

1. $\frac{1}{4}$
2. $\frac{1}{36}$
3. $\frac{1}{2} \times \frac{1}{6}$
 $\frac{1}{12}$
4. $\frac{1}{2} \times \frac{3}{6}$
 $\frac{1}{4}$
5. $\frac{1}{2} \times (\frac{1}{6} + \frac{1}{6})$
 $\frac{1}{2} + \frac{2}{6}$
 $\frac{1}{6}$
6. $\langle x \rangle = v \times \mathbb{P}(X = v)$
 $= 2 \cdot 0.1 + 5 \cdot 0.2 + 6 \cdot 0.2 + 8 \cdot 0.5$
 $= 0.2 + 1 + 1.2 + 4$
 $= 6.4$
7. $\langle x \rangle = v \times \mathbb{P}(X = v)$
 $= 2 \cdot 0.4 + 5 \cdot 0.15 + 6 \cdot 0.35 + 8 \cdot 0.1$
 $= 0.8 + 0.75 + 2.1 + 0.8$
 $= 4.45$
8. $Var(x) = 0.1(2)^2 + 0.2(5)^2 + 0.2(6)^2 + 0.5(8)^2 - 6.4^2$
 $= (0.4 + 5 + 7.2 + 32) - 40.96$
 $= 3.6$
9. $Var(x) = 0.4(2)^2 + 0.15(5)^2 + 0.35(6)^2 + 0.1(8)^2 - 4.45^2$
 $= (1.6 + 3.75 + 12.6 + 6.4) - 19.8025$
 $= 4.6$
10. $P(X = 2 \text{ and } Y = 2)$
 $= 0.1 \cdot 0.4$
 $= 0.04$
11. $P(X = 5 \text{ and } Y = 6)$
 $= 0.2 \cdot 0.35$
 $= 0.07$
12. $P(X = 8 \text{ and } Y = 5)$
 $= 0.5 \cdot 0.15$
 $= 0.075$
13. $|\alpha\rangle = a \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
 $a \begin{pmatrix} 1 & 0 \end{pmatrix}$
14. $|\beta\rangle = a \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
 $b \begin{pmatrix} 1 & 1 \end{pmatrix}$

$$15. \quad |\gamma\rangle = c \begin{pmatrix} 5 \\ 3e^{i\frac{\pi}{4}} \end{pmatrix}$$

$$c \begin{pmatrix} 5 & 3e^{-i\frac{\pi}{4}} \end{pmatrix}$$

$$16. \quad \langle\psi|\psi\rangle = 1$$

$$\langle\alpha|\alpha\rangle = 1$$

$$\alpha = a \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\psi\rangle = \frac{1}{\sqrt{|1|^2+|0|^2}}$$

$$= 1$$

$$17. \quad \langle\beta|\beta\rangle = 1$$

$$|\psi\rangle = \frac{1}{\sqrt{|1|^2+|1|^2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$18. \quad NP - HARD$$

$$19. \quad NP - HARD$$

$$20. \quad NP - HARD$$

$$21. \quad NP - HARD$$