- A bit

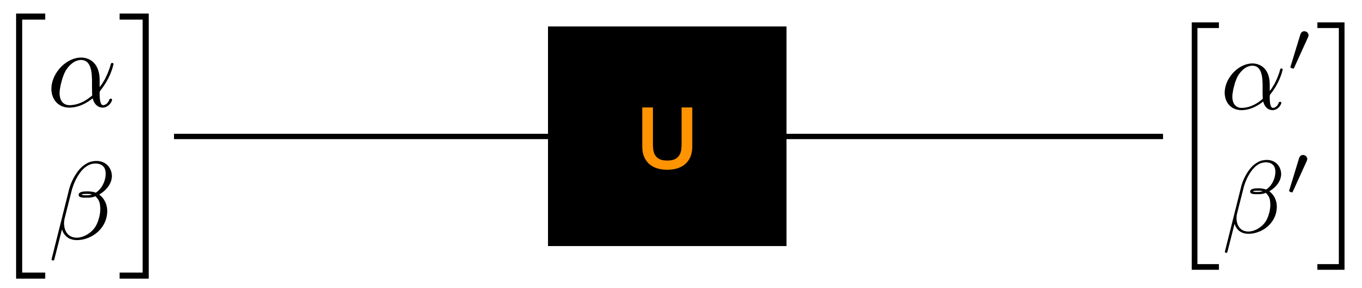
State of a bit is in a set, the set of possible states, {0, 1}

The state of a bit changes through logic operations / logic gates.

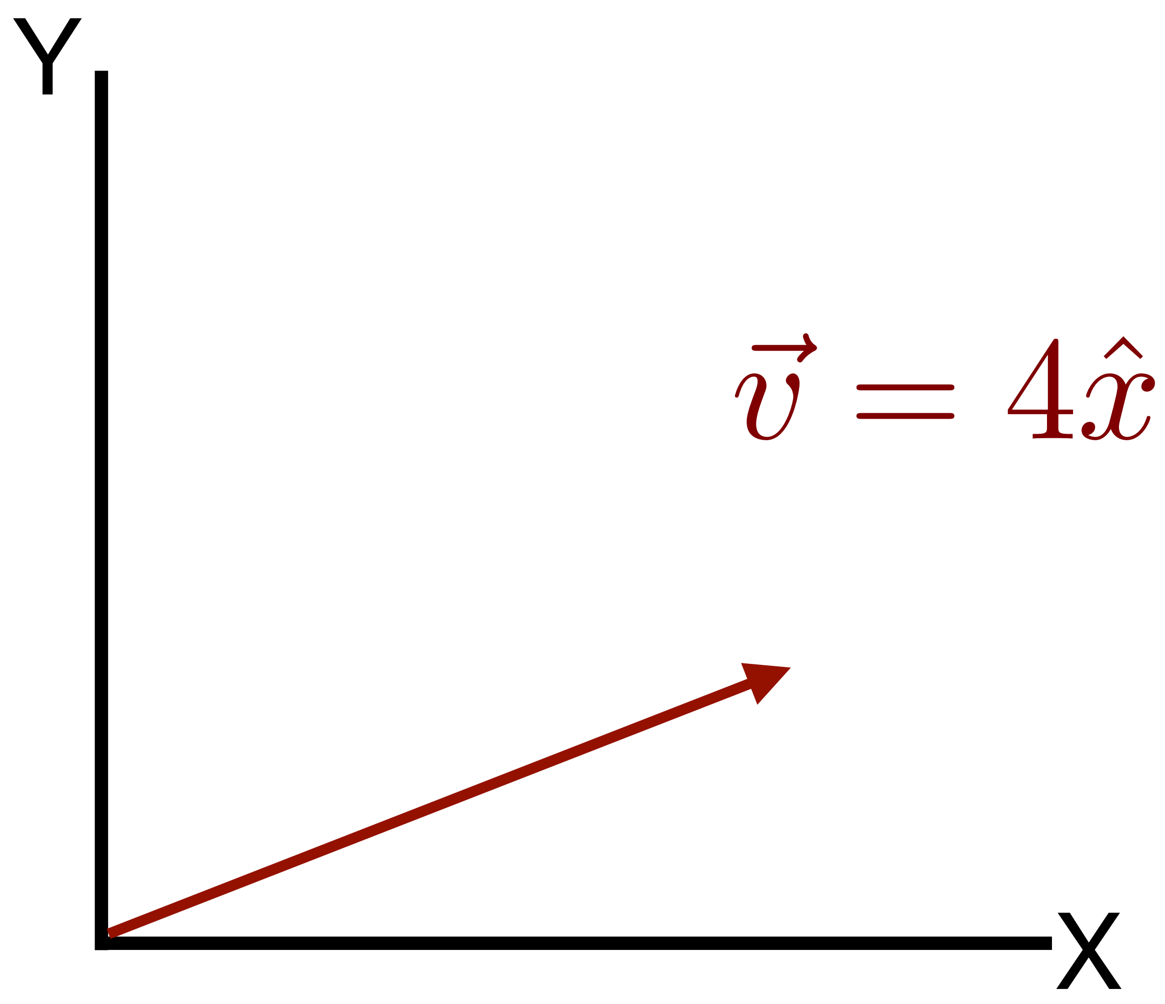
- **A qubit**

State of a qubit is a unit (normalized) vector in a two-dim complex vector space.

The state of a qubit changes through unitary operations / quantum gates.

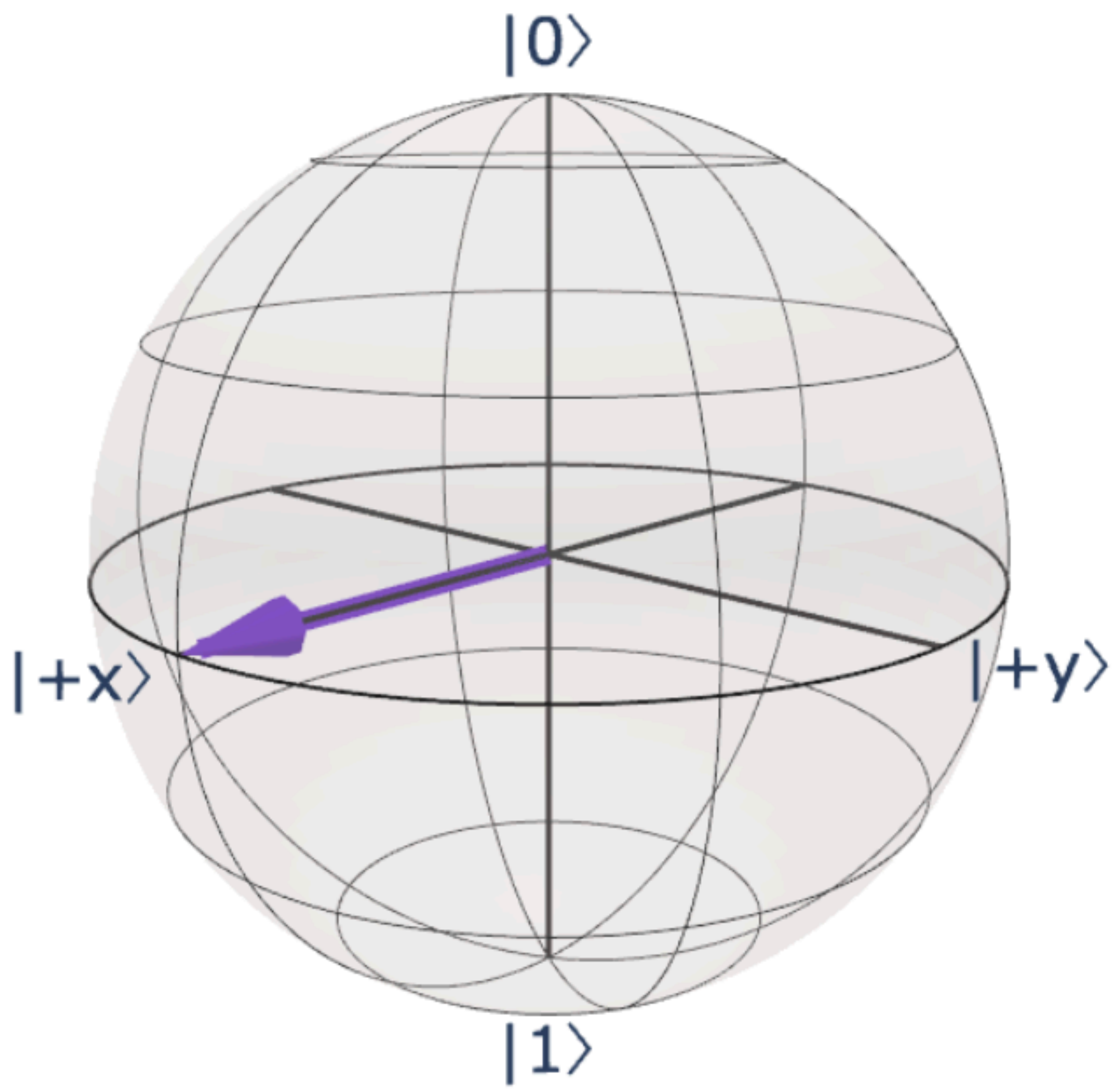


State of a qubit is a unit (normalized) vector in a two-dim complex vector space.



$$\vec{v} = 4\hat{x} + 2\hat{y} = 2(2\hat{x} + \hat{y}) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



