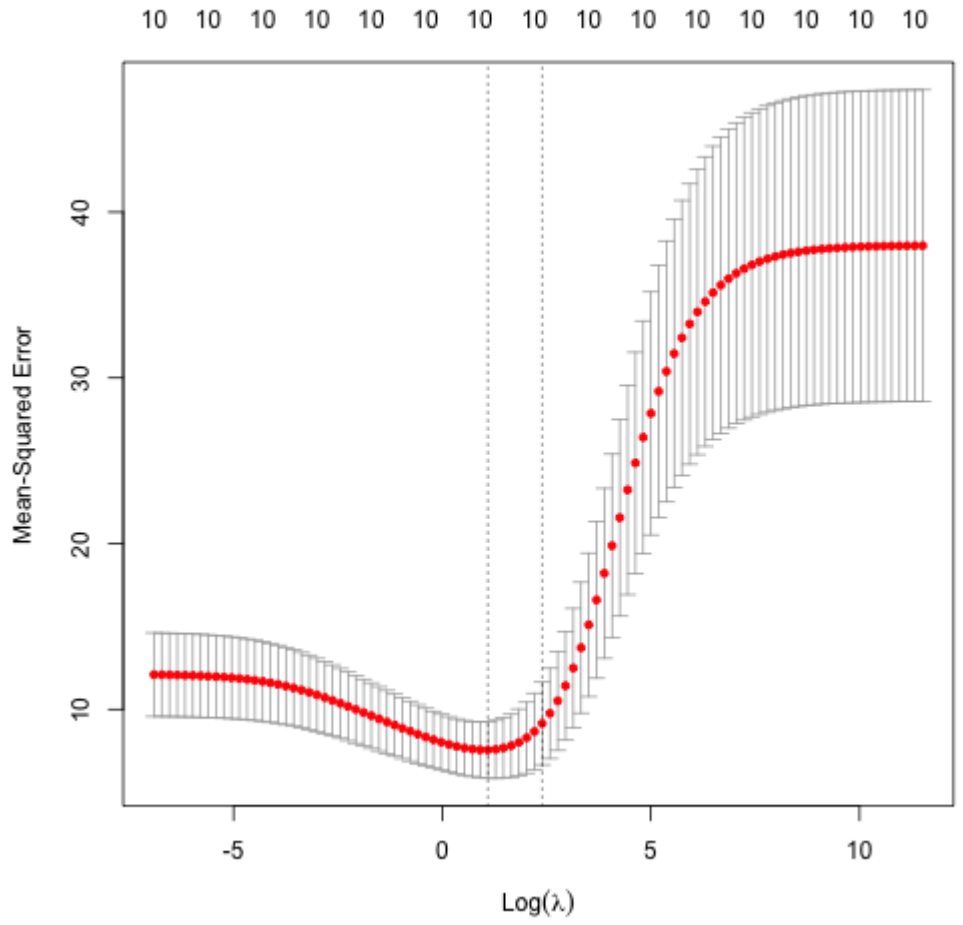
```
y ← mtcars %>% # center and scale y's, glmnet will center and scale Xs
  select(mpg) %>%
  scale(center = TRUE, scale = FALSE) %>%
  as.matrix()
X ← mtcars %>%
  select(-mpg) %>%
  as.matrix()
```

Ridge regression with glmnet

```
lambdas_to_try ← 10^seq(-3, 5, length.out = 100) # penalty parameter grid
ridge_cv ← cv.glmnet(X, y,
                     alpha = 0, # alpha is the elastic net parameter, 0 → ridge
                     lambda = lambdas_to_try, # lambda grid
                     standardize = TRUE, # standardize X's
                     nfolds = 10) # number of CV folds
```

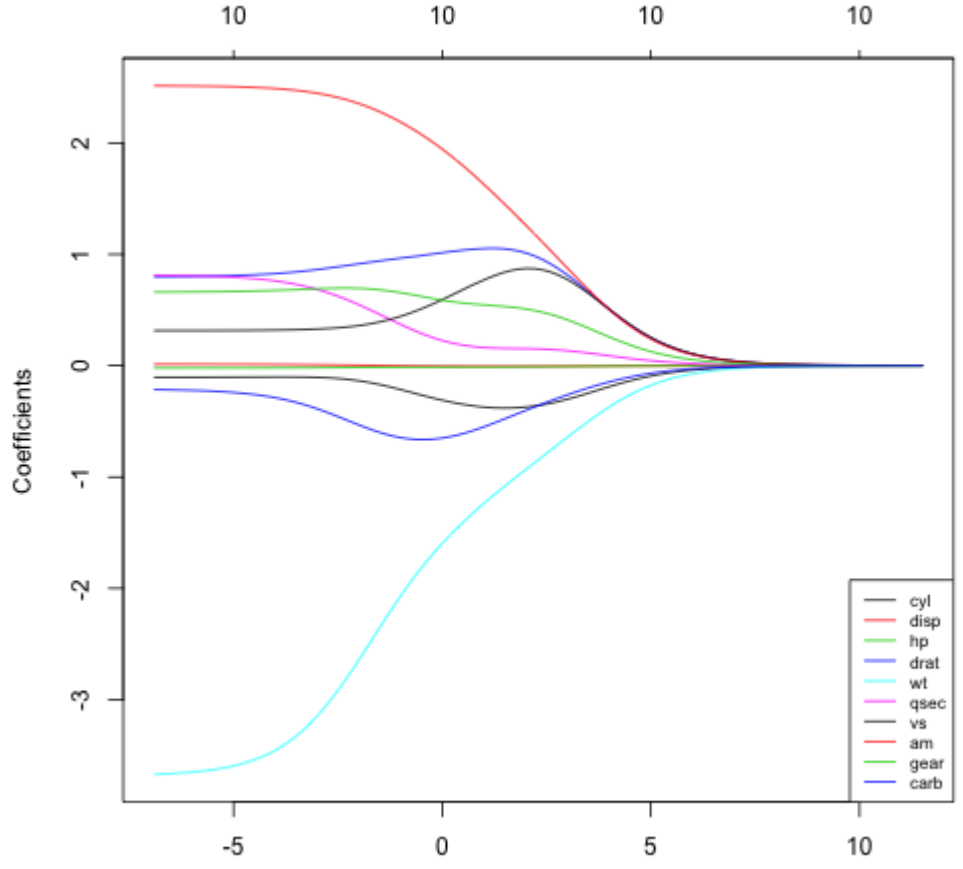
Ridge regression with glmnet

Here's MSE as a function of the choice of $\log(\lambda)$, notice we keep all variables



Ridge regression with glmnet

```
res_ridge ← glmnet(X, y, alpha = 0, lambda = lambdas_to_try, standardize = TRUE)
plot(res_ridge, xvar = "lambda")
legend("bottomright", lwd = 1, col = 1:6, legend = colnames(X), cex = .7)
```

