Matboard Design Report

# Introduction:

For the matboard bridge design, a bridge with the length between 1250 and 1270 mm is to be built using only a sheet of matboard and contact cement. The bridge will be subject to a moving load of 400 N from a train moving from the beginning to the end of the bridgespan. This report first explains the various design iterations that led to the final design choice, along with reasoning behind the choices and the resulting failure load of these choices. The report then provides a detailed description of the construction process of the bridge.

# Major Design Iterations:

## Design 1:

For the first design:

- Two side walls are 75 mm apart, and are glued to the top flange by a fold outwards that meets the top flange's edge. Between the walls, an additional board is fitted there. Together with the side wall's glue connection folds, this layer acts as doubling the thickness of the top flange.
- The top flange has a width of 100 mm.
- Diaphragms are placed 50 mm apart near the edges, then 125mm apart, then 150 mm apart twice, with a 300 mm gap in the middle of the board.
- The side walls are 100 mm tall initially at the supports, and gradually increase the height to 200 mm at 400 mm from the support, then decrease in height after 800 mm from the support back to a height of 100 mm.
- This design does not have a bottom board.

The major design choices for this first design are: the changing cross section that gets lower (with the top flange at the same height level, the height of cross section increases) near the middle, forming a shape similar to a trapezoid below a rectangle; the side walls' folds and the additional board effectively forms a double-layered top flange; and the absence of a bottom board. These design choices were made with the intention to prevent failure of the bridge by compressive flexural stress near the top of the bridge, since the material's compressive strength is significantly lower than its tensile strength. Having a thick top flange and no bottom board will have the effect of raising the height of the centroidal axis and decreasing the distance between top flange and the centroidal axis, therefore reducing the flexural stress experienced by the top flange since $\sigma = \frac{My}{I}$. The varying cross section was also made for this purpose, since increasing the height of the bridge can significantly increase the second moment of inertia (by power of 2 or 3) while having a less significant increase of the "y" value in $\sigma = \frac{My}{I}$. This was done by gradually increasing the height from a relatively short support because the maximum bending moment occurs near the middle of the bridge.

The following is the graphs generated for the different types of failure for this design.

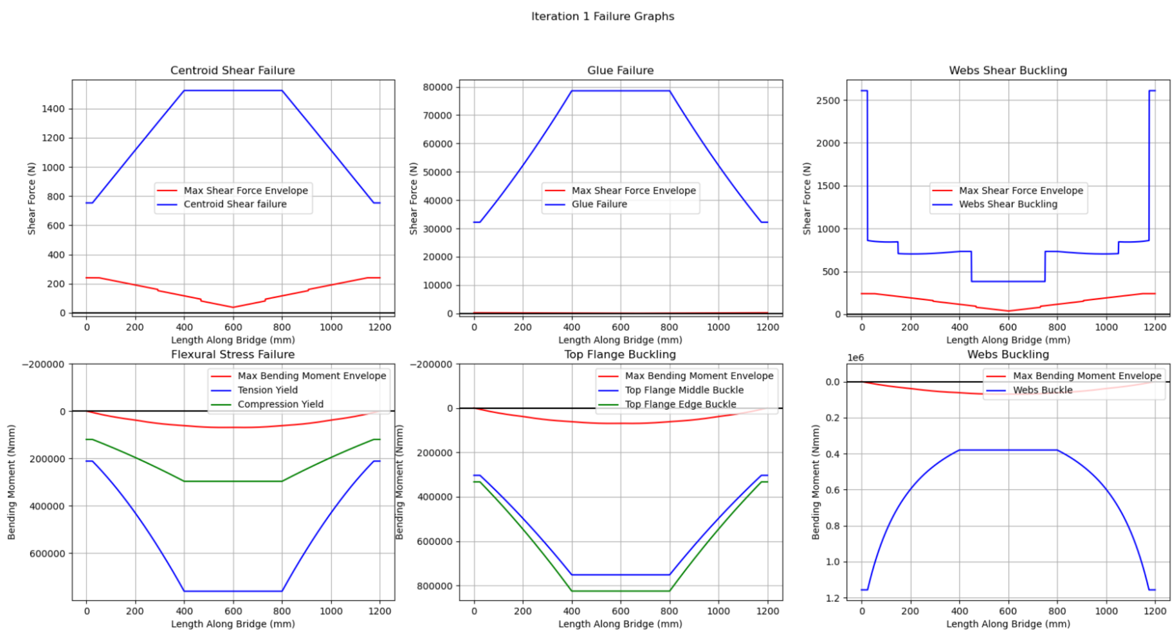



**Figure 1:** Failure Graphs For Design 1

The factor of safety (FOS) for each mode of failure is:

**Table 1:** Factors of Safety for each failure mode for Design 1

| Mode of failure | FOS | Location of failure as a distance along the bridge from the left support (mm) |
|---|---|---|
| Board Shear Failure | 3.14 | 0 |
| Glue Failure | 134 | 0 |
| Shear Buckling | 3.41 | 150 |
| Board Tension Failure | 10.87 | 942 |
| Board Compression Failure | 4.27 | 556 |
| Top Flange Middle Buckling | 10.82 | 556 |
| Top Flange Side Buckling | 11.88 | 556 |
| Webs Buckling | 5.47 | 556 |

The minimum factor of safety is 3.14 from board shear failure, meaning that the theoretical maximum train load this design can take is a train of 1256 N. This mode of failure means that the bridge's material will fail at the centroidal axis due to the shear force being high near the supports. At the same time, the compressive failure is relatively high near the centre of the bridge, showing that having a high centre span of the bridge does increase the factor of safety against compressive failure.

