FINITE ELEMENT MESH USING h-REFINEMENT PROCEDURE

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Abstract

The mesh generation is one of the main step of finite element method. This paper presents an adaptive h-refinement procedure in the finite element method. The representative adaptive methods are the r-refinement procedure which is relocated only the nodal position around the attention point, the h-refinement procedure which is subdivided elements and the p-refinement procedure which is increased the order of elements, and a method which is combined with these methods. As for these three kinds of methods, the r-refinement procedure is not practical because it doesn't expect much to improve accuracy by itself. Thus, it is taken in combination with others. And the h-refinement and p-refinement procedure are applied to subjects well by themselves. Therefore, the purpose of this study is to present an adaptive h-refinement procedure in finite element method.

1 INTRODUCTION

Recently, two or three-dimensional finite element analysis is used for the field of science and engineering owing to development of a computer system. When a phenomena is analyzed, more complicated analytical domain is required the accurate results. Thus, it is important to obtain expectative results on more complicated analytical domain using limited computer. Regarding the finite element method, it is well-known that numerical results are obtained more accurately by remeshing to smaller elements. Moreover, the numerical results depend on the analytical mesh. The mesh is required to analyze accurately for the numerical analysis.

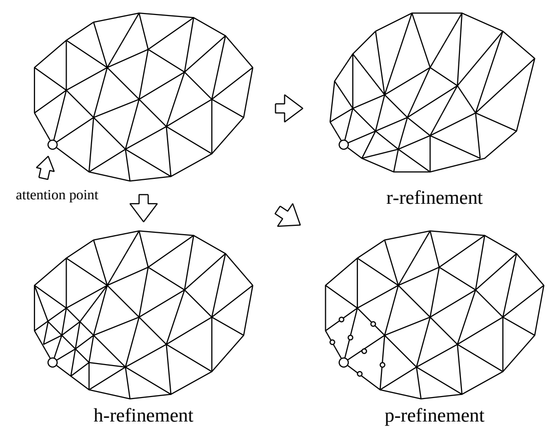
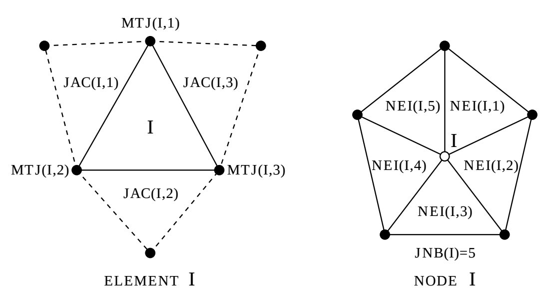


Fig.2. Representative adaptive methods



MTJ : Relation between Element and Nodes
JAC : Relation between Element and Adjacent Elements
JNB : the Number of Element surrounding Node
NEI : Element Number surrounding Node

Fig.1. Needful Data for Mesh Generation

The mesh generation needs the data described in Fig.1. The data relating a certain element need the relation between element and nodes and the relation between element and adjacent element. The data relating a certain nodal point need the element number surrounding node and the number of element surrounding node. It is important for the mesh generation to obtain the adjacent relation of triangle.
As for finite element method, if relative accuracy or error of numerical results about unknown exact solution could be estimated, it would apply to numerical results and be a great convenience. Thus, the various ways of improving or estimating accuracy by means of converting the subdivision of elements or the order of elements based on the error estimator adaptively,

is considered. These ways are known as the adaptive method. The representative adaptive methods are the r-refinement procedure which is relocated only the nodal position around the attention point, the h-refinement procedure which is subdivided elements and the p-refinement procedure which is increased the order of elements (see Fig.2), and a method which is combined with these methods. As for these three kinds of methods, the r-refinement procedure is not practical because it doesn't expect much to improve accuracy by itself. Thus, it is taken in combination with others. And the h-refinement and p-refinement procedure are applied to subjects well by themselves. About the h-refinement procedure, it is described that the way to subdivide initial elements which is designated. Moreover, the algorithm is simple and it is easy to apply to more complicated analytical domain.

Therefore, the purpose of this study is to present an adaptive h-refinement procedure in finite element method. Examples illustrate the behaviour and the efficiency of the approach described.