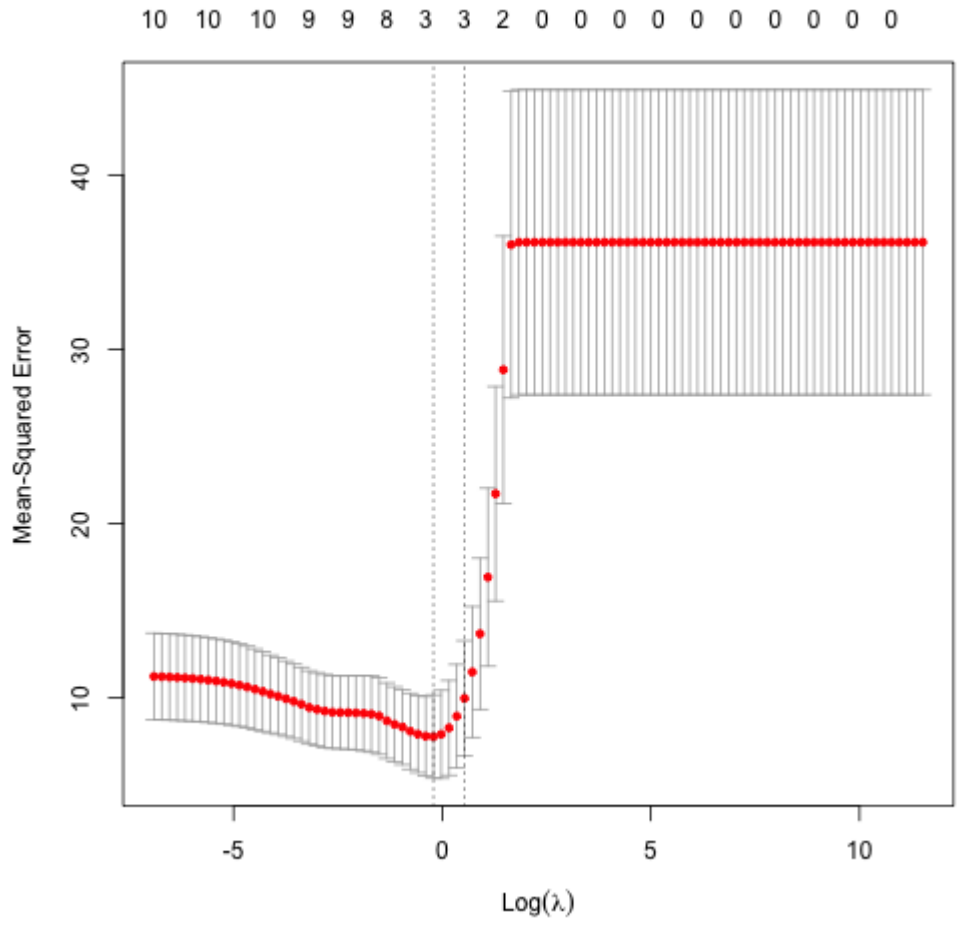


LASSO with glmnet

```
lambdas_to_try ← 10^seq(-3, 5, length.out = 100) # penalty parameter grid
lasso_cv ← cv.glmnet(X, y,
                     alpha = 1, # alpha is the elastic net parameter, 1 → LASSO
                     lambda = lambdas_to_try, # lambda grid
                     standardize = TRUE, # standardize X's
                     nfolds = 10) # number of CV folds
```

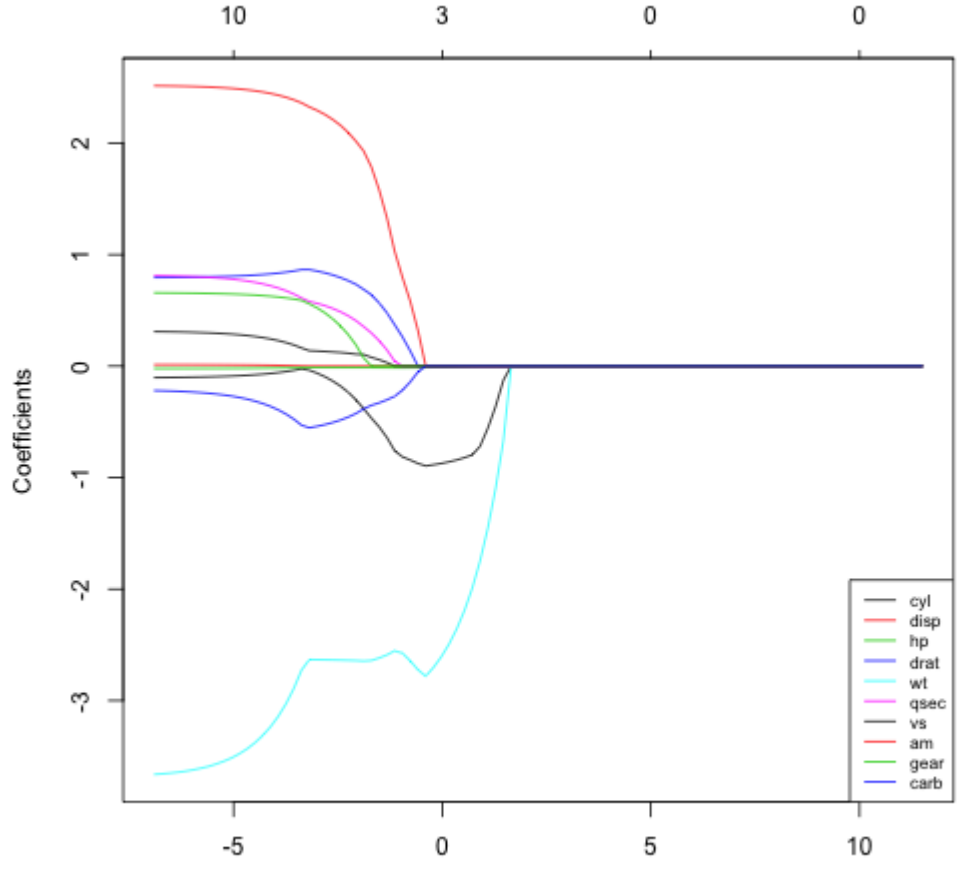
LASSO with glmnet

Here's MSE as a function of the choice of $\log(\lambda)$, LASSO generates sparse solutions



LASSO with glmnet

```
res_lasso ← glmnet(X, y, alpha = 1, lambda = lambdas_to_try, standardize = TRUE)
plot(res_lasso, xvar = "lambda")
legend("bottomright", lwd = 1, col = 1:6, legend = colnames(X), cex = .7)
```

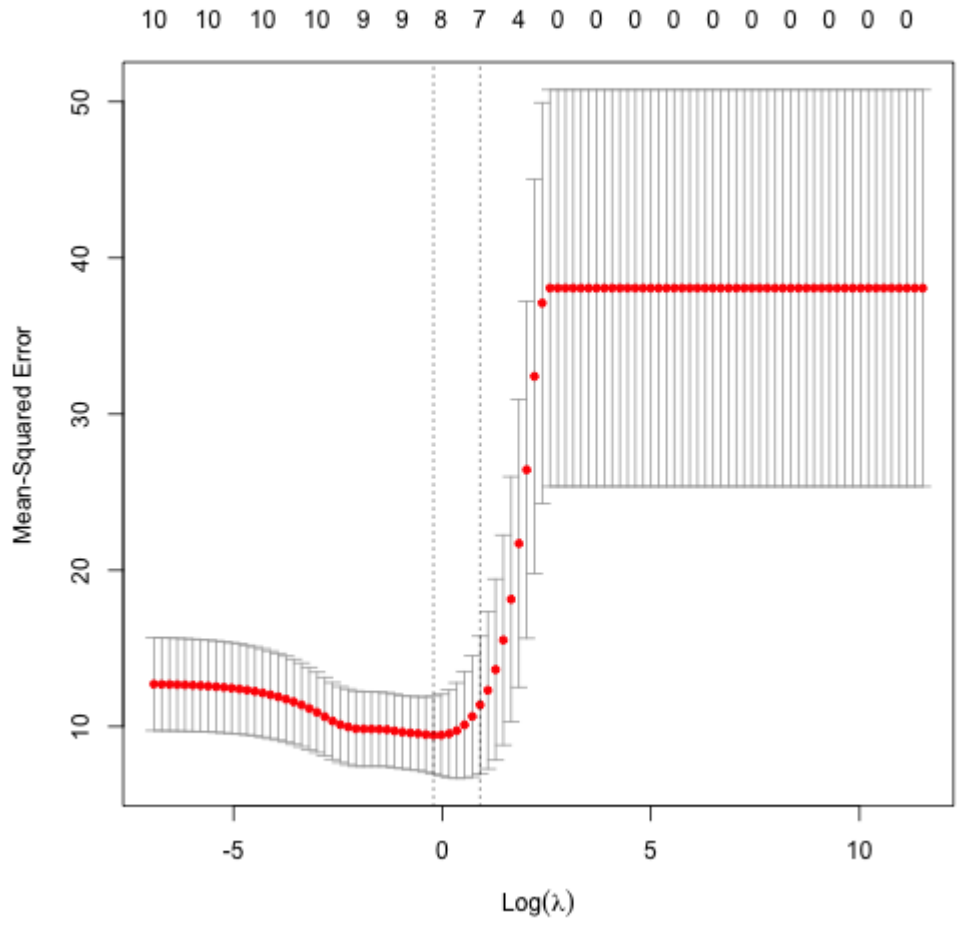


Elastic net with glmnet

```
lambdas_to_try ← 10^seq(-3, 5, length.out = 100) # penalty parameter grid
elastic_net_cv ← cv.glmnet(X, y,
                     alpha = 0.45, # alpha is the elastic net parameter
                     lambda = lambdas_to_try, # lambda grid
                     standardize = TRUE, # standardize X's
                     nfolds = 10) # number of CV folds
```

