

# NCERT Discrete - 10.5.2.19

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**Question 10.5.2.19:** Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000?

**Solution:**

TABLE 0  
INPUT PARAMETERS

Parameter	Value	Description
$x(0)$	5000	Initial Income
$d$	200	Annual Increment (Common Difference)
$x(n)$	$x(0) + nd$	$n^{\text{th}}$ term of the AP

$$7000 = 5000 + (n - 1)(200) \quad (1)$$

$$2000 = (n - 1)(200) \quad (2)$$

$$n = 11 \quad (3)$$

1) Finding  $x(n)$

The series is an arithmetic progression.

$$x(n) = x(0) + nd \quad (4)$$

From Table 0:

$$x(n) = (5000 + 200n)(u(n)) \quad (5)$$

as  $x(n) = 0 \forall n < 0$ .

2) Z-transform of  $x(n)$

Let Z-transform of  $x(n)$  be  $X(z)$ . Let  $U(z)$  be the Z-transform of  $u(n)$ .

$$X(z) = \sum_{n=-\infty}^{\infty} (5000 + 200n)(u(n))(z^{-n}) \quad (6)$$

$$X(z) = 5000 \sum_{n=-\infty}^{\infty} u(n)(z^{-n}) + 200 \sum_{n=-\infty}^{\infty} n(u(n))(z^{-n}) \quad (7)$$

Using differentiation in Z-domain property,

$$Z\{nx(n)\} = -z \frac{d}{dz} X(z) \quad (8)$$

$$X(z) = 5000U(z) + 200(-z) \frac{d}{dz} U(z) \quad (9)$$

$$= \frac{5000}{1 - z^{-1}} + 200(-z) \left( -\frac{1}{z^2(1 - z^{-1})^2} \right) \quad (10)$$

$$= 5000(1 - z^{-1})^{-1} + 200 \frac{z}{(z - 1)^2} \quad (11)$$

$$X(z) = \frac{5000z}{z - 1} + \frac{200z}{(z - 1)^2} \forall |z| > 1 \quad (12)$$

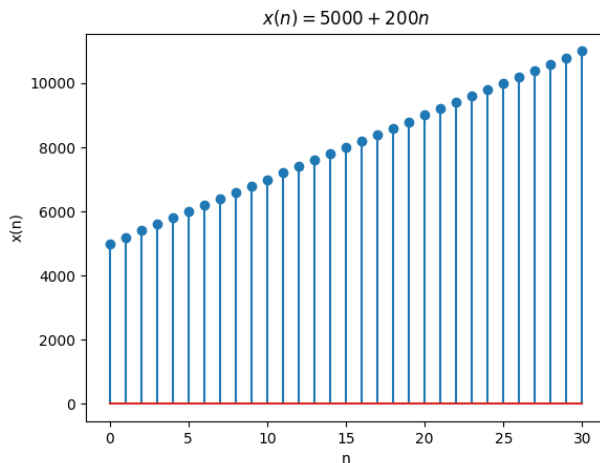


Fig. 1. Plot of  $x(n)$  vs  $n$