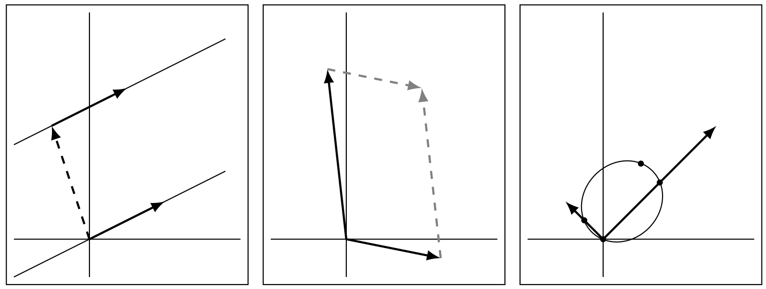
A



Solutions To Selected Problems

Chapter 1

Section 1.1

1. y
2. n
3. y
4. y
5. n
6. n
7. y
8. n
9. y
10. n
11. $x = 1, y = -2$
12. $x = 2, y = \frac{1}{3}$
13. $x = -1, y = 0, z = 2$
14. $x = 1,\ y = 0,\ z = 0$
15. 29 chickens and 33 pigs
16. 12 \$0.30 trinkets, 8 \$0.65 trinkets

Section 1.2

1. $\begin{bmatrix} 3 & 4 & 5 & 7 \\ -1 & 1 & -3 & 1 \\ 2 & -2 & 3 & 5 \end{bmatrix}$
2. $\begin{bmatrix} 2 & 5 & -6 & 2 \\ 9 & 0 & -8 & 10 \\ -2 & 4 & 1 & -7 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 3 & -4 & 5 & 17 \\ -1 & 0 & 4 & 8 & 1 \\ 2 & 3 & 4 & 5 & 6 \end{bmatrix}$
4. $\begin{bmatrix} 3 & -2 & 4 \\ 2 & 0 & 3 \\ -1 & 9 & 8 \\ 5 & -7 & 13 \end{bmatrix}$
5. $\begin{aligned} x_1 + 2x_2 &= 3 \\ -x_1 + 3x_2 &= 9 \end{aligned}$
6. $\begin{aligned} -3x_1 + 4x_2 &= 7 \\ x_2 &= -2 \end{aligned}$
7. $\begin{aligned} x_1 + x_2 - x_3 - x_4 &= 2 \\ 2x_1 + x_2 + 3x_3 + 5x_4 &= 7 \end{aligned}$
8. $\begin{aligned} x_1 &= 2 \\ x_2 &= -1 \\ x_3 &= 5 \\ x_4 &= 3 \end{aligned}$
9. $\begin{aligned} x_1 + x_3 + 7x_5 &= 2 \\ x_2 + 3x_3 + 2x_4 &= 5 \end{aligned}$
10. $\begin{bmatrix} -2 & 1 & -7 \\ 0 & 4 & -2 \\ 5 & 0 & 3 \end{bmatrix}$
11. $\begin{bmatrix} 2 & -1 & 7 \\ 5 & 0 & 3 \\ 0 & 4 & -2 \end{bmatrix}$
12. $\begin{bmatrix} 2 & -1 & 7 \\ 2 & 3 & 5 \\ 5 & 0 & 3 \end{bmatrix}$
13. $\begin{bmatrix} 2 & -1 & 7 \\ 0 & 4 & -2 \\ 5 & 8 & -1 \end{bmatrix}$
14. $\begin{bmatrix} 2 & -1 & 7 \\ 0 & 2 & -1 \\ 5 & 0 & 3 \end{bmatrix}$
15. $\begin{bmatrix} 2 & -1 & 7 \\ 0 & 4 & -2 \\ 0 & 5/2 & -29/2 \end{bmatrix}$
16. $2R_2 \to R_2$
17. $R_1 + R_2 \to R_2$
18. $2R_3 + R_1 \to R_1$
19. $R_1 \leftrightarrow R_2$
20. $-R_2 + R_3 \leftrightarrow R_3$
21. $x = 2, y = 1$
22. $x = -1, y = 3$
23. $x = -1, y = 0$
24. $x = \frac{1}{2}, y = \frac{1}{3}$
25. $x_1 = -2, x_2 = 1, x_3 = 2$
26. $x_1 = 1, x_2 = 5, x_3 = 7$

Section 1.3

1. (a) yes (b) no (c) no (d) yes
2. (a) yes (b) yes (c) no (d) yes
3. (a) no (b) yes (c) yes (d) yes
4. (a) no (b) yes (c) yes (d) yes

5. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$

8. $\begin{bmatrix} 1 & -7/5 \\ 0 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 7 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 5 \end{bmatrix}$

11. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

12. $\begin{bmatrix} 1 & \frac{5}{4} & -\frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

16. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

17. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

18. $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$

19. $\begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

21. $\begin{bmatrix} 1 & 1 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{bmatrix}$

22. $\begin{bmatrix} 1 & -1 & 3 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & -2 & 5 \end{bmatrix}$

Section 1.4

1. $x_1 = 1 - 2x_2$; x_2 is free. Possible solutions: $x_1 = 1, x_2 = 0$ and $x_1 = -1, x_2 = 1$.
2. $x_1 = -3 + 5x_2$; x_2 is free. Possible solutions: $x_1 = 3, x_2 = 0$ and $x_1 = -8, x_2 = -1$
3. $x_1 = 1$; $x_2 = 2$
4. $x_1 = 0$; $x_2 = -1$
5. No solution; the system is inconsistent.
6. No solution; the system is inconsistent.
7. $x_1 = -11 + 10x_3$; $x_2 = -4 + 4x_3$; x_3 is free. Possible solutions: $x_1 = -11, x_2 = -4, x_3 = 0$ and $x_1 = -1, x_2 = 0$ and $x_3 = 1$.
8. $x_1 = -\frac{2}{3} + \frac{8}{9}x_3$; $x_2 = \frac{2}{3} - \frac{5}{9}x_3$; x_3 is free. Possible solutions: $x_1 = -\frac{2}{3}$, $x_2 = \frac{2}{3}$, $x_3 = 0$ and $x_1 = \frac{4}{9}$, $x_2 = -\frac{1}{9}$, $x_3 = 1$
9. $x_1 = 1 - x_2 - x_4$; x_2 is free; $x_3 = 1 - 2x_4$; x_4 is free. Possible solutions: $x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0$ and $x_1 = -2, x_2 = 1, x_3 = -3, x_4 = 2$
10. $x_1 = 3 - x_3 - 2x_4$; $x_2 = -3 - 5x_3 - 7x_4$; x_3 is free; x_4 is free. Possible solutions: $x_1 = 3$, $x_2 = -3, x_3 = 0, x_4 = 0$ and $x_1 = 0$, $x_2 = -5, x_3 = -1, x_4 = 1$
11. No solution; the system is inconsistent.
12. No solution; the system is inconsistent.
13. $x_1 = \frac{1}{3} - \frac{4}{3}x_3$; $x_2 = \frac{1}{3} - \frac{1}{3}x_3$; x_3 is free. Possible solutions: $x_1 = \frac{1}{3}$, $x_2 = \frac{1}{3}, x_3 = 0$ and $x_1 = -1, x_2 = 0$, $x_3 = 1$
14. $x_1 = 1 - 2x_2 - 3x_3$; x_2 is free; x_3 is free. Possible solutions: $x_1 = 1$, $x_2 = 0, x_3 = 0$ and $x_1 = 8, x_2 = 1$, $x_3 = -3$
15. Never exactly 1 solution; infinite solutions if $k = 2$; no solution if $k \neq 2$.
16. Exactly 1 solution if $k \neq 2$; infinite solutions if $k = 2$; never no solution.
17. Exactly 1 solution if $k \neq 2$; no solution if $k = 2$; never infinite solutions.
18. Exactly 1 solution for all k.

