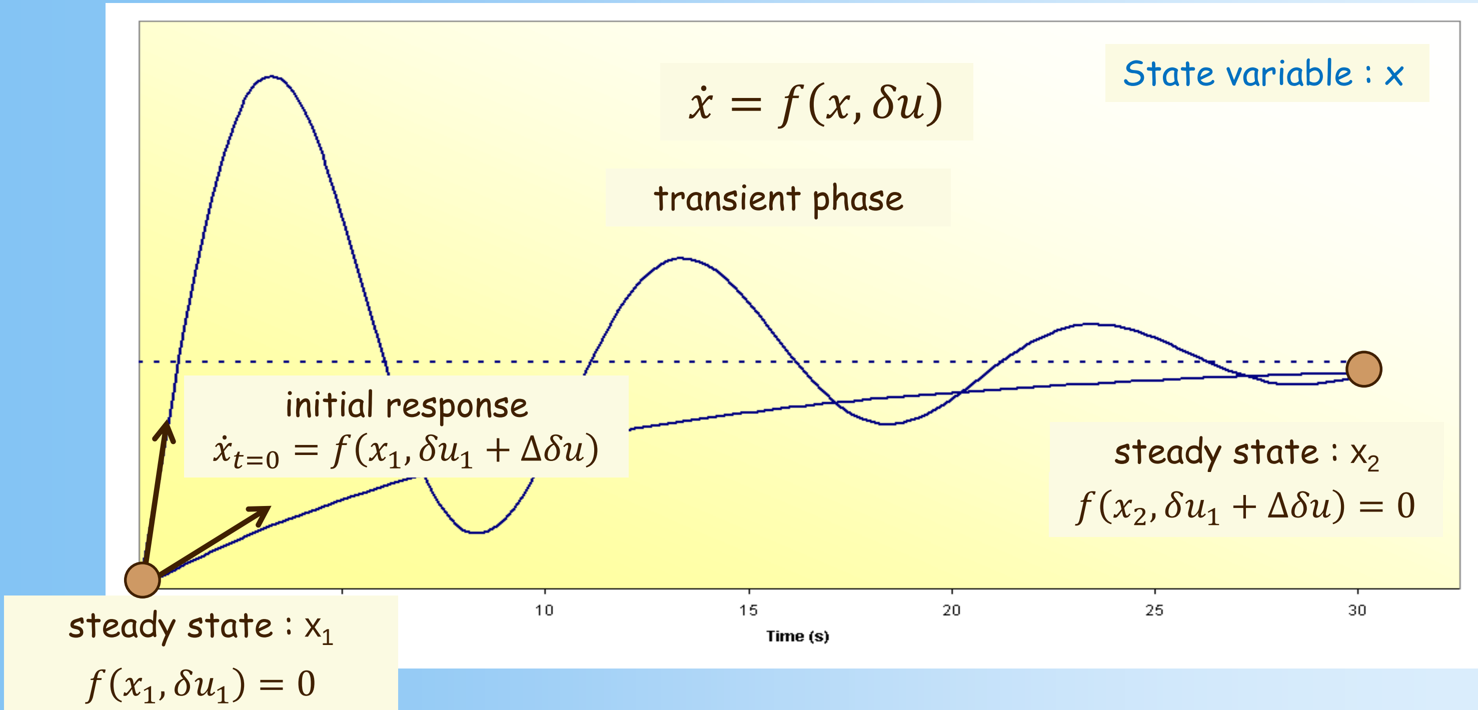
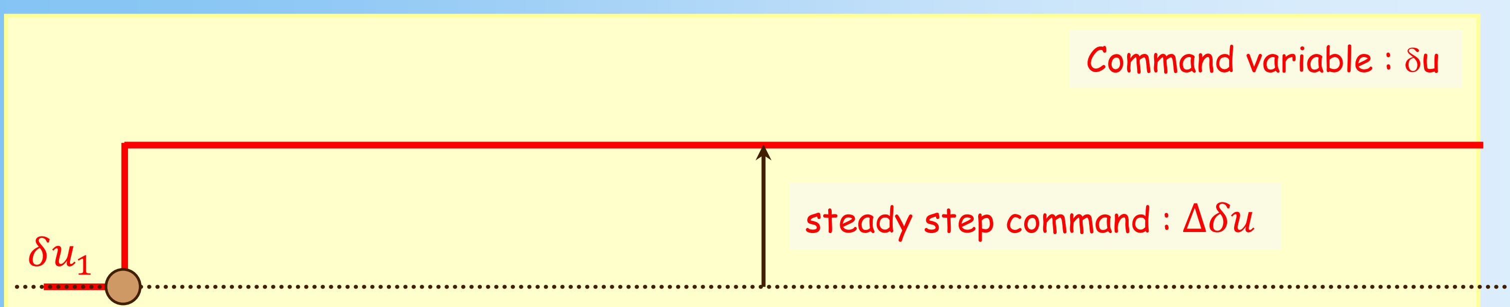
Longitudinal Dynamics Introduction

