General Expression for $d\vec{V}/dt$ & $d\vec{\Omega}/dt$

Lockheed U-2S Dragon Lady

Relation between d/dt and $\vec{\Omega}$

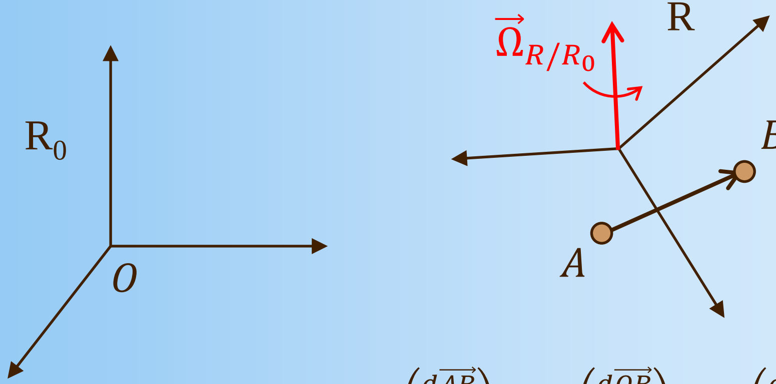
We consider a referential, R_0

We consider another referential R animated with a rotation $\vec{\Omega}_{R/R_0}$

A and B are 2 fix points of the Referential R,

$$\vec{V}_B = \vec{V}_A + \vec{\Omega}_{R/R_0} \wedge \overrightarrow{AB} \rightarrow \left(\frac{d\overrightarrow{AB}}{dt}\right)_{R_0} = \vec{\Omega}_{R/R_0} \wedge \overrightarrow{AB}$$

The velocity field (of a solid) is a torsor with a resultant $\vec{\Omega}$



$$\left(\frac{d\overrightarrow{AB}}{dt}\right)_{R_0} = \left(\frac{d\overrightarrow{OB}}{dt}\right)_{R_0} - \left(\frac{d\overrightarrow{OA}}{dt}\right)_{R_0} = \vec{V}_B - \vec{V}_A$$

Relation between d/dt and $\vec{\Omega}$

We consider a referential, R_0
We consider another referential R animated with a rotation $\vec{\Omega}_{R/R_0}$

If $\vec{X}$ is a fix vector of the Referential R,

$$\rightarrow \left(\frac{d\vec{X}}{dt}\right)_{R_0} = \vec{\Omega}_{R/R_0} \wedge \vec{X}$$

This relation can be generalised for any vector $\vec{X}$

$$\rightarrow \left(\frac{d\vec{X}}{dt}\right)_{R_0} = \left(\frac{d\vec{X}}{dt}\right)_{R} + \vec{\Omega}_{R/R_0} \wedge \vec{X}$$

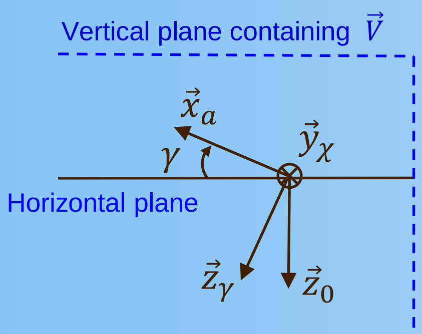
General Expression of $d\vec{V}/dt$

We apply the derivation expression using R_0 and R_a

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \left(\frac{d\vec{V}}{dt}\right)_{R_a} + \vec{\Omega}_{a/0} \wedge \vec{V} = \dot{V}\,\vec{x}_a + V\,\vec{\Omega}_{a/0} \wedge \vec{x}_a$$

but $\vec{\Omega}_{a/0} = \dot{\chi}\,\vec{z}_0 + \dot{\gamma}\,\vec{y}_\chi + \dot{\mu}\,\vec{x}_a$

