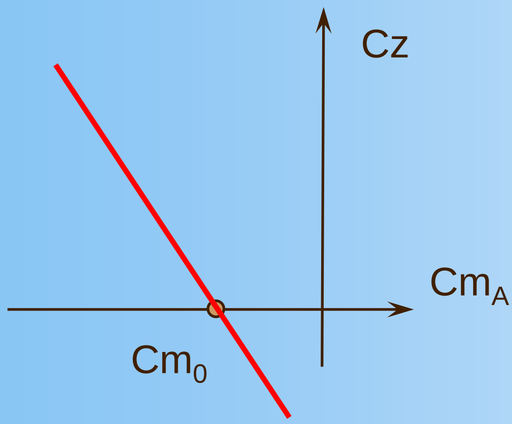
Aerodynamic Center

Boeing 747 Shuttle Carrier

First Assumption

We consider a lifting surface and any point A



$$\rightarrow Cm_A = Cm_0 + \frac{Cm_\alpha^A}{Cz_\alpha} \cdot Cz$$

$$\begin{cases} Cz = Cz_\alpha \cdot (\alpha - \alpha_0) \\ Cm_A = Cm_0 + Cm_\alpha^A \cdot (\alpha - \alpha_0) \end{cases}$$

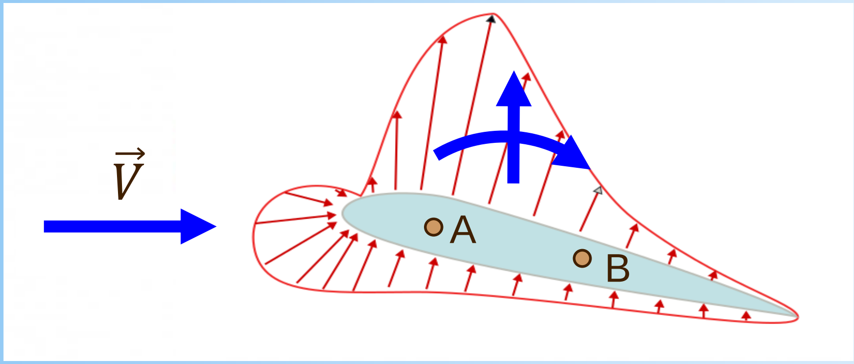
<u>First Assumption</u>

The aerodynamic coefficients (Cz,Cm_A) are linear versus α

Cm_A is a linear function of Cz

Second Assumption

The pressure flowlield along the profile creates a system of aerodynamic force and moment



Second Assumption

The system of aerodynamic forces and moments is a torsor

Any points A,B will be submitted to a force $\vec{F}_a$ and a moment $\vec{M}_a$ such as

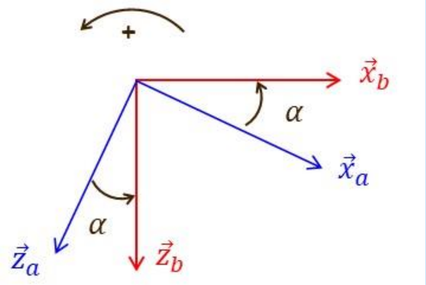
$$\vec{M}_B^a = \vec{M}_A^a + \overrightarrow{BA} \wedge \vec{F}_a$$

Torsor and pitch moment transfer

We projected the vectorial relation with respect to Rb
We assume the Longitudinal Flight and we focus on the y/pitch-component

$$\vec{M}_B^a = \vec{M}_A^a + \overrightarrow{BA} \wedge \vec{F}_a$$

$$_{R_b}\Big|\, Cm_B = {}_{R_b}\Big|\, Cm_A + {}_{R_b}\left|\begin{matrix} \dfrac{x_A - x_B}{L} \\ 0 \\ 0 \end{matrix}\right. \wedge \left[{}_{R_a}\left|\begin{matrix} -Cx \\ 0 \\ -Cz \end{matrix}\right. = {}_{R_b}\left|\begin{matrix} -(Cx\ \cos\alpha - Cz\ \sin\alpha) \\ 0 \\ -(Cx \sin\alpha + Cz \cos\alpha) \end{matrix}\right. \right]$$



$$\begin{cases} \vec{x}_a = \vec{x}_b\ \cos\alpha + \vec{z}_b\ \sin\alpha \\ \vec{z}_a = -\vec{x}_b \sin\alpha + \vec{z}_b \cos\alpha \end{cases}$$

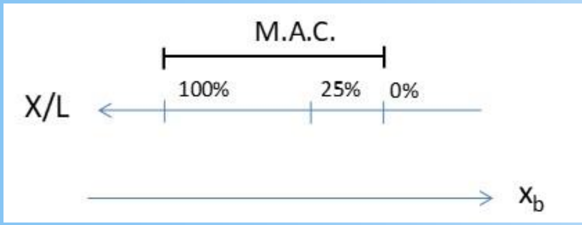
$$\begin{cases} \vec{x}_b = \vec{x}_a\ \cos\alpha - \vec{z}_a\ \sin\alpha \\ \vec{z}_b = \vec{x}_a \sin\alpha + \vec{z}_a \cos\alpha \end{cases}$$