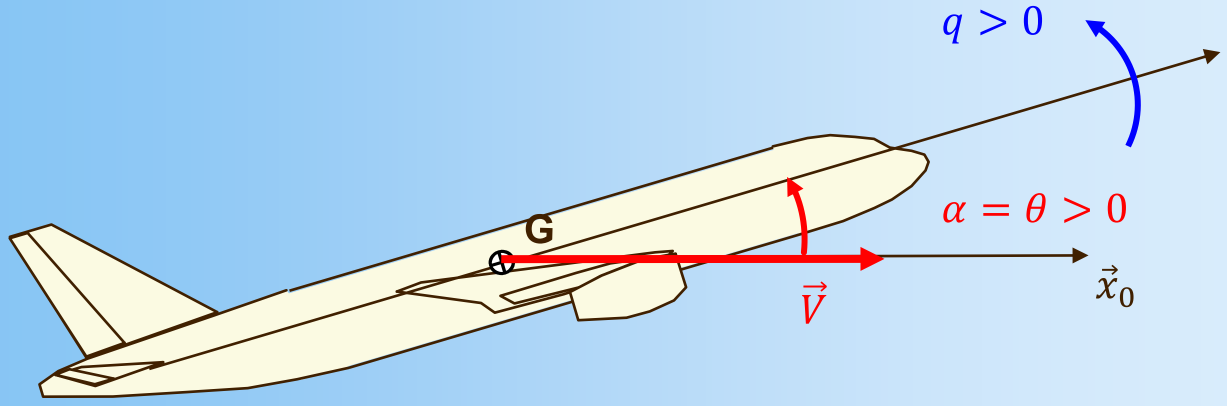
Boeing Blended Wing Body Model

Response to Steady Step Command





Short Period Mode : simplified approach



The aircraft is just allowed to rotate around its Centre of gravity G with a pitch rate q

The velocity $\vec{V}$ is a constant vector.

We assume that G is always in the same horizontal plane

- the aircraft velocity is within the local horizontal plane : $\gamma = 0°$
- the angle of attack follows the pitch angle : $\alpha = \theta \rightarrow \dot{\alpha} = q$

Short Period Mode : simplified approach

We want to solve the differential equations set :

$$\dot{\alpha} = q$$
$$\dot{q} = \frac{\rho V^2 SL}{2B} \cdot Cm_G$$

Starting from an equilibrium, we consider small variations $(\delta\alpha, \delta q = q)$, by differentiation :

$$\delta\dot{\alpha} = q$$
$$\dot{q} = \frac{\rho V^2 SL}{2B} \cdot \delta Cm_G = \frac{\rho V^2 SL}{2B} \cdot \left(Cm_\alpha^G \cdot \delta\alpha + Cm_q \cdot \frac{qL}{V} \right)$$

$$\begin{cases} m_\alpha^G = \dfrac{\rho V^2 SL}{2B} \cdot Cm_\alpha^G \\ m_q = \dfrac{\rho V S L^2}{2B} \cdot Cm_q \end{cases}$$

$$\delta\dot{\alpha} = q$$
$$\dot{q} = m_\alpha^G \cdot \delta\alpha + m_q \cdot q$$

$$\Rightarrow$$

$$\delta\dot{X} = A \cdot \delta X$$

$$\left| \begin{matrix} \delta\dot{\alpha} \\ \dot{q} \end{matrix} \right. = \begin{bmatrix} 0 & 1 \\ m_\alpha^G & m_q \end{bmatrix} \cdot \left| \begin{matrix} \delta\alpha \\ q \end{matrix} \right.$$

Solving a 1st order differential equation

We have to solve a 1st order differential equation with constant coefficients : $\delta\dot{X} = A \cdot \delta X$

The general solution is given by : $\delta X = \delta X_0 \cdot e^{st}$

where $(\delta X_0 \, , s)$ are a couple of complex vector and number.