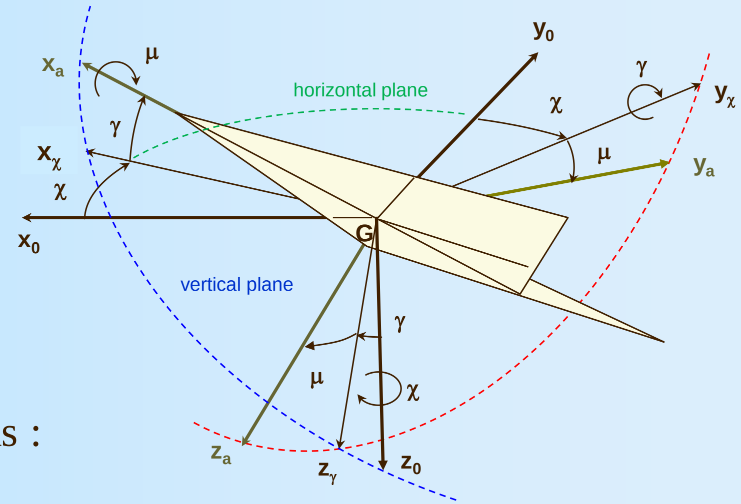
$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \dot{V}\,\vec{x}_a + V\dot{\chi}\,\vec{z}_0 \wedge \vec{x}_a + V\dot{\gamma}\,\vec{y}_\chi \wedge \vec{x}_a$$



$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \dot{V}\,\vec{x}_a + V\cos\gamma\,\dot{\chi}\,\vec{y}_\chi - V\dot{\gamma}\,\vec{z}_\gamma$$

General Remarks

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \dot{V}\,\vec{x}_a + V\cos\gamma\,\dot{\chi}\,\vec{y}_\chi - V\dot{\gamma}\,\vec{z}_\gamma$$



The absolute acceleration includes 3 terms :

1) along the velocity $\vec{V}$
2) within the horizontal plane, perpendicular to $\vec{V}$
3) within the vertical plane containing $\vec{V}$, perpendicular to $\vec{V}$

The 2 last terms correspond to centripetal accelerations for a uniform circular motion :

- within an horizontal plane at a velocity , $V\cos\gamma$ and a rotation rate , $\dot{\chi}$
- within a vertical plane at a velocity , V and a rotation rate , $\dot{\gamma}$

General Expression of $d\vec{\Omega}/dt$

We apply the derivation expression using R_0 and R_b

$$\left(\frac{d\vec{\Omega}}{dt}\right)_{R_0} = \left(\frac{d\vec{\Omega}}{dt}\right)_{R_b} + \vec{\Omega}_{b/0} \wedge \vec{\Omega} = \left(\frac{d\vec{\Omega}}{dt}\right)_{R_b}$$

with $$\vec{\Omega} = \vec{\Omega}_{ac/R_0} = \vec{\Omega}_{R_b/R_0} = {}_{R_b}\left|\begin{matrix} p \\ q \\ r \end{matrix}\right.$$

$$\left(\frac{d\vec{\Omega}}{dt}\right)_{R_0} = \left(\frac{d\vec{\Omega}}{dt}\right)_{R_b} = {}_{R_b}\left|\begin{matrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{matrix}\right.$$

General Remarks

$$\vec{M}_G = [I_G] \cdot \left(\frac{d\vec{\Omega}}{dt}\right)_{R_0} = \begin{bmatrix} A & 0 & \cancel{-E} \\ 0 & B & 0 \\ \cancel{-E} & 0 & C \end{bmatrix}_{R_b} \left| \begin{matrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{matrix} \right.$$

The 2nd Newton law can be expressed easily with respect to Rb

We can also neglect the cross product term E

$$\vec{M}_G = [I_G] \cdot \left(\frac{d\vec{\Omega}}{dt}\right)_{R_0} = {}_{R_b}\left| \begin{matrix} A \cdot \dot{p} \\ B \cdot \dot{q} \\ C \cdot \dot{r} \end{matrix} \right.$$

Pure Longitudinal Flight

Lockheed Constellation

Pure Longitudinal Flight

Pure Longitudinal Flight requires that all the forces acting on the a/c are within the a/c plane of symmetry

