NCERT Analog Assignment

EE23BTECH11007 - Aneesh Kadiyala*

Question 11.14.17: A simple pendulum of length ℓ and having a bob of mass M is suspended in a car. The car is moving in a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?
Solution:

TABLE 0
PARAMETERS

Parameter	Description
v	Speed
R	Radius of circular track
M	Mass of bob
g	Acceleration due to gravity
a_c	Centrifugal acceleration
g_e	Effective gravitational acceleration $\sqrt{g^2+a^2}$

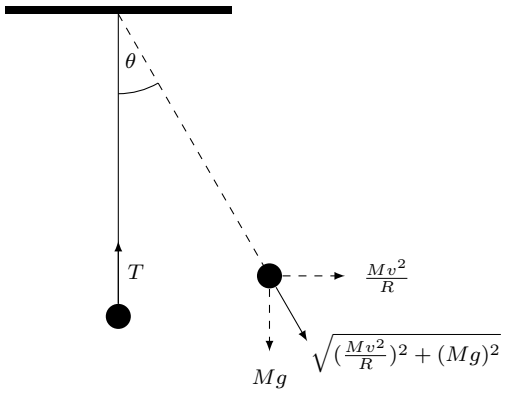

Fig. 0. Free Body Diagram

From the figure, restoring force:

$$F_r = -\left(\sqrt{\left(\frac{Mv^2}{R}\right)^2 + (Mg)^2}\right)\sin\theta(t) \tag{1}$$

For small oscillations, $\theta(t) << 1$.

$$\implies \sin\theta(t) \approx \theta(t) \tag{2}$$

$$\implies F_r \approx -\left(\sqrt{\left(\frac{Mv^2}{R}\right)^2 + (Mg)^2}\right)\theta(t) \tag{3}$$

$$\implies a = -\left(\sqrt{\left(\frac{v^2}{R}\right)^2 + g^2}\right)\theta(t) \tag{4}$$

$$l\frac{d^2\theta(t)}{dt^2} = -\left(\sqrt{\left(\frac{v^2}{R}\right)^2 + g^2}\right)\theta(t) \tag{5}$$

$$\frac{d^2\theta(t)}{dt^2} = \frac{-1}{\ell}\left(\sqrt{\left(\frac{v^2}{R}\right)^2 + g^2}\right)\theta(t) \tag{6}$$

$$\tag{7}$$

Taking Laplace transform:

$$s^2\Theta(s) - s\theta(0) - \theta'(0) = \frac{-1}{\ell}\left(\sqrt{\left(\frac{v^2}{R}\right)^2 + g^2}\right)\Theta(s) \tag{8}$$