NCERT Discrete - 11.9.3.30

EE23BTECH11007 - Aneesh Kadiyala*

Question 11.9.3.30: The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?

Solution:

TABLE 0
INPUT PARAMETERS

Parameter	Value	Description
$x(0)$	30	Initial no. of bacteria
r	2	Ratio of no. of bacteria at end of hour to start of hour (Common Ratio)
$x(n)$	$r^n x(0)u(n)$	n^{th} term of the GP

From the values given in Table 0:

$$x(2) = 30(2^2) = 120 \tag{1}$$
$$x(4) = 30(2^4) = 480 \tag{2}$$
$$x(n) = 30(2^n) \tag{3}$$

Let Z-transform of $x(n)$ be $X(z)$.

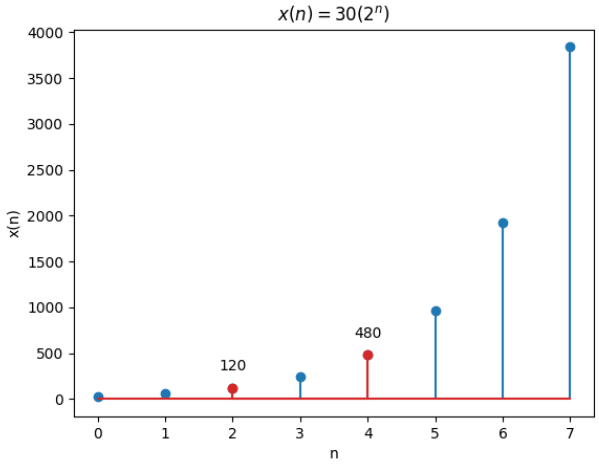

Fig. 0. Plot of $x(n)$ vs n. See Table 0 for details.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)u(n)z^{-n} \tag{4}$$
$$X(z) = \sum_{n=0}^{\infty} x(0)(r^n)(z^{-n}) \tag{5}$$
$$X(z) = x(0) \lim_{n\to\infty} \sum_{i=0}^{n} (\frac{r}{z})^i \tag{6}$$

1) If $|z| > r$:

$$X(z) = \frac{30}{1 - \frac{2}{z}} \tag{7}$$
$$X(z) = \frac{30z}{z - 2} \tag{8}$$

2) If $|z| \leq r$:

$$X(z) \to \infty \tag{9}$$
$$\implies X(z) = \frac{30z}{z - 2} \quad |z| > 2 \tag{10}$$