Method

The brain can be considered as a distributed network of neurons. Brain network structure results in transfer, integration, and analysis of information which constructs the main part of a cognitive process. The sign of this integration and transfer of information is a kind of correlation between the electrical activities in different regions or the same regions of the brain. This correlation shows itself as various forms of neural activities synchrony across different regions of the brain. One type of synchronization in neural activities is across different band frequencies. This type of neural activity synchronization has been demonstrated to have an important role in various cognitive and neural functions. For example, [1] show that there is a synchronization between gamma and alpha band oscillations of temporal cortex regions during coordinate working memory storage. One of the most common types of synchronization across different bands is Phase Amplitude Coupling(PAC).

In PAC, there is a coupling between the phase of low-frequency activity and the amplitude of high frequency. To quantify this coupling for EEG signal, the time-frequency-based method in [2] is used. In this method, we considered that EEG signal is not Wide Sense Stationary(WSS) process. Thus, it is possible to distribute the energy of EEG signal over time and frequency. To do this, Reduced Interference Rihaczek time-frequency(RID-Rihaczek) distribution is used. To calculate RID-Rihaczek distribution, we have

$$C(t,f) = \iint \exp(\frac{-(\theta\tau)^2}{\sigma}) \exp(j\frac{\theta\tau}{\sigma}) A(\theta,\tau) \exp(-j(\theta t + 2\pi f\tau)) d\tau d\theta \tag{1}$$

where $\exp(j\frac{\theta\tau}{\sigma})$ is the kernel function for Rihaczek distribution, $\exp(\frac{-(\theta\tau)^2}{\sigma})$ is Choi-Williams kernel function used to filter cross-term for multi-component signals, $A(\theta,\tau)$ is calculated from EEG signal using the following formula

$$A(\theta,\tau) = \int x(u+\tau/2)x^*(u-\tau/2)\exp(j(\theta u))du \tag{2}$$

where $x(t)$ is an Event Related Potential(ERP) signal for each stimuli in oddball task. To calculate the phase activity of low frequency $\Phi_{fl}(t)$ and the amplitude of high frequency $A_{fh}(t)$, we use the following formula

$$A_{fa}(t) = \int_{fh1}^{fh2} C(f,t)df \tag{3}$$

$$\Phi_{fl} = arg[\frac{c(fl,t)}{|c(fl,t)|}] \tag{4}$$

where $[fh1, fh2]$ is a small range around the high frequency fa, high frequency of interest. Using Mean Vector Length(MVL) to calculate the coupling, we have:

$$MVL(fa, fl) = \Big| \frac{1}{N} \sum_{i=1}^{N} A_{fa}(t_i) \exp(j\Phi_{fl}(t_i)) \Big| \tag{5}$$

References

[1] Jonathan Daume. Phase-amplitude coupling and long-range phase synchronization reveal frontotemporal interactions during visual working memory. *JNEUROSCI*, pages 313–322, 2017.

[2] Tamanna T . K. Munia. Time-frequency based phase-amplitude coupling measure for neuronal oscillations. *Scientific reports*, pages 1–15, 2019.