MAE 109 - ELECTROMAGNETISM

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2020-2021

Part III Plane Wave In Vacuum

I. Introduction

II. Electromagnetism - Maxwell Equation(Spatial/temporal equations)

III. Plane Wave in vacuum

IV. Wave propagation in matter

V. Boundary conditions (Reflection/refraction)

Results I Electromagnetism - Maxwell Equation(Spatial/temporal equations)

(1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ Local Gauss Law (electric flux density)

(2) $\vec{\nabla} \cdot \vec{B} = 0$ General Magnetism Law

(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faraday Law (relationship between Electric and Magnetic Field)

(4) $\overrightarrow{\nabla} \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$ Ampere law (current flow in a wire creating a magnetic field)

E & B Resolution

$$\Delta\vec{E} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0}\vec{\nabla}\rho + \mu_0 \frac{\partial \vec{j}}{\partial t}$$

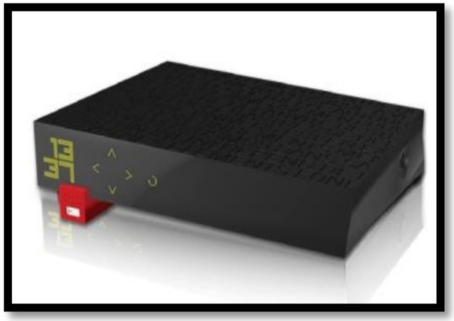
$$\Delta\vec{B} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu_0 \vec{\nabla} \times \vec{j}$$

Electromagnetic Wave Equation?

Summary

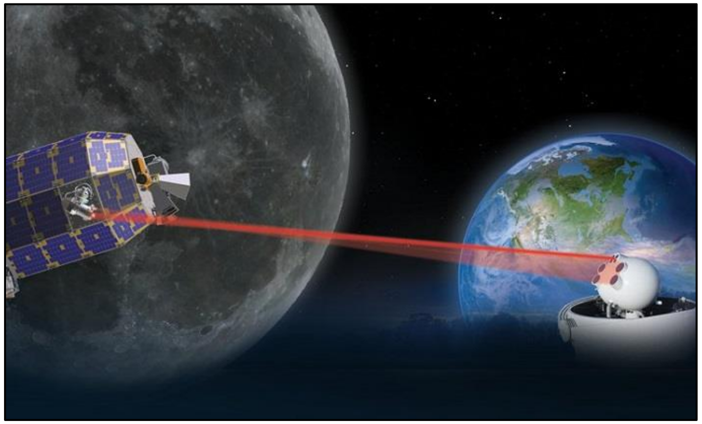
III.1. **From D'Alembert Equation to Helmoltz equation**

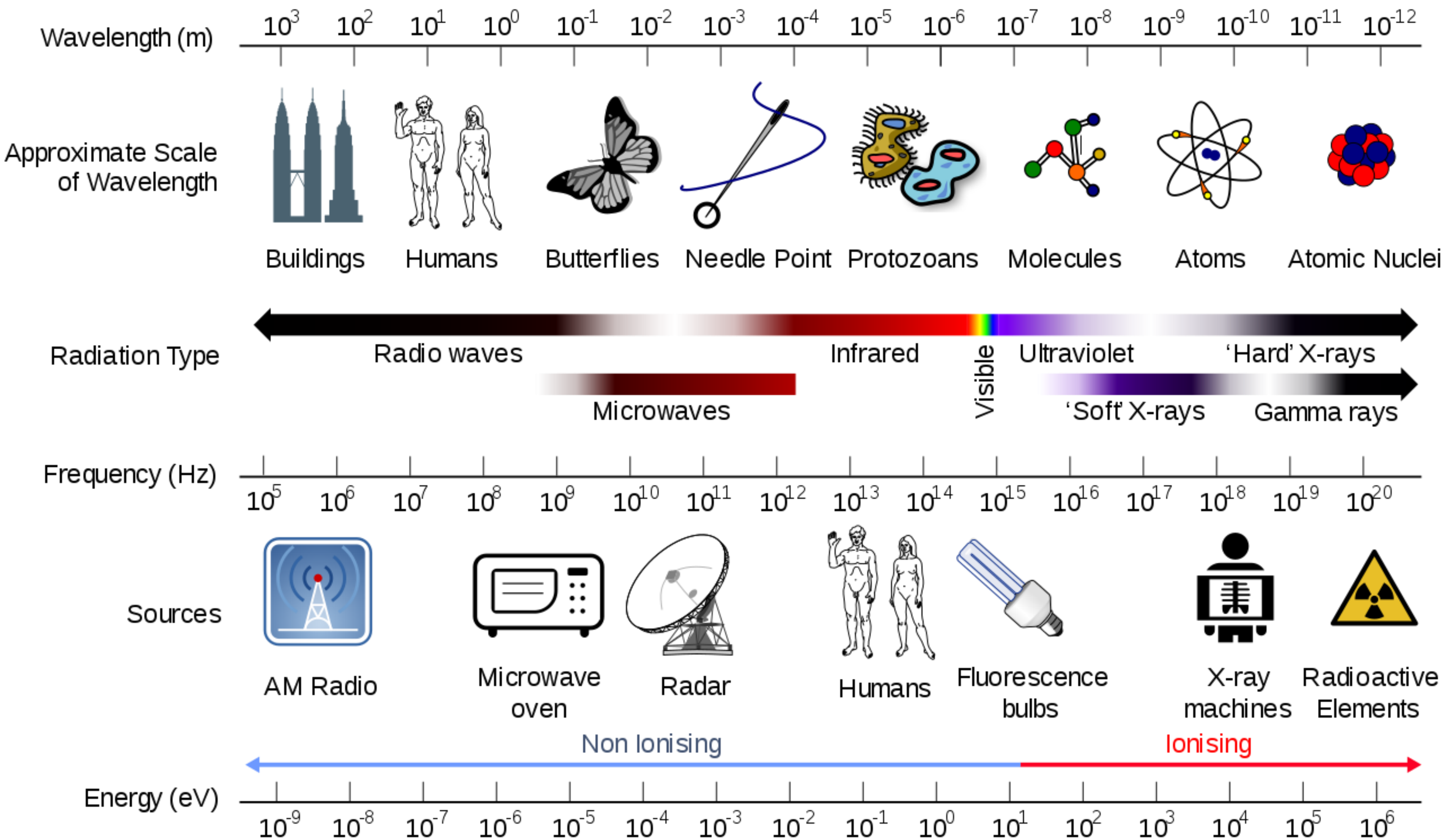
- Wireless radio systems

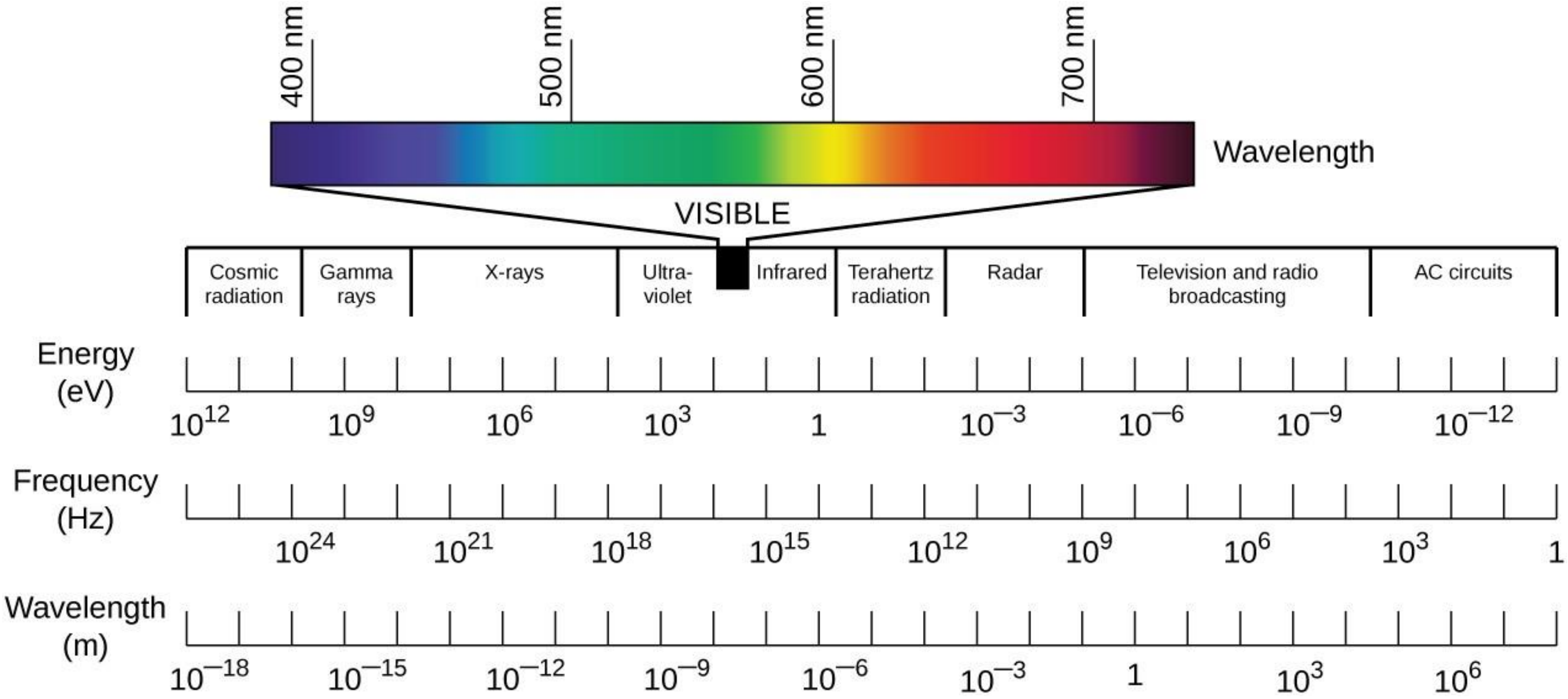


Electromagnetic propagation in the channel

- Free space optics




Wavelength (m)
10^3 10^2 10^1 10^0 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Radiation Type
Radio waves
Infrared
Visible
Ultraviolet
'Hard' X-rays
Microwaves
'Soft' X-rays
Gamma rays
Frequency (Hz)
10^5 10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17 10^18 10^19 10^20
Sources
AM Radio
Microwave oven
Radar
Humans
Fluorescence bulbs
X-ray machines
Radioactive Elements
Non Ionising
Ionising
Energy (eV)
10^-9 10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 10^0 10^1 10^2 10^3 10^4 10^5 10^6