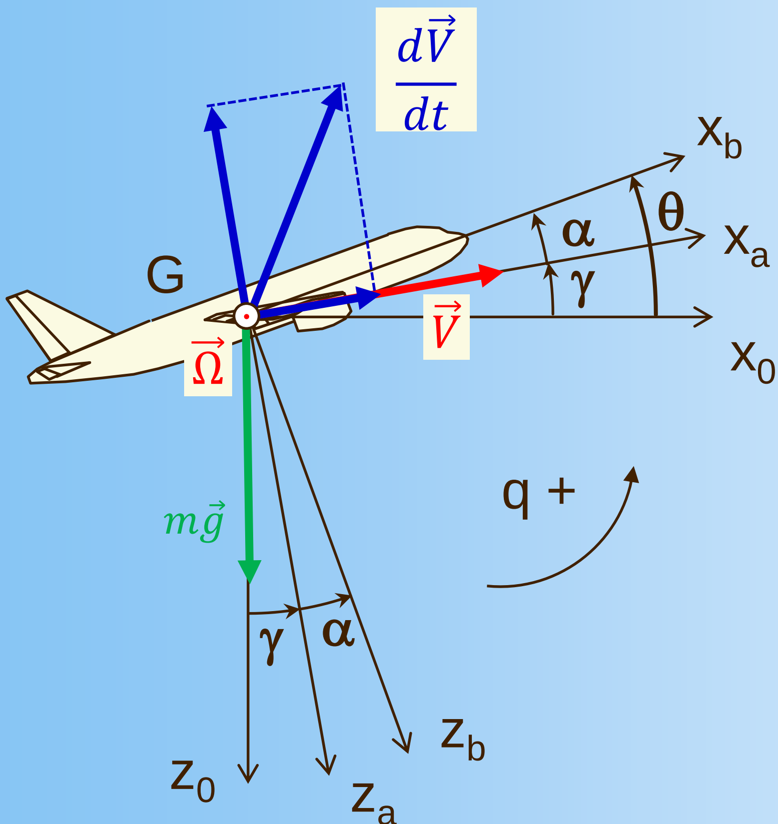
- all forces within the plane (x_b, z_b) : no lateral forces
- the a/c plane of symmetry is vertical because it contains the vector $m\vec{g}$
- the y_b axis is within the horizontal plane
  $\rightarrow \phi_1 = 0 \rightarrow \phi = 0$
- the aircraft velocity is within the airplane plane of symmetry
  $\rightarrow \beta = 0$

- the aircraft acceleration is within the airplane plane of symmetry

$$\rightarrow \left(\frac{d\vec{V}}{dt}\right)_{R_0} = \dot{V}\,\vec{x}_a + V\cos\gamma\,\cancel{\dot{\chi}}\,\vec{y}_\chi - V\dot{\gamma}\,\vec{z}_a$$

- the aircraft can only rotates with respect to the y_b axis : it can only rotates within the vertical plane / plane of symmetry $\rightarrow$ $\vec{\Omega}_{b/0} = \dot{\theta}\,\vec{y}_b \rightarrow q = \dot{\theta}$

Pure Longitudinal Flight



$$\begin{cases} \beta = 0° \\ \phi = 0° \\ \theta = \alpha + \gamma \ (*) \\ q = \dot{\theta} \end{cases}$$

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \dot{V}\,\vec{x}_a - V\dot{\gamma}\,\vec{z}_a$$

$$\vec{\Omega} = q \cdot \vec{y}_b$$

(*) only valid for pure Longitudinal flight, see annex

Steady Turn

Lockheed SR-71 Blackbird

Steady Turn Manoeuvre

The Steady Turn Manoeuvre requires that the rotation vector $\vec{\Omega}$ is a constant vector.

It is a motion performed at constant altitude (steady : no variation).

$$\vec{\Omega}_{a/0} = \vec{\Omega}_{a/b} + \vec{\Omega}_{b/0} = \vec{\Omega}_{b/0} \text{ , because } \vec{\Omega}_{a/b} = \dot{\beta}\,\vec{z}_a - \dot{\alpha}\,\vec{y}_b = \vec{0} \text{ (steady condition)}$$

