Load Factor

Stratolaunch carrier aircraft

Load Factor Vector Definition

$$\vec{F}_a + \vec{F}_t + m\vec{g} = m\frac{d\vec{V}_G}{dt}$$

$$m\left(\vec{g} - \frac{d\vec{V}_G}{dt}\right) = -\left(\vec{F}_a + \vec{F}_t\right) = mg \cdot \vec{n}_G$$

Within the 1st Newton Law, the massic terms are grouped together within the Load Factor vector

Any massic object is submitted to the Load Factor which defines its « apparent gravity »

Load Factor Vector Definition

We assume that the vehicle has a translation motion with respect to the Inertial Referentiel

$$g \cdot \vec{n}_G = \vec{g} - \frac{d\vec{V}_G}{dt}$$

Any fix massic object within the aircraft shares the same load factor $\vec{n}_G$
because it shares the same (absolute) velocity $\vec{V}_G$

$$g \cdot \vec{n}_G = -\frac{\vec{F}_a + \vec{F}_t}{m}$$

for a space vehicle in translation,
only submitted to the gravity,
because $\vec{F}_a + \vec{F}_t = \vec{0}$ then $\vec{n}_G = \vec{0}$

the astronaut inside will share the same load factor
which means that the astronaut is submitted to a null apparent gravity

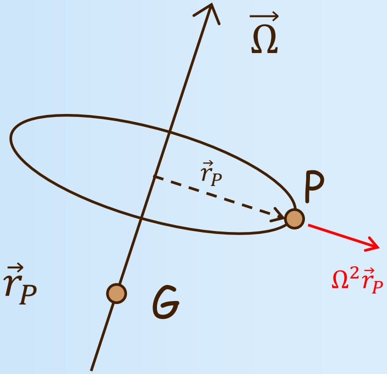
Load Factor Vector Definition

We assume that the vehicle has a rotation motion with respect to the Galilean Referentiel

The absolute motion of a fix point P (within the vehicle) is the addition of 2 motions :

- The motion of the centre of gravity G (of the vehicle)
- The rotation of the vehicle around G

$$\frac{d\vec{V}_P}{dt} = \frac{d\vec{V}_G}{dt} - \Omega^2 \vec{r}_P$$



$$g \cdot \vec{n}_P = \vec{g} - \frac{d\vec{V}_P}{dt} = \vec{g} - \frac{d\vec{V}_G}{dt} + \Omega^2 \vec{r}_P = g \cdot \vec{n}_G + \Omega^2 \vec{r}_P$$

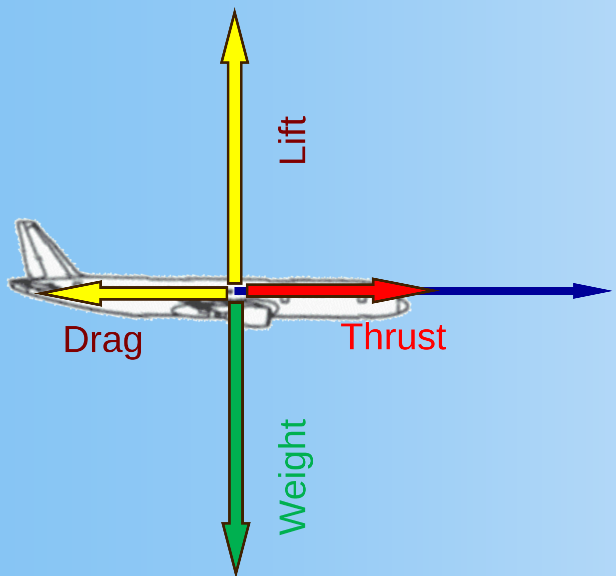
for a space vehicle only submitted to the gravity :

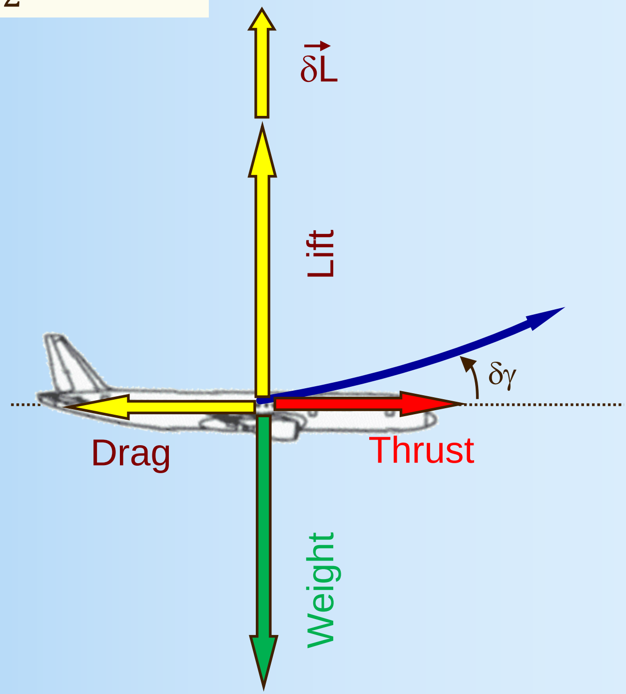
$g \cdot \vec{n}_P = +\Omega^2 \vec{r}_P$ (because $\vec{n}_G = \vec{0}$)

So, the astronaut inside will be submitted to an apparent centrifugal gravity

Load factor and pull-up manoeuvre

$$(z_a) : -mV \cdot \dot{\gamma} = mg - \frac{1}{2}\rho V^2 S \cdot Cz$$





$$(z_a) : mg = \frac{1}{2}\rho V^2 S \cdot Cz$$

$$(z_a) : mg\left(1 + \frac{\dot{\gamma} \cdot V}{g}\right) = \frac{1}{2}\rho V^2 S \cdot (Cz + \delta Cz)$$

$$n_z = 1$$

$$n_z = 1 + \frac{\dot{\gamma} \cdot V}{g}$$

Load factor and pull-up manoeuvre

Any variation of the Lift Force produces a Load Factor variation

$$\delta n_z = \frac{\rho V^2 S}{2mg} \cdot \delta Cz = \frac{\delta Cz}{Cz} = \frac{Cz_\alpha}{Cz} \cdot \delta\alpha$$

