HW8: Robustness Margins and Disk Margins

Exercise 1 True of False? Justify. *(All margins are assumed in dB.)*

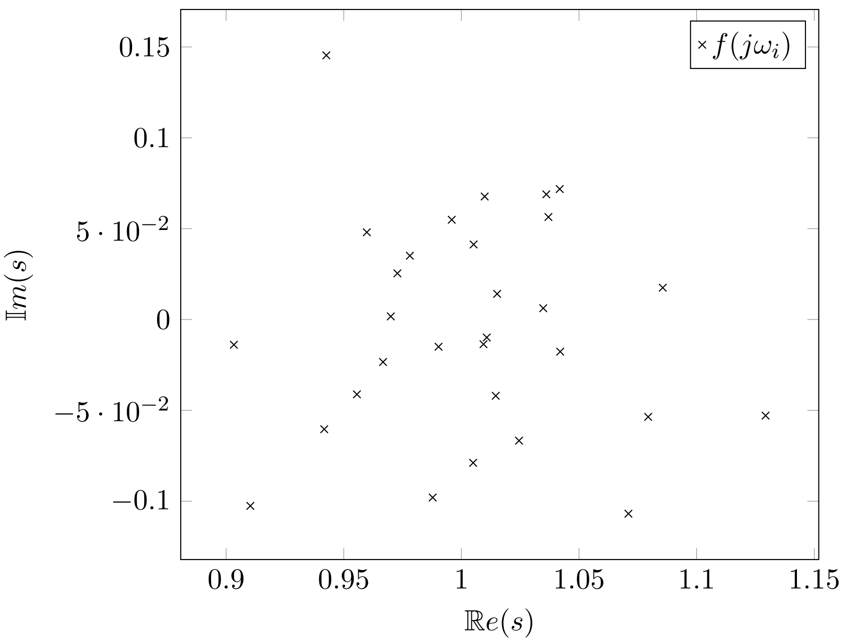
gain margins are just critical points, rlocus will show where the poles crossed.

1. **(1pts)** Any system with a negative gain margin is closed-loop unstable.
2. **(1pts)** Any system with a negative phase margins is closed-loop unstable.
3. **(1pts)** Any system in which MATLAB's `allmargin()` returns positive gain margins is closed-loop stable.

Exercise 2 (MIMO Exam 2021)
Consider the following open-loop stable transfer function for modeling a plant P:

$$L(s) = \frac{s^2 + 2s + 50}{s^3 + 6s^2 + 11s + 6} \tag{1}$$

1. **(3pts)** What are its gain and phase margins?
2. **(3pts)** Plot in MATLAB its T-based disk margin.
3. **(3pts)** Plot in MATLAB its S-T balanced (symmetric) disk margin.
4. **(4pts)** After a frequency sweep test of P, the following experimental Nyquist points for the incertitude $f(s)$ were obtained ($P = f(s)L(s)$) for several distinct ω_i. In view of this data and the previous disk margins, is the system robustly stable in view of real-life uncertainties? Justify.



Exercise 3 (4 pts)

As seen in class, phase margins ($\bar{\phi}$ and $\underline{\phi}$) and gain margins ($\bar{g}$ and $\underline{g}$) are comparable to the T-based disk margin $\bar{\alpha}_T$ by the following formulae:

$$\begin{cases} \bar{g} \geq 1 + \bar{\alpha}_T \\ \underline{g} \leq 1 - \bar{\alpha}_T \\ \bar{\phi} \geq 2\sin^{-1}(\frac{\bar{\alpha}_T}{2}) \\ \underline{\phi} \leq -2\sin^{-1}(\frac{\bar{\alpha}_T}{2}) \end{cases} \tag{2}$$

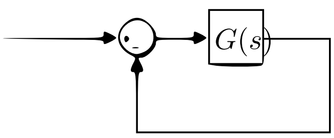
Derive the equivalent inequalities for the S-T balanced (symmetric) disk margin $\bar{\alpha}_{ST}$.

Exercise 4 (0pts + Best Chocolate Bar in Toulouse)

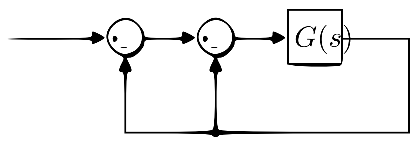
Assume the plant $G(s)$ given by

$$G(s) = \frac{1}{s^3 + 20s^2 + 5s - \pi} \tag{3}$$

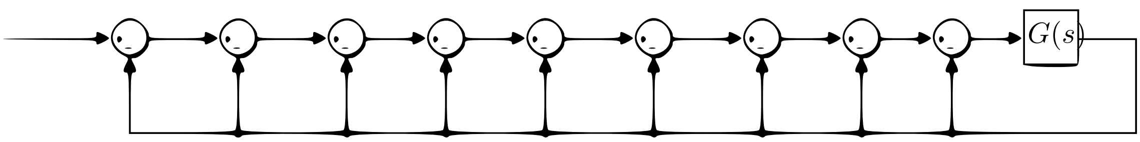
An erratic and unpredictable Control Engineer decides to close an additional loop every 1min on average randomly. If a loop is closed at an instant t_i, it does not affect the probability that a second loop will be closed at $t_{i+1} > t_i$. That is, loops are closed independently. The average rate at which loops are closed is independent of any loop closing occurrences and constant in time (always 1min!). Two loop closing events cannot happen simultaneously, i.e., $t_i \neq t_{i+1}$. (*Hint: This configuration yields a Poisson distribution.*)



System after 1 loop closure.



System after 2 loop closures.



System after 9 loop closures.

1. **(0pts)** What is the probability that the resulting system is stable after **exactly 5min**?
2. **(0pts)** What is the probability that the resulting system is stable after **exactly 1h**?
3. **(0pts)** What is the probability that the resulting system is stable after **exactly 1h40**?