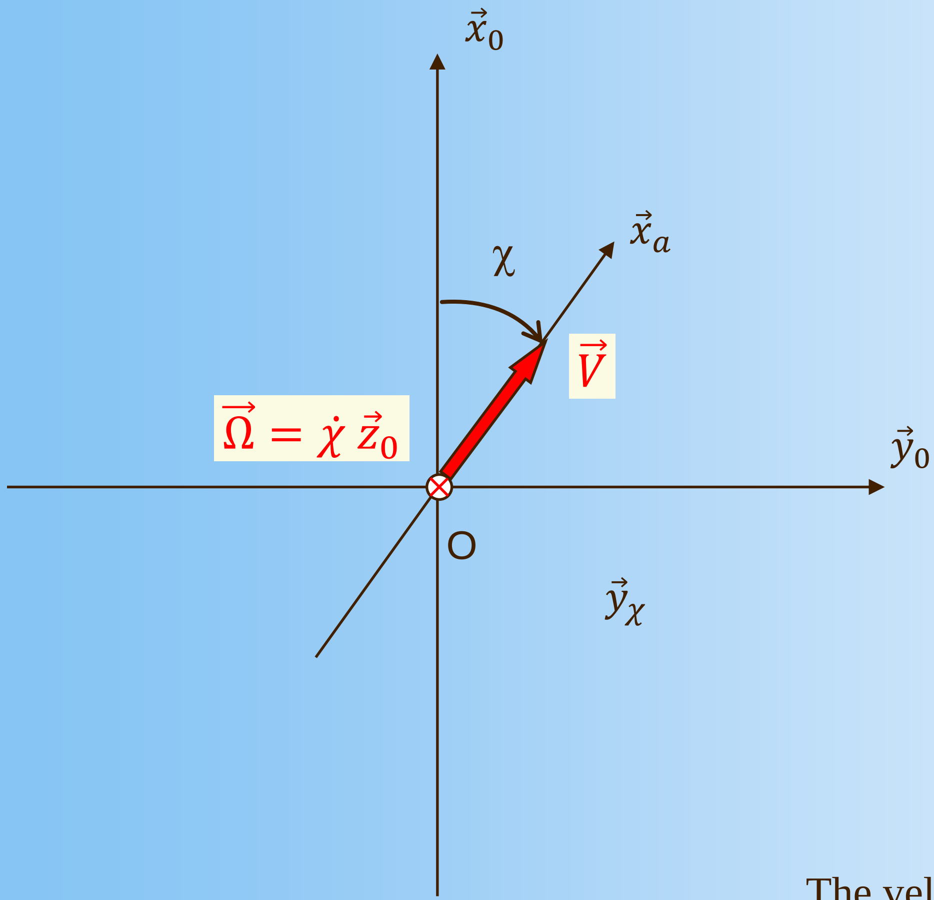
$$\dot{\chi}\,\vec{z}_0 + \dot{\gamma}\,\vec{y}_\chi + \dot{\mu}\,\vec{x}_a = \dot{\psi}\,\vec{z}_0 + \dot{\theta}\,\vec{y}_\psi + \dot{\phi}\,\vec{x}_b$$

By identification : $\dot{\chi} = \dot{\psi} = \Omega$ and $\dot{\gamma} = \dot{\mu} = \dot{\theta} = \dot{\phi} = 0$

The rotation vector $\vec{\Omega}_{a/0} = \vec{\Omega}_{b/0}$ is vertical

$$\boxed{\vec{\Omega} = \Omega\,\vec{z}_0 = \dot{\chi}\,\vec{z}_0 = \dot{\psi}\,\vec{z}_0}$$