**Design 2:**

From design 1, since the bridge failed due to shear near the edges, the main changes to this design is that the initial height of the design has been increased from 100 mm to 150 mm. Reason behind this change is that shear stress $\tau = \frac{VQ}{Ib}$ depends on three main cross-sectional properties: the width of the board, Q, and second moment of inertia.

The solution chosen is to increase the second moment of inertia by increasing the height of the board, because I varies cubically (for the sideboards' moment of inertia) and quadratically (for the top flange's moment of inertia) with increases to side board height, while Q increases only quadratically with increases to side boards' height. Therefore increasing the height can reduce shear stress and thus improve the factor of safety.

For this design:

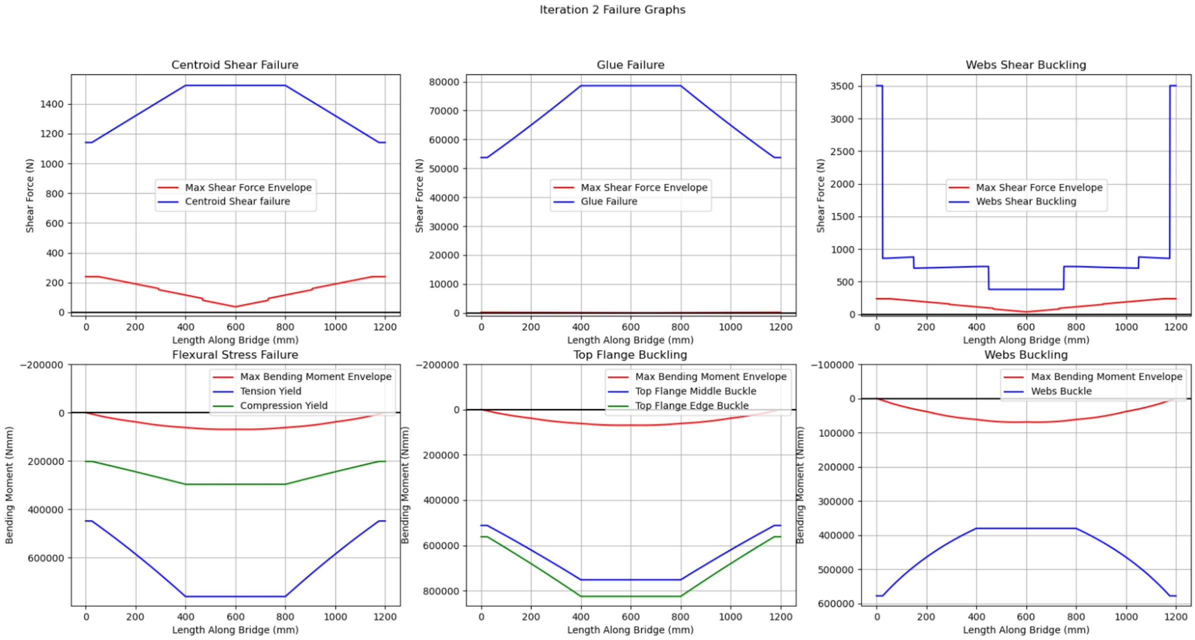



**Figure 2:** Failure Load Graphs of Design 2

The factor of safety (FOS) for each mode of failure is:

**Table 2:** The Factors of Safety For Each Mode of Failure For Design 2

| Mode of failure | FOS | Location of failure as a distance along the bridge from the left support (mm) |
|---|---|---|
| Board Shear Failure | 4.75 | 0 |
| Glue Failure | 223 | 0 |
| Shear Buckling | 3.41 | 150 |
| Board Tension Failure | 10.95 | 556 |
| Board Compression Failure | 4.27 | 556 |
| Top Flange Middle Buckling | 10.82 | 556 |
| Top Flange Side Buckling | 11.88 | 556 |
| Webs Buckling | 5.47 | 556 |

The minimum factor of safety is 3.41 from board shear buckling, meaning that the theoretical maximum train load this design can take is a train of 1364 N. The shear failure has significantly changed from the previous design and is no longer the limiting factor of the bridge. Currently, the failure is caused by shear buckling near the edge of the bridge because the spacing between the diaphragms has changed from 125 mm to 150 mm, therefore increasing diaphragm spacing at that location has resulted in shear buckling near the edge of the bridge as this is where the highest shear force exists.

**Design 3:**

Since increasing the height of the side board improved the factor of safety against shear failure, this next iteration further increased the height of the bridge near the edges as an attempt to reduce shear stress and improve the factor of safety for shear buckling failure. At the same time, since the board failing by tension or compression in the middle has a relatively high factor of safety of 10.95 and 4.27, the height of the bridge in the middle span is reduced to save material.

