ISAE Lab class on Hybrid Systems #2
February 16th, 9:15AM-11:30AM

Consider the implementation of a static controller

$$\kappa : \mathbb{R}^n \to \mathbb{R}^m$$

for the continuous-time plant

$$\dot{z} = f_p(z, u)$$

in a *digital device*, e.g. computer, microcontroller, digital signal processor, etc. This is depicted in Figure 1, where the controller is interfaced with sample-and-hold devices. The sample-and-hold device that samples the state ξ of the plant is referred to as *sampling device* (or analog-to-digital (A/D) converter), while the sample-and-hold device that stores the output of the controller in between computations is referred to as *hold device* (or digital-to-analog (D/A) converter) which is assumed to be of zero-order type, that is, a zero-order hold (ZOH).

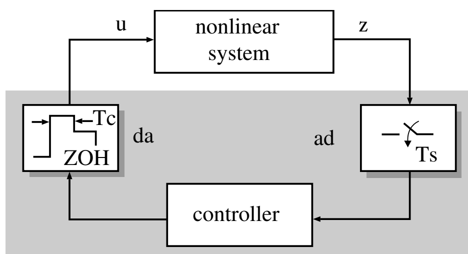

Figure 1: Sample-and-hold control of a nonlinear system.

1. Build a sampling device

1.a) Define each element of a hybrid system model (C, f, D, g) and its state which describe a sampling device.

Remember that the output remains constant and is updated to the value of the input every T_s seconds.

1.b) Build the simulink model of the sampling device and test it with $T_s = 0.25$ and a sinusoidal signal of amplitude 2 and frequency 1 rad/sec.

Do not forget to build also the init file.

2. Build the sample-and-hold closed-loop control system of Figure 1,

2.a) Define each element of a hybrid system model (C, f, D, g) and its state assuming the following:

- The computation of the static feedback law takes no time, i.e., is instantaneous.
- The positive constants T_s and T_c are not necessarily equal.

Build each element then compose the extended state vector to obtain the flow and jump dynamics and associated flow and jump sets

2.b) Explain how the model would change if the computation of the control law takes $\delta > 0$ units of time.

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3. Simulate

Consider a simplified version with only a sampling device (this also corresponds to the case where $T_c = T_s$). Let the system and controller given by:

$$f_p(z, u) = Az + Bu \ , \ \kappa(z_s) = Kz_s$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \ , \ B = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \text{ and } K = \begin{bmatrix} 13 & 7 \end{bmatrix}$$

3.a) Build the simulink model of sample-and-hold control system and test it with $z(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $T_s = 0.2$ then $T_s = 0.28$. Discuss the trajectories.

Consider a simplified version of equations obtained in question 2.a to build the simulink model with the hybrid integrator.

4. Stability analysis

4.a) To study the stability of the system addressed in section 3, what would be an attractor of interest?

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4.b) Let us consider the Lyapunov function

$$V_1(x) = x_z^\top (e^{A_f(T_s - \tau_s)})^\top P e^{A_f(T_s - \tau_s)} x_z$$

where P is a symmetric positive definite matrix solution to the equation $H^\top P H - P < 0$, with $H = e^{A_f T_s} A_g$. Which stability conditions may be proven with this function (UGS, UGAS)?

A_f and A_g correspond to the flow and jump dynamics of the subsystem related to the states of the system and controller. You have to compute $\dot{V}_1(x)$ and $\Delta V_1(x)$ along the trajectories of the hybrid system.

4.c) Let us modify the Lyapunov function as follows

$$V_2(x) = e^{-\sigma \tau_s} V_1(x)$$

with $\sigma > 0$ small enough. Is-it possible to prove the UGAS of the attractor?

Go on computing $\dot{V}_2(x)$ and $\Delta V_2(x)$.