

1.  $\langle A| = (5 \quad 3i)$
2.  $\langle B| = (-7 \quad 4e^{i\frac{\pi}{3}})$
3.  $\langle C| = \begin{pmatrix} 1 \\ e^{-i\frac{\pi}{6}} \end{pmatrix}$
4.  $\langle \psi|\psi\rangle = 1$
5.  $\langle A|A\rangle =$   
 $|A\rangle = (5a \quad 3ia)$   
 $\langle A| = \begin{pmatrix} 5a \\ 3ia \end{pmatrix}$   
 $\begin{pmatrix} 5a \\ 3ia \end{pmatrix} (5a \quad 3ia)$   
 $25a^2 + 9 \times -1^2 \times a^2$   
 $a = \frac{1}{\sqrt{34}}$
6.  $|B\rangle = \begin{pmatrix} 7b \\ 4e^{-i\frac{\pi}{3}} \end{pmatrix}$   
 $\langle B| = (7b \quad 4e^{i\frac{\pi}{3}})$   
 $= \begin{pmatrix} 7b \\ 4e^{-i\frac{\pi}{3}} \end{pmatrix} (7b \quad 4e^{-i\frac{\pi}{3}})$   
 $= 49b^2 + 16 \times 1 \times b^2$   
 $= \frac{1}{\sqrt{65}}$
7.  $|C\rangle = (c \quad e^{i\frac{\pi}{6}})$   
 $\langle C| = \begin{pmatrix} c \\ e^{-i\frac{\pi}{6}} \end{pmatrix}$   
 $= (c \quad e^{i\frac{\pi}{6}}) \begin{pmatrix} c \\ e^{-i\frac{\pi}{6}} \end{pmatrix}$   
 $= c^2 + 1 \times c^2$   
 $= \sqrt{1}$
8.  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
9.  $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
10.  $\langle 0|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \quad 0)$   
 $= 1 + 0$   
 $= 1$
11.  $\langle 0|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (0 \quad 1)$   
 $= 0 + 0 = 0$
12.  $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$   
 $\frac{1}{\sqrt{2}} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$   
 $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$13. \sqrt{\frac{5}{7}}|0\rangle + \sqrt{\frac{2}{7}}|1\rangle$$

$$14. P(0) = |\langle 0|+\rangle|^2 \\ = \left|\frac{1}{\sqrt{2}}\right|^2 \\ = \frac{1}{2}$$

$$15. P(1) = |\langle 1|-\rangle|^2 \\ = \left|-\frac{1}{\sqrt{2}}\right|^2 \\ = \frac{1}{2}$$

$$16. P(0) = |\langle 0|\phi\rangle|^2 \\ = \left|\frac{2}{\sqrt{13}}\right|^2 \\ = \frac{4}{13}$$

$$17. P(1) = |\langle 1|\phi\rangle|^2 \\ = \left|\frac{3}{\sqrt{13}}e^{i\frac{\pi}{4}}\right|^2 \\ = \frac{9}{13}$$

$$18. \langle ++\rangle \\ = \left(\frac{1}{\sqrt{2}}\langle 0| + \frac{1}{\sqrt{2}}\langle 1|\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \\ = \frac{1}{2}\langle 0|0\rangle + \frac{1}{2}\langle 0|1\rangle + \frac{1}{2}\langle 1|0\rangle + \frac{1}{2}\langle 1|1\rangle \\ = \frac{1}{2} + \frac{1}{2} \\ = 1$$

$$19. \langle +|-\rangle \\ = \left(\frac{1}{\sqrt{2}}\langle 0| + \frac{1}{\sqrt{2}}\langle 1|\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \\ = \frac{1}{2}\langle 0|0\rangle - \frac{1}{2}\langle 0|1\rangle + \frac{1}{2}\langle 1|0\rangle - \frac{1}{2}\langle 1|1\rangle \\ = \frac{1}{2} - \frac{1}{2} \\ = 0$$