$$\delta X = \delta X_0 \cdot e^{st} \rightarrow \delta\dot{X} = \delta X_0 \cdot s \cdot e^{st} = s \cdot \delta X$$

$$\delta\dot{X} = A \cdot \delta X = s \cdot \delta X$$

$$\Downarrow$$

$$[A - s \cdot I] \cdot \delta X = 0$$

$$\Downarrow$$

$$[A - s \cdot I] \cdot \delta X_0 \cdot \cancel{e^{st}} = 0$$

$$\Downarrow$$

$$[A - s \cdot I] \cdot \delta X_0 = 0$$

Solving a 1st order differential equation

The general soluti

SUPAERO

The general solution of $\delta\dot{X} = A \cdot \delta X$ is given by $\delta X = \delta X_0 \cdot e^{st}$ where the variable s

is obtained by the resolution of $[A - s \cdot I] \cdot \delta X_0 = 0$

We have a linear system of 2 equations with 2 unknowns to solve
There are 2 possibilities :

- either $det[A - s \cdot I] \neq 0$
  $\rightarrow$ unique solution : $\delta X_0 = 0 \rightarrow \delta X = 0$; obvious solution no interest

- either $det[A - s \cdot I] = 0$
  $\rightarrow$ infinity of solutions : $\delta X_0 \neq 0 \rightarrow \delta X \neq 0$

The condition : $det[A - s \cdot I] = 0$ is equivalent that the s numbers are the eigenvalues of the A matrix and consequently, the δX_0 the associated eigenvectors

Conclusion : solving the differential matricial equation : $\delta\dot{X} = A \cdot \delta X$ consists to find the s-eigenvalues of the A matrix given by

$$det[A - s \cdot I] = 0$$

Short Period Mode : simplified approach

We compute the s eigenvalues of the A matrix :

$$A = \begin{bmatrix} 0 & 1 \\ m_\alpha^G & m_q \end{bmatrix} \Rightarrow det[A - s \cdot I] = det \begin{bmatrix} -s & 1 \\ m_\alpha^G & m_q - s \end{bmatrix} = 0$$

$$det[A - s \cdot I] = 0$$

$$s^2 - m_q \cdot s - m_\alpha^G = 0$$ expressed as $$s^2 + 2\lambda \cdot s + \omega_0^2 = 0 \quad \begin{cases} 2\lambda = -m_q \\ \omega_0^2 = -m_\alpha^G \end{cases}$$

The nature of the s-eigenvalues (real or complex) depends on the sign of the discriminant : Δ'

for G forward, $$\Delta' = \lambda^2 - \omega_0^2 = m_\alpha^G + \frac{m_q^2}{4} = -\omega_n^2 < 0$$

We admit that the discriminant is negative ; both s-eigenvalues are complex conjugate

$$s \,/\, \bar{s} = -\lambda \pm i \cdot \sqrt{-\Delta'} = -\lambda \pm i \cdot \omega_n$$

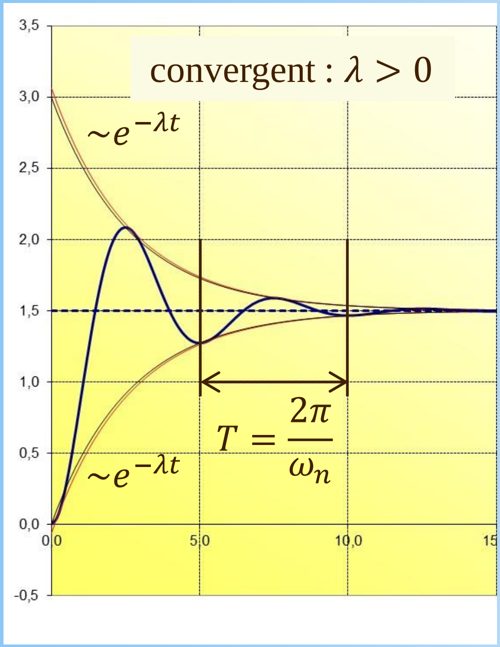
Solving a 1st order differential equation

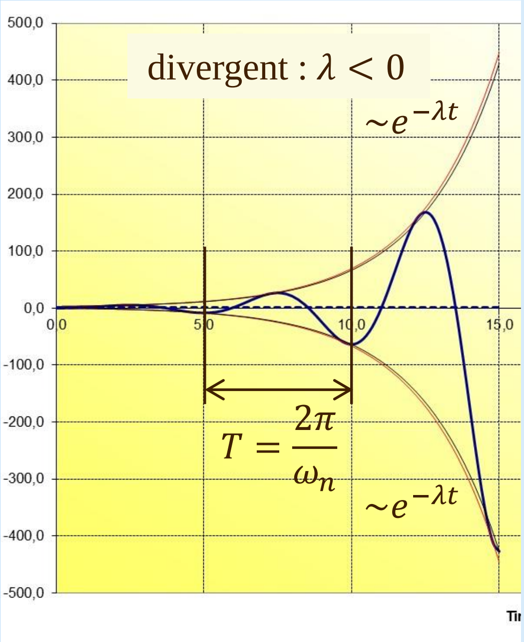
The s-eigenvalues are complex, the solution on the form $\delta X = \delta X_0 \cdot e^{st}$ are periodic function of the time

$$s \,/\, \bar{s} = -\lambda \pm i \cdot \omega_n \Rightarrow \delta X \sim e^{-\lambda t} \cdot (\delta X_1 \cos \omega_n t + \delta X_2 \sin \omega_n t)$$