Torsor and pitch moment transfer

$$ {}_{R_b}\left|\, Cm_B \right. = {}_{R_b}\left|\, Cm_A \right. + {}_{R_b}\left| \begin{matrix} \dfrac{x_A - x_B}{L} \\ 0 \\ 0 \end{matrix}\right. \wedge {}_{R_b}\left| \begin{matrix} -(Cx\ \cos\alpha - Cz\ \sin\alpha) \approx -Cx + Cz\cdot\alpha \\ 0 \\ -(Cx\sin\alpha + Cz\cos\alpha) \approx -Cz \end{matrix}\right. $$

I assume that both the angle of attack and the Cx are small

$$ \boxed{Cm_B = Cm_A + \frac{x_A - x_B}{L}\cdot Cz = Cm_B + \frac{X_B - X_A}{L}\cdot Cz} $$



<u>Conventions between x and X</u> :

x is positive forward / X is positive aft

Aerodynamic centre concept

$$Cm_B = Cm_A + \frac{X_B - X_A}{L}\cdot Cz = Cm_0 + \frac{Cm_\alpha^A}{Cz_\alpha}\cdot Cz + \frac{X_B - X_A}{L}\cdot Cz$$

$$Cm_B = Cm_0 + \left[\underbrace{\frac{Cm_\alpha^A}{Cz_\alpha} + \frac{X_B - X_A}{L}}_{=0}\right]\cdot Cz$$

If we take F = B such as : $\boxed{\dfrac{X_F}{L} = \dfrac{X_A}{L} - \dfrac{Cm_\alpha^A}{Cz_\alpha}}$

The point F has the remarkable property

$$\boxed{Cm_F = Cm_0 = cte}$$

independent of Cz or the angle of attack α

Aerodynamic centre concept

Because of the linearity of Cm and Cz versus α, it exists a point F such that the aerodynamic pitch moment Cm_F is constant, independent of α.

This point F is called <u>the aerodynamic neutral point or center point</u>.

This is a fix point, independent of the angle of attack α, given by

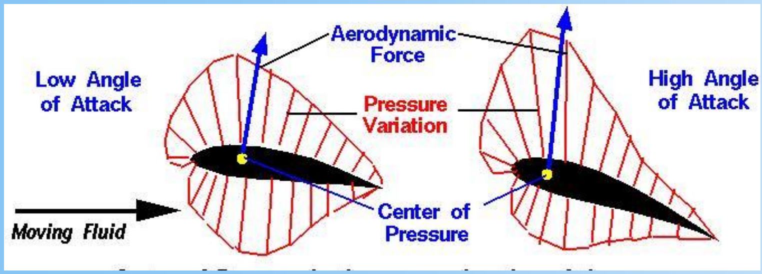
$$\frac{X_F}{L} = \frac{X_A}{L} - \frac{Cm_\alpha^A}{Cz_\alpha}$$

The aerodynamic pitch moment Cm_F is constant, independent of α.

$$Cm_F = Cm_0 = cte \leftrightarrow Cm_\alpha^F = \frac{\partial Cm_F}{\partial \alpha} = 0$$

Aerodynamic centre & Centre of Pressure

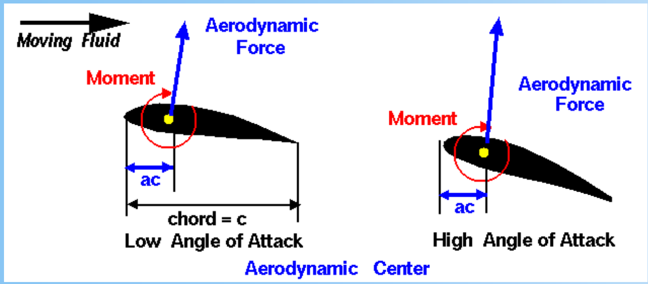
The aerodynamic force on a profile is assumed to be applied at the CoP



CoP (Centre of Pressure)

Point where no aerodynamic moment is applied (only the aerodynamic force is applied)
But CoP position varies with α

