GATE 2023 BM 30

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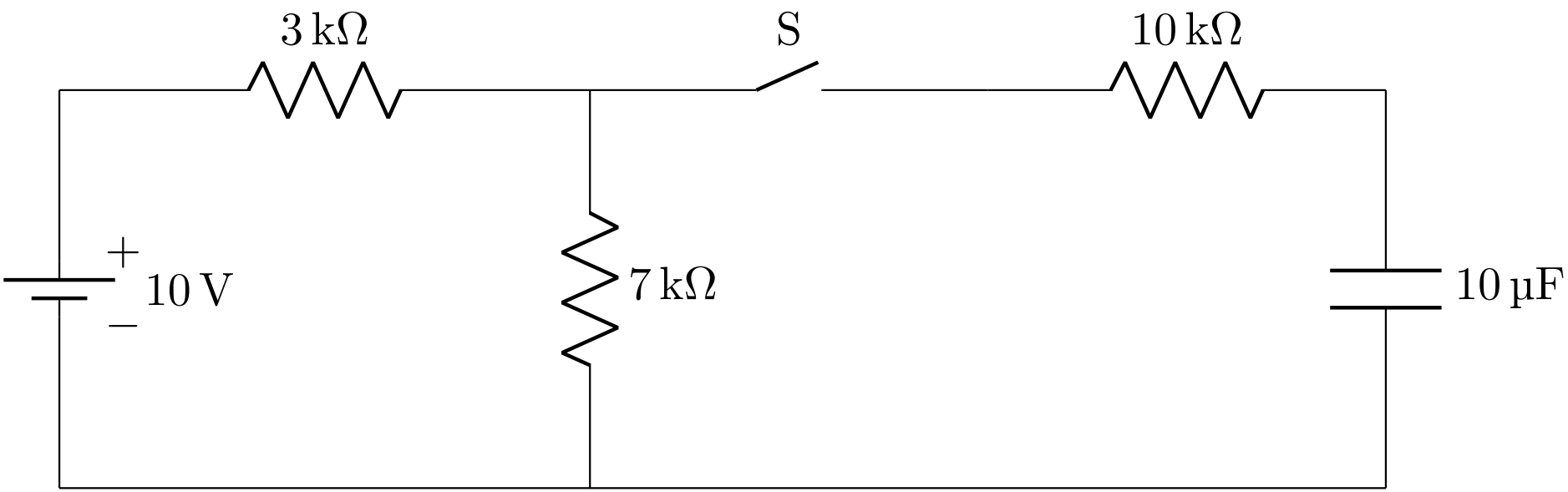
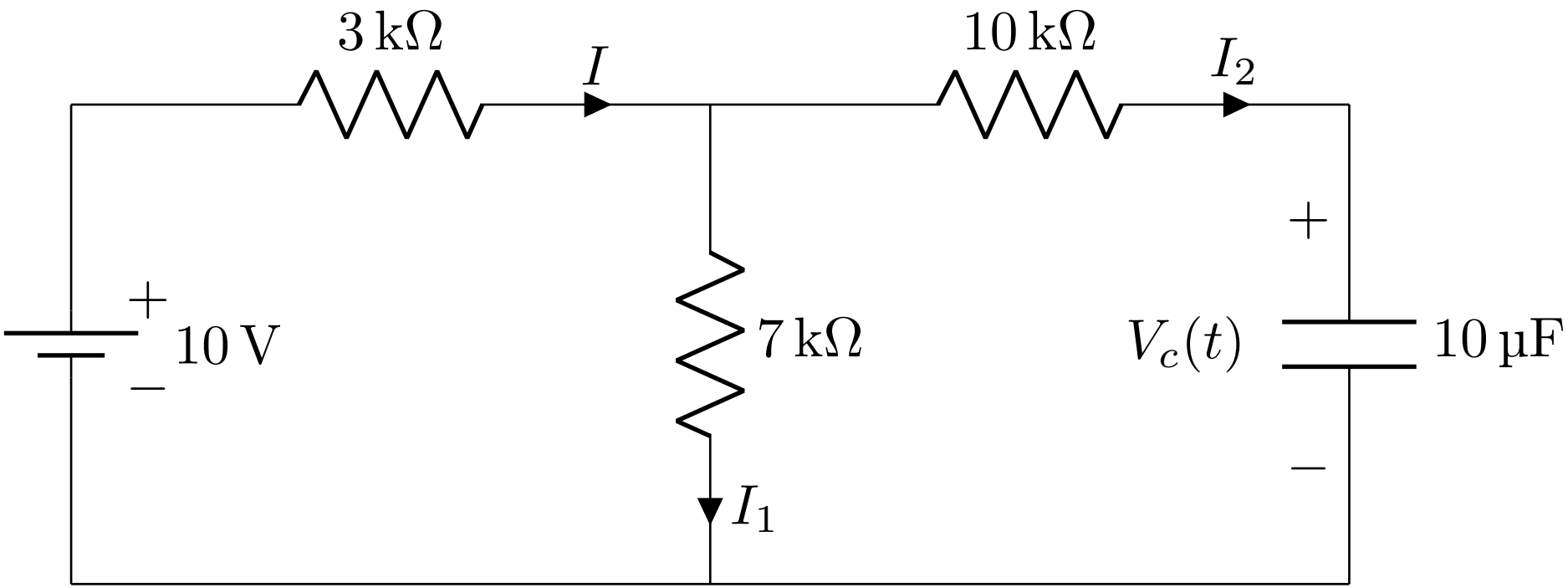
Question

