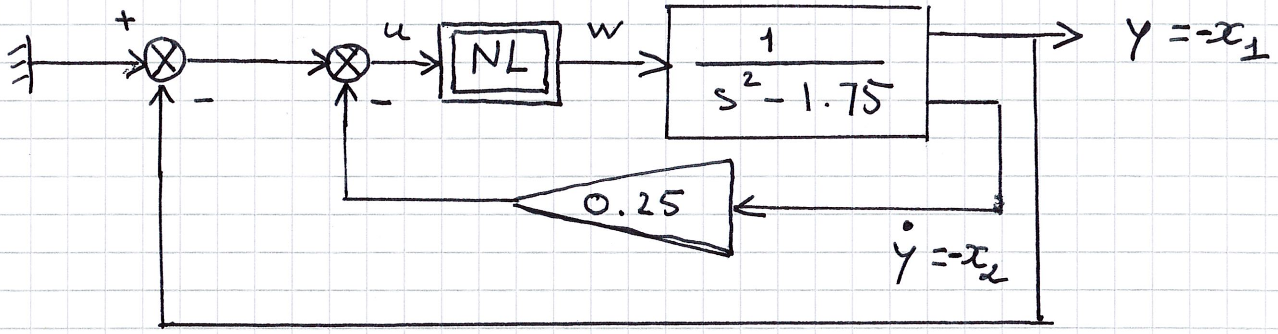
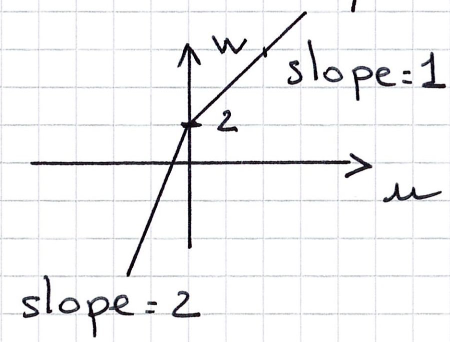
EXERCISE

Consider the following system:



where the nonlinearity is given by:



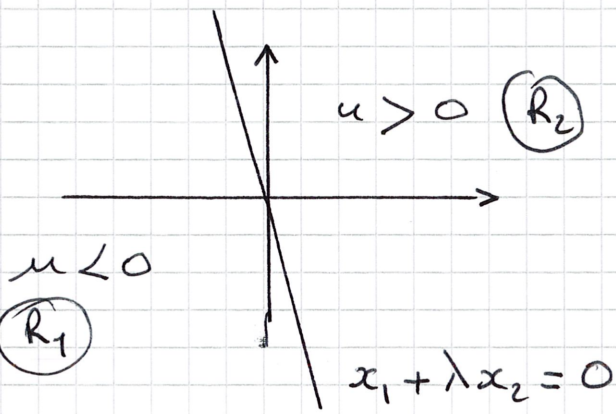
Objectives:

- Plot de phase portrait (x_1, x_2)
- Characterize the stability domain.

① Equations of the nonlinear closed-loop.

- $\frac{y}{w} = \frac{1}{s^2 - 1.75} \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 1.75\, x_1 - w \end{cases}$

- $w = k_i\, u + \ell_i$ with $u = x_1 + 0.25\, x_2$



$$\begin{cases} \ell_1 = \ell_2 = 2 \\ k_1 = 2 \\ k_2 = 1 \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 1.75 - k_i & -0.25\,k_i \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 0 \\ -l_i \end{bmatrix}$$

② Singular points

(R1) $\dot{x} = A_1 x + B_1$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.25 & -0.5 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\Rightarrow x_{s_1} = \begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$\Rightarrow \det(\lambda I - A_1) = \lambda^2 + 0.5\lambda + 0.25$

$\Rightarrow$ stable spiral. $\begin{cases} \omega = 0.5 \\ \zeta = 0.5 \end{cases}$