The final choice made for this iteration has a uniform sidewall all along the bridge with a height of 180 mm.

At the same time, as an attempt to further save material, the additional layer of board added below the top flange has changed to only include a 900 mm long addition at the centre of the board, since the edge of the bridge near the supports does not experience high flexural stress and does not require that much thickness at top board.

For this design:

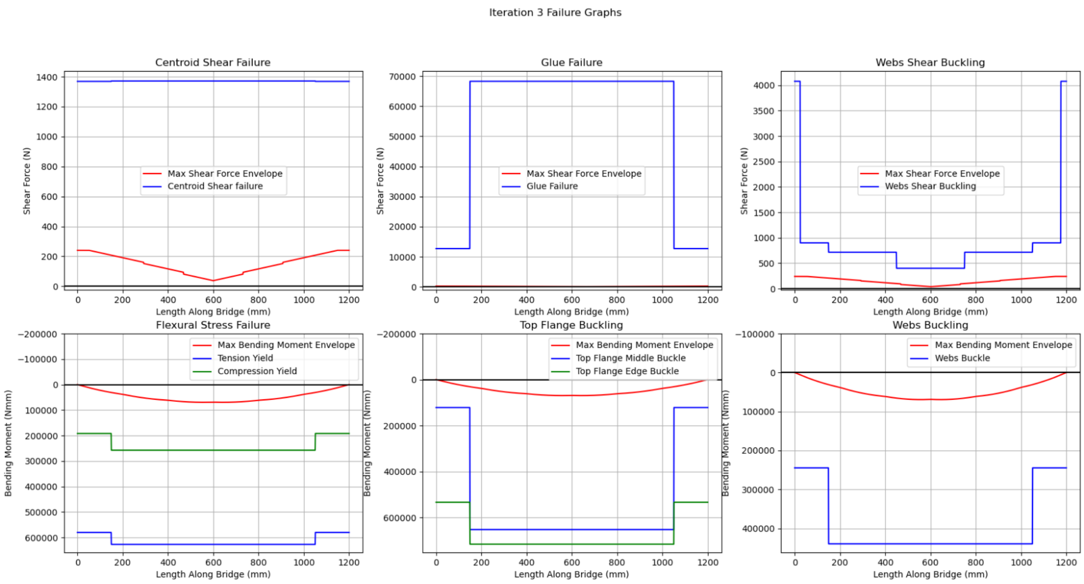



**Figure 3:** Failure Loads of Design 3

The factor of safety (FOS) for each mode of failure is:

**Table 3:** The Factors of Safety For Each Mode of Failure For Design 3

| Mode of failure | FOS | Location of failure as a distance along the bridge from the left support (mm) |
|---|---|---|
| Board Shear Failure | 5.70 | 0 |
| Glue Failure | 52.9 | 0 |
| Shear Buckling | 3.45 | 150 |
| Board Tension Failure | 9.03 | 556 |
| Board Compression Failure | 3.70 | 556 |
| Top Flange Middle Buckling | 3.92 | 1051 |
| Top Flange Side Buckling | 10.30 | 556 |
| Webs Buckling | 6.33 | 556 |

The minimum factor of safety is 3.45 from board shear buckling, meaning that the theoretical maximum train load this design can take is a train of 1380 N. As a result of choosing a constant height for the bridge, the ability of the bridge to resist shear is further improved, both against shear failure and shear buckling, likely due to the further increase of the second moment of inertia at the edge. On the other hand, the bridge's ability to resist compressive and tensile failure at the middle was decreased because the height at the centre of the bridge was decreased, causing a reduction in second moment of inertia and increasing the flexural stress at the middle. As a result of increased flexural stress, the bridge's top flange local buckling factor of safety also decreased slightly. However, flexural stresses are still not the limiting factor, with shear buckling still being the smallest factor of safety, but coming closer to the board's compression failure. The next step would be to try to increase the bridge's resistance against shear buckling to the bridge's compression failure capacity.

