NYU Center for Urban Science + Progress

Capstone Project Report
In partial fullfilment of the requirements for the
Master of Science in
Applied Urban Science & Informatics

Vulnerability of Transportation Networks 2.0

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2018 Capstones

1 ABSTRACT

Transport plays a crucial role in urban conurbations by providing access and mobility to its inhabitants. Disruptions to the regular flow of urban rapid transit systems can lead to major delays with far-reaching impacts. Building on prior research on transport network vulnerability in New York City subway system, this paper adopts the network model approach based on the shortest-path passenger flow simulation to quantify delays to examine the synergistic effects of simultaneous disruptions on the subway network of multiple cities across the world. We seek to identify if a universality of pattern exist in the disruption of subway systems globally, and identify the underlying factors for each observation. Our findings will contribute towards emergency preparation and mitigation plans against potential disruptions in complex transportation networks to enhance system resiliency in cities across the world.

2 INTRODUCTION

2.1 Project Scope

In many urban conurbations, public transport systems serve the critical role of transporting people throughout the city. Time spent in transit becomes a crucial factor determining the quality of public transport, liveability and the quality of life in cities. In order to maximize the effectiveness of rapid transit systems, cities need to be able to anticipate the repercussions of network disruptions. Due to the inherent networked structure of these systems, repercussions of a station (node) failure can have impacts throughout the entire network. While single node failure in rapid transit systems has received considerable attention Jenelius and Mattsson [2015], Ukkusuri and Holguín-Veras, Li and Ozbay [2012] studies involving failure in multiple nodes have been relatively unexplored. M'cleod et al. [2017] examined the cumulative impact of simultaneous failure in multiple nodes in the New York City subway system (a situation likely to occur during a natural disaster or terrorist attack). They highlighted the idea of synergistic effects, revealing that the combined impact of simultaneous disruptions may have more extensive impacts. Our study builds upon their research by extending these methods of synergistic analysis to a variety of major world cities to uncover any underlying patterns that may emerge across systems. Through a system-based analysis, our study incorporates data on service schedule and passenger origin-destination flows from London, San Francisco, Washington, DC, Taipei, and Singapore to study the effect of simultaneous disruptions on the network on multiple levels (geographic, station, platform and line). By including a robust dataset from cities across the world, aim to first design the weighted network before evaluating the model for commuter travel demand through origin-destination flows to assess the synergistic effects of simultaneous node disruptions.

2.2 Literature Review

Much study has been devoted to the structure and vulnerability of transport systems. Utilizing this research as a guide, we attempt to expand upon previously identified patterns found in urban rapid transit systems, particularly those examining synergistic effects of nodes shutting down, and search for possible universal features that emerge across networks in cities of varying size and network shape.

From a comparative framework, Derrible and Kennedy's examined the network of 19 cities using a multivariate regressionDerrible and Kennedy [2009]. The work defines ridership as the annual number of transit trips divided by the population, and measures network design through directness, connectivity and coverage. Ridership, computed as a function of population, incorporates three main factors: population, density, and urban area; these were selected to characterize and standardize the dataset for comparison between the 19 cities selected. The study brings to light many of the issues they faced in terms of data collection and standardizing across such a broad range of cities; with five cities instead of nineteen, our proposed scope is more narrow in focus, and the cities we have selected have generally similar methods of recording their data (see Section 2.1 for discussion), alleviating many of the issues facing Derrible and Kennedy.

Han and Liu examined passenger flow through the Beijing subway system using topological information and individual passenger movement data Han and Liu [2009]. They constructed a directed weighted passenger flow network, treating the subway network as a series of undirected graphs, unweighted edges and vertices. They defined degree distribution as the number of edges connecting to a subway station (node) and explained station degree through a distribution $p(K)$, returning a degree probability of random selected stations. The conclude that the networks under study are resilient to the random loss of nodes but are vulnerable to attacks on high-degree hubs. Their study is useful for us as their approach towards examining multi-nodal shutdowns is similar to our analysis of paired and single node disruptions.