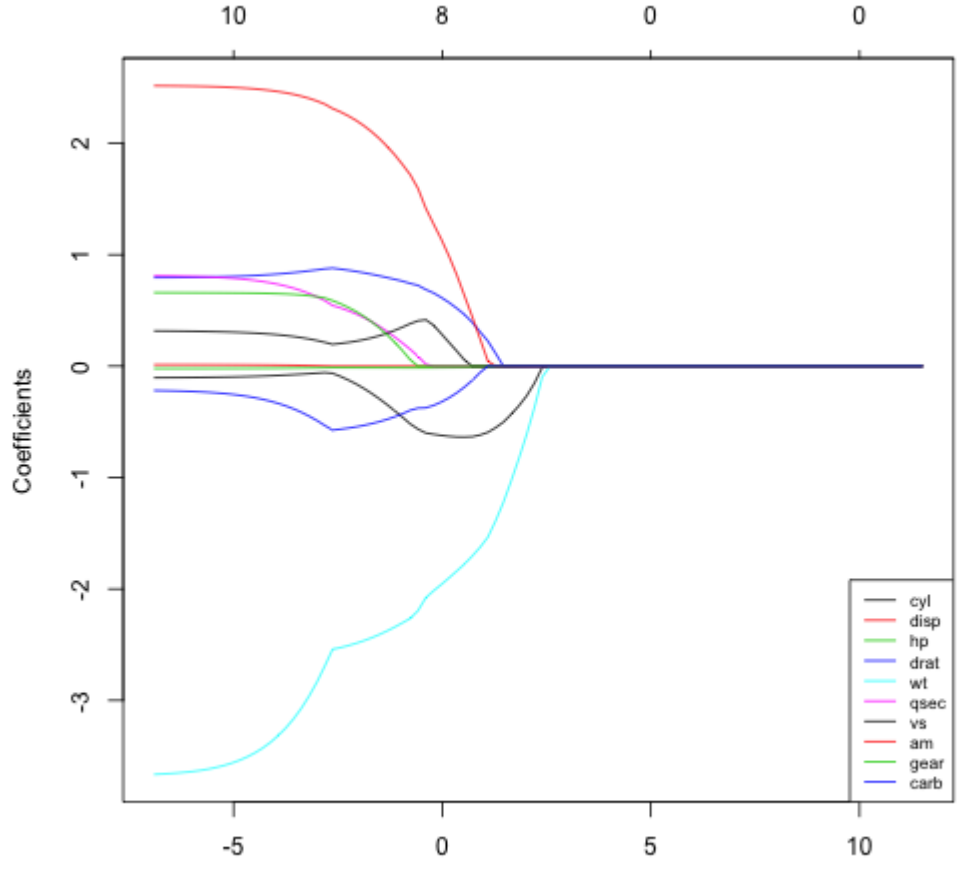
Elastic net with glmnet

Here's MSE as a function of the choice of $\log(\lambda)$, elastic net generates sparse solutions



Elastic net with glmnet

```
res_en ← glmnet(X, y, alpha = 0.45, lambda = lambdas_to_try, standardize = TRUE)
plot(res_en, xvar = "lambda")
legend("bottomright", lwd = 1, col = 1:6, legend = colnames(X), cex = .7)
```



Elastic net with caret

Elastic net has a second hyper-parameter, α that we can tune in addition to λ

`glmnet` doesn't let you tune both, but `caret` does

```
train_control ← trainControl(method = "cv", # use repeated cv
                             number = 10, # number of folds
                             search = "random",
                             verboseIter = TRUE)
```

Elastic net with caret

use `train` to train the model in `caret` using `glmnet`

```
elastic_net_model ← train(mpg ~ .,
                          data = cbind(y, X), # data
                          method = "glmnet", # use glmnet package
                          preProcess = c("center", "scale"), # already centered and scaled
                          tuneLength = 100, # 100 point grid for tuning parameters
                          trControl = train_control)
```

```
## + Fold01: alpha=0.40947, lambda=0.381599
## - Fold01: alpha=0.40947, lambda=0.381599
## + Fold01: alpha=0.01047, lambda=0.004588
## - Fold01: alpha=0.01047, lambda=0.004588
## + Fold01: alpha=0.18385, lambda=0.293152
## - Fold01: alpha=0.18385, lambda=0.293152
## + Fold01: alpha=0.84273, lambda=0.016224
## - Fold01: alpha=0.84273, lambda=0.016224
## + Fold01: alpha=0.23116, lambda=0.668596
## - Fold01: alpha=0.23116, lambda=0.668596
## + Fold01: alpha=0.23910, lambda=0.035556
## - Fold01: alpha=0.23910, lambda=0.035556
## + Fold01: alpha=0.07669, lambda=6.069734
## - Fold01: alpha=0.07669, lambda=6.069734
```

Elastic net with caret

We can figure out the best set of tuning parameters by looking at `bestTune`

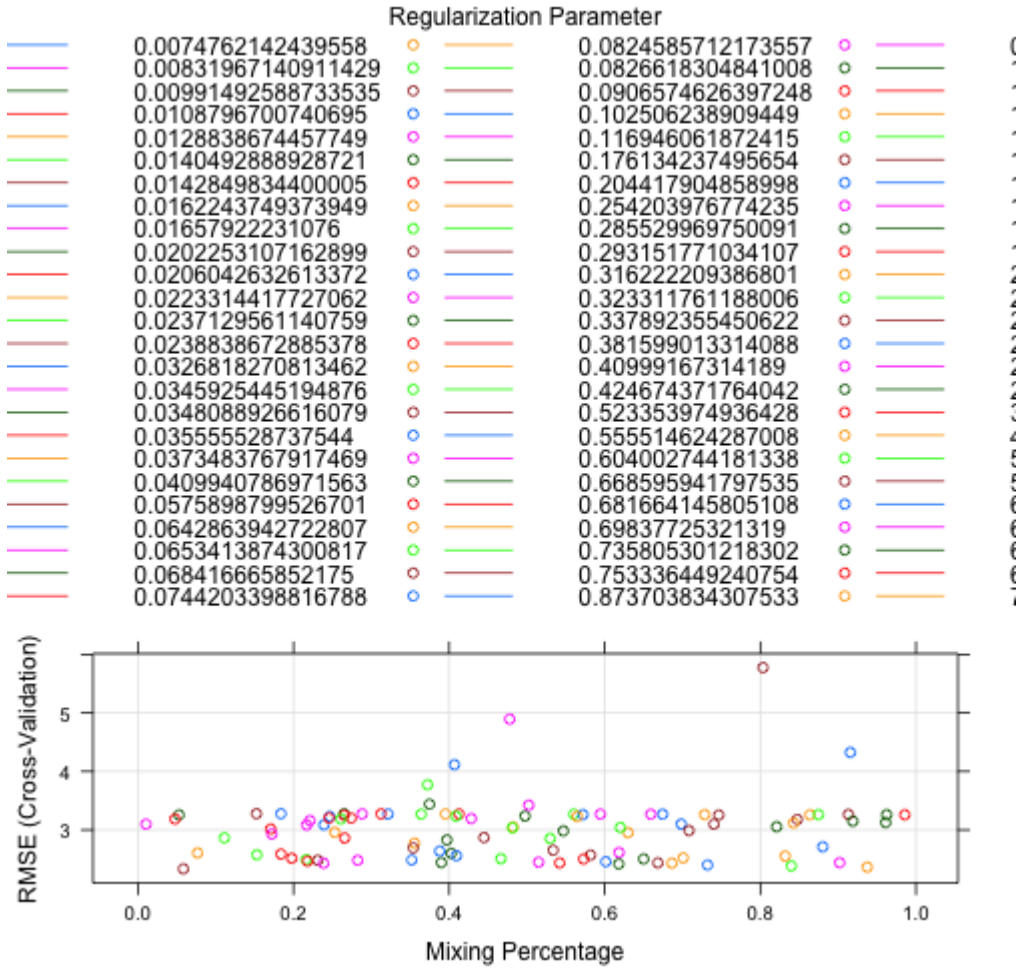
```
elastic_net_model$bestTune
```

```
##        alpha   lambda
## 4 0.05847849 1.793411
```