**Design 4:**

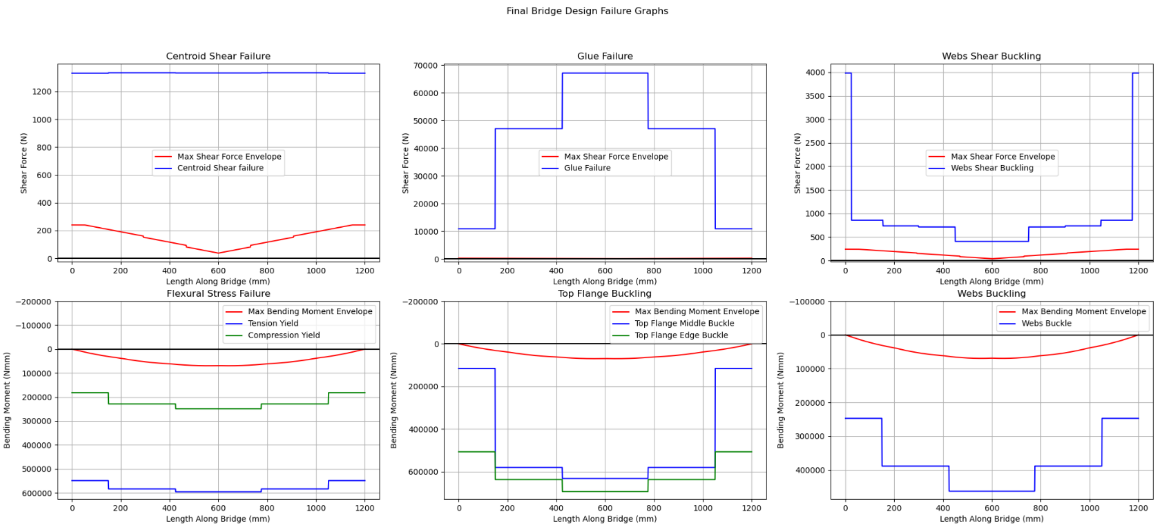
For this design iteration, more emphasis was placed onto the material constraints, since the previous design has proven to be difficult to arrange and cut on the piece of given matboard when that was attempted. Therefore, as an effort to save materials, the height of the bridge was shortened from 180 mm to 175 mm, and the glue connection between sideboards and top flange now folds inwards to save materials for the additional layer below the top flange, since this way the glue fold itself saves board that originally needed to be arranged for that additional layer board. However, this causes the bridge to fail due to compression near the middle since the additional layer is now a bit thinner and slightly reduced the moment of inertia near the centre. To resolve this, two thin pieces of board each with width 12 mm are glued to the side of the sideboard's glue joint to increase moment of inertia at the middle of the bridge. Additionally, the locations of the diaphragms are readjusted as an attempt to increase the factor of safety against shear buckling,

Additional bridge failure modes were also considered for this iteration:

- For failure from compression forces at the supports, calculations of the walls and diaphragms failing due to local buckling were done, and the current design's walls at the support can take a minimum of 4.07 MPa of compressive stress. This stress multiplied by the area of board in contact with the support shows that the supports can take 1299 N of compressive force, which is not a limiting factor for this design. Also, to ensure that the force is evenly distributed across all the area at the support, a bottom piece was added to the supports to prevent any uneven height of board during the construction process.

- For splice connections, a piece of 70 mm by 58 mm board is stuck across the connection at the bottom because for this connection, the limiting factor comes from tensile flexural stress pulling the boards apart, which is located at the bottom of the bridge. Another board that is 70 mm by 20 mm was glued at the midpoint between the bottom splice connector and the top of the bridge for additional security.

For this final design:




**<u>Figure 4:</u>** Failure Graphs For Final Design Iteration

The factor of safety (FOS) for each mode of failure is:

**<u>Table 4:</u>** Factors of Safety For Each Failure Mode For Final Design

| Mode of failure | FOS | Location of failure as a distance along the bridge from the left support (mm) |
|---|---|---|
| Board Shear Failure | 5.55 | 0 |
| Glue Failure | 45.2 | 0 |
| Shear Buckling | 3.56 | 25 |
| Board Tension Failure | 8.58 | 556 |
| Board Compression Failure | 3.59 | 556 |
| Top Flange Middle Buckling | 3.73 | 1051 |
| Top Flange Side Buckling | 9.97 | 556 |
| Webs Buckling | 6.10 | 424 |

The minimum factor of safety is 3.56 from board shear buckling, meaning that the theoretical maximum train load this design can take is a train of 1424 N. At this point, the bridge should have been pushed to the limit because now the bridge's factor of safety for two completely different types of failure at two far apart sections of the bridge are very similar: shear buckling 25 mm from the support with factor of safety 3.56, and compression failure 556 mm from the support with factor of safety 3.59. Also, by arranging the required materials on the matboard, there is almost no more space left for any addition to the bridge simply due to material constraints. The arrangement of the board can be seen in the following figure:

<u>**The Construction Instructions and Photo Evidence:**</u>

To build the bridge, the following process was followed:

1. The bridge's boards are drawn on digital software indicating where the given piece of mat board should be cut, along with where the boards should be folded. This allows the construction process to proceed without the need for further consideration during the construction. The following figure shows the cuts being arranged on the piece of matboard.

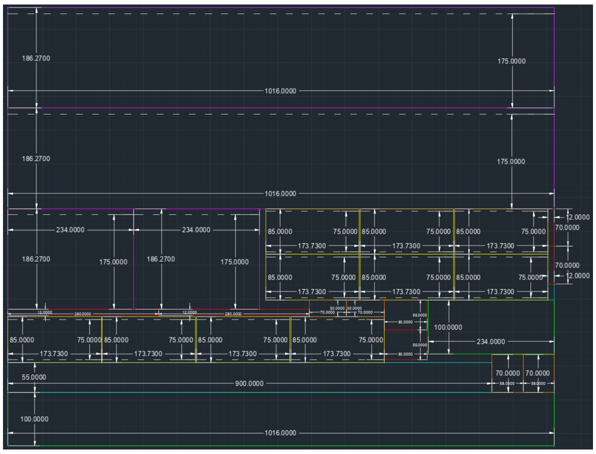


*Figure 1, 2. AutoCad Drawing of how to divide the matboard*

2. Following the cutting lines drawn on software, lines are carefully measured, drawn onto the board, and labelled. This process ensures that any cuts made to the board follows the plan precisely. The following figures show the process of drawing lines and labelling boards, and the piece of board after labelling.