if we assume that the velocity stays constant

Any variation of the Load Factor produces a pitch rate q

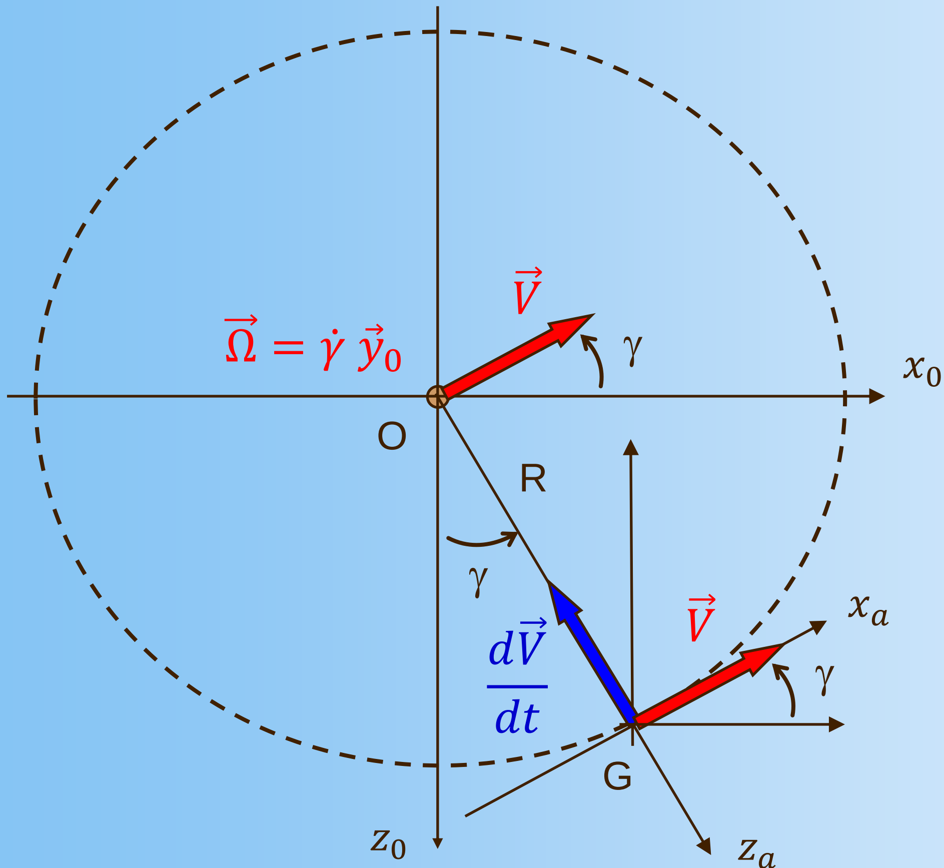
$$\delta n_z = \frac{\dot{\gamma} \cdot V}{g} \approx \frac{q \cdot V}{g}$$

if we assume that $q = \dot{\alpha} + \dot{\gamma} \approx \dot{\gamma}$

Load Factor and pull-up manoeuvre

We assume a pull-up manœuvre with a constant δn_z

The aircraft will perform a uniform circular motion within the local vertical plane



$$\rightarrow \Omega = \dot{\gamma} \approx \frac{g \cdot \delta n_z}{V}$$

$$\rightarrow \quad V = \Omega \cdot R$$

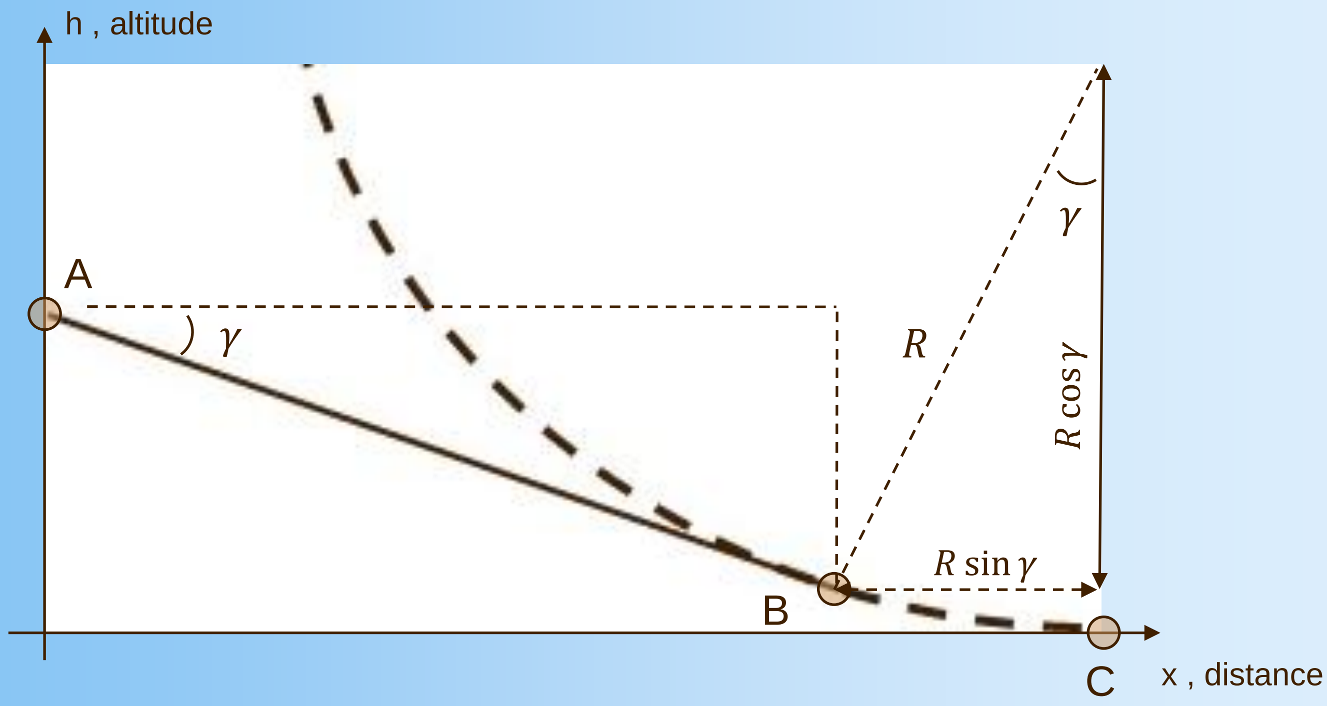
$$\rightarrow \quad R = \frac{V^2}{g \cdot \delta n_z}$$

$$\frac{dV}{dt} = \Omega \cdot V = \Omega^2 \cdot R$$

Exercise / Airborn Landing Field distance

We want to compute the airborn landing field length assuming :

1. we start at 35 ft altitude by a straight segment with $\gamma = -3°$: from A to B
2. then , a circular trajectory assuming a constant $\delta n_z = 0{,}1\ g$: from B to C
3. we touch the ground at C with a 0° slope (the impact velocity $V_z = 0\ ft/s$)
4. we assume a constant velocity V along all the trajectory



Exercise / Airborn Landing Field distance

Point A : $x_A = 0$, $h_A = 35\ ft$

Point B : $x_B = ?$, $h_B = R \cdot (1 - \cos\gamma)$ $\quad with\ \ R = \dfrac{V^2}{g \cdot \delta n_z}$

Point C : $x_C = x_B + R \cdot \sin\gamma$, $h_C = 0\ ft$

Then, the undetermination about x_B can be drawn because

$$tan\,\gamma = \frac{h_A - h_B}{x_B - x_A} \rightarrow x_B = (h_A - h_B) \cdot tan\,\gamma$$

The total airborn landing field length is given by :

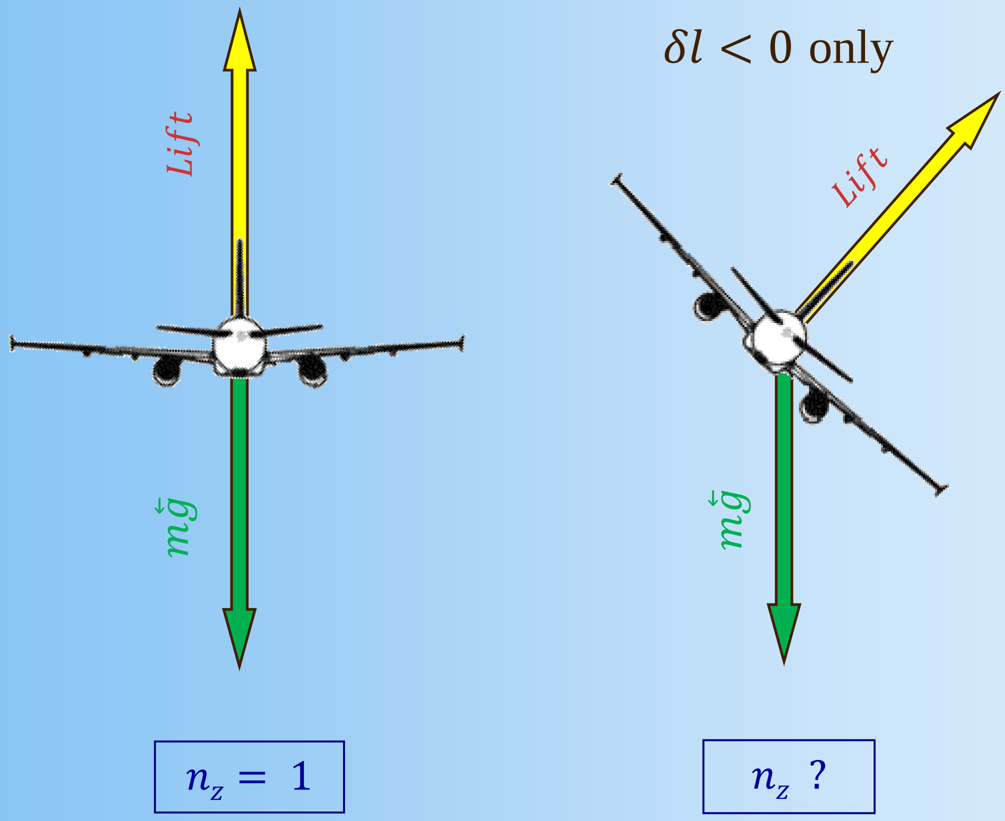
$$\boxed{x_C = [h_A - R \cdot (1 - \cos\gamma)] \cdot tan\,\gamma + R \cdot \sin\gamma}$$