Yin et al also analysed disruptions on the Beigin rail transit system Yin et al. [2016]. However, unlike Han et al, they examined betweenness based on shortest path, reasonable alternative paths, shortest paths per line or per stations, and passenger flow; these three criteria are used to determine global efficiency in the event that a disruption were to occur on the rail transit system. Their method of evaluating line and station importance is relevant to our research in measuring station centrality, betweenness and criticality.

Sun constructed the network of Shanghai Metro system using a proposed vulnerability evaluation model Sun et al. [2015]. Both Yin and Sun found that the subway systems are robust to random disruptions, but more vulnerable to intentional nodal based disruptions. This suggests that certain combination of nodal disruptions may have larger impact on the network. Kim identified individual critical nodal disruptions of the Washington transit system by evaluating network reliability and system flow loss by using origin and destination passenger dataKim et al. [2015]. Kim highlighted that hubs may not be the most important nodes in the network. In contrast to traditional network vulnerability, he suggests that non-transfer stations with high volumes of passenger flow or bridge stations to hubs may be more critical in terms of network vulnerability.

3 DATA

3.1 Data sources

3.1.1 London, United Kingdom Data is readily available from the London Data Store, broad data on station entry and exit for weekday and weekends can be used to identify general travel patterns in the city. More granular data from the Oyster card containing individual entry and exit in the London Underground may be accessed. While the data available includes 5 percent of commute in November 2009, the granularity of the data will be useful to better model travel demand. Station locations are available via a geocoded KML [1], and schedule information can be extracted from the Transport for London live feed [2].

3.1.2 Singapore Data containing commuter travel data is recorded through the ez-link card. This dataset is not openly available, but may be obtained directly from the Land Transport Authority. Approval is currently being sorted for two weeks of data in January 2018, one for a randomly selected week, and another for a week coinciding with a public holiday. Station locations are available in the form of a shapefile, and schedule information can also be extracted from a live feed available from the SMRT Connect App [3]

3.1.3 Taipei, Taiwan Data containing commuter travel data is recorded through the

[1] https://api-portal.tfl.gov.uk/docs

[2] https://api.tfl.gov.uk/line/24/timetable/490000036s

[3] https://www.mytransport.sg/content/mytransport/home/dataMall.html

easycard, a contactless travel card on the MRT network. The dataset is openly available from Data.Taipei, containing origin-destination passenger flows and counts for each hour of the day. The records for the month of February 2017 will be used. Station locations are available in the form of a shapefile [4]

3.1.4 Washington, D.C., United States Data for Washington, D.C. was collected from two primary sources, a publicly available dataset on Kaggle [5] which lists weekday ridership during four specified time-slots (AM Peak = opening to 9:30am, Midday = 9:30am to 3:00pm, PM Peak = 3:00pm to 7:00pm, Evening = 7:00pm to midnight, Late-Night Peak = Friday and Saturday nights only - midnight to closing) during the month of May 2012; while this is hardly comprehensive data, we believe that it will sufficiently serve our purposes for identifying the general network flow between stations in the city. To acquire structural data for the Washington D.C. Metro, we used the Washington Metropolitan Area Transit Authority (WMATA) API [6], compiling station location, line structure, and distance metrics into a central dataframe to be combined with the passenger-flow data, which provides the weightings to the network structure.

3.1.5 San Francisco Bay Area, United States Real time arrivals, schedules, delays, and station area information are available through Bay Area Rapid Transit (BART) API [7]. Geospatial data includes station locations, entrances, etc are available in KML or KMZ format. Ridership data is also available [8], compiled yearly on the site. We will be using data from 2017 (as it is the most recently available). This data contains origin and destination points and a timestamp (rounded to the nearest hour) for each ride; by aggregating these datapoints, we will achieve an nearly identical data-format to Washington, D.C. data.