For HQ, the Aerodynamic Center (Neutral Point) is more convenient

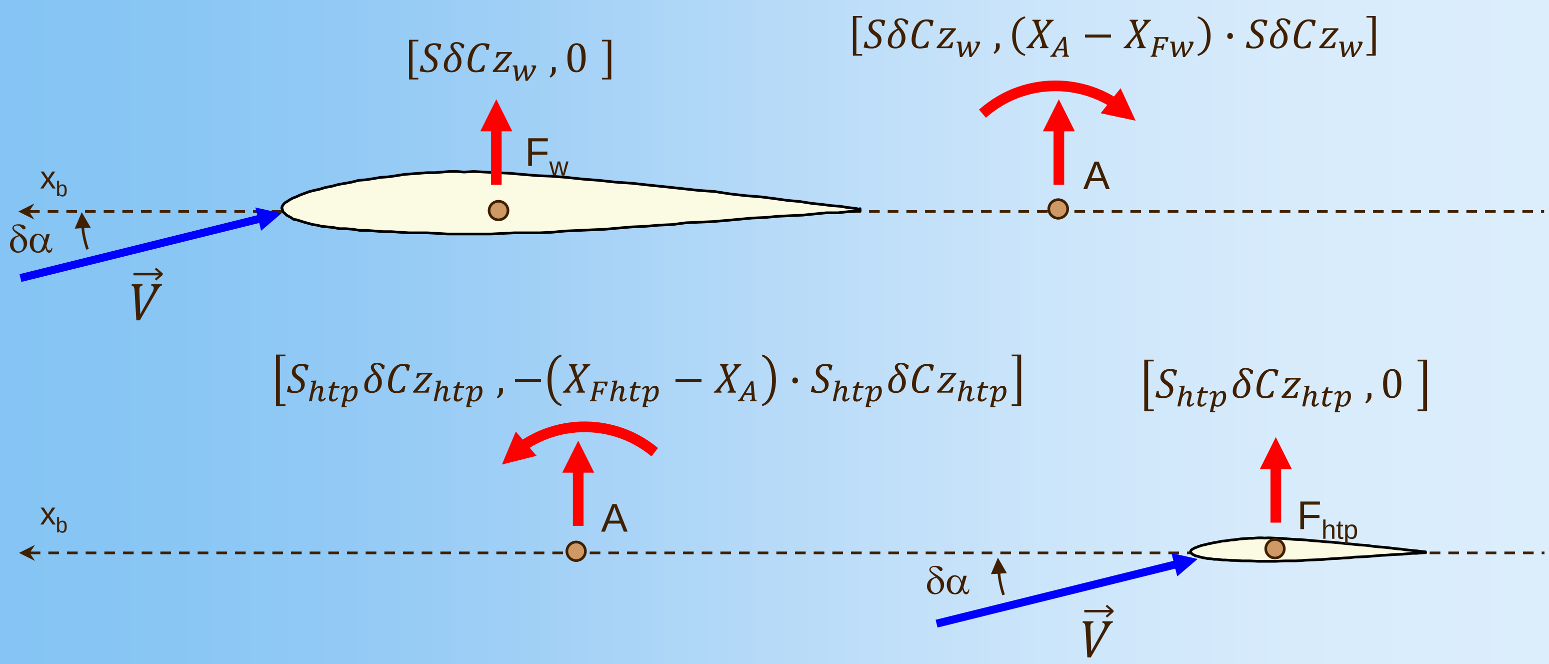


F (Neutral / Center Point)

Point where the aerodynamic moment is constant
And F position is constant with α

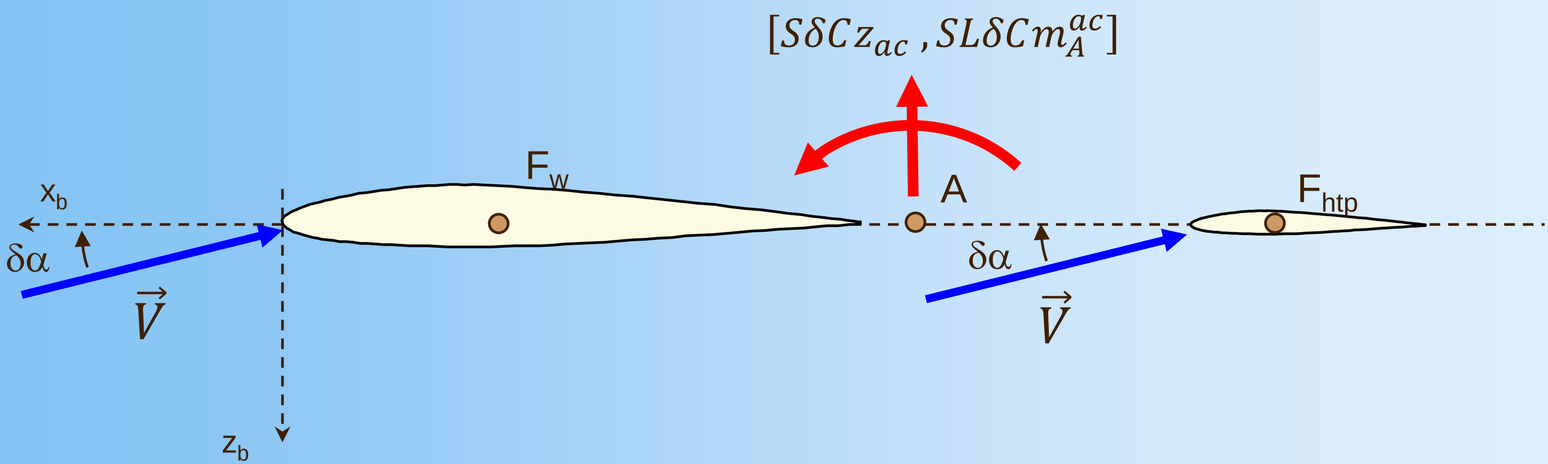
Aero. Center for complete aircraft

$$\text{at F} : \delta\alpha \rightarrow \delta Cm_F = 0$$



An aircraft can be considered as a system constituted by 2 lifting surfaces : the Wing and the Horizontal Tail Plane.
Considered separately, each lifting surface has its own Aerodynamic Centre where only applies a lift force variation for a given angle of attack a variation.