with $\boxed{R = \dfrac{V^2}{g \cdot \delta n_z}}$

Boeing E-3A AWACS

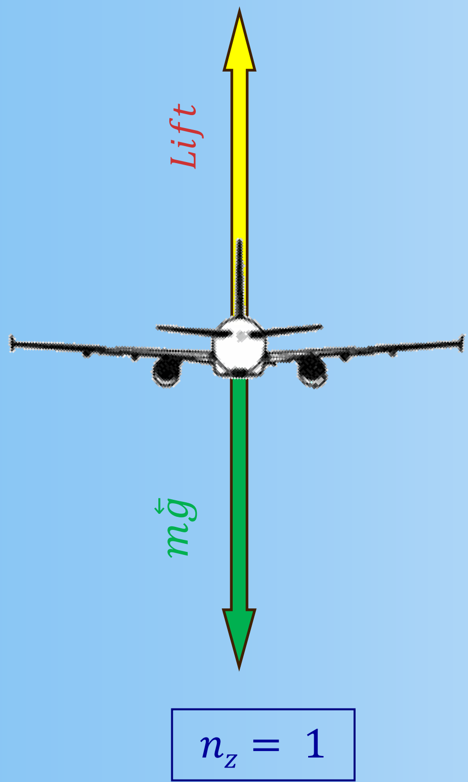
Load Factor : Steady Turn

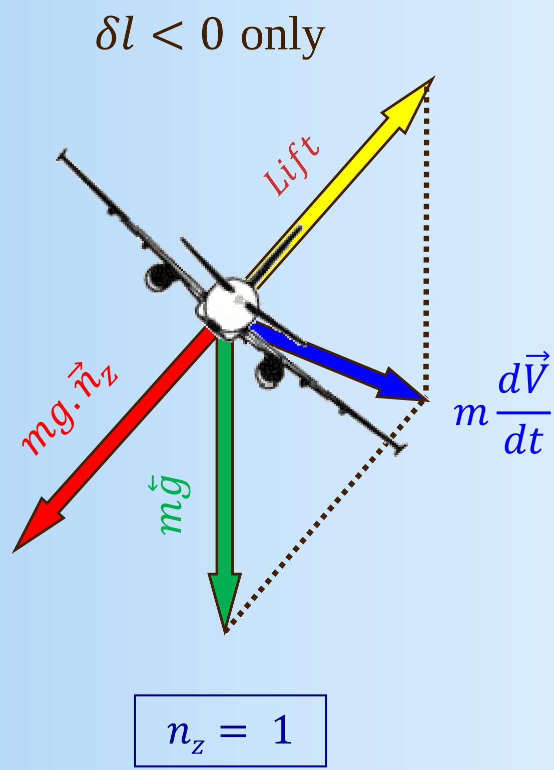
Load factor and Turn Manoeuvre



The pilot bank the aircraft without touching to the δm nor the δx commands,
What happens to the load factor n_z ?

Load factor and Turn Manoeuvre





Why is theaccel

If the pilot doesn't touch to the δm nor the δx commands,

- the angle of attack stays constant, the lift module stays constant, the load factor is unchanged
- the aircraft is loosing altitude and its velocity is increasing

Starting from an initial longitudinal flight trim situation,

- The pilot applies only a roll command $\delta l < 0$: the aircraft is banking on the right
- The Lift Force follows the a/c plane of symmetry (keeping the same module)
- If you do not touch to δx nor δm, the resulting force is directed downwards
- You are loosing altitude ($\gamma < 0$) and the velocity increases : $\dot{V} = -g \cdot \sin\gamma$

When I say that the angle of attack doesn't change
because I do not command any elevator δm, is it correct ?

α is not an angle between $\vec{x}_a$ and $\vec{x}_b$

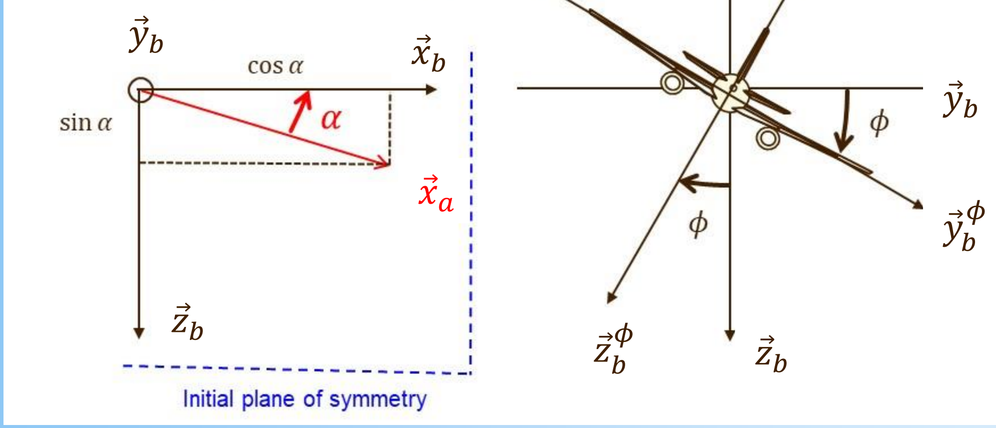
SUPAERO

The initial trim is a longitudinal flight with $\alpha > 0°$ and $\beta = 0°$. The question is what happens to the angle of attack when the aircraft is rolled by an angle ϕ ?

