

Laboratory 04

Finite Element method for the diffusion-reaction equation in 2D: convergence analysis

Exercise 1.

Let $\Omega = (0, 1) \times (0, 1)$, and let us consider the following diffusion-reaction problem with homogeneous Dirichlet boundary conditions:

$$\begin{cases} -\nabla \cdot (\mu \nabla u) + \sigma u = f & \mathbf{x} \in \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1a)$$

$$(1b)$$

where $\mathbf{x} = (x, y)^T$, $\mu(\mathbf{x}) = 1$, $\sigma = 1$ and

$$f(\mathbf{x}) = (20\pi^2 + 1) \sin(2\pi x) \sin(4\pi y) .$$

The exact solution to this problem is

$$u_{\text{ex}}(x, y) = \sin(2\pi x) \sin(4\pi y) .$$

1.1. Write the weak formulation, the Galerkin formulation and the finite element formulation of (1).

1.2. Starting from the code of Laboratory 3, implement a finite element solver for problem (1). The solver should read the mesh from file (four differently refined meshes are provided as `mesh/mesh-square-*.msh`).

1.3. Using the four meshes provided, study the convergence of the solver for polynomials of degree $r = 1$ and of degree $r = 2$. Plot the error in the L^2 and H^1 norms against h , knowing that for every mesh file `mesh/mesh-square- N .msh`, the mesh size equals $h = 1/N$.

Exercise 2.

Let Ω be the domain depicted in Figure 1, contained in the files `mesh/mesh-u-*.msh`. The boundaries of the mesh are labelled as shown in Figure 1 (i.e. Γ_0 is labelled 0, Γ_1

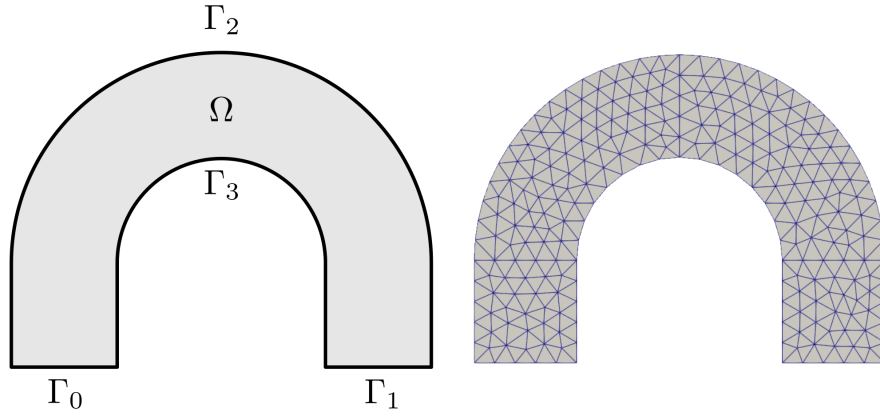


Figure 1: Domain for Exercise 2 (left), and a triangular mesh over it (right), corresponding to the file `mesh/mesh-u-5.msh`.

is labelled 1, and so on). Consider the problem:

$$\begin{cases} -\nabla \cdot (\mu \nabla u) = 0 & \text{in } \Omega , \\ u = 0 & \text{on } \Gamma_0 , \\ u = 1 & \text{on } \Gamma_1 , \\ \mu \nabla u \cdot \mathbf{n} = 0 & \text{on } \Gamma_2 \cup \Gamma_3 . \end{cases}$$

2.1. Starting from the previously implemented code, solve (2) using linear finite elements.