400 nm
500 nm
600 nm
700 nm
Wavelength
VISIBLE
Cosmic radiation
Gamma rays
X-rays
Ultra-violet
Infrared
Terahertz radiation
Radar
Television and radio broadcasting
AC circuits
Energy (eV)
10^{12} 10^{9} 10^{6} 10^{3} 1 10^{-3} 10^{-6} 10^{-9} 10^{-12}
Frequency (Hz)
10^{24} 10^{21} 10^{18} 10^{15} 10^{12} 10^{9} 10^{6} 10^{3} 1
Wavelength (m)
10^{-18} 10^{-15} 10^{-12} 10^{-9} 10^{-6} 10^{-3} 1 10^{3} 10^{6}

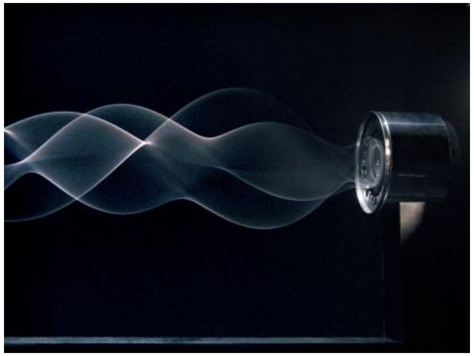
III.1 From D'Alembert Equation to Helmoltz equation

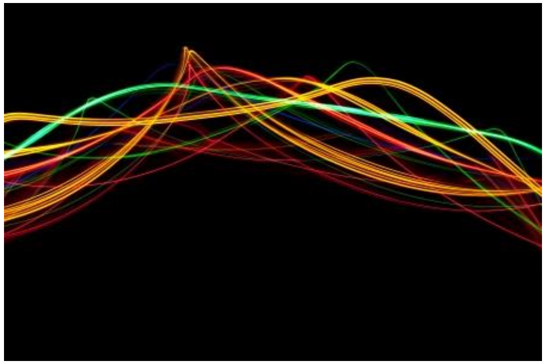
What is an Electromagnetic Field and Electromagnetic WAVE?

How to demonstrate the propagation?

Different kind of waves :

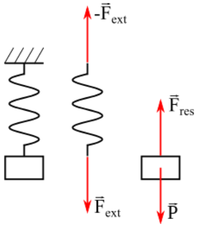
- Mechanical wave (fluid, acoustic)
- **Electromagnetic Wave (Radio, Optics)**
- Matter waves (De Broglie Wave / Wave nature of the matter).

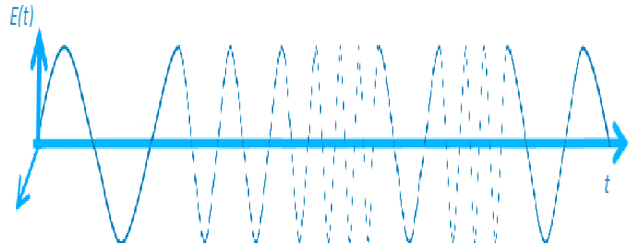




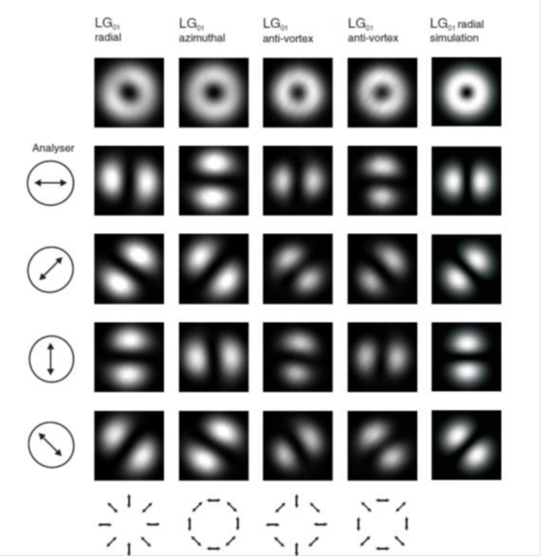
III.1 Analogy between Electromagnetic wave and mecanical waves
Longitudinal & tranverse Wave

- Longitudinal Wave as a Spring





- Transverse Wave – Vibrating-Wire



Part III.1 Wave Equation

Harp string moving: D'Alembert Equation

Wave / stationary (SteadyState) phenomenon

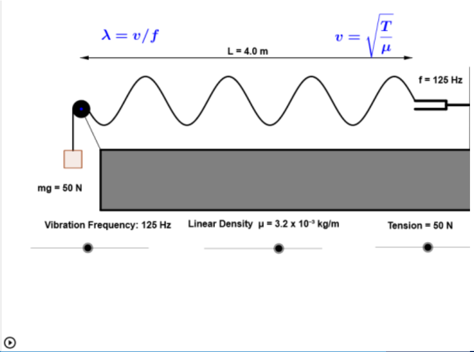
D'Alembert Equation (1730) for a vibrating string :

$$\Delta\vec{p} = \frac{1}{c^2}\frac{\partial^2\vec{p}}{\partial t^2}$$

$\vec{p}$ is a vector corresponding to the pressure in acoustic

c is the speed of the sound in the air

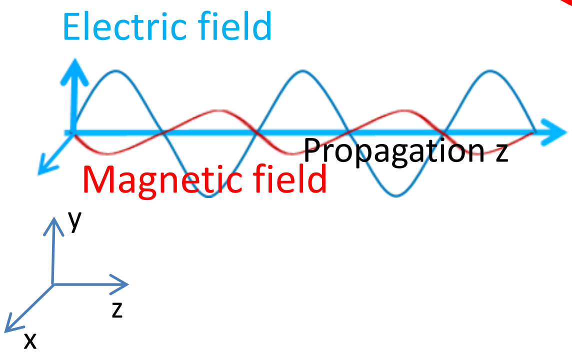
- [Geogebra Activity on LMS: Vibrating String]



What about the Electromagnetism?

Part III.1 Plane wave in Vaccum: Vacuum Propagation

No charge, No current, Permittivity ε_0 and Permeability μ_0



$$\Delta\vec{E} - \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} = \cancel{\frac{1}{\varepsilon_0}\vec{\nabla}\rho} + \mu_0\cancel{\frac{\partial\vec{j}}{\partial t}}$$

$$\Delta\vec{B} - \varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2} = -\mu_0\cancel{\vec{\nabla}\times\vec{j}}$$

General Wave equation

$$\Delta\vec{E} - \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\Delta\vec{B} - \varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2} = 0$$

Travelling Wave

$$\Delta\vec{E} = \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Delta\vec{B} = \varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\boxed{\Delta\vec{\boldsymbol{p}} = \frac{\mathbf{1}}{\boldsymbol{c}^2}\frac{\partial^2\vec{\boldsymbol{p}}}{\partial t^2}}$$

III.1 Plane wave in Vaccum: Wave Equation

$$\Delta\vec{E} = \varepsilon_0\mu_0 \frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Delta\vec{B} = \varepsilon_0\mu_0 \frac{\partial^2\vec{B}}{\partial t^2}$$

$$\boldsymbol{\Delta\vec{A} = \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}}$$

D'Alembert Equation of E, B Field

$$\boxed{\boldsymbol{c = \frac{1}{\sqrt{(\varepsilon_0\mu_0)}}}}$$

$\varepsilon_0 = 8{,}85418782 \times 10^{-12}$ F m^{-1} $(or\ A^2.s^4.kg^{-1}.m^{-3})$,
$\mu_0 = 4\pi \cdot 10^{-7}\ \ H/m\ (or\ kg \cdot m \cdot A^{-2} \cdot s^{-2})$

c is a velocity (m/s)
Which Velocity?

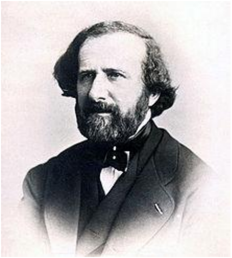
The same for E and B Field.

E & B are Coupled and propagate with the same Velocity

III.1. Velocity of Electromagnetic Wave : speed of the light

Huygens
Jupiter moon
eclipse (1675)
220000km/s

Bradley Light
Aberration (1729)
301000km/s



Fizeau

Foucault

Rotating mirrors (1849)
298000km/s

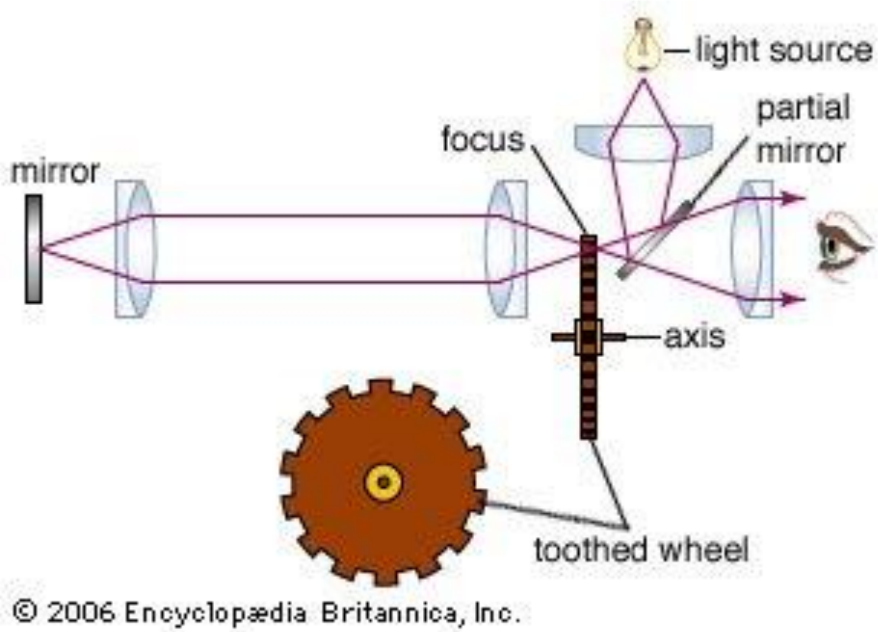
Michelson
Interferometer
(1926)
299796km/s