Steady Turn manoeuvre

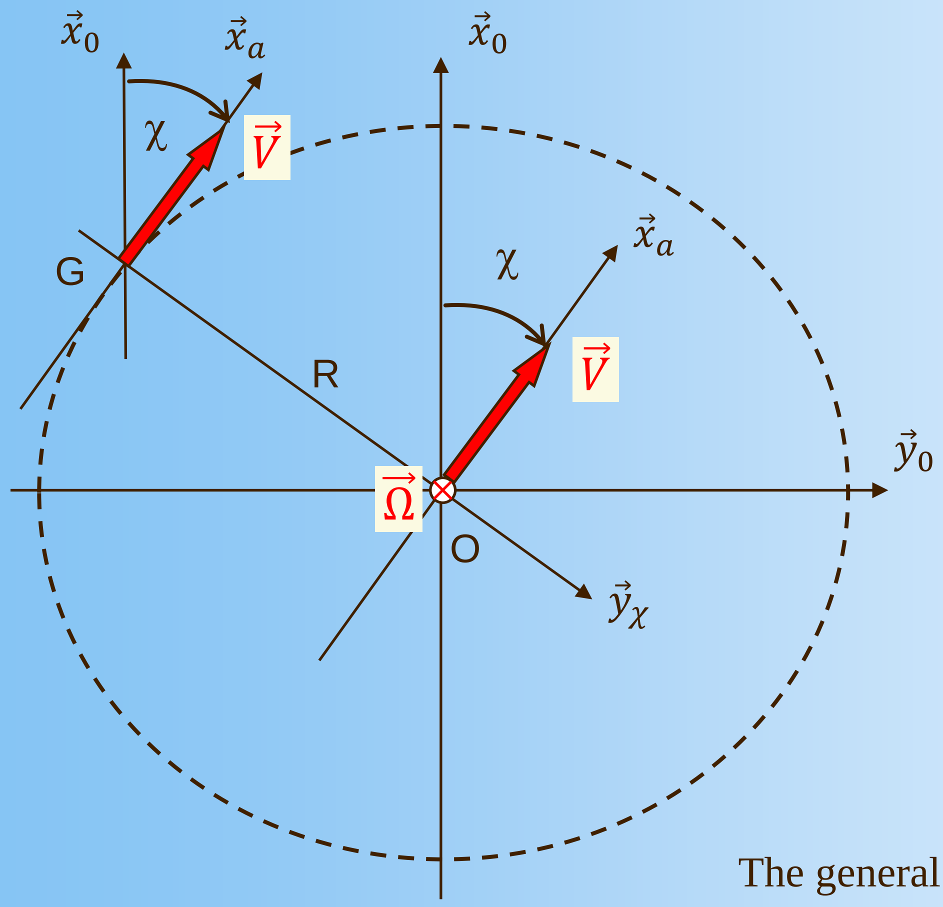


$\rightarrow \quad \gamma = 0$

$\rightarrow \quad \dot{\chi} = \Omega$

The velocity vector is rotating at a constant rate Ω within an horizontal plane

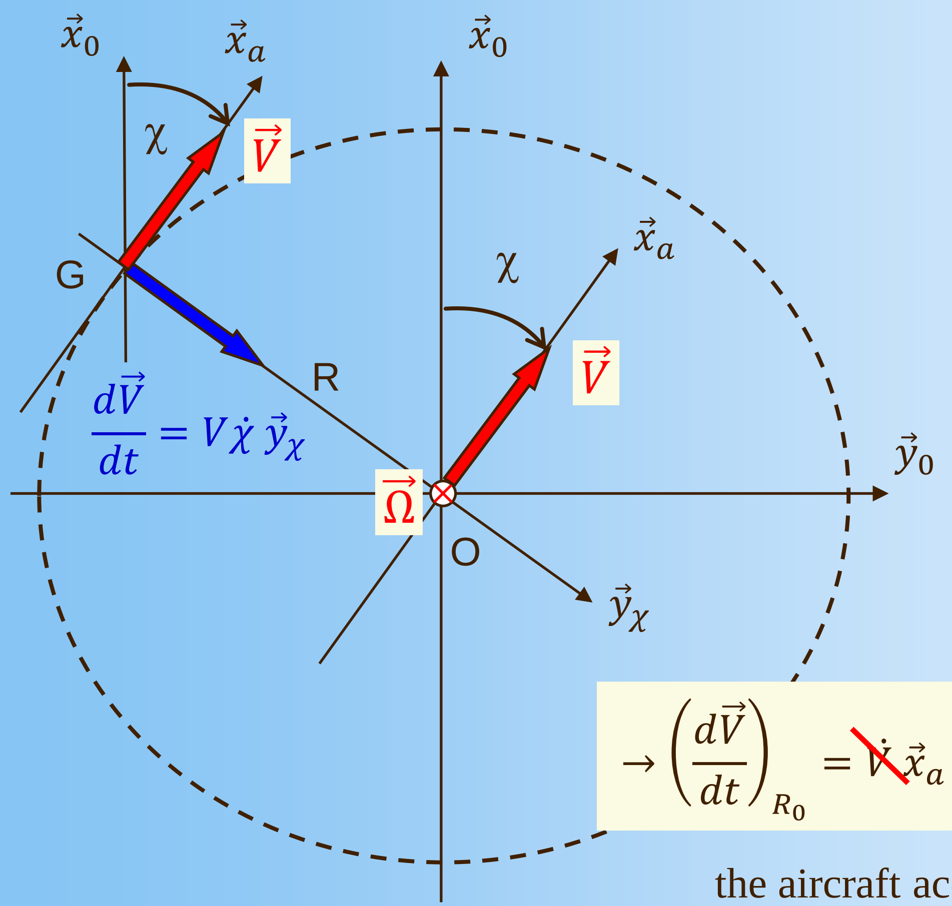
Steady Turn manoeuvre



$$\rightarrow \quad V = \Omega \cdot R$$

