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# Applications of Machine Learning to Mechanical Systems

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**ICAMS**

INTERDISCIPLINARY CENTRE FOR  
ADVANCED MATERIALS SIMULATION

**RUHR  
UNIVERSITÄT  
BOCHUM**

**RUB**

# Outline

## I. Lecture

1. Machine Learning (ML) methods
2. Applications as surrogate models for indentation
3. Learning microstructure-property relationships

## II. Tutorials

1. ML-Classification
2. ML-Regression

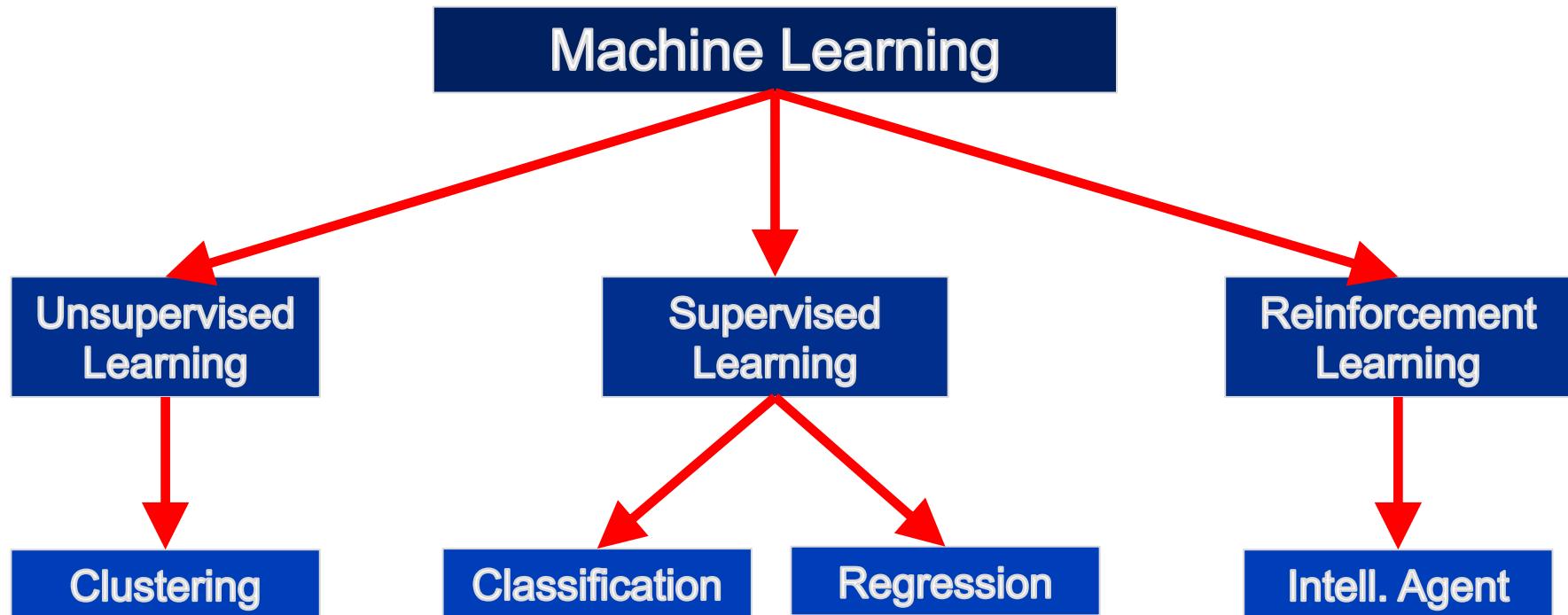
Use tutorials on [MyBinder.org](https://mybinder.org)



Installation from GitLab repository:

<https://gitlab.ruhr-uni-bochum.de/hartmajg/ml-tutorial.git>

# Machine Learning (ML)

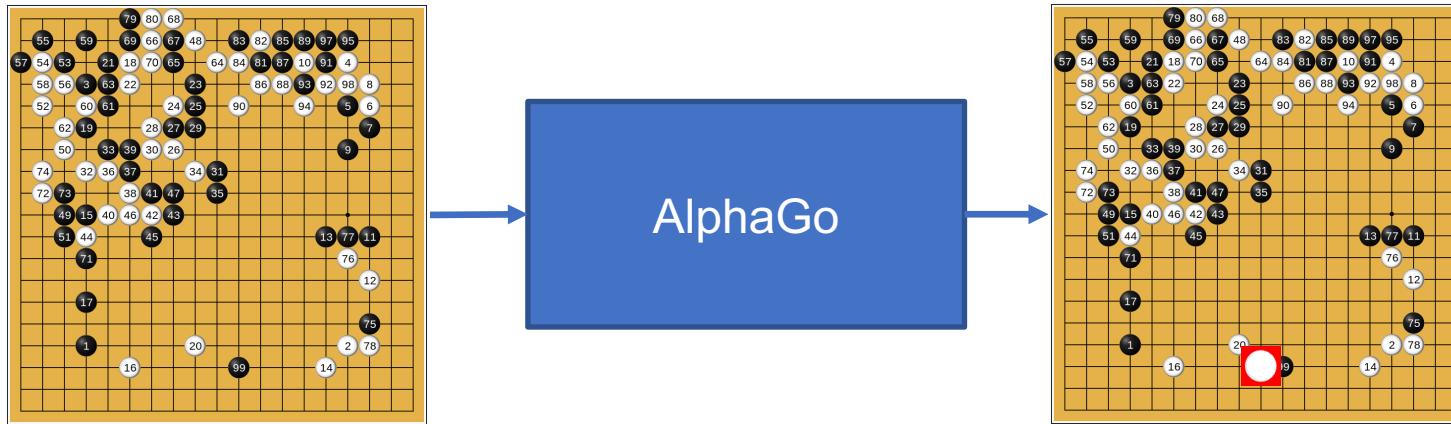


All examples of this lecture have been performed with scikit-learn  
(<https://scikit-learn.org/stable/>)



# Reinforcement Learning: Intelligent Agents

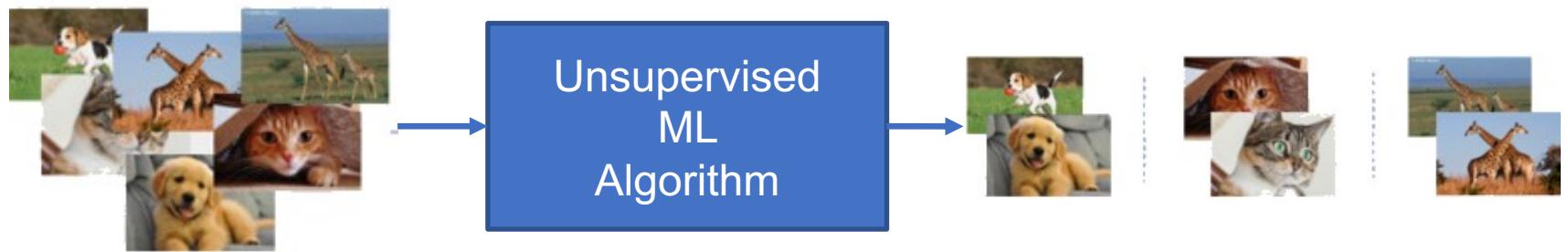
Task: Create a computer code that can play “Go”



Source: Wikipedia  
Wikimedia Commons, CC BY-SA 3.0

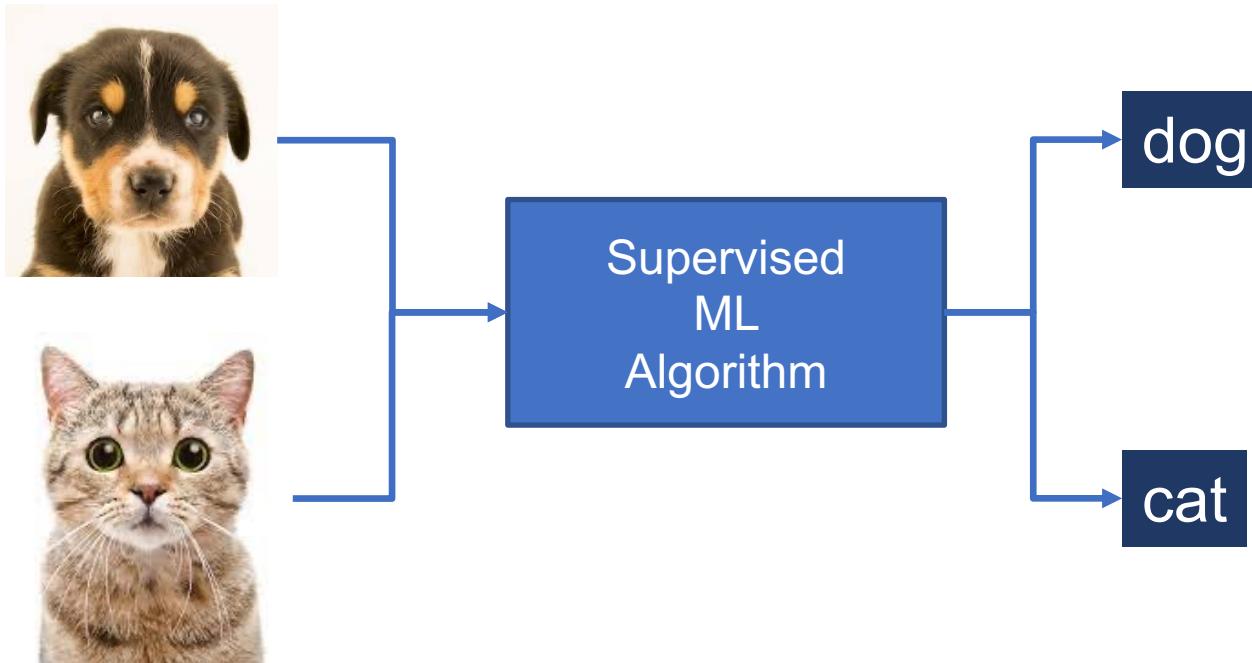
# Unsupervised Learning: Clustering

**Task: Sort pictures of same animals into groups (clustering)**



# Supervised Learning: Classification

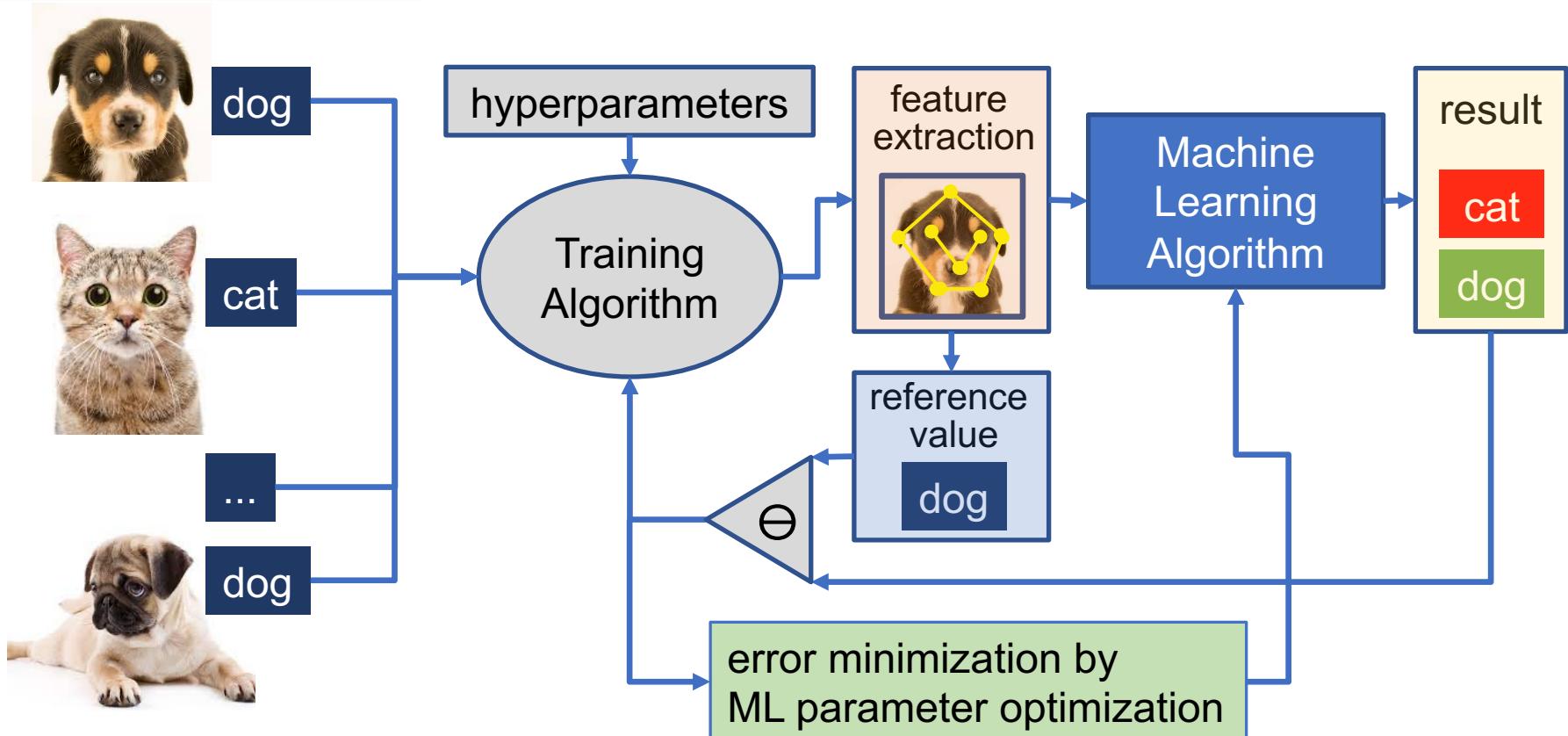
**Task: Identify pictures of cats and dogs (classification)**



