NCERT Discrete - 11.9.3.30

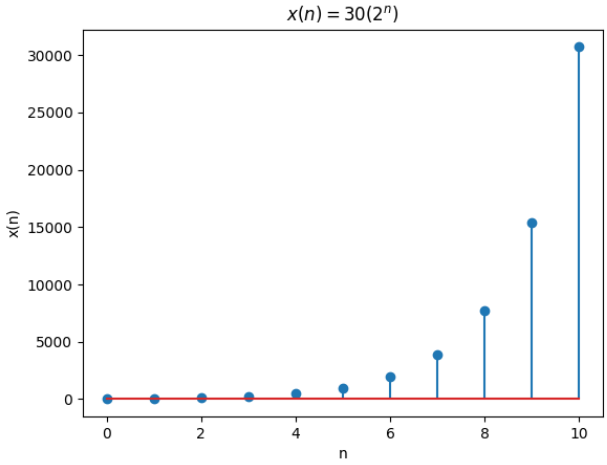
EE23BTECH11007 - Aneesh Kadiyala*

Question 11.9.3.30: The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour 4^{th} hour and n^{th} hour?

Solution:

TABLE 0
INPUT PARAMETERS

Parameter	Value	Description
$x(0)$	30	Initial no. of bacteria
$x(n)/x(n-1)$	2	Ratio of no. of bacteria at end of hour to start of hour



1) **Finding** $x(n)$
Let number of bacteria initially be $x(0) = 30$
Let number of bacteria at the end of n^{th} hour be $x(n)$.
Since number of bacteria doubles every hour,

$$x(n) = 2x(n-1) \tag{1}$$
$$= 2(2x(n-2)) \tag{2}$$
$$\dots$$
$$x(n) = 2^n x(0) \tag{3}$$
$$\implies x(n) = 30(2^n) \tag{4}$$

Therefore, number of bacteria at the end of the 2^{nd} hour is 120, 4^{th} hour is 480, and n^{th} hour is $30(2^n)$.

2) **Z-transform of** $x(n)$
Let Z-transform of $x(n)$ be $X(z)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)u(n)z^{-n} \tag{5}$$
$$X(z) = \sum_{n=0}^{\infty}(30)(2^n)(z^{-n}) \tag{6}$$
$$X(z) = 30\lim_{n\to\infty}\sum_{i=0}^{n}(\frac{2}{z})^i \tag{7}$$

a) If $|z| > 2$:

$$X(z) = \frac{30}{1-\frac{2}{z}} \tag{8}$$
$$X(z) = \frac{30z}{z-2} \tag{9}$$

b) If $|z| \le 2$:

$$X(z) \to \infty \tag{10}$$

$$\implies X(z) = \frac{30z}{z-2} \forall |z| > 2 \tag{11}$$