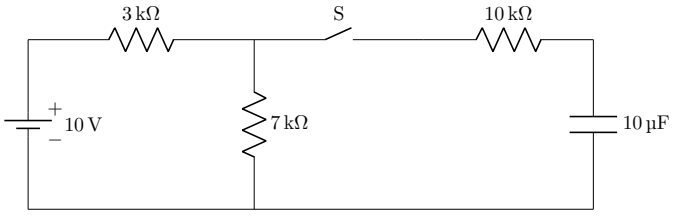
GATE 2023 BM 30

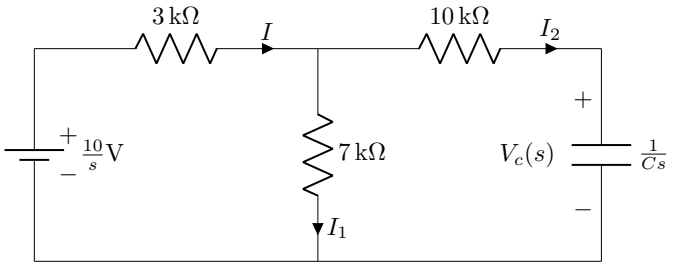
EE23BTECH11007 - Aneesh Kadiyala*

Question: In the following circuit, the switch S is open for $t < 0$ and closed for $t \geq 0$. What is the steady state voltage (in Volts) across the capacitor when the switch is closed?



Solution:

1) In s-domain:



$$\implies I(s) = \frac{\frac{10}{s}\text{V}}{3\text{k}\Omega + \frac{(7\text{k}\Omega)(10\text{k}\Omega + \frac{1}{sC})}{17\text{k}\Omega + \frac{1}{sC}}} \tag{1}$$

$$I = I_1 + I_2 \tag{2}$$

$$I_1(7\text{k}\Omega) = I_2\left(10\text{k}\Omega + \frac{1}{sC}\right) \tag{3}$$

$$I_2(s) = \frac{7\text{k}\Omega}{17\text{k}\Omega + \frac{1}{sC}} I(s) \tag{4}$$

$$\implies I_2(s) = \frac{7\left(10^{-5}\right)}{0.121s + 1} \tag{5}$$

$$V_c(s) = I_2(s)\frac{1}{sC} \tag{6}$$

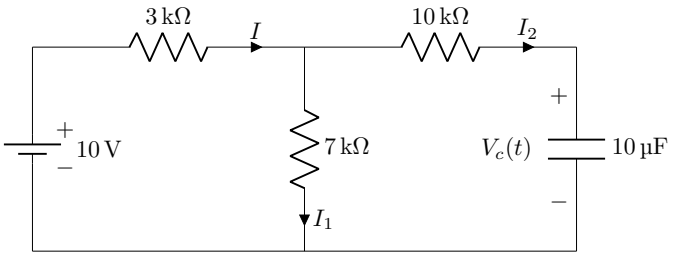
$$= \frac{7}{s(0.121s + 1)} \tag{7}$$

$$= 7\left(\frac{1}{s} - \frac{1}{s + \frac{1}{0.121}}\right) \tag{8}$$

Taking inverse Laplace transform:

$$V_c(t) = 7u(t)\left(1 - e^{-\frac{t}{0.121}}\right) \tag{9}$$

2) For $t > 0$:



$$(7\text{k}\Omega)I_1 = 10\text{V} - (3\text{k}\Omega)I \tag{10}$$

$$(10\text{k}\Omega)I_1 = 10\text{V} - (3\text{k}\Omega)I_2 \tag{11}$$

$$\implies I_1 = \frac{10\text{V} - (3\text{k}\Omega)I_2}{10\text{k}\Omega} \tag{12}$$

$$= 1\text{mA} - 0.3I_2 \tag{13}$$

$$(7\text{k}\Omega)I_1 = (10\text{k}\Omega)I_2 + V_c \tag{14}$$

$$7\text{V} - (2.1\text{k}\Omega)I_2 = (10\text{k}\Omega)I_2 + V_c \tag{15}$$

$$\implies V_c + 12100I_2 = 7 \tag{16}$$

$$I_2 = (10\mu\text{F})\frac{dV_c}{dt} \tag{17}$$

$$\implies V_c + 0.121\frac{dV_c}{dt} = 7 \tag{18}$$

$$\frac{dV_c}{dt} + \frac{V_c}{0.121} = \frac{7}{0.121} \tag{19}$$

$$V_c = 7\left(1 + \frac{c}{0.121}e^{-\frac{t}{0.121}}\right) \tag{20}$$

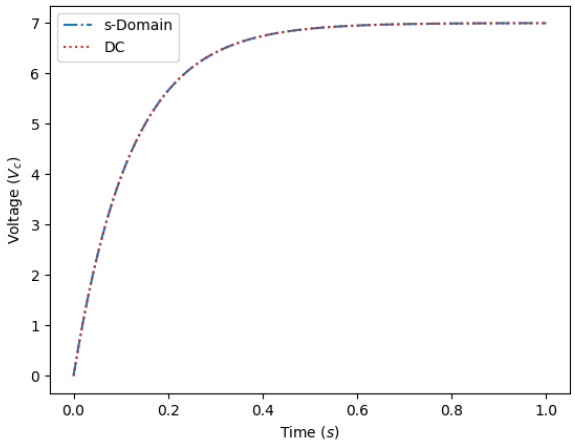
where c is the integration constant. Since $V_c = 0$, $c = -0.121$.

$$V_c = 7(1 - e^{-\frac{t}{0.121}}) \tag{21}$$

In steady state,

$$t \to \infty \tag{22}$$

$$V_c = 7\text{V} \tag{23}$$


s-Domain
DC
Voltage (V_c)
Time (s)
0
1
2
3
4
5
6
7
0.0
0.2
0.4
0.6
0.8
1.0