In the following circuit, the switch S is open for $t < 0$ and closed for $t \geq 0$. What is the steady state voltage (in Volts) across the capacitor when the switch is closed? (GATE 2023 BM)



Circuit Diagram

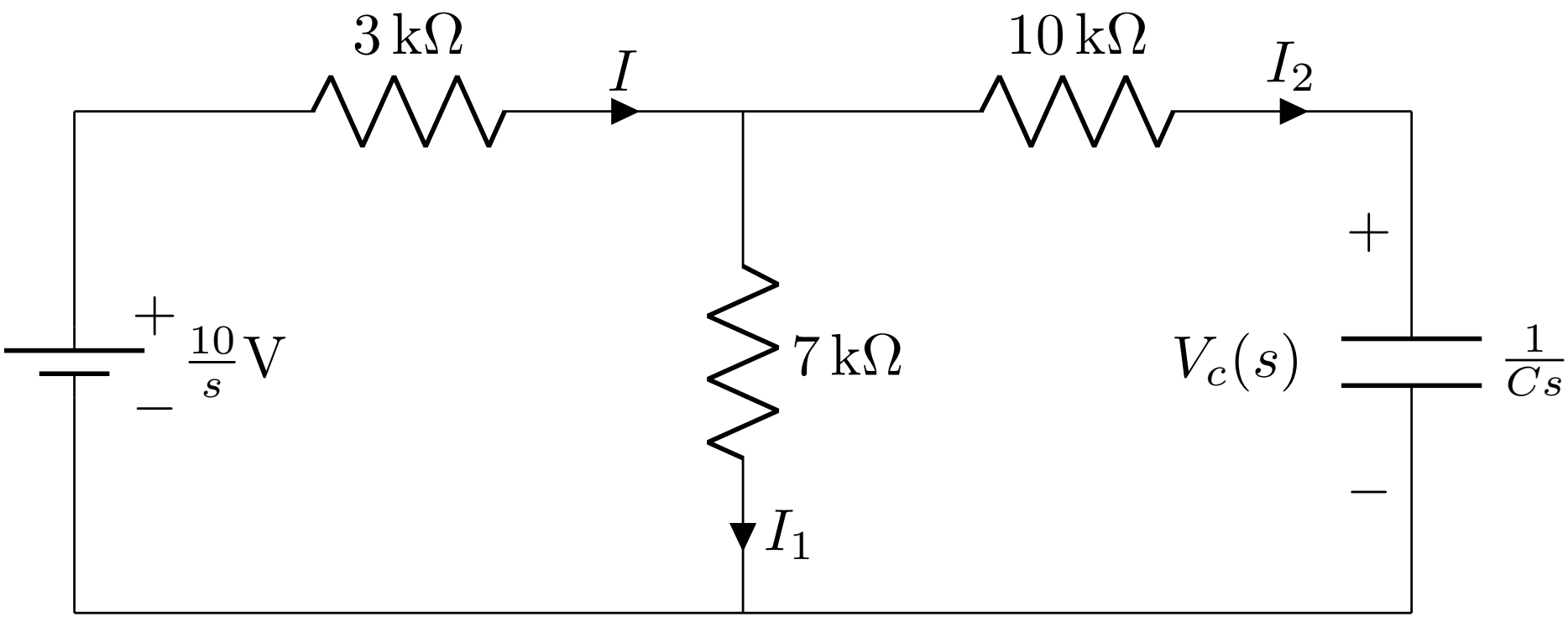


Solution

In steady state, no current flows through the capacitor.

$$I_2 = 0 \tag{1}$$
$$V_c = (7\text{k}\Omega)\, I_1 \tag{2}$$
$$= (7\text{k}\Omega)\, I \tag{3}$$
$$= (7\text{k}\Omega)\, \frac{10\text{V}}{10\text{k}\Omega} \tag{4}$$
$$\implies V_c = 7\text{V} \tag{5}$$

S-domain Circuit Diagram



S-domain Analysis

$$\implies I(s) = \frac{\frac{10}{s}\text{V}}{3\text{k}\Omega + \frac{(7\text{k}\Omega)(10\text{k}\Omega + \frac{1}{sC})}{17\text{k}\Omega + \frac{1}{sC}}} \tag{6}$$

$$I = I_1 + I_2 \tag{7}$$

$$I_1(7\text{k}\Omega) = I_2\left(10\text{k}\Omega + \frac{1}{sC}\right) \tag{8}$$

$$I_2(s) = \frac{7\text{k}\Omega}{17\text{k}\Omega + \frac{1}{sC}} I(s) \tag{9}$$

