# Applications of Machine Learning in Mechanics of Materials


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# Literature

**Course material including numerical examples:**

Link to Binder

GitHub repository

PDF's of lecture slides under docs.

**Further reading:**

- Nemat-Nasser, Hori:
  Micromechanics: Overall Properties of the Heterogeneous Materials
- Shaofan, Wang:
  Introduction to Micromechanics and Nanomechanics

# Outline

**Applications of Machine Learning in Mechanics of Materials**

**Part I**
- Theoretical homogenization rules
- Micromechanical modeling

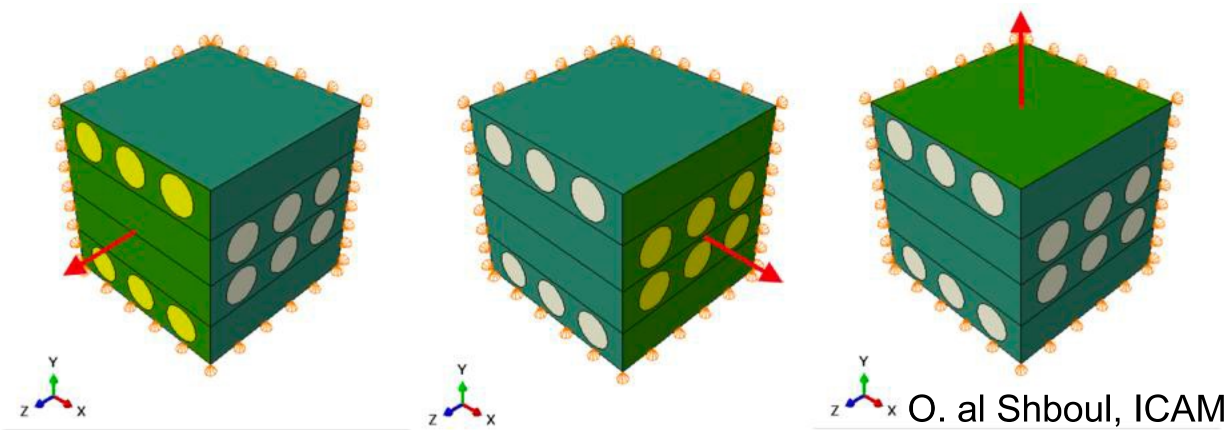
**Part II**
- Theory of Finite Element Analysis (FEA)
- Data generation

**Part III**
- Training of machine learning models
- Analysis of results

# Theoretical homogenization rules

## Motivation

- Effective properties of heterogeneous multiphase and polycrystalline materials can be calculated numerically by micromechanical modeling
- There exist also theoretical homogenization rules to estimate effective material properties of heterogeneous materials
- Theoretical homogenization rules are typically applied to composite materials with two distinctly different phases, e.g. glass or carbon fiber-reinforced plastics




O. al Shboul, ICAMS

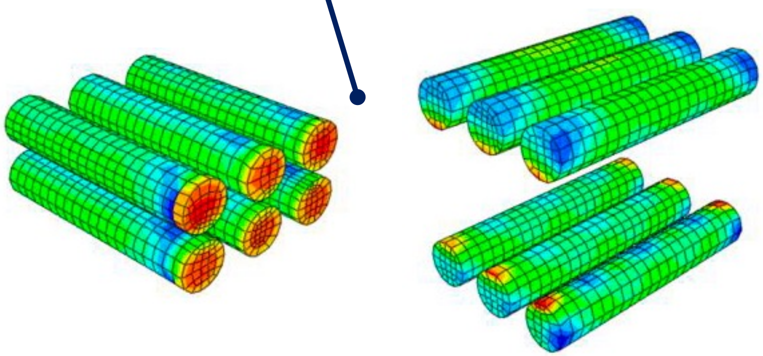
# Example: Homogenization of density

## Density

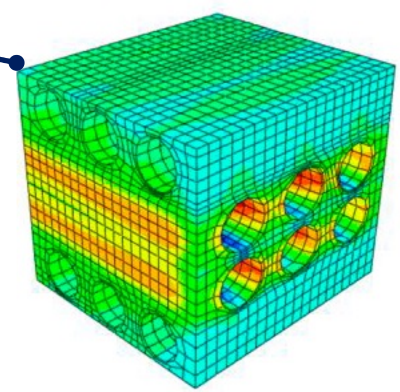
Mass: $m_c = m_f + m_m$

$$\rho_c = \frac{m_c}{V_c} = \frac{m_f + m_m}{V_c} = \frac{\rho_f V_f + \rho_m V_m}{V_c}$$

$$\boldsymbol{\rho_c = \rho_f f_f + \rho_m (1 - f_f)}$$ **(homogenization rule for density)**



fibers / filler



matrix

# Theoretical homogenization rules

## Elastic constants

- Rule of mixture for long fiber reinforced materials (iso-strain or Voigt model)
- Appropriate for properties along fiber direction

$$\varepsilon_f = \varepsilon_m = \varepsilon_c = \frac{\Delta L}{L} \quad \text{(iso-strain)}$$

$$F_c = F_f + F_m$$

Definition stress: $\sigma = F/A$

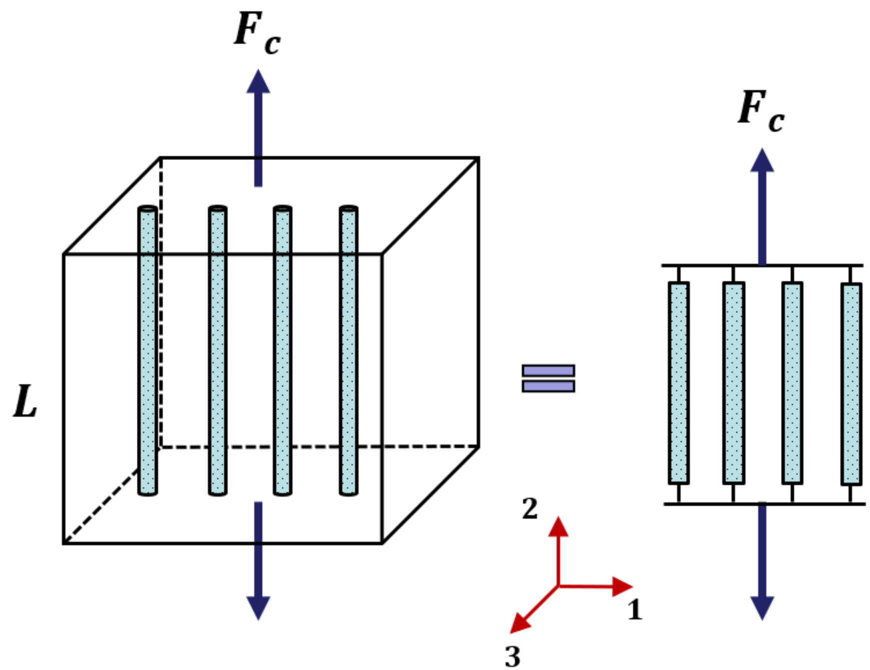
$$\sigma_c^l A_c = \sigma_f A_f + \sigma_m A_m$$

Assumption: $f_f = V_f/V_c \approx A_f/A_c$

Hooke's law and iso-strain:

$$\frac{\sigma_c^l}{\varepsilon_c^l} = E_c^l = \frac{\sigma_f\ f_f + \sigma_m f_m}{\varepsilon_c^l}$$

$$E_c^l = E_f f_f + E_m\left(1 - f_f\right) = E_{11}$$




(homogenization rule for elastic stiffness under iso-strain conditions)

# Theoretical homogenization rules

## Elastic constants

- Rule of mixture for long fiber reinforced materials (iso-stress or Reuss model)
- Rigorous derivation for laminates, lower bound for fiber reinforced materials

$\sigma_c^t = \sigma_f = \sigma_m$ (iso-stress)

