NCERT Discrete - 11.9.3.30

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Question 11.9.3.30: The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour 4^{th} hour and n^{th} hour?

Solution:

TABLE 0
INPUT PARAMETERS

Parameter	In terms of x(n)	Value
Initial number of bacteria (a_0)	x(0)	30
Ratio of bacteria at the end of the hour to the start of the hour (r)	$x(n)/x(n-1)$	2

1) Let number of bacteria initially be $a_0 = 30$
Let number of bacteria at the end of n^{th} hour be a_n.
Since number of bacteria doubles every hour,

$$a_n = 2a_{n-1}$$
$$a_n = 2(2a_{n-2})$$
$$\dots$$
$$a_n = 2^n a_0 = 2^n(30)$$
$$\implies a_2 = 2^2(30) = 120 \text{ and } a_4 = 2^4(30) = 480$$

Therefore, number of bacteria at the end of the 2^{nd} hour is 120, 4^{th} hour is 480, and n^{th} hour is $30(2^n)$.

2) **Finding** $x(n)$
The series is a geometric progression.

$$x(n) = x(0)(r^n)$$

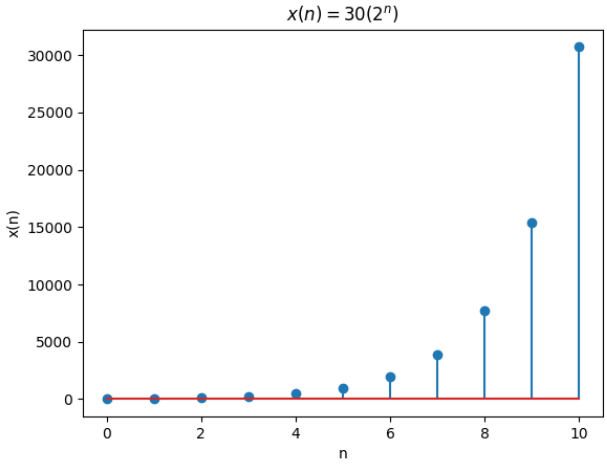
where r is the common ratio.
It is given that $x(0) = 30$, $r = 2$ (see table 0).

$$\implies x(n) = 30(2^n)(u(n))$$

as $x(n) = 0 \forall n < 0$.

3) **Z-transform of** $x(n)$
Let Z-transform of $x(n)$ be $X(z)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)u(n)z^{-n}$$



$$X(z) = \sum_{n=0}^{\infty} (30)(2^n)(z^{-n})$$

$$X(z) = 30 \lim_{n\to\infty} \sum_{i=0}^{n} (\frac{2}{z})^i$$

a) If $|z| > 2$:

$$X(z) = \frac{30}{1 - \frac{2}{z}}$$

$$X(z) = \frac{30z}{z-2}$$

b) If $|z| \leq 2$:

$$X(z) \to \infty$$

$$\implies X(z) = \frac{30z}{z-2} \forall |z| > 2$$