Maxwell 1864: Dynamical Theory of the Electromagnetic Field (Nature of Light and Electromagnetics): $\boldsymbol{c = 31074000 m.s^{-1} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$

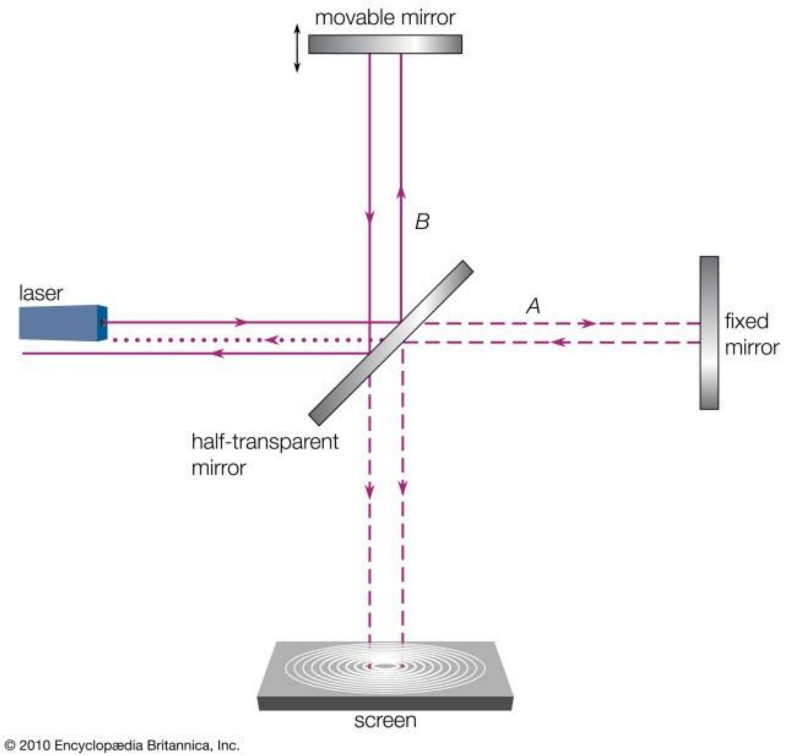
Today *(General Conference on weights and measures – CGPM)* ***299792.458 km/s***

We assume $\boldsymbol{c \approx 3 \cdot 10^8 m/s}$

III.1 speed of the light Experiments



Armand Fuzeau 1849 (5% difference)
Léon Foucault 1850 (1% difference)



A.A Michelson Late 1870's (0,02 % difference)

III. 1 From D'Alembert to Helmholtz Equation: **Travelling Wave Equation**

$$\Delta\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Delta\vec{B} = \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$$

Propagation of the E and B Field at the same velocity

$\Rightarrow$ **the speed of the light c**

Light is an Electromagnetic Wave (UV, Visible or Infrared)

D'Alembert Equation resolution in Electromagnetism to Helmholtz Equation (Harmonic Solution)

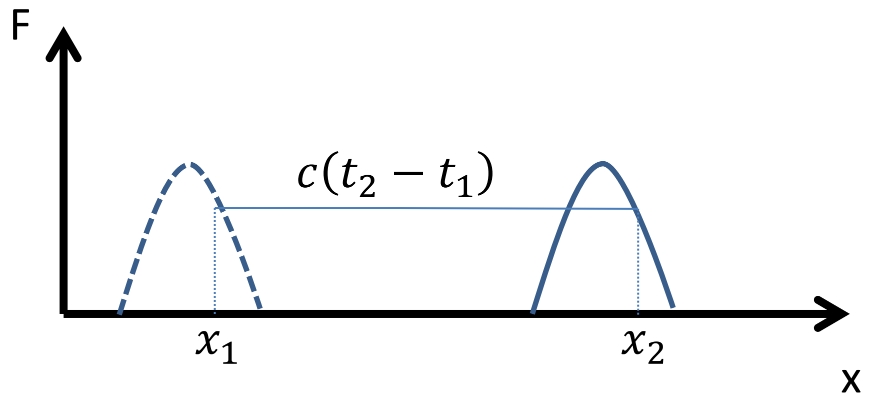
III. 2. Propagation of the EM plane wave in vacuum: One dimensions Wave Equation Resolution

1 Dimension wave Equation : $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 f}{\partial t^2}$

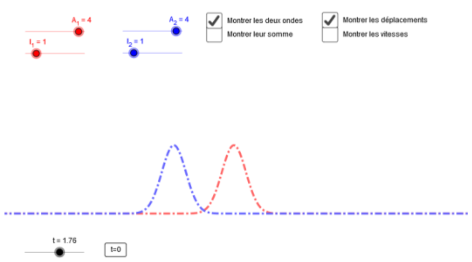
General solution

$$f(x,t) = F(ct - x) + G(ct + x)$$

F is a propagating signal (x direction) and G is a contrapropagating signal (-x direction)



[Geogebra activity on LMS]



III. 2. Propagation of the EM plane wave in vacuum: Practice

Verify that $\boldsymbol{f(x,t) = F(ct - x) + G(ct + x)}$ is a solution of Wave Equation if

$$\frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$$

We denote :

$X = ct - x$ and $Y = ct + x$

Thus

$x = 1/2(X - Y)$ and $ct = \frac{1}{2}(X + Y)$

Method:

Compute $\frac{\partial^2 f}{\partial x^2}$ and $\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial f}{\partial Y}\frac{\partial Y}{\partial x} = -\frac{\partial f}{\partial X} + \frac{\partial f}{\partial Y}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{\partial f}{\partial X} + \frac{\partial f}{\partial Y}\right)$$

$$= \frac{\partial}{\partial X}\left(-\frac{\partial f}{\partial X}\right)\underbrace{\frac{\partial X}{\partial x}}_{\boxed{-1}} + \frac{\partial}{\partial Y}\left(-\frac{\partial f}{\partial X}\right)\underbrace{\frac{\partial X}{\partial x}}_{\boxed{-1}} + \frac{\partial}{\partial X}\left(\frac{\partial f}{\partial Y}\right)\underbrace{\frac{\partial Y}{\partial x}}_{\boxed{1}} + \frac{\partial}{\partial Y}\left(\frac{\partial f}{\partial Y}\right)\underbrace{\frac{\partial Y}{\partial x}}_{\boxed{1}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial X^2} + \frac{\partial^2 f}{\partial Y^2} + 2\frac{\partial^2 f}{\partial X \partial Y}$$

III. 2. Propagation of the EM plane wave in vacuum: Practice

$$**** \frac{\partial f}{\partial ct} = \frac{\partial f}{\partial X} + \frac{\partial f}{\partial Y}$$

$$\frac{\partial^2 f}{\partial (ct)^2} = \frac{\partial}{\partial (ct)}\frac{\partial f}{\partial (ct)} = \frac{\partial}{\partial (ct)}\left(\frac{\partial f}{\partial X} + \frac{\partial f}{\partial Y}\right)$$

$$= \frac{\partial}{\partial X}\left(\frac{\partial f}{\partial X}\right)\frac{\partial X}{\partial (ct)} + \frac{\partial}{\partial Y}\left(\frac{\partial f}{\partial X}\right)\frac{\partial X}{\partial (ct)} + \frac{\partial}{\partial X}\left(\frac{\partial f}{\partial Y}\right)\frac{\partial Y}{\partial (ct)} + \frac{\partial}{\partial Y}\left(\frac{\partial f}{\partial Y}\right)\frac{\partial Y}{\partial (ct)}$$

$$\frac{\partial^2 f}{\partial (ct)^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{\partial X^2} + \frac{\partial^2 f}{\partial Y^2} + 2\frac{\partial^2 f}{\partial X \partial Y}$$

$$\boxed{\textbf{Thus}\quad \frac{\partial^2 f}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0}$$

III. 2. Propagation of the EM plane wave in vacuum: Electromagnetic Plane Wave

1 D solution of Wave equation in vacuum

Ttravelling wave in (z) Direction $\Rightarrow$ $E(x,y,z,t) = E(x,y)(z-ct)$

Thus $B(z,t) = B(x,y)(z-ct)$

Plane $z = z_0 \; at \; t = t_0$:

- identity of $\vec{E}$
- identity of $\vec{B}$

Plane Wave Definition

From Mawxell equation in Vacuum (no charges):

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \& \overrightarrow{\nabla} \cdot \vec{B} = 0$$

E&B-Field No Component tangential to the propagation axis

$\Leftrightarrow$ $\boldsymbol{E_z = 0}$ **and** $\boldsymbol{B_z = 0}$

III. 2. Propagation of the EM plane wave in vacuum: EM Plane Wave

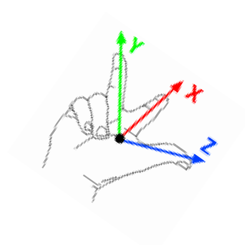
$$\overrightarrow{E(x,y,z,t)} = \overrightarrow{E_0(x,y,z)}\,(z-ct)$$

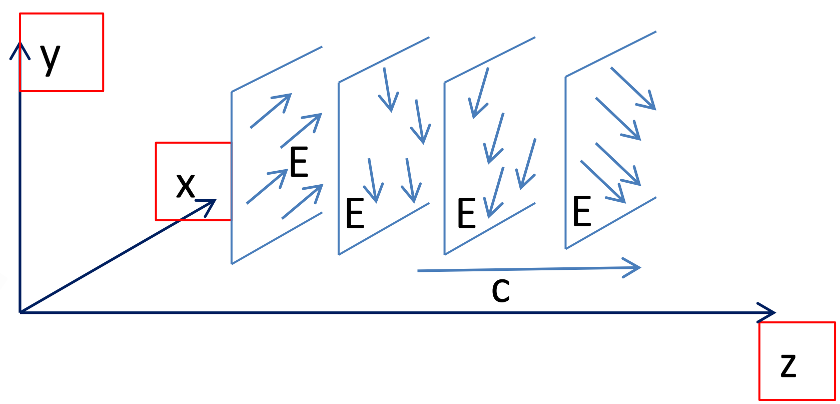
According to the figure propagation along z axis

$$\overrightarrow{E(x,y,z)}(z-ct) = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} E_{0x}(z-ct) \\ E_{0y}(z-ct) \\ 0 \end{pmatrix}$$