2 h-REFINEMENT PROCEDURE

The h-refinement procedure is one of the adaptive methods. It is described that the way to subdivide initial elements which is designated. Moreover, the algorithm is simple and it is easy to apply to complicated analytical domain. A h-refinement procedure is illustrated as follows. The basic procedure of the subdivision is described in Fig.3. In short, it applies the way of generating three nodal points to subdi-

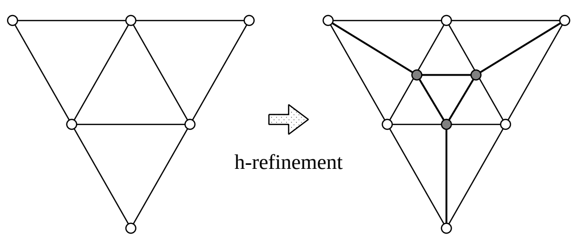


Fig.3. Basic Procedure of Subdivision

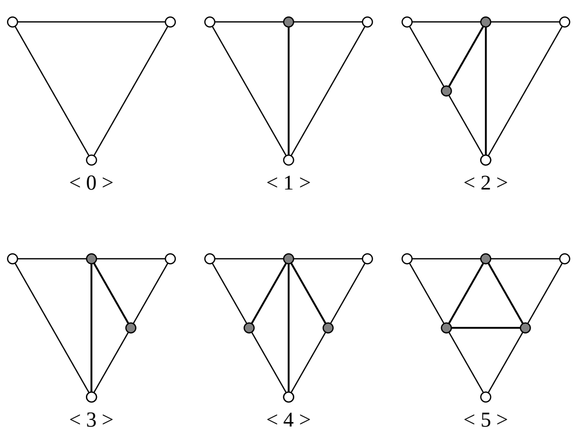


Fig.4. Pattern of Subdivision

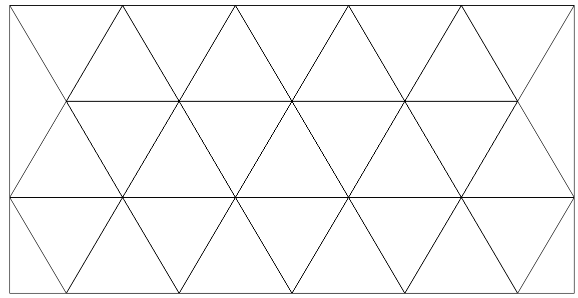


Fig.5. First Step : Designated

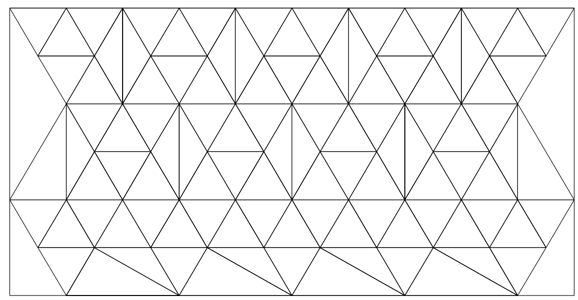


Fig.6. Second Step : Subdivided

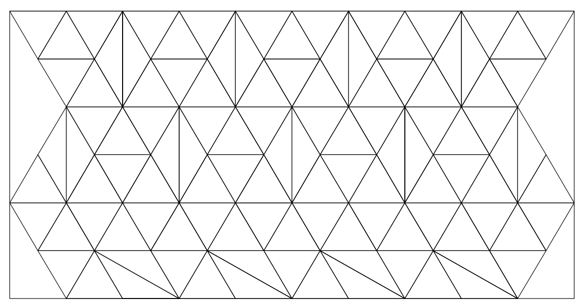


Fig.7. Third Step : Modified

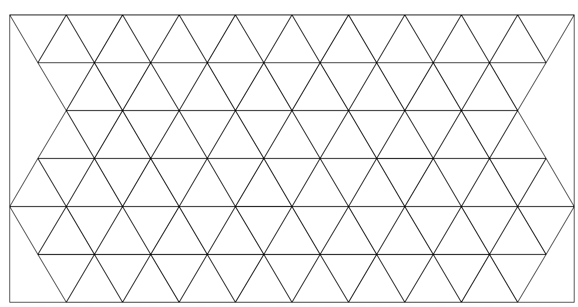


Fig.8. Last Step : Swapping

vide the designated element. If the designated elements are subdivided according to the basic procedure, the elements represent the patterns described in Fig.4. The patterns is 6 in all. The procedure of subdivision is illustrated by means of simple model. First of all, the elements which is subdivided is designated (see Fig.5). The elements is provided the pattern number 0. Secondly, the elements is subdivided according to the basic procedure (see Fig.6). The elements is provided the adequate pattern number. Thirdly, the constrained elements is modified (see Fig.7). The elements of the pattern number 1, 2 and 3 is modified to configuration of the pattern number 4. Finally, the elements is arranged by means of swapping the edge. The elements of the pattern number 4 is arranged to expectative configuration of the pattern number 5 by means of swapping the edge (see Fig.8). Therefore the initial mesh is subdivided into the mesh described in Fig.8 using a h-refinement procedure. The characteristic of a h-refinement procedure is able to subdivide only the designated elements.

3 NUMERICAL EXAMPLE

The efficiency of the approach is described. It is ascertained whether the mesh is subdivided locally and globally using a h-refinement procedure in this study. The mesh described in Fig.8 is applied to the initial mesh. Kasumigaura Lake described in Fig.14 is applied to the initial mesh so as to ascertain whether the arbitrary domain is subdivided regularly.

3.1 Global Subdivision

It is ascertained whether the mesh is subdivided globally using a h-refinement procedure. Figs.9 and 10 show the global subdivision. The mesh described in Fig.8 is subdivided into that in Fig.9 globally. The mesh described in Fig.9 is subdivided into that in Fig.10 globally. From the results, the mesh is able to be subdivided globally and regularly using a h-refinement procedure.

3.2 Local Subdivision

It is ascertained whether the mesh is subdivided locally using a h-refinement procedure. Figs.11,12 and 13 show the local subdivision. The center of the mesh described in Fig.9 is subdivided into that in Fig.11 locally. The side of the mesh described in Fig.9 is subdivided into that in Fig.12 locally. The