How

Depending on the sign of λ, the oscillation will be convergent or divergent





Short Period Mode : simplified approach

The Short Period Mode is an eigen mode of the Longitudinal Dynamics linked to (α, q)

With this simplified approach, for G forward

- The 2 eigenvalues are complex and conjugate
- The motion is oscillatory :

Why can we negate lambda

The period is **directly** linked to the aircraft stability (*) : $T = \dfrac{2\pi}{\omega_n} \approx \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{\sqrt{-m_\alpha^G}}$

The damping process is linked to the Cm_q coefficient : $\lambda = -\dfrac{m_q}{2} > 0$

- The motion is a convergent oscillation

(*) the aircraft is stable : $m_\alpha^G < 0$

Short Period Mode : simplified approach

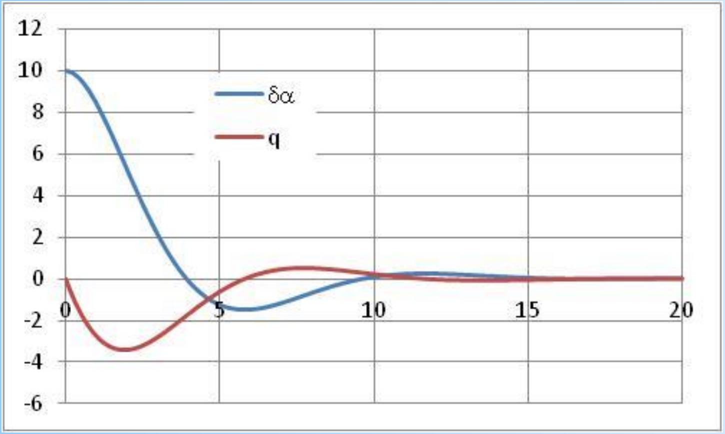
For a stable aircraft (G forward to F) the mode is oscillating / convergent : the more stable the aircraft , the smaller the period T , the higher the pulsation

$$T \approx 2\pi/\omega_0 = 2\pi/\sqrt{-m_\alpha^G}$$

The damping process is coming from the Cm_q (rotation around G).