$\Delta t_c = \Delta t_f + \Delta t_m$

Definition strain: $\varepsilon = \Delta t / t$

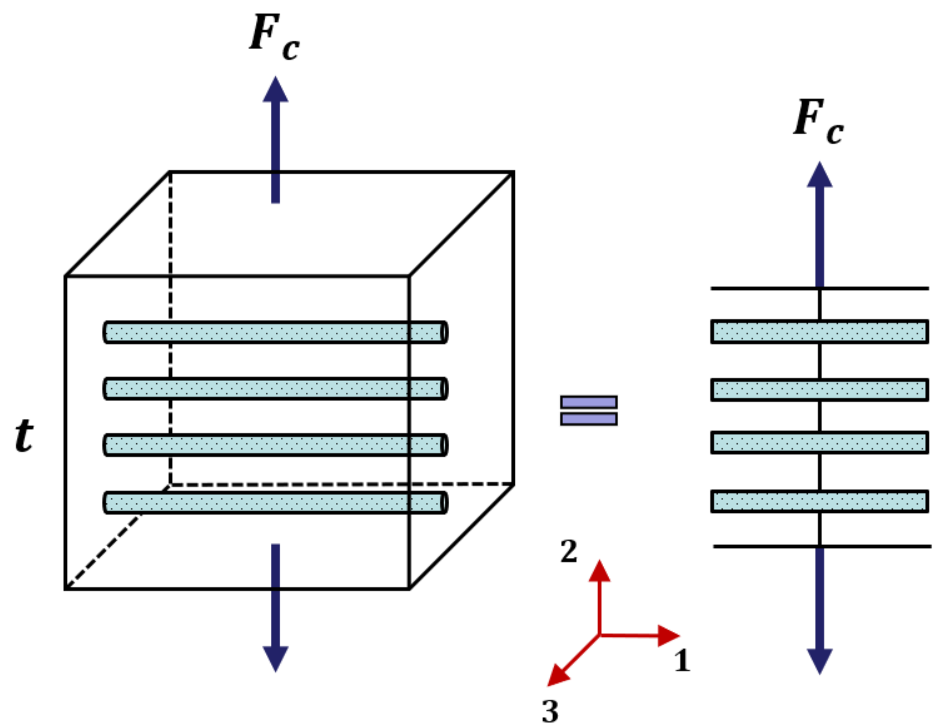
$\varepsilon_c^t t_c = \varepsilon_f t_f + \varepsilon_m t_m$

Assumption: $f_f = V_f / V_c \approx t_f / t_c$

Hooke's law and iso-stress:

$$\frac{\varepsilon_c^t}{\sigma_c^t} = \frac{1}{E_c^t} = \frac{\varepsilon_f \ f_f + \varepsilon_m f_m}{\sigma_c^t}$$

$$\frac{1}{E_c^t} = \frac{f_f}{E_f} + \frac{1 - f_f}{E_m} = \frac{1}{E_{22}}$$

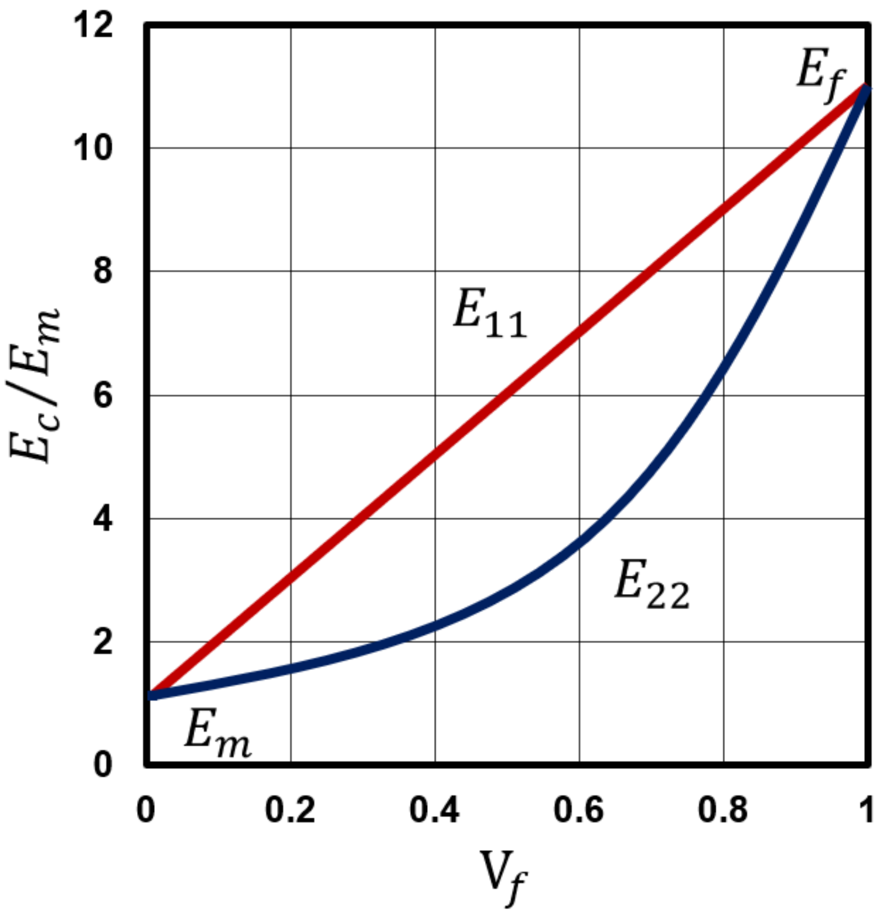



(homogenization rule for elastic stiffness under iso-stress conditions)

# Theoretical homogenization rules

## Rule of mixture

Variation of effective Young's modulus ($E_c$) *in* longitudinal ($E_{11}$) and transversal direction ($E_{22}$) with fiber volume fraction ($v_f$)

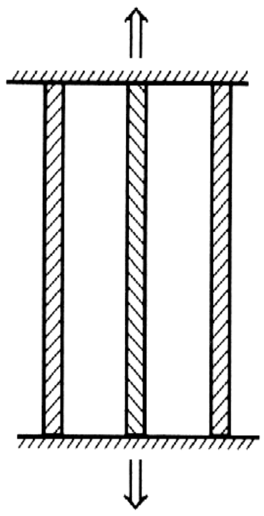


# General form for all aggregates

Upper bound (Taylor model)
Iso-strain assumption

$\varepsilon^{(i)} = \varepsilon_{tot}$ $\sigma_0 = \sum_{i=1}^{N} f_i \sigma^{(i)}(\varepsilon_{tot})$

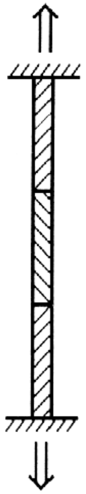
Real case is softer than upper bound
and harder than lower bound



(b)

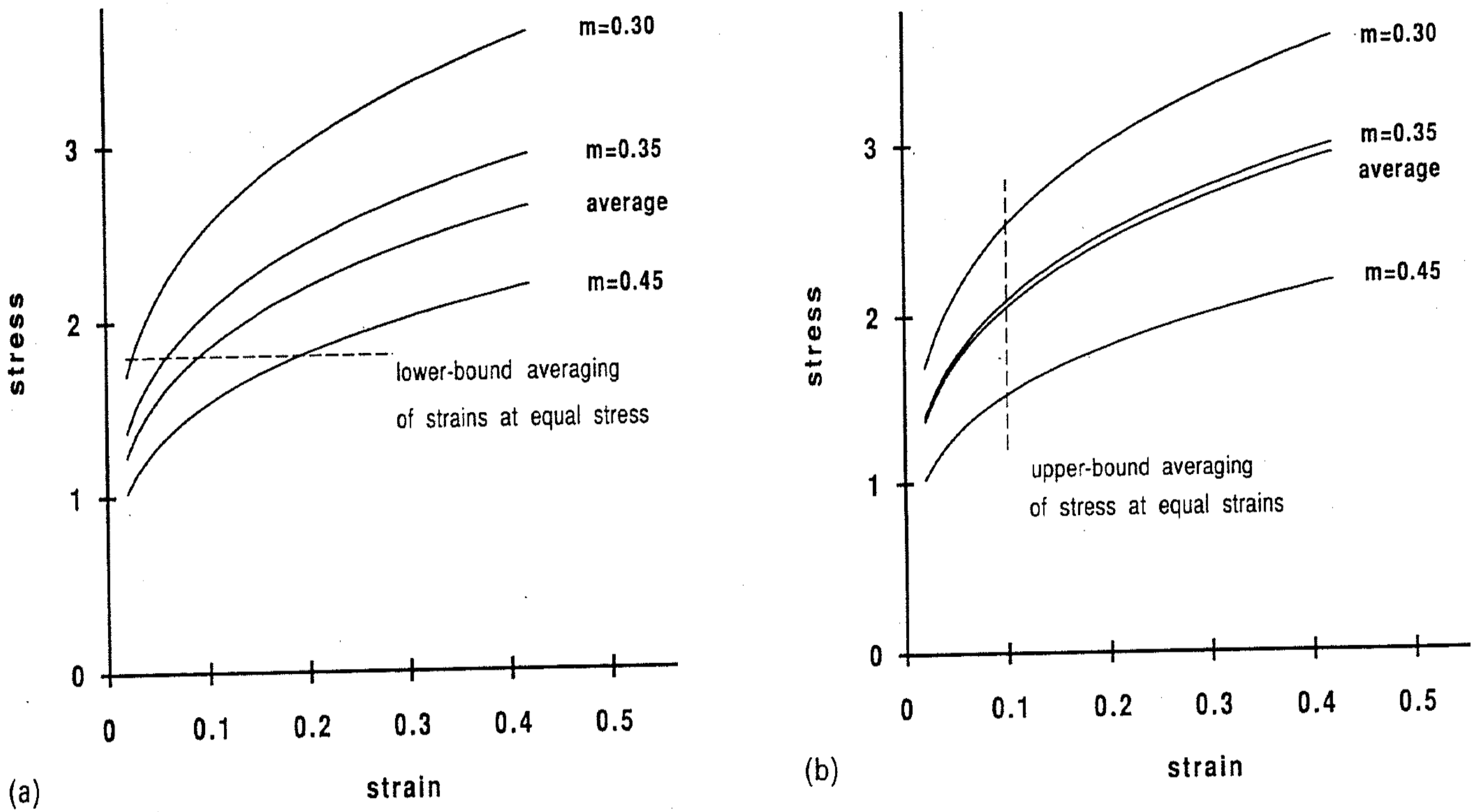
Lower bound (Sachs model)
Iso-stress assumption