Here we selected something close to ridge regression

Elastic net with caret

```
plot(elastic_net_model)
```



Non-parametric regression: Regression Trees

Regression trees sequentially split the feature space into subspaces where the function is estimated as the average outcome for units with features in that subspace

Non-parametric regression: Regression Trees

Regression trees sequentially split the feature space into subspaces where the function is estimated as the average outcome for units with features in that subspace

These are called **trees** because the splitting is sequential, one feature at a time, so when you plot all the splits it begins to look like an upside down tree where

Each split is called a **node**, and the first split is called your **root node**

Each terminal point of your tree is called a **leaf node**

Non-parametric regression: Regression Trees

Regression trees sequentially split the feature space into subspaces where the function is estimated as the average outcome for units with features in that subspace

These are called **trees** because the splitting is sequential, one feature at a time, so when you plot all the splits it begins to look like an upside down tree where

Each split is called a **node**, and the first split is called your **root node**

Each terminal point of your tree is called a **leaf node**

Trees effectively partition the space into a bunch of hyperrectangles in a way that reduces RSS

Growing a regression tree

How do we grow our regression tree?

1. Let $g(x) = \bar{y}$ and let the sum of squared errors be

$$Q(g) = \sum_{i=1}^{N}(y_i - g(x_i))^2 = \sum_{i=1}^{N}(y_i - \bar{y})^2$$

2. For a feature j and split point s consider splitting the data depending on whether $x_{i,j} \leq s$ or $x_{i,j} > s$, and let $\bar{y}_{left}$ and $\bar{y}_{right}$ be the average values in the two subspaces

3. If $x_j \leq t$ let $g_{j,t}(x) = \bar{y}_{left}$ else $g_{j,t}(x) = \bar{y}_{right}$

4. Find the $j^*, s^* = argmin_{j,s} Q(g_{j,s}(\cdot))$

Growing a regression tree

This gives us the covariate j^* to split, and where to split it into separate subspaces s^* in order to minimize the sum of squared errors

This first split will end up being our root node

We then continue this process for each of the subspaces, splitting on the best covariates and creating new nodes and growing our tree

Growing a regression tree

This gives us the covariate j^* to split, and where to split it into separate subspaces s^* in order to minimize the sum of squared errors

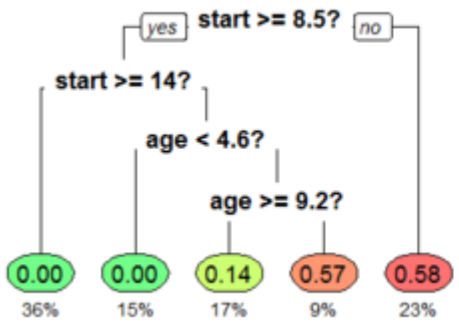
This first split will end up being our root node

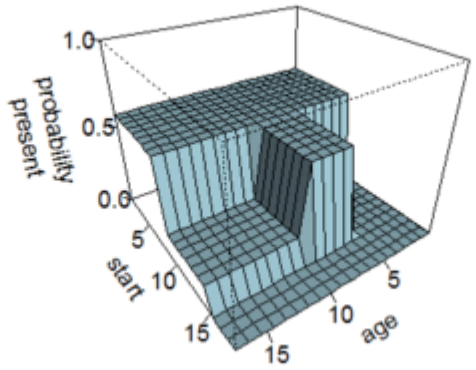
We then continue this process for each of the subspaces, splitting on the best covariates and creating new nodes and growing our tree

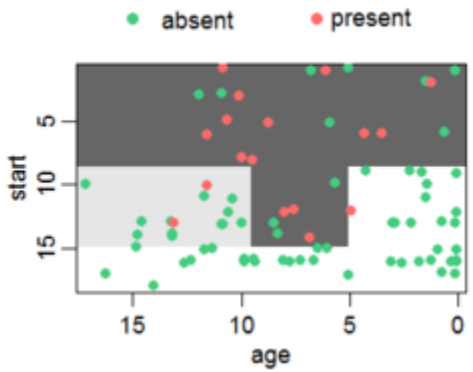
This is called a **greedy** approach because we are selecting the best split at each step instead of looking ahead

What do regression trees look like?

Whats the probability of kyphosis after surgery given age and the starting vertabrae?







The left shows the tree diagram

The middle shows the actual regression function

The right shows a 2d projection of the regression function where darker colors are higher probabilities

Regularizing regression trees

If we just followed the regression tree algorithm we could minimize error by splitting until there is just one observation in each feature subspace, this will have perfect in-sample prediction but terrible out-of-sample prediction

Regularizing regression trees

If we just followed the regression tree algorithm we could minimize error by splitting until there is just one observation in each feature subspace, this will have perfect in-sample prediction but terrible out-of-sample prediction

We solve this problem similar to how we did linear regression: we penalize complexity (the number of leaves)

$$Q(g) + \lambda \cdot \#leaves$$

The penalty (if large enough) will keep the tree from having too many nodes

Cross-validating trees

How do we choose λ? Basically the same way as we did for linear regression

- Create a grid of λs
- For each λ:
  - Split data into k mutually-exclusive folds of about equal size, usually choose $k = 5, 10$
  - For $j = 1, \ldots, k$
    - Grow the tree using all folds but fold j
    - Predict out-of-sample on fold j
  - Compute squared prediction error across the k folds: $Q(\lambda) = \sum_{j=1}^{k} \sum_{i \in \text{fold j}} (y_i - g(j, \lambda))$
- Choose $\hat{\lambda}_{min} = argmin_{\lambda} Q(\lambda)$

Pruning trees

Using this simple cross-validation approach may stop growing the tree too early, one split may not help immediately, but it may help us find future profitable splits

```
- This is a drawback of a greedy algorithm
```

This suggests that one way we can improve is by **pruning** the tree

1. Grow a big tree, select some rule to stop like 5 observations per leaf, or a very small λ
2. Prune branches or leaves that don't improve the prediction by a sufficient amount

Pruning trees: examples

The simplest way to prune is called **reduced error pruning**

It works as follows

1. Starting at the leaves, remove each node
2. Check if prediction accuracy on the validation sample is the same or better
3. If 2 is true, remove the node
4. Continue until we cannot improve the tree any further

This is simple and fast

There are other more complex ways to prune (e.g. cost complexity)

Bagging predictors

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Bagging is easy:

1. Bootstrap resample B datasets
2. Estimate a tree on each dataset (can use data-driven cross-validation, pruning, whatever)
3. Average all the predictions: $\frac{1}{B}\sum_{j=1}^{B} g_j(\boldsymbol{x})$

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This only matters because trees are non-linear,
so bagging smooths out the end predictions

Random forests

The problem with bagging is that the B bagged estimated are correlated

Important regressors will always appear near the top of the tree in the bootstrapped samples

This means all the trees will look similar

Predictions won't actually be as good as you might think

How can we break this correlation?