Aero. Center for complete aircraft



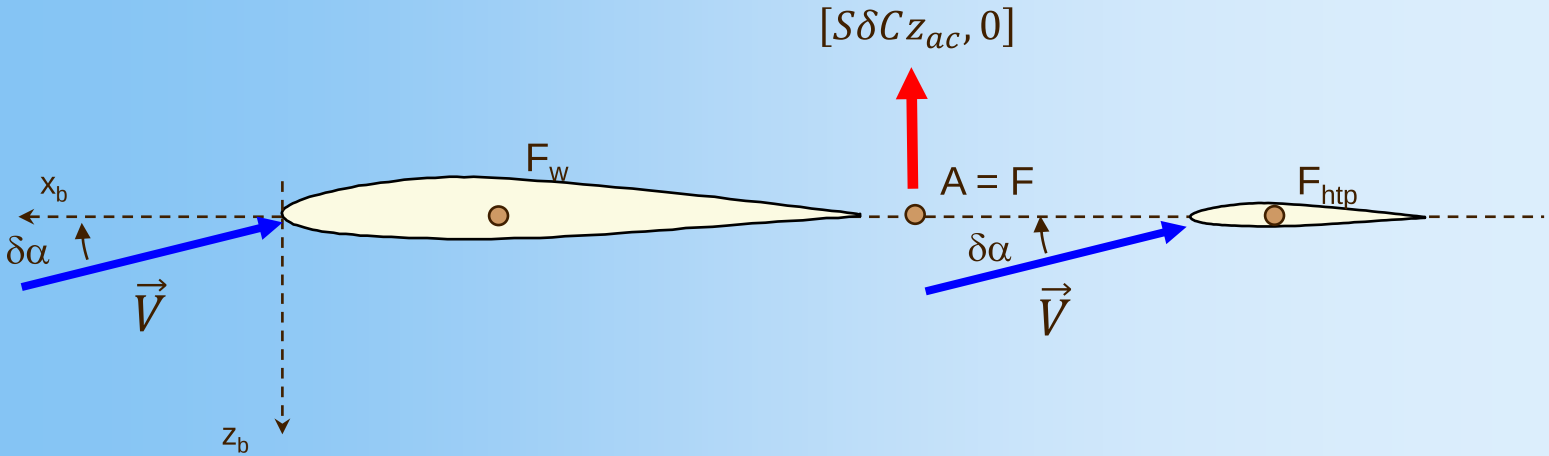
$$S\delta Cz_{ac} = S\delta Cz_w + S_{htp}\delta Cz_{htp}$$

$$SL\delta Cm_A^{ac} = (X_A - X_{Fw}) \cdot S\delta Cz_w - \left(X_{Fhtp} - X_A\right) \cdot S_{htp}\delta Cz_{htp}$$

Considered as a whole, any point of the aircraft is submitted to the sum of the torsors produced by the 2 lifting surfaces

Aero. Center for complete aircraft

$$\text{at F} : \delta\alpha \rightarrow \delta Cm_F = 0$$



$$\delta Cm_F^{ac} = 0 \Rightarrow (X_F - X_{Fw}) \cdot S\delta Cz_w - \left(X_{Fhtp} - X_F\right) \cdot S_{htp}\delta Cz_{htp} = 0$$

$$X_F = \frac{X_{Fw} \cdot S\delta Cz_w + X_{Fhtp} \cdot S_{htp}\delta Cz_{htp}}{S\delta Cz_w + S_{htp}\delta Cz_{htp}} = \frac{X_{Fw} \cdot SCz_\alpha^w + X_{Fhtp} \cdot S_{htp}Cz_\alpha^{htp}}{SCz_\alpha^w + S_{htp}Cz_\alpha^{htp}}$$

The point F is the Aerodynamic Center of the complete aircraft : $\delta Cm_F = 0$

Aero. Center for complete aircraft

An aircraft can be considered as a system constituted by 2 lifting surfaces : the wing and the empennage.

Each lifting surface has its own Aerodynamic Centre where will be only applied a lift force variation for a given angle of attack α variation.