Initially, the velocity is within the aircraft plane of symmetry $(\vec{x}_b , \vec{z}_b)$: the angle of attack is α , no side slip. Then, the aircraft rolls by an angle ϕ

$$\vec{x}_a = \cos\alpha \cdot \vec{x}_b + \sin\alpha \cdot \vec{z}_b$$

$$\vec{z}_b = \sin\phi \cdot \vec{y}_b^{\phi} + \cos\phi \cdot \vec{z}_b^{\phi}$$

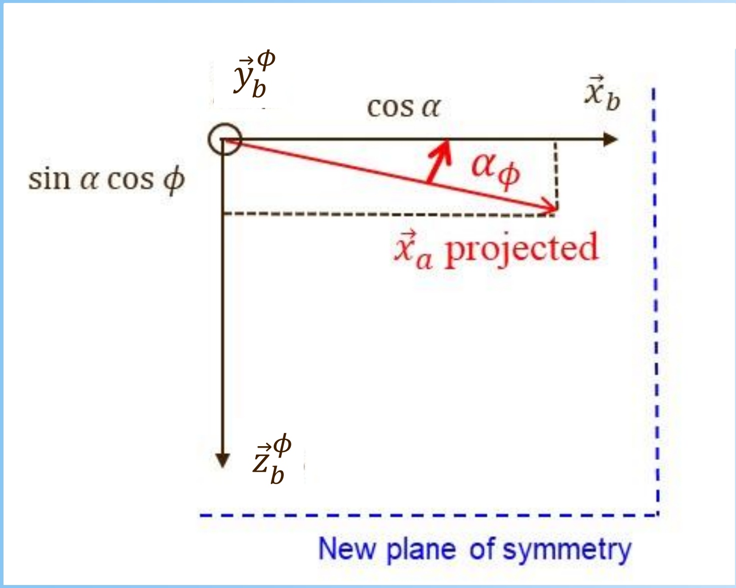


α is not an angle between $\vec{x}_a$ and $\vec{x}_b$

The aircraft plane of symmetry is also rotating aroud $\vec{x}_b$: what is the projection of $\vec{x}_a$ within the new plane of symmetry $(\vec{x}_b\,,\vec{z}_b^{\phi})$?

Expressing $\vec{x}_a$ with respect to $(\vec{x}_b\,,\vec{y}_b^{\phi},\vec{z}_b^{\phi})$

$$\vec{x}_a = \cos\alpha\cdot\vec{x}_b + \sin\alpha\sin\phi\cdot\vec{y}_b^{\phi} + \sin\alpha\cos\phi\cdot\vec{z}_b^{\phi}$$

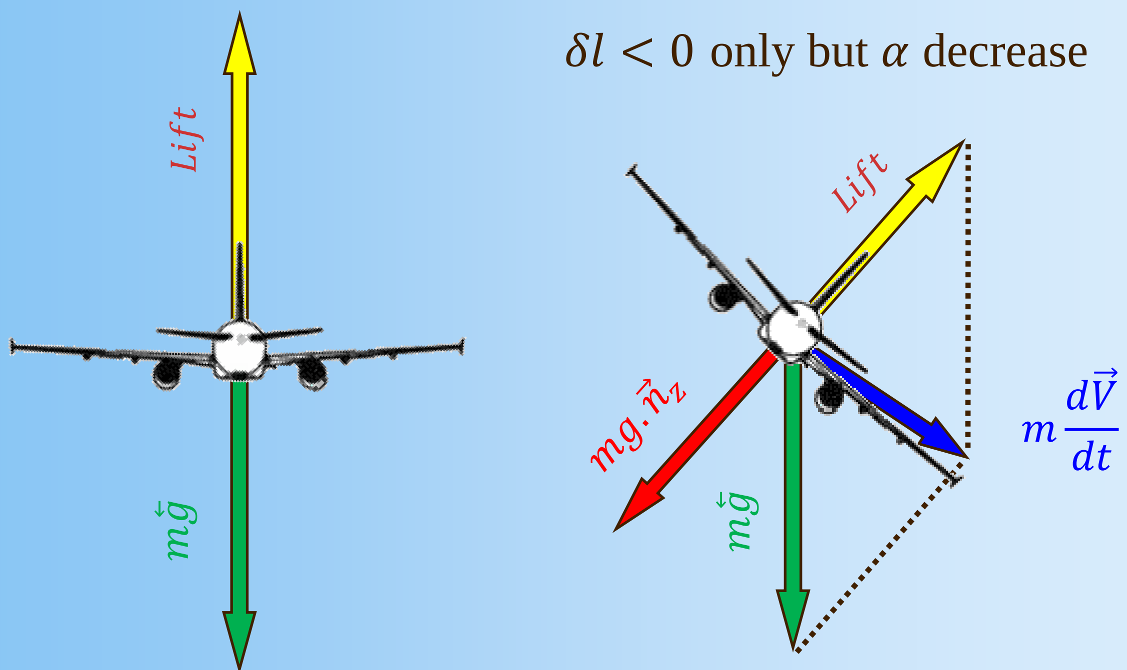


$$\tan\alpha_\phi = \frac{\sin\alpha\cos\phi}{\cos\alpha}$$

$$\tan\alpha_\phi = \tan\alpha\cos\phi$$

$$\boxed{\alpha_\phi \approx \alpha\cos\phi}$$

The bank manœuvre decreases « geometrically » the angle of attack

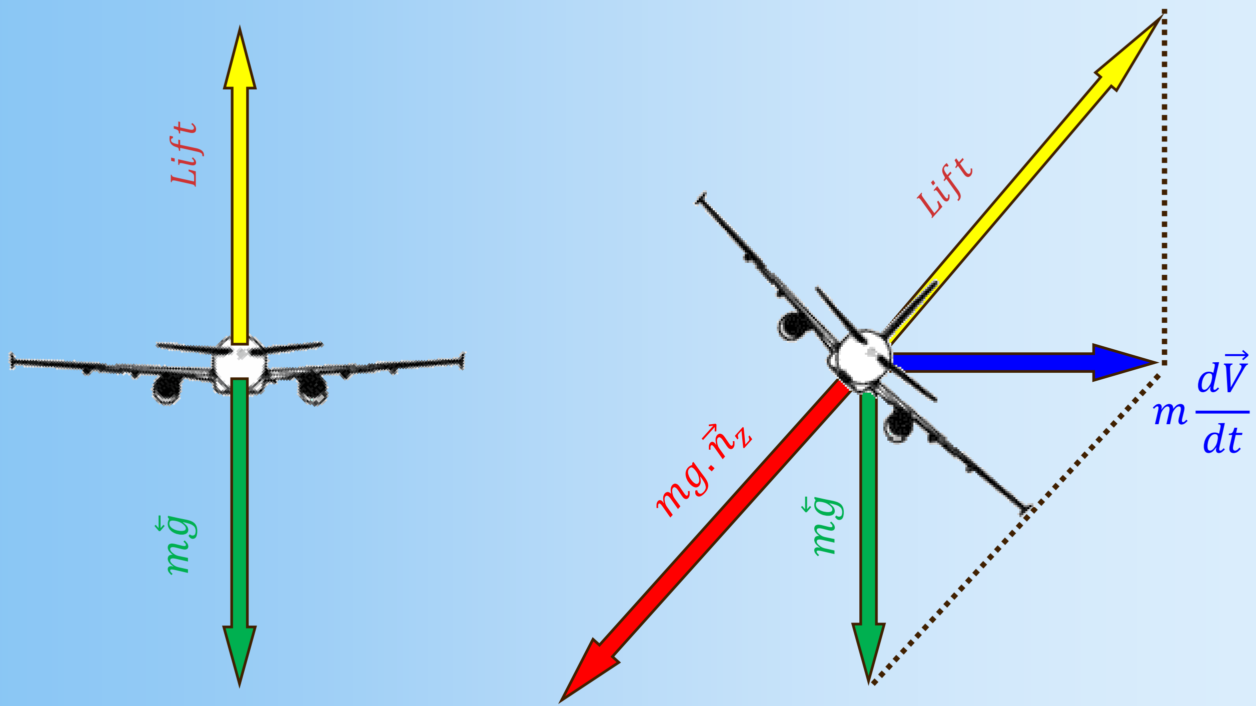


$n_z = 1$ $n_z < 1$

If the pilot doesn't touch to the δm nor the δx commands,

- the angle of attack decreases by a $\cos\phi$ factor
- The lift is decreasing in module, the load factor decreases
- the aircraft is loosing altitude and its velocity is increasing