$\sigma^{(i)} = \sigma_0$ $\varepsilon_{tot} = \sum_{i=1}^{N} f_i \varepsilon^{(i)}(\sigma_0)$



(a)

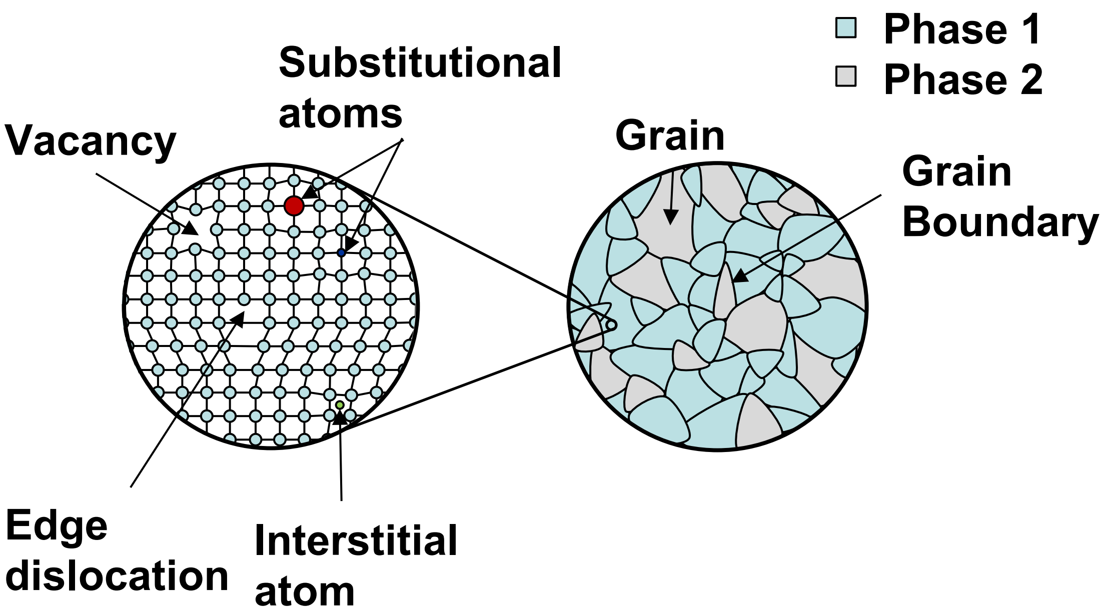
# Polycrystalline aggregates: Analytical homogenization



# Micromechanical Modeling

**Elements of Microstructure:**

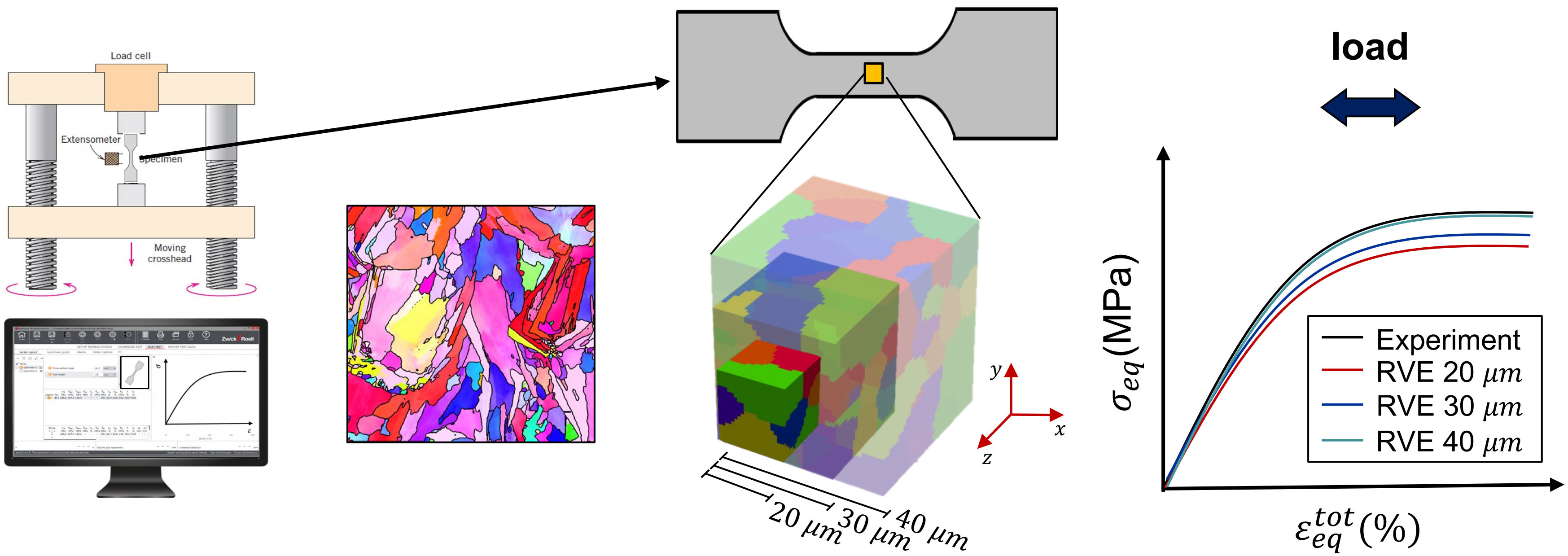
- Phases
- Grains
- Grain boundaries
- Dislocations
- Atomic defects



# Micromechanical Modeling

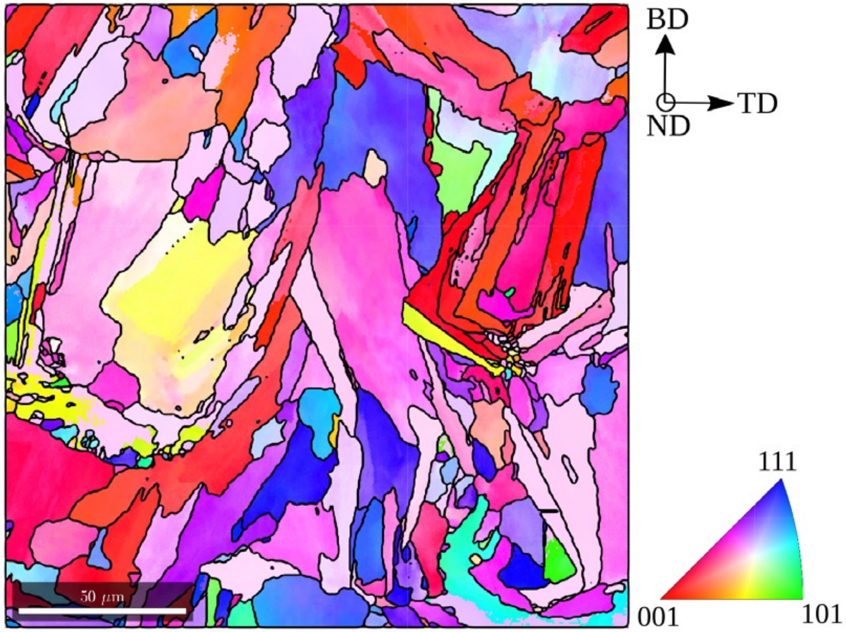
**RVE (Representative Volume Element): Microstructure model**