- Computational basis states of a qubit

Two special quantum states corresponding to the 0 and 1 states of a classical bit

$$|0\rangle := \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

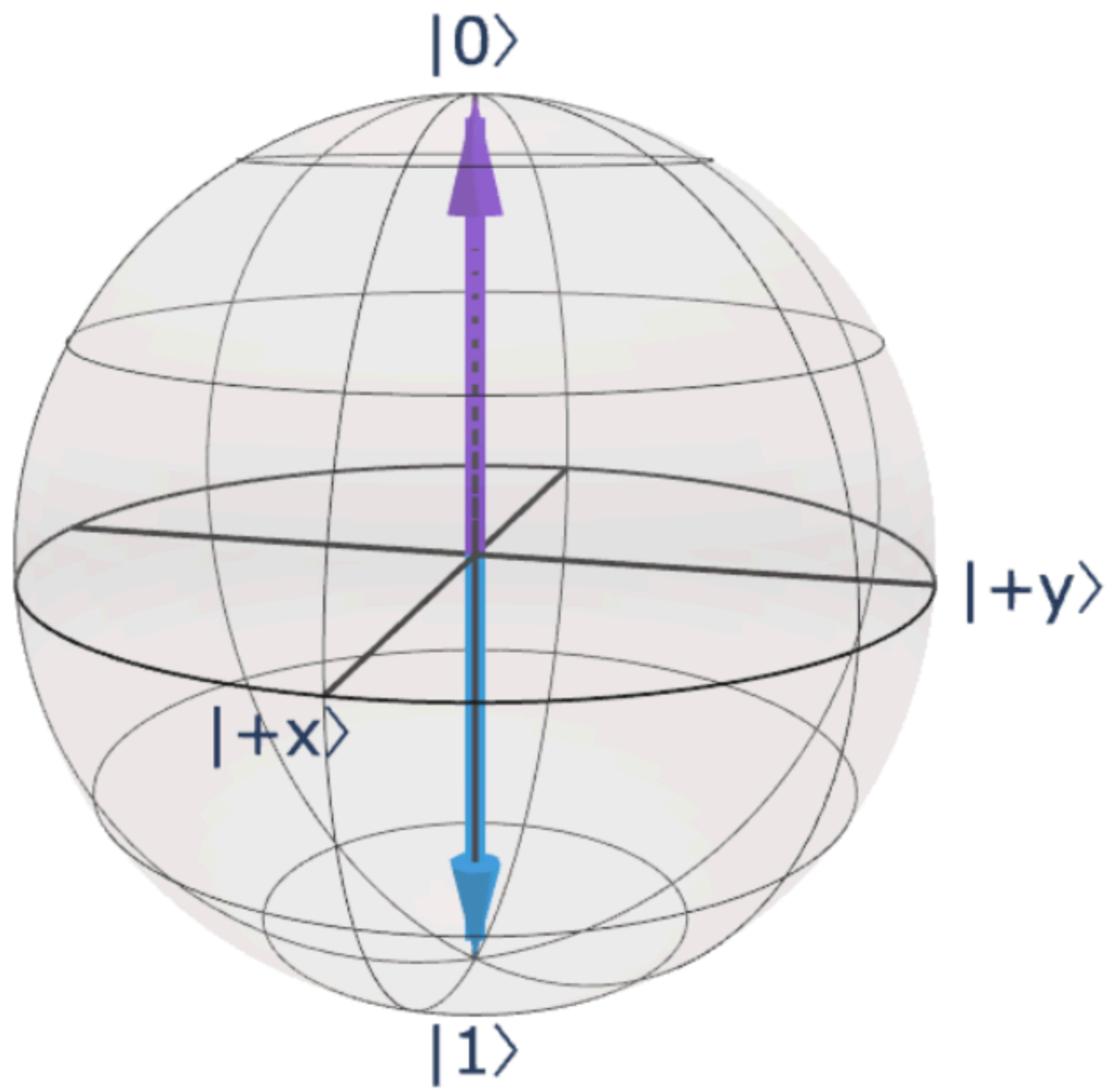
★ $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \neq 0$

- General states of a qubit (**superposition**)

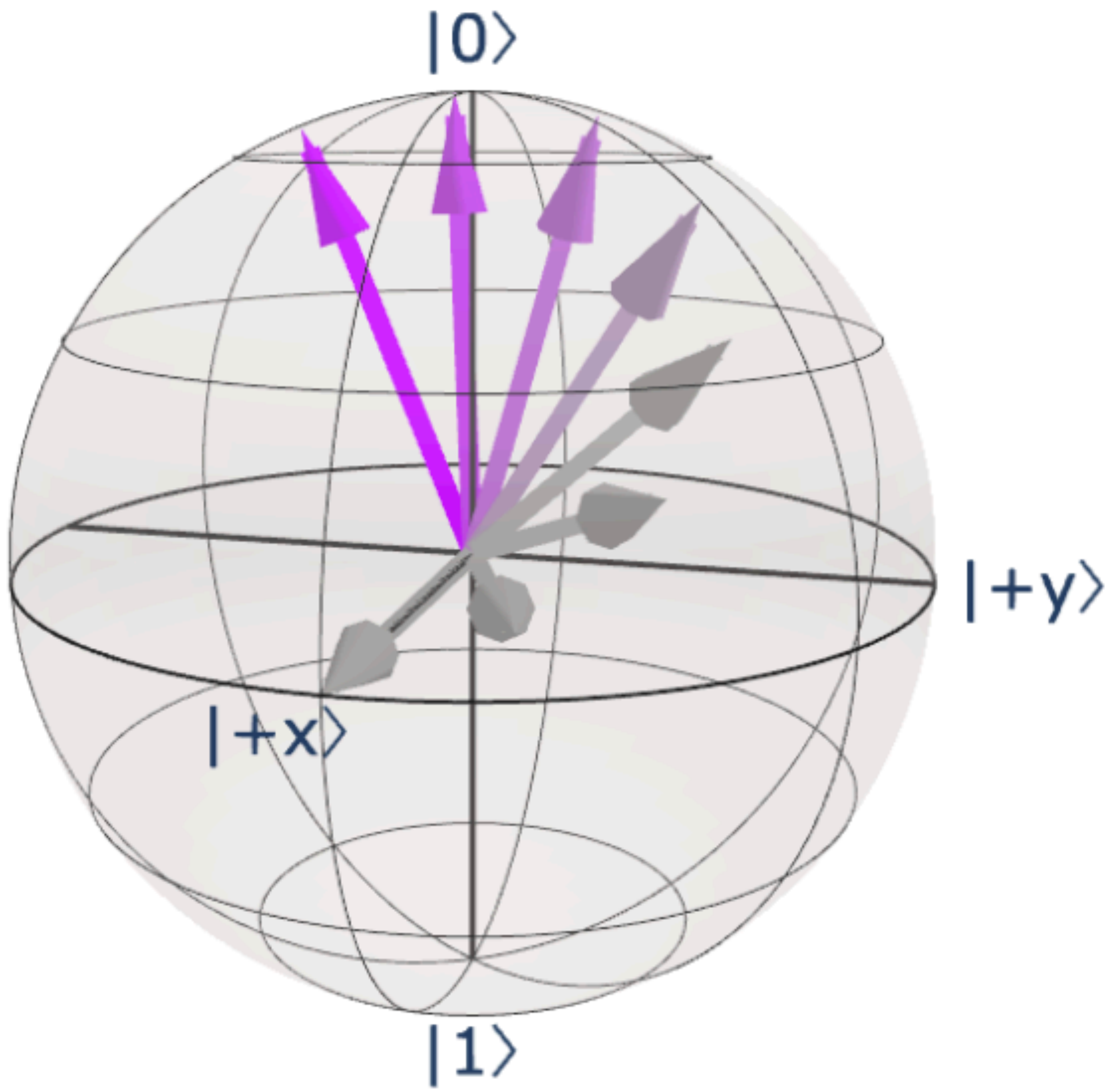
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \qquad \alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1$$

Superposition is one of the key attributes of the quantum computer.

- A bit



- A qubit

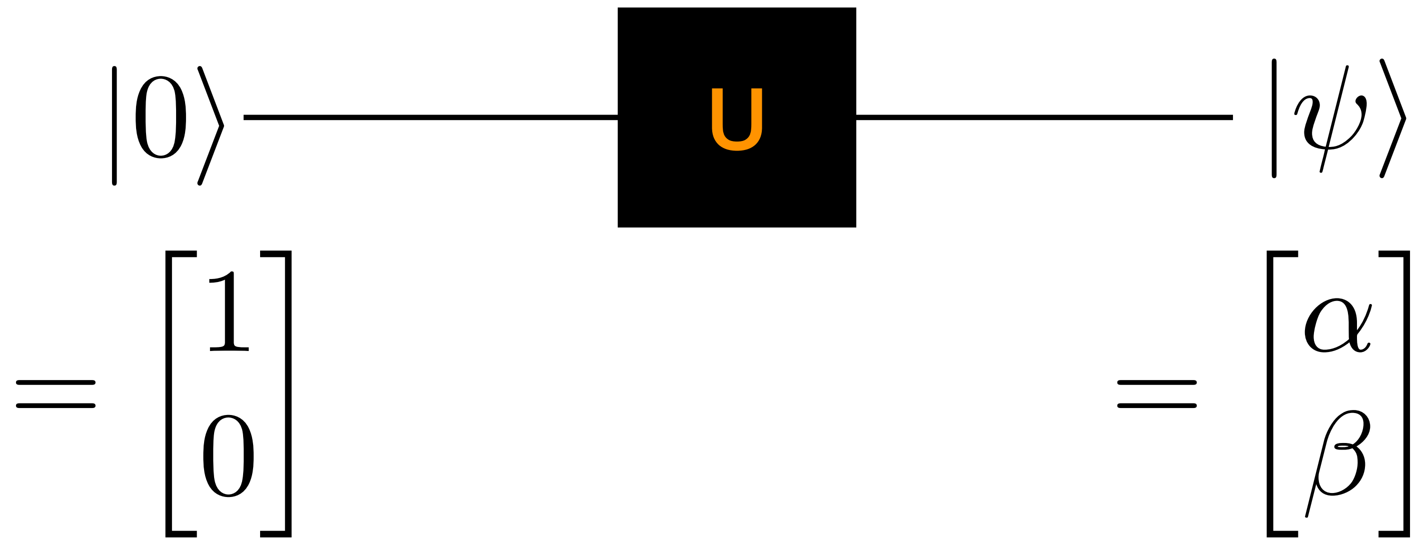


- A bit

Encode 1 on a bit:

- **A qubit**

Encode $|\psi\rangle (= \alpha|0\rangle + \beta|1\rangle)$ on a qubit :



- Unitary Operation / Quantum Gate

Generalization of real rotations in two-dim complex vector space

$$U^\dagger = U^{-1}, \quad UU^\dagger = U^\dagger U = I$$

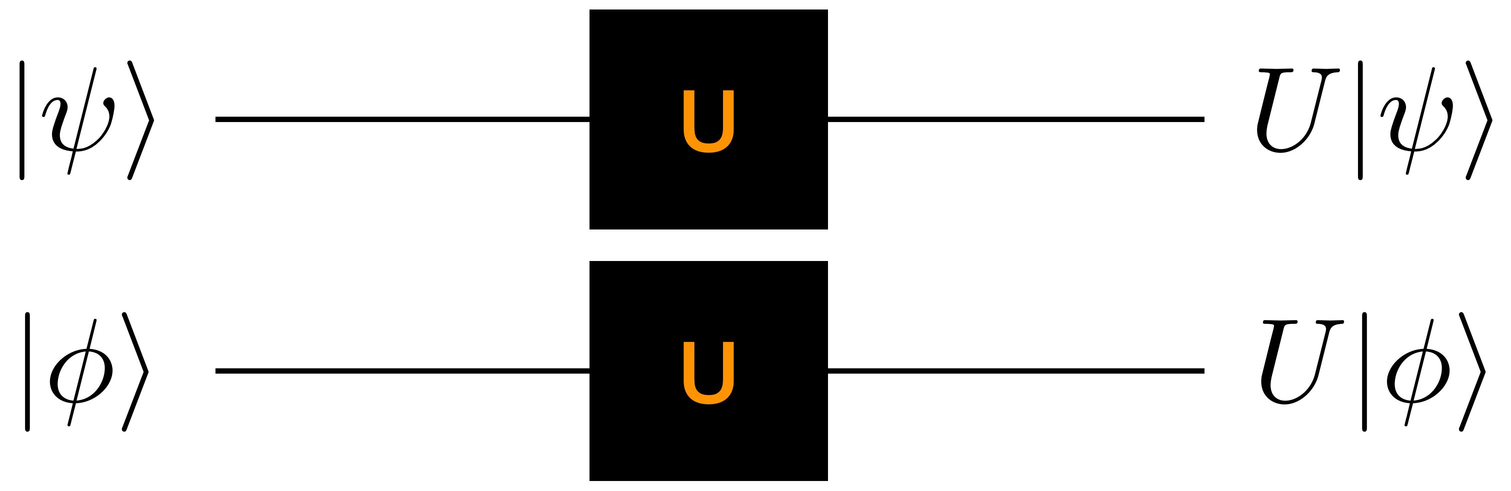
Linear

Reversible

Preserves the logical relation between the states

e.g. length preserving

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\phi\rangle = \gamma|0\rangle + \delta|1\rangle \qquad \langle\phi|\psi\rangle = \begin{bmatrix}\gamma^* & \delta^*\end{bmatrix}\begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$



$$\langle\phi|U^\dagger U|\psi\rangle = \langle\phi|\psi\rangle$$

- Unitary Operation / Quantum Gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\quad \boxed{X} \quad} |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