3.2 Final dataset

We incorporate two measures to evaluate subway network vulnerability. The first is an indicator of economic value measured by travel time, and second, an indicator of demand measured by passenger flow between nodes Jenelius [2009], M'cleod et al. [2017].

Our study is can be divided into two parts. First, modeling travel demand flows within the subway network, and second evaluating the synergistic effect of simultaneous disruptions based on node criticality. Two types of passenger travel data will be required for our analysis.

Type 1 data refers to turnstile entries and exit counts at each station. This data is commonly available in cities with the turnstile system such as New York City and Paris. Type 1 data does not include information on passenger travel

[4] http://data.taipei/opendata/datalist/listDataset

[5] https://www.kaggle.com/jmataya/metrorail2012/data

[6] https://developer.wmata.com/docs/services/5476364f031f590f38092507/operations/5476364f031f5909e4fe330e

[7] https://www.bart.gov/schedules/developers/geo

[8] https://www.bart.gov/about/reports/ridership

flows through the network. Therefore, some preprocessing is required to model the directed passenger flows. M'cleod et al. [2017] used commuter home-to-work data from the American Association of State Highway (AASHTO) to model passenger flow between network nodes and verified the model using MTA turnstile data.

Type 2 data refers to individual passenger entrance and exit data collected through the smart card system (RFID card). Type 2 data is only available from a few selected cities due to the high infrastructural outlay required for smart card systems, and the complexities of obtaining this data from the relevant authorities.

Our study uses Type 2 data available from 5 cities to evaluate the effects of simultaneous node disruptions on the subway network and use the data to train a mobility model to further extrapolate to other cities with only Type 1 data available.

In addition to passenger data, information on the physical network and train schedules are also required. The locations of subway stations can be obtained either through the official transit websites in shapefile or geojson format or extracted from OpenStreetMaps. The train schedules can either be obtained from official subway websites in the General Transit Feed Specification (GTFS) format or through extracting from transit apps. Using these datasets, a directed weighted subway network will be constructed, consisting of major nodes and subnodes, and other attributes such as transfer and travel times.

Given that our study incorporates datasets from 5 major cities across the globe, it is crucial that the data is standardized in a consistent format for comparative and scalable analysis.

3.3 Exploratory Data Analysis

We conducted exploratory data analysis for London, Singapore, Taipei and Washington, DC using the origin-destination flow data to better understand the travel patterns in each city. The data was aggregated to reflect the number of people entering and exiting each station during the morning (AM peak) and evening (PM peak) rush hours. AM peak was defined as the time between 5am to 10am, while evening peak was defined as between 3pm to 7pm.

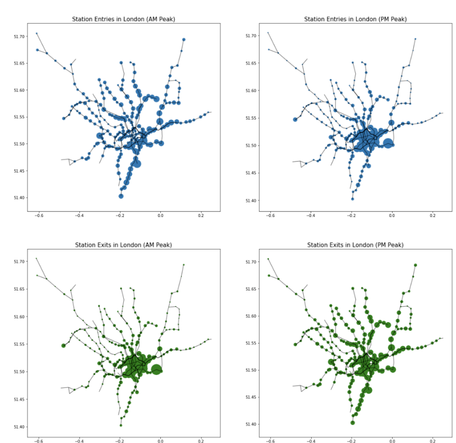


Figure 1. Commuter volumes at each station during AM and PM peak in London

3.3.1 London Figure 1 shows the entries and exits during both AM and PM peak travel times in London. During the AM peak, we observe a clear even pattern of commuter entries across the whole subway network. From the plot of station exits at AM peak hours, we find the highest volume in Central London where people go to work. The pattern of station entries at PM peak hours is very similar to the exits at AM peak hours when commuters heading home. The pattern of station exits at PM peak hours is similar to morning entries. However, some differences are observed in middle and west of Central London where might be popular tourist destinations.