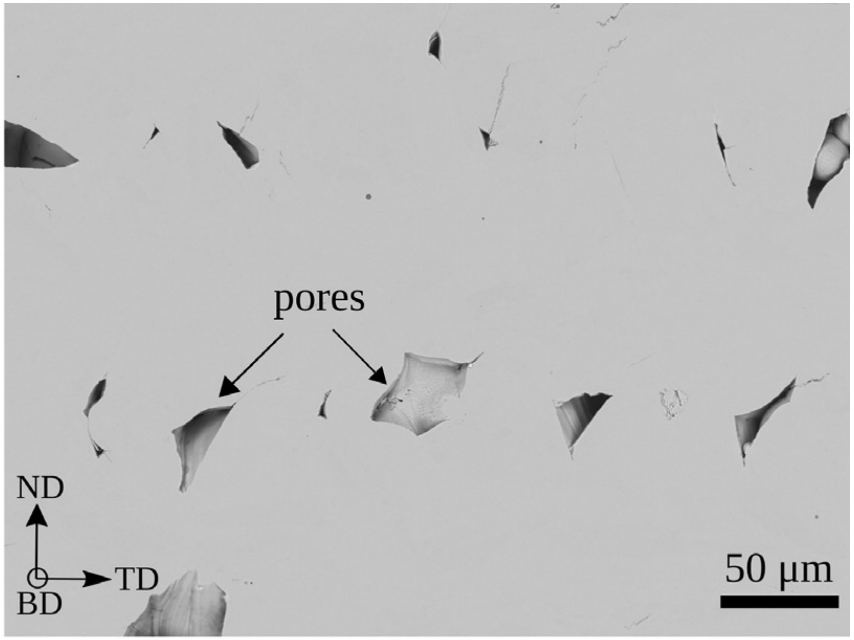
- RVE captures microstructural features in statistical sense (volume fraction of phases, size and shape distribution of grains)
- RVE size influences the efficiency and accuracy of mechanical property prediction




Experimental Test | EBSD Scan | RVE Reconstruction (Kanapy) | Finite Element Simulation (Abaqus)

# Micromechanical Modeling

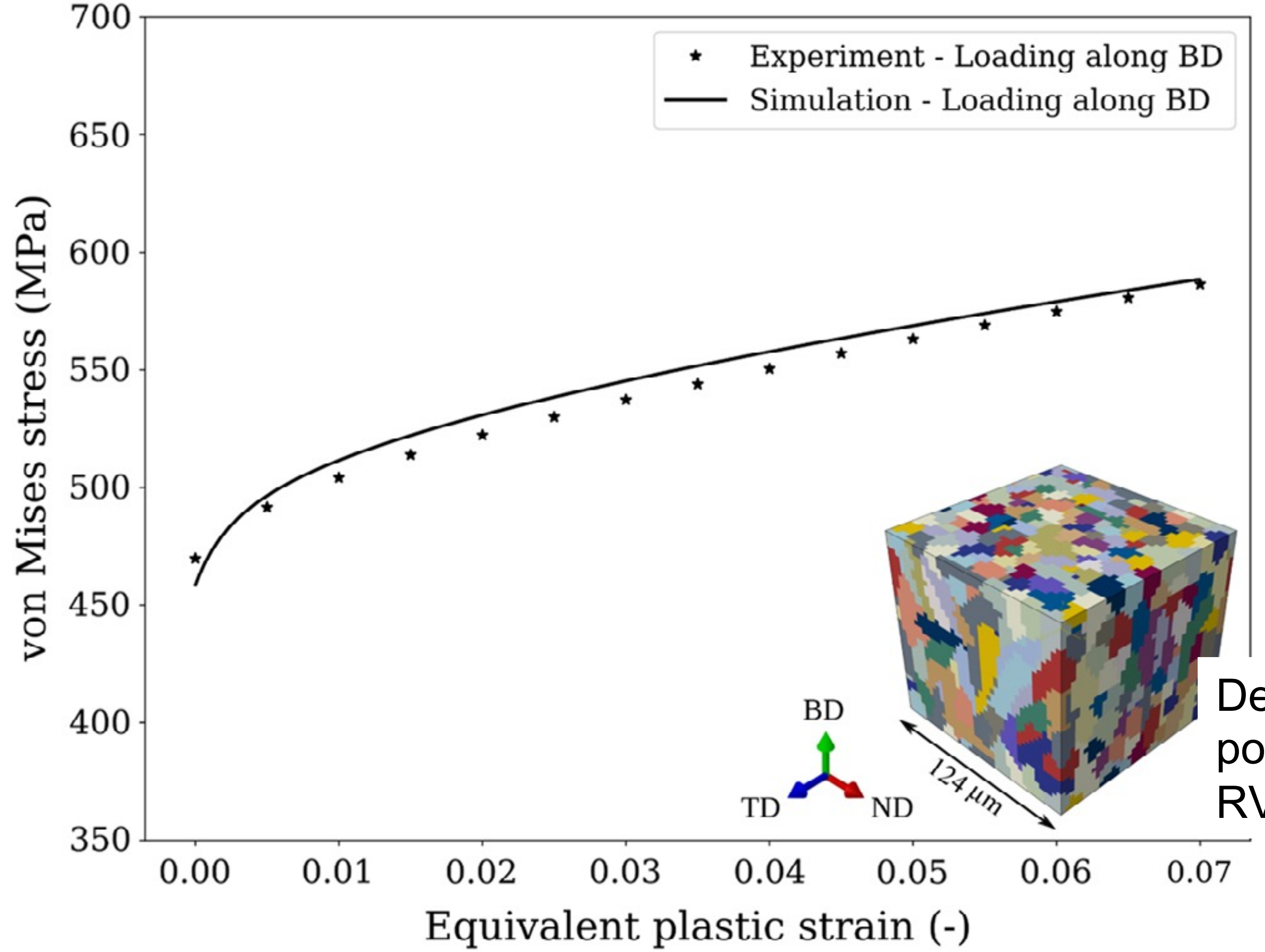



Additively (PBF-LB/M) manufactured sample of austenitic steel (316L): elongated grains and porosity

MRG Prasad, A Biswas, K Geenen, W Amin, S Gao, J Lian, A Röttger, N Vajragupta, A Hartmaier, Adv. Eng. Mater. 2020, 2000641
[DOI: 10.1002/adem.202000641](https://doi.org/10.1002/adem.202000641)

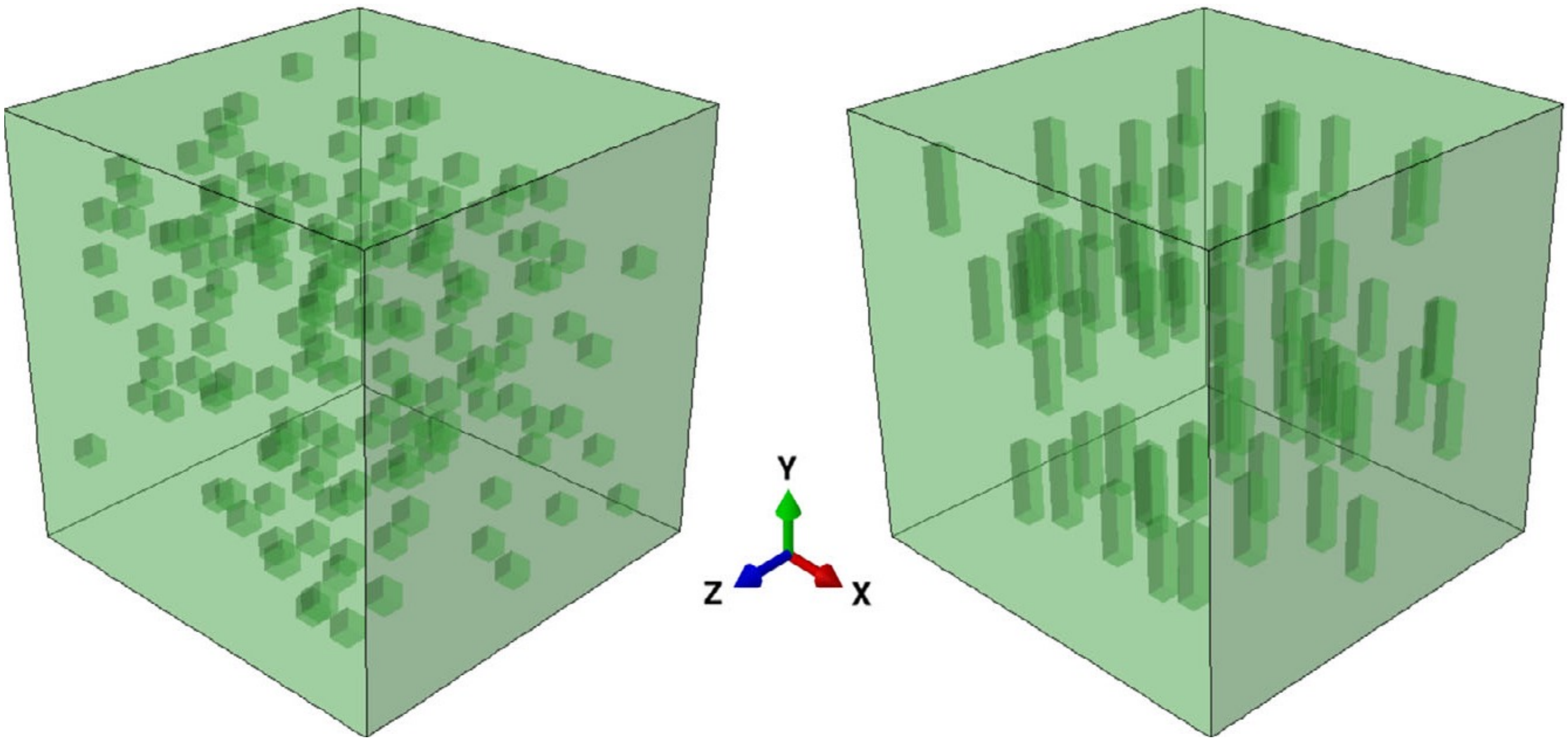
# Micromechanical Modeling




Dense polycrystalline RVE

- Homogenized stress-strain curve of dense 316L polycrystal
- Experimental data used to determine material parameters