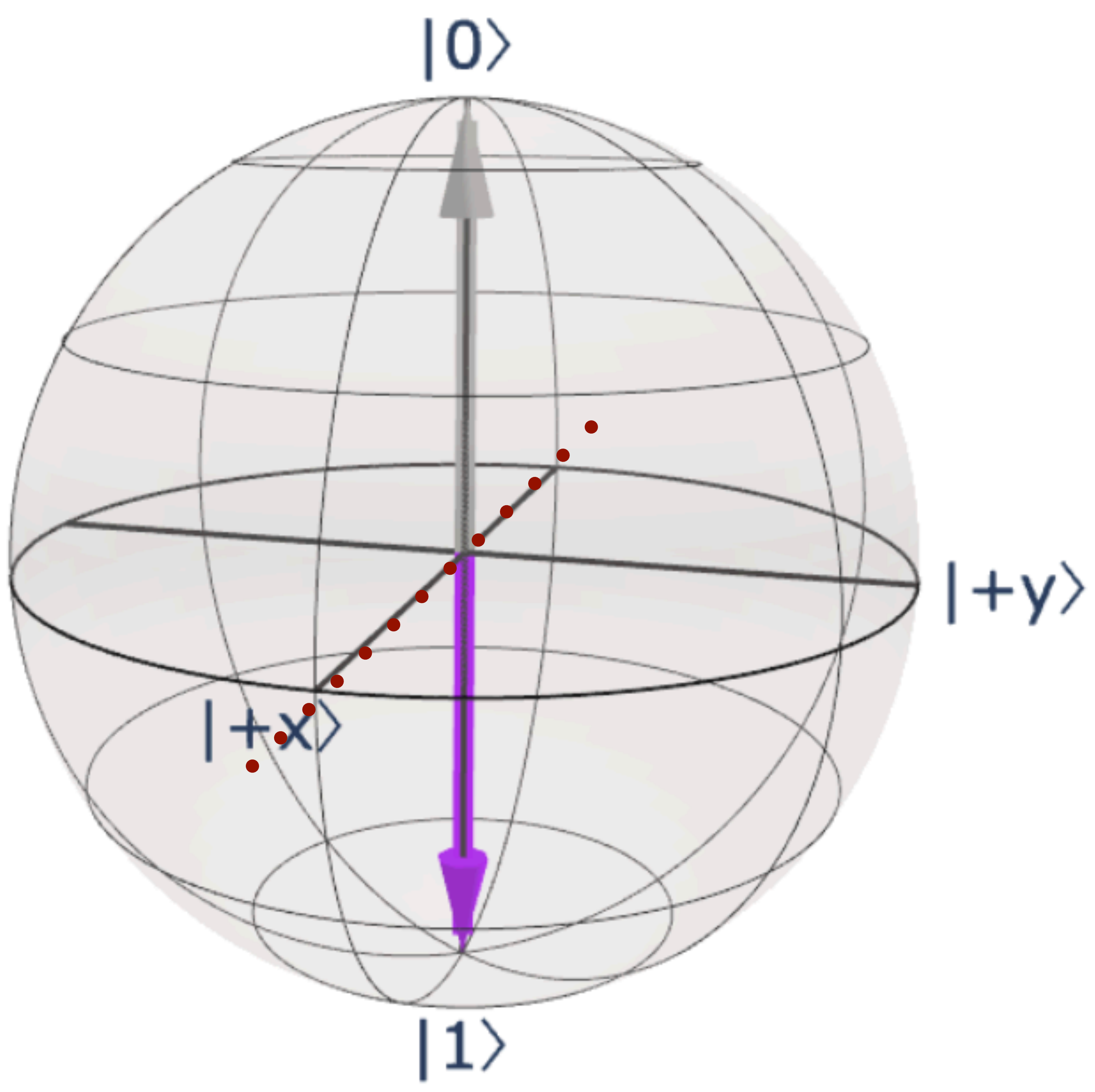
$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\quad \boxed{X} \quad} |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

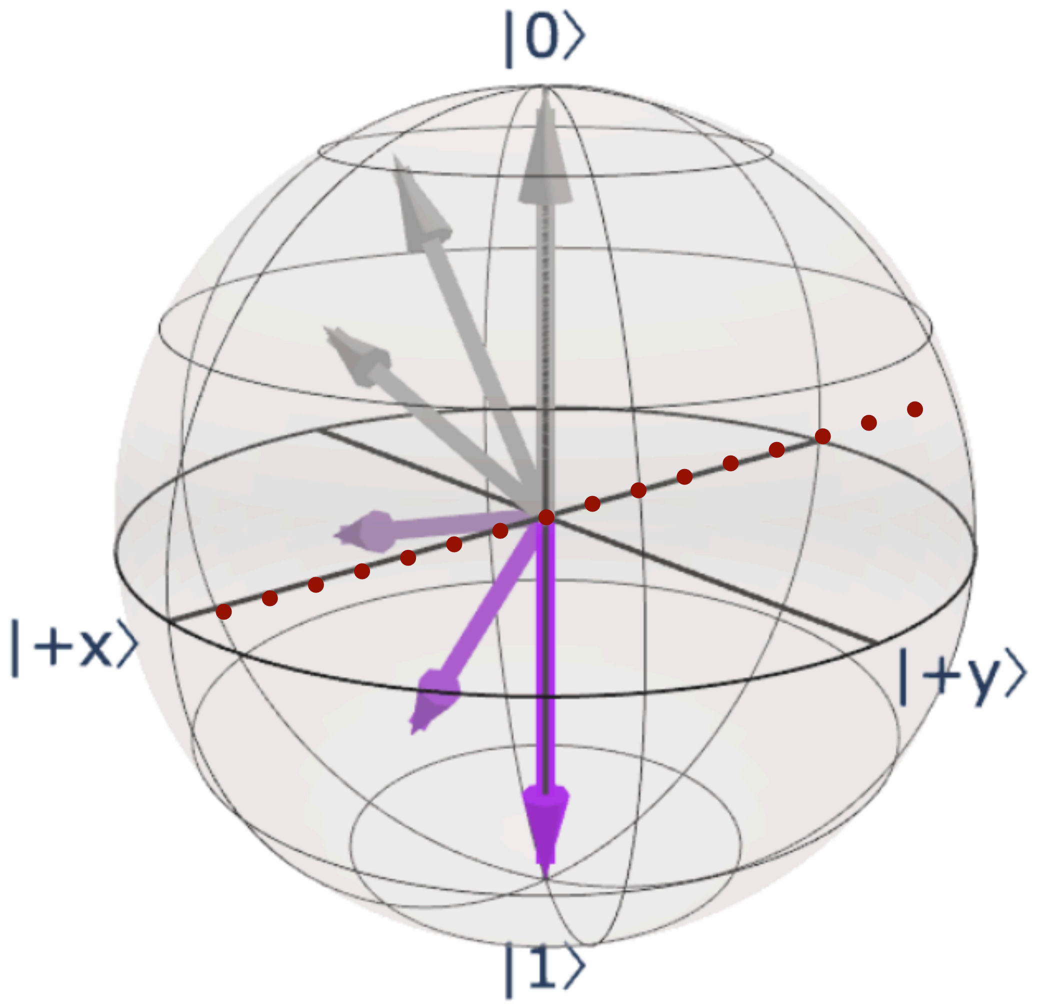
$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\quad \boxed{H} \quad} |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\quad \boxed{H} \quad} |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

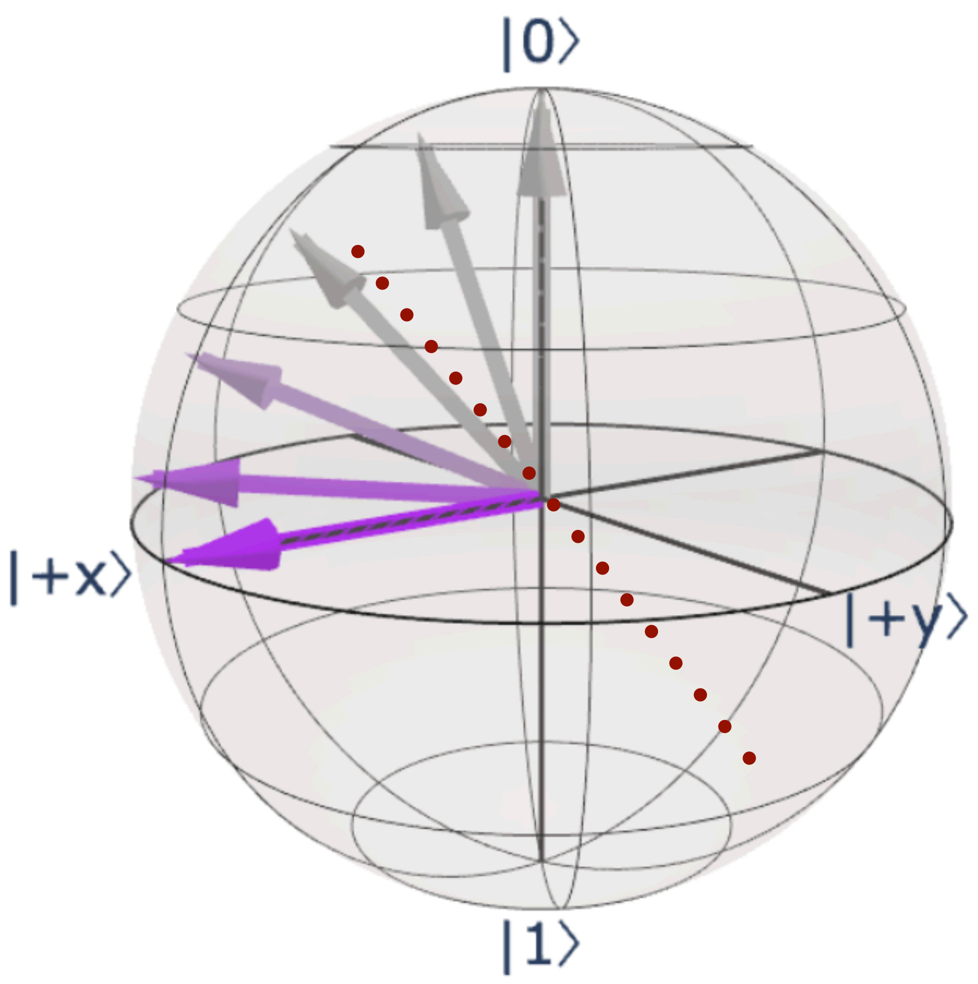
$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