Load Factor and Steady Turn Manoeuvre



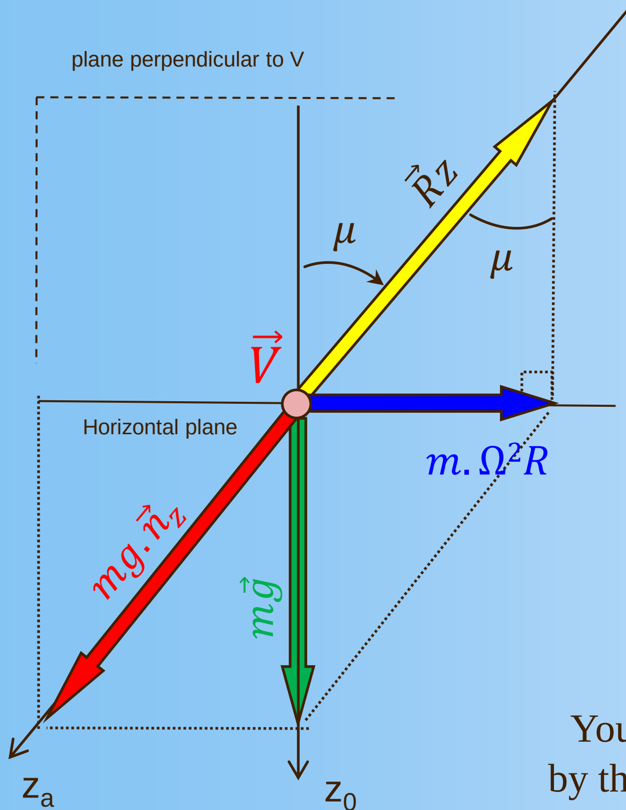
$n_z = 1$

$n_z > 1$

The steady Turn manœuvre imposes that the acceleration is within the horizontal plane,

- The lift force shall be increased until the acceleration lies within the horizontal plane
- As a result, the load factor is increased

How is the load factor increased?



This phenomenon should have been true in the previous case as well where the acceleration was not horizontal

- $\gamma = 0°$: $\vec{V}$ is within the local horizontal plane
- no lateral forces

The z_a axis is rotating around x_a by an angle μ, the aerodynamic roll angle

$$\cos\mu = \frac{mg}{Rz} = \frac{mg}{mg \cdot n_z} \rightarrow n_z = \frac{1}{\cos\mu} > 1$$

$$\tan\mu = \frac{m\Omega^2 R}{mg} = \frac{m\Omega \cdot V}{mg} \rightarrow \Omega = \frac{g \cdot \tan\mu}{V}$$

You can approximate the aerodynamic roll angle μ by the bank angle ϕ_1 (exact when $\gamma = 0°$ and $\beta = 0°$)

$$n_z \approx \frac{1}{\cos\phi_1} \qquad \Omega \approx \frac{g \cdot \tan\phi_1}{V}$$

From Longitudinal Flight to Steady Turn

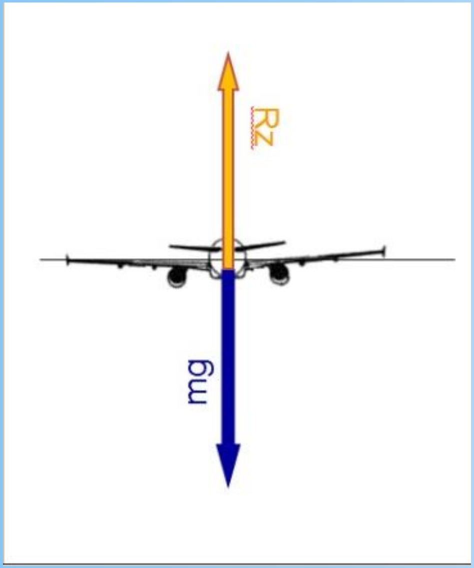
Starting from an initial Longitudinal Flight trim situation (Trim n°1),
we bank the aircraft by an angle ϕ_1 up to the Steady Turn equilibrium (Trim n°2)
without using the Longitudinal command δm / δx

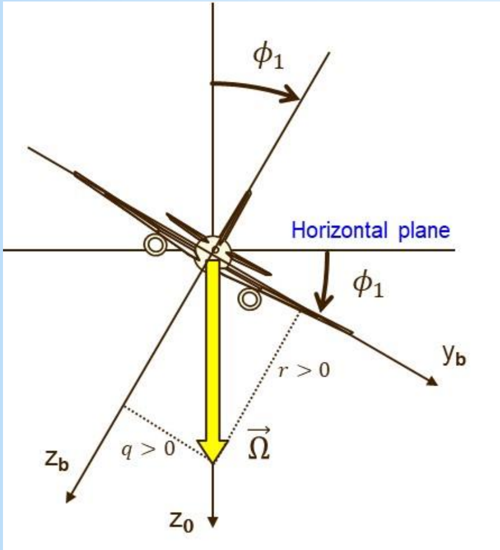
Trim n°1

$$\begin{cases} n_z = 1 \\ q = 0 \end{cases}$$

Trim n°2

$$\begin{cases} n_z = 1/\cos\phi_1 > 1 \\ q = \Omega \sin\phi_1 > 0 \end{cases}$$





From Longitudinal Flight to Steady Turn

$\Delta\delta x = 0 / \Delta\delta m = 0°$

Situation at t=0 : Trim n°1

$$
\begin{cases}
-\frac{1}{2}\rho V_1^2 SCx + F_0(\rho, V_1)\cdot \delta x_1 - mg\cdot\gamma_1 = 0 \\
-\frac{1}{2}\rho V_1^2 S\cdot Cz_1 + mg = 0 \\
Cm_0 + Cm_{\alpha,G}(\alpha_1 - \alpha_0) + Cm_{\delta m}\delta m_1 = 0 \\
q_1 = 0
\end{cases}
$$

Situation after a certain time: Trim n°2

$$
\begin{cases}
-\frac{1}{2}\rho V_2^2 SCx + F_0(\rho, V_2)\cdot \delta x_1 - mg\cdot\gamma_2 = 0 \\
-\frac{1}{2}\rho V_2^2 S\cdot Cz_2 + mg\cdot \boldsymbol{nz_2} = 0 \\
Cm_0 + Cm_{\alpha,G}(\alpha_2 - \alpha_0) + Cm_q\cdot \boldsymbol{q_2} L/V + Cm_{\delta m}\delta m_1 = 0 \\
\boldsymbol{q_2} > 0
\end{cases}
$$

Turning with no Longitudinal commands

$$Cm_\alpha^G \cdot \Delta\alpha + Cm_q \frac{\boldsymbol{q_2} L}{V} = 0 \rightarrow \Delta\alpha = -\frac{\overset{-}{Cm_q}}{\underset{-}{Cm_\alpha^G}} \frac{\overset{+}{\boldsymbol{q_2}} L}{V} < 0 \rightarrow \alpha \text{ decreases}$$

Pitch Moment equation

$$mg \cdot (\boldsymbol{nz_2} - 1) = \frac{1}{2}\rho S \Delta(V^2 Cz) \rightarrow \Delta(V^2 Cz) > 0 \quad \rightarrow V \text{ increases}$$

Lift Force equation

$$mg \cdot \Delta\gamma = \Delta\left\{F_0(\rho, V) \cdot \delta x_1 - \frac{1}{2}\rho V^2 S Cx\right\}$$

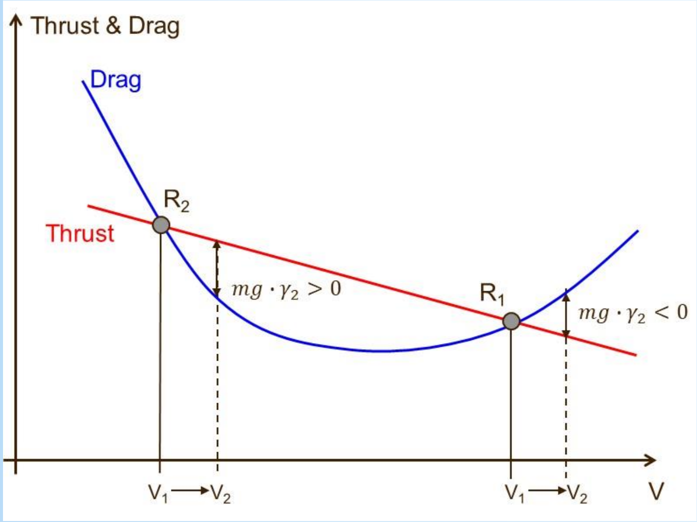
Drag Force equation

if First Flight regime

→ γ decreases

we obtain a Steady Turn solution with a descending spiral trajectory

How is this different from the previous situation?



