# Origami

## Computing Veechgroups of origamis

1.0

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**Pascal Kattler**

**Sergio Siccha**

**Andrea Thevis**

**Pascal Kattler**
Email: kattler@math.uni-sb.de
Homepage: http://www.math.uni-sb.de/ag/weitze/
Address: AG Weitze-Schmithüsen
FR 6.1 Mathematik
Universität des Saarlandes
D-66041 Saarbrücken

**Sergio Siccha**
Email: siccha@mathb.rwth-aachen.de
Homepage: https://www.mathb.rwth-aachen.de/Mitarbeiter/siccha.php
Address: Lehrstuhl B für Mathematik RWTH - Aachen
Pontdriesch 10-16
52062 Aachen
Germany

**Andrea Thevis**
Email: thevis@math.uni-sb.de
Homepage: http://www.math.uni-sb.de/ag/weitze/
Address: AG Weitze-Schmithüsen
FR 6.1 Mathematik
Universität des Saarlandes
D-66041 Saarbrücken

# Contents

# Chapter 1

# The Origami object

## 1.1 The action on the Origami

### 1.1.1 ActionOfSl

▷ `ActionOfSl(word, Origami)` (function)

**Returns:** the Origami Object word.Origami

This lets act a word in the free group $Group(S,T)$ ,representing an element of $Sl_2(\mathbb{Z})$ on an Origami and returns $word.Origami$.

### 1.1.2 ActionOfF2ViaCanonical

▷ `ActionOfF2ViaCanonical(word, Origami)` (function)

**Returns:** the Origami Object word.Origami

This lets act a word in the free group $Group(S,T)$, representing an element of $Sl_2(\mathbb{Z})$, on an Origami and returns word.Origami. But in contrast to "ActionOfSl" the result is stored in the canonical representation.

### 1.1.3 RightActionOfF2ViaCanonical

▷ `RightActionOfF2ViaCanonical(word, Origami)` (function)

**Returns:** the Origami Object Origami.word

This lets act a word in the free group $Group(S,T)$ on an Origami from right and returns $Origami.word = word^{-}1.Origami$, where the left action is the common action of $Sl_2(\mathbb{Z})$ on 2 mannifolds. This action has the same Veechgroup and orbits as the left action. In contrast to "ActionOfSl" the result is stored in the canonical representation.

## 1.2 The Origami object

### 1.2.1 CanonicalOrigami

▷ `CanonicalOrigami(Origami)` (function)

**Returns:** An Origami

This calculates a canonical representation of an origami, represented as record rec(d := * , x := *, y := *). Two origamis are equal if they are described by the same permutations in their canonical representation.

### 1.2.2 VerticalPerm (for IsOrigami)

▷ `VerticalPerm(Origami)` (attribute)

**Returns:** a permutation

This returns the horizontal permutation $\sigma_x$ of the Origami.

### 1.2.3 HorizontalPerm (for IsOrigami)

▷ `HorizontalPerm(Origami)` (attribute)

**Returns:** a permutation

This returns the vertical permutation $\sigma_y$ of the Origami.

### 1.2.4 DegreeOrigami (for IsOrigami)

▷ `DegreeOrigami(Origami)` (attribute)

**Returns:** an integer

This returns the degree of an Origami.

### 1.2.5 Stratum (for IsOrigami)

▷ `Stratum(Origami)` (attribute)

**Returns:** a list of integers

This calculates the stratum of an Origami. That is a list of the orders of the singularities.

### 1.2.6 VeechGroup (for IsOrigami)

▷ `VeechGroup(Origami)` (attribute)

**Returns:** a ModularSubgroup object

This calculates the Veechgroup of an Origami. This is a subgroup of $Sl_2(\mathbb{Z})$ of finite degree. The group is stored as ModularSubgroup from the ModularSubgroup package. The Veechgroup is represented as the coset permutations $\sigma_S$ and $\sigma_T$ with respect to the generators $S$ and $T$. This means if $i$ is the integer associated to the rigth coset $G$ (Cosets( O ) [ i ] VeechGroup = H) then we have for the coset H, associated to $\sigma_S(i)$, that $SG = H$. Dito for $\sigma_T$.

### 1.2.7 Cosets (for IsOrigami)

▷ `Cosets(Origami)` (attribute)

**Returns:** a list of words in the Free group, generated by $S$ and $T$.

This Calculates the right cosets of the Veechgroup of an Origami.

### 1.2.8 Equals

▷ `Equals(Origami1, Origami2)` (function)

**Returns:** true or false

This tests wether Origami1 is equal to Origami2 up to numbering of the squares.

### 1.2.9 ExampleOrigami

▷ `ExampleOrigami(d)` (function)

**Returns:** a random origami

This creates a random origami of degree d.

### 1.2.10 CalcVeechGroup

▷ `CalcVeechGroup(Origami)` (function)

**Returns:** A list with tree entrys

This function is used to calculate some attributes. It calculates the Veechgroup of a given origami and . the veechgroup is stored as ModularGroup Object from the ModularGroup package. The cosets of the veechgroup is stored in a list of words in the generators S and T of the matrix group Sl_2(Z).

### 1.2.11 CalcStratum

▷ `CalcStratum(Origami)` (function)

**Returns:** nothing

Calculates the stratum of an object and sets its attribute. The stratum is stored as list of integers.

### 1.2.12 ToRec

▷ `ToRec(Origami)` (function)

**Returns:** record of the form rec(d := * , x := *, y := *) Describtion This calculates a record representation for an origami object.

# Chapter 2

# Introduction

This package provides calculations with Origamis. An Origami can be obtained in the following way from two permutations $\sigma_a, \sigma_b \in S_d$. We take $d$ Squares $Q_1, \ldots, Q_d$ and clue the lower side of $Q_i$ to the upper side of $Q_{\sigma_y(i)}$ and the right side of $Q_i$ to the left side of $Q_{\sigma_x(i)}$. So in this Package we identify an Origami with a pair of permutations, witch acts transitive on $\{1 \ldots d\}$ up to simultan conjugation. We store an Origami as Origami object. We are mainly interested in the Veechgroup of an Origami. It can be shown that the Veechgroup of an Origami is a subgroup of $Sl_2(\mathbb{Z})$ of finite index. So we fix two generators

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

and

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

## 2.1 The Free Group

In this package we fix the Free Group $F$ generated by $S$ and $T$.

# Index