

GATE 2021 EC 23

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Question: A speech signal, band limited to 4 kHz, is sampled at 1.25 times the Nyquist rate. The speech samples, assumed to be statistically independent and uniformly distributed in the range -5 V to +5 V, are subsequently quantized in an 8-bit uniform quantizer and then over a voice-grade AWGN telephone channel. If the ratio of transmitted signal power to channel noise power is 26 dB, the minimum channel bandwidth required to ensure reliable transmission of the signal with arbitrarily small probability of transmission error (*rounded off to one decimal place*) is _____ kHz.

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Solution:

TABLE 0
INPUT PARAMETERS

Parameter	Value	Description
B_0	4 kHz	Bandwidth of signal
R_N	$2B_0$	Nyquist Rate
f_s	$1.25R_N$	Sampling Frequency
R	nf_s	Data Rate
C	$B \log_2 \left(1 + \frac{P}{N}\right)$	Capacity of AWGN Channel with bandwidth B
$10 \log_{10} \frac{P}{N}$	26 dB	Signal to Noise Ratio

The signal is band limited to 4 kHz. Let the bandwidth of the signal be B_0 .

$$B_0 = 4\text{kHz} \quad (1)$$

$$\text{Nyquist Rate } R_N = 2B_0 \quad (2)$$

$$\Rightarrow R_N = 8\text{kHz} \quad (3)$$

$$\text{Sampling Frequency } f_s = 1.25R_N \quad (4)$$

$$\Rightarrow f_s = 10\text{kHz} \quad (5)$$

$$\text{Data Rate } R = nf_s \quad (6)$$

$$= (8)(10\text{kHz}) \quad (7)$$

$$\Rightarrow R = (8)(10^4) \text{ bits/second} \quad (8)$$

Channel capacity for an Additive White Gaussian Noise channel is

$$C = B \log_2 \left(1 + \frac{P}{N}\right) \text{ bits/second} \quad (9)$$

where P is the maximum channel power and N is the noise power and B is the channel bandwidth. Given, signal power to channel noise power is 26 dB.

$$\Rightarrow 10 \log_{10} \frac{P}{N} = 26\text{dB} \quad (10)$$

$$\Rightarrow \frac{P}{N} = 10^{2.6} \quad (11)$$

$$\approx 398.107 \quad (12)$$

For reliable transmission:

$$R \leq C \quad (13)$$

$$8(10^4) \leq B \log_2 399.107 \quad (14)$$

$$B \geq \frac{8(10^4)}{\log_2 399.107} \quad (15)$$

$$\Rightarrow B \geq 9258.58\text{Hz} \quad (16)$$

\therefore the minimum channel bandwidth required to ensure reliable transmission of the signal is ≈ 9.26 kHz.