Section 1.5

1. 29 chickens and 33 pigs
2. 12 \$0.30 trinkets, 8 \$0.65 trinkets
3. 42 grande tables, 22 venti tables
4. 35 blue, 40 green, 20 red, 5 yellow
5. 30 men, 15 women, 20 kids
6. $f(x) = 6x - 3$
7. $f(x) = -2x + 10$
8. $f(x) = -x^2 + x + 5$
9. $f(x) = \frac{1}{2}x^2 3x + 1$
10. $f(x) = 3x - 5$
11. $f(x) = 3$
12. $f(x) = -x^3 + x^2 - x + 1$
13. $f(x) = x^3 + 1$
14. $f(x) = x^2 + 1$
15. $f(x) = \frac{3}{2}x + 1$
16. (a) Substitution yields the equations $2 = ae^b$ and $4 = ae^{2b}$; these are not linear equations.

    (b) $y = ae^{bx}$ implies that $\ln y = \ln(ae^{bx}) = \ln a + \ln e^{bx} = \ln a + bx$.

    (c) Plugging in the points for x and y in the equation $\ln y = \ln a + bx$, we have equations

    $$\begin{array}{ccccc} \ln a & + & b & = & \ln 2 \\ \ln a & + & 2b & = & \ln 4 \end{array}.$$

    To solve,

    $$\begin{bmatrix} 1 & 1 & \ln 2 \\ 1 & 2 & \ln 4 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \ln 2 \end{bmatrix}.$$

    Therefore $\ln a = 0$ and $b = \ln 2$.

    (d) Since $\ln a = 0$, we know that $a = e^0 = 1$. Thus our exponential function is $f(x) = e^{x \ln 2}$.

17. The augmented matrix from this system is $\begin{bmatrix} 1 & 1 & 1 & 1 & 8 \\ 6 & 1 & 2 & 3 & 24 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix}$. From this we find the solution

$$\begin{aligned} t &= \frac{8}{3} - \frac{1}{3}f \\ x &= \frac{8}{3} - \frac{1}{3}f \\ w &= \frac{8}{3} - \frac{1}{3}f. \end{aligned}$$

The only time each of these variables are nonnegative integers is when $f = 2$ or $f = 8$. If $f = 2$, then we have 2 touchdowns, 2 extra points and 2 two point conversions (and 2 field goals); this doesn't make sense since the extra points and two point conversions follow touchdowns. If $f = 8$, then we have no touchdowns, extra points or two point conversions (just 8 field goals). This is the only solution; all points were scored from field goals.

18. The augmented matrix from this system is $\begin{bmatrix} 1 & 1 & 1 & 1 & 8 \\ 6 & 1 & 2 & 3 & 29 \\ 0 & 1 & -1 & 0 & 2 \end{bmatrix}$. From this we find the solution

$$\begin{aligned} t &= 4 - \frac{1}{3}f \\ x &= 3 - \frac{1}{3}f \\ w &= 1 - \frac{1}{3}f. \end{aligned}$$

The only time each of these variables are nonnegative integers is when $f = 0$ or $f = 3$. If $f = 0$, then we have 4 touchdowns, 3 extra points and 1 two point conversions (no field goals). If $f = 3$, then we have 3 touchdowns, 2 extra points and no two point conversions (and 3 field goals).

19. Let x_1, x_2 and x_3 represent the number of free throws, 2 point and 3 point shots taken. The augmented matrix from this system is $\begin{bmatrix} 1 & 1 & 1 & 30 \\ 1 & 2 & 3 & 80 \end{bmatrix}$. From this we find the solution

$$\begin{aligned} x_1 &= -20 + x_3 \\ x_2 &= 50 - 2x_3. \end{aligned}$$

In order for x_1 and x_2 to be nonnegative, we need $20 \leq x_3 \leq 25$. Thus there are 6 different scenerios: the "first" is where 20 three point shots are taken, no free throws, and 10 two point shots; the "last" is where 25 three point shots are taken, 5 free throws, and no two point shots.

20. Let x_1, x_2 and x_3 represent the number of free throws, 2 point and 3 point shots taken. The augmented matrix from this system is $\begin{bmatrix} 1 & 1 & 1 & 70 \\ 1 & 2 & 3 & 110 \end{bmatrix}$. From this we find the solution

$$\begin{aligned} x_1 &= 30 + x_3 \\ x_2 &= 40 - 2x_3. \end{aligned}$$

In order for x_2 to be nonnegative, we need $x_3 \leq 20$. Thus there are 21 different scenerios: the "first" is where 0 three point shots are taken ($x_3 = 0$, 30 free throws and 40 two point shots; the "last" is where 20 three point shots are taken, 50 free throws, and no two point shots.

21. Let $y = ax + b$; all linear functions through (1,3) come in the form $y = (3 - b)x + b$. Examples: $b = 0$ yields $y = 3x$; $b = 2$ yields $y = x + 2$.

22. Let $y = ax + b$; all linear functions through (2,5) come in the form $y = (2.5 - \frac{1}{2}b)x + b$. Examples: $b = 1$ yields $y = 2x + 1$; $b = -1$ yields $y = 3x - 1$.

23. Let $y = ax^2 + bx + c$; we find that $a = -\frac{1}{2} + \frac{1}{2}c$ and $b = \frac{1}{2} - \frac{3}{2}c$. Examples: $c = 1$ yields $y = -x + 1$; $c = 3$ yields $y = x^2 - 4x + 3$.

24. Let $y = ax^2 + bx + c$; we find that $a = 2 - \frac{1}{2}c$ and $b = -1 + \frac{1}{2}c$. Examples: $c = 0$ yields $y = 2x^2 - x$; $c = -2$ yields $y = 3x^2 - 2x - 2$.

Chapter 2

Section 2.1

1. $\begin{bmatrix} -2 & -1 \\ 12 & 13 \end{bmatrix}$
2. $\begin{bmatrix} 11 & -8 \\ -1 & -19 \end{bmatrix}$
3. $\begin{bmatrix} 2 & -2 \\ 14 & 8 \end{bmatrix}$
4. $\begin{bmatrix} -14 \\ -5 \\ -9 \end{bmatrix}$
5. $\begin{bmatrix} 9 & -7 \\ 11 & -6 \end{bmatrix}$
6. $\begin{bmatrix} -2 & 1 \\ 12 & 13 \end{bmatrix}$
7. $\begin{bmatrix} -14 \\ 6 \end{bmatrix}$
8. $\begin{bmatrix} -12 \\ 2 \end{bmatrix}$
9. $\begin{bmatrix} -15 \\ -25 \end{bmatrix}$
10. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
11. $X = \begin{bmatrix} -5 & 9 \\ 2 & -14 \end{bmatrix}$
12. $X = \begin{bmatrix} 0 & -22 \\ -7 & 17 \end{bmatrix}$
13. $X = \begin{bmatrix} -5 & -2 \\ -9/2 & -19/2 \end{bmatrix}$
14. $X = \begin{bmatrix} 8 & 12 \\ 10 & 2 \end{bmatrix}$
15. $a = 2$, $b = 1$
16. $a = -1$, $b = 1/2$
17. $a = 5/2 + 3/2b$
18. $a = 5$, $b = 0$
19. No solution.
20. $a = -1$, $b = 1$
21. No solution.

Section 2.2

1. -22
2. 2
3. 0
4. 1
5. 5
6. -21
7. 15
8. -8
9. -2
10. 23
11. Not possible.
12. Not possible.

13. $AB = \begin{bmatrix} 8 & 3 \\ 10 & -9 \end{bmatrix}$

$BA = \begin{bmatrix} -3 & 22 \\ 4 & -10 \end{bmatrix}$

14. $AB = \begin{bmatrix} 24 & -24 \\ 17 & -17 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

15. $AB = \begin{bmatrix} -1 & -2 & 12 \\ 10 & 4 & 32 \end{bmatrix}$

BA is not possible.

16. $AB = \begin{bmatrix} 3 & 8 \\ -5 & -8 \\ -8 & -32 \end{bmatrix}$

BA is not possible.

