1. $\langle A| = \begin{pmatrix} 5 & 3i \end{pmatrix}$
2. $\langle B| = \begin{pmatrix} -7 & 4e^{i\frac{\pi}{3}} \end{pmatrix}$
3. $\langle C| = \begin{pmatrix} 1 \\ e^{-i\frac{\pi}{6}} \end{pmatrix}$
4. $\langle \psi|\psi\rangle = 1$
5. $\langle A|A\rangle =$
   $|A\rangle = \begin{pmatrix} 5a & 3ia \end{pmatrix}$
   $\langle A| = \begin{pmatrix} 5a \\ 31a \end{pmatrix}$
   $\begin{pmatrix} 5a \\ 3ia \end{pmatrix} \begin{pmatrix} 5a & 3ia \end{pmatrix}$
   $25a^2 + 9 \times -1^2 \times a^2$
   $a = \frac{1}{\sqrt{34}}$
6. $|B\rangle = \begin{pmatrix} 7b \\ 4e^{-i\frac{\pi}{3}} \end{pmatrix}$
   $\langle B| = \begin{pmatrix} 7b & 4e^{i\frac{\pi}{3}} \end{pmatrix}$
   $= \begin{pmatrix} 7b \\ 4e^{-i\frac{\pi}{3}} \end{pmatrix} \begin{pmatrix} 7b & 4e^{-i\frac{\pi}{3}} \end{pmatrix}$
   $= 49b^2 + 16 \times 1 \times b^2$
   $= \frac{1}{\sqrt{65}}$
7. $|C\rangle = \begin{pmatrix} c & e^{i\frac{\pi}{6}} \end{pmatrix}$
   $\langle C| = \begin{pmatrix} c \\ e^{-i\frac{\pi}{6}} \end{pmatrix}$
   $= \begin{pmatrix} c & e^{i\frac{\pi}{6}} \end{pmatrix} \begin{pmatrix} c \\ e^{-i\frac{\pi}{6}} \end{pmatrix}$
   $= c^2 + 1 \times c^2$
   $= \sqrt{1}$
8. $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
9. $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
10. $\langle 0|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix}$
    $= 1 + 0$
    $= 1$
11. $\langle 0|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix}$
    $= 0 + 0 = 0$
12. $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
    $\frac{1}{\sqrt{2}} \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$
    $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

13. $\sqrt{\frac{5}{7}}\left|0\right\rangle + \sqrt{\frac{2}{7}}\left|1\right\rangle$

14. $P(0) = \left|\langle 0|+\rangle\right|^2$
$= \left|\frac{1}{\sqrt{2}}\right|^2$
$= \frac{1}{2}$

15. $P(1) = \left|\langle 1|-\rangle\right|^2$
$= \left|-\frac{1}{\sqrt{2}}\right|$
$= \frac{1}{2}$

16. $P(0) = \left|\langle 0|\phi\rangle\right|$
$= \left|\frac{2}{\sqrt{13}}\right|^2$
$= \frac{4}{13}$

17. $P(1) = \left|\langle 1|\phi\rangle\right|$
$= \left|\frac{3}{\sqrt{13}}e^{i\frac{\pi}{4}}\right|$
$= \frac{9}{13}$

18. $\langle +|+\rangle$
$= \left(\frac{1}{\sqrt{2}}\langle 0| + \frac{1}{\sqrt{2}}\langle 1|\right)\left(\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle\right)$
$= \frac{1}{2}\cancel{\langle 0|0\rangle} + \frac{1}{2}\langle 0|1\rangle + \frac{1}{2}\langle 1|0\rangle + \frac{1}{2}\cancel{\langle 1|1\rangle}$
$= \frac{1}{2} + \frac{1}{2}$
$= 1$

19. $\langle +|-\rangle$
$= \left(\frac{1}{\sqrt{2}}\langle 0| + \frac{1}{\sqrt{2}}\langle 1|\right)\left(\frac{1}{\sqrt{2}}\left|0\right\rangle - \frac{1}{\sqrt{2}}\left|1\right\rangle\right)$
$= \frac{1}{2}\cancel{\langle 0|0\rangle} - \frac{1}{2}\langle 0|1\rangle + \frac{1}{2}\langle 1|0\rangle - \frac{1}{2}\cancel{\langle 1|1\rangle}$
$= \frac{1}{2} - \frac{1}{2}$
$= 0$