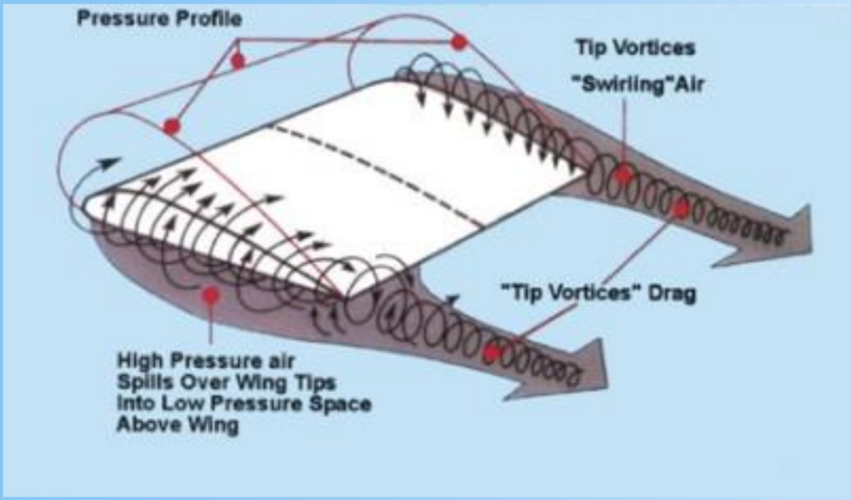
Now, if you consider the whole system, any point A will be submitted to both the sum of the lift force and pitch moment variations produced by the 2 lifting surfaces.
The general condition : $Cm_F = Cte$ / $\delta Cm_F = 0$ defines the Aerodynamic Centre for the system.

The Aerodynamic Centre for a system of several lifting surfaces has the property of a Barycentre where the ponderation is given by the lift efficiency of each lifting surface.

<u>Remark</u> : if you want to move the Aerodynamic Centre for the whole aircraft forward or backward, you just have to modify the lift efficiency of the empennage

- If you increase the empennage area $\rightarrow$ F moves backward
- If you decrease the empennage area $\rightarrow$ F moves forward

Tip Vortex principle

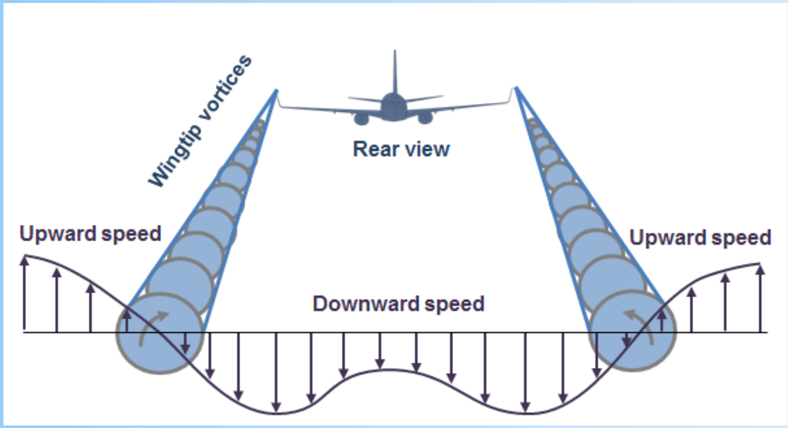


Learjet 75

Vortices are created because of the difference in pressure between the upper and lower surfaces of a wing. The tendency is for particles of air to move from the lower wing surface around the wing tip to the upper surface (from the region of high pressure to the region of low pressure). The combination with oncoming free-stream flow leads to the formation of Tip vortices.

The stronger the wing lift, the stronger the pressure differential, the stronger the Tip vortices

From Tip Vortex tp Downwash



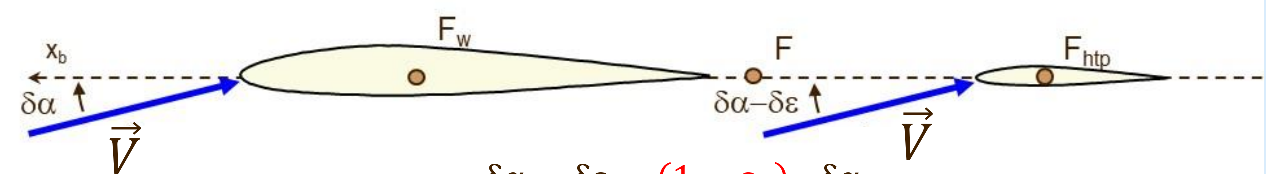
As far as we move aft from the wing, this produces a general downward flowfield between both vortices. As a result, there is a decreasing of the angle of attack :
The wing sees an angle of attack equal to α , the Horizontal Tail Plane sees $\alpha - \varepsilon$

The downwash angle ε is a function of α : $\varepsilon = \varepsilon_0 + \varepsilon_\alpha \cdot \alpha > 0$

$\varepsilon_\alpha > 0$: more α, more lift, more tip vortices, more downwash

Aero. Center for complete aircraft

Strictly speaking, when you consider the aircraft as a whole system, the angle of attack seen by the empennage is no more equal to the angle of attack seen by the wing.