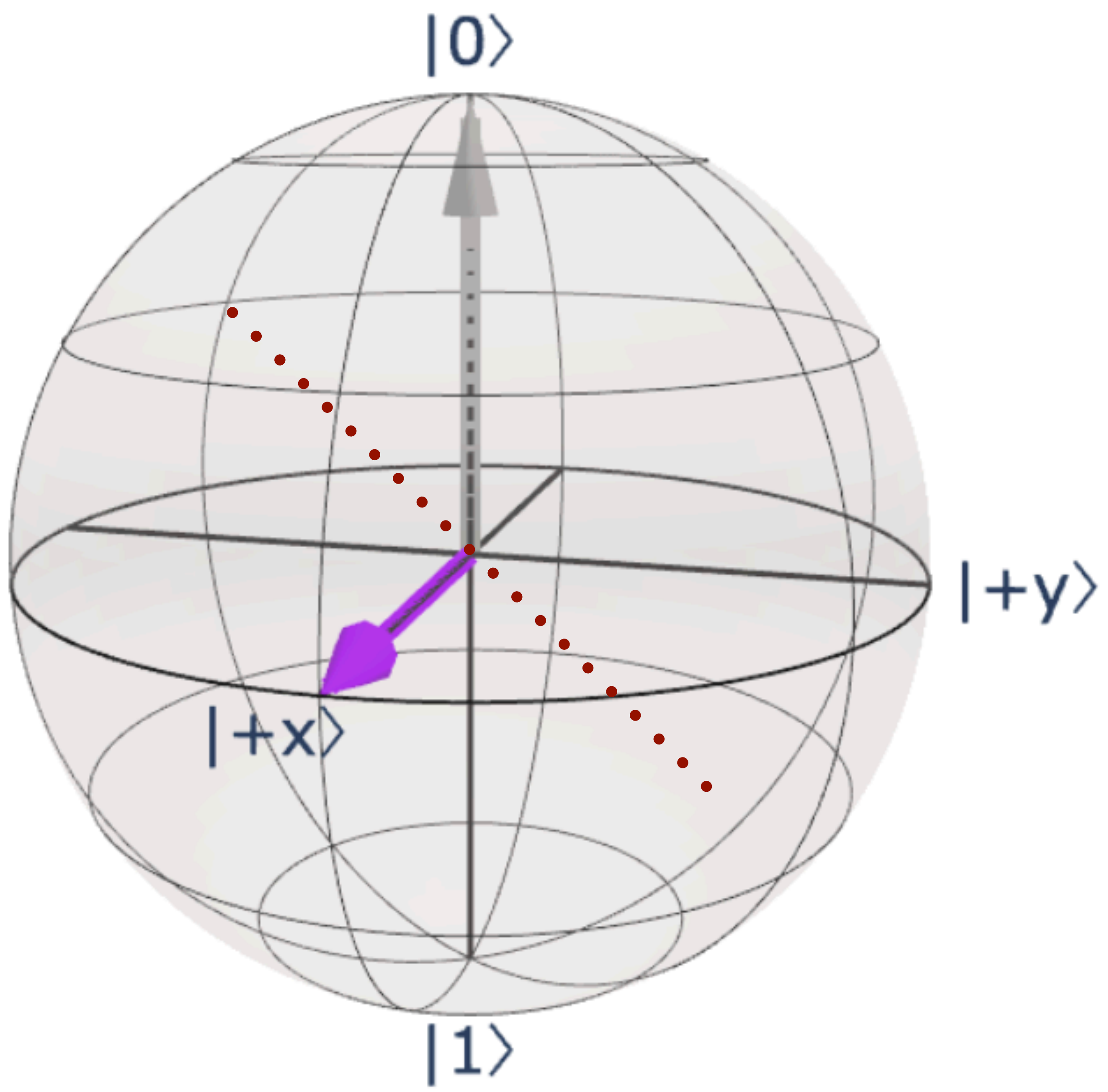
- X gate

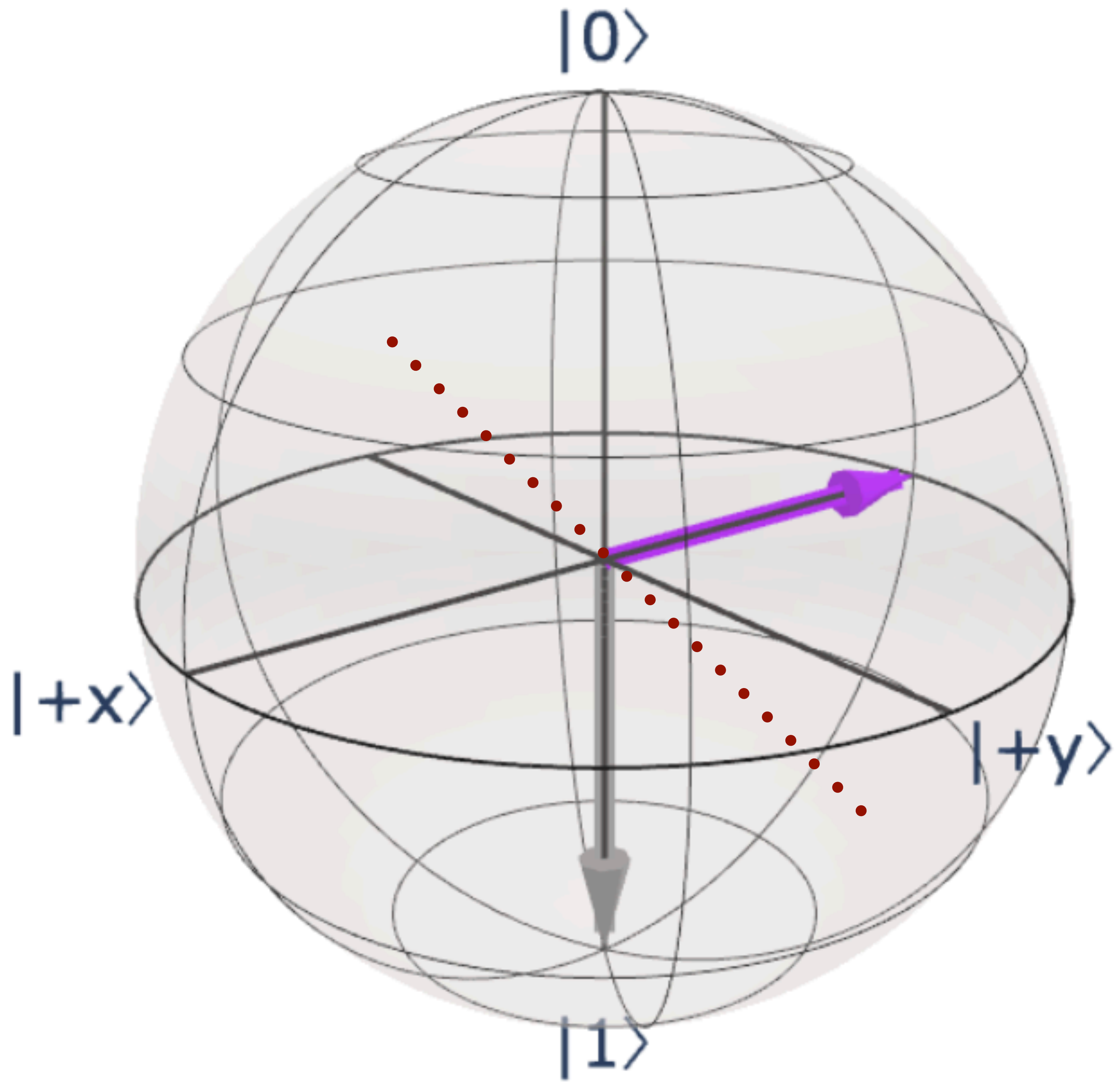




- H gate







- Unitary Operation / Quantum Gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$