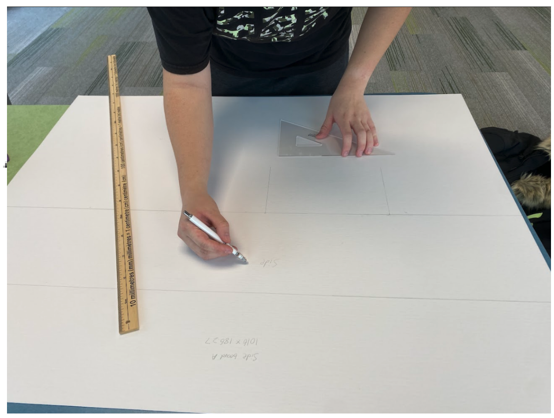
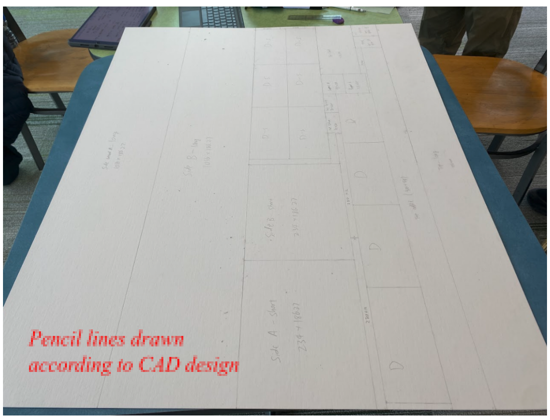




*Figure 3, 4. Drawing and Labelling Lines on the Matboard*

3. After drawing the lines, the board was cut using an Exacto knife along the lines. The following figures show the process of cutting the board, as well as pieces of the boards after being cut.

*Figure 5, 6. Cutting the board*

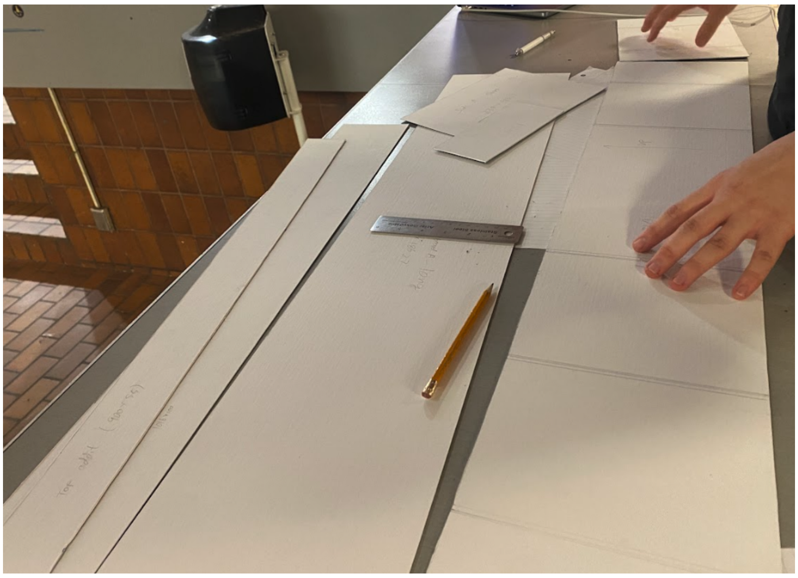


*Figure 7, 8. Board pieces after cutting*

4. The next step was to draw lines on the sideboard pieces indicating the locations where diaphragms will be glued. This is to facilitate the process of glueing the diaphragms

and ensure that the diaphragms are glued to the precise locations. The following figure shows the side boards with lines marked on them.