Turning with no Longitudinal commands

Starting from an initial longitudinal flight trim situation,

- The aircraft is banked by an angle ϕ_1
- The Lift Force follows the a/c plane of symmetry
- If you do not touch to δx nor δm, the resulting force is directed downwards
- You are loosing altitude ($\gamma < 0$) and the velocity increases : $\dot{V} = -g \cdot \sin\gamma$

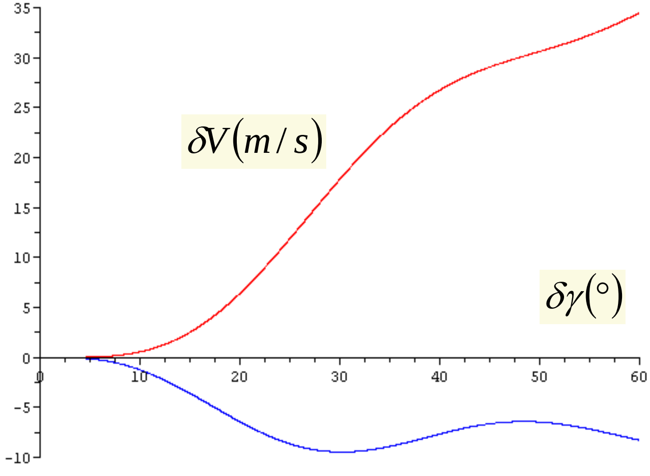
After a certain time, if the aircraft is stable (spiral mode stability)
you recover a new trim situation**, the Steady Turn solution**, the increase of velocity has made growing the lift force, until the acceleration is within the horizontal plane

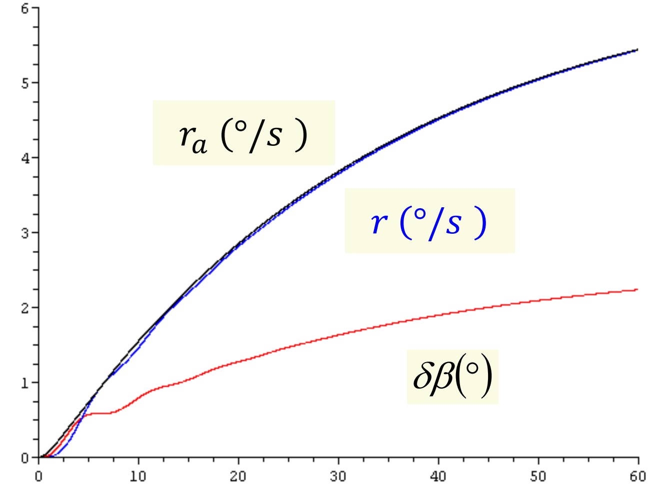
How are we sure that Lift is increasing since alpha also decreases

The positive pitch rate, associated to the Steady Turn, produces a decrease of the angle of attack

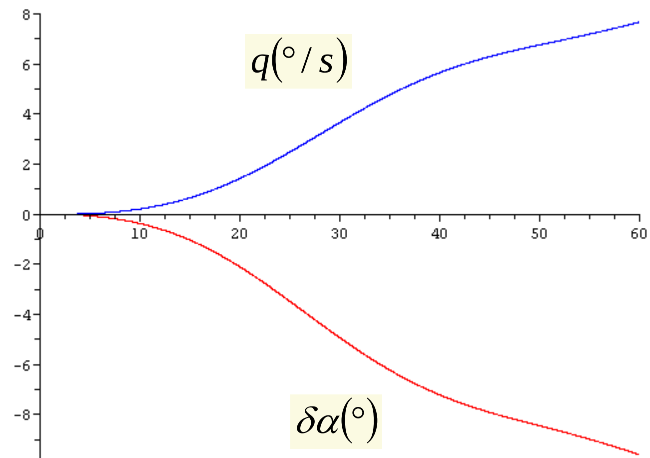
And, if the aircraft velocity is sufficiently high (first regime), the velocity increase produces an unbalance in the drag equation associated to a negative flight slope

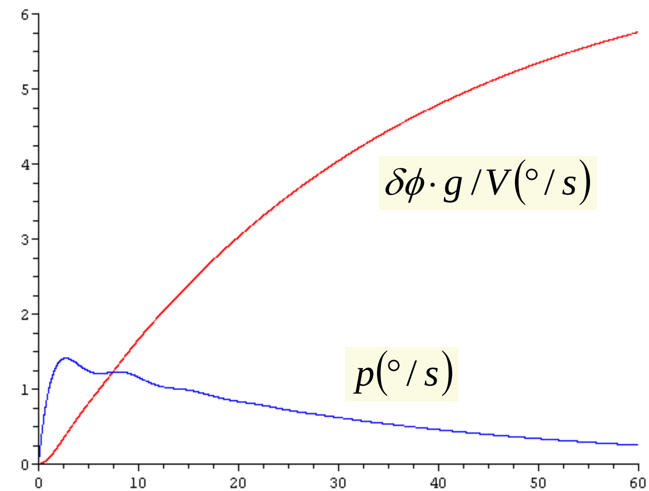
you have reached the Steady Turn solution with a descending spiral trajectory ($\gamma < 0$)





Δδl aileron step input : longi/lateral coupling





Turning with Longitudinal commands

Hence, it is possible to obtain the Steady Turn manœuvre but :

- It takes a certain time
- You loose altitude
- And the velocity is increased

Certainly, not an operationnal way for turning !

The pilot wants to turn :

- Quickly
- By keeping the same altitude
- And the same velocity

He has no choice : he must increase quickly the lift force .
So, he pulls on the stick : the aircraft is pitching up, the angle of attack increases and the lift coefficient Cz, as well, until the acceleration is within the horizontal plane

The increase of lift produces an increase of drag ; if the pilot doesn't want to loose altitude, he has no choice : he must increase the thrust in order to keep the balance of the drag equation with a zero flight slope angle.