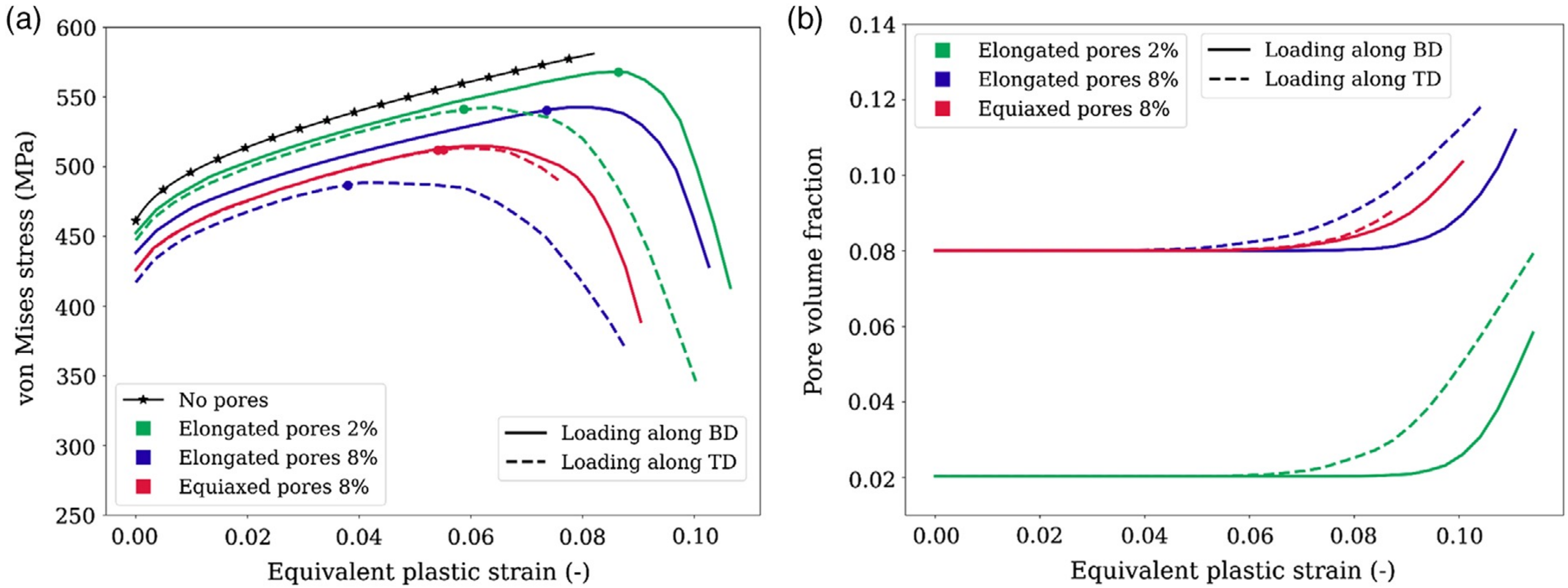
# Micromechanical Modeling




equiaxed pores | elongated pores

Representative volume elements (RVE) with porosity as second phase

# Micromechanical Modeling

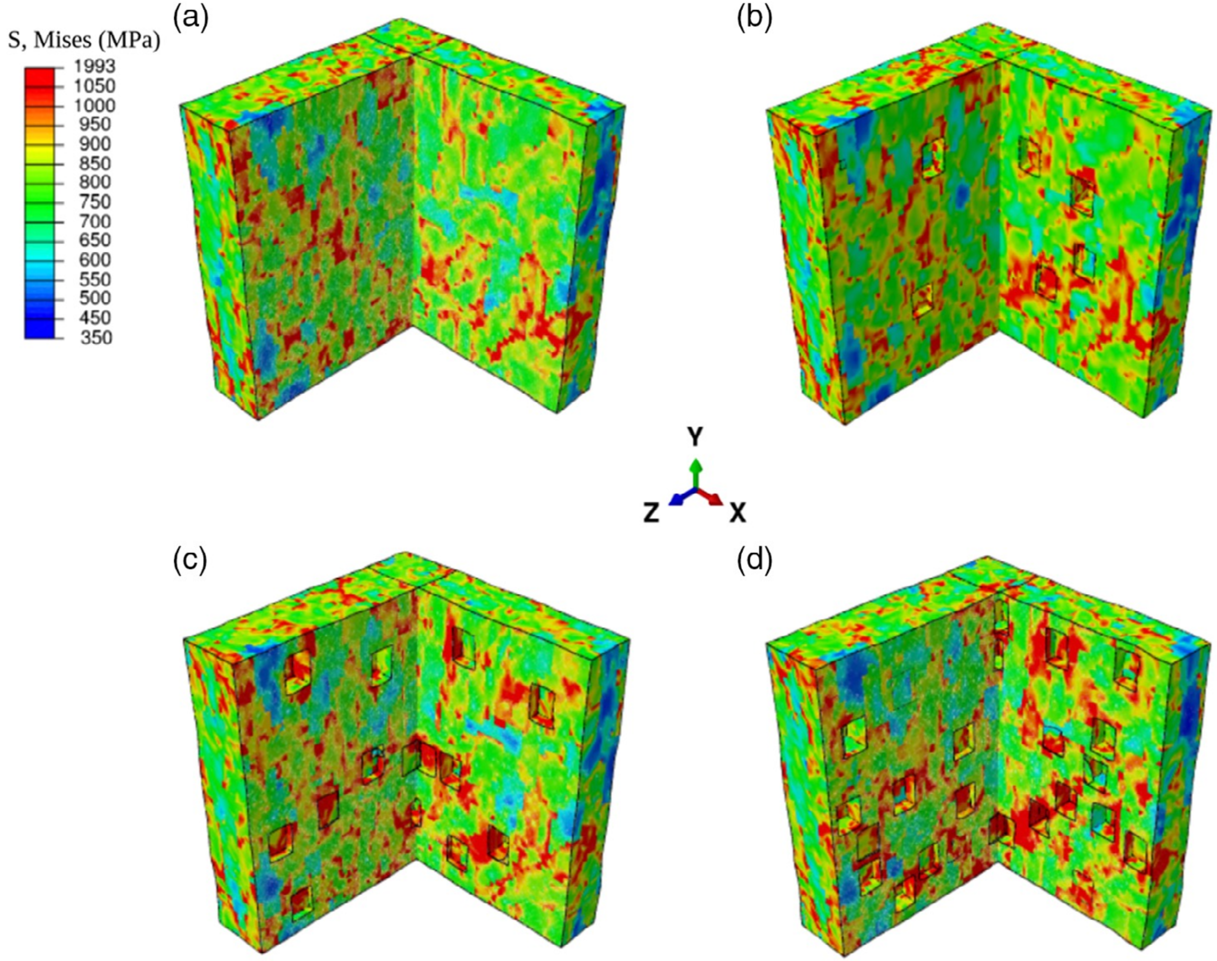



homogenized
stress vs. plastic strain curves

pore volume fraction as
function of plastic strain

- Comparison of RVEs with different pore shapes and volume fractions
- Point of instability calculated using the Considère criterion (filled circles)

# Micromechanical Modeling




Cross-sectional views of stress contours inside the RVEs
a) nonporous, b) 2% porosity, c) 5% porosity, d) 8% porosity

# Micromechanical material modeling

**Micromechanical modeling** = calculating the mechanical response of a simplified representation of a materials microstructure to mechanical loads

↓

representative volume elements (RVE)
for elastic properties also analytical homogenization methods