$$\implies I_2(s) = \frac{7\left(10^{-5}\right)}{0.121s + 1} \tag{10}$$

S-domain Analysis

$$V_c(s) = I_2(s)\frac{1}{sC} \tag{11}$$

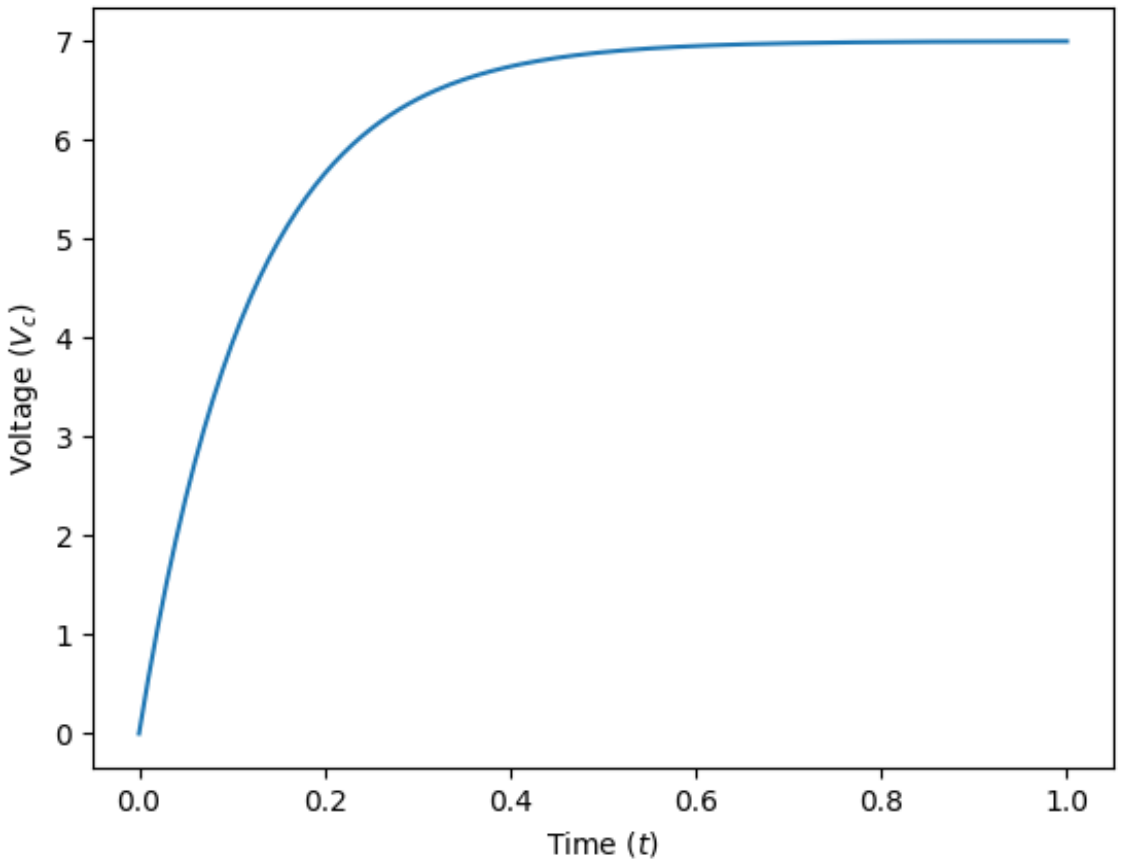
$$= \frac{7}{s(0.121s+1)} \tag{12}$$

$$= 7\left(\frac{1}{s} - \frac{1}{s+\frac{1}{0.121}}\right) \tag{13}$$

Taking inverse Laplace transform:

$$V_c(t) = 7u(t)\left(1 - e^{-\frac{t}{-0.121}}\right) \tag{14}$$

Plot



Code

```
#ifndef EE1205_EE1205_H
#define EE1205_EE1205_H

#include <stdlib.h>

// Bare basic version of Python's numpy.linspace(...)
double* linspace(double start, double stop, size_t count) {
    double* result = (double*) malloc(sizeof(double) * count);
    for (size_t i = 0; i < count; i++) {
        result[i] = start + (stop - start) * i / (count - 1);
    }
    return result;
}

#endif
```

Code

```
#include "ee1205.h"
#include <stdio.h>
#include <math.h>

// Function that calculates v_c(t) = 7(1 — e^(—t/0.121))
double v_c(double t) {
    return 7. * (1. — exp(—t / .121));
}

int main () {
    // x—axis
    double* t = linspace(0, 1, 201);

    // Open file.
    FILE* out;
    fopen_s(&out, "gate23bm30cout.txt", "w");

    for (size_t i = 0; i < 201; i++) {
        // Write v_c(t) to output file.
        fprintf(out, "%lf ", v_c(t[i]));
    }

    // Close file.
    fclose(out);
    // Free memory allocated for t.
    free(t);

    return 0;
}
```

Code

```
import matplotlib.pyplot as plt
import numpy as np

# X-axis
t = np.linspace(0, 1, 201)

# Y-axis
V_c = np.loadtxt("gate23bm30cout.txt")

plt.plot(t, V_c)

plt.xlabel("Time ($t$)")
plt.ylabel("Voltage ($V_c$)")

plt.savefig("plot.png")
```

In steady state $t \to \infty$. From (14):

$$\lim_{t \to \infty} V_c(t) = 7\text{V} \tag{15}$$