# Supervised Learning: Training of ML Model

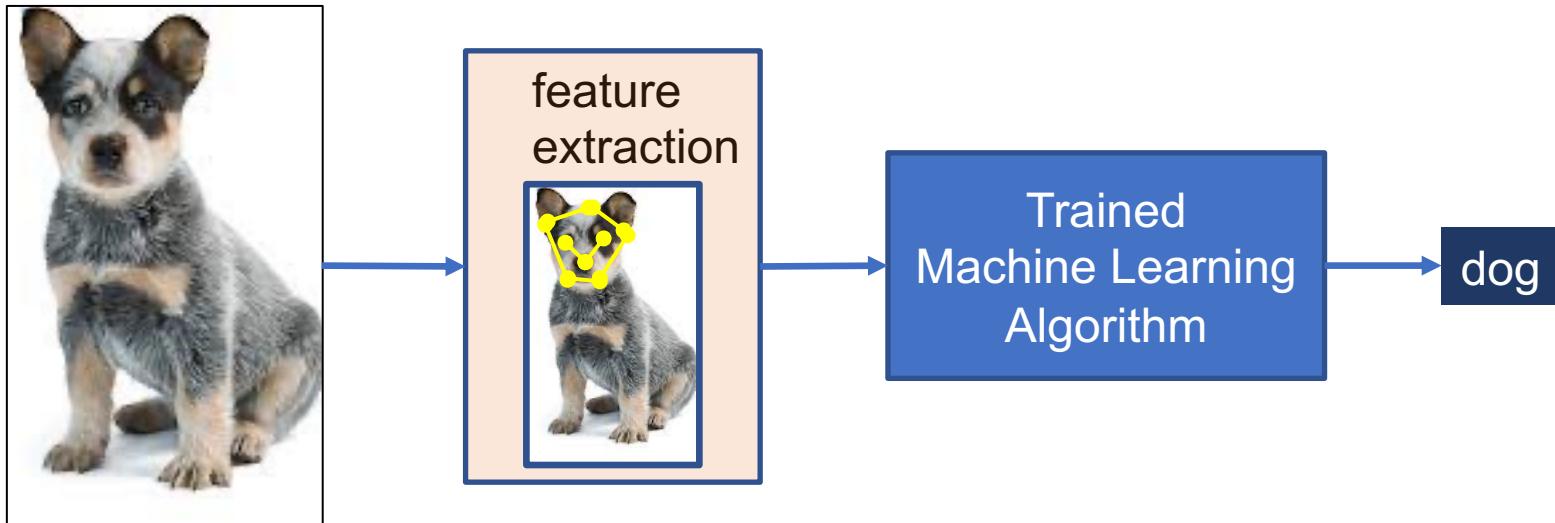
- choice of ML algorithm
- feature design
- hyperparameter optimization

Raw Data with Label



# Supervised Learning: Validation

## Validation with unseen data



# Supervised Learning: Feature Extraction



## Automated feature extraction from raw data:

- convolutions of raw data (CNN)
- autoencoder
- Fast Fourier Transform
- N-point-statistics/auto correlation
- Principle Component Analysis
- Singular Value Decomposition

## Feature extraction based on domain knowledge:

- extraction of physical quantities
- correlation analysis
- experience with similar tasks

# Supervised Learning: Regression

**input vector  
“features”**

Selection of features (or descriptors)  
determines the physics of the ML model

**output vector  
“label” / result**

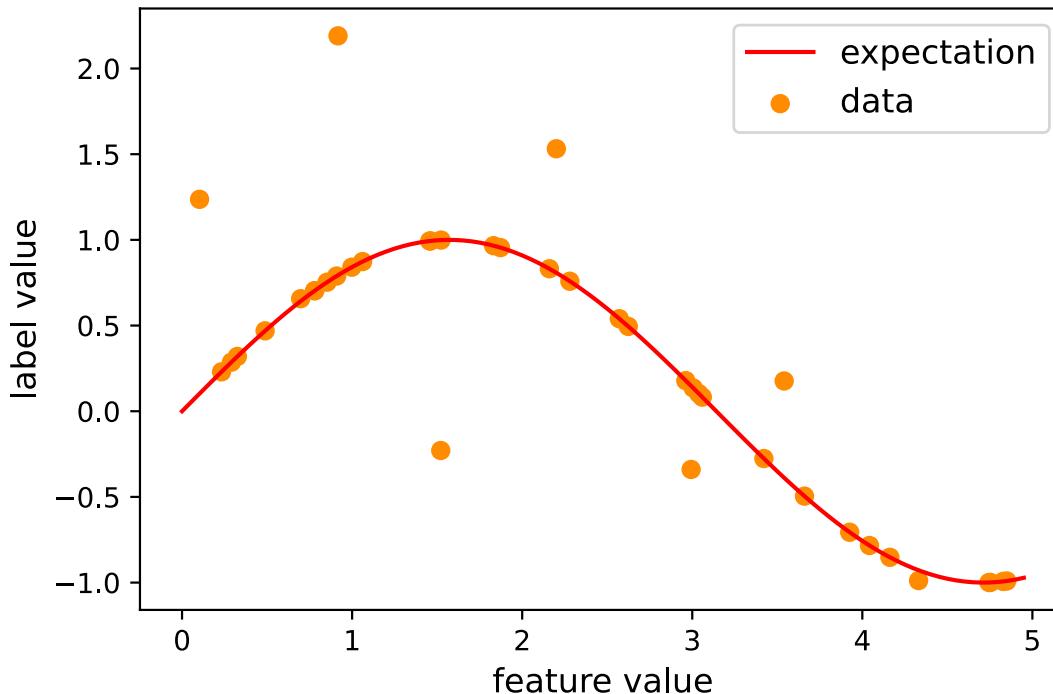


## Training Procedure

Find ML parameters that minimize deviation  
between result of ML model and known data  
point (ground truth).

# Supervised Learning: Regression

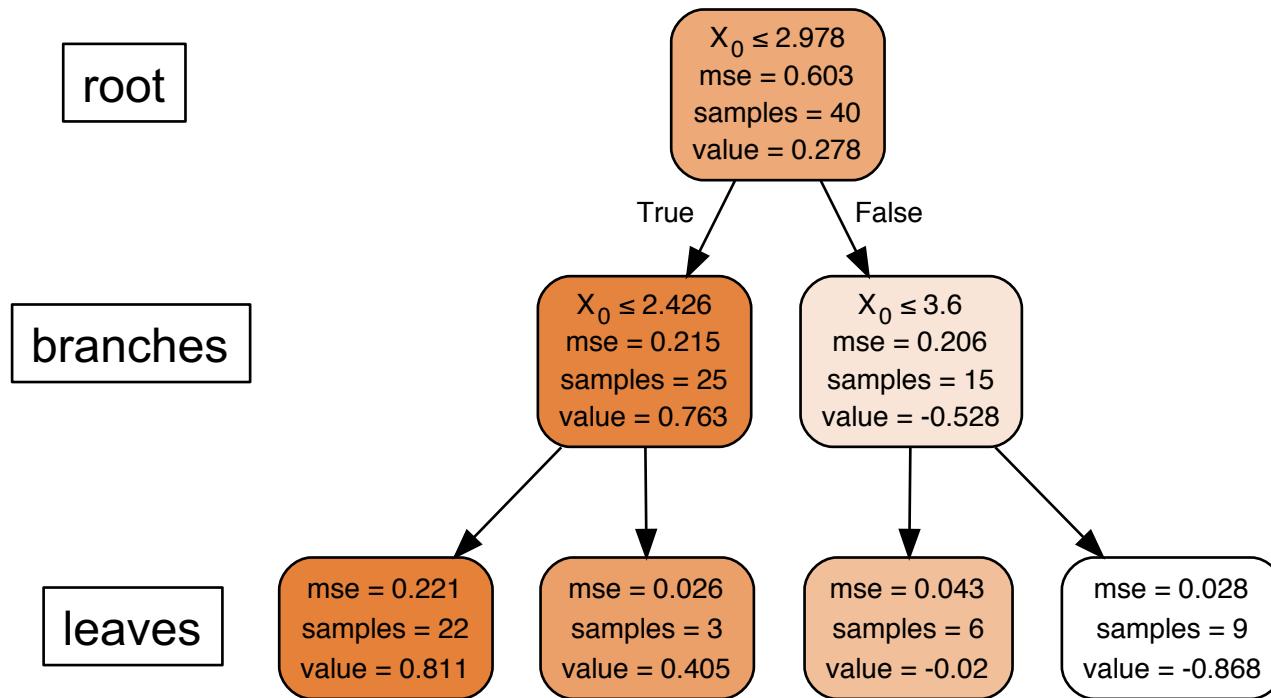
1-d example: noisy sine function



$$y = f(x) = \sin x$$
$$0 \leq x \leq 5$$

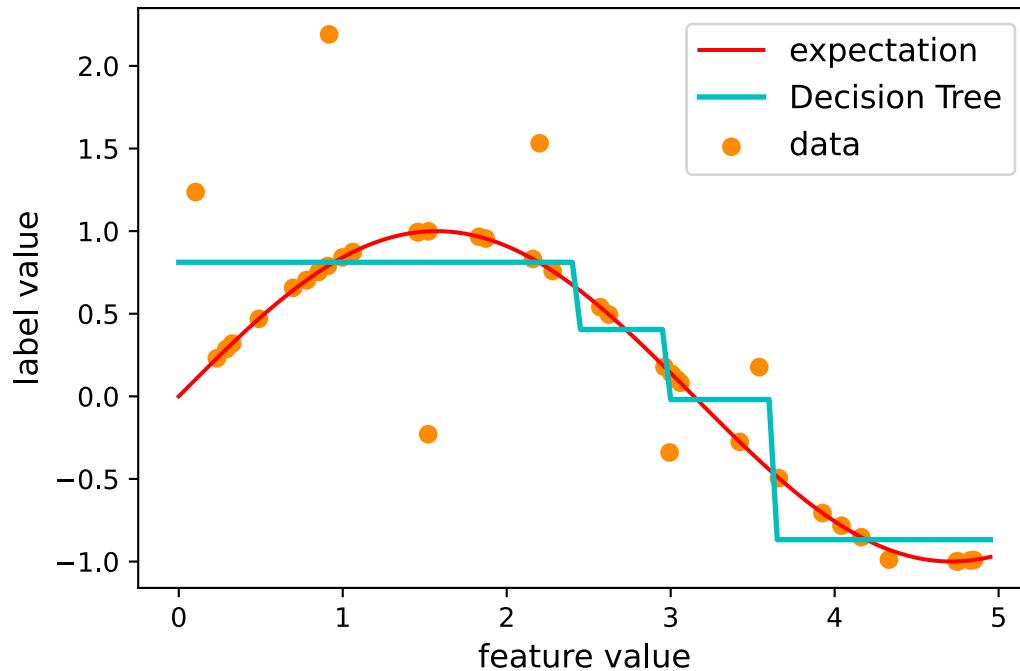
# Decision Tree Regression

Succession of if-clauses leads to final result in “leaves”



# Supervised Learning: Decision Tree Regression

1-d example: noisy sine function

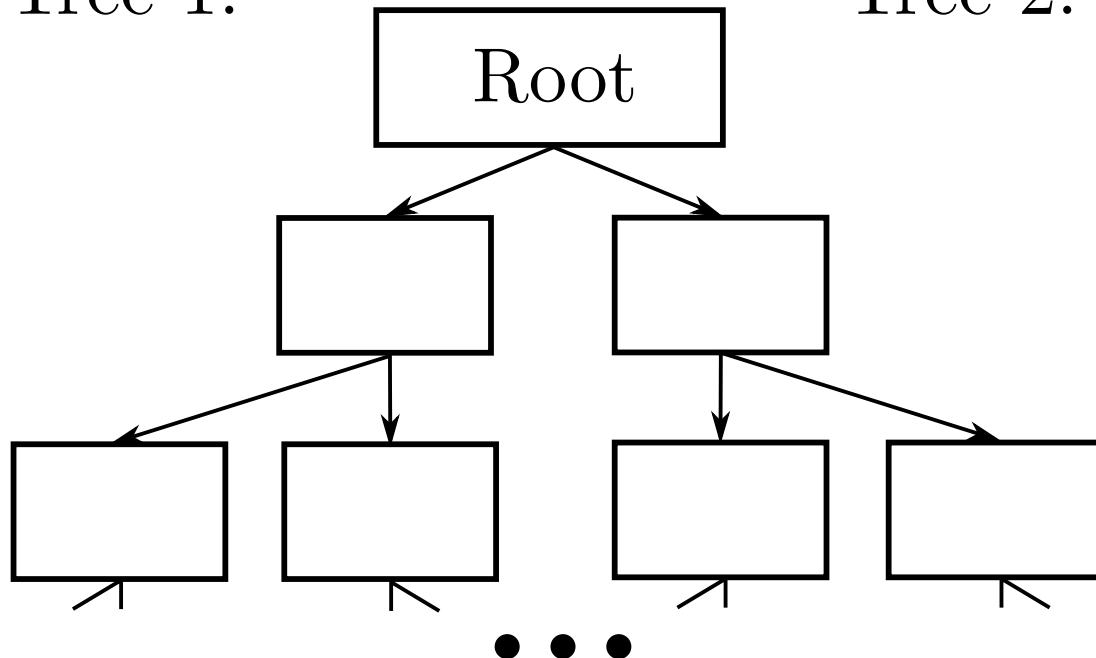


**Decision Tree**  
hyperparameters:  
depth = 2

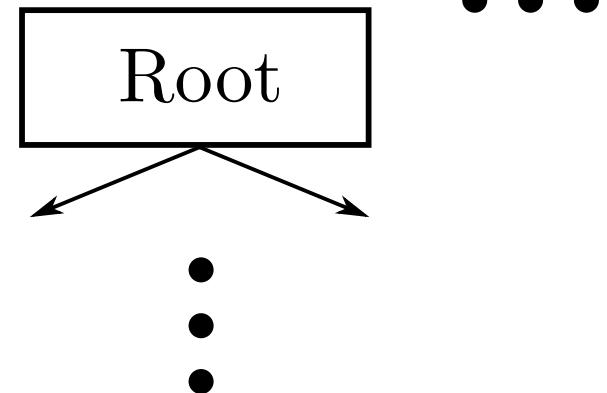
$$y = f(x) = \begin{cases} 0.811 & \text{if } x \leq 2.426 \\ 0.405 & \text{if } 2.426 < x \leq 2.978 \\ -0.02 & \text{if } 2.978 < x \leq 3.6 \\ -0.868 & \text{if } x > 3.6 \end{cases}$$