G forward wrt F : aircraft stable

$m_\alpha^G < 0$

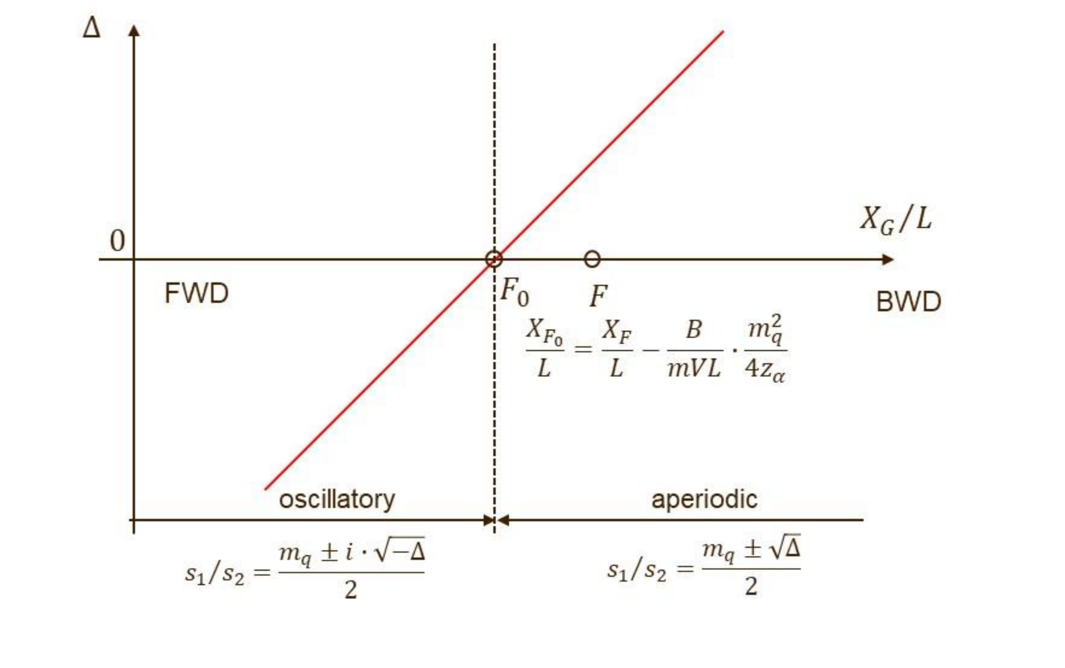


Short Period mode versus CG

$$\boxed{s^2 - m_q \cdot s - m_\alpha^G = 0}$$

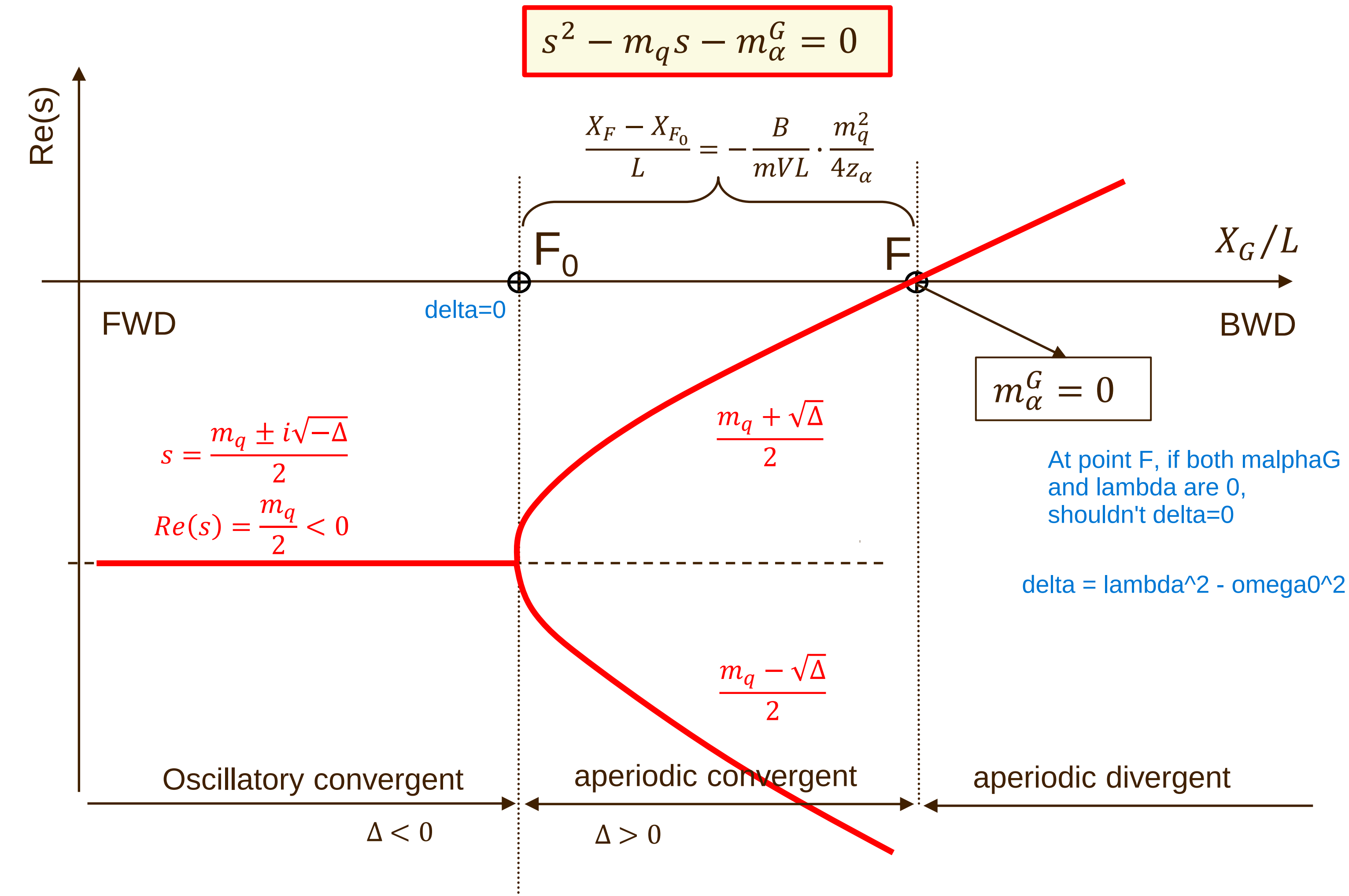
$$\boxed{\Delta = m_q^2 + 4 \cdot m_\alpha^G}$$

$$\begin{cases} m_\alpha^G = \dfrac{\rho V^2 SL}{2B} \cdot Cm_\alpha^G = \dfrac{mVL}{B} \cdot \dfrac{X_G - X_F}{L} \cdot z_\alpha \\ m_q = \dfrac{\rho VSL^2}{2B} \cdot Cm_q \end{cases}$$