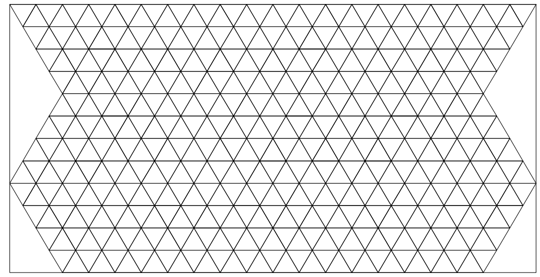


Fig.9. Global Subdivision 1

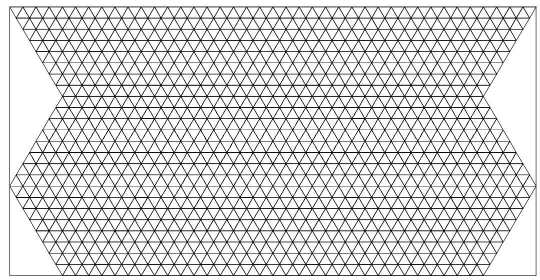


Fig.10. Global Subdivision 2

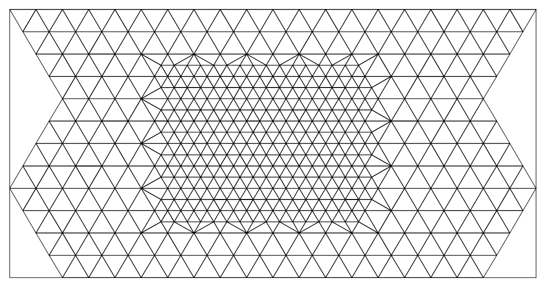


Fig.11. Local Subdivision 1

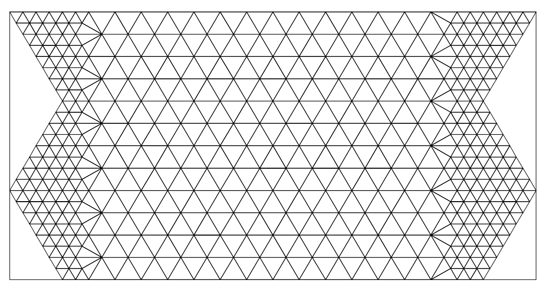


Fig.12. Local Subdivision 2

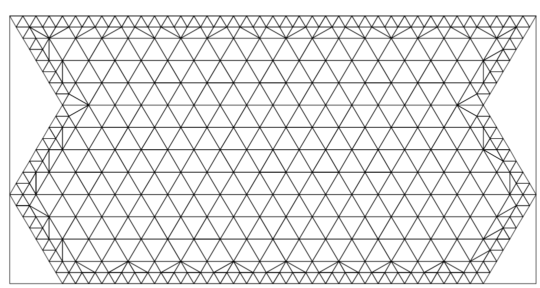


Fig.13. Local Subdivision 3

boundary of the mesh described in Fig.9 is subdivided into that in Fig.13 locally. From the results, the mesh is able to be subdivided locally using a h-refinement procedure.

3.3 Kasumigaura Lake

Fig.14 shows Kasumigaura Lake which is arbitrary domain. This mesh is the modified result using the Radiation Datum Line Method (RDLM) proposed by Matsumoto. The RDLM is one of the node generation methods. This RDLM is able to generate nodes symmetrically from the datum line in the object domain. This node generation is able to be performed the triangulation which is similar in shape to an equilateral triangle. However, the RDLM doesn't necessarily generate expectative shape, especially near the boundary. It is necessary to modify the mesh near the boundary. Thus, Fig.14 shows the result of modification. The elements which is comparatively big in area is subdivided into the fine mesh described in Fig.15 adaptively. From the result, the arbitrary domain is able to be subdivided adaptively and regularly using a h-refinement procedure.