17. AB is not possible.

$BA = \begin{bmatrix} 27 & -33 & 39 \\ -27 & -3 & -15 \end{bmatrix}$

18. $AB = \begin{bmatrix} 10 & -18 & 11 \\ -45 & 24 & -21 \\ -15 & 12 & -9 \end{bmatrix}$

$BA = \begin{bmatrix} 52 & -21 \\ 45 & -27 \end{bmatrix}$

19. $AB = \begin{bmatrix} -32 & 34 & -24 \\ -32 & 38 & -8 \\ -16 & 21 & 4 \end{bmatrix}$

$BA = \begin{bmatrix} 22 & -14 \\ -4 & -12 \end{bmatrix}$

20. $AB = \begin{bmatrix} -10 & 19 & -32 \\ 10 & 31 & -28 \\ 20 & 37 & -26 \end{bmatrix}$

$BA = \begin{bmatrix} -5 & -14 \\ 45 & 0 \end{bmatrix}$

21. $AB = \begin{bmatrix} -56 & 2 & -36 \\ 20 & 19 & -30 \\ -50 & -13 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} -46 & 40 \\ 72 & 9 \end{bmatrix}$

22. $AB = \begin{bmatrix} -7 & 3 & 7 & -15 \\ -5 & -1 & -17 & 5 \end{bmatrix}$

BA is not possible.

23. $AB = \begin{bmatrix} -15 & -22 & -21 & -1 \\ 16 & -53 & -59 & -31 \end{bmatrix}$

BA is not possible.

24. $AB = \begin{bmatrix} 3 & 4 & 0 \\ 1 & 4 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 0 & 0 & 4 \\ -3 & 6 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

25. $AB = \begin{bmatrix} 0 & 0 & 4 \\ 6 & 4 & -2 \\ 2 & -4 & -6 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & -2 & 6 \\ 2 & 2 & 4 \\ 4 & 0 & -6 \end{bmatrix}$

26. $AB = \begin{bmatrix} 0 & -44 & 18 \\ -20 & -1 & -18 \\ -5 & -21 & -3 \end{bmatrix}$

$BA = \begin{bmatrix} -25 & -5 & -25 \\ 25 & -11 & 2 \\ -15 & 19 & 32 \end{bmatrix}$

27. $AB = \begin{bmatrix} 21 & -17 & -5 \\ 19 & 5 & 19 \\ 5 & 9 & 4 \end{bmatrix}$

$BA = \begin{bmatrix} 19 & 5 & 23 \\ 5 & -7 & -1 \\ -14 & 6 & 18 \end{bmatrix}$

28. $DA = \begin{bmatrix} 6 & -4 \\ 18 & -8 \end{bmatrix}$

$AD = \begin{bmatrix} 6 & 12 \\ -6 & -8 \end{bmatrix}$

29. $DA = \begin{bmatrix} 4 & -6 \\ 4 & -6 \end{bmatrix}$

$AD = \begin{bmatrix} 4 & 8 \\ -3 & -6 \end{bmatrix}$

30. $DA = \begin{bmatrix} -1 & 4 & 9 \\ -4 & 10 & 18 \\ -7 & 16 & 27 \end{bmatrix}$

$AD = \begin{bmatrix} -1 & -2 & -3 \\ 8 & 10 & 12 \\ 21 & 24 & 27 \end{bmatrix}$

31. $DA = \begin{bmatrix} 2 & 2 & 2 \\ -6 & -6 & -6 \\ -15 & -15 & -15 \end{bmatrix}$

$AD = \begin{bmatrix} 2 & -3 & 5 \\ 4 & -6 & 10 \\ -6 & 9 & -15 \end{bmatrix}$

32. $DA = \begin{bmatrix} d_1a & d_1b \\ d_2c & d_2d \end{bmatrix}$

$AD = \begin{bmatrix} d_1a & d_2b \\ d_1c & d_2d \end{bmatrix}$

33. $DA = \begin{bmatrix} d_1a & d_1b & d_1c \\ d_2d & d_2e & d_2f \\ d_3g & d_3h & d_3i \end{bmatrix}$

$AD = \begin{bmatrix} d_1a & d_2b & d_3c \\ d_1d & d_2e & d_3f \\ d_1g & d_2h & d_3i \end{bmatrix}$

34. $A\vec{x} = \begin{bmatrix} 35 \\ -5 \end{bmatrix}$

35. $A\vec{x} = \begin{bmatrix} -6 \\ 11 \end{bmatrix}$

36. $A\vec{x} = \begin{bmatrix} 8 \\ 7 \\ 3 \end{bmatrix}$

37. $A\vec{x} = \begin{bmatrix} 5 \\ 1 \\ -21 \end{bmatrix}$

38. $A\vec{x} = \begin{bmatrix} 2x_1 - x_2 \\ 4x_1 + 3x_2 \end{bmatrix}$

39. $A\vec{x} = \begin{bmatrix} x_1 + 2x_2 + 3x_3 \\ x_1 + 2x_3 \\ 2x_1 + 3x_2 + x_3 \end{bmatrix}$

40. $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $A^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

41. $A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$; $A^3 = \begin{bmatrix} 8 & 0 \\ 0 & 27 \end{bmatrix}$

42. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$;
$A^3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

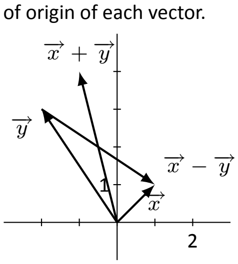
43. $A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$;
$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

44. $A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$;
$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

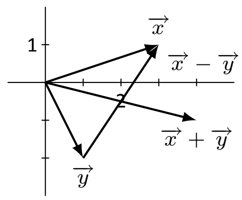
45. (a) $\begin{bmatrix} 0 & -2 \\ -5 & -1 \end{bmatrix}$
(b) $\begin{bmatrix} 10 & 2 \\ 5 & 11 \end{bmatrix}$
(c) $\begin{bmatrix} -11 & -15 \\ 37 & 32 \end{bmatrix}$
(d) No.
(e) $(A+B)(A+B) = AA + AB + BA + BB = A^2 + AB + BA + B^2$.

Section 2.3

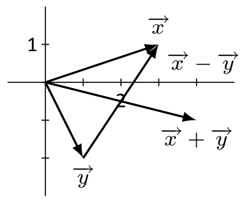
1. $\vec{x} + \vec{y} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$,
$\vec{x} - \vec{y} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$
Sketches will vary depending on choice of origin of each vector.



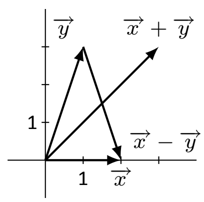
2. $\vec{x} + \vec{y} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $\vec{x} - \vec{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
Sketches will vary depending on choice of origin of each vector.



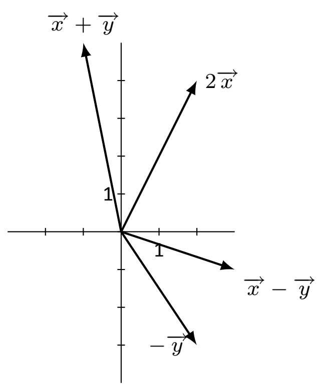
3. $\vec{x} + \vec{y} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$,
$\vec{x} - \vec{y} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
Sketches will vary depending on choice of origin of each vector.



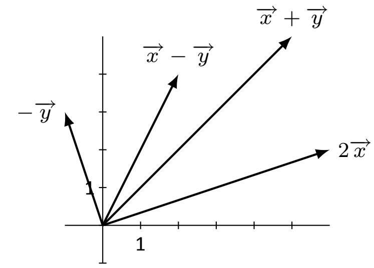
4. $\vec{x} + \vec{y} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, $\vec{x} - \vec{y} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$
Sketches will vary depending on choice of origin of each vector.



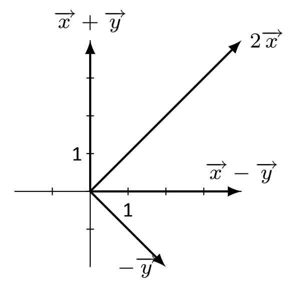
5. Sketches will vary depending on choice of origin of each vector.