Random forests

Randomly select only L out of K features: feature bagging

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Not obvious, no real theoretical guidance

For classification problems $\sqrt{K}$ is recommended

For regression $K/3$ is recommended

Boosted trees

Boosting is another method to improve prediction from weak learners (better than random chance predictors)

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Let $g(x|X, Y)$ be a simple regression tree

Define the residual as $\varepsilon_{1i} = Y_i - g_1(X_i|X, Y)$

With a boosted tree we then estimate a regression tree on the new data (X, ε_1)

Boosted trees

Repeat this process many times to get a set of gs

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These give you an additive approximation to the actual regression tree:

$$\sum_{m=1}^{M} g_m(x|X, \varepsilon_{m-1}) = \sum_{k=1}^{K} h_k(x_k) \text{ where } \varepsilon_0 = Y$$

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We generally fix tree depth but select number of trees in a quasi-cross-validation procedure

Trees examples: Preliminaries

We need `ISLR` for our tree data, `tree` to do the regression tree, `randomForest` to estimate a random forest, and `gbm` to estimate a boosted forest

We will be working with the carseats dataset

```
if (!require("pacman")) install.packages("pacman")
pacman::p_load(ISLR, tree, randomForest, gbm, tidyverse)
set.seed(123)
```

Supervised learning examples: Preliminaries

```
carseats ← Carseats %>% as_tibble()
carseats
```

```
## # A tibble: 400 x 11
##    Sales CompPrice Income Advertising Population Price ShelveLoc   Age Education
##    <dbl>     <dbl>  <dbl>       <dbl>      <dbl> <dbl> <fct>     <dbl>     <dbl>
##  1  9.5        138     73          11        276   120 Bad          42        17
##  2 11.2        111     48          16        260    83 Good         65        10
##  3 10.1        113     35          10        269    80 Medium       59        12
##  4  7.4        117    100           4        466    97 Medium       55        14
##  5  4.15       141     64           3        340   128 Bad          38        13
##  6 10.8        124    113          13        501    72 Bad          78        16
##  7  6.63       115    105           0         45   108 Medium       71        15
##  8 11.8        136     81          15        425   120 Good         67        10
##  9  6.54       132    110           0        108   124 Medium       76        10
## 10  4.69       132    113           0        131   124 Medium       76        17
## # … with 390 more rows, and 2 more variables: Urban <fct>, US <fct>
```

Supervised learning example: estimating a tree

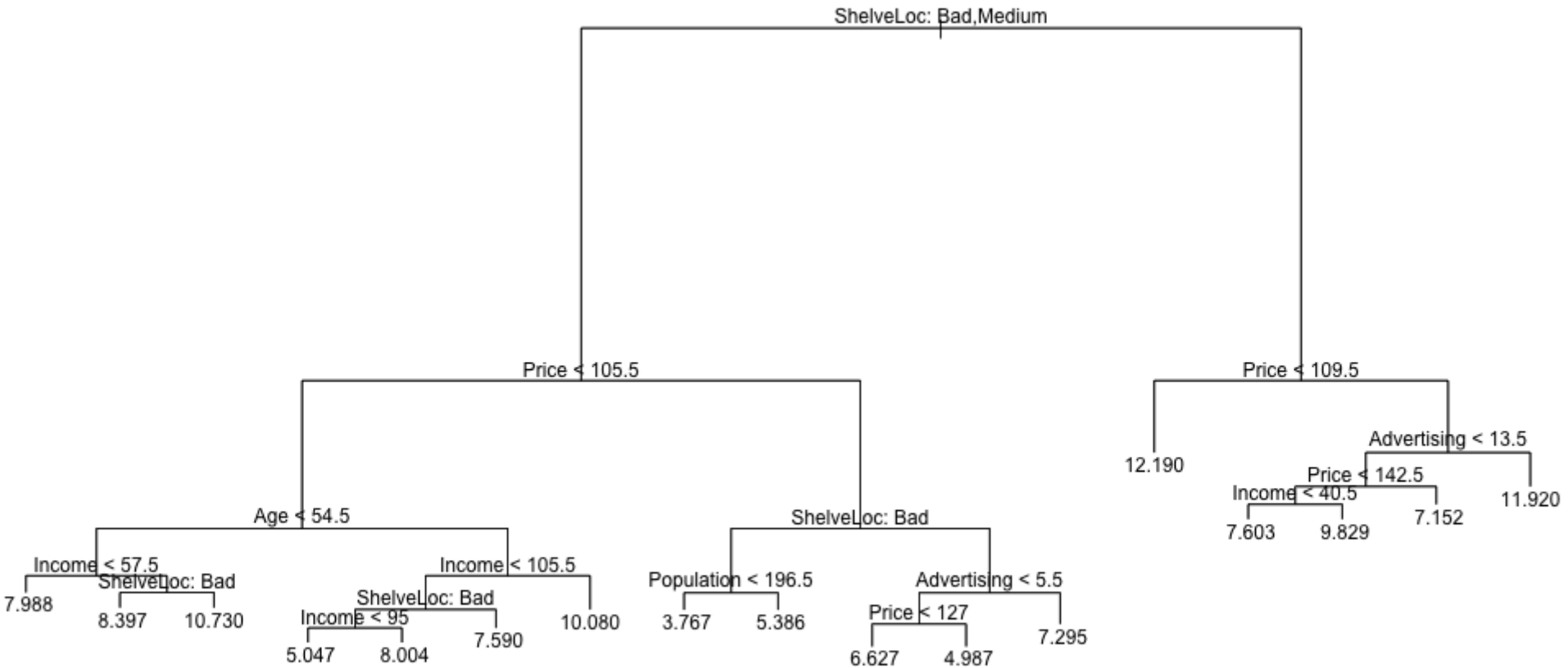
Lets estimate our regression tree with car sales as the outcome

```
tree_carseats ← tree(Sales ~ ., data = carseats)
summary(tree_carseats)
```

```
##
## Regression tree:
## tree(formula = Sales ~ ., data = carseats)
## Variables actually used in tree construction:
## [1] "ShelveLoc"   "Price"       "Age"         "Income"      "Population"
## [6] "Advertising"
## Number of terminal nodes:  17
## Residual mean deviance:  2.878 = 1102 / 383
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -4.98700 -1.23000 -0.06125  0.00000  1.22500  4.75400
```

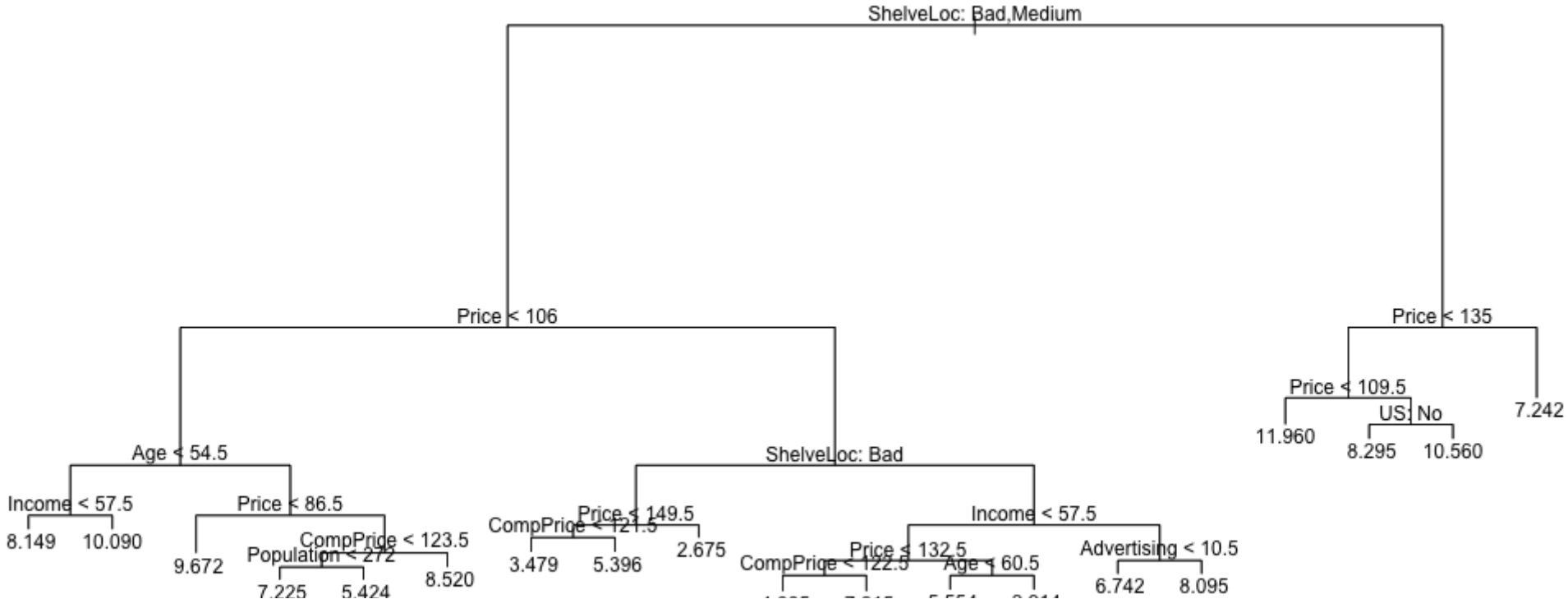
Supervised learning example: the tree

```
plot(tree_carseats)
text(tree_carseats, pretty = 0)
```