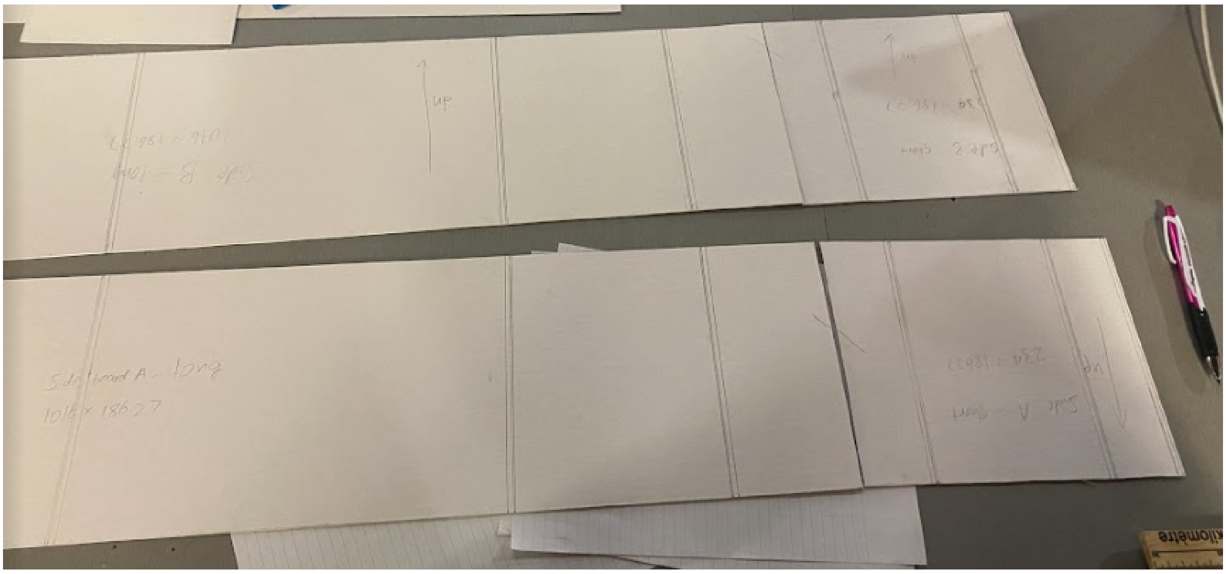
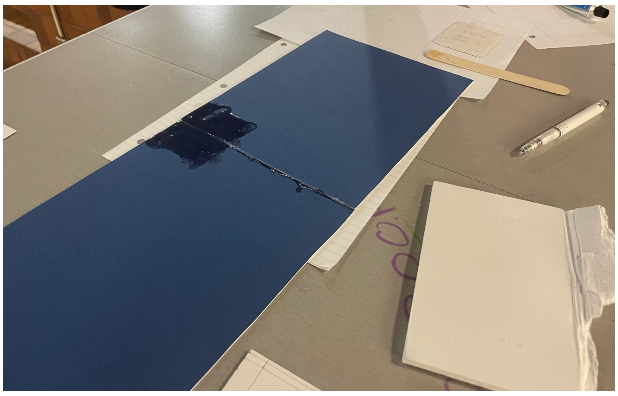


*Figure 9. Side boards with lines marked on them*

5. Then, the side boards are glued together along with the splice connectors. The side boards are first pushed into each other to weakly bond to each other, then the position of the splice connectors are traced and then glued onto the sideboards. The following figures show the process of glueing on the splice connectors.



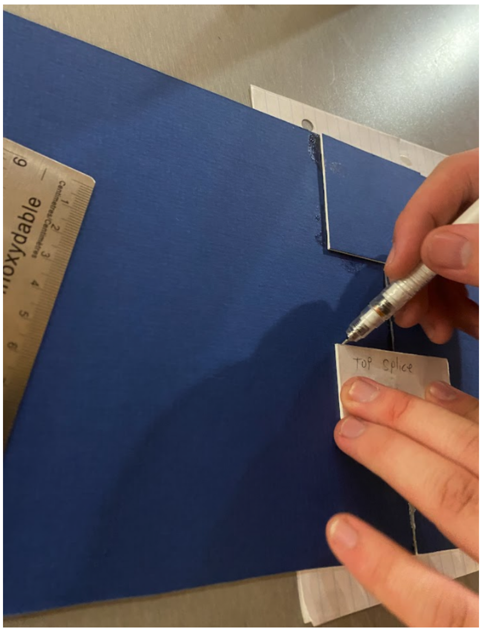



*Figure 10, 11. How the Splice Connectors were Applied to the Side Board*

6. The diaphragms are then folded to produce a surface for the glue to be applied on. The figures show the process of drawing lines on diaphragms to indicate the location of fold, and the folded diaphragms.

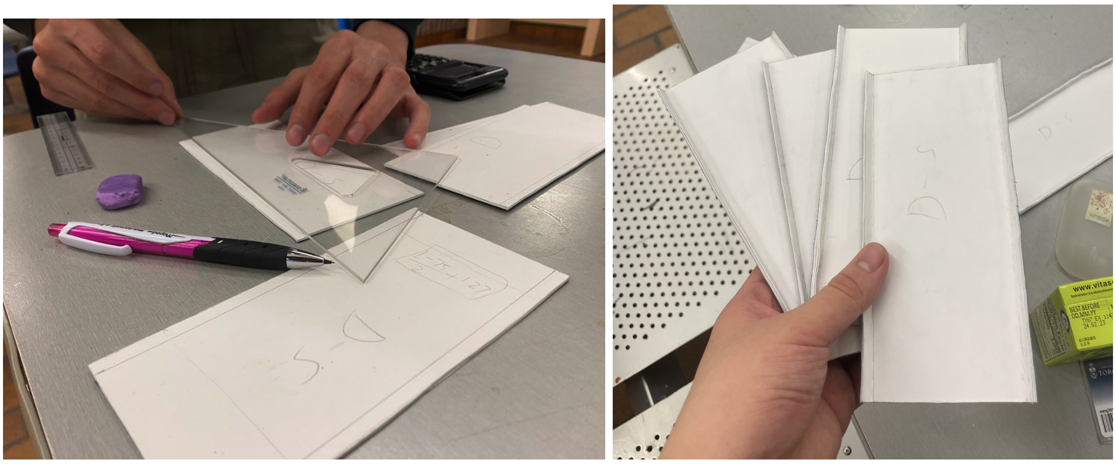


*Figure 12, 13. The Process of Creasing and Folding the Diaphragms*

7. Afterwards, the sidewalls are folded where the glue connection with the top flange will be, and the diaphragms are glued onto the side walls one by one. The figures show the process of glueing on the diaphragms and the final sideboard with all diaphragms glued onto it.

