

# NCERT Discrete - 11.9.3.30

EE23BTECH11007 - Aneesh Kadiyala\*

**Question 11.9.3.30:** The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2<sup>nd</sup> hour 4<sup>th</sup> hour and  $n^{\text{th}}$  hour?

**Solution:**

- 1) Let number of bacteria initially be  $a_0 = 30$   
Let number of bacteria at the end of  $n^{\text{th}}$  hour be  $a_n$ .

Since number of bacteria doubles every hour,

$$a_n = 2a_{n-1}$$

$$a_n = 2(2a_{n-2})$$

...

$$a_n = 2^n a_0 = 2^n (30)$$

$$\Rightarrow a_2 = 2^2(30) = 120 \text{ and } a_4 = 2^4(30) = 480$$

Therefore, number of bacteria at the end of the 2<sup>nd</sup> hour is 120, 4<sup>th</sup> hour is 480, and  $n^{\text{th}}$  hour is  $30(2^n)$ .

- 2) **Finding  $x(n)$**

The series is a geometric progression.

$$x(n) = x(0)(r^n)$$

where  $r$  is the common ratio.

From the solution,  $x(0) = 30$ ,  $r = 2$ .

$$\Rightarrow x(n) = 30(2^n)(u(n))$$

as  $x(n) = 0 \forall n < 0$ .

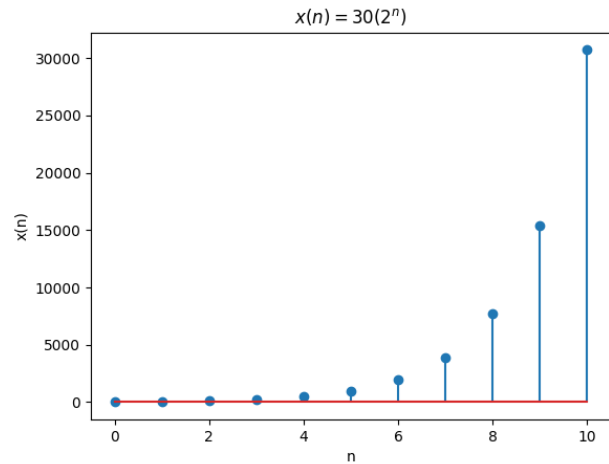
- 3) **Z-transform of  $x(n)$**

Let Z-transform of  $x(n)$  be  $X(z)$ .

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)u(n)z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (30)(2^n)(z^{-n})$$

$$X(z) = 30 \lim_{n \rightarrow \infty} \sum_{i=0}^n \left(\frac{2}{z}\right)^i$$



- a) If  $|z| > 2$ :

$$X(z) = \frac{30}{1 - \frac{2}{z}}$$

$$X(z) = 30z(z - 2)^{-1}$$

- b) If  $|z| \leq 2$ :

$$X(z) \rightarrow \infty$$

Region of Convergence (ROC) of  $z$  is  $|z| > 2$ .