# Random Forest Regression

Tree 1:



Tree 2:



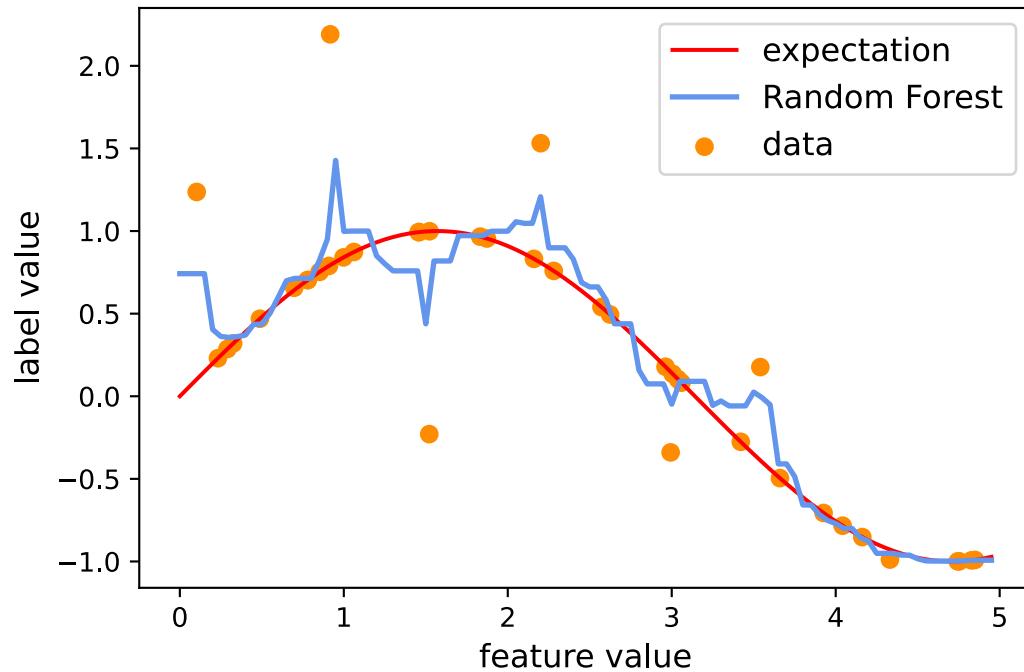
• • •

**Goal:** Create model that predicts output value for given input data by learning simple decision rules

- Number of trees = 100 ... 500
- Leaves contain either 1 or 0 samples
- Final result is average of the leave values obtained from all trees

# Supervised Learning: Random Forest Regression

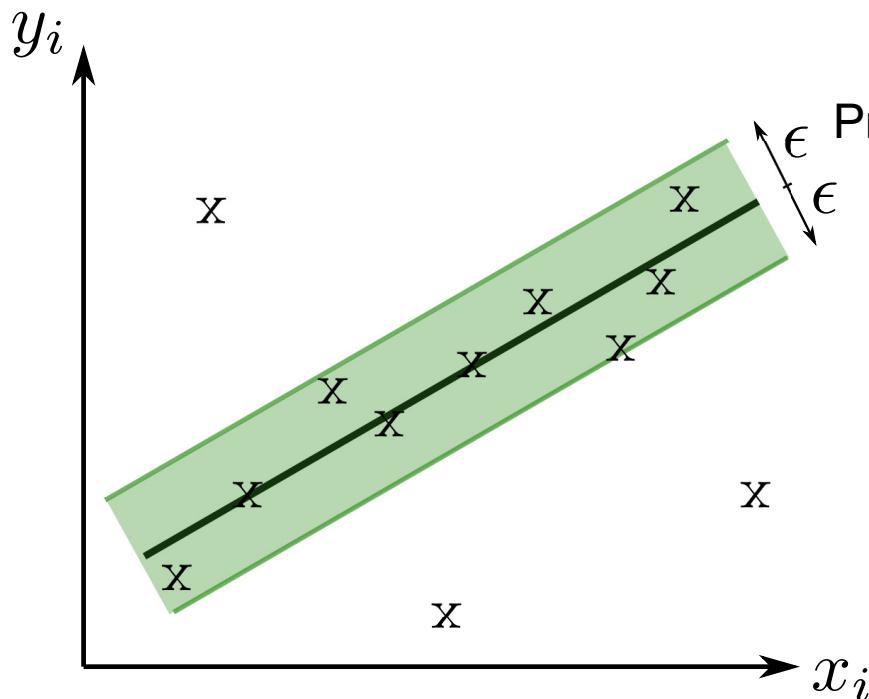
1-d example: noisy sine function



**Random Forest**  
hyperparameters:  
max. depth = 5  
 $N_{\text{tree}} = 20$

$$y = f(x) = \frac{1}{N} \sum_{i=1}^N f_{DT}^{(i)}(x)$$

# Support Vector Machine (Regression/Classification)



Precision  $\epsilon = 1\%$

SVM function:

$$f(x) = \sum_{k=1}^n y_k a_k K(x_k^{(SV)}, x) + \rho$$

RBF kernel:

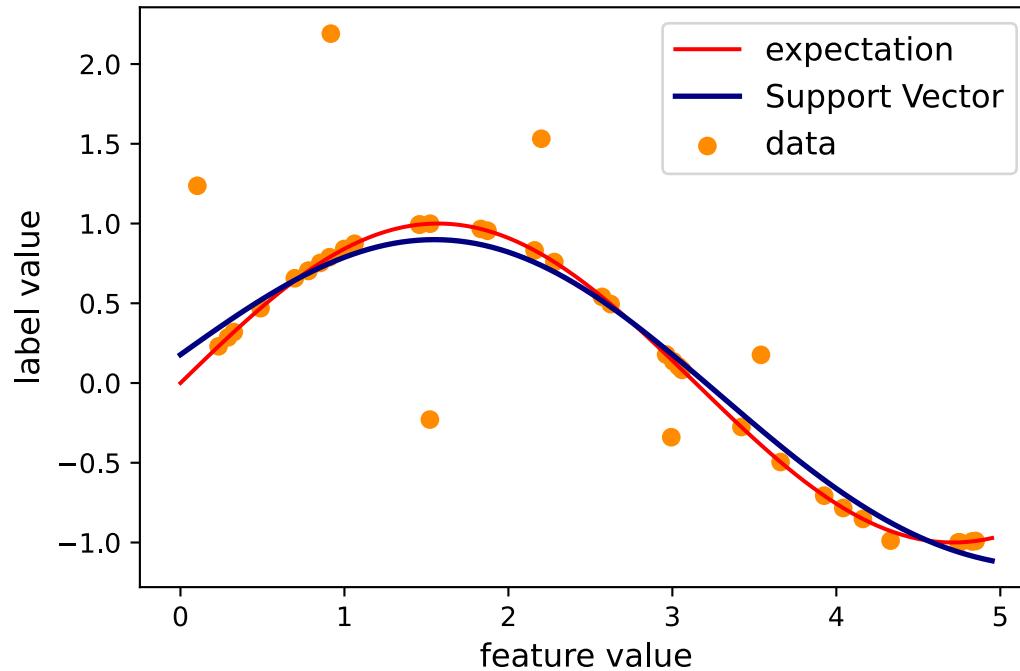
$$K(x_k^{(SV)}, x) = \exp(-\gamma \|x - x_k^{(SV)}\|^2)$$

**Goal:** Find a function such that data points lie within a corridor of  $\pm\epsilon$  (function as flat as possible, actual error unimportant, penalty for outliers)

- Linear or Gaussian kernel for interpolation between support vectors
- Support vectors determined during training function (data points closest to target function)

# Supervised Learning: Support Vector Regression

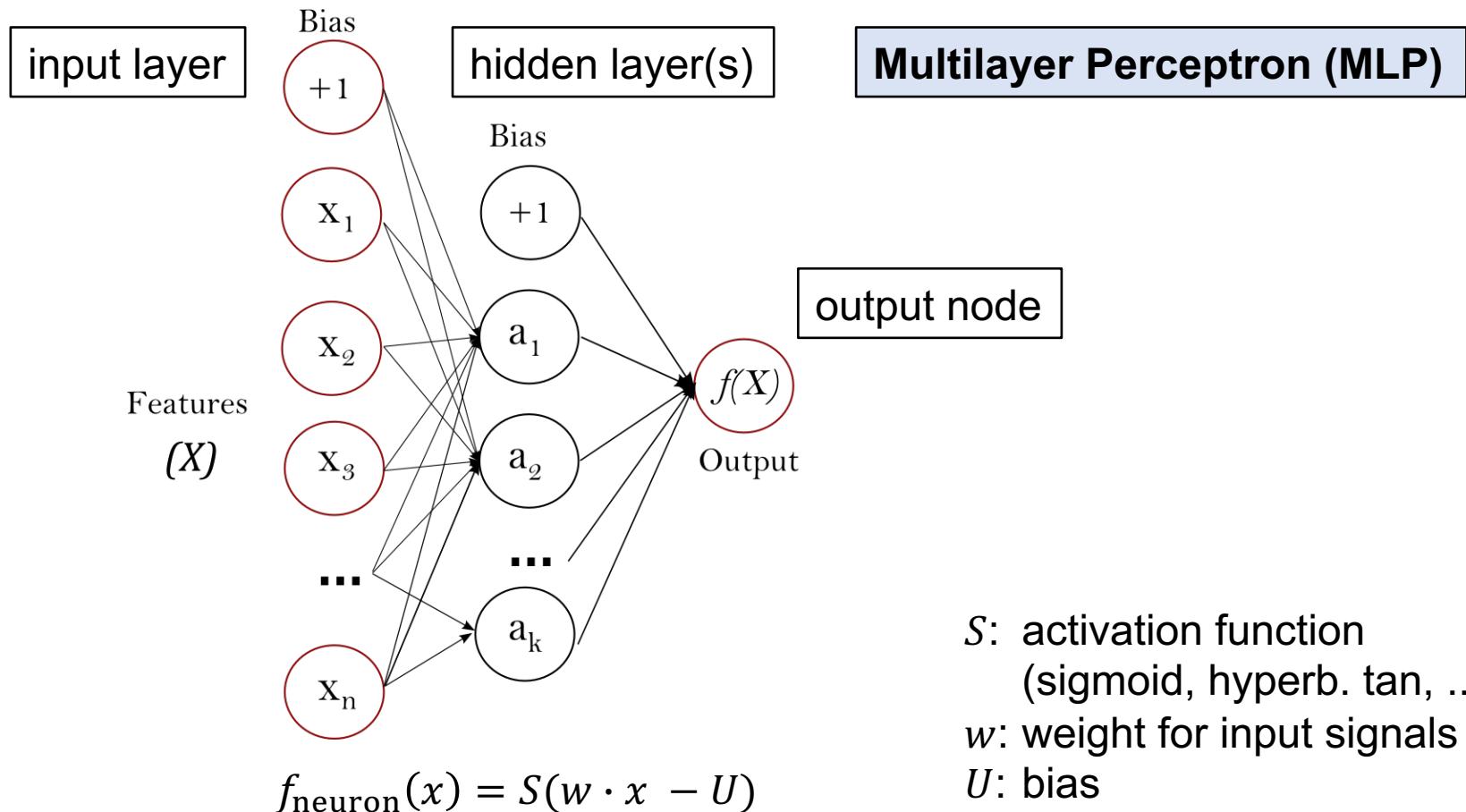
1-d example: noisy sine function



**Support Vector Mach.**  
hyperparameters:  
 $C = 10$   
 $\text{gamma} = 0.1$   
kernel: radial basis fct.

$$f(x) = \sum_{k=1}^n y_k a_k K\left(x_k^{(SV)}, x\right) + \rho$$

# Neural Networks (Regression/Classification)

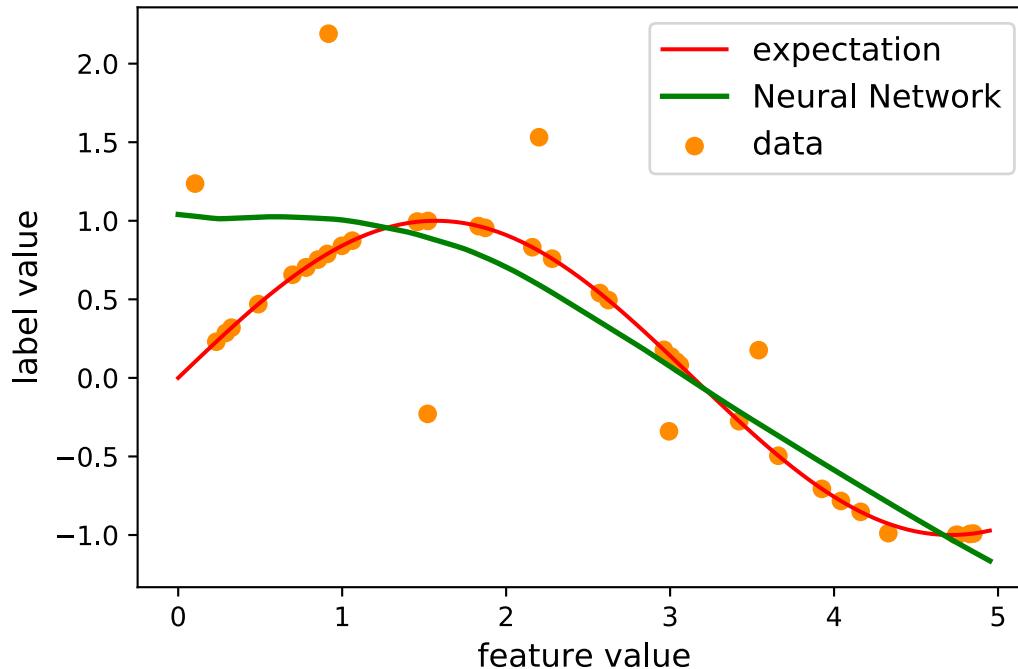


**Goal:** Find bias values and weights for activation functions that describe training data best. – *Deep learning*: multiple hidden layers.