The Short Period mode ceases to be oscillatory for a CG before the F point

Short Period mode versus CG



Short Period mode versus CG

The upper real branch $\frac{m_q+\sqrt{\Delta}}{2}$ cuts the x-absciss when $m_\alpha^G = 0$, when $G = F$

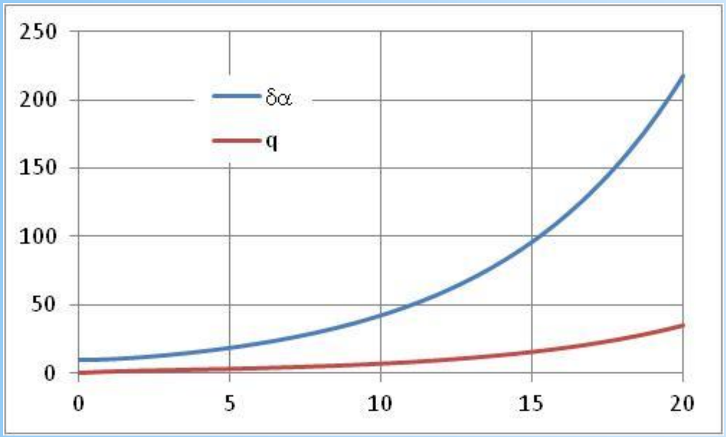
The Short Period mode becomes divergent when $G = F$

The neutral point F is the limit of the convergence for the Short Period mode

The divergence of the Short Period mode is non-periodic

G backward wrt F : aircraft unstable

$m_\alpha^G > 0$



Longitudinal Dynamics Results

Tupolev 160 “Blackjack”

Small state variables variation principle

$$\dot{V} = -g \cdot \gamma - \frac{\rho V^2 S}{2m} \cdot Cx + \frac{F}{m}$$

$$\dot{\gamma} = -\frac{g}{V} + \frac{\rho V S}{2m} \cdot Cz$$

$$\dot{\alpha} = q - \dot{\gamma} = q + \frac{g}{V} - \frac{\rho V S}{2m} \cdot Cz$$

$$\dot{q} = \frac{\rho V^2 S L}{2B} \cdot Cm_G$$

Starting from an equilibrium, we consider small variations ($\delta V, \delta\gamma, \delta\alpha, \delta q = q$) obtained by small command variations ($\Delta\delta x, \Delta\delta m$) ; this is obtained by differentiating the flight equations :

$$\delta\dot{\gamma} = \boxed{\frac{g}{V^2}}\delta V + \boxed{\frac{\rho S Cz}{2m}} \cdot \delta V + \frac{\rho V S}{2m} \cdot \delta Cz = \frac{2g}{V^2}\delta V + \frac{\rho V S}{2m} \cdot \delta Cz$$

$$mg = \frac{1}{2}\rho V^2 S Cz \rightarrow \frac{\rho S Cz}{2m} = \frac{g}{V^2}$$

Small state variables variation principle

$$\delta\dot{V} = -\frac{\rho VSCx}{m}\cdot\delta V - g\cdot\delta\gamma - \frac{\rho V^2 S}{2m}\cdot\delta Cx + \frac{F_V\cdot\delta V + F_0\cdot\Delta\delta x}{m}$$

$$\delta\dot{\gamma} = \frac{2g}{V^2}\delta V + \frac{\rho VS}{2m}\cdot\left[Cz_\alpha\cdot\delta\alpha + Cz_q\cdot\frac{qL}{V} + Cz_{\delta m}\cdot\Delta\delta m\right]$$

$$\delta\dot{\alpha} = -\frac{2g}{V^2}\delta V + q - \frac{\rho VS}{2m}\cdot\left[Cz_\alpha\cdot\delta\alpha + Cz_q\cdot\frac{qL}{V} + Cz_{\delta m}\cdot\Delta\delta m\right]$$

$$\dot{q} = \frac{\rho V^2 SL}{2B}\cdot\delta Cm_G = \frac{\rho V^2 SL}{2B}\cdot\left[Cm_\alpha^G\cdot\delta\alpha + Cm_q\cdot\frac{qL}{V} + Cm_{\delta m}\cdot\Delta\delta m\right]$$