$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$


$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$


$$|\psi'\rangle = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

$$= \alpha|0\rangle + \beta|1\rangle$$

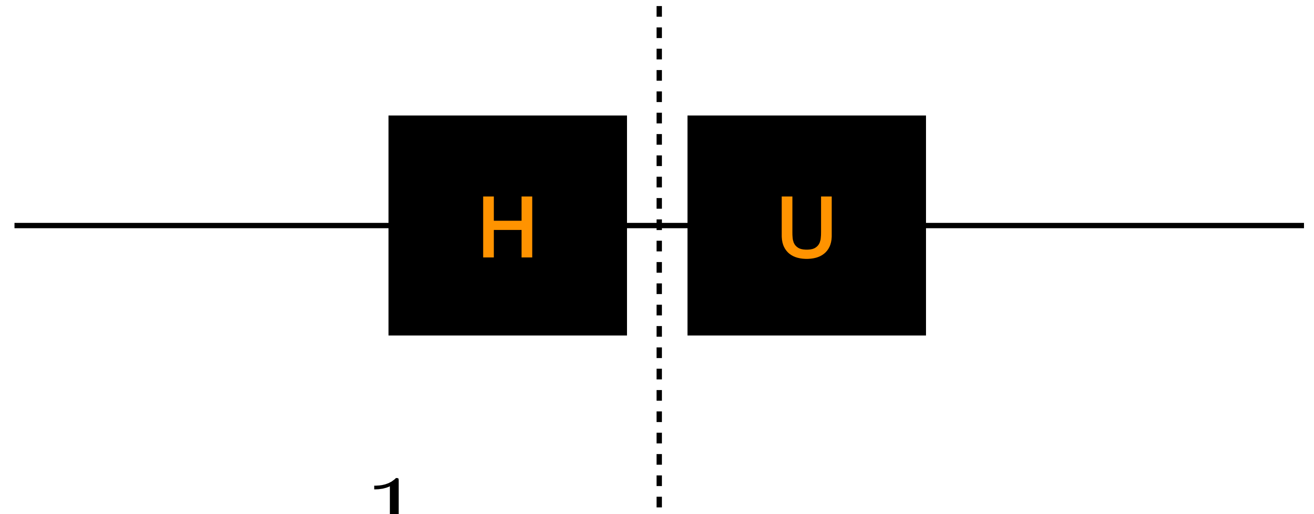
$$= \alpha|1\rangle + \beta|0\rangle$$

$$= \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

-Superposition and Quantum Parallelism

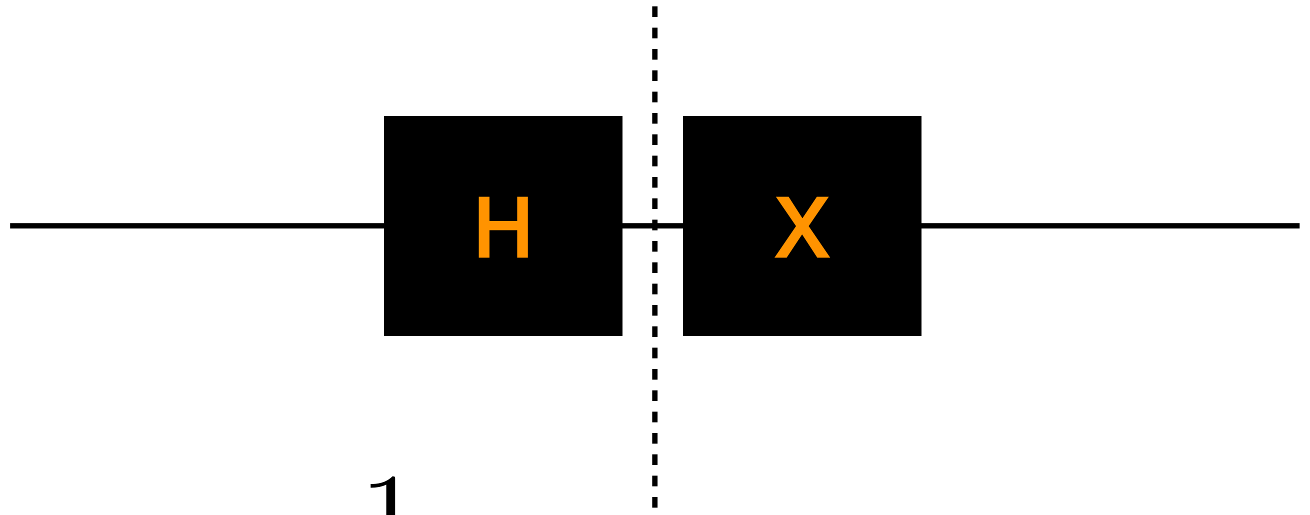
$|0\rangle$



$$\frac{1}{\sqrt{2}}(U|0\rangle + U|1\rangle)$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$|0\rangle$



$$\frac{1}{\sqrt{2}}(X|0\rangle + X|1\rangle) = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- **Measurements** (non - unitary)

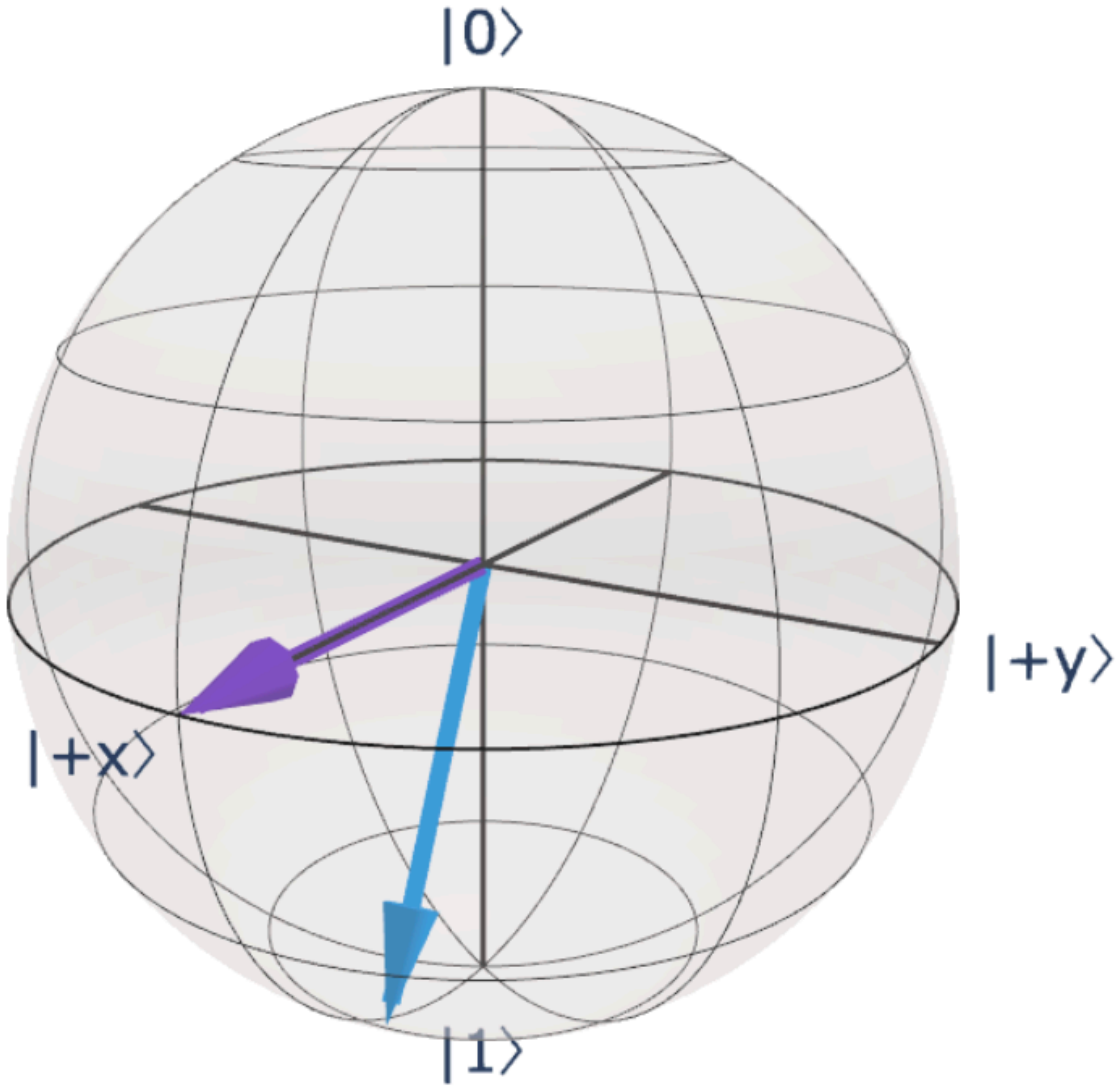
State of a qubit is not directly observable.

Obtain the information encoded in qubits by measurement in the computational basis.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \qquad \alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1$$

$P_{|0\rangle} = |\alpha|^2$ If get 0 $\Rightarrow$ $|\psi\rangle = |0\rangle$

$P_{|1\rangle} = |\beta|^2$ If get 1 $\Rightarrow$ $|\psi\rangle = |1\rangle$



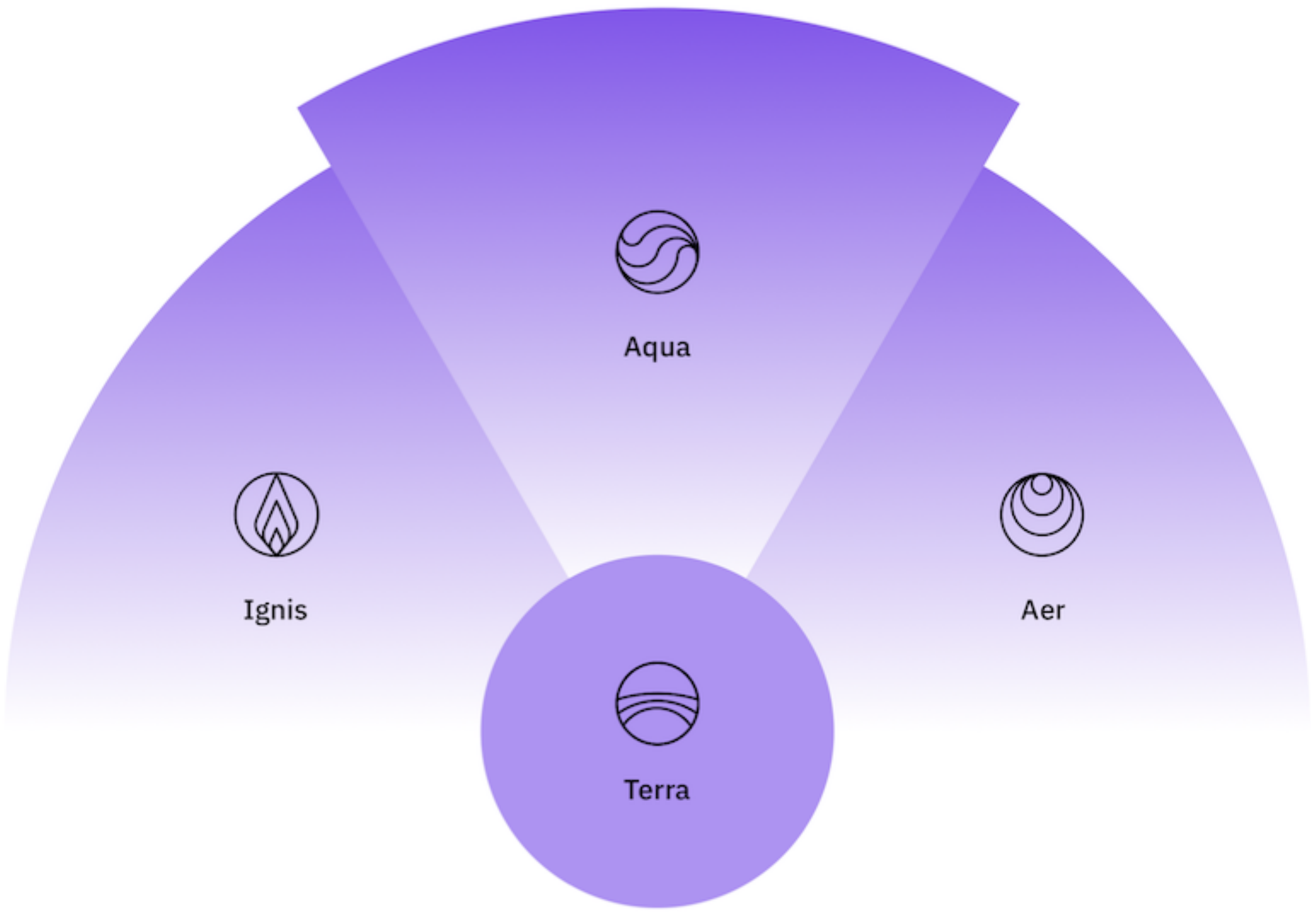
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$P_{|0\rangle} = P_{|1\rangle} = \frac{1}{2}$$

$$|\psi\rangle = (0.16 + i0.19)|0\rangle + (0.54 + i0.8)|1\rangle$$

$$= \begin{bmatrix}0.16 + i0.19\\0.54 + i0.8\end{bmatrix}$$

$$P_{|0\rangle} = 0.06 \qquad P_{|1\rangle} = 0.9$$



Terra

- Build and optimize circuits.
- Execute on devices and post process results.

