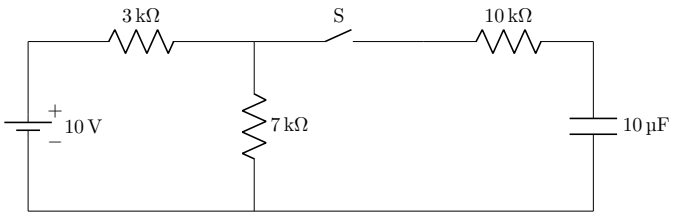
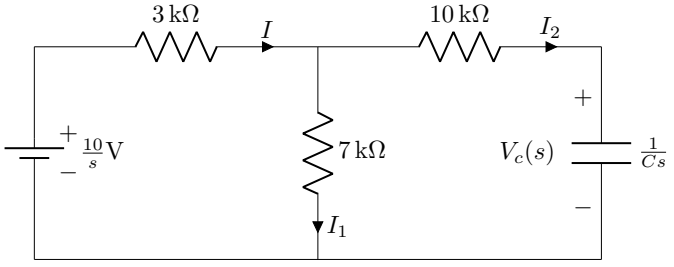
GATE 2023 BM 30

EE23BTECH11007 - Aneesh Kadiyala*

Question: In the following circuit, the switch S is open for $t < 0$ and closed for $t \geq 0$. What is the steady state voltage (in Volts) across the capacitor when the switch is closed?



Solution:
In s-domain:



$$\implies I(s) = \frac{\frac{10}{s}\text{V}}{3\text{k}\Omega + \frac{(7\text{k}\Omega)(10\text{k}\Omega + \frac{1}{sC})}{17\text{k}\Omega + \frac{1}{sC}}} \tag{1}$$

$$I = I_1 + I_2 \tag{2}$$

$$I_1(7\text{k}\Omega) = I_2\left(10\text{k}\Omega + \frac{1}{sC}\right) \tag{3}$$

$$I_2(s) = \frac{7\text{k}\Omega}{17\text{k}\Omega + \frac{1}{sC}} I(s) \tag{4}$$

$$\implies I_2(s) = \frac{7\left(10^{-5}\right)}{0.121s + 1} \tag{5}$$

$$V_c(s) = I_2(s)\frac{1}{sC} \tag{6}$$

$$= \frac{7}{s(0.121s + 1)} \tag{7}$$

$$= 7\left(\frac{1}{s} - \frac{1}{s + \frac{1}{0.121}}\right) \tag{8}$$

Taking inverse Laplace transform:

$$v_c(t) = 7u(t)\left(1 - e^{-\frac{t}{0.121}}\right) \tag{9}$$