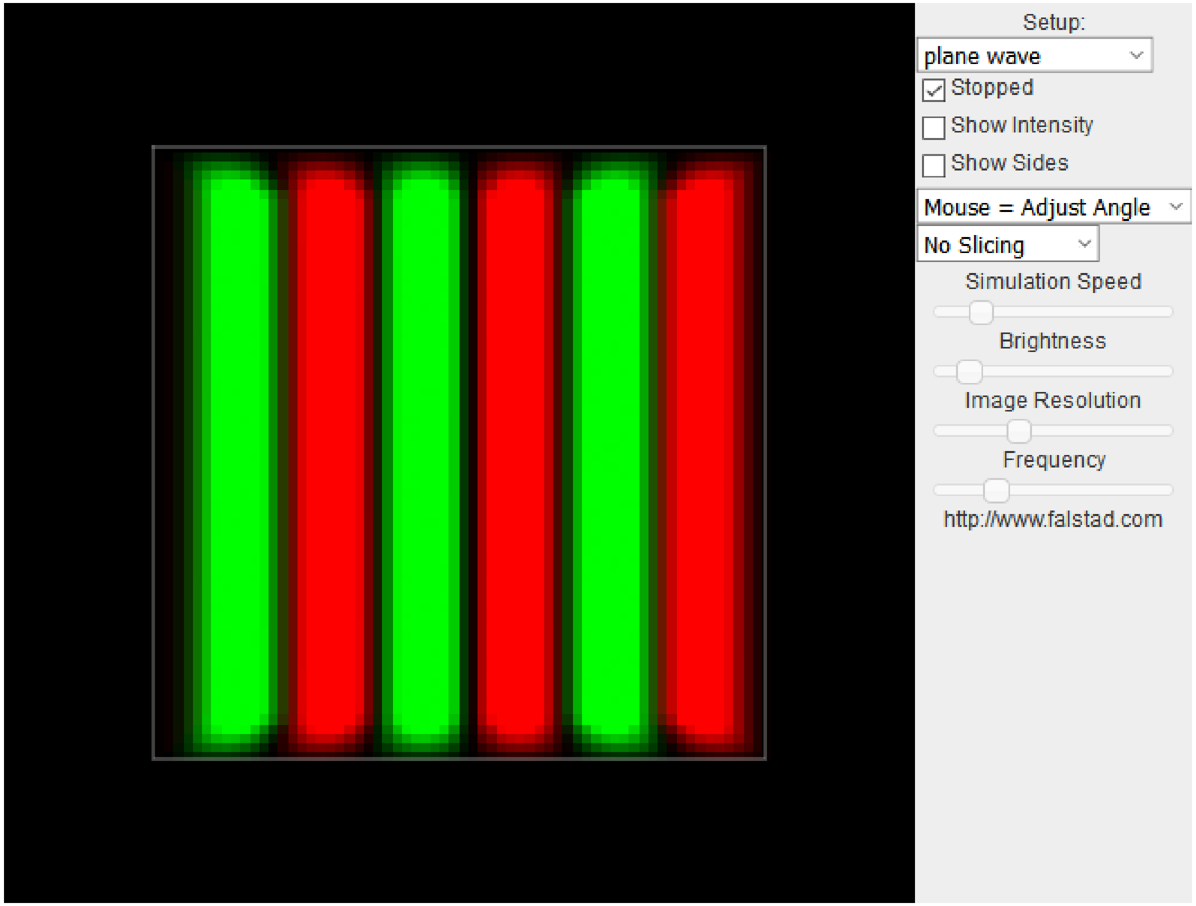
$$\overrightarrow{B(x,y,z)}(z-ct) = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} B_{0x}(z-ct) \\ B_{0y}(z-ct) \\ 0 \end{pmatrix}$$

Draw $\vec{B}$ on this figure





III. 2. Propagation of the EM plane wave in vacuum: Activity



to play the animation on wavebox of the plane wave according to the frequency

III. 2. Propagation of the EM plane wave in vacuum:

Faraday Equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

CLICK HERE

See MemOperator $\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \cdot \overrightarrow{u_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \cdot \overrightarrow{u_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \cdot \overrightarrow{u_z}$

$$\vec{\nabla} \times \overrightarrow{E(x,y,z)} = \begin{pmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{pmatrix} = \begin{pmatrix} -\frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} \\ 0 \end{pmatrix} = \begin{pmatrix} -E_y \\ E_x \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{\partial B_x}{\partial t} \\ -\frac{\partial B_y}{\partial t} \\ 0 \end{pmatrix} = \begin{pmatrix} cB_x \\ cB_y \\ 0 \end{pmatrix}$$

III. 2. Propagation of the EM plane wave in vacuum:

$E_x = cB_y$ and $E_y = -cB_x$

➔ $|E| = c\,|B|$ & $\vec{E} \cdot \vec{B} = 0$ $\Leftrightarrow$ $\vec{E} \perp \vec{B}$

FOR A PLANE WAVE IN VACUUM $\Rightarrow$ $\boldsymbol{\vec{E} \times \vec{B} = cB^2 \overrightarrow{u_z} = \frac{E^2}{c} \overrightarrow{u_z}}$

$\Rightarrow$ **E & B are in the x,y plane**

$\Rightarrow$ **E & B are orthogonals**

III. 2. Propagation of the EM plane wave in vacuum:

Propagation of a transversal wave along increasing **x axis** in vacuum :

$$\overrightarrow{E(x,y,z,t)} = \overrightarrow{E_0(x,y,z)}\,(x-ct) \qquad \overrightarrow{B(x,y,z,t)} = \overrightarrow{B_0(x,y,z)}\,(x-ct)$$

$$\overrightarrow{E(x,y,z)} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ E_y \\ E_z \end{pmatrix}$$ and $$\overrightarrow{B(x,y,z)} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ B_y \\ B_z \end{pmatrix}$$

Relationship between E & B Maxwell Faraday $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

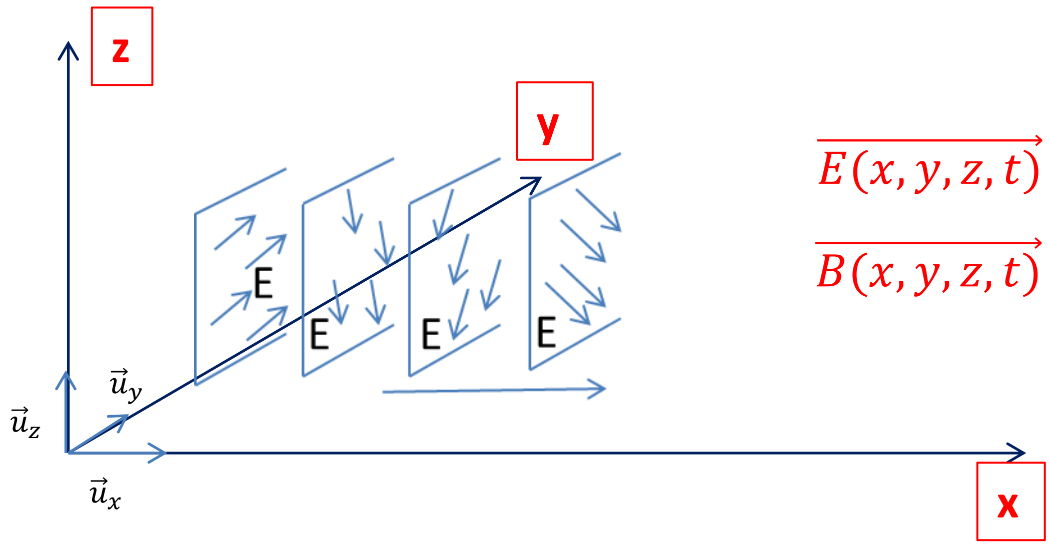
$-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$ and $-\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$ $\Rightarrow E_z = -cB_y$ and $E_y = -cB_z$

$\vec{E}\cdot\vec{B} = 0$

$\Rightarrow |\vec{E}| = c|\vec{B}|$ and $\vec{E}\times\vec{B} = cE^2\overrightarrow{u_x}$

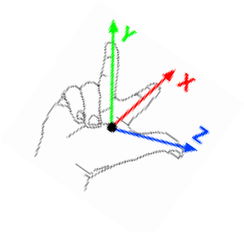
III. 2. Propagation of the EM plane wave in vacuum: Practice

- Give the axis on the cartesian system
- Draw $\vec{B}$ on this figure



$$\overrightarrow{E(x,y,z,t)} = \overrightarrow{E_0(x,y,z)}\,(x-ct)$$

$$\overrightarrow{B(x,y,z,t)} = \overrightarrow{B_0(x,y,z)}\,(x-ct)$$



$\vec{u}_x \times \vec{u}_y = \vec{u}_z$

III.3. Harmonic Solution of the EM Plane Wave in Vacuum

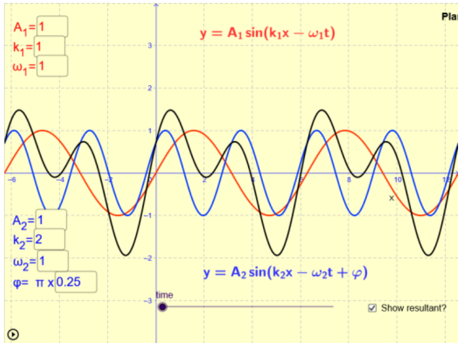
Assuming an harmonic solution allowing:

- Generalization: Fourier domain (linearized), combination of waveform

$$f(x,t) = f_0 cos\left[\omega\left(\pm\frac{x}{c} - t\right)\right] = f_0\, cos[(\pm kx - \omega t\,)]$$

  - ω is the pulsation
  - k is the Wave Number with $k = \frac{\omega}{c}$

[Geogebra activity on LMS: Superposition of two plane wave]



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III.3. Harmonic Solution of the EM Plane Wave in Vacuum

Harmonic solution / Maxwell equation: complex notation

$$f(x,t) = f_0\, e^{[i(\omega t - kx)]} + f_0\, e^{[i(\omega t + kx)]} = f_0 e^{\left[-2i\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)\right]}$$