Turning with Longitudinal commands

If the pilot wants to maintain the same speed V :

$$mg \cdot (\boldsymbol{nz_2} - 1) = \frac{1}{2}\rho S \Delta(V^2 Cz) \rightarrow \Delta Cz = \frac{2mg}{\rho V^2 S} \cdot (\boldsymbol{nz_2} - 1) > 0 \rightarrow \Delta\alpha > 0$$

Lift Force equation

→ The pilot shall increase the aircraft angle of attack with the right $\Delta\delta m$

$$Cm_\alpha^G \cdot \Delta\alpha + Cm_q \frac{\boldsymbol{q_2} L}{V} + Cm_{\delta m} \cdot \Delta\delta m = 0 \rightarrow \Delta\delta m = -\frac{Cm_q}{Cm_{\delta m}} \frac{\boldsymbol{q_2} L}{V} - \frac{Cm_\alpha^G}{Cm_{\delta m}} \Delta\alpha < 0$$

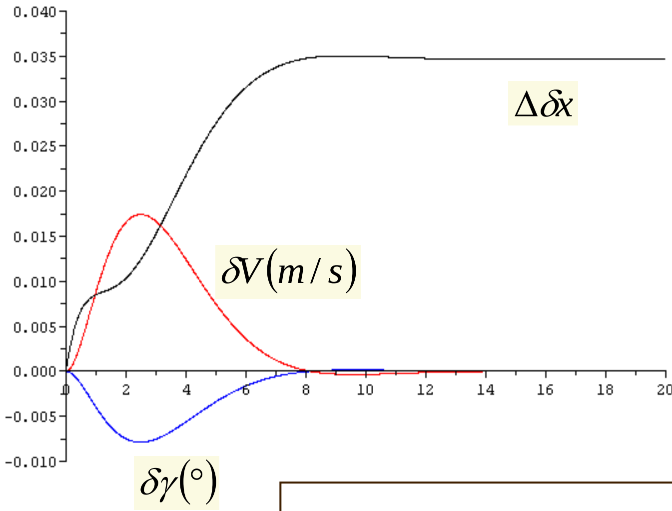
Pitch Moment equation

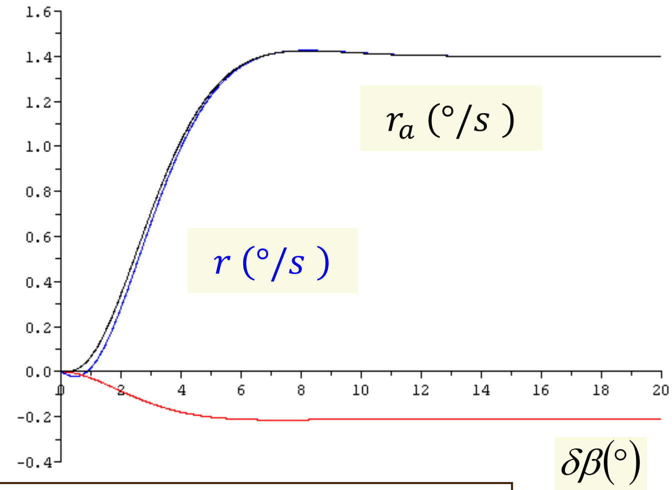
If the pilot wants to maintain the same altitude :

$$\Delta\left\{F_0(\rho, V) \cdot \delta x_1 - \frac{1}{2}\rho V^2 S Cx\right\} = mg \cdot \Delta\gamma = 0 \rightarrow \Delta\delta x = \frac{\rho V^2 S \cdot \Delta(Cx)}{2 \cdot F_0(\rho, V)} > 0$$

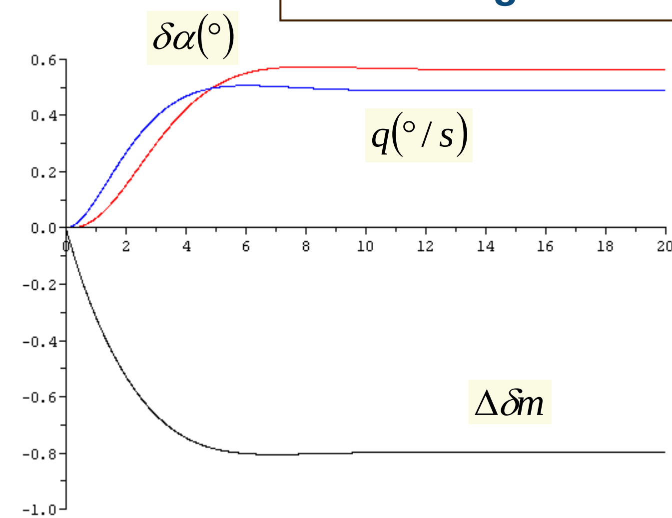
Drag Force equation

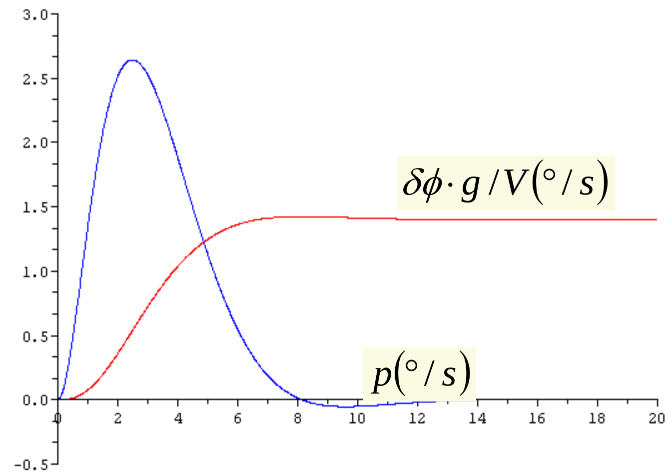
→ The pilot shall increase the engine thrust with the right $\Delta\delta x$





Turning with maintaining speed and altitude





NOT in PROGRAM

Boeing 787 Dreamliner

Generalisation of the Steady Turn : Steady Spiral

Steady Turn : generalisation with $\gamma \neq 0°$

NOT in PROGRAM

The generalisation of the steady turn leads to a steady spiral motion :

- The lift force is rolling by an angle μ

- within the horizontal plane, the center of gravity is still moving along a circular motion by a radius R
  - with a uniform velocity : $V\cos\gamma = \Omega R$
  - and a centripetal acceleration : $\Omega^2 R = \Omega V \cos\gamma$

- with respect to the z_0-axis, the center of gravity is moving by a uniform velocity : $V\sin\gamma$

- within the plane perpendicular to $\vec{V}$, the load factor vector is given by :

$$mg \cdot \vec{n} = mg \cdot (\vec{n}_y + \vec{n}_z) = m\left[\vec{g} - \frac{d\vec{V}}{dt}\right]_{\perp\vec{V}}$$