The general motion is a uniform circular motion within an horizontal plane

Steady Turn manoeuvre

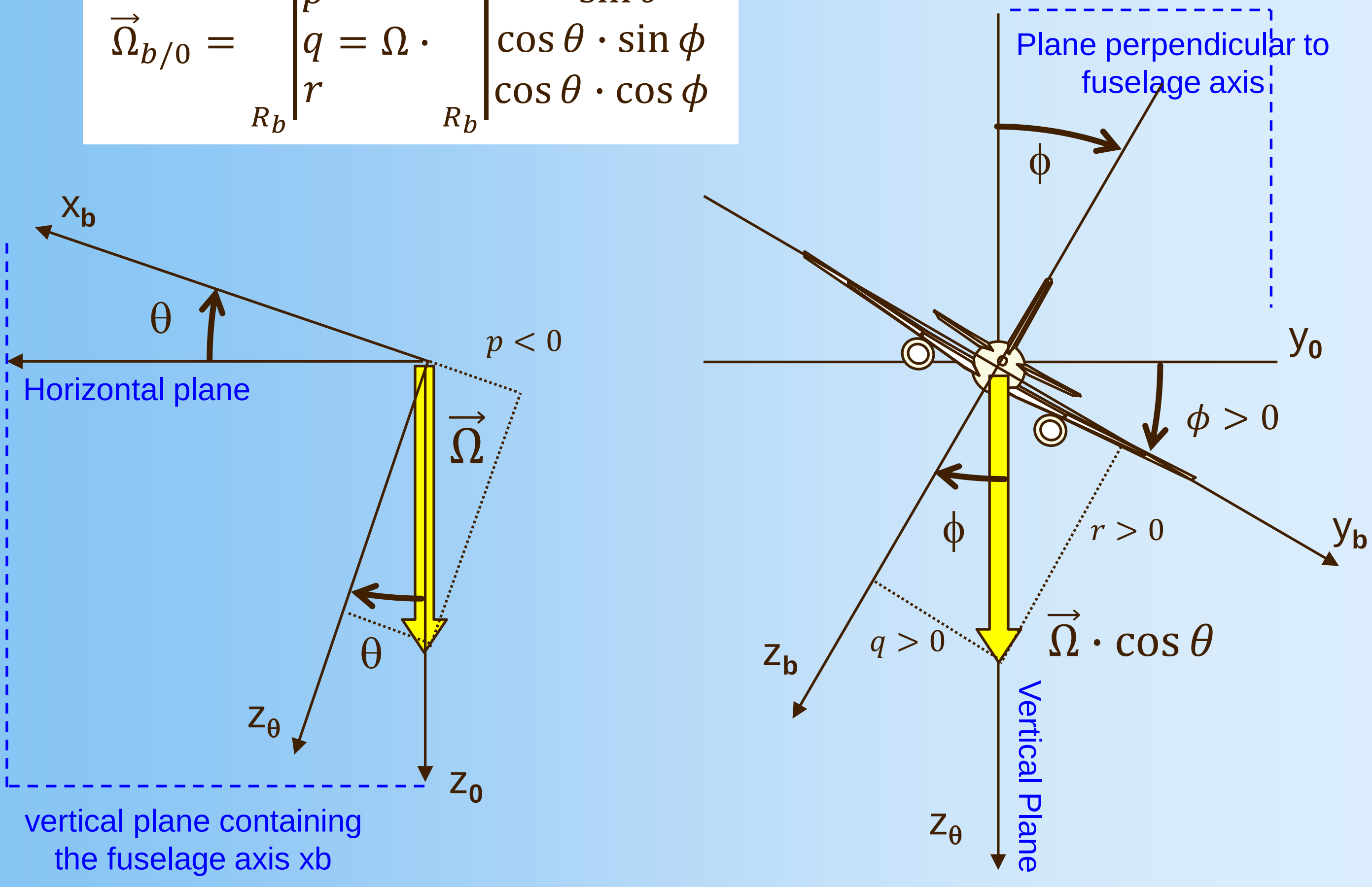


$$\rightarrow \left(\frac{d\vec{V}}{dt}\right)_{R_0} = \cancel{\dot{V}}\,\vec{x}_a + V\dot{\chi}\,\vec{y}_\chi - \cancel{V\dot{\gamma}}\,\vec{z}_\gamma = \Omega V\,\vec{y}_\chi$$

