for the case when $V_0 = 0$ assume ~~mass~~ diffusion only.

$$C_{lr} = C_{l_0}\left[1 + \frac{\alpha a}{Db}\ln\frac{r_x}{r_0}\right]$$

$V_0 = 0$

when ~~$V_0 \neq 0$~~ $C_{avg} =$

$$C_{avg} = C_0\left[\left(1 - \frac{1}{2}\frac{\alpha R_0}{Db}\right) + R_x^2\left(\frac{\alpha R_0}{Db}\right)\frac{\ln\frac{R_x}{R_0}}{R_x^2 - R_0^2}\right]^{-1}$$

$\frac{C_{avg}}{C_0} = [\;]^{-1}$

$P_c$ $C_l$

when $V_0 \neq 0$

$$C_{avg} = C_0 V_0\left[\alpha + (V_0 - \alpha)\left(\frac{2}{2-\gamma}\right)\left(\frac{\left(\frac{r_x}{r_0}\right)^{2-\gamma} - 1}{\left(\frac{r_x}{r_0}\right)^2 - 1}\right)\right]^{-1}$$

$\alpha = I_{max}/(k_m + C_0)$ $\qquad \frac{r_x}{r_0}$ = ~~Relative~~ Root Ratio

$\gamma_2 = \frac{r_0 v}{Db}$ $\qquad V_0 \neq 0$

$\gamma = \frac{r_0 V_0}{Db}$ $\qquad V_0 \neq 0$

$P_c C_{avg} = C_0$

$\frac{U_{est}}{\Delta t} = 2\pi r_0 L \alpha P_c C_{avg} \Delta$ where $C_{avg}$ is Conc a soln

~~$C_{avg}$~~ $= P_c\left[\left(1 - \frac{1}{2}\gamma\right) + \frac{R_x^2 \alpha \gamma \ln\frac{R_x}{R_0}}{R_x^2 - R_0^2}\right]^{-1}$ $\qquad V_0 = 0$

$$P_c = \frac{\alpha + V_0\left[\alpha + (V_0 - \alpha)\left(\frac{2}{2-\gamma}\right)\left(\frac{\left(r_x/r_0\right)^{2-\gamma} - 1}{(r_x/r_0)^2 - 1}\right)\right]^{-1}}{}$$