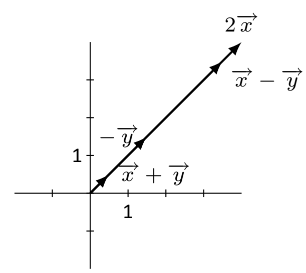
6. Sketches will vary depending on choice of origin of each vector.



7. Sketches will vary depending on choice of origin of each vector.



8. Sketches will vary depending on choice of origin of each vector.



9. $||\vec{x}|| = \sqrt{5}$; $||a\vec{x}|| = \sqrt{45} = 3\sqrt{5}$. The vector $a\vec{x}$ is 3 times as long as $\vec{x}$.

10. $||\vec{x}|| = \sqrt{65}$; $||a\vec{x}|| = \sqrt{260} = 2\sqrt{65}$. The vector $a\vec{x}$ is 2 times as long as $\vec{x}$.

11. $||\vec{x}|| = \sqrt{34}$; $||a\vec{x}|| = \sqrt{34}$. The vectors $a\vec{x}$ and $\vec{x}$ are the same length (they just point in opposite directions).

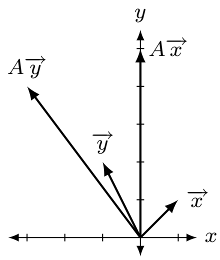
12. $||\vec{x}|| = \sqrt{90} = 3\sqrt{10}$; $||a\vec{x}|| = \sqrt{10}$. The vector $a\vec{x}$ is one-third the length of $\vec{x}$; equivalently, $\vec{x}$ is 3 times as long as $a\vec{x}$.

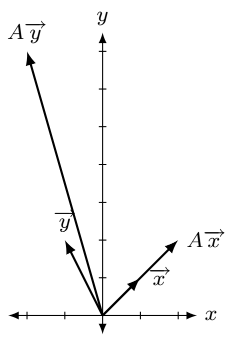
13. (a) $||\vec{x}|| = \sqrt{2}$; $||\vec{y}|| = \sqrt{13}$; $||\vec{x} + \vec{y}|| = 5$.
(b) $||\vec{x}|| = \sqrt{5}$; $||\vec{y}|| = 3\sqrt{5}$; $||\vec{x} + \vec{y}|| = 4\sqrt{5}$.

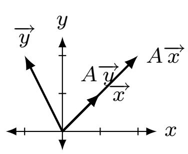
(c) $||\vec{x}|| = \sqrt{10}$; $||\vec{y}|| = \sqrt{29}$; $||\vec{x}+\vec{y}|| = \sqrt{65}$.

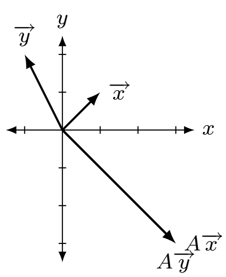
(d) $||\vec{x}|| = \sqrt{5}$; $||\vec{y}|| = 2\sqrt{5}$; $||\vec{x}+\vec{y}|| = \sqrt{5}$.

The equality holds sometimes; only when $\vec{x}$ and $\vec{y}$ point along the same line, in the same direction.

14. 

15. 

16. 

17. 

Section 2.4

1. Multiply $A\vec{u}$ and $A\vec{v}$ to verify.
2. Multiply $A\vec{u}$ and $A\vec{v}$ to verify.
3. Multiply $A\vec{u}$ and $A\vec{v}$ to verify.
4. Multiply $A\vec{u}$ and $A\vec{v}$ to verify.
5. Multiply $A\vec{u}$ and $A\vec{v}$ to verify.
6. Multiply $A\vec{u}$ and $A\vec{v}$ to verify.
7. Multiply $A\vec{u}$, $A\vec{v}$ and $A(\vec{u}+\vec{v})$ to verify.
8. Multiply $A\vec{u}$, $A\vec{v}$ and $A(\vec{u}+\vec{v})$ to verify.
9. Multiply $A\vec{u}$, $A\vec{v}$ and $A(\vec{u}+\vec{v})$ to verify.
10. (a) $\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$
11. (a) $\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} 2/5 \\ -13/5 \end{bmatrix}$
12. (a) $\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} -10 \\ -5 \end{bmatrix}$
13. (a) $\vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} -2 \\ -9/4 \end{bmatrix}$
14. (a) $\vec{x} = x_2 \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$

    (b) No solution.
15. (a) $\vec{x} = x_3 \begin{bmatrix} 5/4 \\ 1 \\ 1 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5/4 \\ 1 \\ 1 \end{bmatrix}$
16. (a) $\vec{x} = x_3 \begin{bmatrix} -33 \\ 7 \\ 1 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -33 \\ 7 \\ 1 \end{bmatrix}$
17. (a) $\vec{x} = x_3 \begin{bmatrix} 14 \\ -10 \\ 0 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 14 \\ -10 \\ 0 \end{bmatrix}$
18. (a) $\vec{x} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} 3/16 \\ 0 \\ 21/16 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$
19. (a) $\vec{x} =$ $x_3 \begin{bmatrix} 2 \\ 2/5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 2/5 \\ 0 \\ 1 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} -2 \\ 2/5 \\ 0 \\ 0 \end{bmatrix} +$ $x_3 \begin{bmatrix} 2 \\ 2/5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 2/5 \\ 0 \\ 1 \end{bmatrix}$
20. (a) $\vec{x} =$ $x_3 \begin{bmatrix} -3/4 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3/2 \\ -5 \\ 0 \\ 1 \end{bmatrix}$

    (b) $\vec{x} = \begin{bmatrix} -1/4 \\ -2 \\ 0 \\ 0 \end{bmatrix} +$ $x_3 \begin{bmatrix} -3/4 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3/2 \\ -5 \\ 0 \\ 1 \end{bmatrix}$
21. (a) $\vec{x} = x_2 \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} +$ $x_4 \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 13/2 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

(b) $\vec{x} =$

$$\begin{bmatrix} -5 \\ 0 \\ 3/2 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} +$$

$$x_4 \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 13/2 \\ 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

22. (a) $\vec{x} =$

$$x_4 \begin{bmatrix} 8 \\ -12 \\ 10 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -15 \\ 68/3 \\ -20 \\ 0 \\ 1 \end{bmatrix}$$

(b) $\vec{x} = \begin{bmatrix} 2 \\ -16/3 \\ 7/2 \\ 0 \\ 0 \end{bmatrix} +$

$$x_4 \begin{bmatrix} 8 \\ -12 \\ 10 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -15 \\ 68/3 \\ -20 \\ 0 \\ 1 \end{bmatrix}$$

23. (a) $\vec{x} =$

$$x_4 \begin{bmatrix} 1 \\ 13/9 \\ -1/3 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

(b) $\vec{x} = \begin{bmatrix} 1 \\ 1/9 \\ 5/3 \\ 0 \\ 0 \end{bmatrix} +$

$$x_4 \begin{bmatrix} 1 \\ 13/9 \\ -1/3 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

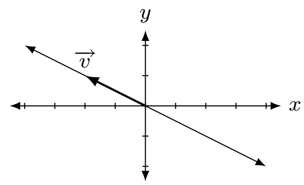
24. (a) $\vec{x} =$

$$x_4 \begin{bmatrix} 3 \\ -1 \\ -6 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -17 \\ 12 \\ 44 \\ 0 \\ 1 \end{bmatrix}$$

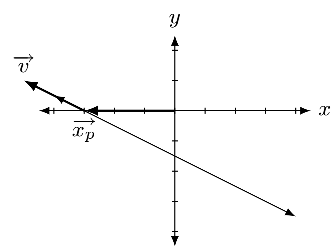
(b) $\vec{x} = \begin{bmatrix} 7 \\ -6 \\ -19 \\ 0 \\ 0 \end{bmatrix} +$

$$x_4 \begin{bmatrix} 3 \\ -1 \\ -6 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -17 \\ 12 \\ 44 \\ 0 \\ 1 \end{bmatrix}$$