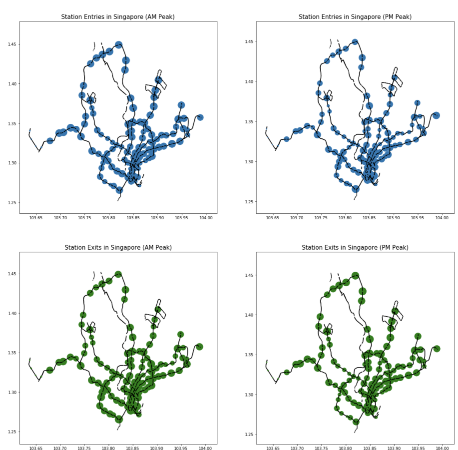


Figure 2. Commuter volumes at each station during AM and PM peak in Singapore

3.3.2 Singapore Figure 2 shows the entries and exits during both AM and PM peak travel times in Singapore. Unlike the other cities, Singapore has a relatively even pattern of entries and exits throughout the whole system. This may attributed to the planning policy in Singapore, resulting in a even distribution of residential and economic activities across the city state. Nevertheless, from the AM peak entries and exits plot, we can observe a movement of people towards the central business district. However, during the PM peak, the pattern is less pronounced, possibly due to the longer working hours or availability of lifestyle and post-work activities in the central business district.

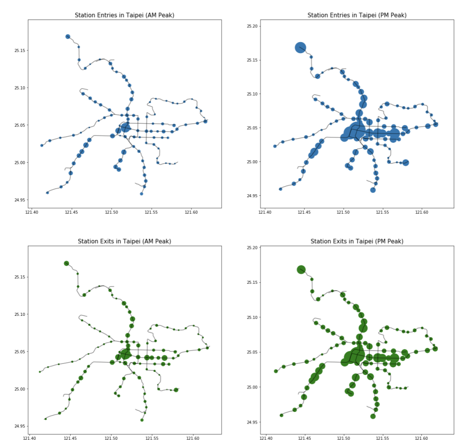


Figure 3. Commuter volumes at each station during AM and PM peak in Taipei

3.3.3 Taipei Figure 3 shows the entries and exits during both AM and PM peak travel times in Taipei. During the AM peak, we observe a relatively even pattern of commuter entries into stations across the entire subway network. In comparison, more commuters are exiting the stations along the Tamsui-Xinyi line, and the stations in downtown Taipei near the Zhongzheng District. This pattern may be reflective of the underlying land use in the city. On the other hand, during the PM peak, there are relatively large volumes of commuters entering the stations on multiple lines across the city, but the highest volumes are observed near the Zhongzheng District, as well as Tamsui station which is situated in a popular seaside district. A similar pattern can be observed for station exits during the PM peak where most stations along all lines have relatively similar volumes of commuters exiting the station, with the exception of again the Zhongzheng District where large volumes of commuters are exiting. This suggests that this district may be a popular night destination.

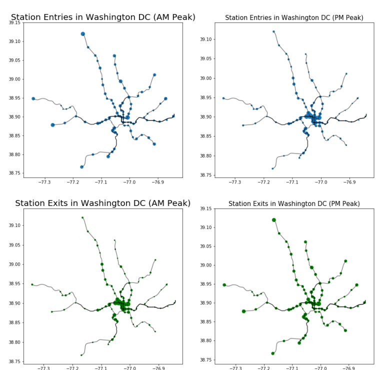


Figure 4. Commuter volumes at each station during AM and PM peak in Washington, DC

3.3.4 Washington, DC Figure 4 shows the entries and exits during both AM and PM peak travel times in Washington, DC. The pattern of exits during AM peak and entry during PM peak are very similar, reflecting a possible underlying pattern of home-work flows in the subway system.

3.3.5 Power Law of Passenger Flow The passenger flow within the subway network across four cities was compared by taking a linear regression with logarithmic transformation (Figure 5). The results reveal that passenger flow in all four cities largely follow the power law.

