GATE 2021 EC 23

EE23BTECH11007 - Aneesh Kadiyala*

Question: A speech signal, band limited to 4 kHz, is sampled at 1.25 times the Nyquist rate. The speech samples, assumed to be statistically independent and uniformly distributed in the range -5 V to +5 V, are subsequently quantized in an 8-bit uniform quantizer and then over a voice-grade AWGN telephone channel. If the ratio of transmitted signal power to channel noise power is 26 dB, the minimum channel bandwidth required to ensure reliable transmission of the signal with arbitrarily small probability of transmission error (*rounded off to one decimal place*) is ____ kHz.

(GATE 2021 EC)

Solution:
The signal is band limited to 4 kHz. Let the bandwidth of the signal be B_0.

$$B_0 = 4\text{kHz} \tag{1}$$
$$\text{Nyquist Rate } R_N = 2B_0 \tag{2}$$
$$\implies R_N = 8\text{kHz} \tag{3}$$
$$\text{Sampling Frequency } f_s = 1.25R_N \tag{4}$$
$$\implies f_s = 10\text{kHz} \tag{5}$$
$$\text{Data Rate } R = nf_s \tag{6}$$
$$= (8)(10\text{kHz}) \tag{7}$$
$$\implies R = (8)\left(10^4\right) \text{ bits/second} \tag{8}$$

Channel capacity for an Additive White Gaussian Noise channel is

$$C = B\log_2\left(1 + \frac{P}{N}\right) \text{bits/second} \tag{9}$$

where P is the maximum channel power and N is the noise power and B is the channel bandwidth. Given, signal power to channel noise power is 26 dB.

$$\implies 10\log_{10}\frac{P}{N} = 26\text{dB} \tag{10}$$
$$\implies \frac{P}{N} = 10^{2.6} \tag{11}$$
$$\approx 398.107 \tag{12}$$

For reliable transmission:

$$R \leq C \tag{13}$$
$$8\left(10^4\right) \leq B\log_2 399.107 \tag{14}$$
$$B \geq \frac{8\left(10^4\right)}{\log_2 399.107} \tag{15}$$
$$\implies B \geq 9258.58\text{Hz} \tag{16}$$

$\therefore$ the minimum channel bandwidth required to ensure reliable transmission of the signal is ≈ 9.26 kHz.