$$\delta\alpha - \delta\varepsilon = (1-\varepsilon_\alpha)\cdot\delta\alpha$$

The different variations of the Lift Forces can be expressed :

$$\begin{cases} \delta Cz_w = Cz_\alpha^w \cdot \delta\alpha \\ \delta Cz_{htp} = Cz_\alpha^{htp} \cdot (\delta\alpha - \delta\varepsilon) = Cz_\alpha^{htp} \cdot (1-\varepsilon_\alpha)\cdot\delta\alpha \end{cases}$$

$$\boxed{X_F = \frac{X_{Fw}\cdot SCz_\alpha^w + X_{Fhtp}\cdot S_{htp}Cz_\alpha^{htp}\cdot(1-\varepsilon_\alpha)}{S\delta Cz_w + S_{htp}Cz_\alpha^{htp}\cdot(1-\varepsilon_\alpha)}}$$

The HTP lift gradient is decreased by the ratio $(1-\varepsilon_\alpha)$

Aerodynamic centre F and CG point

Let us consider the Centre of Gravity G and apply the pitch moment transport between F and G

$$Cm_G = Cm_F + \frac{X_G - X_F}{L} \cdot Cz = Cm_0 + \frac{X_G - X_F}{L} \cdot Cz_\alpha(\alpha - \alpha_0)$$

$$\Downarrow$$

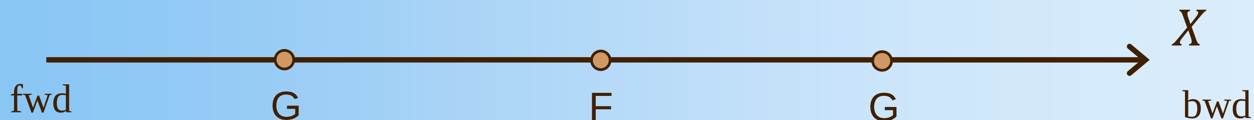
$$\boxed{Cm_\alpha^G = \frac{X_G - X_F}{L} \cdot Cz_\alpha}$$

G fwd

$$X_G < X_F \rightarrow Cm_\alpha^G < 0$$

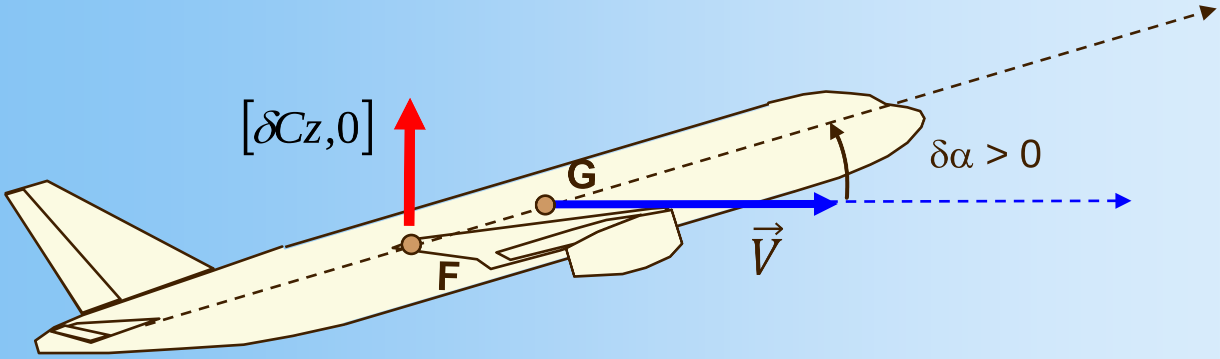
G bwd

$$X_G > X_F \rightarrow Cm_\alpha^G > 0$$



Aircraft Static Stability : $Cm_\alpha^G < 0$

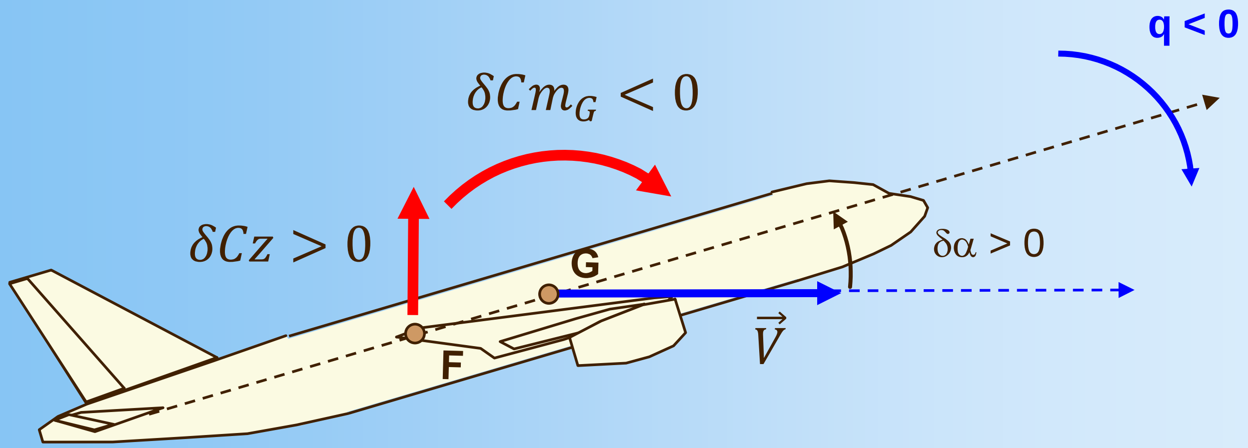
$$\boxed{\text{at F} : \delta\alpha \rightarrow \delta Cm_F = 0}$$