Supervised learning example: the tree

```
set.seed(101)
train ← sample(1:nrow(carseats), 320)
tree_carseats ← tree(Sales ~ ., carseats, subset = train)
plot(tree_carseats)
text(tree_carseats, pretty = 0)
```



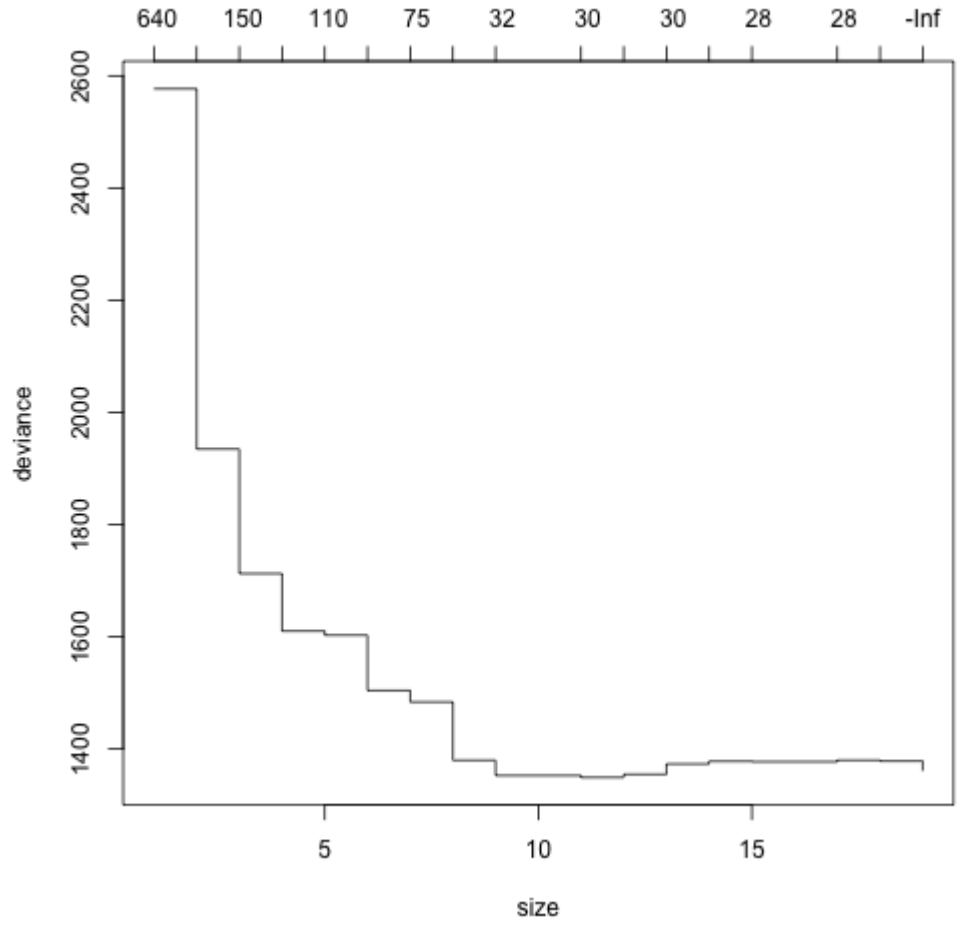
Supervised learning example: the tree's error

```
tree_pred ← predict(tree_carseats, carseats[-train,])
mse ← mean((carseats[-train,]$Sales - tree_pred)^2)
mse
```

```
## [1] 5.040445
```

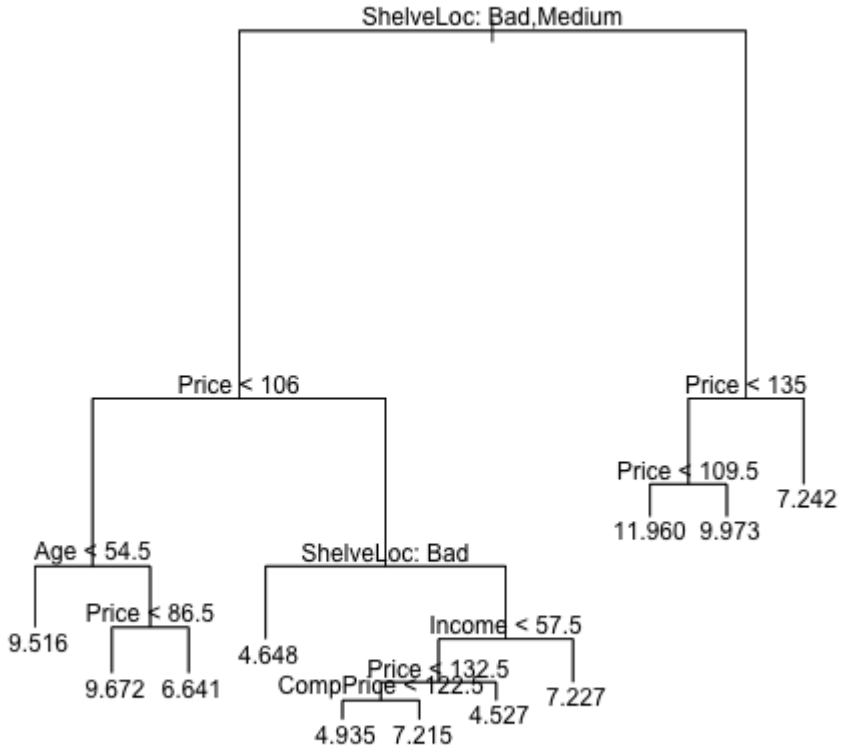
Supervised learning example: cross-validation

```
cv_carseats = cv.tree(tree_carseats)
plot(cv_carseats)
```



Supervised learning example: pruning

```
set.seed(123)
prune_carseats ← prune.tree(tree_carseats, best = 10)
plot(prune_carseats)
text(prune_carseats, pretty = 0)
```



Supervised learning example: pruning

```
tree_pred_prune = predict(prune_carseats, carseats[-train,])
mse_prune ← mean((carseats[-train,]$Sales - tree_pred_prune)^2)
mse
```

```
## [1] 5.040445
```

```
mse_prune
```

```
## [1] 5.905862
```

Supervised learning example: random forests

```
set.seed(101)
train = sample(1:nrow(carseats), 320)
rf_carseats = randomForest(Sales~., data = carseats, subset = train)
rf_carseats
```

```
##
## Call:
##  randomForest(formula = Sales ~ ., data = carseats, subset = train)
##                Type of random forest: regression
##                      Number of trees: 500
## No. of variables tried at each split: 3
##
##           Mean of squared residuals: 2.779889
##                     % Var explained: 64.79
```

```
mse
```