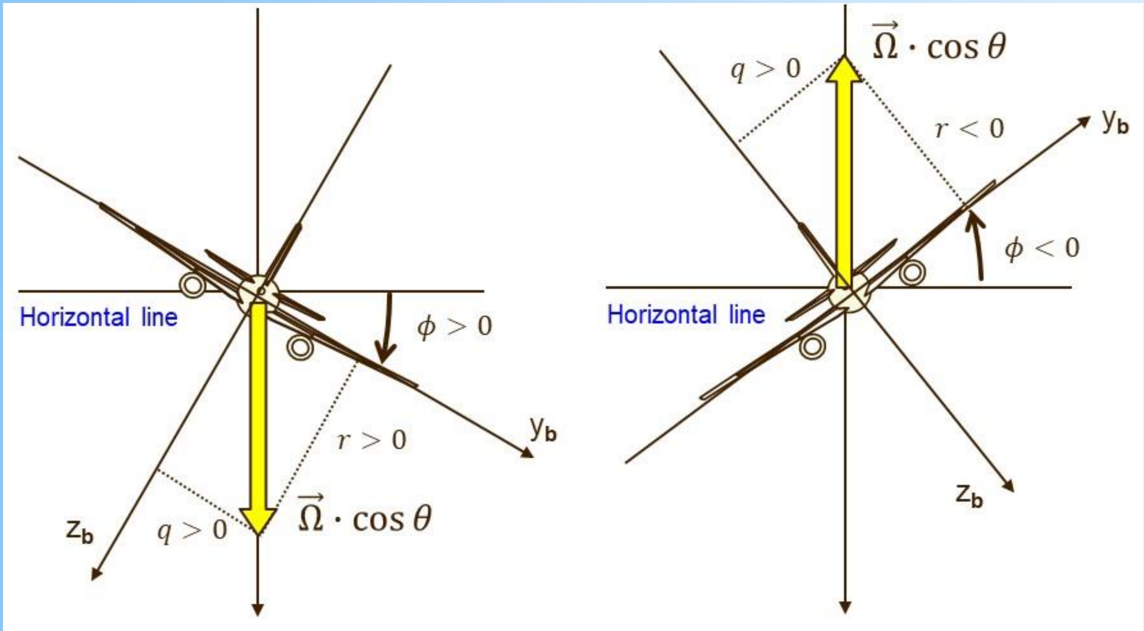
the aircraft acceleration is perpendicular to $\vec{V}$ within the horizontal plane

Steady Turn : expression of $\vec{\Omega}_{b/0}$

$$\vec{\Omega}_{b/0} = \left|\begin{matrix} p \\ q \\ r \end{matrix}\right._{R_b} = \Omega \cdot \left|\begin{matrix} -\sin\theta \\ \cos\theta \cdot \sin\phi \\ \cos\theta \cdot \cos\phi \end{matrix}\right._{R_b}$$



Steady Turn : positive, pitch up rate, q



During a steady turn manoeuvre, there is a steady pitch-up rate :

$$q = \Omega \cos\theta \cdot \sin\phi = \Omega \cdot \sin\phi_1 > 0$$

General Remarks

Pure Longitudinal Flight / Steady Turn Manoeuvre correspond to trimmed situations.

➢ If the aircraft is stable (longitudinal stability) and if the pilot doesn't apply any roll / yaw command, at the end, we will converge to a trimmed situation corresponding to the Pure Longitudinal Flight

➢ If the aircraft is stable (spiral stability) and if the pilot applies a roll / yaw command, at the end, we will converge to a trimmed situation corresponding to the Steady Turn Manœuvre

These notions of « stability » and « convergence » are linked to the dynamic properties of the aircraft (aircraft eigen modes) that we will study latter in this course.