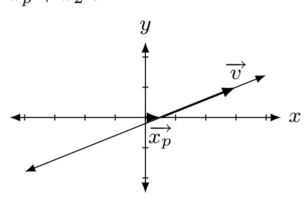
25. $\vec{x} = x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = x_2 \vec{v}$



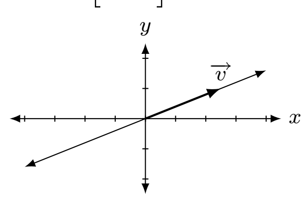
26. $\vec{x} = \begin{bmatrix} -3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \vec{x_p} + x_2 \vec{v}$



27. $\vec{x} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2.5 \\ 1 \end{bmatrix} = \vec{x_p} + x_2 \vec{v}$



28. $\vec{x} = x_2 \begin{bmatrix} 2.5 \\ 1 \end{bmatrix} = x_2 \vec{v}$



Section 2.5

1. $X = \begin{bmatrix} 1 & -9 \\ -4 & -5 \end{bmatrix}$

2. $X = \begin{bmatrix} 3 & -7 \\ -3 & 1 \end{bmatrix}$

3. $X = \begin{bmatrix} -2 & -7 \\ 7 & -6 \end{bmatrix}$

4. $X = \begin{bmatrix} 0 & -2 \\ -8 & 6 \end{bmatrix}$

5. $X = \begin{bmatrix} -5 & 2 & -3 \\ -4 & -3 & -2 \end{bmatrix}$

6. $X = \begin{bmatrix} -1 & 2 & -4 \\ -6 & -2 & 3 \end{bmatrix}$

7. $X = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$

8. $X = \begin{bmatrix} -1/4 & 1/2 \\ 3/4 & -1/2 \end{bmatrix}$

9. $X = \begin{bmatrix} 3 & -3 & 3 \\ 2 & -2 & -3 \\ -3 & -1 & -2 \end{bmatrix}$

10. $X = \begin{bmatrix} 6 & 1 & -1 \\ -1 & -1 & 7 \\ -5 & 7 & -4 \end{bmatrix}$

11. $X = \begin{bmatrix} 5/3 & 2/3 & 1 \\ -1/3 & 1/6 & 0 \\ 1/3 & 1/3 & 0 \end{bmatrix}$

12. $X = \begin{bmatrix} -1/2 & -1/2 & 0 \\ -1/2 & -1 & 1/2 \\ -1/2 & -3/4 & 3/4 \end{bmatrix}$

Section 2.6

1. $\begin{bmatrix} -24 & -5 \\ 5 & 1 \end{bmatrix}$

2. $\begin{bmatrix} -3 & 4 \\ -1 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 1/3 & 0 \\ 0 & 1/7 \end{bmatrix}$

4. $\begin{bmatrix} -4/7 & 5/7 \\ 3/7 & -2/7 \end{bmatrix}$

5. A^{-1} does not exist.

6. $\begin{bmatrix} -2 & 7/2 \\ 1 & -3/2 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} -5/13 & 3/13 \\ 1/13 & 2/13 \end{bmatrix}$

10. $\begin{bmatrix} 1/4 & 1/4 \\ -9/8 & -5/8 \end{bmatrix}$

11. $\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$

12. A^{-1} does not exist.

13. $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -3 \\ 6 & 10 & -5 \end{bmatrix}$

14. A^{-1} does not exist.

15. $\begin{bmatrix} 1 & 0 & 0 \\ 52 & -48 & 7 \\ 8 & -7 & 1 \end{bmatrix}$

16. $\begin{bmatrix} 1 & -9 & 4 \\ 5 & -26 & 11 \\ 0 & -2 & 1 \end{bmatrix}$

17. A^{-1} does not exist.

18. $\begin{bmatrix} 91 & 5 & -20 \\ 18 & 1 & -4 \\ -22 & -1 & 5 \end{bmatrix}$

19. $\begin{bmatrix} 25 & 8 & 0 \\ 78 & 25 & 0 \\ -30 & -9 & 1 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 0 & 0 \\ 5 & -3 & -8 \\ -4 & 2 & 5 \end{bmatrix}$

21. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

22. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

23. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & -1 & 0 & -4 \\ -35 & -10 & 1 & -47 \\ -2 & -2 & 0 & -9 \end{bmatrix}$

24. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -11 & 1 & 0 & -4 \\ -2 & 0 & 1 & -4 \\ -4 & 0 & 0 & 1 \end{bmatrix}$

25. $\begin{bmatrix} 28 & 18 & 3 & -19 \\ 5 & 1 & 0 & -5 \\ 4 & 5 & 1 & 0 \\ 52 & 60 & 12 & -15 \end{bmatrix}$

26. $\begin{bmatrix} 1 & 28 & -2 & 12 \\ 0 & 1 & 0 & 0 \\ 0 & 254 & -19 & 110 \\ 0 & -67 & 5 & -29 \end{bmatrix}$

27. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

28. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & -1/4 \end{bmatrix}$

29. $\vec{x} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

30. $\vec{x} = \begin{bmatrix} -7 \\ -7 \end{bmatrix}$

31. $\vec{x} = \begin{bmatrix} -10 \\ -1 \end{bmatrix}$

32. $\vec{x} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

33. $\vec{x} = \begin{bmatrix} -7 \\ 1 \\ -1 \end{bmatrix}$

34. $\vec{x} = \begin{bmatrix} -7 \\ -7 \\ 9 \end{bmatrix}$

35. $\vec{x} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$

36. $\vec{x} = \begin{bmatrix} -9 \\ 10 \\ -4 \end{bmatrix}$

Section 2.7

1. $(AB)^{-1} = \begin{bmatrix} -13/440 & 3/88 \\ 19/440 & -1/88 \end{bmatrix}$

2. $(AB)^{-1} = \begin{bmatrix} 5/77 & 17/231 \\ -1/308 & 1/77 \end{bmatrix}$

3. $(AB)^{-1} = \begin{bmatrix} -1/56 & 1/84 \\ -11/252 & -1/126 \end{bmatrix}$

4. $(AB)^{-1} = \begin{bmatrix} -9/352 & -7/352 \\ 1/352 & -9/352 \end{bmatrix}$

5. $A^{-1} = \begin{bmatrix} -5/39 & -4/39 \\ -2/13 & 1/13 \end{bmatrix}$,
$(A^{-1})^{-1} = \begin{bmatrix} -3 & -4 \\ -6 & 5 \end{bmatrix}$

6. $A^{-1} = \begin{bmatrix} -1/7 & -3/7 \\ 1/14 & -2/7 \end{bmatrix}$,
$(A^{-1})^{-1} = \begin{bmatrix} -4 & 6 \\ -1 & -2 \end{bmatrix}$

7. $A^{-1} = \begin{bmatrix} 4/25 & 1/25 \\ -7/50 & -4/25 \end{bmatrix}$,
$(A^{-1})^{-1} = \begin{bmatrix} 8 & 2 \\ -7 & -8 \end{bmatrix}$

8. $A^{-1} = \begin{bmatrix} 1/9 & 0 \\ -7/81 & 1/9 \end{bmatrix}$,
$(A^{-1})^{-1} = \begin{bmatrix} 9 & 0 \\ 7 & 9 \end{bmatrix}$

9. Solutions will vary.

10. Likely some entries that should be 0 will not be exaclty 0, but rather very small values.

11. Likely some entries that should be 0 will not be exactly 0, but rather very small values.

Chapter 3

Section 3.1

1. $\begin{bmatrix} -7 & 4 \\ 4 & -6 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -7 \\ 1 & 8 \end{bmatrix}$