*Figure 14, 15. Glueing the Diaphragms to the Side Board*

8. The other side board is then glued onto the diaphragms, connecting both side boards together. The figures show the glue applied to the second sideboard, and the half-finished product after this step.

*Figure 16, 17. How The Glue was Applied to the Sideboard and the Final Product*

9. The additional piece of board below the top flange near the middle of the board was glued onto the diaphragms between the sideboards' glue connection folds.
10. Glue was then applied to the sideboards' glue connection folds and the board from the previous step to glue the top flange to the bridge's body. The Figures below show the application of glue and the bridge being flipped upside down to press on the glue connection for the top flange.

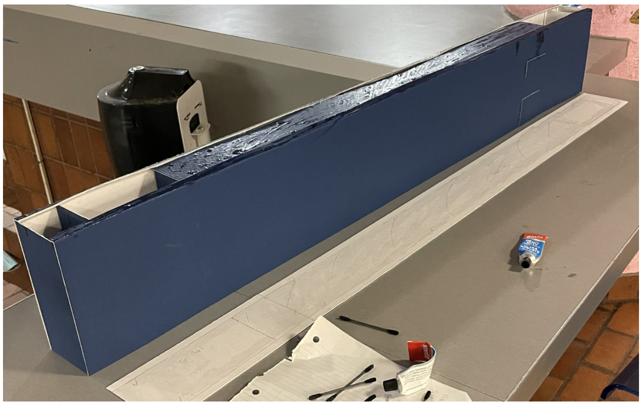



*Figure 18, 19. The Process of Attaching the Sideboard*

11. Since the diaphragms were not perfectly aligned with the bottom of the bridge, small pieces of leftover mat board are added to the diaphragm near the support location to make sure that section is aligned, so the compressive force at supports can be evenly distributed amongst the matboards. The Figures 20 and 21 show the small pieces of boards added to the second diaphragms from both sides.

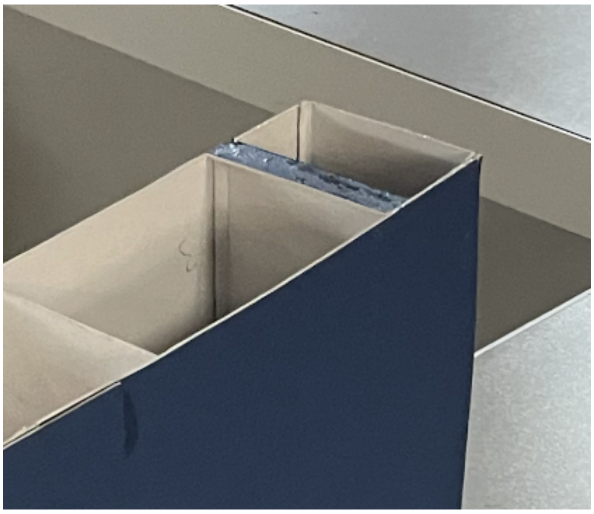



*Figure 20, 21. The Additional Layer Added onto the Diaphragms*

12. Bottom boards are then glued to the supports. Figure 22 shows the bridge after both bottom boards are glued.

*Figure 22. The Addition of the Bottom Boards*

13. Two thin strips of matboard were glued to the outer edge of the top flange at the middle of the bridge. The Figures below show the pieces after the pieces were glued on.