Source: scikit-learn

# Supervised Learning: Neural Network

1-d example: noisy sine function



**Neural Network**  
hyperparameters:  
hidden layers: 3  
neurons: 100  
activation function: relu  
init. learning rate: 0.001  
...

$$f(x) = \sum_{i=1}^{N_{\text{neuron}}} w^{(i)} f_{\text{neuron}}^{(i)}$$

# Summary Machine Learning Methods

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## Common supervised Machine Learning (ML) methods

- Support Vector Machines  
(robust and moderate data requirements, only 2 hyperparameters)
- Neuronal Networks  
(prone to overfitting, large volumes of training data required, many hyperparameters)
- Gaussian Process  
(not covered here)
- Random Forrest  
(simple and robust reference method, prone to overfitting, only 2 hyperparameters)

**Feature Selection** determines physics of the trained model.

**Hyperparameter tuning** decisive for success of training.

**Independent Validation** necessary to ensure validity of trained model.

# Outline

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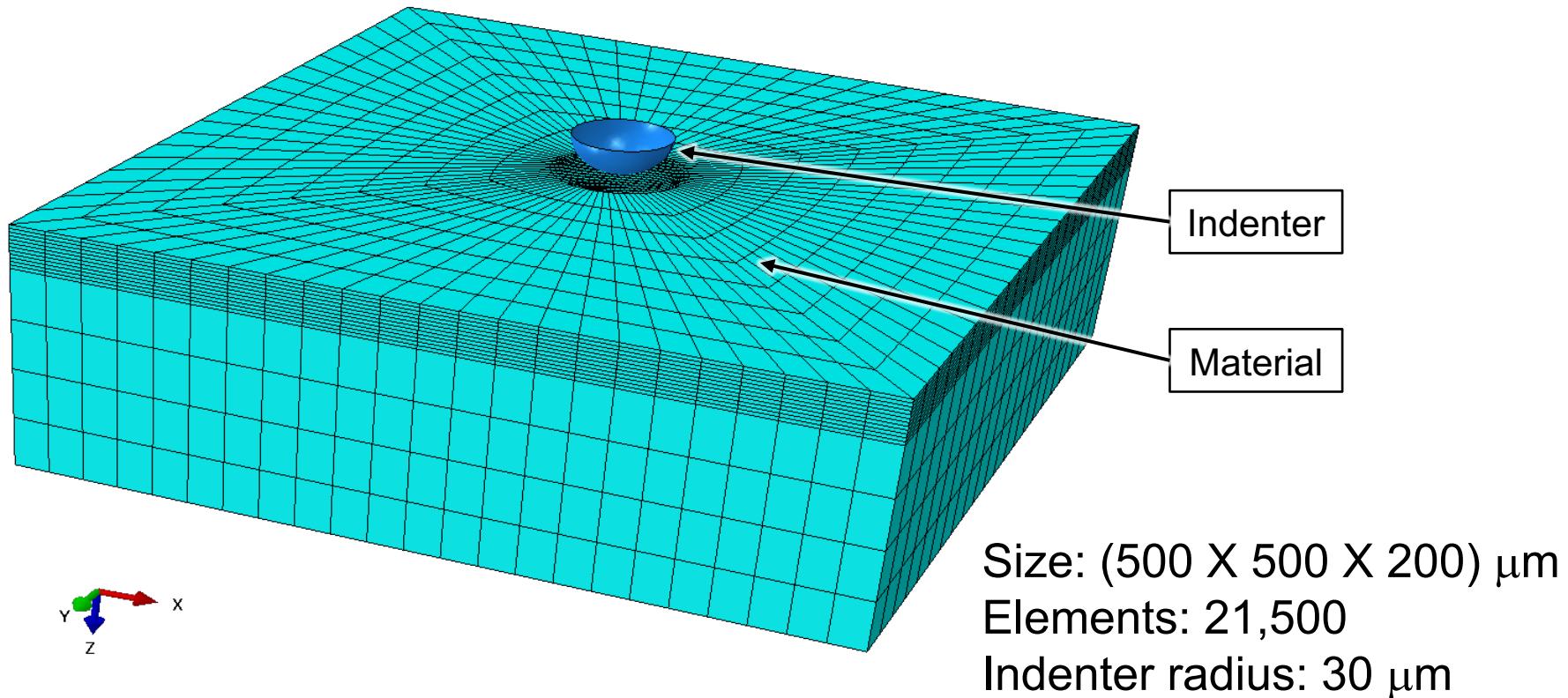
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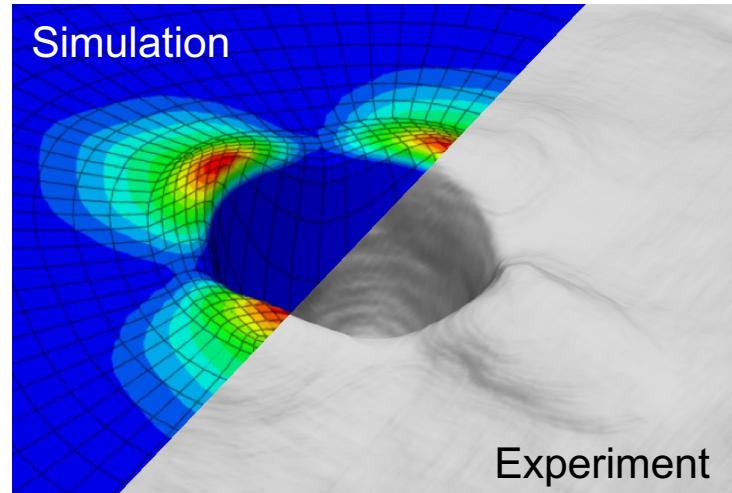
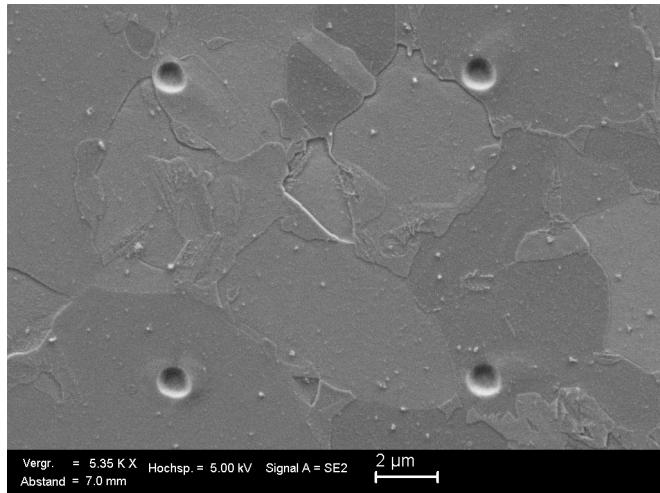
# Finite Element Model of indentation



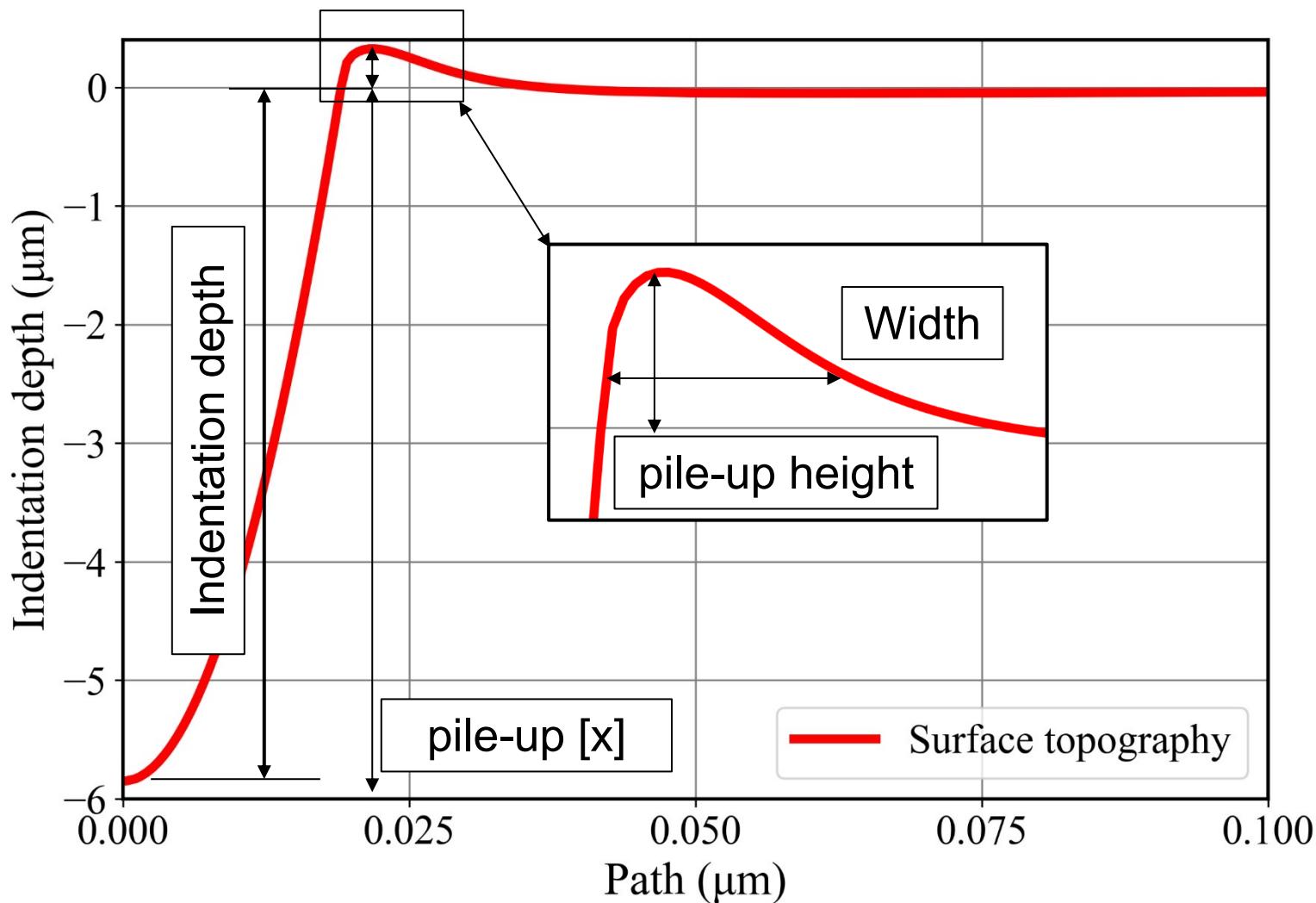
- Finite element model gives accurate description of indentation process
- Simulation times hours to days, depending on model size and constitutive model

# Finite Element Model of indentation

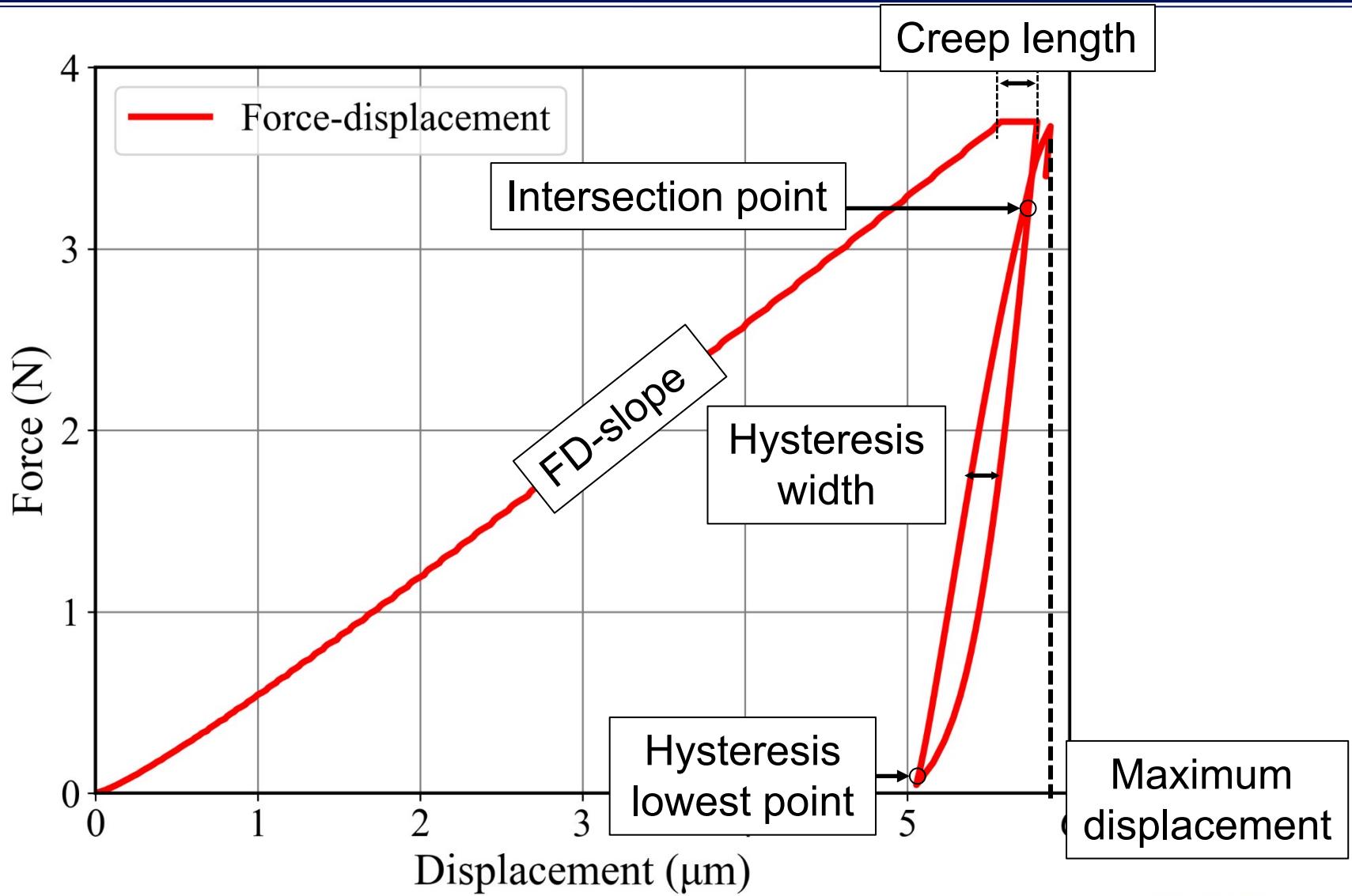
- Spherical nanoindentation into individual ferrite grains (ARMCO iron)
- Indenter tip radius: 800nm
- Max. load: 2.5 mN



