NCERT Discrete Assignment

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Question 10.5.2.19: Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000?
Solution:

1) Let Subba Rao's initial salary a_0 = Rs. 5000
   Let Subba Rao's increment d = Rs. 200
   Subba Rao's salary after n years = a_n = $a_0 + (n-1)d$

$$7000 = 5000 + (n-1)(200)$$

$$2000 = (n-1)(200)$$

$$n = 11$$

$\implies$ Subba Rao's income reaches Rs. 7000 eleven years after he started working.
Therefore, Subba Rao's income reached Rs. 7000 in the year 2006.

2) **Finding** $x(n)$
   The series is an arithmetic progression.

$$x(n) = x(1) + (n-1)d$$

where d is the common difference.
It is given that initial value $x(1)$ is 5000 and common difference d is 200.

$$\implies x(n) = 5000 + (n-1)(200)$$

$$\implies x(n) = 4800 + 200n$$

3) **Z-transform of** $x(n)$
   Let Z-transform of $x(n)$ be $X(z)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$X(z) = \sum_{n=1}^{\infty} (4800 + 200n)z^{-n}$$

$$X(z) = 4800 \lim_{n\to\infty} \sum_{i=1}^{n} z^{-i} + 200 \lim_{n\to\infty} \sum_{i=1}^{n} iz^{-i}$$

a) If $|z| > 1$:

$$X(z) = \frac{4800z^{-1}}{1 - z^{-1}} + 200(\frac{z^{-1}}{1 - z^{-1}} + \frac{z^{-1}}{(1 - z^{-1})^2})$$

$$X(z) = \frac{4800}{z - 1} + 200(\frac{1}{z - 1} + \frac{z}{(z - 1)^2})$$

$$X(z) = 5000(z - 1)^{-1} + 200z(z - 1)^2$$

b) If $|z| \leq 1$:

$$X(z) \to \infty$$

Question 11.9.3.30: The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour 4th hour and n^{th} hour?
Solution:

1) Let number of bacteria initially be a_0 = 30
   Let number of bacteria at the end of nth hour be a_n.
   Since number of bacteria doubles every hour,

$$a_n = 2a_{n-1}$$

$$a_n = 2(2a_{n-2})$$

$$...$$

$$a_n = 2^n a_0 = 2^n(30)$$

$$\implies a_2 = 2^2(30) = 120 \text{ and } a_4 = 2^4(30) = 480$$

Therefore, number of bacteria at the end of the 2nd hour is 120, 4th hour is 480, and n^{th} hour is $30(2^n)$.

2) **Finding** $x(n)$
   The series is a geometric progression.

$$x(n) = x(1)(r^{n-1})$$

where r is the common ratio.
From the solution, $x(1) = 30$, $r = 2$.

$$\implies x(n) = 30(2^{n-1})$$

$$\implies x(n) = 15(2^n)$$

3) **Z-transform of** $x(n)$

Let Z-transform of $x(n)$ be $X(z)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$X(z) = \sum_{n=1}^{\infty} (15)(2^n)(z^{-n})$$

$$X(z) = 15 \lim_{n\to\infty} \sum_{i=1}^{n} (\frac{2}{z})^i$$

a) If $|z| > 2$:

$$X(z) = 15\frac{\frac{2}{z}}{1 - \frac{2}{z}}$$

$$X(z) = 30(z - 2)^{-1}$$

b) If $|z| \leq 2$:

$$X(z) \to \infty$$