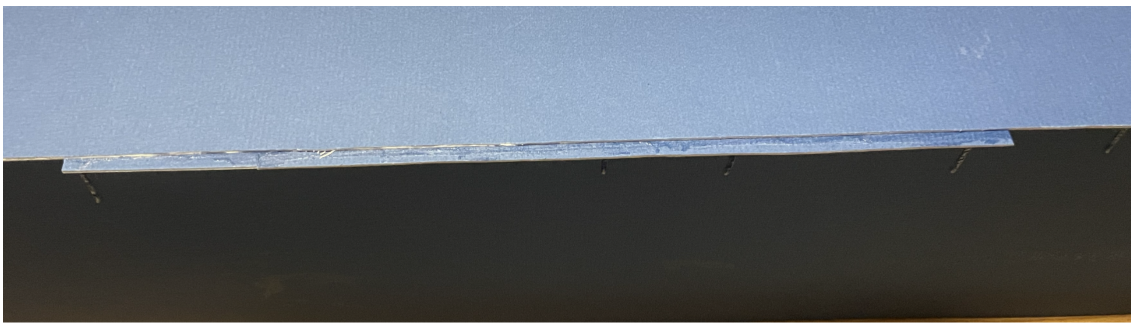


*Figure 23. Thin Pieces Added to the Flange*

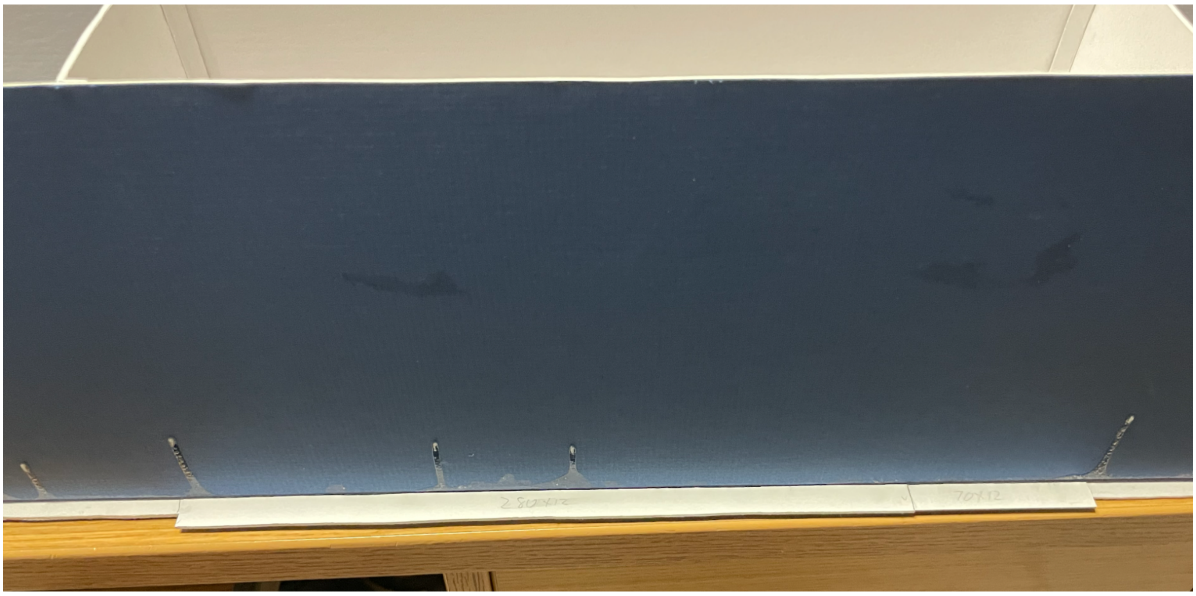


*Figure 24 The Flanges On the Outer Edge of the Board*

After the bridge construction, there are only small pieces of leftover matboards. The orientation and size of these pieces makes it hard to find a place to incorporate them in a way that helps increase the factor of safety by a noticeable degree.

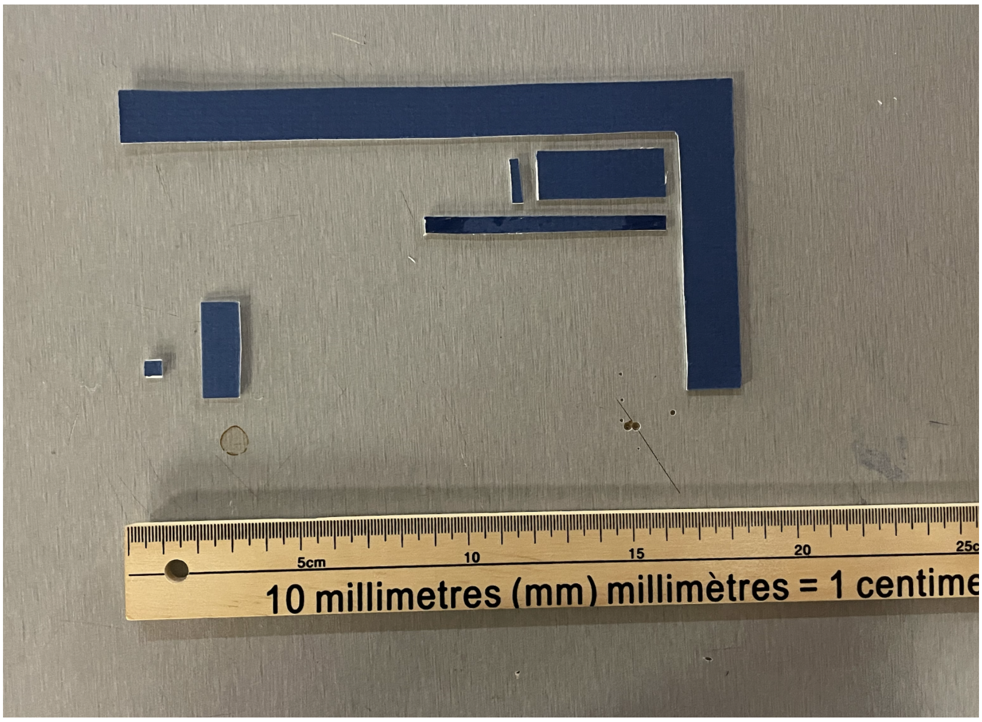



*Figure 25. Pieces of Leftover Matboard*