3. A is diagonal, as is A^T. $\begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

4. A is symmetric. $\begin{bmatrix} 13 & -3 \\ -3 & 1 \end{bmatrix}$

5. $\begin{bmatrix} -5 & 3 & -10 \\ -9 & 1 & -8 \end{bmatrix}$

6. $\begin{bmatrix} -2 & 1 & 9 \\ 10 & -7 & -2 \end{bmatrix}$

7. $\begin{bmatrix} 4 & -9 \\ -7 & 6 \\ -4 & 3 \\ -9 & -9 \end{bmatrix}$

8. $\begin{bmatrix} 3 & -10 \\ -10 & -2 \\ 0 & -3 \\ 6 & 1 \end{bmatrix}$

9. $\begin{bmatrix} -7 \\ -8 \\ 2 \\ -3 \end{bmatrix}$

10. $\begin{bmatrix} -9 \\ 8 \\ 2 \\ -7 \end{bmatrix}$

11. $\begin{bmatrix} -9 & 6 & -8 \\ 4 & -3 & 1 \\ 10 & -7 & -1 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 1 & 9 \\ -5 & 5 & 2 \\ 2 & 9 & 3 \end{bmatrix}$

13. A is symmetric. $\begin{bmatrix} 4 & 0 & -2 \\ 0 & 2 & 3 \\ -2 & 3 & 6 \end{bmatrix}$

14. A is symmetric. $\begin{bmatrix} 0 & 3 & -2 \\ 3 & -4 & 1 \\ -2 & 1 & 0 \end{bmatrix}$

15. $\begin{bmatrix} 2 & 5 & 7 \\ -5 & 5 & -4 \\ -3 & -6 & -10 \end{bmatrix}$

16. A is skew symmetric. $\begin{bmatrix} 0 & -6 & 1 \\ 6 & 0 & 4 \\ -1 & -4 & 0 \end{bmatrix}$

17. $\begin{bmatrix} 4 & 5 & -6 \\ 2 & -4 & 6 \\ -9 & -10 & 9 \end{bmatrix}$

18. A is lower triangular and A^T is upper triangular; $\begin{bmatrix} 4 & -2 & 4 \\ 0 & -7 & -2 \\ 0 & 0 & 5 \end{bmatrix}$

19. A is upper triangular; A^T is lower triangular. $\begin{bmatrix} -3 & 0 & 0 \\ -4 & -3 & 0 \\ -5 & 5 & -3 \end{bmatrix}$

20. A is upper triangular; A^T is lower triangular. $\begin{bmatrix} 6 & 0 & 0 \\ -7 & -8 & 0 \\ 2 & -1 & 1 \\ 6 & 0 & -7 \end{bmatrix}$

21. A is diagonal, as is A^T. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

22. A is symmetric. $\begin{bmatrix} 6 & -4 & -5 \\ -4 & 0 & 2 \\ -5 & 2 & -2 \end{bmatrix}$

23. A is skew symmetric. $\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -4 \\ -2 & 4 & 0 \end{bmatrix}$

24. A is upper and lower triangular; it is diagonal; it is both symmetric and skew symmetric. It's got it all. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Section 3.2

1. 6
2. 1
3. 3
4. 3
5. -9
6. -5
7. 1
8. 10
9. Not defined; the matrix must be square.
10. Not defined; the matrix must be square.
11. -23
12. 0
13. 4
14. n
15. 0
16. (a) tr(A)=-5; tr(B)=-4; tr($A+B$)=-9
    (b) tr(AB) = 23 = tr(BA)
17. (a) tr(A)=8; tr(B)=-2; tr($A+B$)=6
    (b) tr(AB) = 53 = tr(BA)
18. (a) tr(A)=0; tr(B)=-12; tr($A+B$)=-12
    (b) tr(AB) = 86 = tr(BA)
19. (a) tr(A)=-1; tr(B)=6; tr($A+B$)=5
    (b) tr(AB) = 201 = tr(BA)

Section 3.3

1. 34
2. 41
3. -44

4. -74
5. -44
6. -100
7. 28
8. 11
9. (a) The submatrices are $\begin{bmatrix} 7 & 6 \\ 6 & 10 \end{bmatrix}$, $\begin{bmatrix} 3 & 6 \\ 1 & 10 \end{bmatrix}$, and $\begin{bmatrix} 3 & 7 \\ 1 & 6 \end{bmatrix}$, respectively.
   (b) $C_{1,2} = 34$, $C_{1,2} = -24$, $C_{1,3} = 11$
10. (a) The submatrices are $\begin{bmatrix} -6 & 8 \\ -3 & -2 \end{bmatrix}$, $\begin{bmatrix} -10 & 8 \\ 0 & -2 \end{bmatrix}$, and $\begin{bmatrix} 10 & -6 \\ 0 & -3 \end{bmatrix}$, respectively.
   (b) $C_{1,2} = 36$, $C_{1,2} = -20$, $C_{1,3} = -30$
11. (a) The submatrices are $\begin{bmatrix} 3 & 10 \\ 3 & 9 \end{bmatrix}$, $\begin{bmatrix} -3 & 10 \\ -9 & 9 \end{bmatrix}$, and $\begin{bmatrix} -3 & 3 \\ -9 & 3 \end{bmatrix}$, respectively.
   (b) $C_{1,2} = -3$, $C_{1,2} = -63$, $C_{1,3} = 18$
12. (a) The submatrices are $\begin{bmatrix} -6 & -4 & 6 \\ -8 & 0 & 0 \\ -10 & 8 & -1 \end{bmatrix}$, $\begin{bmatrix} -8 & 0 \\ -10 & -1 \end{bmatrix}$, and $\begin{bmatrix} -8 & 0 \\ -10 & 8 \end{bmatrix}$, respectively.
   (b) $C_{1,2} = 0$, $C_{1,2} = -8$, $C_{1,3} = -64$
13. -59
14. 250
15. 15
16. -52
17. 3
18. 0
19. 0
20. 1
21. 0
22. 2
23. -113
24. 179
25. Hint: $C_{1,1} = d$.

Section 3.4

1. 84
2. 48
3. 0
4. 60
5. 10
6. -36
7. 24
8. 72
9. 175
10. 0
11. -200
12. 57
13. 34
14. 29
15. (a) $\det(A) = 41$; $R_2 \leftrightarrow R_3$
    (b) $\det(B) = 164$; $-4R_3 \rightarrow R_3$
    (c) $\det(C) = -41$; $R_2 + R_1 \rightarrow R_1$
16. (a) $\det(A) = 90$; $2R_1 \rightarrow R_1$
    (b) $\det(B) = 45$; $10R_1 + R_3 \rightarrow R_3$
    (c) $\det(C) = 45$; $C = A^T$
17. (a) $\det(A) = -16$; $R_1 \leftrightarrow R_2$ then $R_1 \leftrightarrow R_3$
    (b) $\det(B) = -16$; $-R_1 \rightarrow R_1$ and $-R_2 \rightarrow R_2$
    (c) $\det(C) = -432$; $C = 3 * M$
18. (a) $\det(A) = -120$; $R_1 \leftrightarrow R_2$ then $R_1 \leftrightarrow R_3$ then $R_2 \leftrightarrow R_3$
    (b) $\det(B) = 720$; $2R_2 \rightarrow R_2$ and $3R_3 \rightarrow R_3$
    (c) $\det(C) = -120$; $C = -M$
19. $\det(A) = 4$, $\det(B) = 4$, $\det(AB) = 16$
20. $\det(A) = 7$, $\det(B) = -17$, $\det(AB) = -119$
21. $\det(A) = -12$, $\det(B) = 29$, $\det(AB) = -348$
22. $\det(A) = 11$, $\det(B) = 0$, $\det(AB) = 0$
23. -59
24. 250
25. 15
26. -52
27. 3
28. 0
29. 0
30. 1

Section 3.5

1. (a) $\det(A) = 14$, $\det(A_1) = 70$, $\det(A_2) = 14$
   (b) $\vec{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$
2. (a) $\det(A) = -43$, $\det(A_1) = 215$, $\det(A_2) = 0$
   (b) $\vec{x} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$
3. (a) $\det(A) = 0$, $\det(A_1) = 0$, $\det(A_2) = 0$
   (b) Infinite solutions exist.
4. (a) $\det(A) = 54$, $\det(A_1) = -162$, $\det(A_2) = -54$
   (b) $\vec{x} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$
5. (a) $\det(A) = 16$, $\det(A_1) = -64$, $\det(A_2) = 80$
   (b) $\vec{x} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$
6. (a) $\det(A) = 0$, $\det(A_1) = -52$, $\det(A_2) = 26$
   (b) No solution exists.
7. (a) $\det(A) = -123$, $\det(A_1) = -492$, $\det(A_2) = 123$, $\det(A_3) = 492$
   (b) $\vec{x} = \begin{bmatrix} 4 \\ -1 \\ -4 \end{bmatrix}$

8. (a) det $(A) = 0$, det $(A_1) = 0$, det $(A_2) = 0$, det $(A_3) = 0$
   (b) Infinite solutions exist.