(R2) $\dot{x} = A_2 x + B_2$

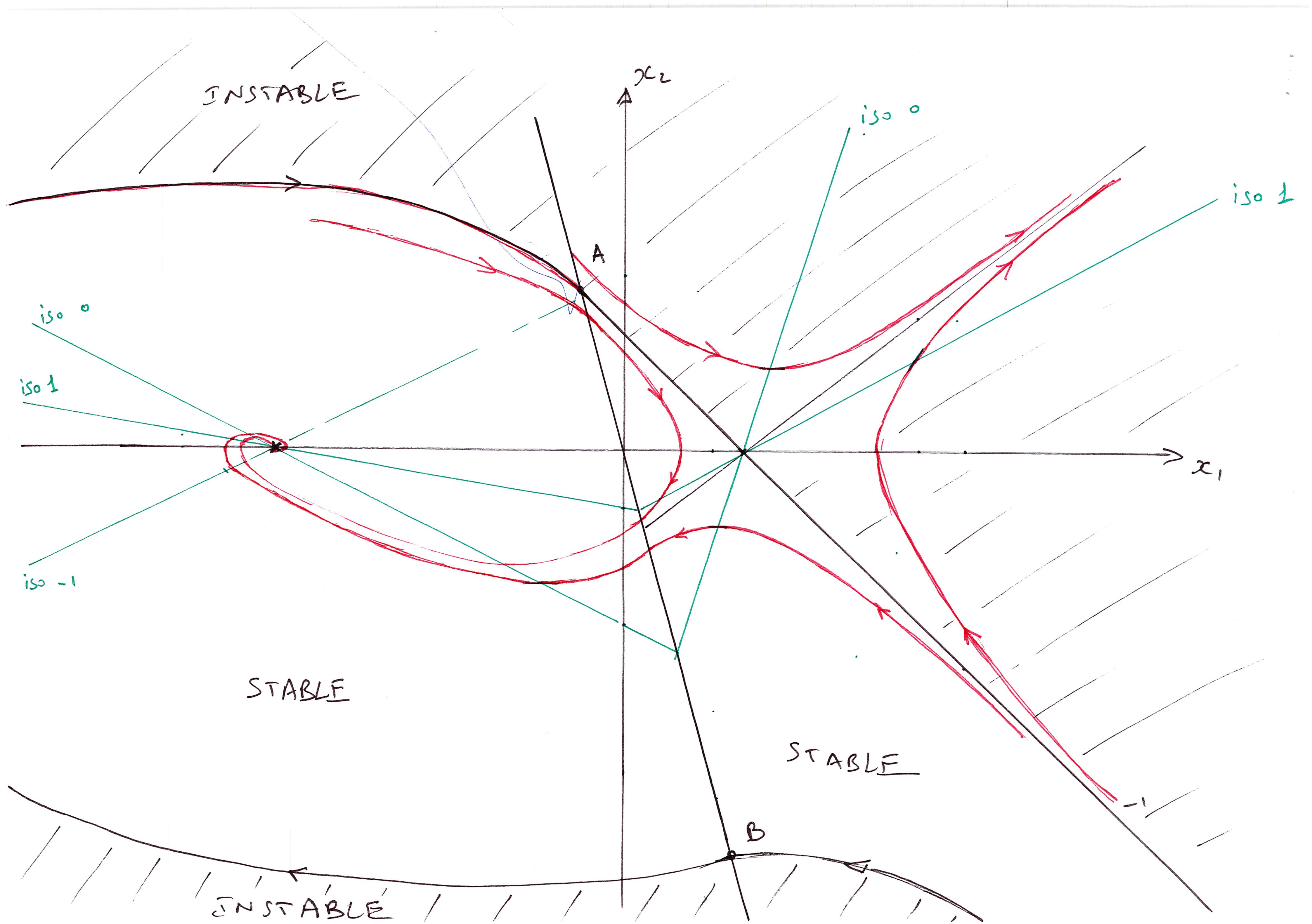
$$A_2 = \begin{bmatrix} 0 & 1 \\ 0.75 & -0.25 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\Rightarrow x_{s_2} = \begin{bmatrix} 0 \\ 2.67 \end{bmatrix}$$

$\Rightarrow \det(\lambda I - A_2) = \lambda^2 + 0.25\lambda - 0.75$
$= (\lambda + 1)(\lambda - 0.75)$

$\Rightarrow$ saddle point

③ Phase portrait
$\Rightarrow$ next page


INSTABLE
x_2
iso 0
iso 1
A
iso 0
iso 1
iso -1
x_1
STABLE
STABLE
-1
B
INSTABLE