Aer

- High-performance simulators.

Ignis

- Noise verification, validation, and mitigation.

Aqua

- Quantum applications and algorithms.

- Simulating Quantum Circuits

Can simulate quantum systems using classical computers.

Limited to ~50 qubits

Qubits	State	Memory
2	$\alpha\|00\rangle + \beta\|01\rangle + \gamma\|10\rangle + \delta\|11\rangle$	64 bytes
4	16 terms	256 bytes
8	256 terms	4096 bytes (4 KB)
16	65356 terms	1048576 bytes (1MB)
32	4.3 billion	68719476736 bytes (64 GB)
50	1.2 quadrillion	18014398509481984 bytes (16 EB)

- IBM Quantum Experience (IQX)

The IQX is a cloud-based platform for using quantum computers.

You can program quantum computers using qiskit in a Jupyter Notebook environment or explore and execute circuits graphically using the circuit Composer.

- Phase

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$= |\alpha|e^{i\theta_0}|0\rangle + |\beta|e^{i\theta_1}|1\rangle$$

$$|\psi\rangle = e^{i\theta_0}\left(\, |\alpha||0\rangle + |\beta|e^{i\phi}|1\rangle \, \right), \;\; \phi = \theta_1 - \theta_0$$

Global phase

Relative phase

Global phase difference : Not detectable

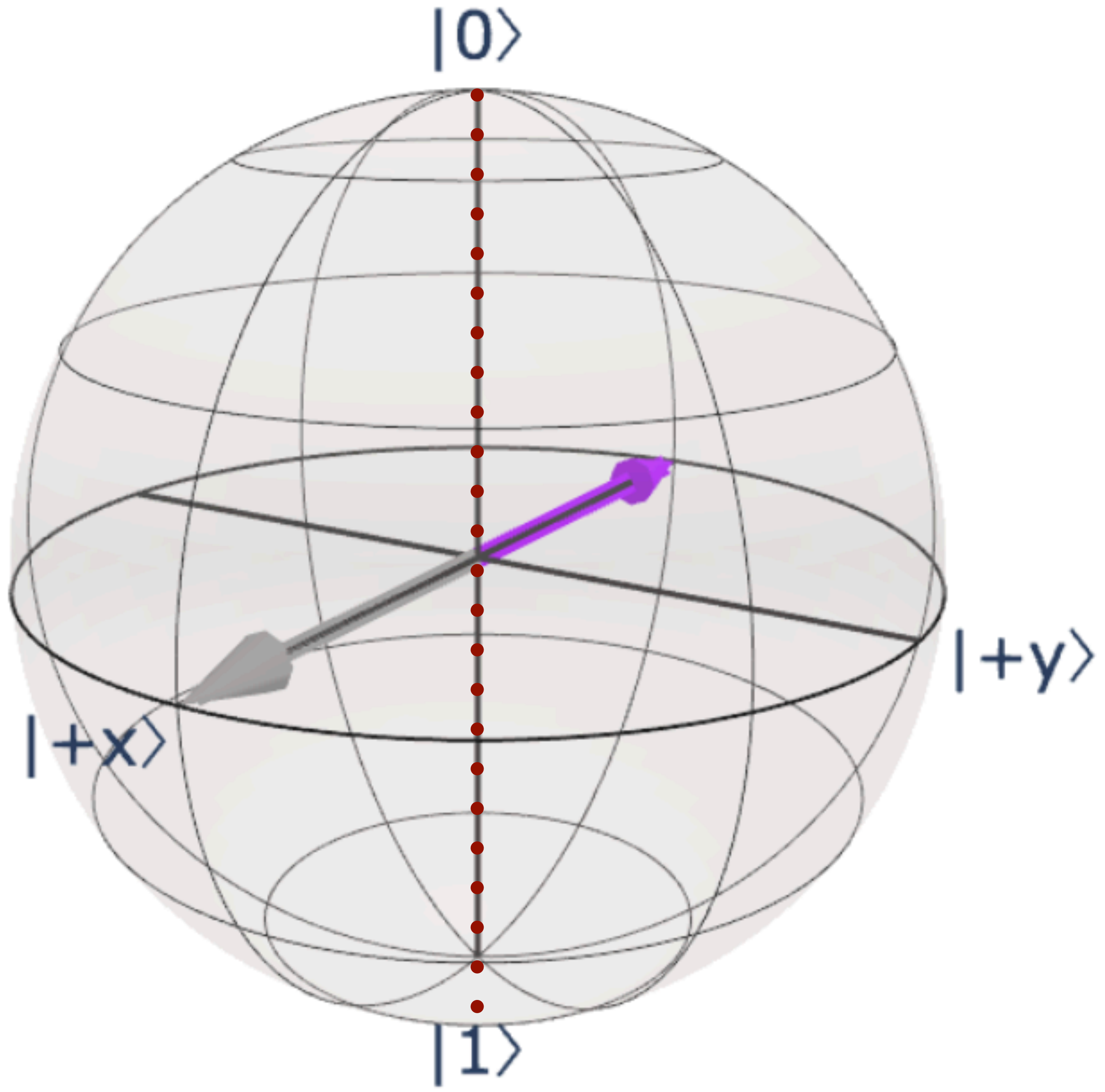
Relative phase difference : Detectable

- Phase gate Z

: π rotation around z axis

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \quad \text{—}\boxed{Z}\text{—} \quad e^{i0}|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \quad \text{—}\boxed{Z}\text{—} \quad e^{i\pi}|1\rangle = \begin{bmatrix}0\\-1\end{bmatrix}$$



$$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

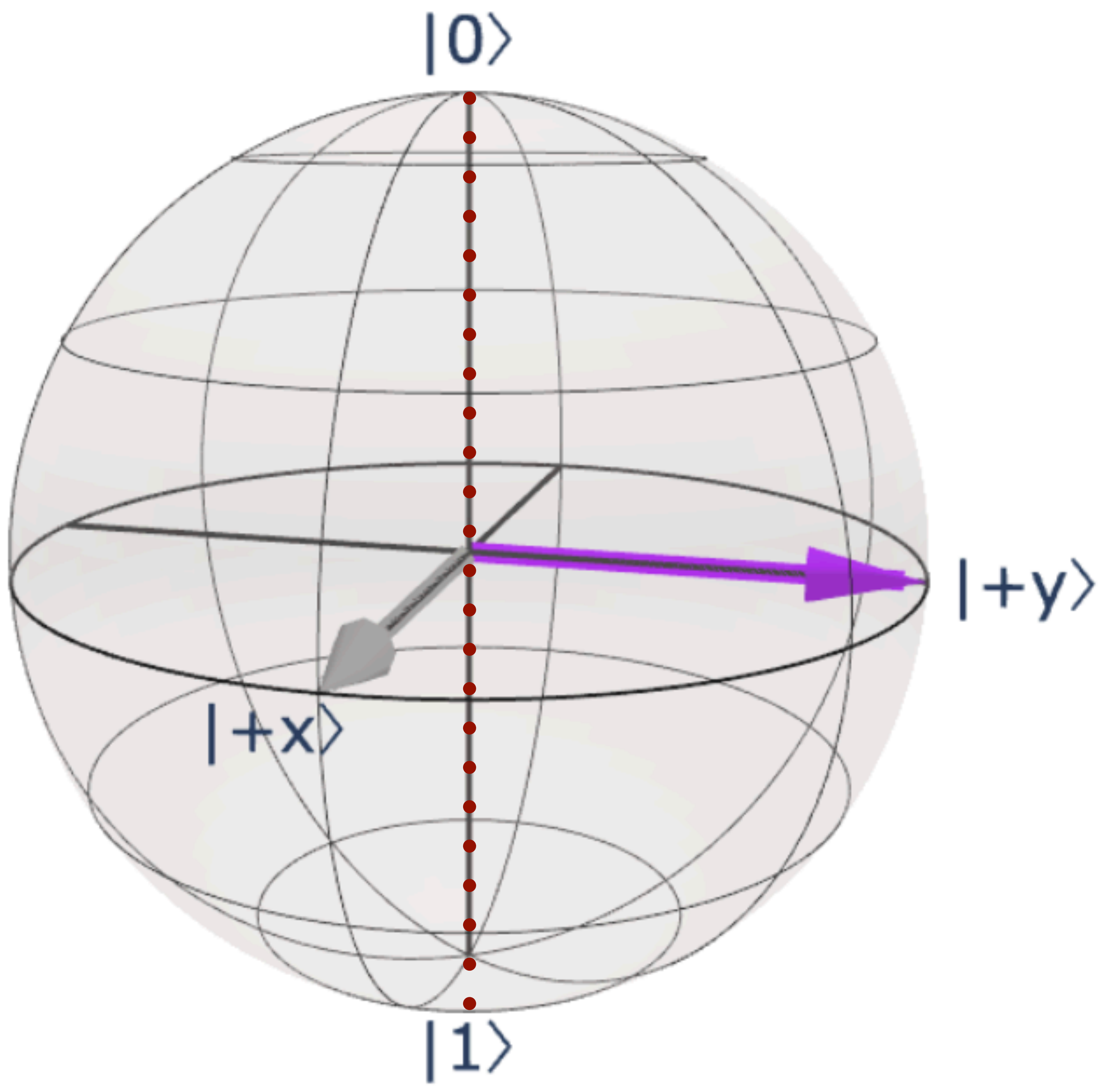
$$\begin{aligned}\underline{Z|+\rangle} &= \frac{1}{\sqrt{2}}Z(|0\rangle + |1\rangle)\\ &= \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi}|1\rangle)\\ &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \underline{|-\rangle = \begin{bmatrix}1\\-1\end{bmatrix}}\end{aligned}$$

- Phase gate S

: $\frac{\pi}{2}$ rotation around z axis

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \text{ —— } \boxed{S} \text{ —— } e^{i0}|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \text{ —— } \boxed{S} \text{ —— } e^{i\frac{\pi}{2}}|1\rangle = \begin{bmatrix}0\\i\end{bmatrix}$$