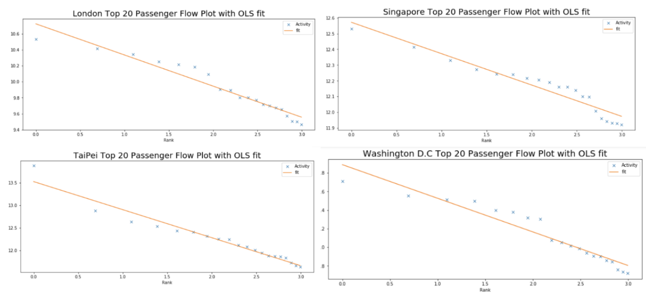


Figure 5. Top 20 passenger flow in London, Singapore, Taipei and Washington, DC

4 METHODOLOGY

Our research method can be divided into three major components. First modeling the subway network as a directed, weighted network. Second, collecting Type 2 individual passenger origin-destination travel data. Thirdly, assessing the criticality of single and paired nodes and examine the synergistic effects to provide a comprehensive understanding of the vulnerability of the network

4.1 Constructing Network

Firstly, we modeled the subway system for each city within the study as a directed, weighted network; stations will be denoted as master nodes, and transfer stations with multiple platforms for different lines will be denoted as sub-nodes (Figure 6). The connections between all origin and destination pairs of stations will be given a weight represented by the travel time, transfer walk time, and waiting time. Using the origin-destination travel data derived from smart card data (Type 2), passenger demand within the subway system will be measured. Our study has identified five cities where Type 2 data is available.

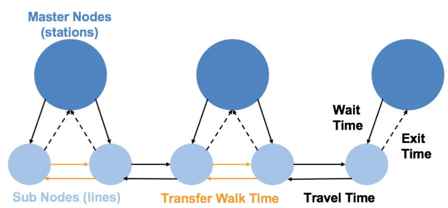


Figure 6. Directed and Weighted Network Structure

4.2 Modeling Travel Demand

However, as Type 1 data is more widely available in most cities without individual passenger entry and exit data, our study aims to model the directed passenger flow through the subway network by combining Type 1 data with commuter home-to-work data. The home to work data is available from three sources: American Association of State Highway and Transportation Officials (ASSHTO) [9] where subway commuter origin-destination matrices on the census tract level is available. Detailed employment information at the block level can also be obtained from the US Census Longitudinal Employer-Household Dynamics (LEHD) [10].

There are several approaches to model Type 2 data from Type 1 data. Gravity law assumes that the number of trips is related to the population residing at the origin and destination, and inversely related with distance Zipf [1946], Erlander and Smith [1990]. Conversely, Stouffer's theory of intervening opportunities suggests that distance has no effect on destination choice, instead the number of opportunities play are more crucial role in determining location choices Stouffer [1940]. The radiation law Simini et al. [2012], Yang et al. [2014]is similar to the gravity law, but is more dynamic in considering the costs and benefits of travel.

We aim to compare the different travel flow models to estimate the travel patterns of subway commuters, testing the trip distribution laws against Type 1 commuting data from one of 5 cities in our study. With available Type 2 data, we will be able to validate our model, providing a more accurate measure of commuter demand for cities. Overall, through modeling Type 2 data from the more readily available Type 1 data, we aim to contribute to research by expanding the applicability of our study to a wider range of cities around the world. We demonstrate the flexibility of our model by validating our model on multiple cities with both data types.

4.3 Criticality and Synergistic Effects

After building the network structures and obtaining travel demand for each city, the travel time and delays on each subway journey from node to node was computed using the Dijkstra algorithm Dijkstra [1959]. In applying the Dijkstra

[9] https://www.transportation.org

[10] https://lehd.ces.census.gov/data/

algorithm, we calculated the shortest network path by assuming that commuters are perfectly rational and have perfect information on the fastest route from origin to destination. We reran the algorithm for travel between all nodes while constantly removing nodes to simulate travel under disruption. This allows us to measure the extent of travel delay depending on which part of the system is disrupted.

