

NCERT Discrete - 10.5.2.19

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Question 10.5.2.19: Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000?

Solution:

TABLE 0
INPUT PARAMETERS

| Parameter | Value | Description |
|-----------|-------------|--------------------------------------|
| $x(0)$ | 5000 | Initial Income |
| d | 200 | Annual Increment (Common Difference) |
| $x(n)$ | $x(0) + nd$ | n^{th} term of the AP |

From the values given in Table 0:

$$7000 = 5000 + (n - 1)(200) \quad (1)$$

$$\Rightarrow 2000 = (n - 1)(200) \quad (2)$$

$$\therefore n = 11 \quad (3)$$

1) Finding $x(n)$

The series is an arithmetic progression.

$$x(n) = (x(0) + nd)(u(n)) \quad (4)$$

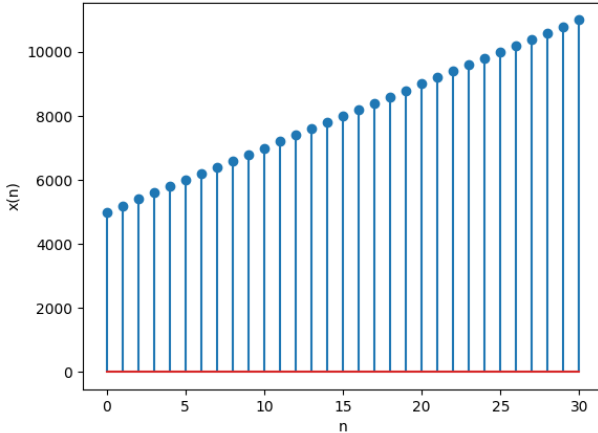


Fig. 1. Plot of $x(n)$ vs n . See Table 0 for details.

2) Z-transform of $x(n)$

Let Z-transform of $x(n)$ be $X(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

$$U(z) = \frac{1}{1 - z^{-1}} \quad \forall \quad |z| > 1 \quad (5)$$

$$\frac{d}{dz}U(z) = \frac{d}{dz}\left(\sum_{n=-\infty}^{\infty} u(n)z^{-n}\right) \quad (6)$$

$$= \sum_{n=-\infty}^{\infty} u(n) \frac{d}{dz} z^{-n} \quad (7)$$

$$= \sum_{n=-\infty}^{\infty} u(n)(-n)z^{-n-1} \quad (8)$$

$$= -z^{-1} \sum_{n=-\infty}^{\infty} nu(n)z^{-n} \quad (9)$$

$$= -z^{-1}Z\{nu(n)\} \quad (10)$$

$$\Rightarrow Z\{nu(n)\} = -z \frac{d}{dz}U(z) \quad (11)$$

$$= \frac{z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \quad (12)$$

$$X(z) = x(0)U(z) + dZ\{nu(n)\} \quad (13)$$

$$= \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \quad (14)$$

Using the values from Table 0:

$$X(z) = \frac{5000}{1 - z^{-1}} + \frac{200z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \quad (15)$$