↓

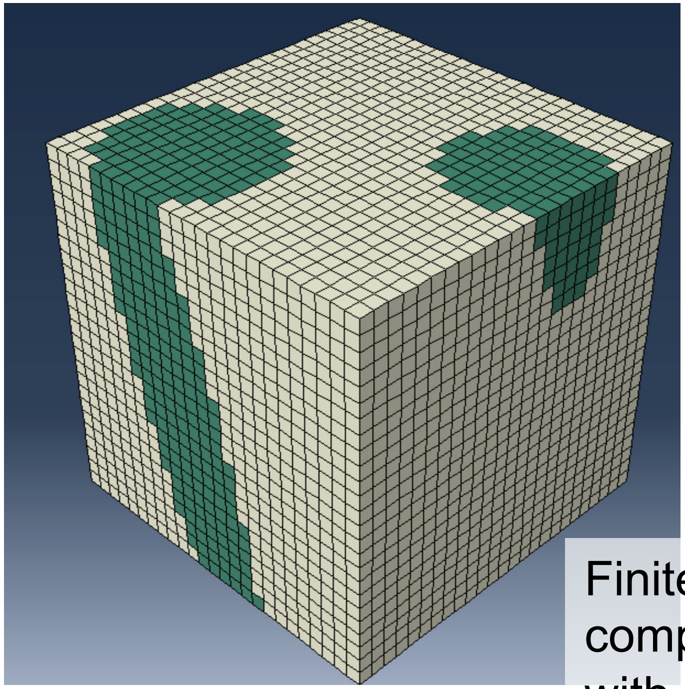
extrapolation to **macroscopic quantities** is achieved by **homogenization** methods (involving statistical treatments)

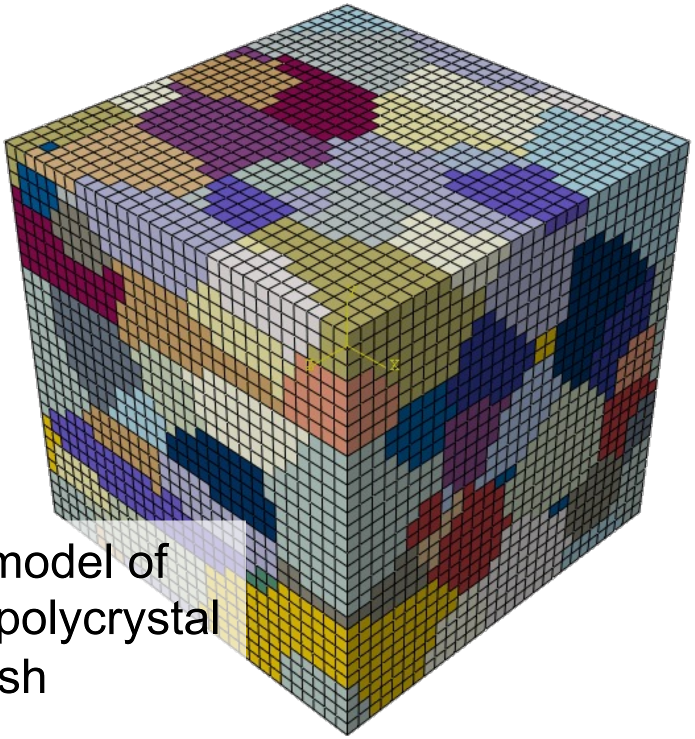
Recent advances in microscopic material description (crystal plasticity, cohesive zone models, etc.) and increasing computer power enables RVE's of reasonable sizes.

# Introduction into the Finite Element Analysis

**Basics of Finite Element Analysis (FEA):**

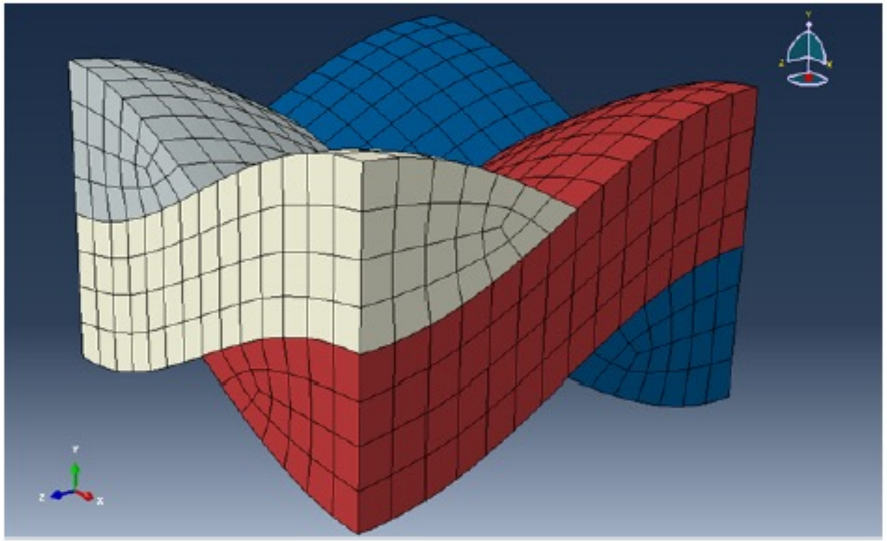
- Numerical method to calculate the **deformed shape** of a solid structure under mechanical boundary conditions (forces or distortions)
- **Mechanical equilibrium** is assumed: All forces on the structure are in balance, i.e. the sum of all forces acting on the surface is zero
- Discretization of the structure with **finite elements**





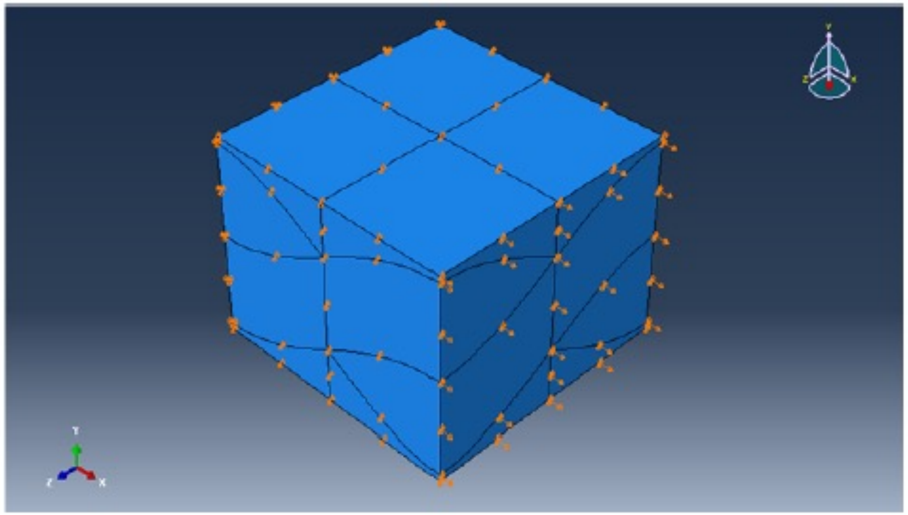
Finite element model of composite and polycrystal with regular mesh

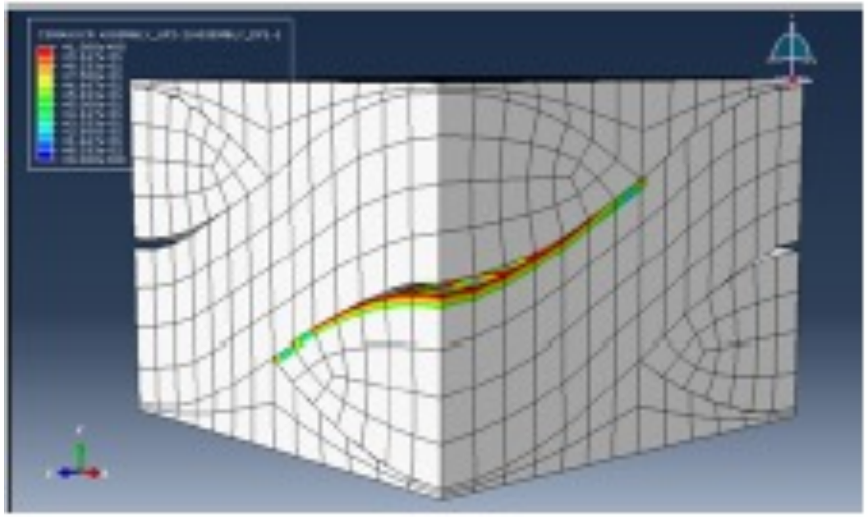
# FEA of fiber reinforced polymers



RVE of plain weave composite

- plasticity in matrix
- damage at interfaces



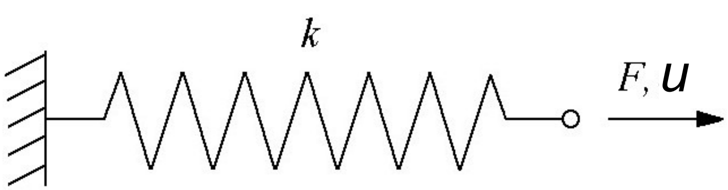


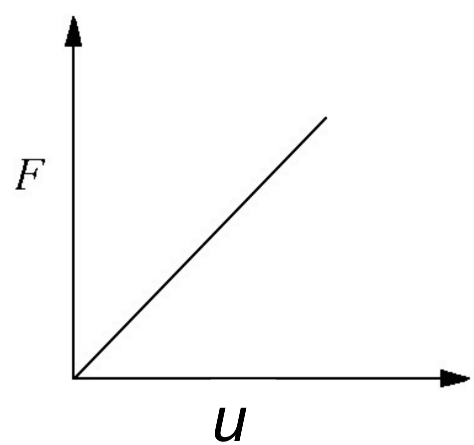
# Introduction into the Finite Element Analysis

