NCERT Discrete - 10.5.2.19

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Question 10.5.2.19: Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000?

Solution:

TABLE 0
INPUT PARAMETERS

Parameter	Value	Description
$x(0)$	5000	Initial Income
d	200	Annual Increment (Common Difference)
$x(n)$	$x(0) + nd$	n^{th} term of the AP

From the values given in Table 0:

$$7000 = 5000 + (n-1)(200) \tag{1}$$

$$\implies 2000 = (n-1)(200) \tag{2}$$

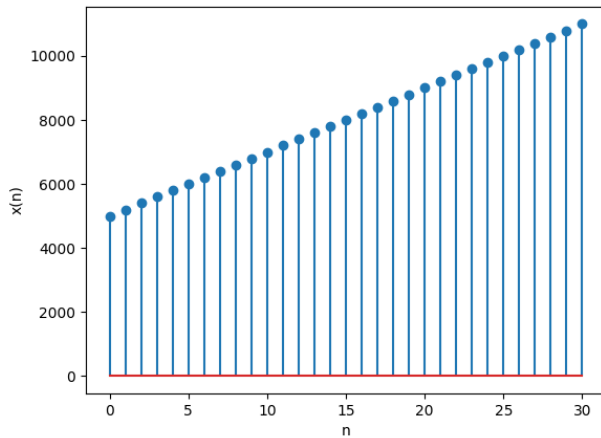$$\therefore n = 11 \tag{3}$$

$$x(n) = (x(0) + nd)(u(n)) \tag{4}$$

Fig. 0. Plot of $x(n)$ vs n. See Table 0 for details.

Let Z-transform of $x(n)$ be $X(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

$$X(z) = x(0)U(z) + dZ\{nu(n)\} \tag{5}$$

$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \tag{6}$$

Using the values from Table 0:

$$X(z) = \frac{5000}{1 - z^{-1}} + \frac{200z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \tag{7}$$