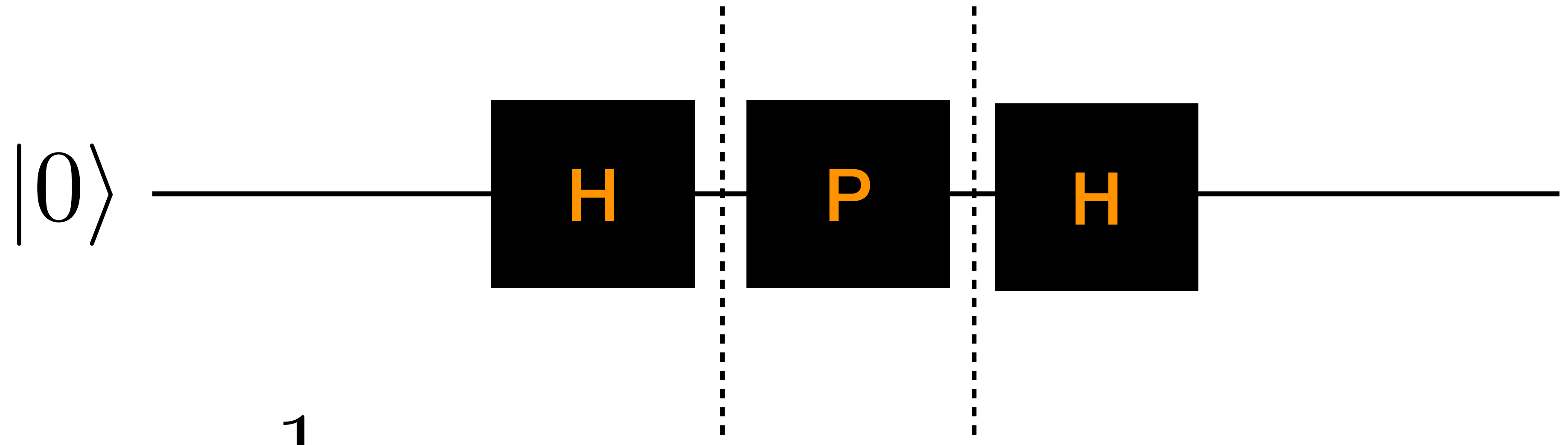


$$S = \begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$$

$$\begin{aligned} S|+\rangle &= \frac{1}{\sqrt{2}}S(|0\rangle + |1\rangle) \\ &= \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/2}|1\rangle) \\ &= \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = \begin{bmatrix}1\\i\end{bmatrix} \end{aligned}$$

- Interference

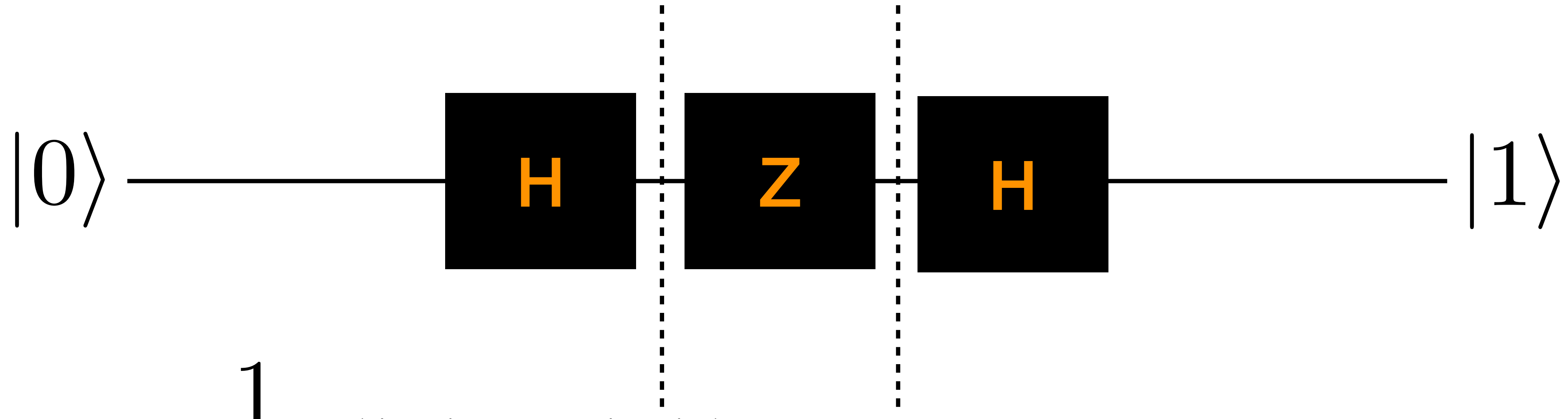
$$\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)$$



$$e^{i\frac{\phi}{2}}(\cos\frac{\phi}{2}|0\rangle - i\sin\frac{\phi}{2}|1\rangle)$$

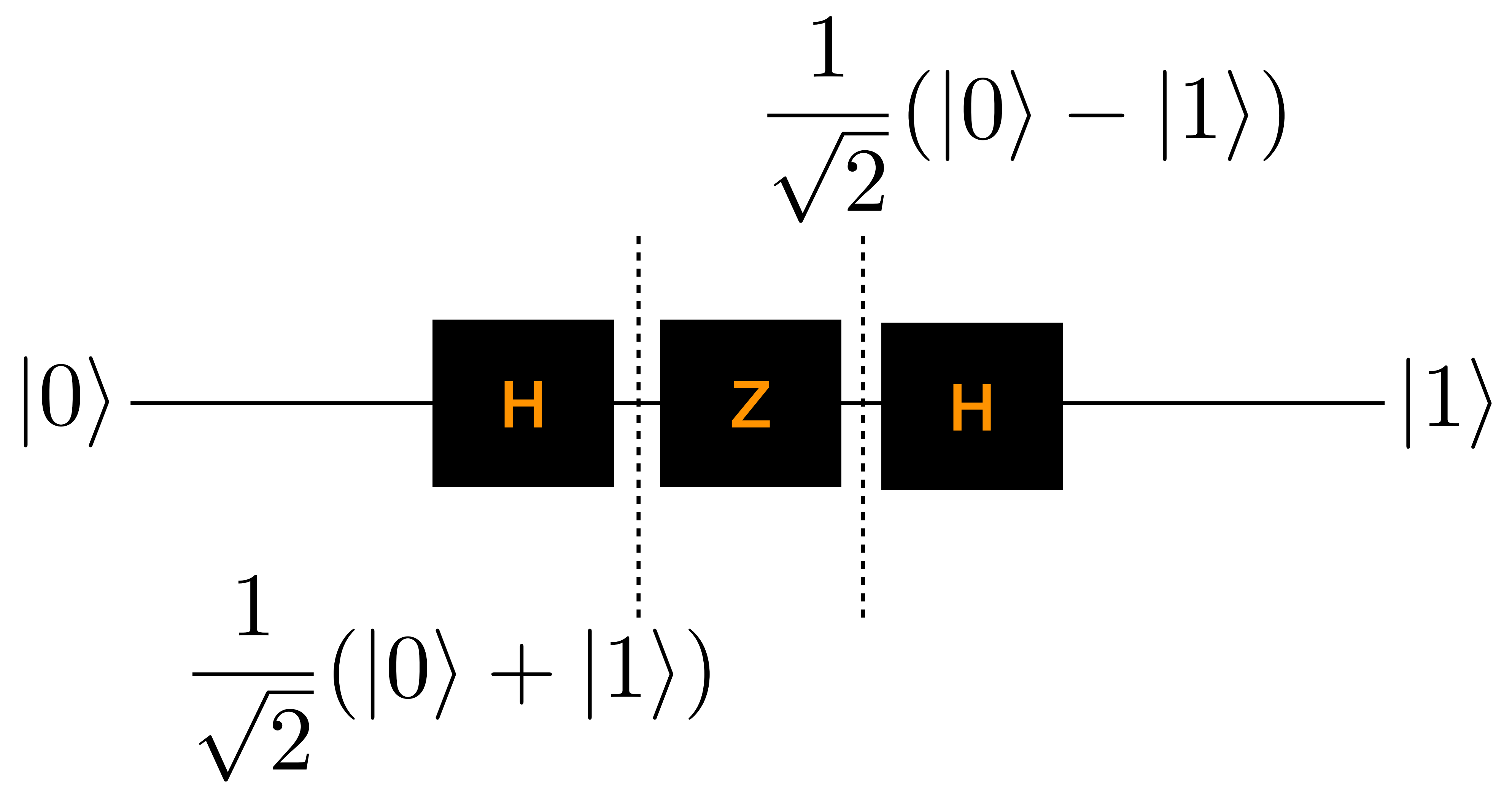
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

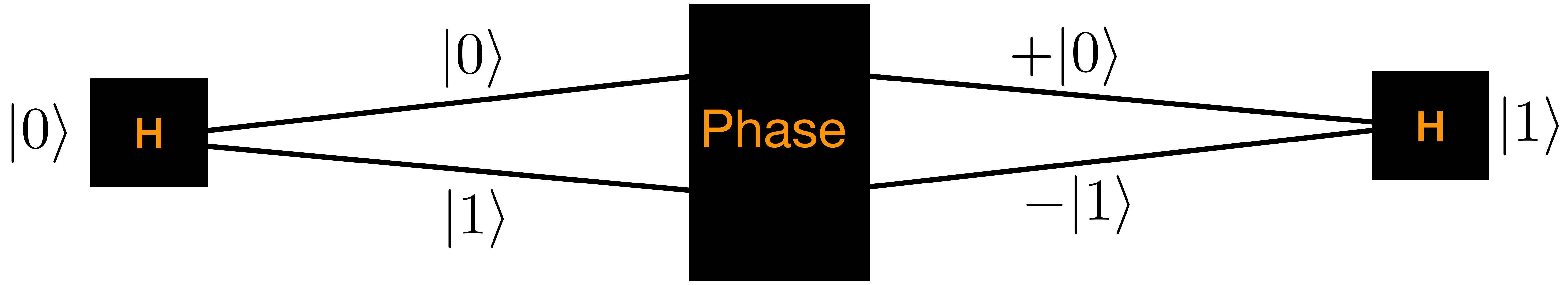
$$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- Interference





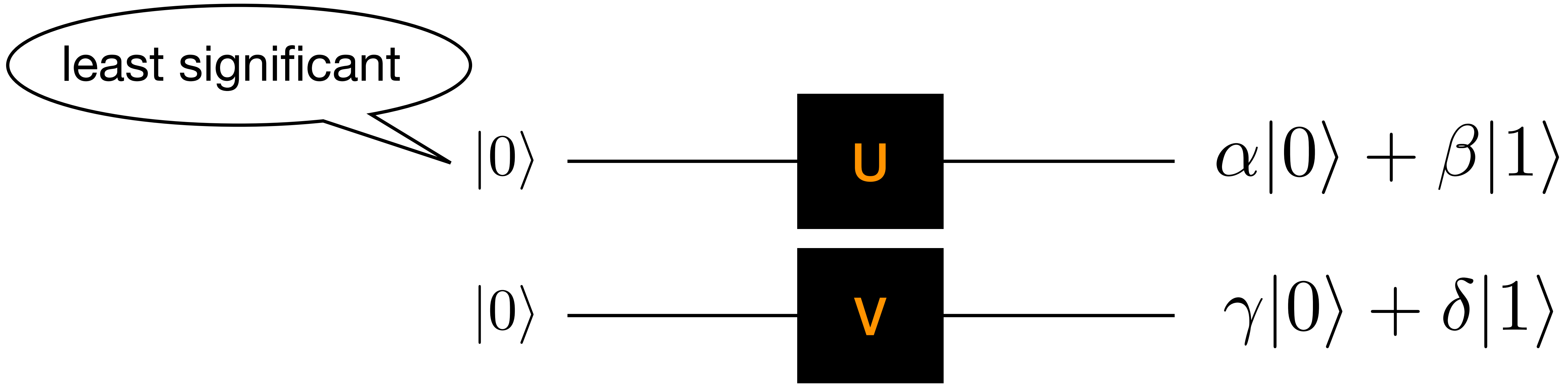
- Composite System : 2 qubit case

State of 2 qubit system is a unit (normalized) vector in a 2^2 dim complex vector space.

