## Nonlinear Dynamic inversion applied to helicopter control ISAE

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We consider an helicopter motion in the vertical plan. Figure (1) shows the forces that are considered. Aerodynamic forces are neglected. Our objective is to control the helicopter posi-

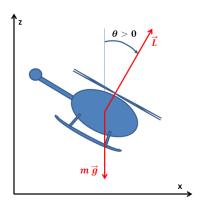


Figure 1: Helicopter

tion. We will first consider the control of the accelerations  $\gamma_x$  and  $\gamma_z$ . A feedback linearization approach will be used for that purpose. Then loops will be added to control the velocities  $v_x$  and  $v_z$ , and finally the positions x and z.

### 1 Modeling

The dynamic model is made of three parts

Kinematic	Forces equation	Torques equation
$\int \dot{x} = v_x$	$\int m \dot{v}_x = L \sin \theta$	$\int \dot{\theta} = q$
$\dot{z} = v_z$	$ \begin{cases}     m \dot{v}_z = L \cos \theta - m g \end{cases} $	$\int J\dot{q} = -f q + T$

Numerical values are as follows:

m	mass	10	[kg]
J	inertia	0.2	$[kg  m^2]$
f	damping	0.1	$Nm (rad/s)^{-1}$
g	gravity	9.81	$[m  s^{-2}]$

Inputs are the thrust (or lift force when hovering) L and the torque T. The lift is limited to  $L_{max} = 200 [N]$ . The torque satisfies  $|T| \le T_{max}$  with  $T_{max} = 0.035 [N m]$ . Both actuators may be considered as first order systems with transfer function  $1/(1+\tau s)$ , where  $\tau = 0.1$ .

#### Simulation tool: The following files are provided

- 1. A simulink model called *simulation\_model\_1.slx*, with three blocks: a reference generator, an *actuators* block that models the dynamic of the actuators, and a block *helicopter* that contains the helicopter dynamics.
  - The configuration parameters of have been set to:  $stop\ time = T_{sim}$ ,  $solver\ option\ type = Fixed step$ , and Fixed-step  $size = DT_{sim}$ .
    - The inputs of the *helicopter* block are the thrust L and the torque T. Outputs are the position, velocity and acceleration  $(x, v_x, \gamma_x, z, v_z, \gamma_z)$  and the pitch angle and the rotation rate  $(\theta, q)$ .
    - The *actuators* block simulates the actuators dynamic taking into account for position saturations.
    - The references block computes the demands. For this openloop model the demands  $d_1$  and  $d_2$  are applied on the inputs L and T. These signals are steps with a starting time, a stop time, and an amplitude.
- 2. A script file called  $simulation\_script.m$ . All the simulation parameters are defined in this file. The file also run the simulation and plots the demands  $(L_d, T_d)$ , the controls (L, T), and the accelerations  $(\gamma_x, \gamma_z)$ .

Run the script file in order to check the open loop behavior.

## 2 Feedback linearization

The outputs to be controlled are the accelerations  $\gamma_x$  and  $\gamma_z$ . A feedback linearization approach will be used for that purpose.

Your job: is to define, implement and test this control law.

- 1. Remark that the characteristic orders of the accelerations are equal to zero (i.e. there exists a direct feedthrough), and that the decoupling matrix is not inversible.
- 2. Show that thanks to (two) state augmentations, it is possible to get a non singular decoupling matrix  $\Delta$ . The new inputs will be denoted  $(v_L, v_T)$ . Implement this first part of the control law in a block called *state augmentation*. It is better to do this in a copy  $simulation\_model\_2.slx$  of the initial model.
- 3. Show that product  $\Delta^{-1} \Delta_0$  has a fairly simple expression. Then give the expression of the control law that yields to an input/output behavior that is linear and decoupled, with all poles fixed at 0. The new inputs will be denoted  $(w_x, w_z)$ .

- 4. Implement this second part of the control law in a block called feedback linearization.
- 5. Run the simulation with the control law. Find a simple mean to check that the feedback linearization behave as expected.
- 6. Then design a stabilizing state feedback to control  $\gamma_x$  and  $\gamma_z$ . and ensures their convergence towards references  $\gamma_x^*$  and  $\gamma_z^*$ . The expected response time is one second on both axis.
- 7. Implement this third part of the control law in a block called *acceleration control*. It is better to do this in a copy *simulation\_model\_3.slx* of the previous model. Complete the script *simulation\_script.m* by the computation of the feedback gains to be used.
- 8. Run the simulation script. Be happy... or not!

## 3 Velocity and position control

Now it is possible for you to implement a speed and position control loop. First make a copy of the previous simulink model and call it *simulation\_model\_4.mdl*. Then do the job!

#### Some guidelines:

- 1. The required response time for the speed control is 3 s. Thus you can assume than the acceleration control loop is very fast (i.e. the acceleration is equal to its demand). And a simple P controller can be used to regulate the speed.
- 2. The required response time for the position control is 10 s. Thus you can assume than the speed control loop is very fast (i.e. the speed is equal to its demand). And a simple PI controller can be used to regulate the position. The integral control is mandatory if wind is present (but there are no in our simulation).
- 3. Show than if the expected response times (for speed and position) are decreased too short then oscillations appear. Could you find a way to reduce them by maintaining a fast response?