# Definition of labels: Residual imprints



# Definition of labels: Force-displacement curve



# Data generation for training of ML surrogate model

## Isotropic hardening

$$R = Q(1 - e^{-b\varepsilon_{eq}})$$

## Non-linear kinematic hardening

$$\kappa = \sum_i^n \kappa_i; d\kappa_i = \frac{2}{3} C_i d\varepsilon_p - g_i \kappa_i d\varepsilon_{eq}$$

## Creep/time-dependent deformation

$$\begin{aligned}\dot{\varepsilon}^{cr} &= A \tilde{q}^n t^m \\ &= A_0 \left(\frac{q}{q_0}\right)^n \left(\frac{t}{t_0}\right)^m\end{aligned}$$

**1000** different combinations of material parameters are generated randomly from defined ranges.

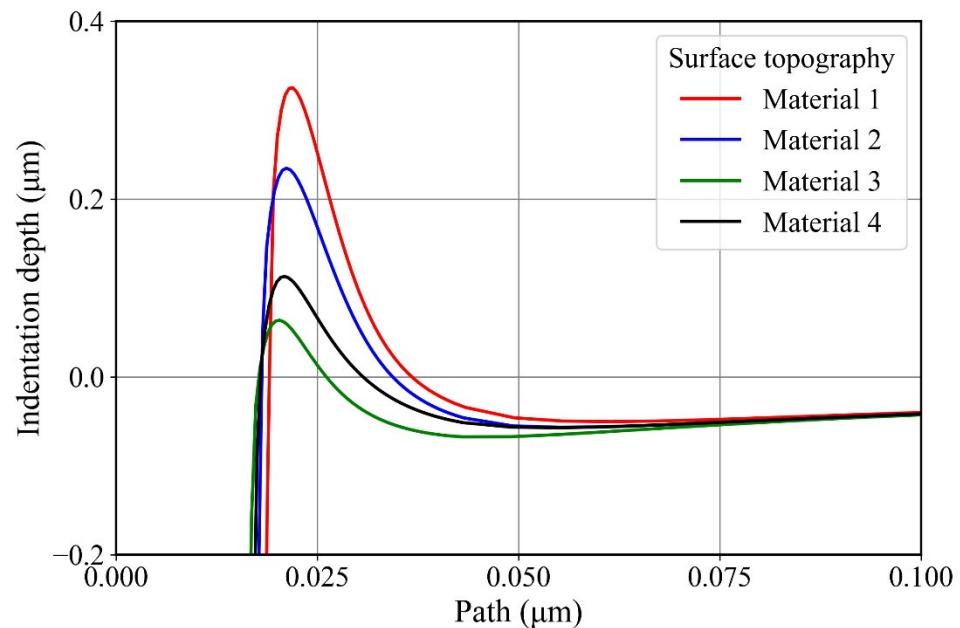
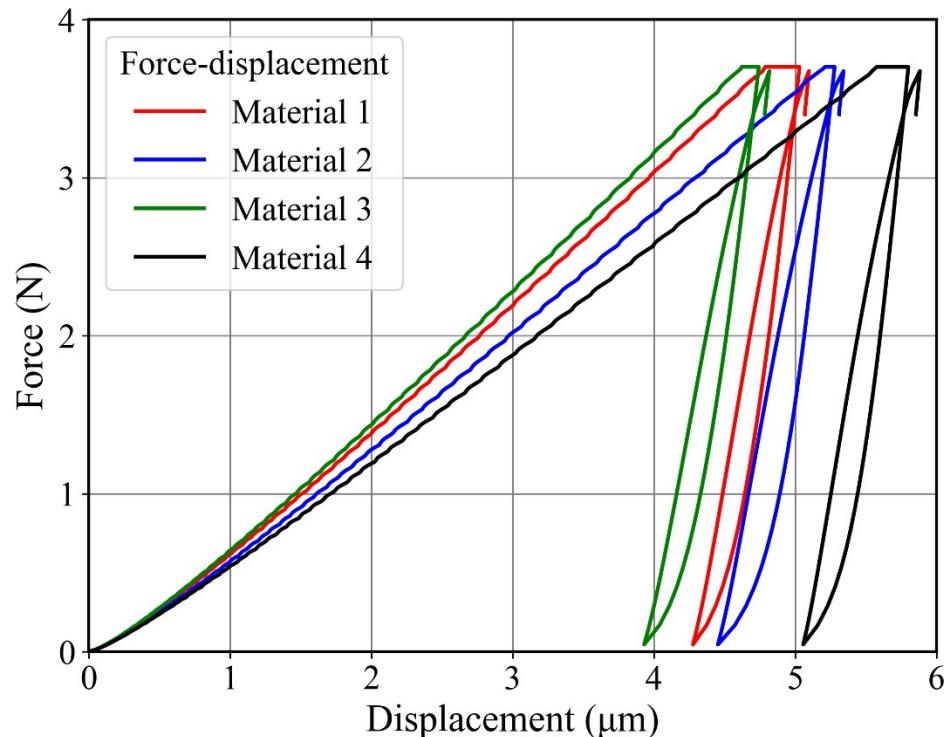
## Material Parameter Ranges

Parameter	Min	Max
$A_0$ , 1/s	1E-07	1E-05
$n$ , -	1.75	3.0
$m$ , -	-0.95	-0.5
$C_1$ , MPa	125000	225000
$g_1$ , -	350	550
$C_2$ , MPa	3000	5500
$Q$ , MPa	-350	-1750
$b$ , -	0.5	25

$$A = A_0 (750 \text{ MPa})^{-n} (100 \text{ s})^{-m}$$

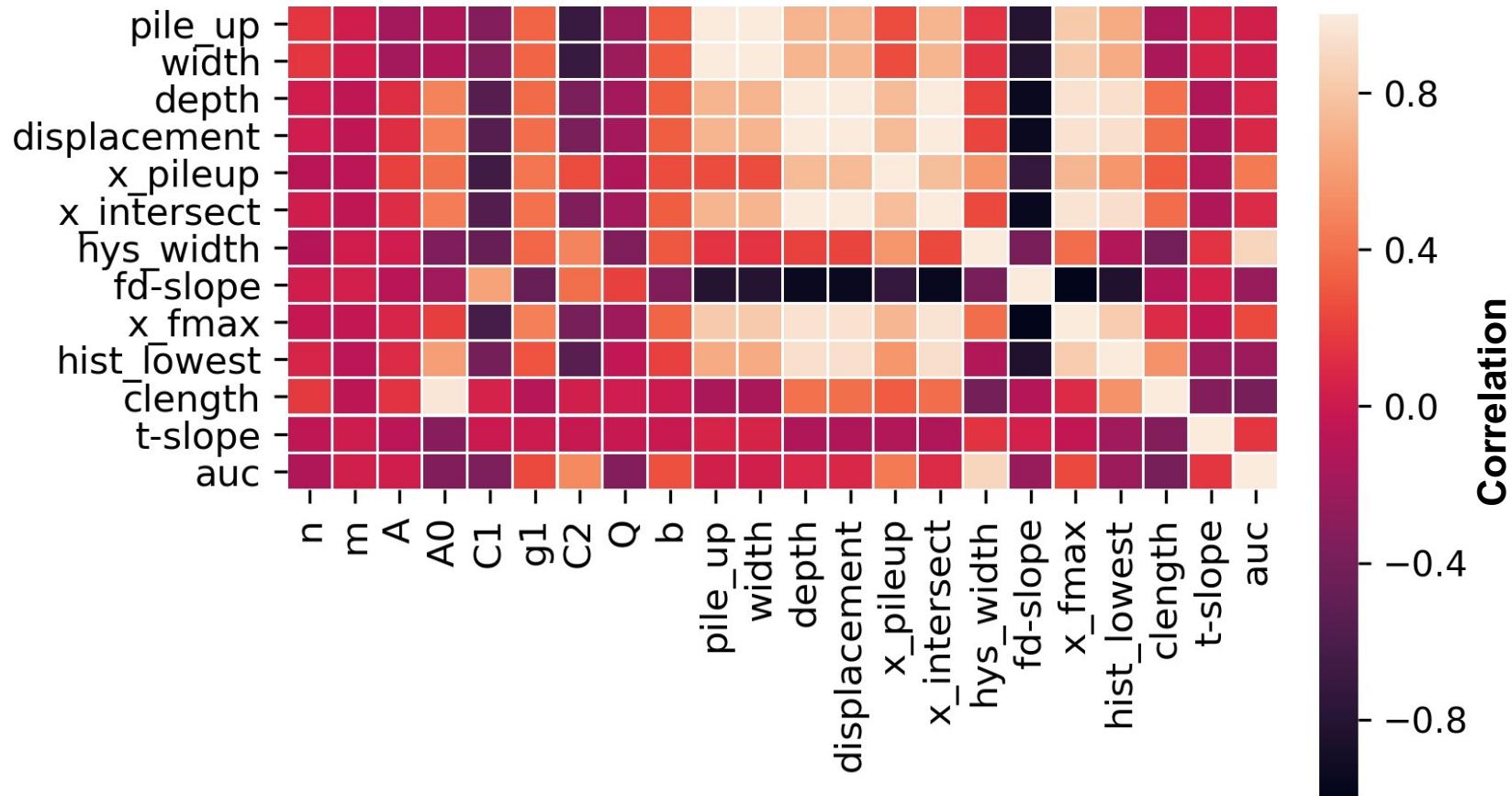
# Database generation

## Simulation output



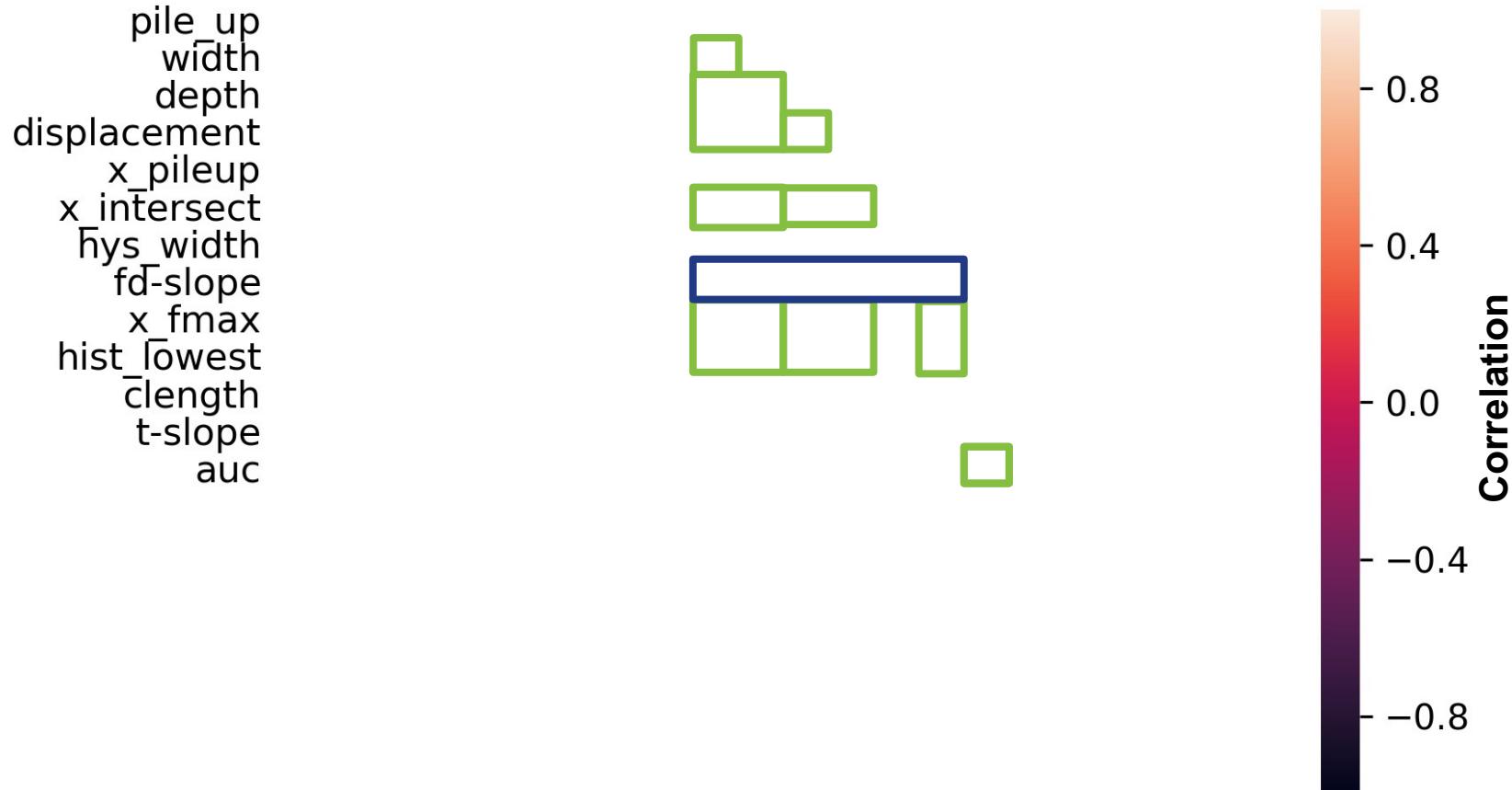
# Feature selection

## Heat map of extracted features and labels



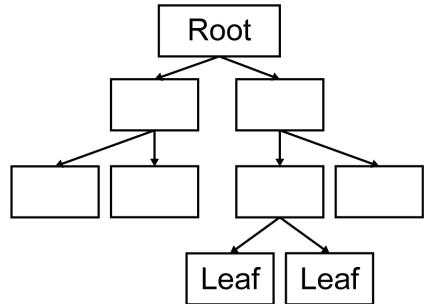
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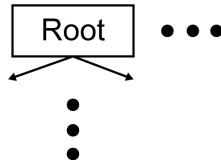