We obtain the state matrix A and the command matrix B

$$\begin{bmatrix}\delta\dot{V}\\ \delta\dot{\gamma}\\ \delta\dot{\alpha}\\ \dot{q}\end{bmatrix} = \begin{bmatrix} x_V & x_\gamma & x_\alpha & x_q \\ z_V & 0 & z_\alpha & z_q \\ -z_V & 0 & -z_\alpha & 1-z_q \\ 0 & 0 & m_\alpha^G & m_q \end{bmatrix}\cdot\begin{bmatrix}\delta V\\ \delta\gamma\\ \delta\alpha\\ q\end{bmatrix} + \begin{bmatrix} x_{\delta x} & 0 \\ 0 & z_{\delta m} \\ 0 & -z_{\delta m} \\ 0 & m_{\delta m}\end{bmatrix}\cdot\begin{bmatrix}\Delta\delta x\\ \Delta\delta m\end{bmatrix}$$

$$\delta\dot{X} = A\cdot\delta X + B\cdot\delta U$$

Short Period Mode : general approach

The Short Period Mode is the eigen mode of the Longitudinal Dynamics linked to the variables (α, q)

$$\begin{bmatrix} \delta\dot{V} \\ \delta\dot{\gamma} \\ \boxed{\begin{matrix}\delta\dot{\alpha} \\ \dot{q}\end{matrix}} \end{bmatrix} = \begin{bmatrix} x_V & x_\gamma & x_\alpha & x_q \\ z_V & 0 & z_\alpha & z_q \\ -z_V & 0 & -z_\alpha & 1-z_q \\ 0 & 0 & m_\alpha^G & m_q \end{bmatrix} \cdot \begin{bmatrix} \delta V \\ \delta\gamma \\ \boxed{\begin{matrix}\delta\alpha \\ q\end{matrix}} \end{bmatrix}$$

By considering the associated sub matrix :

$$A = \begin{bmatrix} -z_\alpha & 1-z_q \\ m_\alpha^G & m_q \end{bmatrix}$$

$$\begin{cases} z_\alpha = \dfrac{\rho V S}{2m} \cdot C z_\alpha \\ z_q = \dfrac{\rho S L}{2m} \cdot C z_q \end{cases} \qquad \begin{cases} m_\alpha^G = \dfrac{\rho V^2 S L}{2B} \cdot C m_\alpha^G \\ m_q = \dfrac{\rho V S L^2}{2B} \cdot C m_q \end{cases}$$

$$det[A - s \cdot I] = 0$$

$$det \begin{bmatrix} -z_\alpha - s & 1 - z_q \\ m_\alpha^G & m_q - s \end{bmatrix} = 0 \quad \Rightarrow \quad s^2 - (m_q - z_\alpha)s - (1 - z_q) \cdot m_\alpha^G - m_q z_\alpha = 0$$

$$s^2 + 2\lambda \cdot s + \omega_0^2 = 0$$

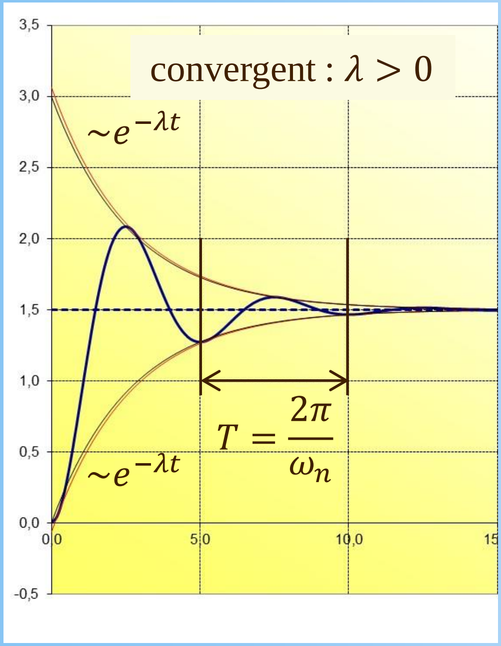
for G forward : $\Delta' = \lambda^2 - \omega_0^2 = -\omega_n^2 < 0$ $\qquad \Delta' \approx m_\alpha^G + m_q z_\alpha + \dfrac{(m_q - z_\alpha)^2}{4} < 0$

the eigenvalues are complex and conjugate : $s \,/\, \bar{s} = -\lambda \pm i \cdot \sqrt{-\Delta'} = -\lambda \pm i \cdot \omega_n$