We assessed the level of impact generated from any disruption by measuring the amount of "Passenger Hours"lost when any one node, pairs or sets of nodes are removed from the network. "Passengers Hoursïs defined as the total time loss as a result of a network disruption, measured as the product of the number of passengers traveling on that route, and the travel delay computed through the Dijkstra algorithm.

$C(X) = \sum_{a,b} D(a, b, X) \times P(a, b)$

This allow us to compute the criticality scores of each node, pairs or set of nodes. With these criticality scores, we aim to assess the synergistic effects of simultaneous disruptions, by assessing the extent in which the criticality scores of simultaneous paired disruptions differs from the criticality scores from the sum of separate disruptions on single nodes.

$S(x_1,\ x_2) = C(x_1,\ x_2) - (C(x_1) + C(x_2))$

4.4 Single Node Disruption

We constructed the subway network for London, Singapore, Taipei, and Washington, D.C. The structure of each network and the levels of complexities are different. After building the networks, the Dijkstra algorithm was applied to calculate the shortest network path, and travel under disruption was simulated by removing single nodes. The resultant travel duration was computed and compared with that under normal conditions, criticality scores are then computed by multiply the delay time by passenger flow demand. The constructed networks and top stations with highest single node criticality scores for each city are shown below.

4.4.1 London Network The London network (Figure 7) comprises 270 master nodes and 390 sub nodes. There are total 660 nodes and 2274 edges. The average in and out degree is 3.4455. It is by far the most complex network in our study. The top 5 stations with highest criticality score during AM peak hours on weekdays are shown in Table 1. The entry and exit counts are also listed. The relatively small number of passenger counts compare to other cities are because that the demand data is a 5% sample. The criticality score by station and a zoomed in plot for Central London are shown in Figure 8.

Station	Entry	Exit	Criticality
King's Cross St.Pancras	4176	2781	1560166
Euston	2040	1036	1177994
Stockwell	402	1522	1135985
Baker Street	2864	1025	1059683
Finsbury Park	600	2179	1032780

Table 1. Stations with top 5 criticality scores

It is observed that major transfer hubs such as King's Cross St.Pancras, Euston,

Figure 7. Nodes and Edges connected in London network

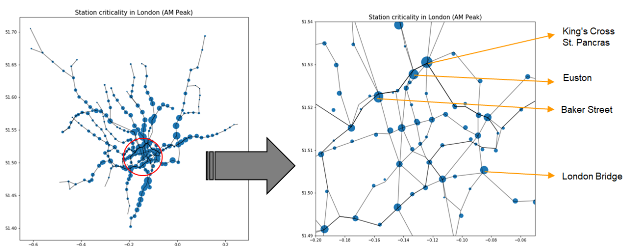


Figure 8. Criticality scores by station in London

and London Bridge are identified with the highest single node criticality score, this can be attributed to the high passenger activity in these stations.

4.4.2 Singapore Network The Singapore network comprises 118 master nodes and 177 subnodes. As a relatively complex network, there are a total of 295 nodes and 835 edges. The completed network shows all the subnodes connected by transfer edges and edges travelled by trains (Figure 9). The top 5 stations with the highest criticality score during the AM peak hours are shown in Table 2.