Propagative travelling wave (x>0)

Contra-propagative travelling wave (x<0)

⚠ According to the sign convention fixed by the observer

$\lambda = \frac{2\pi}{k} = cT$ spatial period or wavelength

$T = \frac{2\pi}{\omega}$ is the period

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

III.3. Harmonic Solution of the EM Plane Wave in Vacuum

$\vec{E} = \overrightarrow{E_0}\cos(kz - \omega t)$ $\vec{B} = \vec{B}_0\cos(kz - \omega t)$

$\overrightarrow{E_0}$ and $\overrightarrow{B_0}$ Constant & Real such as $\overrightarrow{E_0} = \begin{pmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{pmatrix}$ and $\overrightarrow{B_0} = \begin{pmatrix} B_{0x} \\ B_{0y} \\ B_{0z} \end{pmatrix}$

By considering 1 component of the E-Field (1 Dimension) = E-Field Exist only in one direction (x):

$$\vec{E} = \overrightarrow{E_0}\cos(kz - \omega t) = \begin{pmatrix} E_{0x}cos(kz - \omega t) \\ 0 \\ 0 \end{pmatrix}$$

Computation of the wave equation: $\boldsymbol{\Delta\vec{E} - \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} = \vec{0}}$

III.3. Harmonic Solution of the EM Plane Wave in Vacuum

Wave equation $\Delta\vec{E} - \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} = \vec{0}$

$$\Delta\vec{E} = \left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right)\overrightarrow{u_x} + \left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right)\overrightarrow{u_y} + \left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}\right)\overrightarrow{u_z}$$

$$\Delta\vec{E} = \frac{\partial^2 E_x}{\partial z^2}\overrightarrow{u_x} = \mathrm{E_{0x}}\frac{\partial}{\partial z}\left(\frac{\partial cos(kz-\omega t)}{\partial z}\right)\overrightarrow{u_x} = -kE_{0x}\frac{\partial sin\big((kz-\omega t)\big)}{\partial z}\overrightarrow{u_x} = -\mathrm{k}^2\mathrm{E_{0x}}\cos(kz-\omega t)\overrightarrow{u_x}$$

$$\frac{\partial^2\vec{E}}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial\vec{E}}{\partial t}\right) = \frac{\partial}{\partial t}\begin{pmatrix}\frac{\partial E_{0x}cos(kz-\omega t)}{\partial t}\\ 0\\ 0\end{pmatrix} = \omega E_{0x}\frac{\partial sin(kz-\omega t)}{\partial t}\overrightarrow{u_x} = \omega^2\mathrm{E_{0x}}\cos(kz-\omega t)\overrightarrow{u_x}$$

III.3. Harmonic Solution of the EM Plane Wave in Vacuum

$$\vec{\nabla} \times \vec{E} = \begin{pmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\partial E_x}{\partial z} \\ 0 \end{pmatrix}$$

$$\frac{\partial \vec{B}}{\partial t} = -\begin{pmatrix} B_{0x} \frac{\partial cos(kz-\omega t)}{\partial t} \\ B_{0y} \frac{\partial cos(kz-\omega t)}{\partial t} \\ B_{0z} \frac{\partial cos(\omega t-kz)}{\partial t} \end{pmatrix} = \omega sin(kz-\omega t) \begin{pmatrix} B_{0x} \\ B_{0y} \\ B_{0z} \end{pmatrix}$$

$B_{0x} = 0$ and $B_{0x} = 0$

$$\vec{B} = \begin{pmatrix} 0 \\ B_{0y} cos(kz-\omega t) \\ 0 \end{pmatrix}$$

$$\vec{E} \cdot \vec{B} = 0$$

$$E_{0x} \cdot k \cdot sin(kz-\omega t) = B_{0y} \cdot \omega \cdot sin(kz-\omega t)$$

E & B are in Phase

III.3. Harmonic Solution of the EM Plane Wave in Vacuum

E $\perp$ & B$\perp$ To the propagation Axis z

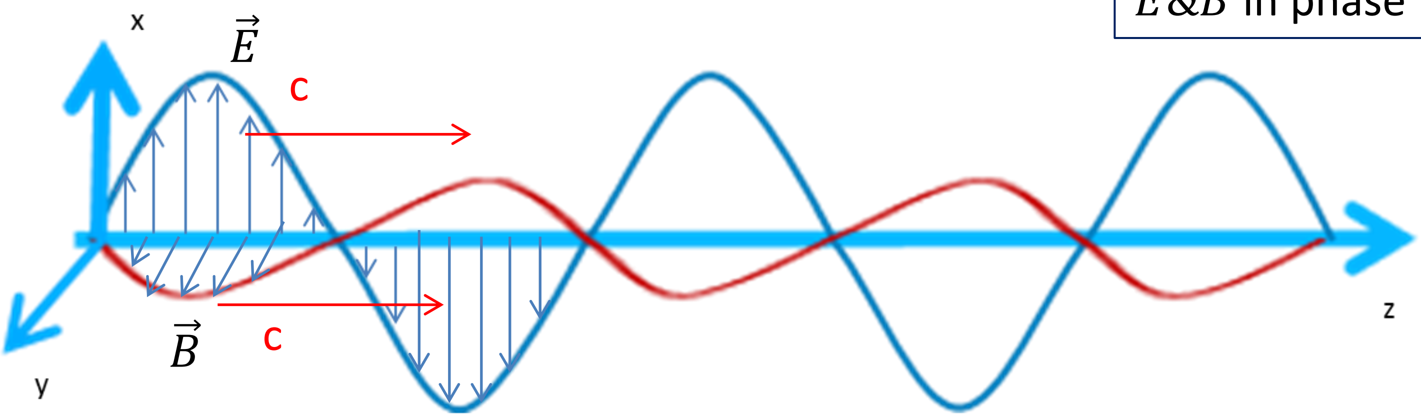
$\vec{E} \times \vec{B}$ in the direction of the propagation Axis z

$$\frac{\omega}{k}|\vec{B}| = |\vec{E}|$$

$$k = \frac{\omega}{c}$$

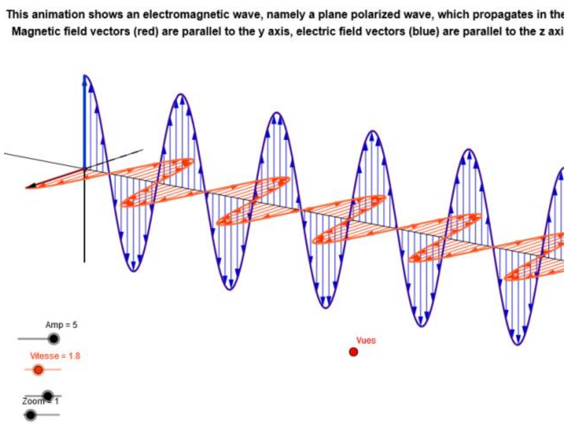
$$c|\vec{B}| = |\vec{E}|$$

$\vec{E}$&$\vec{B}$ in phase



- Geogebra activity

CLICK HERE



- Wave Impedance in vacuum definition : η_0

$$c|\vec{B}| = |\vec{E}| \quad \rightarrow \quad c\mu_0|\vec{H}| = |\vec{E}| \quad \rightarrow \quad \eta_0 = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

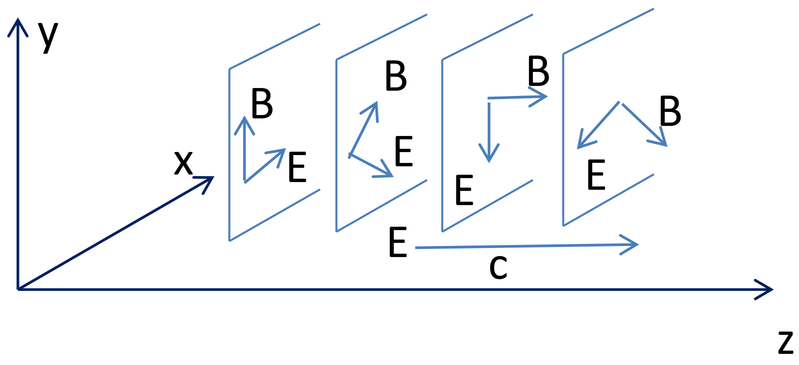
$\varepsilon_0 = 8{,}85418782 \times 10^{-12}$ F m^{-1} $(or\ A^2.s^4.kg^{-1}.m^{-3})$, $\mu_0 = 4\pi \cdot 10^{-7}\ \ H/m\ (or\ kg \cdot m \cdot A^{-2} \cdot s^{-2})$

$$\eta_0 \approx 377\sqrt{\frac{kg{\cdot}m{\cdot}A^{-2}{\cdot}s^{-2}}{A^2.s^4.kg^{-1}.m^{-3}}} \approx 377\ \ m^2 \cdot kg\ \cdot s^{-3} \cdot A^{-2} \approx 377\Omega$$

III.3. Harmonic Solution of the EM Plane Wave in Vacuum

Evolution of the EM field Between 2 Wave Planes (Wavefront) : the behavior of $\vec{E}$ & $\vec{B}$ Field in a transverse plan (x,y) along the propagation axis.

$$E_x \neq 0, E_y \neq 0, B_x, \neq 0\ B_y \neq 0\ , E_z, = 0\ B_z = 0$$



Monochromatic Plane Wave Propagation

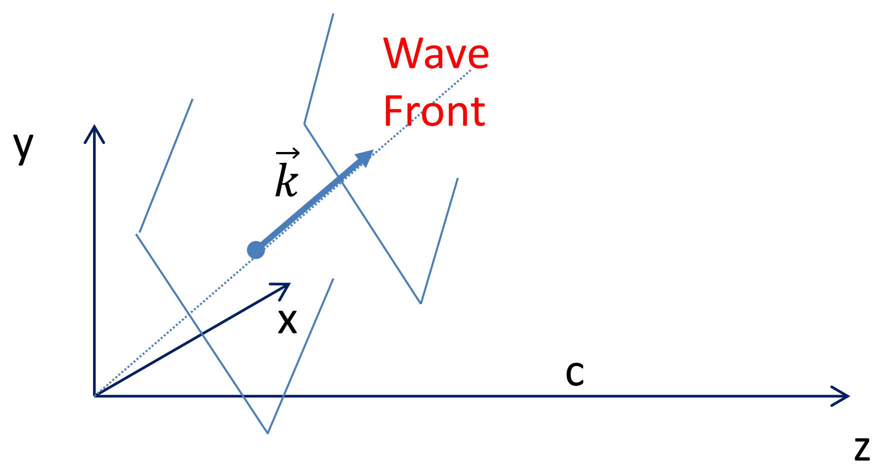
Plane Wave: The Phase of the Wave is the same at all points, on each plane ⊥ to the propagation wave.