# Training and testing of ML algorithms

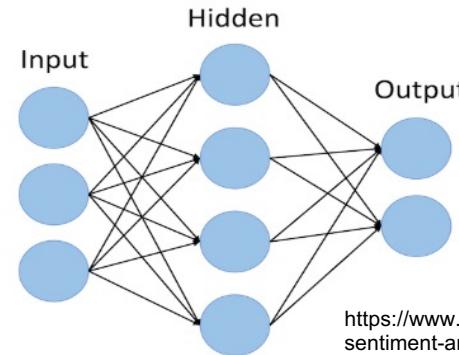
Tree1:



Tree2:

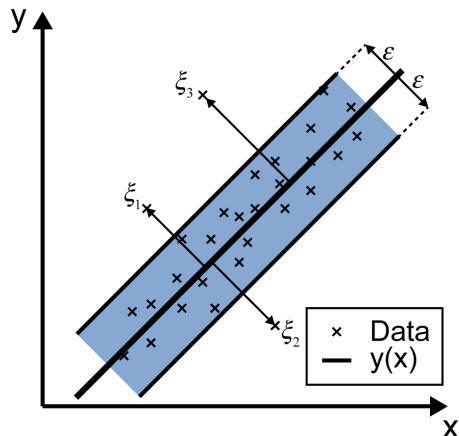


Random forest regression



<https://www.oreilly.com/content/perform-sentiment-analysis-with-lstm-using-tensorflow/>

Artificial neural networks

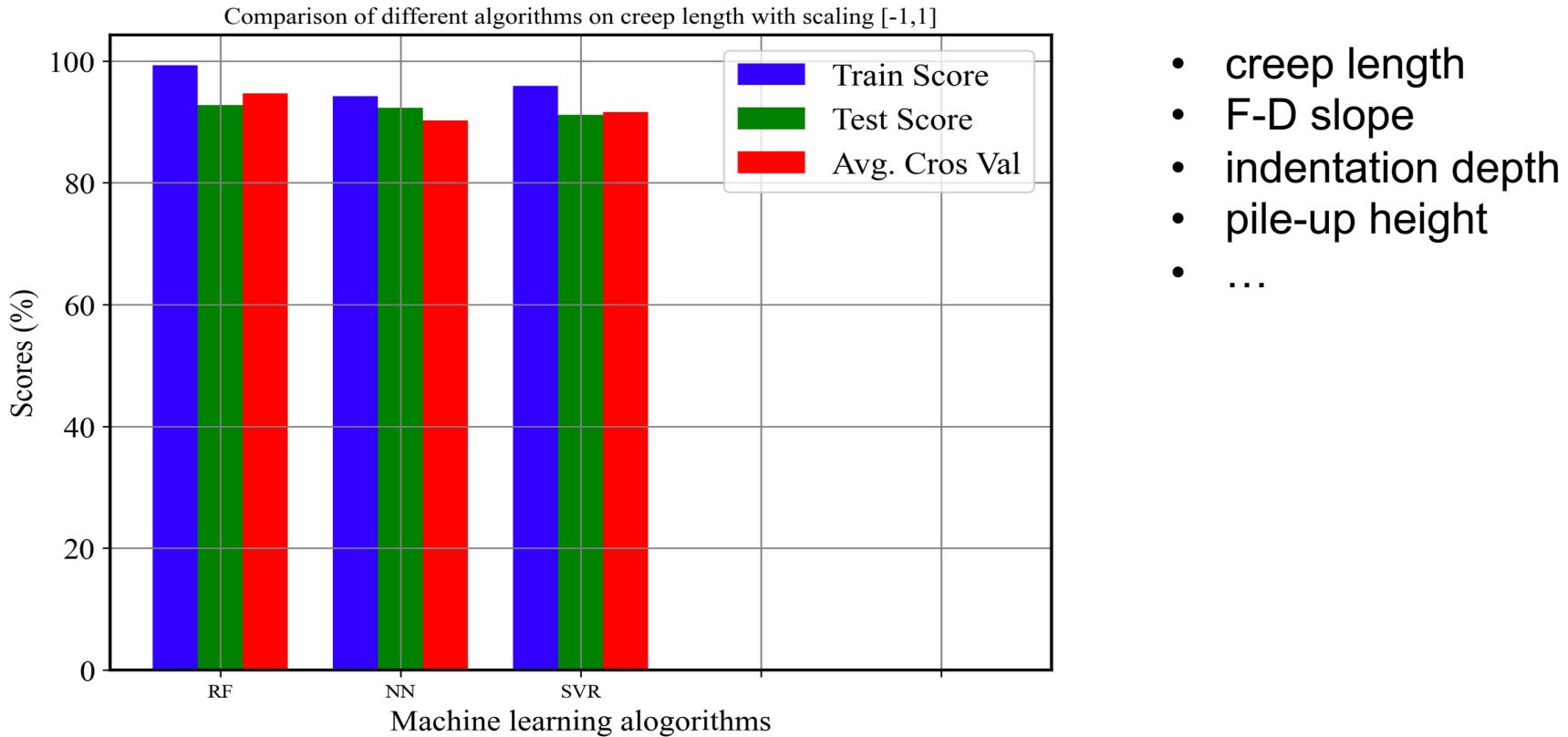


Support vector regression

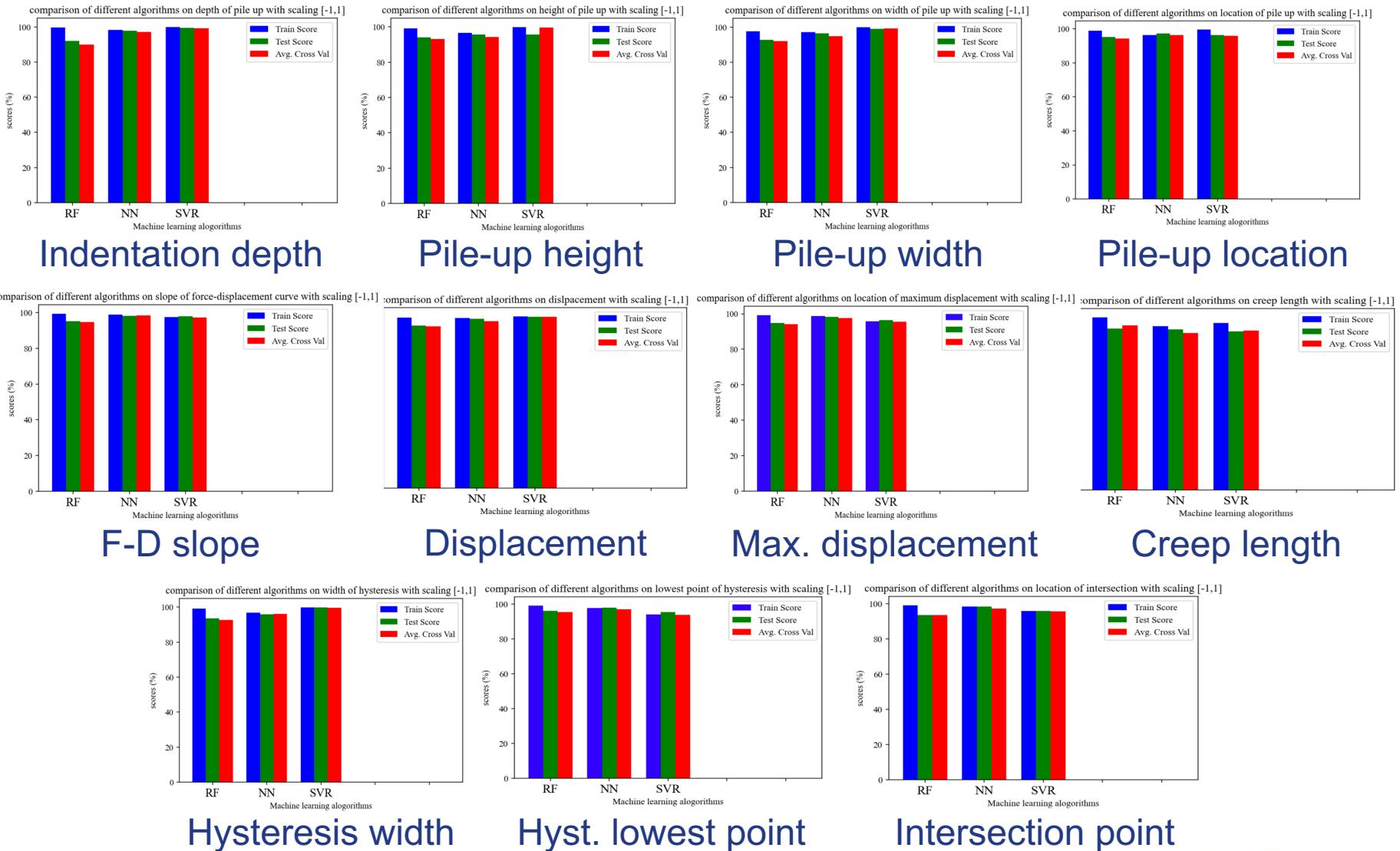
- Machine learning library on **Scikit Learn**
- Random Forest Regression (**RFR**), Support Vector Regression (**SVR**), and Neural Networks (**NN**) are chosen.
- 75% training data and 25% testing data
- Grid search for determining hyperparameters

# ML Regression for Characteristic Quantities

Function relating material parameters to characteristic quantities of indentation process:



# Training and testing results



# Validation of surrogate model

	Pile up (µm)	x-pile up (µm)	Width (µm)	Depth (µm)	Displace- ment (µm)	X- intersect (µm)	Hys- width (µm)	Hys- lowest (µm)	Creep length (µm)	F-D slope (N/µm)
<b>FEM</b>	0.244	22.17	0.977	5.476	5.512	5.411	0.360	4.407	0.052	0.684
<b>NN</b>	0.230	22.22	0.953	5.568	5.576	5.572	0.353	4.686	0.062	0.676
<b>NN Rel. dif. (%)</b>	3.50%	0.23%	2.51%	1.68%	1.16%	2.97%	2.06%	6.32%	18.14%	1.28%
<b>SVR</b>	0.236	21.83	0.957	5.441	5.471	5.461	0.330	4.610	0.054	0.671
<b>SVR Rel. dif. (%)</b>	6.00%	1.52%	2.04%	0.64%	0.73%	0.91%	8.37%	4.60%	2.84%	1.96%
<b>RFR</b>	0.226	21.58	0.899	5.254	5.298	5.230	0.332	4.454	0.044	0.704
<b>RFR Rel. dif. (%)</b>	7.53%	2.65%	8.04%	4.04%	3.87%	3.35%	7.82%	1.07%	14.26%	2.87%

Trained ML models are fed with unknown material parameters and compared with FEM simulation for validation

Relative difference:

$$\text{Rel. diff.} = \left| \frac{\text{FEM-predicted value}}{\text{FEM}} \right| * 100$$

# Summary – Surrogate model

- Finite Element (FE) simulations can model mechanical problems with high accuracy
- For repeated tasks, as for example for inverse methods or optimization problems, the numerical cost of FE simulations poses a severe restriction
- Trained ML models can be used as numerically efficient surrogate models for such tasks – training effort only occurs once

**Co-authors:** H.M. Sajjad, Z. Hamzeh, P. Nooshmer, N. Vajragupta

unpublished work

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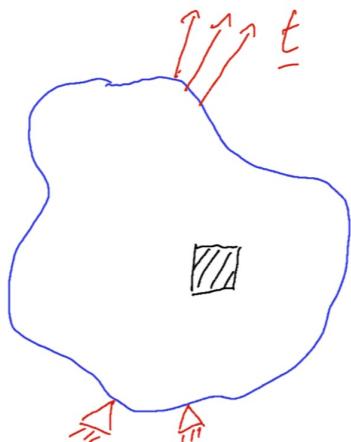
1. ML-Classification
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# Constitutive modeling (State-of-the-Art)