20. $\langle \phi|+\rangle$
$= \left(\frac{2}{\sqrt{13}}\langle 0| + \frac{3}{13}e^{-i\frac{\pi}{4}}\langle 1|\right)\left(\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle\right)$
$= \frac{\sqrt{2}}{13}\cancel{\langle 0|0\rangle} + \frac{\sqrt{2}}{13}\langle 0|1\rangle + \frac{3\sqrt{2}}{26}e^{-i\frac{\pi}{4}}\langle 1|0\rangle + \frac{3\sqrt{2}}{26}e^{-i\frac{\pi}{4}}\cancel{\langle 1|1\rangle}$
$= \frac{1}{\sqrt{26}}\left(2 + 3e^{-i\frac{\pi}{4}}\right)$

21. $\langle -|\phi\rangle$
$\left(\frac{1}{\sqrt{2}}\langle 0| - \frac{1}{\sqrt{2}}\langle 1|\right)\left(\frac{2}{\sqrt{13}}\left|0\right\rangle + \frac{3}{13}e^{-i\frac{\pi}{4}}\left|1\right\rangle\right)$
$= -\frac{1}{\sqrt{26}}\left(2 + 3e^{-i\frac{\pi}{4}}\right)$

22. $\sigma_x\left|0\right\rangle = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}$
$= \begin{pmatrix}0+0\\ 1+0\end{pmatrix}$
$= \begin{pmatrix}0\\ 1\end{pmatrix}$
$= \left|1\right\rangle$

23. $\sigma_x |1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$= \begin{pmatrix} 0+1 \\ 0+0 \end{pmatrix}$
$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$= |0\rangle$

24. $\sigma_y |0\rangle =$
$= \begin{pmatrix} 0-i \\ 0+0 \end{pmatrix}$
$= \begin{pmatrix} -i0 \end{pmatrix}$
$= i |0\rangle$

25. $\sigma_y |1\rangle =$
$= \begin{pmatrix} 0-1 \\ 0+0 \end{pmatrix}$
$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$= i |0\rangle$

26. $\sigma_z |0\rangle =$
$= \begin{pmatrix} 1+0 \\ 0+0 \end{pmatrix}$
$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$= |1\rangle$

27. $\sigma_z |1\rangle =$
$= \begin{pmatrix} 0+0 \\ 0-1 \end{pmatrix}$
$= \begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$= - |1\rangle$

28. $H |0\rangle =$
$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1+0 \\ 1-0 \end{pmatrix}$
$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$= |+\rangle$

29. $H |1\rangle =$
$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0+1 \\ 0-1 \end{pmatrix}$
$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
$= |-\rangle$

30. $\sigma_x |+\rangle =$
$= \sigma_x \left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right)$
$= \frac{1}{\sqrt{2}} (\sigma_x |0\rangle + \sigma_x |1\rangle)$
$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$
$= |+\rangle$

31. $\sigma_x |-\rangle =$
$= \sigma_x \left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right)$
$= \frac{1}{\sqrt{2}} (\sigma_x |0\rangle - \sigma_x |1\rangle)$
$= \frac{1}{\sqrt{2}} (|1\rangle + |0\rangle)$
$= |+\rangle$

32. $H |+\rangle =$
$= \frac{1}{\sqrt{2}} (H |0\rangle + H |1\rangle)$
$= \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle)$
$= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} |0\rangle + \cancel{\frac{1}{\sqrt{2}} |1\rangle} + \frac{1}{\sqrt{2}} |0\rangle - \cancel{\frac{1}{\sqrt{2}} |1\rangle} \right)$
$= |0\rangle$

33. $\sigma_x |+\rangle =$
$= \sigma_x \left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right)$
$= \frac{1}{\sqrt{2}} (\sigma_x |0\rangle + \sigma_x |1\rangle)$
$= \frac{1}{\sqrt{2}} (|1\rangle - |0\rangle)$
$= |-\rangle$

34. $H\sigma_x |0\rangle =$
$= H (\sigma_x |0\rangle)$
$= H(|1\rangle)$
$= |-\rangle$

35. $\sigma_x H |0\rangle =$
$= \sigma_x (H |0\rangle)$
$= \sigma_x (|+\rangle)$
$= |+\rangle$

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