

NCERT Discrete - 11.9.3.30

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Question 11.9.3.30: The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour 4th hour and n^{th} hour?

Solution:

TABLE 0
INPUT PARAMETERS

Parameter	Value	Description
$x(0)$	30	Initial no. of bacteria
r	2	Ratio of no. of bacteria at end of hour to start of hour (Common Ratio)
$x(n)$	$(x(0))(2^n)$	n^{th} term of the GP

1) Finding $x(n)$

$$x(n) = 2x(n-1) \quad (1)$$

$$= 2(2x(n-2)) \quad (2)$$

...

$$x(n) = 2^n x(0) \quad (3)$$

$$\Rightarrow x(n) = 30(2^n) \quad (4)$$

2) Z-transform of $x(n)$

Let Z-transform of $x(n)$ be $X(z)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)u(n)z^{-n} \quad (5)$$

$$X(z) = \sum_{n=0}^{\infty} (30)(2^n)(z^{-n}) \quad (6)$$

$$X(z) = 30 \lim_{n \rightarrow \infty} \sum_{i=0}^n \left(\frac{2}{z}\right)^i \quad (7)$$

a) If $|z| > 2$:

$$X(z) = \frac{30}{1 - \frac{2}{z}} \quad (8)$$

$$X(z) = \frac{30z}{z-2} \quad (9)$$

b) If $|z| \leq 2$:

$$X(z) \rightarrow \infty \quad (10)$$

$$\Rightarrow X(z) = \frac{30z}{z-2} \quad \forall |z| > 2 \quad (11)$$

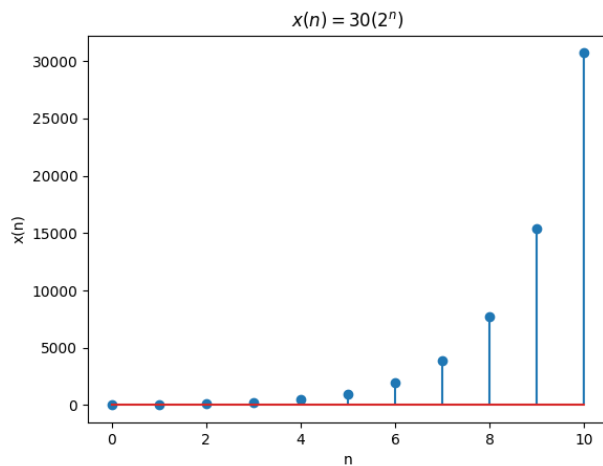


Fig. 1. Plot of $x(n)$ vs n