At the Aerodynamic Center of the Aircraft, there will be only applied a lift force variation for a given angle of attack $\delta\alpha$ variation

Aircraft Static Stability : $Cm_\alpha^G < 0$

$$\delta Cm_G = \frac{X_G - X_F}{L} \cdot \delta Cz = Cm_\alpha^G \cdot \delta\alpha$$



If G is forward with respect to F

→ $Cm_\alpha^G < 0$, the aircraft is statically stable

Aircraft Static Stability : $Cm_{\alpha}^{G} < 0$

Starting from an initial trimmed situation, there is a small perturbation : $\delta\alpha > 0$

If we consider the variations due to the δα perturbation :

- At F, a lift force variation is applied : $1/2\,\rho V^2 S \cdot \delta Cz > 0$
- At F, no pitch moment variation is applied : $1/2\,\rho V^2 SL \cdot \delta Cm_F = 0$

(By definition , at the aircraft Aerodynamic Center F, the pitch moment is constant whatever α)

If G is located forward w.r.t. to F, the lift-up force variation applied at F produces a pitch down moment variation at G : the aircraft is rotating down around G and α is decreasing. Naturally, the aircraft returns to its initial equilibrium after an initial perturbation : this is the definition of a system which is statically stable.

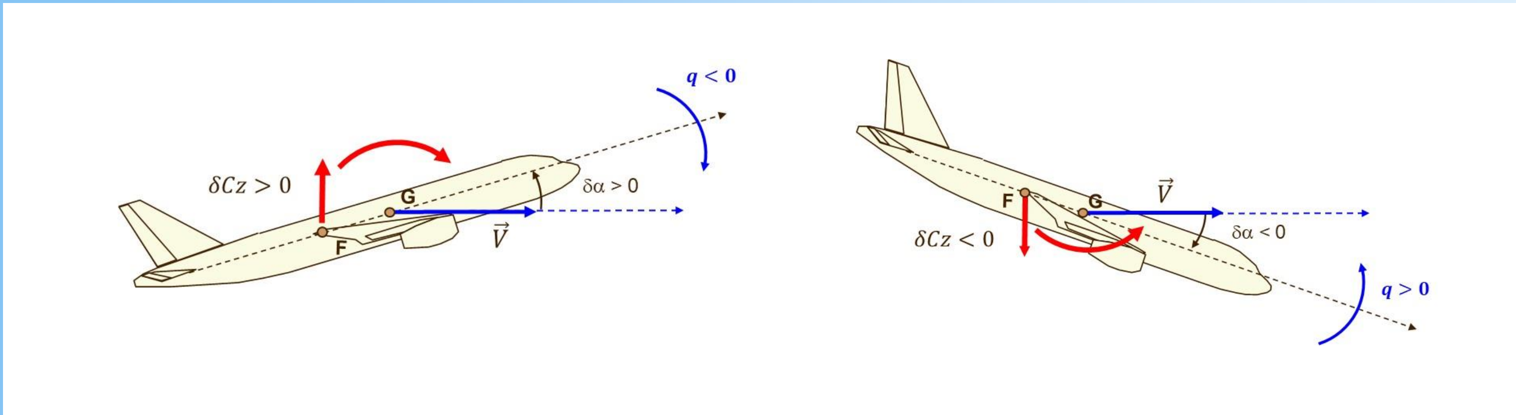
This condition of (Statical) Stability (with respect to α) is inherently associated to :

G located forward with respect to F : $X_G < X_F \rightarrow Cm_{\alpha}^{G} < 0$