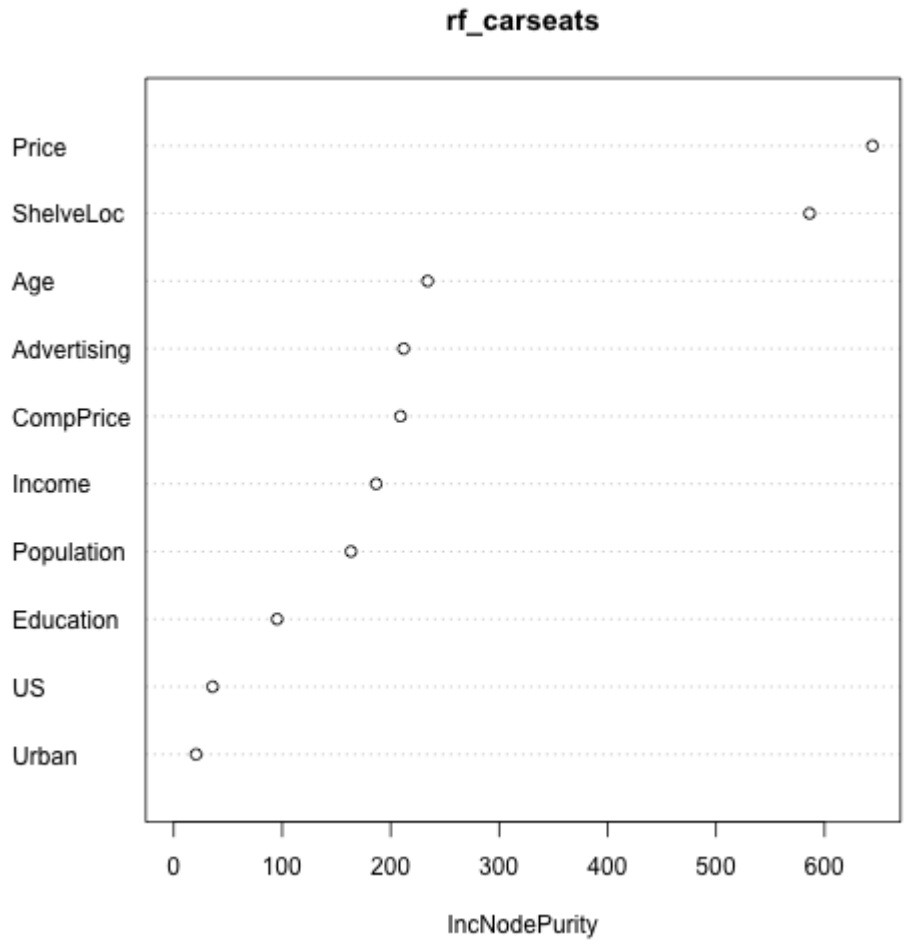
```
## [1] 5.040445
```

```
mse_prune
```

```
## [1] 5.905862
```

Random forests: variable importance

```
varImpPlot(rf_carseats)
```

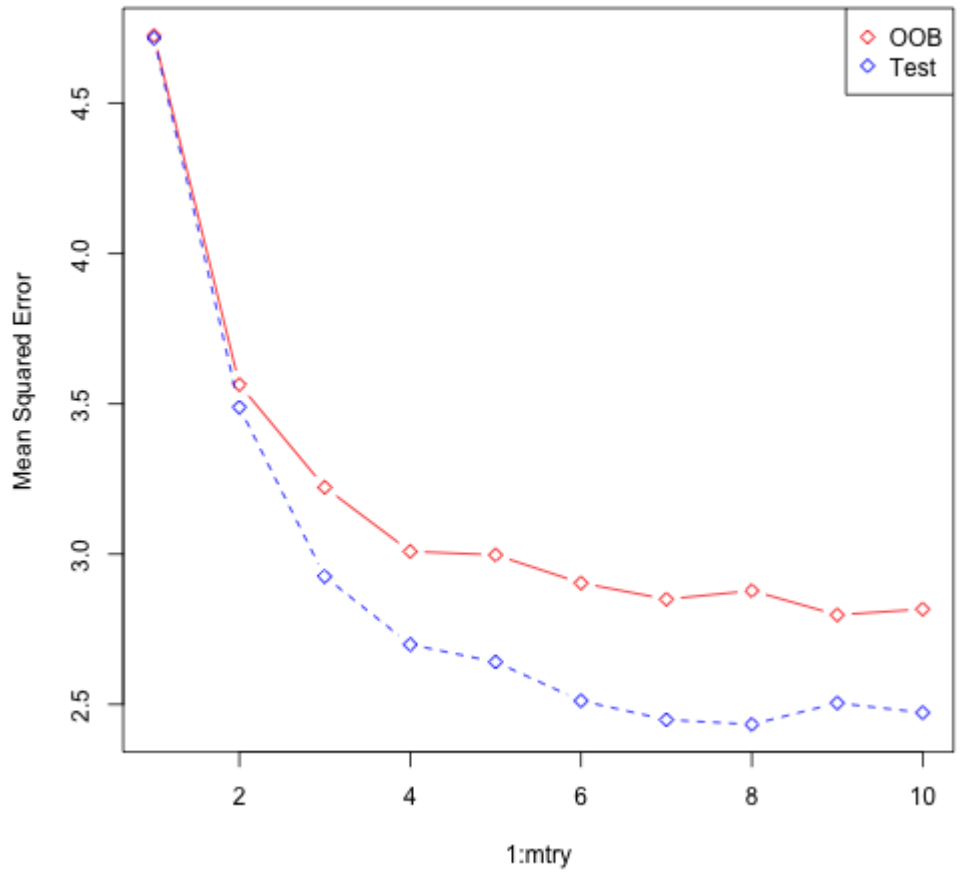


Random forests: tuning feature bagging

```
oob_err = double(10)
test_err = double(10)
for (mtry in 1:10) {
  set.seed(101)
  fit = randomForest(Sales~., data = carseats, subset = train, mtry = mtry, ntree = 350)
  oob_err[mtry] = mean(fit$mse)
  pred = predict(fit, carseats[-train,])
  test_err[mtry] = with(carseats[-train,], mean( (Sales - pred)^2 ))
}
```

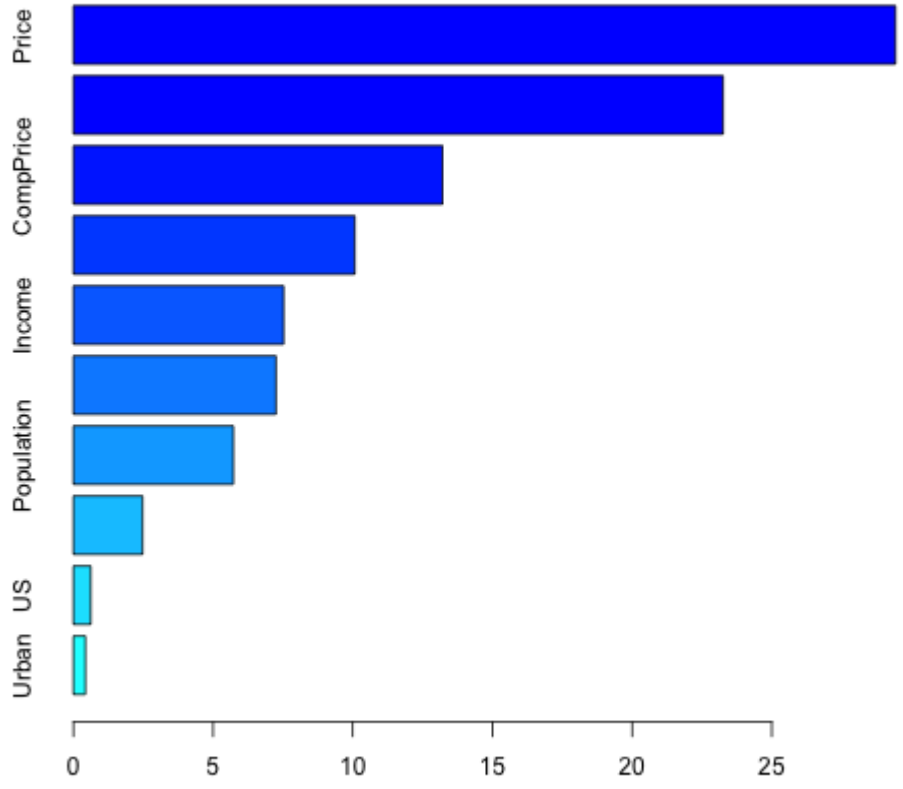
Random forests: tuning feature bagging

```
matplot(1:mtry, cbind(test_err, oob_err), pch = 23, col = c("red", "blue"), type = "b", ylab = "Mean Squared Error")
legend("topright", legend = c("OOB", "Test"), pch = 23, col = c("red", "blue"))
```



