Welcome to Homework 17!

Use the Homework 17 Jupyter notebook (posted in the Week 17 module) to complete this assignment. If you cannot access the notebook or IBM Quantum Experience for any reason, remember that you can instead watch the week 17 homework review session (or the recording) in order to complete the homework.

Instructions:

1. Download the HW17 Jupyter Notebook from Canvas (it is posted in the week 17 module)
2. Create an IBM Q Experience account if you don't already have one
3. Upload the HW17 Jupyter Notebook into the IBM Q Experience using the import button in the "Quantum Lab" tab. For detailed instructions, see the Quantum Experience guide.
4. Try to complete all of the exercises in the notebook!
5. To submit the homework assignment on Canvas, refer to your work in the Jupyter notebook.
6. Submit your Canvas quiz.

Question 1:
Solve the following tensor product by hand for the state Psi, which of the following represents the correct numpy statevector of Psi:
(Hint: Reference Activity 1 on the notebook)

$$|\Psi\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

a) Psi = np.array([[1],[0],[0],[1]])
b) Psi = np.array([[0],[1],[1],[0]])
c) Psi = np.array([[0],[0],[1],[0]])
d) Psi = np.array([[1],[0],[0],[0]])

Question 2:
Solve the following matrix multiplication by hand for Phi, use the Psi from Question 1. Which of the following represents the resulting numpy statevector of Phi:
(Hint: Reference Activity 1 on the notebook)

$$|\Phi\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} |\Psi\rangle$$

a) Psi = np.array([[1],[0],[0],[1]])
b) Psi = np.array([[0],[0],[0],[1]])
c) Psi = np.array([[0],[0],[1],[0]])
d) Psi = np.array([[1],[1],[1],[1]])

Question 3:
Using numpy to solve for Psi, which of the following represents the resulting numpy statevector of Psi:

$$|\Psi\rangle = \left(\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(Hint: Reference Activity 2a on the notebook)

a) Psi = np.array([[0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]])
b) Psi = np.array([[0], [1], [0], [0])

c) Psi = np.array([[0], [0], [0], [0], [1], [0], [0], [0]])
d) Psi = np.array([[0], [0], [1], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]])

Question 4:
Give the value of the statevector of Phi:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$U = (I \otimes I) \otimes CNOT \qquad |\Phi\rangle = U|\Psi\rangle$$

(Hint: Reference Activity 2c from the notebook, if you get stuck read the prompt of question 5 – it gives a clue as to what the state of Phi should be!)

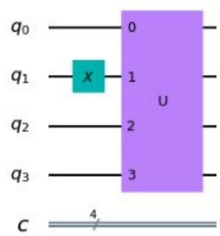
a) Psi = np.array([[0], [0], [0], [1], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]])
b) Psi = np.array([[0], [0], [0], [1], [0], [1], [0], [1], [0], [0], [0], [0], [0], [1], [0], [0]])
c) Psi = np.array([[0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [1]])
d) Psi = np.array([[0], [0], [1], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]])

Question 5:
Using the following definitions and the knowledge that that:
starting state Psi = |0010> and final state Phi = |0011>
Remember also that we are right indexing so Qubit 0 is on the right, Qubit 3 is on the left.
Therefore, |0001> means that Qubit 0 is 1
Which of these circuits is functionally equivalent to the U circuit shown in the question:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

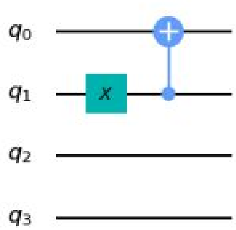
$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$U = (I \otimes I) \otimes CNOT \qquad |\Phi\rangle = U|\Psi\rangle$$

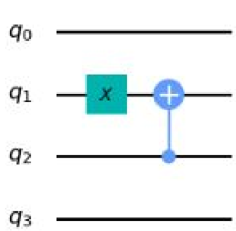


This is a hard question!
(Hint: Reference Activity 2c and 4b and if in doubt try each of the circuits)

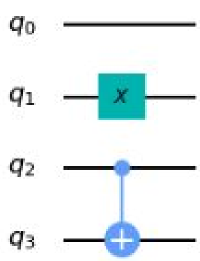
a) *Circuit A*



b) *Circuit B*



c) Circuit C



d) Circuit D