**Basics of Finite Element Analysis (FEA):**
In a simple 1-d finite element model, we can envision the finite elements as linear springs.
In mechanical equilibrium under given boundary conditions, the forces and the deformed shape of the system can be calculated.
**Basic equation of FEA**

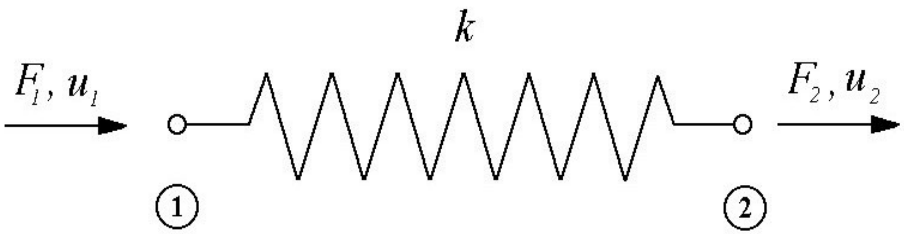
$$\underline{F} = \underline{\underline{K}}\,\underline{u}$$

Discretization (spring model)



## Discretization (spring model)

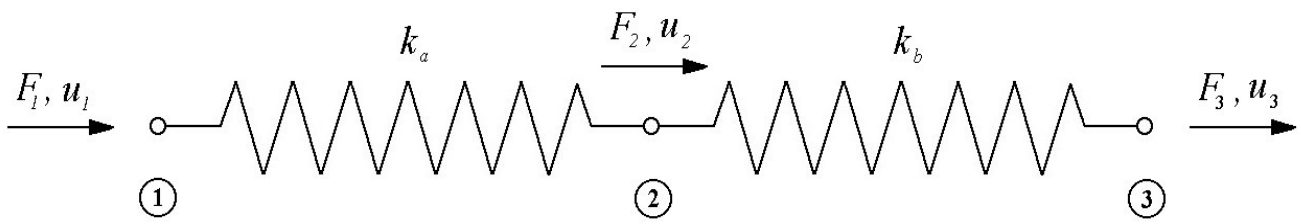



$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Element stiffness matrix for one-spring-system

$$[K] = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

# Introduction into the Finite Element Analysis




Element stiffness matrix

Element 1

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_a & -k_a & 0 \\ -k_a & k_a & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

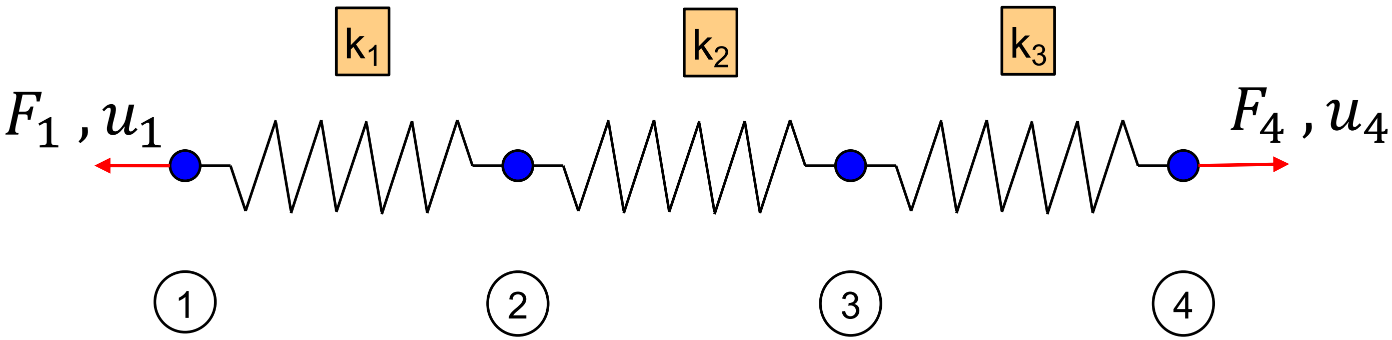
Element 2

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_b & -k_b \\ 0 & -k_b & k_b \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

system stiffness matrix

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_a & -k_a & 0 \\ -k_a & k_a + k_b & -k_b \\ 0 & -k_b & k_b \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

# Basics of Finite Element Analysis




$$\{F\} = \begin{Bmatrix} F^{(1)} \\ F^{(2)} \\ F^{(3)} \\ F^{(4)} \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{Bmatrix} u^{(1)} \\ u^{(2)} \\ u^{(3)} \\ u^{(4)} \end{Bmatrix} = [K]\{u\}$$

# Introduction into the Finite Element Analysis

Definition of stiffness matrix

[K]{u}={F}

Theoretical analysis of mechanical equilibrium yields

$$[K] = \frac{1}{2} \iiint_V [B]^T [C][B] dV$$

[C]: elasticity tensor, material property
[B]: strain-displacement-matrix: geometry, interpolation function

- Element stiffness matrix is calculated by volume integration over element
- System Stiffness matrix is assembled from element stiffness matrices

## Computation of strain field

- Strain is calculated by using shape functions to interpolate nodal displacements $u_j$ into finite element
- Conversion of displacement to strain is simple algebraic operation (B-matrix)

$$\varepsilon_\alpha(x,y,z) = B_{\alpha j}(x,y,z)\, u_j$$

$\alpha = 1,2,\ldots 6$ (tensor components in Voigt notation)

$j = 1,2,\ldots N$ (index over nodes of FE)

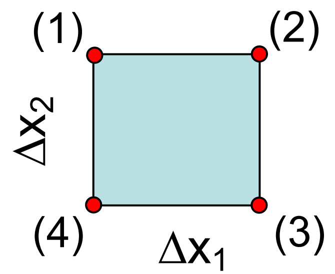
## stress field (linear elasticity: Hooke's law)

$$\sigma_\alpha(x,y,z) = C_{\alpha\beta}\, \varepsilon_\alpha(x,y,z)$$

$C_{\alpha\beta}$ (stiffness tensor)

# Strain tensor

- infinitesimally small volume element
- small strains




$\underline{u}^{(1)}$ displacement of point $\underline{x}^{(1)}$

$$\varepsilon_{11} = \lim_{\Delta x_1 \to 0} \frac{u_1^{(2)} - u_1^{(1)}}{\Delta x_1} = \frac{\partial u_1}{\partial x_1} = u_{1,1}$$

$$\gamma_{12} = \lim_{\Delta x_1 \to 0} \frac{u_2^{(2)} - u_2^{(1)}}{\Delta x_1} + \lim_{\Delta x_2 \to 0} \frac{u_1^{(1)} - u_1^{(4)}}{\Delta x_2} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} = u_{2,1} + u_{1,2}$$

general:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad \left(\gamma_{12} = 2\varepsilon_{12}\right)$$

### Computation of strain fields

- derivation of interpolation function with respect to spatial coordinate
- linear elements $\rightarrow$ constant strain within elements

$$\varepsilon_\alpha^{(j)} = \begin{pmatrix} \frac{\partial}{\partial x} & & \\ & \frac{\partial}{\partial y} & \\ & & \frac{\partial}{\partial z} \\ & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & \end{pmatrix} \begin{pmatrix} u_x^{(j)} \\ u_y^{(j)} \\ u_z^{(j)} \end{pmatrix} = L_{\alpha i} u_i^{(j)}$$

$$\alpha = 1,\ldots,6 \quad \text{(Voigt notation)}$$

$$j = 1,\ldots,N$$

$$i = 1,\ldots,3$$

Shape functions (interpolation functions) connect the displacement field within an element with the nodal displacements.

$$u_i(x,y,z) = N_{ij}(x,y,z)\, u_j \qquad i = 1,2,3 \qquad j = 1,\ldots,N$$

$u_j$ is (1xN)-matrix of nodal displacements: $\{u\}=(u_x^{(1)}, u_y^{(1)}, \ldots, u_z^{(M)})$

M: number of nodes, N: degrees of freedom (3N)