## Component

Mechanical / thermal / environmental loading conditions

total strain rate  $\dot{\varepsilon}_{\text{tot}}$   
temperature  $T$   
deformation history  $\varepsilon_{\text{pl}}$



## Constitutive models

History dependent evolution equations for state variables

$$\begin{aligned}\dot{\sigma} &= \dot{\sigma}(\sigma, \varepsilon_{\text{pl}}, \dot{\varepsilon}_{\text{tot}}, D, T) \\ \dot{\varepsilon}_{\text{pl}} &= \dot{\varepsilon}_{\text{pl}}(\sigma, \varepsilon_{\text{pl}}, \dot{\varepsilon}_{\text{tot}}, D, T) \\ \dot{D} &= \dot{D}(\sigma, \varepsilon_{\text{pl}}, \dot{\varepsilon}_{\text{tot}}, D, T)\end{aligned}$$

## Material response

Time dependent state variables

stress tensor  $\sigma = \sigma(t)$   
plastic strain  $\varepsilon_{\text{pl}} = \varepsilon_{\text{pl}}(t)$   
damage  $D = D(t)$

## Material parameters

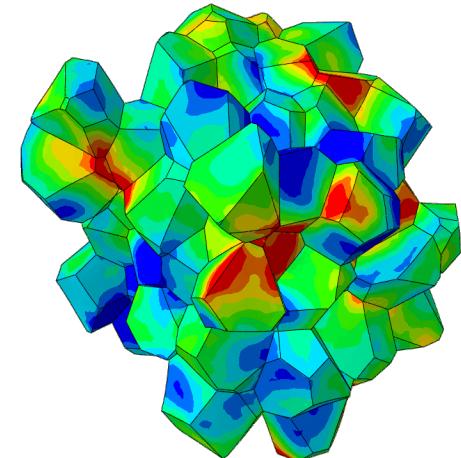
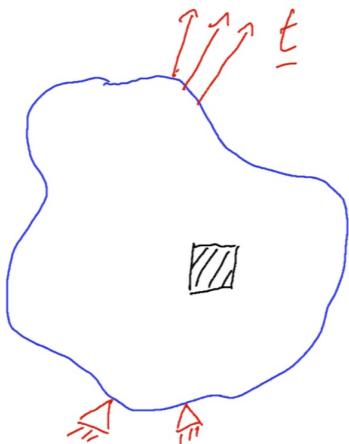
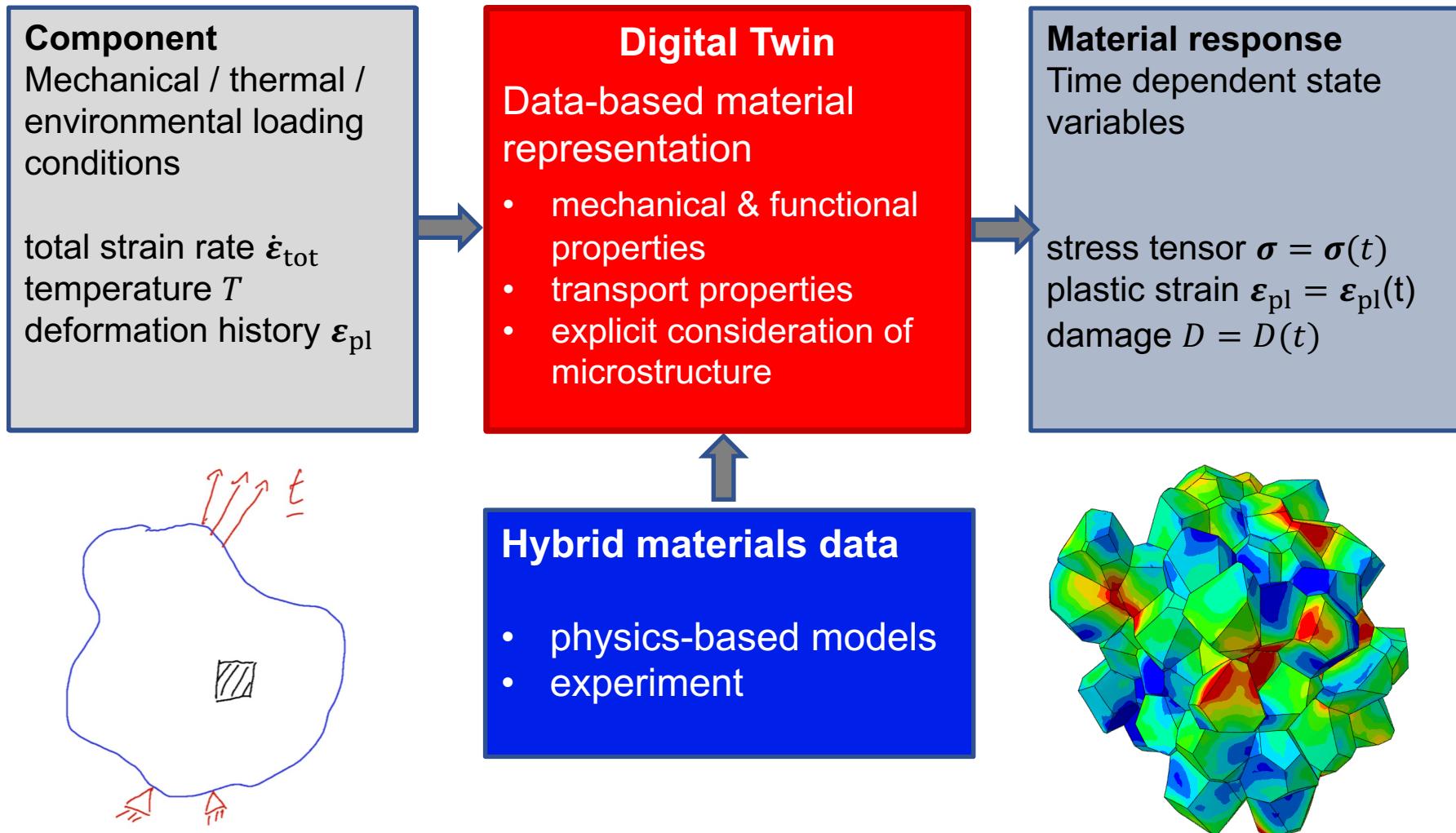
Material specific input to constitutive models

elastic tensor  $C_{ijkl}$   
yield strength  $\sigma_y$   
work hardening rate  $\partial\sigma/\partial\varepsilon_{\text{pl}}$   
damage onset  $\varepsilon_{\text{pl}}^{\text{crit}}$

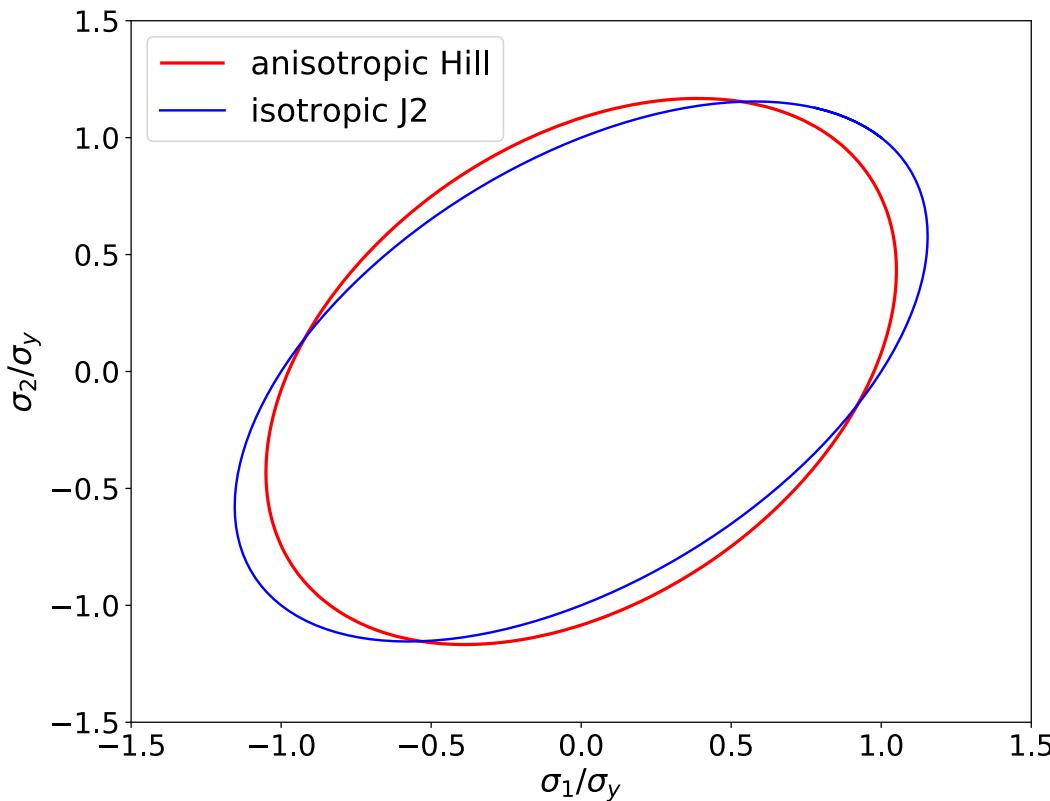
## Problem

- Solutions involve rather intricate mathematical formalism
- Explicit consideration of microstructure is difficult
- Material parameters vary with microstructure

# Constitutive Modeling: Digital Twin of Material



# Continuum plasticity



Yield loci of isotropic J2 and anisotropic Hill material definitions in cross-section of principle stress space

## Yield function

$$f(\boldsymbol{\sigma}) = \sigma_{\text{eq}} - \sigma_y$$

## Yield locus $f(\boldsymbol{\sigma}) = 0$

- elasticity inside  $f(\boldsymbol{\sigma}) < 0$
- plasticity on yield locus
- material does not support stresses outside yield locus

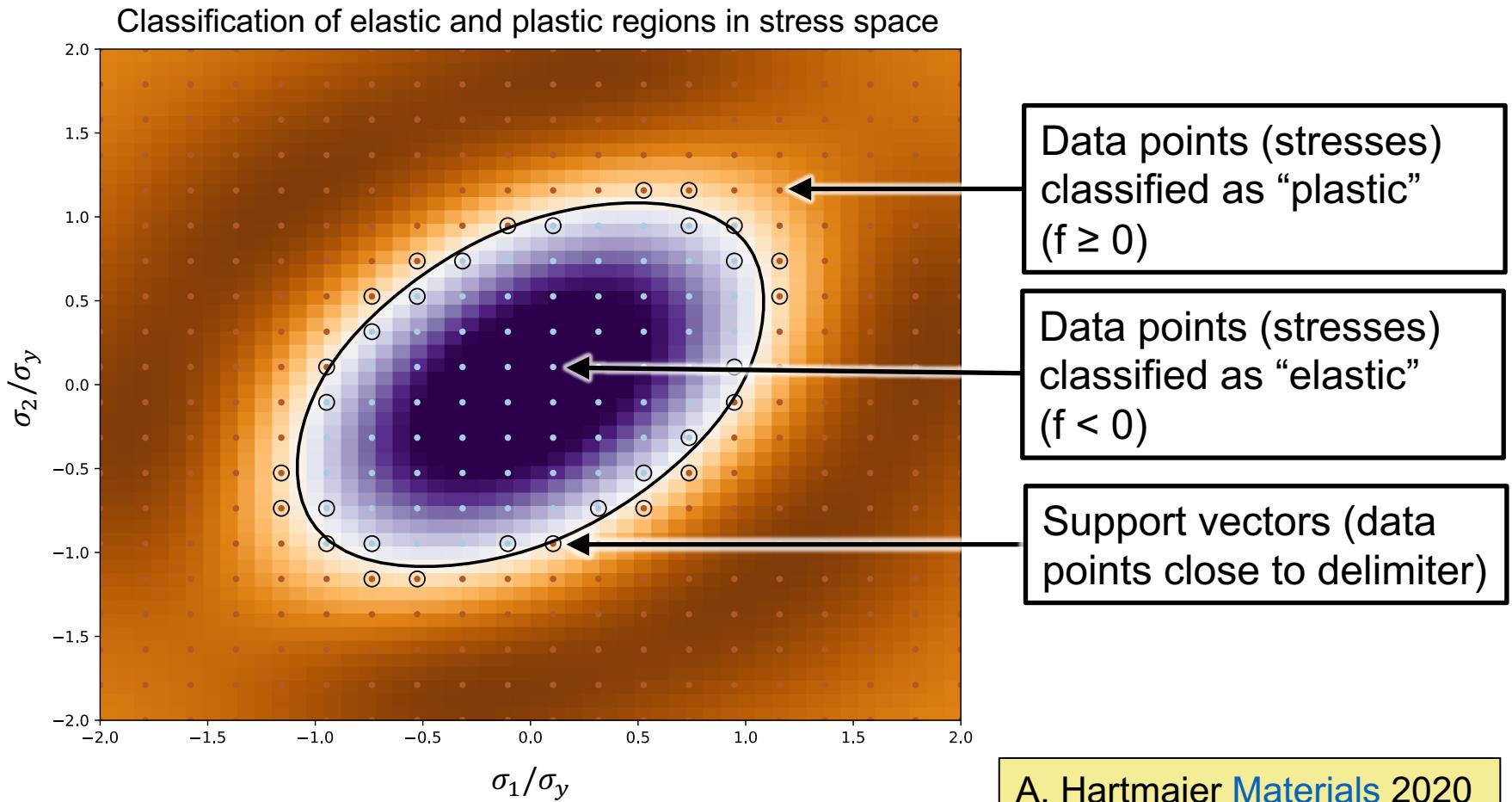
## Plastic strain rate (flow rule)