$$20. \langle \phi|+\rangle \\ = \left(\frac{2}{\sqrt{13}}\langle 0| + \frac{3}{13}e^{-i\frac{\pi}{4}}\langle 1|\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \\ = \frac{\sqrt{2}}{13}\langle 0|0\rangle + \frac{\sqrt{2}}{13}\langle 0|1\rangle + \frac{3\sqrt{2}}{26}e^{-i\frac{\pi}{4}}\langle 1|0\rangle + \frac{3\sqrt{2}}{26}e^{-i\frac{\pi}{4}}\langle 1|1\rangle \\ = \frac{1}{\sqrt{26}}(2 + 3e^{-i\frac{\pi}{4}})$$

$$21. \langle -|\phi\rangle \\ = \left(\frac{1}{\sqrt{2}}\langle 0| - \frac{1}{\sqrt{2}}\langle 1|\right)\left(\frac{2}{\sqrt{13}}|0\rangle + \frac{3}{13}e^{-i\frac{\pi}{4}}|1\rangle\right) \\ = -\frac{1}{\sqrt{26}}(2 + 3e^{-i\frac{\pi}{4}})$$

$$22. \sigma_x|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ = \begin{pmatrix} 0+0 \\ 1+0 \end{pmatrix} \\ = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ = |1\rangle$$

$$\begin{aligned}
23. \quad \sigma_x |1\rangle &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 0+1 \\ 0+0 \end{pmatrix} \\
&= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= |0\rangle
\end{aligned}$$

$$\begin{aligned}
24. \quad \sigma_y |0\rangle &= \\
&= \begin{pmatrix} 0-i \\ 0+0 \end{pmatrix} \\
&= (-i0) \\
&= i|0\rangle
\end{aligned}$$

$$\begin{aligned}
25. \quad \sigma_y |1\rangle &= \\
&= \begin{pmatrix} 0-1 \\ 0+0 \end{pmatrix} \\
&= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= i|0\rangle
\end{aligned}$$

$$\begin{aligned}
26. \quad \sigma_z |0\rangle &= \\
&= \begin{pmatrix} 1+0 \\ 0+0 \end{pmatrix} \\
&= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= |1\rangle
\end{aligned}$$

$$\begin{aligned}
27. \quad \sigma_z |1\rangle &= \\
&= \begin{pmatrix} 0+0 \\ 0-1 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ -1 \end{pmatrix} \\
&= -|1\rangle
\end{aligned}$$

$$\begin{aligned}
28. \quad H |0\rangle &= \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} 1+0 \\ 1-0 \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
&= |+\rangle
\end{aligned}$$

$$\begin{aligned}
29. \quad H |1\rangle &= \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} 0+1 \\ 0-1 \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\
&= |-\rangle
\end{aligned}$$

$$\begin{aligned}
30. \quad \sigma_x |+\rangle &= \\
&= \sigma_x \left( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right) \\
&= \frac{1}{\sqrt{2}} (\sigma_x |0\rangle + \sigma_x |1\rangle) \\
&= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \\
&= |+\rangle \\
31. \quad \sigma_x |-\rangle &= \\
&= \sigma_x \left( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right) \\
&= \frac{1}{\sqrt{2}} (\sigma_x |0\rangle - \sigma_x |1\rangle) \\
&= \frac{1}{\sqrt{2}} (|1\rangle + |0\rangle) \\
&= |+\rangle \\
32. \quad H |+\rangle &= \\
&= \frac{1}{\sqrt{2}} (H |0\rangle + H |1\rangle) \\
&= \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle) \\
&= \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle + \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \right) \\
&= |0\rangle \\
33. \quad \sigma_x |+\rangle &= \\
&= \sigma_x \left( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right) \\
&= \frac{1}{\sqrt{2}} (\sigma_x |0\rangle + \sigma_x |1\rangle) \\
&= \frac{1}{\sqrt{2}} (|1\rangle - |0\rangle) \\
&= |-\rangle \\
34. \quad H \sigma_x |0\rangle &= \\
&= H (\sigma_x |0\rangle) \\
&= H(|1\rangle) \\
&= |-\rangle \\
35. \quad \sigma_x H |0\rangle &= \\
&= \sigma_x (H |0\rangle) \\
&= \sigma_x (|+\rangle) \\
&= |+\rangle
\end{aligned}$$