Steady Turn : generalisation with $\gamma \neq 0°$

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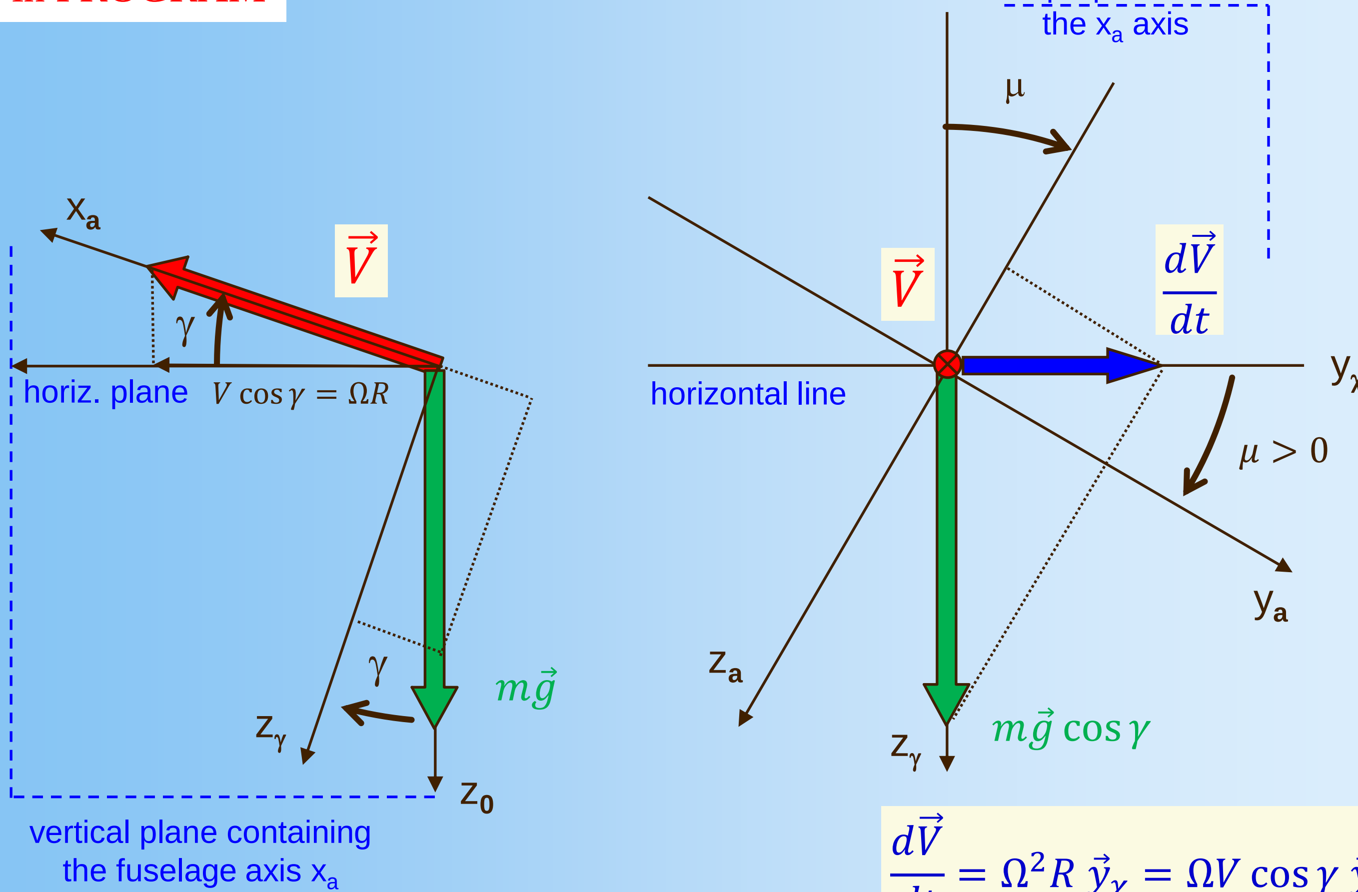


$$\frac{d\vec{V}}{dt} = \Omega^2 R\,\vec{y}_\chi = \Omega V\cos\gamma\,\vec{y}_\chi$$

Steady Turn : generalisation with $\gamma \neq 0°$

NOT in PROGRAM

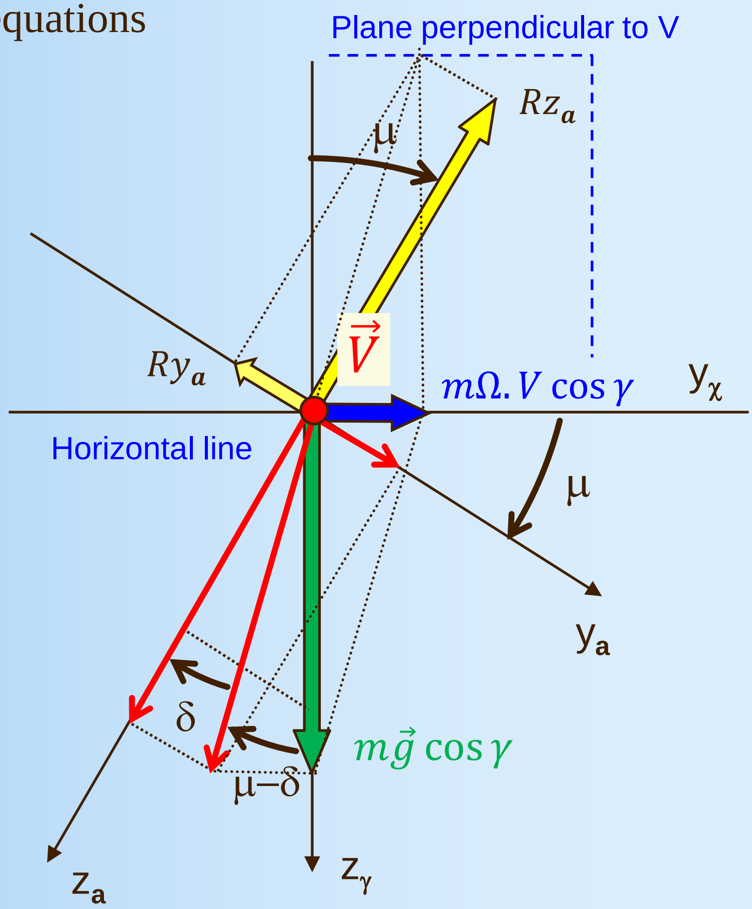
Projection of Forces equations

$$mg \cdot \vec{n} = m\left[\vec{g} - \frac{d\vec{V}}{dt}\right]_{\perp \vec{V}}$$

$$\begin{cases} y_\chi : \; mg \cdot n \sin(\mu - \delta) = m\Omega V \cos\gamma \\ z_\gamma : \; mg \cdot n \cos(\mu - \delta) = mg \cos\gamma \end{cases}$$

$$\begin{cases} y_\chi : \Omega = \dfrac{g}{V \cos\gamma} \cdot n \sin(\mu - \delta) \\ z_\gamma : \; n = \dfrac{\cos\gamma}{\cos(\mu - \delta)} \end{cases}$$

$$\begin{cases} y_\chi : \Omega = \dfrac{g}{V} \cdot \tan(\mu - \delta) \\ z_\gamma : \; n = \dfrac{\cos\gamma}{\cos(\mu - \delta)} \end{cases}$$



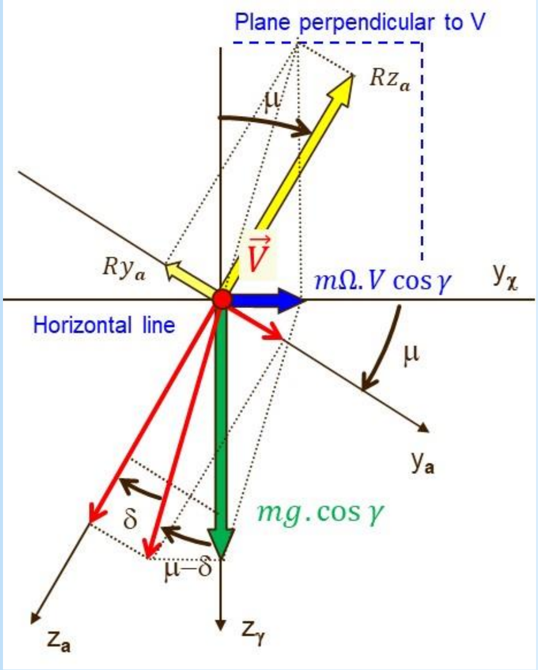
NOT in PROGRAM

The generalisation of the steady turn leads to a steady spiral motion

Assuming lateral forces : $Ry_a \neq 0$

$$\tan\delta = \frac{Ry_a}{Rz_a}$$

$$\begin{cases} \Omega = \dfrac{g}{V} \cdot \tan(\mu - \delta) \\ n = \dfrac{\cos\gamma}{\cos(\mu - \delta)} \end{cases}$$



NOT in PROGRAM

The generalisation of the steady turn leads to a steady spiral motion

Assuming no lateral forces : $Ry_a = 0$

$$\begin{cases} \Omega = \dfrac{g}{V} \cdot \tan \mu \\ n_z = \dfrac{\cos \gamma}{\cos \mu} \end{cases}$$