1. considering of $\vec{E}$ & $\vec{B}$ Field in a transverse plan (x,y) along the propagation axis.

$E_x \neq 0, \quad E_y \neq 0, \quad B_x \neq 0, \quad B_y \neq 0, \quad E_z = 0, \quad B_z = 0$.

$\vec{E} = \overrightarrow{E_0}\cos(kz - \omega t)$ & $\vec{B} = \vec{B}\cos(kz - \omega t)$

1. In a real system, the plane waves are constructed with arbitrary direction of propagation.



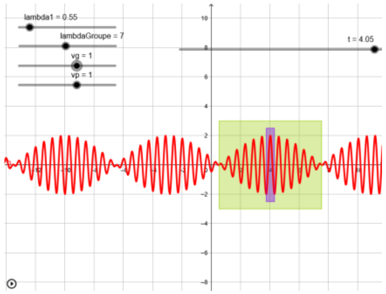
What is the velocity of the travelling wave?

- **Phase velocity:** velocity of the crest of the travelling

- **Group velocity:** velocity of the envelope
  - Due to superposition of travelling waves with different wavelengths

- Geogebra activity on LMS
  - [Phase and group velocity]

CLICK HERE



  - [Group velocity]

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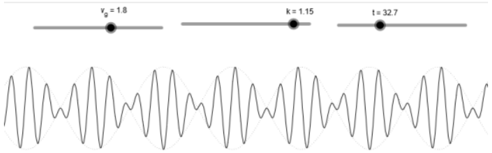


Velocity of the travelling wave : Phase Velocity

Phase velocity: velocity of the crest of the travelling wave

$$\vec{E} = \overrightarrow{E_0}\cos(kz - \omega t)$$

$\omega t - kz$ is constant such as $\frac{d(kz-\omega t)}{dt} = 0$

$k\frac{dz}{dt} - \omega\frac{dt}{dt} = \frac{\omega}{c}\frac{dz}{dt} - \omega = 0$ $\rightarrow \frac{dz}{dt}$ $=\boldsymbol{v_\varphi}$= c

In vacuum, $\boldsymbol{v_\varphi = c}$

For Two travelling waves with different wavelenghts

$\overrightarrow{E_1} = \overrightarrow{E_0}\cos(k_1 z - \omega_1 t)$ with $v_\varphi = \frac{\omega_1}{k_1}$

$\overrightarrow{E_2} = \overrightarrow{E_0}\cos(k_2 z - \omega_2 t)$with $v_\varphi = \frac{\omega_2}{k_2}$

III.4. Velocity of the travelling wave

Group velocity : velocity of the envelope due to the superposition of travelling waves with different wavelenghts

$$\overrightarrow{E_T} = \overrightarrow{E_1} + \overrightarrow{E_2} = \overrightarrow{E_0}\cos(k_1 z - \omega_1 t) + \overrightarrow{E_0}\cos(k_2 z - \omega_2 t)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$\overrightarrow{E_T} = \overrightarrow{E_1} + \overrightarrow{E_2} = 2\overrightarrow{E_0}\cos\left(\frac{k_1 - k_2}{2} z - \frac{\omega_1 - \omega_2}{2} t\right)\cos\left(\frac{k_1 + k_2}{2} z - \frac{\omega_1 + \omega_2}{2} t\right)$$

III.4. Velocity of the travelling wave

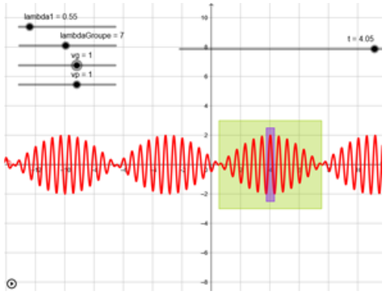
envelope carrier

$$\overrightarrow{E_T} = \overrightarrow{E_1} + \overrightarrow{E_2} = 2\overrightarrow{E_0}\cos\left(\frac{k_1 - k_2}{2}z - \frac{\omega_1 - \omega_2}{2}t\right)\cos\left(\frac{k_1 + k_2}{2}z - \frac{\omega_1 + \omega_2}{2}t\right)$$

Assume $k_1 \approx k_2 \approx k$ and $\omega_1 \approx \omega_2 \approx \omega$ ➔ 2 velocities : $v_\varphi = \frac{\omega}{k}$ and $v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{d\omega}{dk}$

The carrier is the higher frequency signal : $f_1 + f_2$

The envelop is the lower frequency signal: $f_1 - f_2$



III.4. Velocity of the travelling wave

What is the velocity of the travelling wave?

$$\overrightarrow{E_T} = \overrightarrow{E_1} + \overrightarrow{E_2} = 2\overrightarrow{E_0}\cos\left(\frac{k_1-k_2}{2}z - \frac{\omega_1-\omega_2}{2}t\right)\cos\left(\frac{k_1+k_2}{2}z - \frac{\omega_1+\omega_2}{2}t\right)$$

Assume $k_1 \approx k_2 \approx k$ and $\omega_1 \approx \omega_2 \approx \omega$ ➔ 2 velocities : $v_\varphi = \frac{\omega}{k}$ and $v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{d\omega}{dk}$

In the current case, v_φ is the velocity of the carrier and v_g is the velocity of the modulating signal.

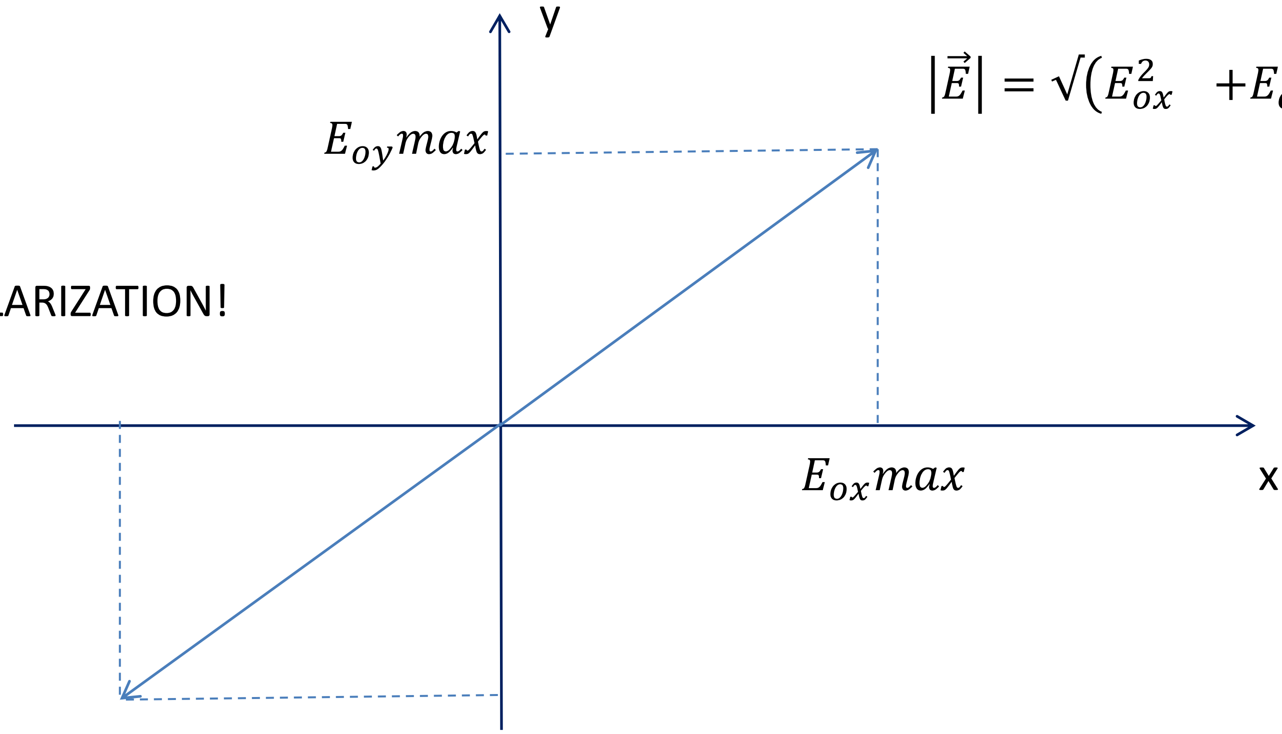
v_g is the velocity of the group (or packet) and corresponds to the energy flow.