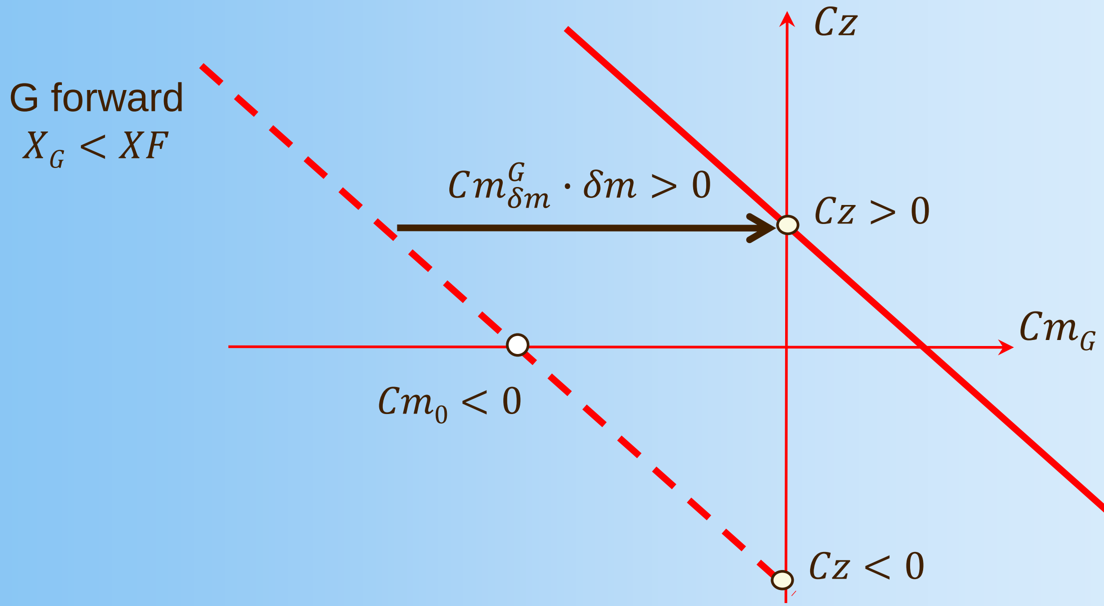
Aircraft Static Stability : $Cm_{\alpha}^{G} < 0$

The rotation around G doesn't stop when α is recovering its initial value because the aircraft is animated with a certain rotation speed. As soon as the angle of attack becomes negative, the force applied at F becomes a lift-down force and the moment at G a pitch-up moment which again forces the aircraft to rotate towards its initial situation …

Hence, the aircraft is oscillating around its initial equilibrium which is a characteristic of a stable system. The more the aircraft will be stable, the more the restoring moment will be high and the more the period of the oscillation will be short (or the pulsation of the oscillation will be high)



Stable aircraft & Trim constraint



A stable aircraft with $Cm_0 < 0$ has a negative Cz (with $\delta m = 0$) at trim ($Cm_G = 0$)

For flying, a stable aircraft with $Cm_0 < 0$ must be trimmed with a $Cm_{\delta m} \cdot \delta m > 0$

Stable Conventional configuration

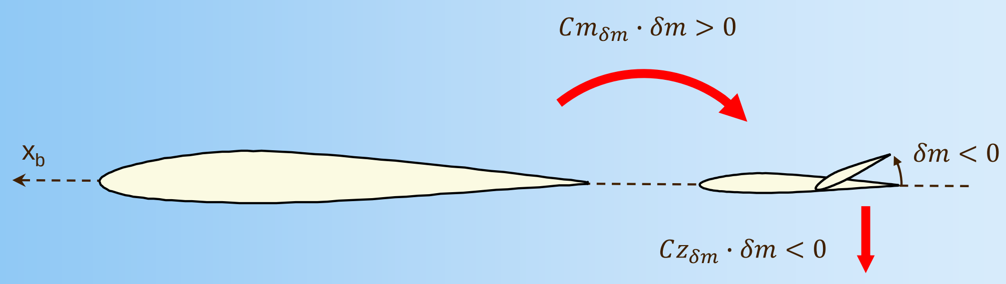
We consider a stable aircraft ; we plot the curve Cz as a function of Cm_G :

- At $Cz = 0$, Cm is the same for any point, equal to Cm_0 generally negative
- The aircraft is stable so the slope $dCm_G/dCz < 0$

The trim condition for this stable aircraft : $Cm_G = 0$ leads to a negative Cz which is obviously non acceptable. For getting a trim with a positive Cz, you shall translate the curve such that :

$Cm_G = Cm_0 + \frac{dCm_G}{dCz} \cdot Cz + \delta Cm_G$ with $\delta Cm_G > 0$

For a Conventional configuration, a pitch-up moment $\delta Cm_G = Cm_{\delta m} \cdot \delta m > 0$ corresponds to a negative δm which produces a lift-down force resulting in a decrease of the total lift of the aircraft. For maintaining the initial aircraft lift, you must increase the angle of attack for increasing the wing lift. This will result in an increase of the initial drag.



Stable Canard configuration

Trimming a stable Conventional aircraft has a price which is to overcome an additional drag (called the trim drag) ; you shall increase the thrust if you want to maintain your altitude

<u>Remark</u> : for a Canard, a pitch-up moment $\delta Cm_G = Cm_{\delta m} \cdot \delta m > 0$ corresponds to a positive δm which produces a lift-up force resulting in an increase of the total lift of the aircraft. For maintaining the initial aircraft lift, you must decrease the angle of attack for decreasing the wing lift. This will result in a decrease of the initial drag.

Rutan Long-EZ G-WILY