The state of 2 qubits changes through unitary operations / quantum gates.

Ex) n = 2 : 4-dim $\quad |00\rangle, \quad |01\rangle, \quad |10\rangle, \quad |11\rangle$

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

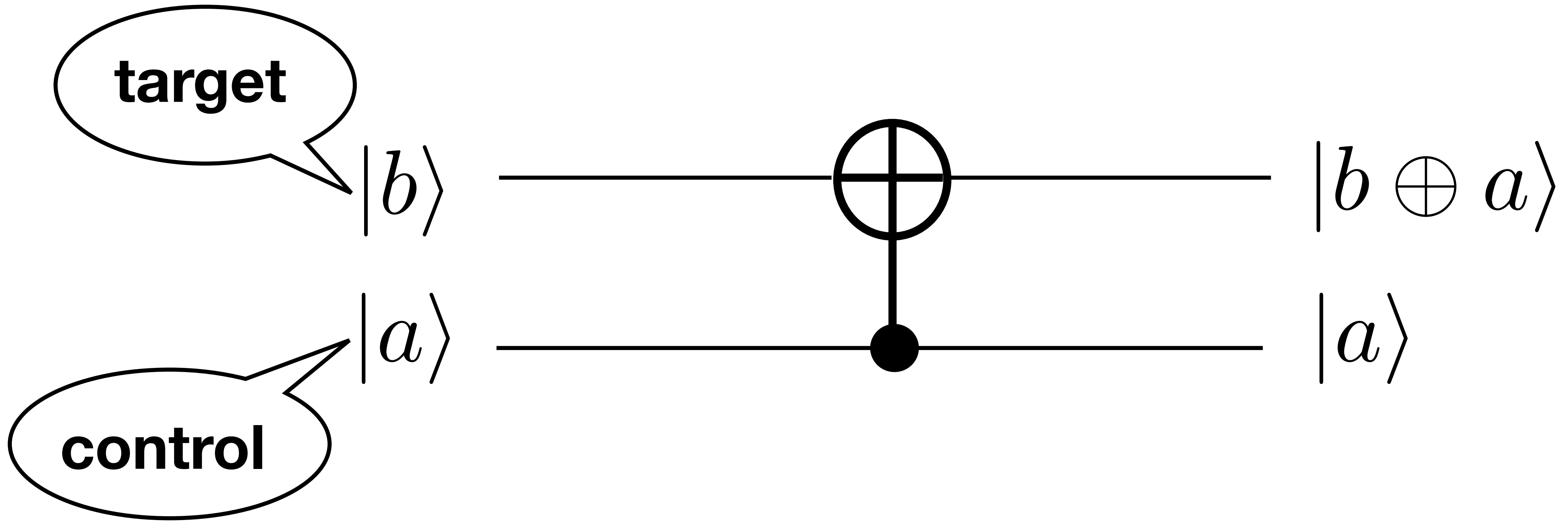


$$(\gamma|0\rangle + \delta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle) = \gamma\alpha|00\rangle + \gamma\beta|01\rangle + \delta\alpha|10\rangle + \delta\beta|11\rangle$$

: Product state, each sub system can be prepared independently

- Two-Qubit Gate, CNOT

: **controlled-not**, or controlled-x gate



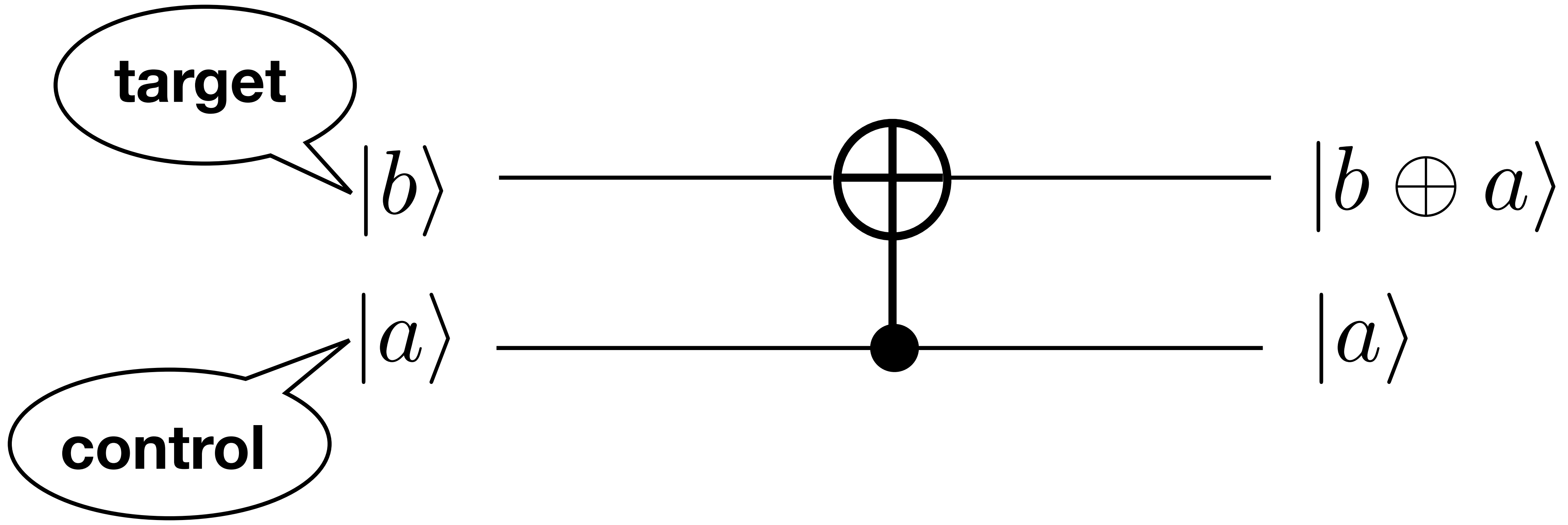
$$U_{CN}|ab\rangle = |a\ b \oplus a\rangle$$

$$|00\rangle \rightarrow |00\rangle; \quad |01\rangle \rightarrow |01\rangle; \quad |10\rangle \rightarrow |11\rangle; \quad |11\rangle \rightarrow |10\rangle$$

What happens if $|a\rangle = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ and $|b\rangle = |0\rangle$?

- Two-Qubit Gate, CNOT

: **controlled-not**, or controlled-x gate



$$U_{CN}|ab\rangle = |a\ b \oplus a\rangle$$

$$|00\rangle \rightarrow |00\rangle; \quad |01\rangle \rightarrow |01\rangle; \quad |10\rangle \rightarrow |11\rangle; \quad |11\rangle \rightarrow |10\rangle$$

What happens if $|a\rangle = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ and $|b\rangle = |0\rangle$?

$$\begin{aligned} U_{CN}|ab\rangle &= \frac{1}{\sqrt{2}} U_{CN}(|0\rangle + |1\rangle)|0\rangle \\ &= \frac{1}{\sqrt{2}}(U_{CN}|00\rangle + U_{CN}|10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \end{aligned}$$

- Entanglement

Non-classical correlation between quantum systems

Like superposition, **entanglement** is a key resource for quantum computation.

Entangled states can be utilized as a medium to perform computational tasks that are impossible for classical systems : teleportation, superdense coding

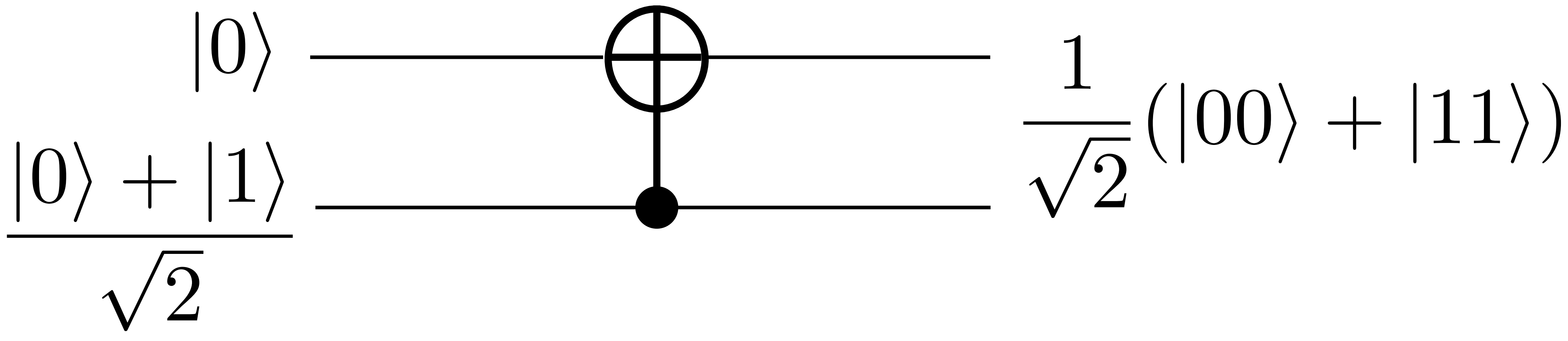
ex) $$\underbrace{|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)}$$ Bell State

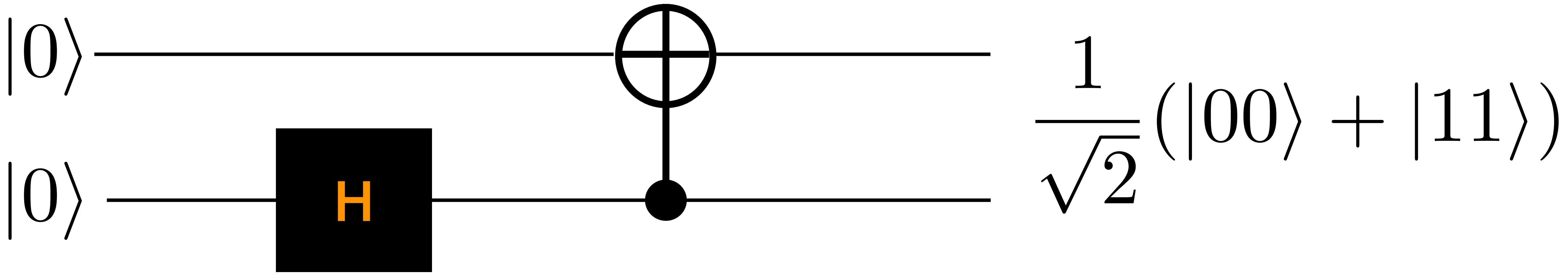
: maximally entangled state, cannot be written as a product state.

describe the composite system completely

nothing is known about the individual subsystems

Quantum Circuit to construct bell state





Quantum teleportation