# GATE 2023 BM 30

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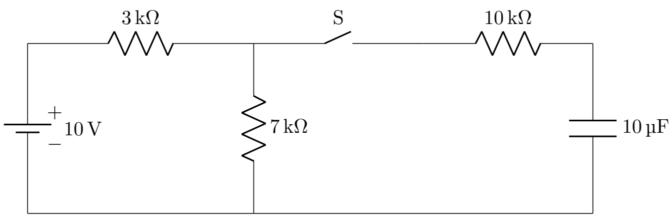
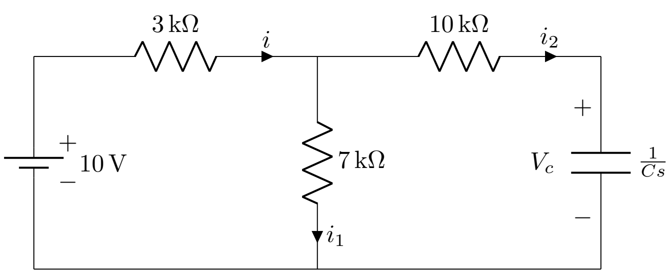
**Question:** In the following circuit, the switch S is open for $t < 0$ and closed for $t \geq 0$. What is the steady state voltage (in Volts) across the capacitor when the switch is closed?




**Solution:**
In steady state, no current flows through capacitor.

$$i_2 = 0 \tag{1}$$
$$\implies i = i_1 \tag{2}$$
$$v_c = (7\text{k}\Omega)(i_1) \tag{3}$$
$$10\text{V} = (10\text{k}\Omega)\, i_1 \tag{4}$$
$$\implies i_1 = 1\text{mA} \tag{5}$$
$$v_c = 7\text{V} \tag{6}$$

In s-domain:




$$(7\text{k}\Omega)i_1 = 10\text{V} - (3\text{k}\Omega)i \tag{7}$$
$$(10\text{k}\Omega)i_1 = 10\text{V} - (3\text{k}\Omega)i_2 \tag{8}$$
$$\implies i_1 = \frac{10\text{V} - (3\text{k}\Omega)i_2}{10\text{k}\Omega} \tag{9}$$
$$= 1\text{mA} - 0.3 i_2 \tag{10}$$
$$(7\text{k}\Omega)i_1 = (10\text{k}\Omega)i_2 + v_c \tag{11}$$
$$7\text{V} - (2.1\text{k}\Omega)i_2 = (10\text{k}\Omega)i_2 + v_c \tag{12}$$
$$\implies v_c + 12100 i_2 = 7 \tag{13}$$

where $i_2 = C\frac{dv_c}{dt}$.

$$\implies v_c + 12100C\frac{dv_c}{dt} = 7 \tag{14}$$
$$v_c + 0.121\frac{dv_c}{dt} = 7 \tag{15}$$
$$\frac{dv_c}{dt} + \frac{v_c}{0.121} = \frac{7}{0.121} \tag{16}$$
$$v_c e^{\frac{t}{0.121}} = \frac{7}{0.121}\int e^{\frac{t}{0.121}}\,dt \tag{17}$$
$$v_c e^{\frac{t}{0.121}} = \frac{7}{0.121}\left(0.121 e^{\frac{t}{0.121}} + c\right) \tag{18}$$

where $c$ is the integration constant. Since $v_c = 0$, $c = -0.121$.

$$v_c = 7(1 - e^{-\frac{t}{0.121}}) \tag{19}$$