$$\dot{\boldsymbol{\varepsilon}}_{\text{pl}} = \lambda \boldsymbol{n}$$

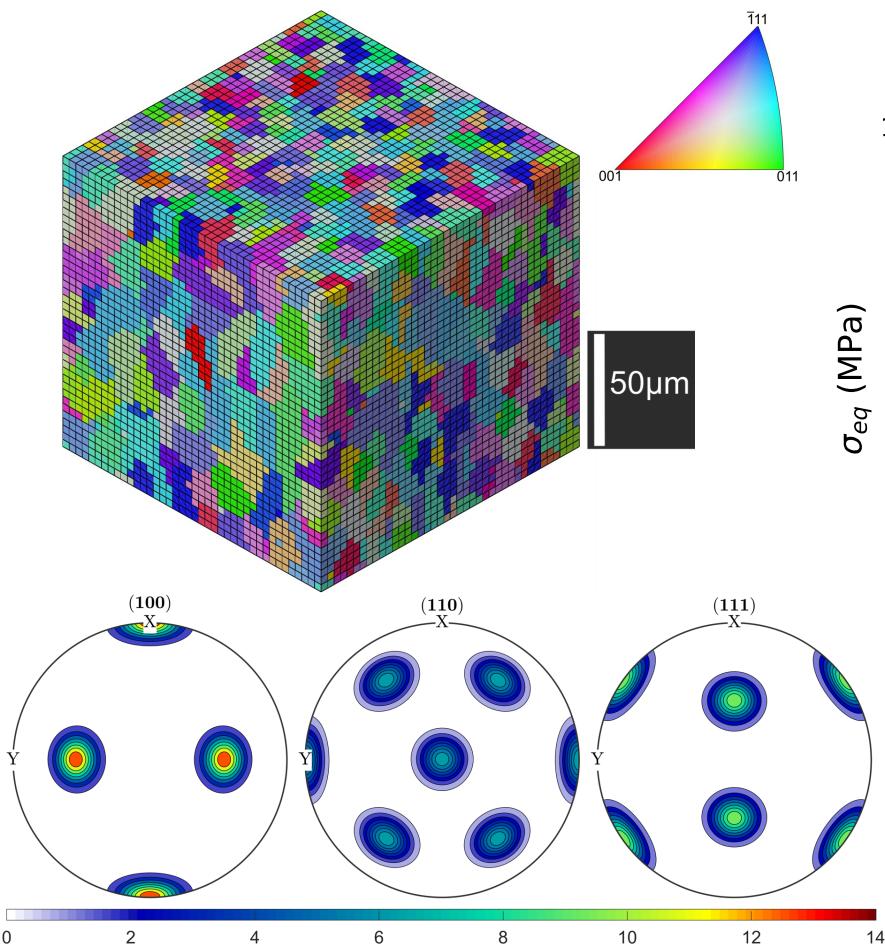
- $\lambda$ : plastic strain multiplier obtained from return mapping algorithm
- $\boldsymbol{n}$ : normal to yield locus

# Data-based model for plastic yielding

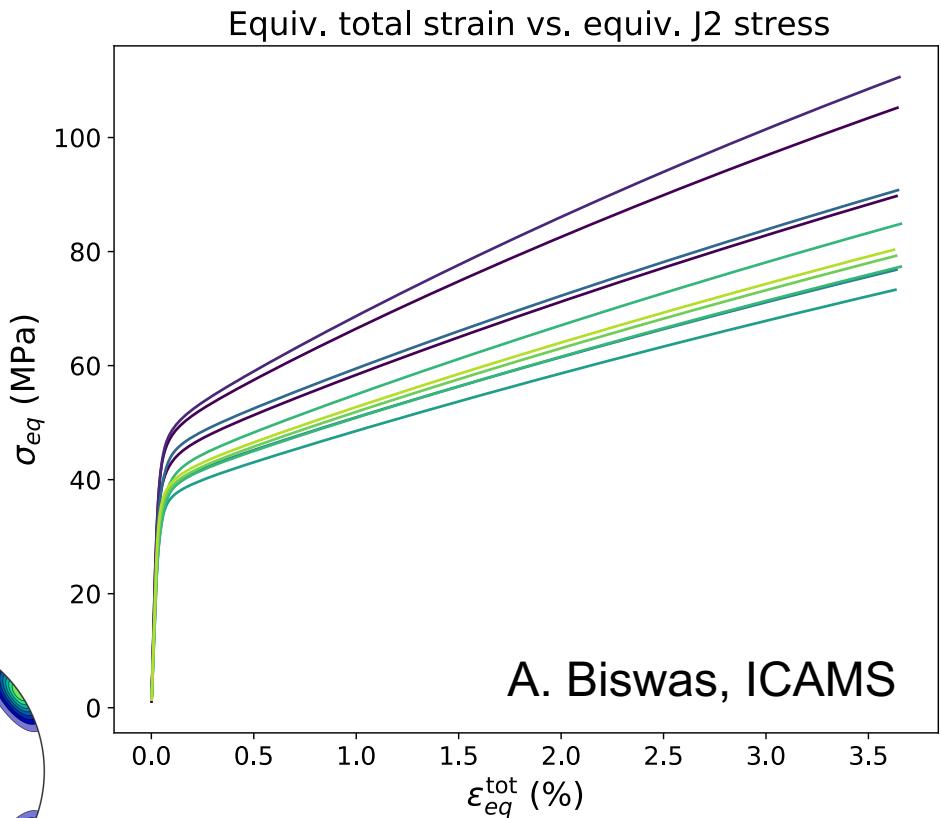
## Trained Support Vector Classification



# Training Data Generation by Micromechanical Model



Micromechanical model with  $\sim 2,200$  grains and Goss texture

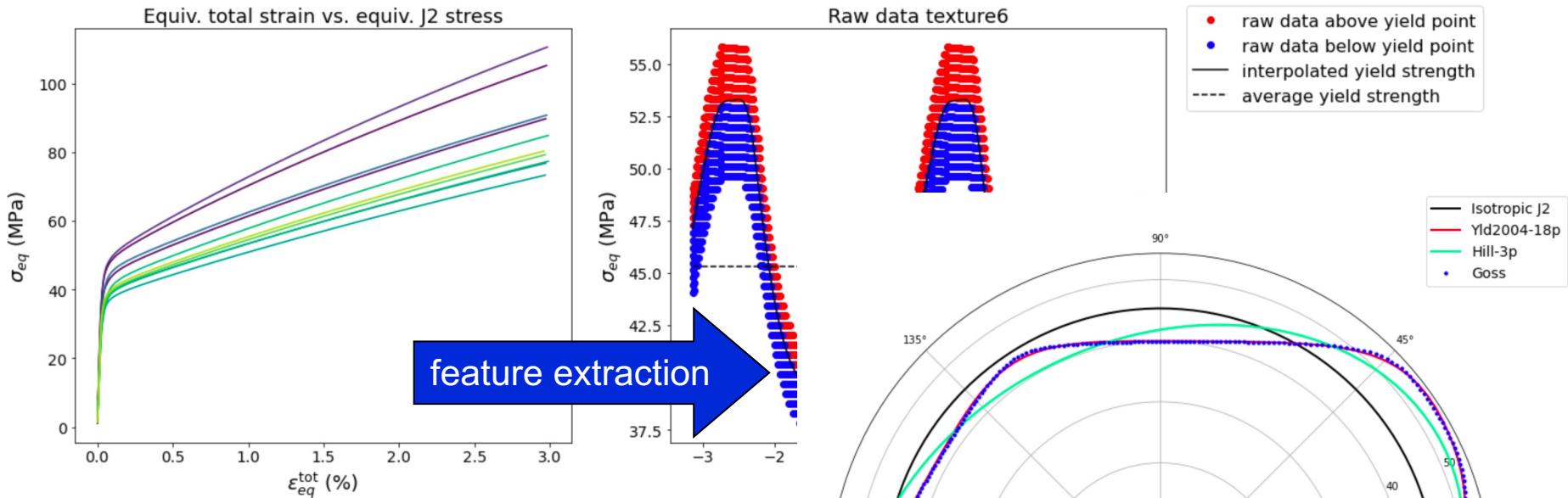


## Resulting stress-strain curves

- Crystal plasticity model
- Different deviatoric load cases

Anisotropic yield strength → data

# Training of Classifier for Elastic / Plastic Regions

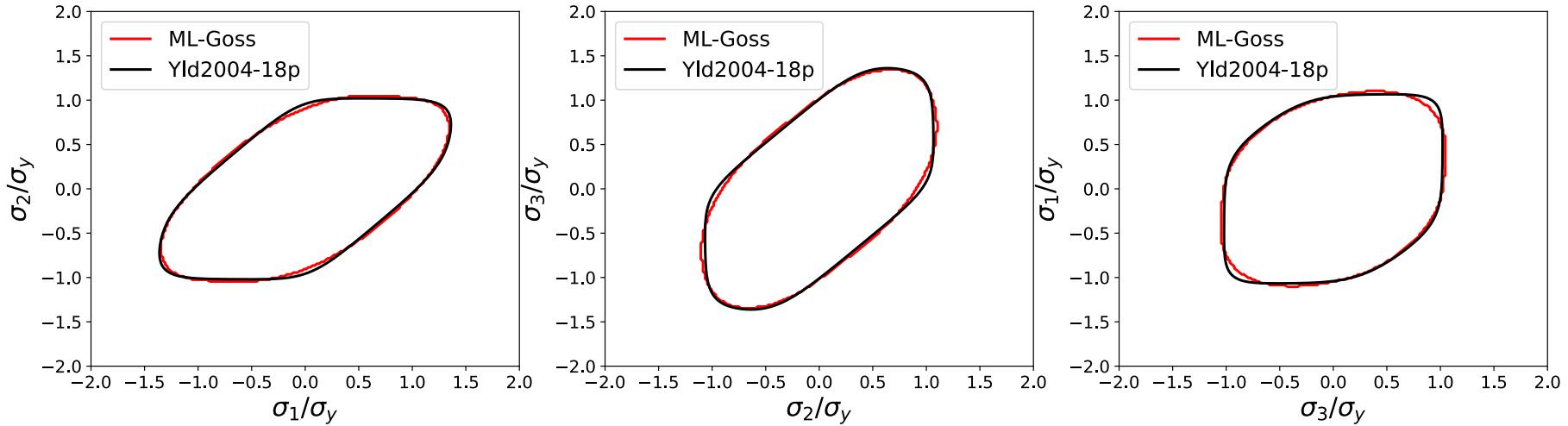


Digital Twin: Microstructure sensitive data-based description of plastic material properties

## Experimental data

- validation of micromechanical model
- hybrid mechanical data bases

# Trained Machine Learning Flow Rule

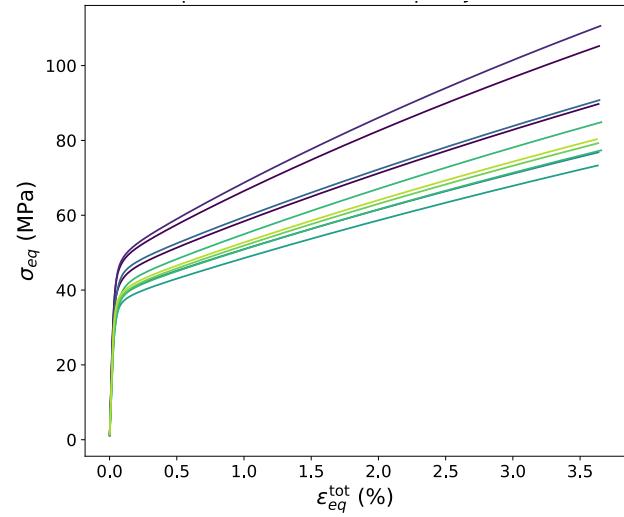
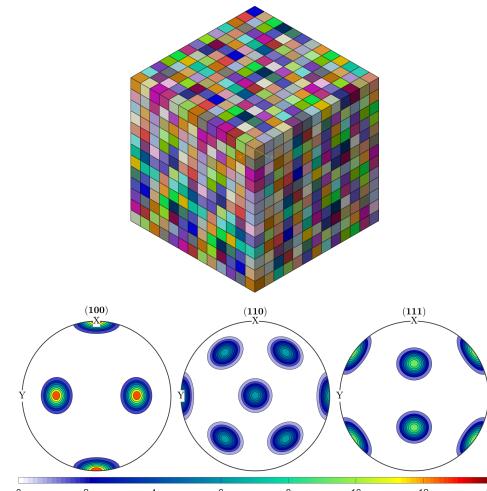
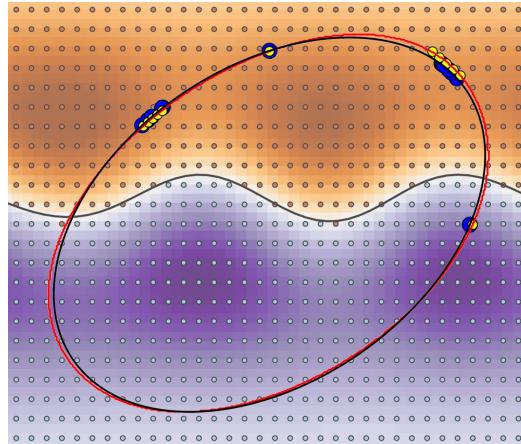


## Yield loci in different cross-sections of principle stress space

- Mechanical data obtained from micromechanical model
- Training of Machine Learning flow rule
- Barlat Yld2004-18p yield function fitted to same data

# Summary

- **Data-oriented material descriptions based on ML models** can replace classical constitutive rules and their parameters in finite element modeling  
→ Advantage: **Explicit consideration of microstructure** is possible
- Micromechanical modeling (synthetic RVEs, crystal plasticity, damage) is a powerful tool to generate **data for microstructure-property relationships**
- Fully parameterized and validated micromechanical models can complement experimental data to **hybrid mechanical data**



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