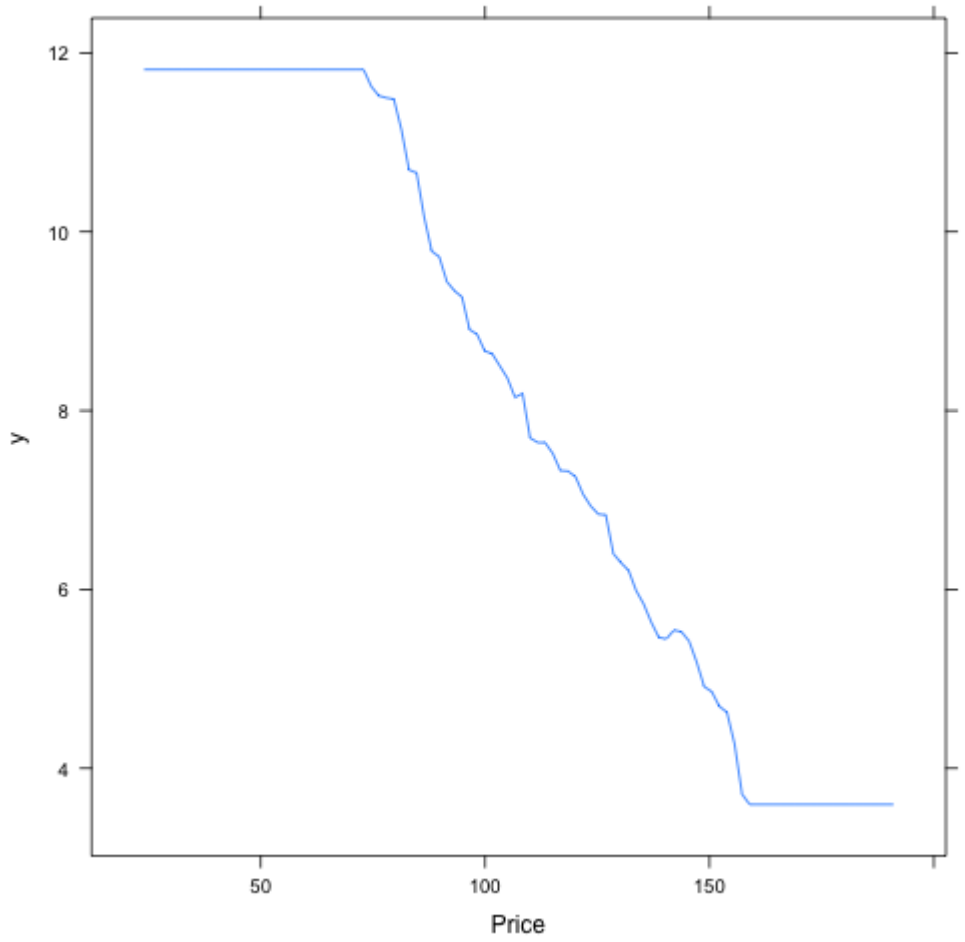
Boosted trees

```
boost_carseats = gbm(Sales~., data = carseats[train,],
                     distribution = "gaussian", n.trees = 10000, shrinkage = 0.01, interaction.depth = 4)
summary(boost_carseats)
```



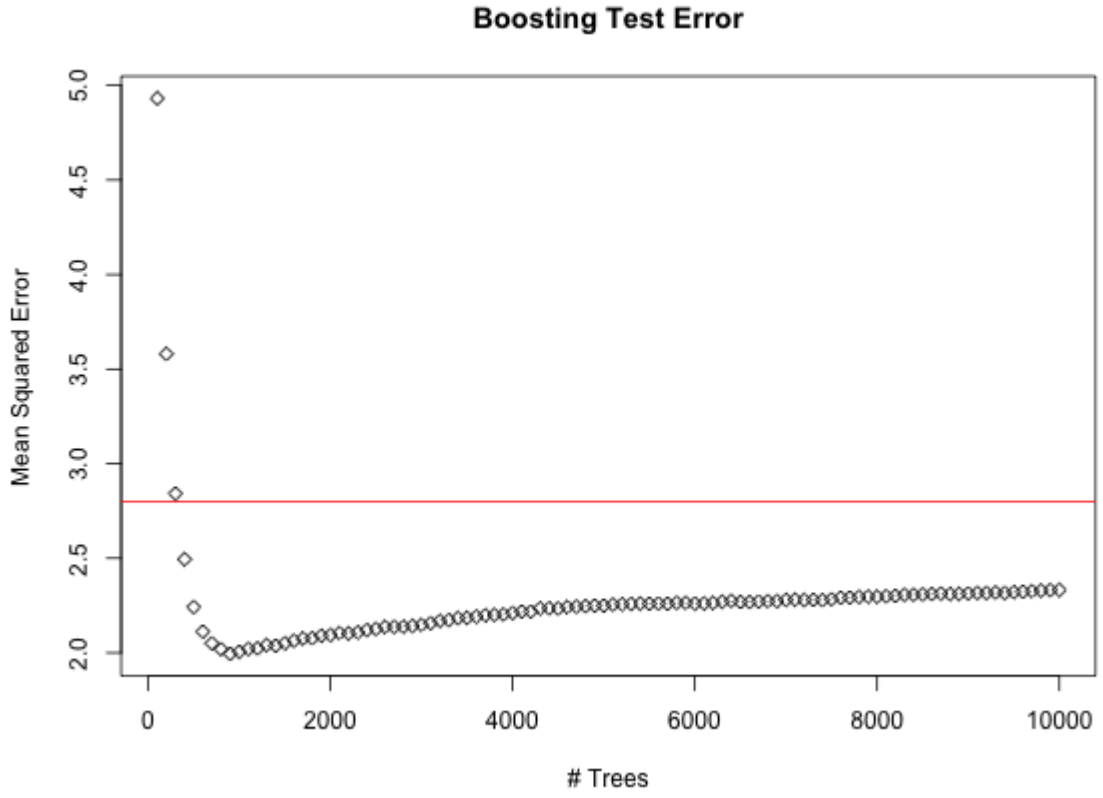
Boosted trees, the important variables

```
plot(boost_carseats, i = "Price")
```



Boosted trees, prediction error

```
n_trees = seq(from = 100, to = 10000, by = 100)
predmat = predict(boost_carseats, newdata = carseats[-train,], n.trees = n_trees)
boost_err = with(carseats[-train,], apply( (predmat - Sales)^2, 2, mean) )
plot(n_trees, boost_err, pch = 23, ylab = "Mean Squared Error", xlab = "# Trees", main = "Boosting Test Error")
abline(h = min(test_err), col = "red")
```



Econ-specific stuff

Often times we may want to predict using FEs

Aproblem with LASSO is that it may only select a few of them (recall they're just a vector of dummy variables)

How do we force LASSO to either select all or none?

Group LASSO