III.5 Polarization of the EM Wave: Linear Polarization

$$\vec{E} = \begin{pmatrix} E_{0x}\cos(\omega t - kz) \\ E_{0y}\cos(\omega t - kz + \varphi) \\ 0 \end{pmatrix}$$

$\varphi = 0$

LINEAR POLARIZATION!



$$\left|\vec{E}\right| = \sqrt{\left(E_{ox}^2 \quad +E_{oy}^2 \quad\right)}$$

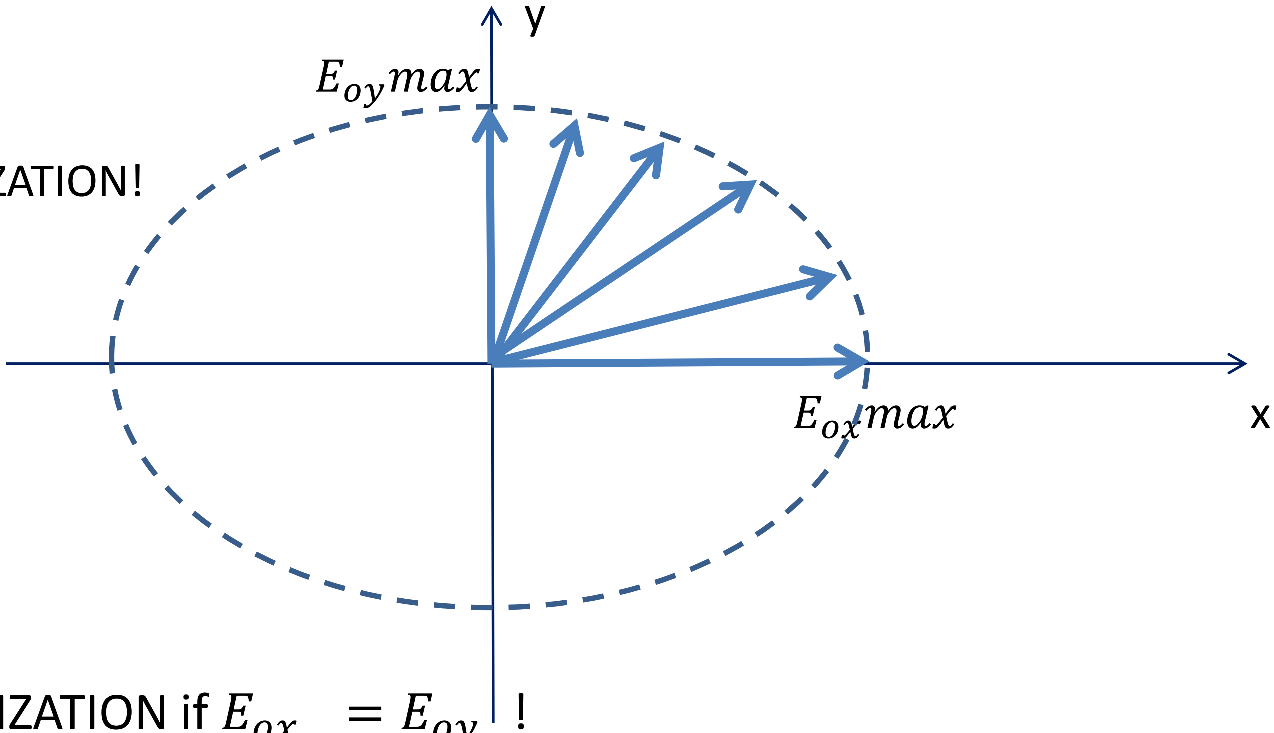
III.5 Polarization of the EM Wave: Elliptical Polarization

$$\varphi = \frac{\pi}{2}$$

$$\vec{E} = E_{ox}\ \ cos(\omega t - kz)\overrightarrow{e_x} + E_{oy}\ \ cos\left(\omega t - kz + \frac{\pi}{2}\right)\overrightarrow{e_y}$$

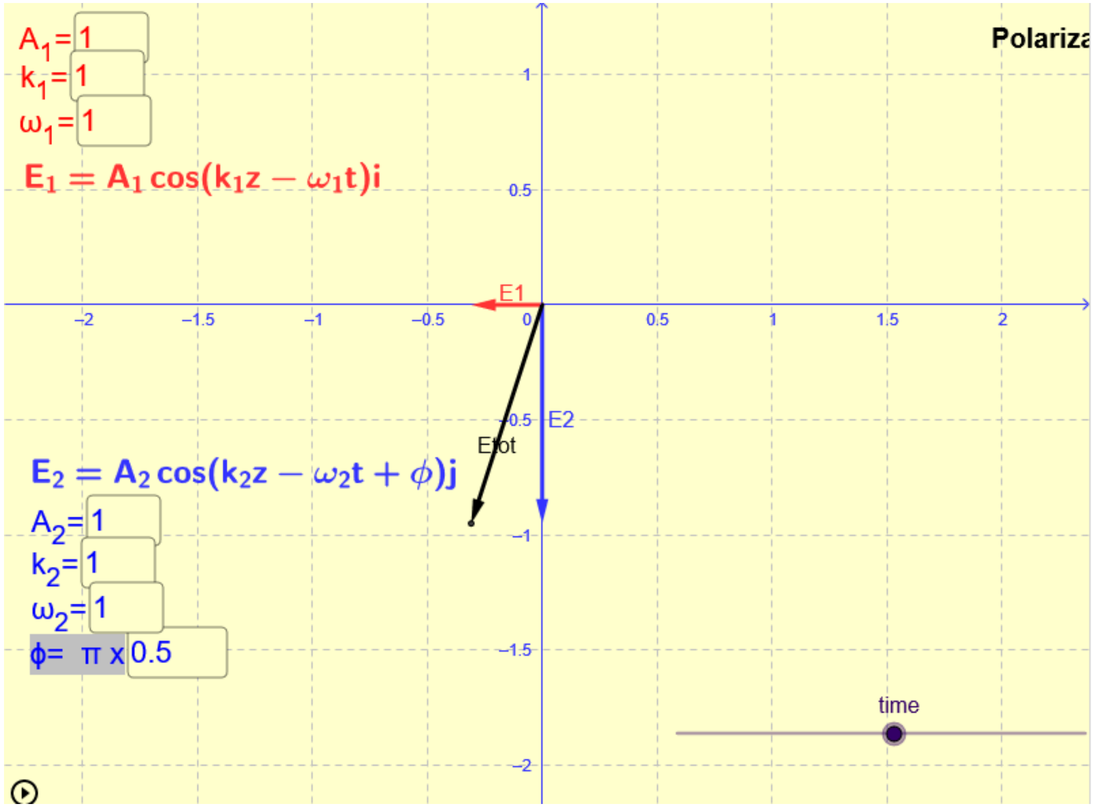
$$\vec{E} = E_{ox}\ \ cos(\omega t - kz)\overrightarrow{e_x} + E_{oy}\ \ sin(\omega t - kz)\overrightarrow{e_y}$$

ELLIPTICAL POLARIZATION!



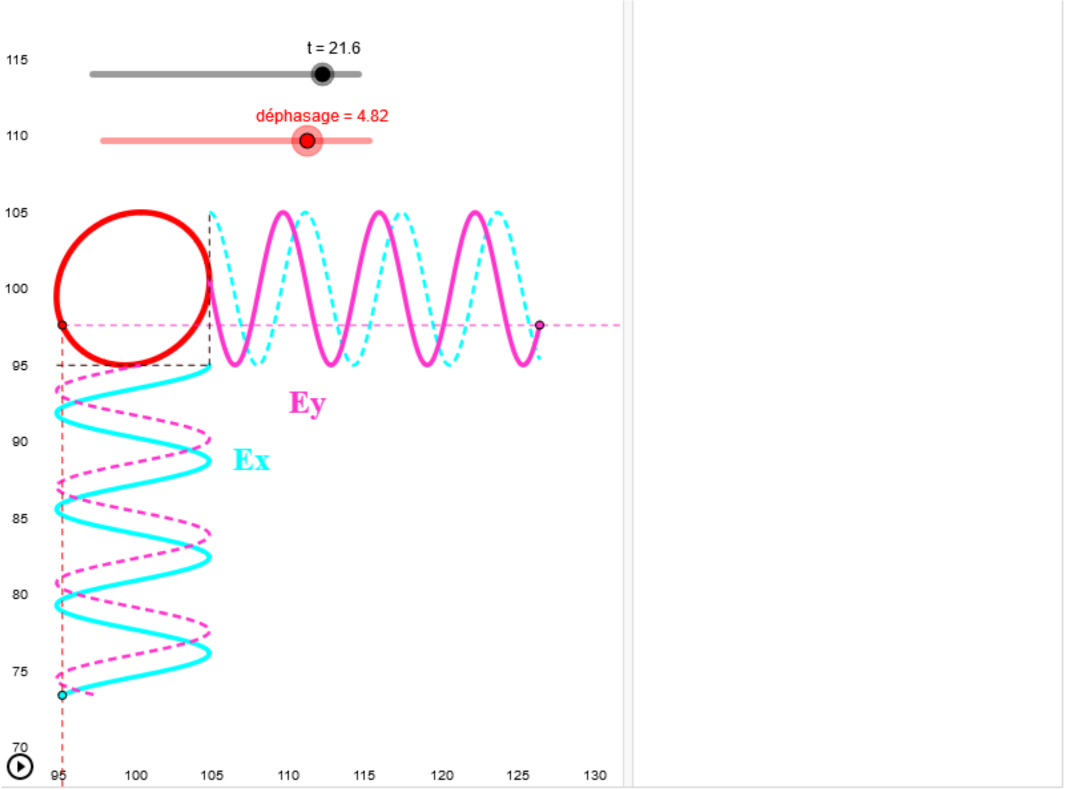
CIRCULAR POLARIZATION if $E_{ox} = E_{oy}$!

- Geogebra activity on LMS : Change the value of ϕ and observe the animation according to the time



III.5 Polarization of the EM Wave: Activity

- Geogebra activity, change the phase between the component X and Y of the E field to draw linear, elliptical and circular wave

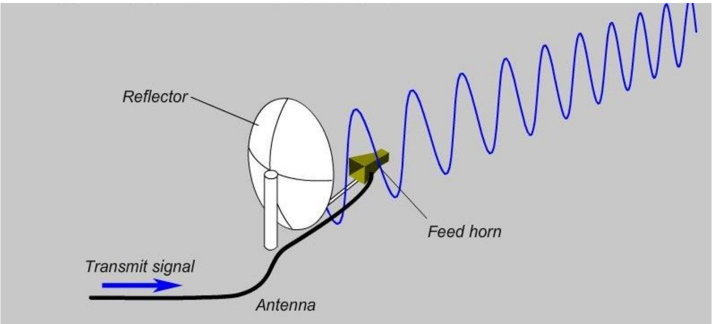


III.5 Polarization of the EM Wave: Application in Microwave

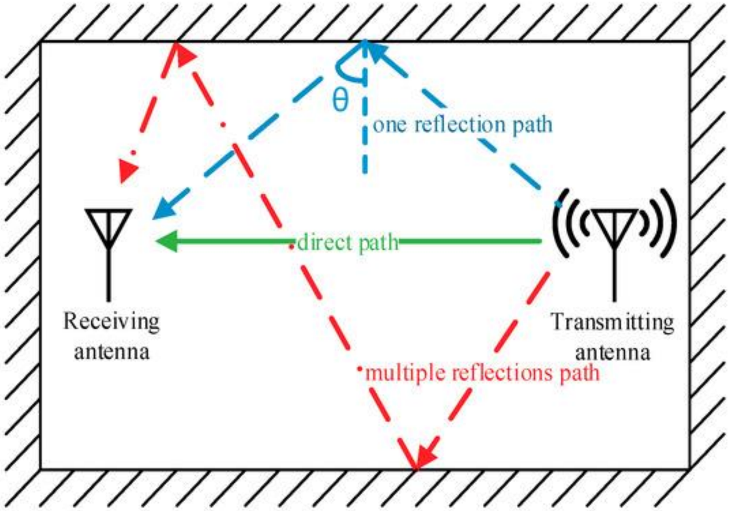
Wave's polarisation

- **When use LP or/and CP plane wave in RF?**

➢ **LP (Linearly polarized)**
Emitting and receiving antenna systems fixed.