9. (a) det $(A) = 56$, det $(A_1) = 224$, det $(A_2) = 0$, det $(A_3) = -112$
   (b) $\vec{x} = \begin{bmatrix} 4 \\ 0 \\ -2 \end{bmatrix}$

10. (a) det $(A) = 96$, det $(A_1) = -960$, det $(A_2) = 768$, det $(A_3) = 288$
    (b) $\vec{x} = \begin{bmatrix} -10 \\ 8 \\ 3 \end{bmatrix}$

11. (a) det $(A) = 0$, det $(A_1) = 147$, det $(A_2) = -49$, det $(A_3) = -49$
    (b) No solution exists.

12. (a) det $(A) = 77$, det $(A_1) = -385$, det $(A_2) = -154$, det $(A_3) = -154$
    (b) $\vec{x} = \begin{bmatrix} -5 \\ -2 \\ -2 \end{bmatrix}$

Chapter 4

Section 4.1

1. $\lambda_1 = -5$ with $\vec{x_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 2$ with $\vec{x_2} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

2. $\lambda_1 = 4$ with $\vec{x_1} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 5$ with $\vec{x_2} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$

3. $\lambda_1 = -4$ with $\vec{x_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$;
   $\lambda_2 = -2$ with $\vec{x_2} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

4. $\lambda_1 = -3$ with $\vec{x_1} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 5$ with $\vec{x_2} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

5. $\lambda_1 = -4$ with $\vec{x_1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 4$ with $\vec{x_2} = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$

6. $\lambda_1 = 2$ with $\vec{x_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 4$ with $\vec{x_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

7. $\lambda_1 = -5$ with $\vec{x_1} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$;
   $\lambda_2 = 5$ with $\vec{x_2} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

8. $\lambda_1 = -1$ with $\vec{x_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$;
   $\lambda_2 = -3$ with $\vec{x_2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

9. $\lambda_1 = -1$ with $\vec{x_1} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$;
   $\lambda_2 = 1$ with $\vec{x_2} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
   $\lambda_3 = 3$ with $\vec{x_3} = \begin{bmatrix} 5 \\ -8 \\ 2 \end{bmatrix}$

10. $\lambda_1 = 3$ with $\vec{x_1} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$;
    $\lambda_2 = 4$ with $\vec{x_2} = \begin{bmatrix} -5 \\ -1 \\ 1 \end{bmatrix}$
    $\lambda_3 = 5$ with $\vec{x_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

11. $\lambda_1 = -5$ with $\vec{x_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$;
    $\lambda_2 = -2$ with $\vec{x_2} = \begin{bmatrix} -12 \\ -8 \\ 3 \end{bmatrix}$
    $\lambda_3 = 5$ with $\vec{x_3} = \begin{bmatrix} 15 \\ 3 \\ 5 \end{bmatrix}$

12. $\lambda_1 = -5$ with $\vec{x_1} = \begin{bmatrix} 24 \\ 13 \\ 8 \end{bmatrix}$;
    $\lambda_2 = -2$ with $\vec{x_2} = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$
    $\lambda_3 = 3$ with $\vec{x_3} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

13. $\lambda_1 = -4$ with $\vec{x_1} = \begin{bmatrix} -6 \\ 1 \\ 11 \end{bmatrix}$;
    $\lambda_2 = -1$ with $\vec{x_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
    $\lambda_3 = 5$ with $\vec{x_3} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

14. $\lambda_1 = -2$ with $\vec{x_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$;
    $\lambda_2 = 1$ with $\vec{x_2} = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix}$
    $\lambda_3 = 5$ with $\vec{x_3} = \begin{bmatrix} 28 \\ 7 \\ 1 \end{bmatrix}$

15. $\lambda_1 = 2$ with $\vec{x_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$;
    $\lambda_2 = 3$ with $\vec{x_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$
    $\lambda_3 = 7$ with $\vec{x_3} = \begin{bmatrix} -1 \\ 15 \\ 10 \end{bmatrix}$

Section 4.2

1. (a) $\lambda_1 = 1$ with $\vec{x_1} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 4$ with $\vec{x_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
   (b) $\lambda_1 = 1$ with $\vec{x_1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 4$ with $\vec{x_2} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$
   (c) $\lambda_1 = 1/4$ with $\vec{x_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 4$ with $\vec{x_2} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$
   (d) 5
   (e) 4

2. (a) $\lambda_1 = -4$ with $\vec{x_1} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$;
   $\lambda_2 = 5$ with $\vec{x_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$
   (b) $\lambda_1 = -4$ with $\vec{x_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$;
   $\lambda_2 = 5$ with $\vec{x_2} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

(c) $\lambda_1 = -1/4$ with $\overrightarrow{x_1} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$;

$\lambda_2 = 1/5$ with $\overrightarrow{x_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(d) 1

(e) -20

3. (a) $\lambda_1 = -1$ with $\overrightarrow{x_1} = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$;

$\lambda_2 = 0$ with $\overrightarrow{x_2} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$

(b) $\lambda_1 = -1$ with $\overrightarrow{x_1} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$;

$\lambda_2 = 0$ with $\overrightarrow{x_2} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(c) $A$is not invertible.

(d) -1

(e) 0

4. (a) $\lambda_1 = 4$ with $\overrightarrow{x_1} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$;

$\lambda_2 = 5$ with $\overrightarrow{x_2} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$

(b) $\lambda_1 = 4$ with $\overrightarrow{x_1} = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$;

$\lambda_2 = 5$ with $\overrightarrow{x_2} = \begin{bmatrix} -1 \\ 9 \end{bmatrix}$

(c) $\lambda_1 = 1/4$ with $\overrightarrow{x_1} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$;

$\lambda_2 = 1/5$ with $\overrightarrow{x_2} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$

(d) 9

(e) 20

5. (a) $\lambda_1 = -4$ with $\overrightarrow{x_1} = \begin{bmatrix} -7 \\ -7 \\ 6 \end{bmatrix}$;

$\lambda_2 = 3$ with $\overrightarrow{x_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_3 = 4$ with $\overrightarrow{x_3} = \begin{bmatrix} 9 \\ 1 \\ 22 \end{bmatrix}$

(b) $\lambda_1 = -4$ with $\overrightarrow{x_1} = \begin{bmatrix} -1 \\ 9 \\ 0 \end{bmatrix}$;

$\lambda_2 = 3$ with $\overrightarrow{x_2} = \begin{bmatrix} -20 \\ 26 \\ 7 \end{bmatrix}$

$\lambda_3 = 4$ with $\overrightarrow{x_3} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

(c) $\lambda_1 = -1/4$ with $\overrightarrow{x_1} = \begin{bmatrix} -7 \\ -7 \\ 6 \end{bmatrix}$;

$\lambda_2 = 1/3$ with $\overrightarrow{x_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_3 = 1/4$ with $\overrightarrow{x_3} = \begin{bmatrix} 9 \\ 1 \\ 22 \end{bmatrix}$

(d) 3

(e) -48

6. (a) $\lambda_1 = -5$ with $\overrightarrow{x_1} = \begin{bmatrix} -5 \\ 1 \\ 2 \end{bmatrix}$;