Figure 10 shows the criticality scores by station in Singapore. Criticality scores are highest at key junctions such as Jurong East, Botanic Gardens and Kovan,

Figure 9. Nodes and Edges connected in Singapore network

Station	Entry	Exit	Criticality
Jurong East	86244	190391	2624991000
Serangoon	139871	51251	2124856000
Chinese Garden	31065	11030	1844449000
Lakeside	112132	34485	1832155000
Kovan	52484	21433	1552727000

Table 2. Stations with top 5 criticality scores

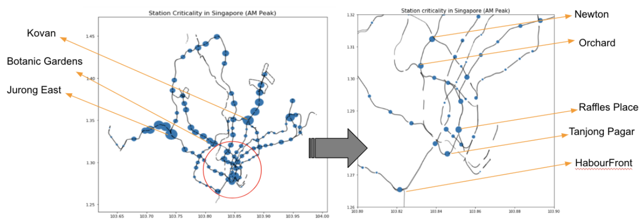


Figure 10. Criticality scores by station in Singapore

where more than one line intersect. In the central business district in downtown Singapore, the highest criticality scores are again observed in key transfer stations, Raffles Place and Newton, as well as Orchard and Tanjong Pagar where there are high commuter volumes. From table 2, we observe that stations with the highest criticality scores are situated at the western and northeastern part of the network. And while Jurong East, Serangoon and Lakeside have very high passenger volumes and high criticality scores, Kovan and Chinese Garden have comparatively lower passenger volumes but similarly high criticality scores. This observation is similar to that highlighted by Kim et al. [2015] in his study on Washington, where he highlighted that non-transfer stations with high passenger flow volumes and bridge stations may be more critical.

the highest criticality scores in Taipei is shown in Table 3.

Figure 11. Nodes and Edges connected in Taipei network

Station	Entry	Exit	Criticality Score
Mingquan W Rd	68174	96708	4880281000
Taipei Main Station	521072	543824	4051424000
Ximen	196396	196272	4030400000
Guting	94130	113934	3695281000
Yuanshan	54325	51475	2970461000

Table 3. Stations with top 5 criticality scores

4.4.3 Taipei The Taipei network comprises 106 master nodes and 167 subnodes. With a land area smaller than that of Singapore, it has a relatively wide coverage, with a total of 273 nodes and 759 edges (Figure 11). The top 5 stations with

Figure 12 shows the criticality scores by station in Taipei. Similar to the other cities, the stations with the highest criticality scores are mainly those at key junctions serving more than one line of service. Other stations with relatively higher

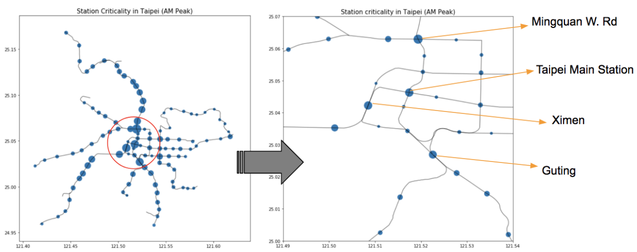


Figure 12. Criticality scores by station in Taipei

criticality scores are those with high commuter volumes. Notably, Mingquan W. Rd station has the highest criticality score despite having lower passenger volume compared to Taipei Main Station and Ximen station. This may be because Mingquan W. Rd station provides key connectivity on the Tamsui-Xinyi line, allowing for transfer opportunities to other parts of the city.

4.4.4 Washington D.C The Washington D.C. network is the smallest with 233 nodes and 696 edges. Despite it being much smaller in scale compared to London, it demonstrates very similar patterns previously in terms of commuter home-to-work travel flows. The highest criticality scores are observed in station downtown (Table 4), where most people are exiting during the AM peak, signaling the role of these stations as gateway to work opportunities. From Table 4, we observe that similar to Taipei, L'Enfant Plaza has the highest criticality score despite having a comparatively low passenger flow volume. However, the highest criticality scores are only observed at two junctions, with other junctions with lower criticality scores (Figure 13).