4 CONCLUSION

In this study, a h-refinement procedure in finite element method is presented. The finite element mesh is able to be subdivided locally, globally, adaptively, and regularly using a h-refinement procedure. However, the mesh using a h-refinement procedure must be applied to the analytical domain so as to analyze the adaptive finite element method. Therefore, to ascertain the efficiency of a h-refinement procedure, the adaptive finite element method using the error estimation of the Zienkiewicz-Zhu's method would like to be presented.

REFERENCES

[1] Junichi Matsumoto, Tsuyoshi Umetsu and Mutsuto Kawahara: Incompressible Viscous Flow Analysis and Adaptive Finite Element Method Using Linear Bubble Function, 1999.

[2] O.C. Zienkiewicz and J.Z. Zhu: A simple error estimator and adaptive procedure for practical engineering analysis, Int. J. Numer.Methods. Eng., 24, 337-357(1987)

[3] J.T. Oden, L. Demkowicz, W. Rachowicz and T.A. Westermann: Toward A Universal *h-p* Adaptive Finite Element Strategy, Part 2. A Posteriori Error Estimation, Computer Methods in Applied Mechanics and Engineering 77 (1989) 113-180.

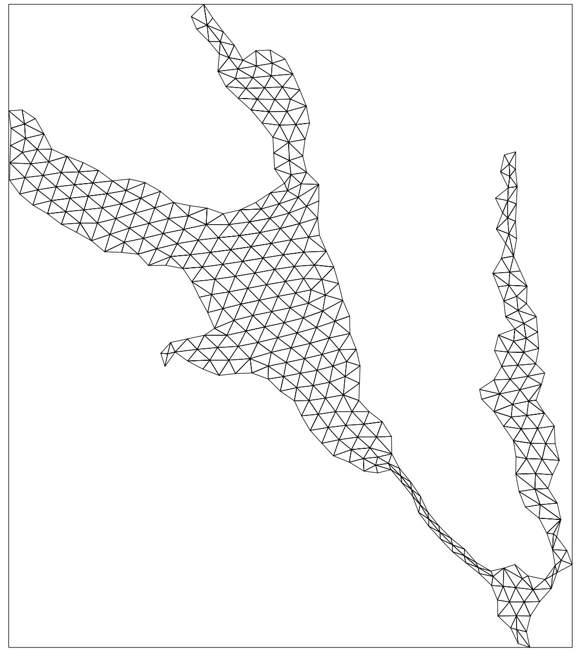


Fig.14. Modified Result using the RDLM

Fig.15. Result of a h-refinement Procedure