$\lambda_2 = -3$ with $\overrightarrow{x_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_3 = 5$ with $\overrightarrow{x_3} = \begin{bmatrix} 20 \\ 4 \\ 3 \end{bmatrix}$

(b) $\lambda_1 = -5$ with $\overrightarrow{x_1} = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$;

$\lambda_2 = -3$ with $\overrightarrow{x_2} = \begin{bmatrix} 1 \\ -11 \\ 8 \end{bmatrix}$

$\lambda_3 = 5$ with $\overrightarrow{x_3} = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$

(c) $\lambda_1 = -1/5$ with $\overrightarrow{x_1} = \begin{bmatrix} -5 \\ 1 \\ 2 \end{bmatrix}$;

$\lambda_2 = -1/3$ with $\overrightarrow{x_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_3 = 1/5$ with $\overrightarrow{x_3} = \begin{bmatrix} 20 \\ 4 \\ 3 \end{bmatrix}$

(d) -3

(e) 75

Chapter 5

Section 5.1

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2. $A = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

4. $A = \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$

5. $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

7. $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$

8. $A = \begin{bmatrix} 0.707 & -0.707 \\ 0.354 & 0.354 \end{bmatrix}$

9. $A = \begin{bmatrix} 0 & -1 \\ -1 & -1 \end{bmatrix}$

10. $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

11. Yes, these are the same; the transformation matrix in each is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

12. No, these are different. The first produces a transformation matrix $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$, while the second produces $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

13. Yes, these are the same. Each produces the transformation matrix $\begin{bmatrix} 1/2 & 0 \\ 0 & 3 \end{bmatrix}$.

14. Yes, these are the same. Each produces the transformation matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Section 5.2

1. Yes

2. No; cannot have a squared term.

3. No; cannot add a constant.

4. No; cannot add a constant.

5. Yes.

6. $[\,T\,] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

7. $[\,T\,] = \begin{bmatrix} 1 & 2 \\ 3 & -5 \\ 0 & 2 \end{bmatrix}$

8. $[\,T\,] = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 1 & 0 & 4 \\ 0 & 5 & 1 \end{bmatrix}$

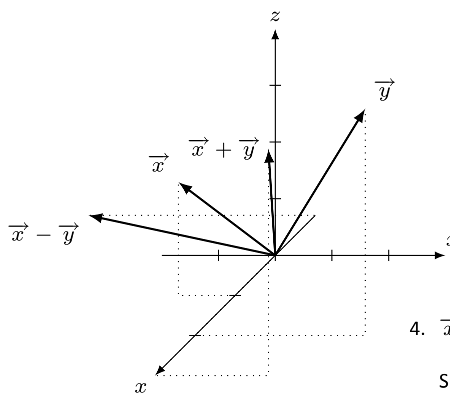
9. $[\,T\,] = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

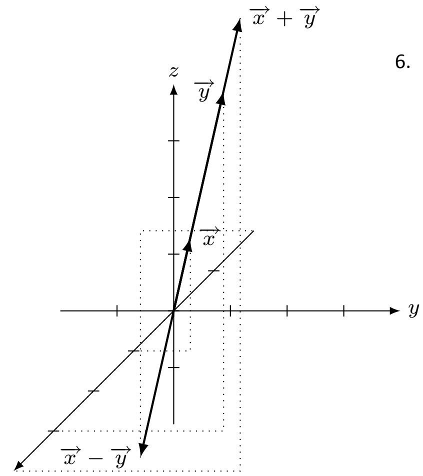
10. $[\,T\,] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

11. $[\,T\,] = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

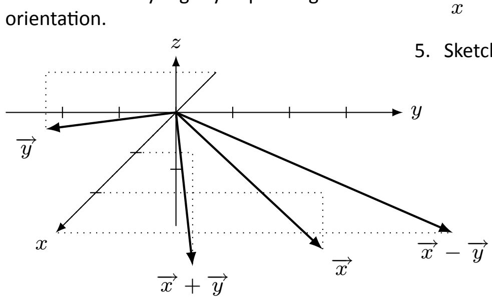
Section 5.3

1. $\vec{x}+\vec{y}=\begin{bmatrix}3\\2\\4\end{bmatrix}$,
$\vec{x}-\vec{y}=\begin{bmatrix}-1\\-4\\0\end{bmatrix}$
Sketches will vary slightly depending on orientation.



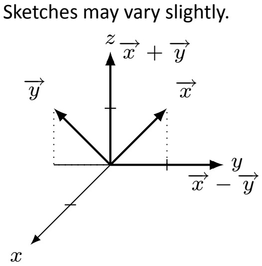


2. $\vec{x}+\vec{y}=\begin{bmatrix}1\\1\\-2\end{bmatrix}$,
$\vec{x}-\vec{y}=\begin{bmatrix}3\\7\\0\end{bmatrix}$
Sketches will vary slightly depending on orientation.

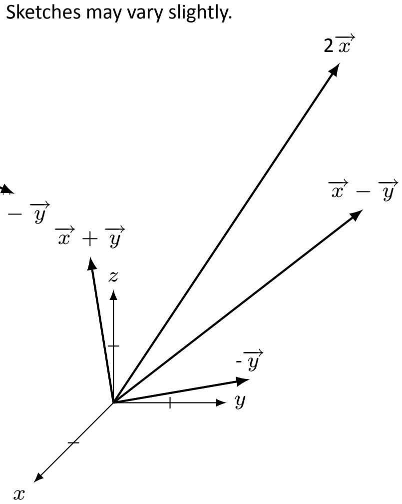


3. $\vec{x}+\vec{y}=\begin{bmatrix}4\\4\\8\end{bmatrix}$,
$\vec{x}-\vec{y}=\begin{bmatrix}-2\\-2\\-4\end{bmatrix}$
Sketches will vary slightly depending on orientation.

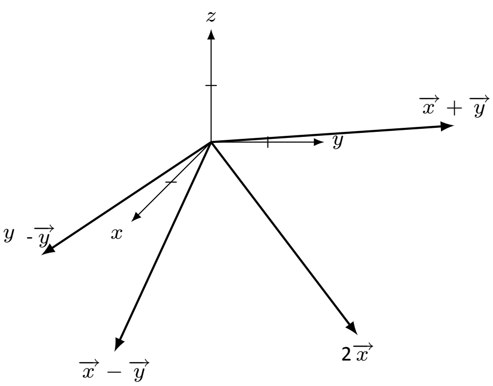
4. $\vec{x}+\vec{y}=\begin{bmatrix}0\\0\\2\end{bmatrix}$, $\vec{x}-\vec{y}=\begin{bmatrix}0\\2\\0\end{bmatrix}$
Sketches may vary slightly.



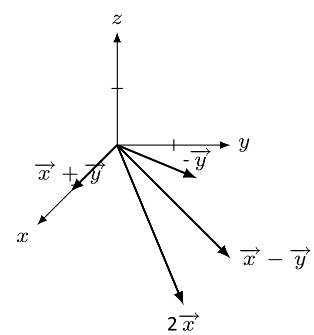
5. Sketches may vary slightly.



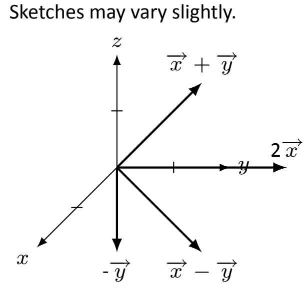
6. Sketches may vary slightly.



7. Sketches may vary slightly.



8. Sketches may vary slightly.



9. $||\vec{x}||=\sqrt{30}$,
$||a\vec{x}||=\sqrt{120}=2\sqrt{30}$

10. $||\vec{x}||=\sqrt{34}$, $||a\vec{x}||=\sqrt{34}$

11. $||\vec{x}||=\sqrt{54}=3\sqrt{6}$,
$||a\vec{x}||=\sqrt{270}=15\sqrt{6}$

12. $||\vec{x}||=3$, $||a\vec{x}||=27$