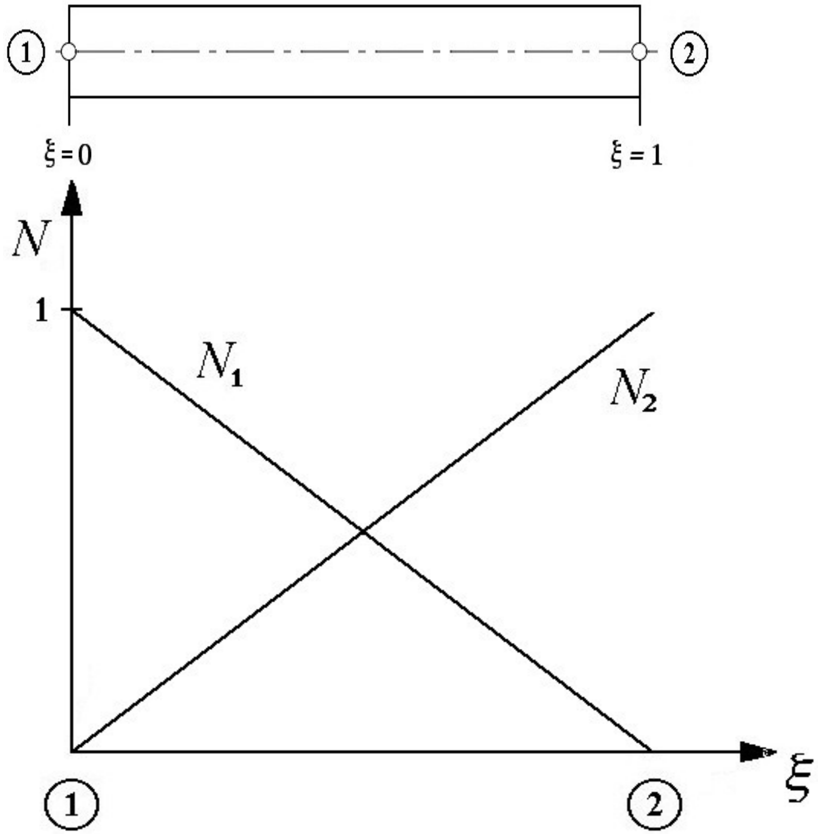
$u_i(x,y,z)$ is $i$-th component of continuous displacement field ($i$=1,2,3)

shape functions have the property

$$N_{ij}(x^{(k)}, y^{(k)}, z^{(k)}) = \delta_{jk} = \begin{cases} 1 & \text{für } j = k \\ 0 & \text{für } j \neq k \end{cases}$$

**Linear (one-dimensional) shape functions**

$$N_{11} = \frac{(\xi - 1)}{(0 - 1)} = 1 - \xi \qquad \text{und} \qquad N_{12} = \frac{(\xi - 0)}{(1 - 0)} = \xi$$

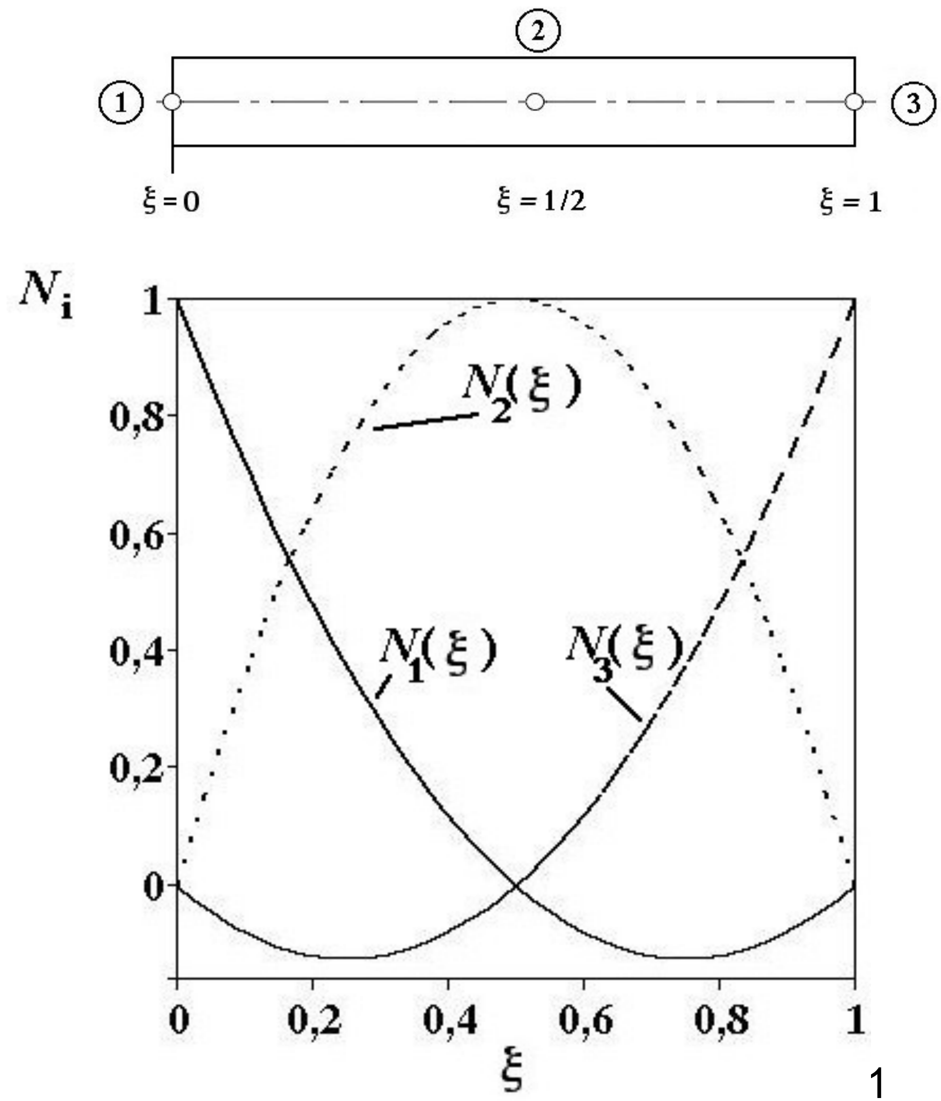


## Quadratic (one-dimensional) shape functions

$$N_{11} = \frac{(\xi - 1/2)(\xi - 1)}{(0 - 1/2)(0 - 1)} = (1-\xi)(1-2\xi) = 1 - 3\xi + 2\xi^2$$

$$N_{12} = \frac{(\xi - 0)(\xi - 1)}{(1/2 - 0)(1/2 - 1)} = 4\xi(1-\xi)$$

$$N_{13} = \frac{(\xi - 0)(\xi - 1/2)}{(1 - 0)(1 - 1/2)} = -\xi(1-2\xi)$$



**Computation of strain fields**

- derivation of interpolation function with respect to spatial coordinate
- linear elements $\rightarrow$ constant strain within elements

$$\varepsilon_\alpha(x,y,z) = L_{\alpha k} N_{kj}\, u_j = B_{\alpha j}\, u_j \qquad \alpha = 1,\ldots,6 \quad \text{(Voigt notation)}$$

$$j = 1,\ldots,N; k = 1,2,3$$

**stress field (linear elasticity: Hooke's law)**

$$\sigma_\alpha(x,y,z) = C_{\alpha\beta}\, \varepsilon_\beta = C_{\alpha\beta}\, B_{\beta i}\, u_i$$

# Elasticity

Hooke‘s law
(relation between stress and strain tensor)

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$$

in Voigt Notation:
(NOTE: no rotations possible!)

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma 23 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma 23 \end{pmatrix}$$

# Elasticity

(Voigt) Elasticity matrix (isotropic or cubic)

$$C_{\alpha\beta} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & & & \\ C_{12} & C_{11} & C_{12} & & & \\ C_{12} & C_{12} & C_{11} & & & \\ & & & C_{44} & & \\ & & & & C_{44} & \\ & & & & & C_{44} \end{pmatrix}$$

in case of isotropy: $C_{44} = \dfrac{C_{11} - C_{12}}{2}$

Normal strains create pure normal stresses
Shear strains create pure sheer stresses
No interference of normal/shear stresses and strains

# Micromechanical Modeling

**Micromechanical modeling is:**

- Microstructure-based prediction of mechanical properties of multiphase and polycrystalline materials
- Virtual mechanical lab based on finite element analysis (or other solvers)

**Micromechanical modeling requires:**

- Microstructure description in form of Representative Volume Elements (RVE)
- Representation of volume fraction and geometrical features of all phases and grains, including crystallographic orientations
- Constitutive models and parameters for all phases and interfaces

**Micromechanical modeling yields:**

- Evolution of local stress, plastic strain and damage within microstructure
- Global values of mechanical and physical quantities through homogenization
- Insights into deformation and failure mechanisms of different microstructures