➢ **CP (Circularly polarized)**
Receiver mobile, in multipath and fading environments and in communication with space vehicles above the earth's ionosphere (see later in dispersive media).

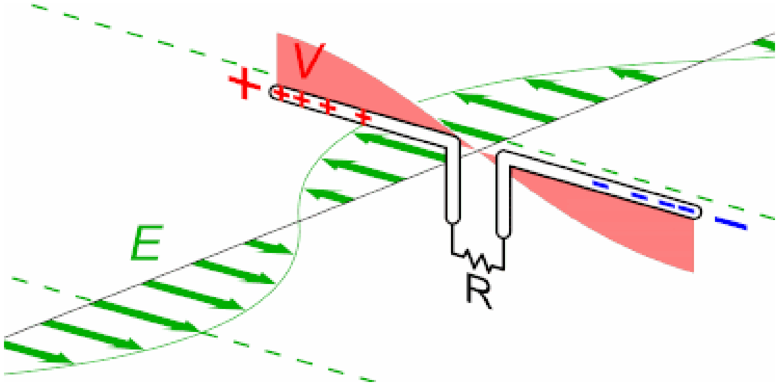


➢ **Several polarizations**: Polarization reconfigurability to improve the quality of the wireless link, optimize the link reliability

Wave's polarisation

How is created the electromagnetic wave?

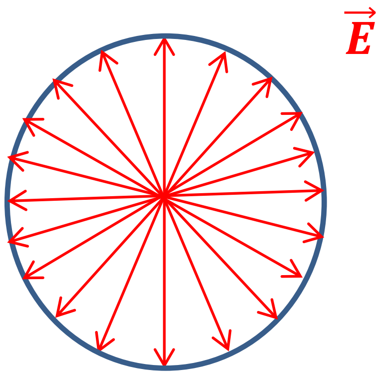
- **By antenna (to be studied the next semester)**

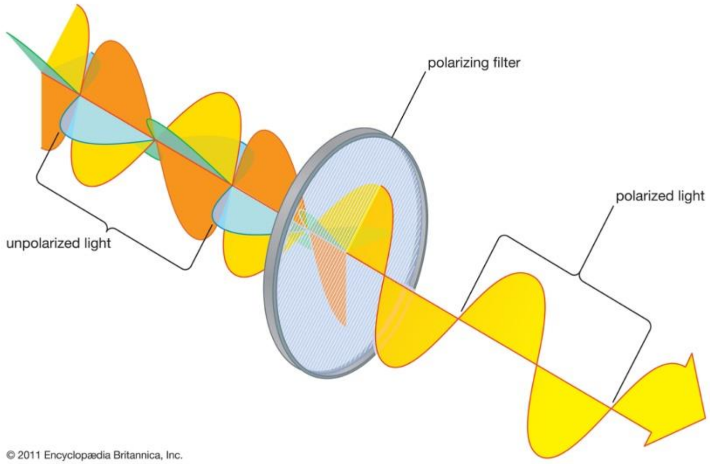


Linearly polarized antenna

https://fr.wikipedia.org/wiki/Antenne_dipolaire

A light source is not polarized. To control the direction of the E field, polarizer (film or crystal) and polarisation controller (fiber optics or crystal).





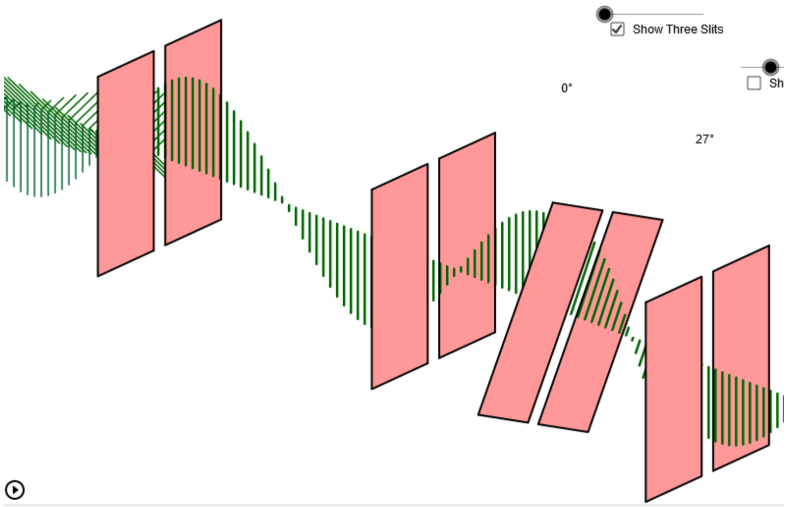
https://www.britannica.com/science/light/Unpolarized-light

The polarization of the light is very sensitive whatever the source and the application. A control of polarization is required to maintain the properties of the optical beam but has an impact on the total field energy propagation especially in photonics (fiber optics communication, quantum optics, optical sensing).

III.5 Polarization of the EM Wave: Activity

What is the polarization of the light if the second polarizer is horizontal ?

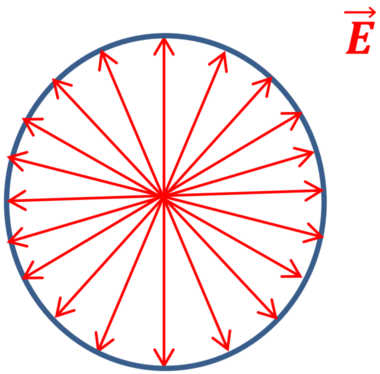
- [Check you answer with geogebra activit]

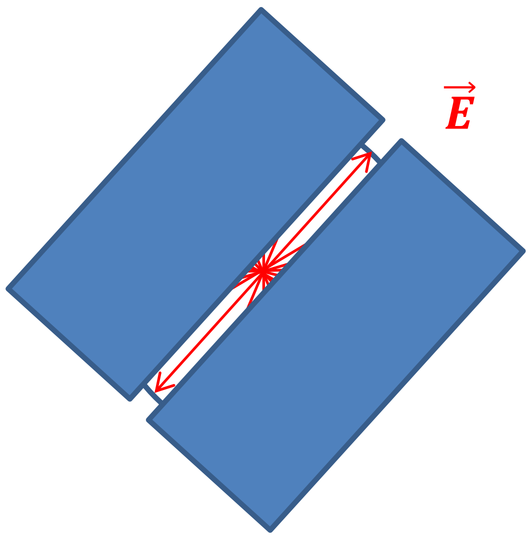


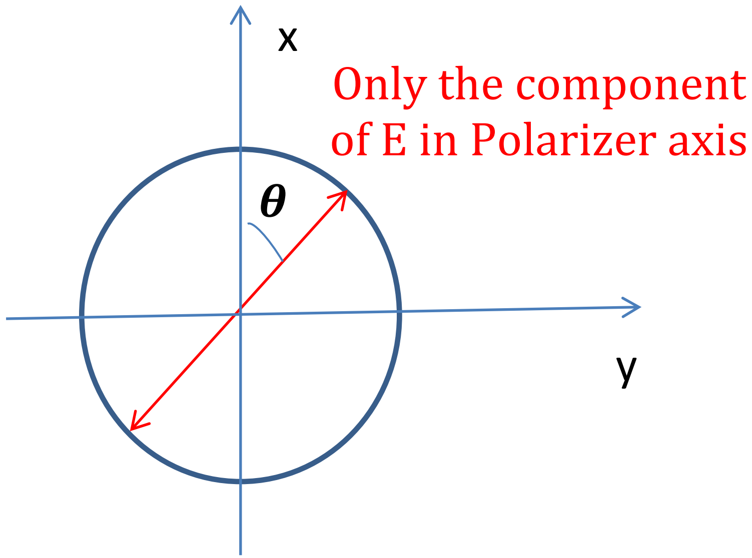
Fiber optic polarization controller
(twist the fiber)

III.5 Polarization of the EM Wave: Application in optics

The polarized optical field follows the Malus Law:







$|E| = E_0 \cos\theta$

The intensity of the field is : $I = |E|^2$

$\boxed{\text{From the Malus Law: } I \sim {E_0}^2 cos^2\theta}$

III. Conclusion **Plane Wave in Vacuum**

From Maxwell Equation in vacuum (without charge and current), **Wave Equation**

$$\Delta\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Delta\vec{B} = \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$$

IN VACUUM

The wave number is $k = \frac{\omega}{c}$

From Maxwell Equation : $\vec{E}\perp\vec{k}$, $\vec{B}\perp\vec{k}$

$\vec{E}\perp\vec{B}$

$c|\vec{B}| = |\vec{E}|$

Wave impedance in vacuum

$\eta_0 = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$

Harmonic wave

Wavelength $\lambda = \frac{2\pi}{k} = cT$

Period $T = \frac{2\pi}{\omega}$

Frequency $f = \frac{\omega}{2\pi}$

Velocity

Light speed : $c \approx 3 \cdot 10^8 \mathrm{m/s}$

Phase velocity $v_\varphi = \frac{\omega}{k}$

Group velocity $v_g = \frac{d\omega}{dk}$

Polarization

Linear polarization: $\varphi = 0 \ or \ \pi$

Circular polarization :

$\varphi = \pm\frac{\pi}{2}$ and $E_{ox} = E_{oy}$

Elliptical polarization: each other

III. Conclusion plane Wave in Vacuum

Polarization

Malus Law, Intensity $I \sim {E_0}^2 cos^2\theta$

How can we propagate energy with an electromagnetic Field?

Part IV