Station	Entry	Exit	Criticality Score
L'Enfant Plaza	68174	96708	15603077
Pentagon	521072	543824	15159951
Pentagon City	196396	196272	13868940
Medical Center	94130	113934	11562871
Takoma	54325	51475	11476806

Table 4. Stations with top 5 criticality scores

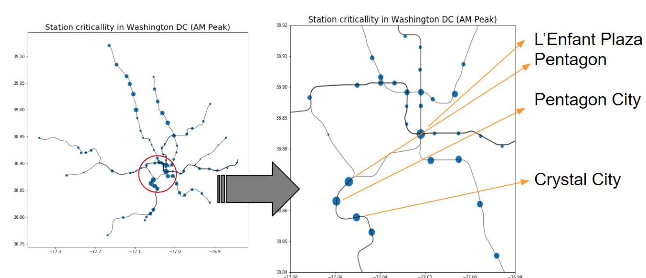


Figure 13. Washington D.C Single Node Disruption Critical Score

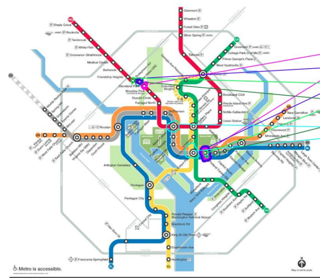
Overall, results from the single node disruption from these four cities reveal that transfer stations which serve more than one line are most commonly stations with the highest criticality scores, as they provide key connectivity for commuters moving within the network. In addition, stations with high commuter volumes also have high criticality scores. However, while London, Taipei and Washington D.C. reflect a similar trend where the highest criticality scores are observed downtown in the city center, Singapore has a relatively different pattern. The highest criticality scores were observed in transfer stations to the west and north-eastern parts of the island which serve key industrial and residential activities. The relatively even distribution of criticality scores across the network may be attributed to the underlying distribution of the population and the planning policies guiding land use in Singapore, and its unique position as a city-state.

4.5 Pair Node Disruption

To gain additional insight on the existence of synergies within the network, pair node disruption was simulated and the criticality scores was compared to the sum of two single node disruption. Pair node disruption for simulated all pairs of stations in Washington DC, given the relatively small network size. The criticality of each pair of nodes is computed using the same method, multiplying the sum of delay by passenger demand. This allows us to examine the existence of 'synergy' in the network by measuring the degree in which the criticality of a pair of nodes exceeds the sum of the criticality of two single disruptions. It was previously found that most synergy values cluster around 0, and only 13 percent of all station pairs in New York City was found to have postive synergy M'cleod et al. [2017].

Our results in Washington, DC reveal that only 5 percent of all station pairs have positive synergies (Figure 14).

The difference in scale of the network in New York City and Washington, DC,



pair_station_name	synergies
[Cleveland Park_0, Federal Center SW_0]	19592801.0
[Federal Center SW_0, Van Ness-UDC_0]	18767641.0
[Federal Center SW_0, Tenleytown-AU_0]	18014452.0
[Capitol South_0, Van Ness-UDC_0]	17888584.0
[L'Enfant Plaza_0, Metro Center_0]	17739865.0

Figure 14. Positive synergies observed in stations in Washington, DC

suggest a possibility that a relationship may exist between the scale of a network and the existence of positive synergies in the network. We aim to uncover any underlying patterns through our study of synergistic effects in 5 cities.

5 CONCLUSIONS

Our study involving 5 major cities from North America, Europe, and Asia contributes to existing research by providing insight on the differences in subway structure and travel patterns in different cities across the globe. Different insights can be gleaned from each component of our study. The first component on designing the weighted network provides a more focused understanding of the structure and topological characteristics of subway networks around the world. Our second component modeling and evaluating travel demand contributes to existing research on travel demand forecasting and trip distributions. Lastly, our third component on assessing criticality and synergistic effects may reveal important universalities and other city specific patterns through the distribution of disruptions.

Our findings on synergistic effects will help inform city planners and agencies in improving the security and resilience of the city. In subsequent analysis, we aim to uncover any universalities and patterns which may explain the synergistic effects observed within the city. This may provide additional insights on our existing understanding of the vulnerability of urban transportation networks.

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