Short Period Mode : general approach

The s-eigenvalues are complex, the solution on the form $\delta X = \delta X_0 \cdot e^{st}$ is periodic function of the time

$$s\,/\,\bar{s} = -\lambda \pm i \cdot \omega_n \quad \Rightarrow \quad \delta X \sim e^{-\lambda t} \cdot (\delta X_1 \cos \omega_n t + \delta X_2 \sin \omega_n t)$$



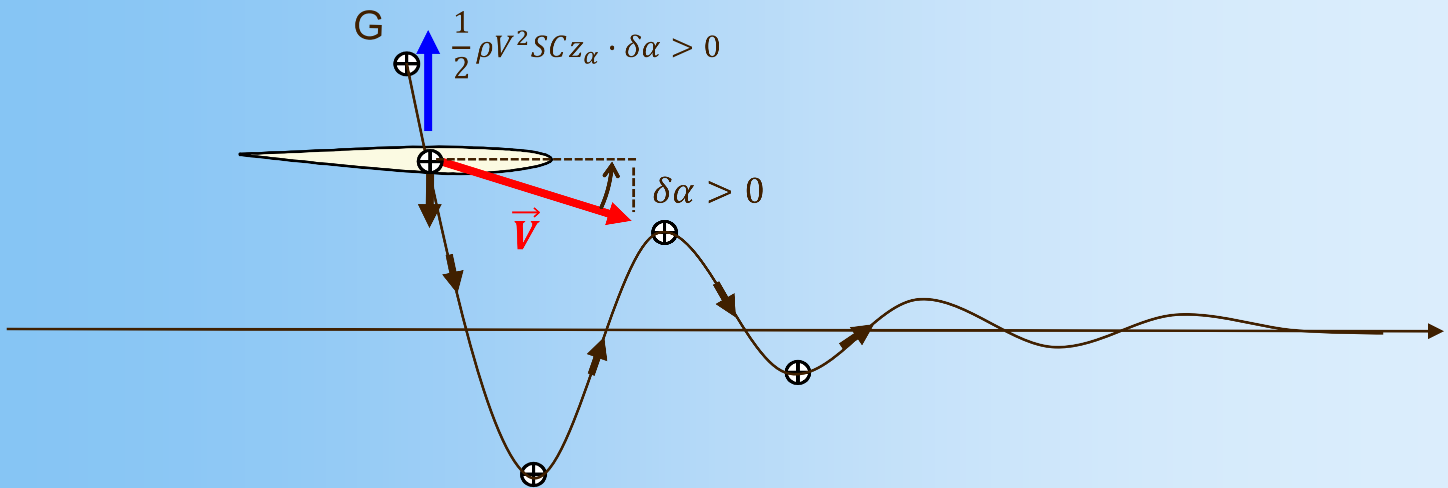
The damping process is improved

$$-\lambda = \frac{m_q - z_\alpha}{2} < 0$$

The period is linked to the aircraft stability

$$T = \frac{2\pi}{\omega_n} \approx \frac{2\pi}{\sqrt{-m_\alpha^G}}$$

Short Period Mode : Wave motion



The aircraft rotation around G produces an oscillation of the angle of attack oscillation ; this generates a lift force oscillation which is responsible of a vertical translation motion, known as the Wave motion.

This Wave motion has its own damping process link to the aircraft Lift efficiency

When the aircraft is moving down, the wing sees a velocity coming from downwards and reacts with a positive Lift up forces which opposes to the Wave motion

Short Period mode : damping process

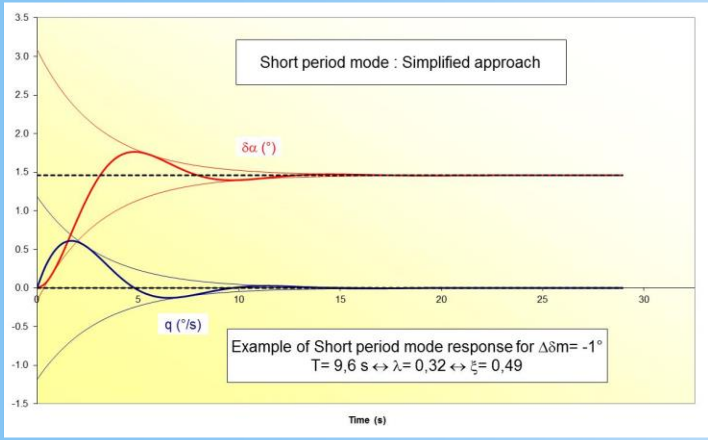
The Short Period mode is the sum of 2 motions :

- a rotation around the aircraft around G
- a vertical oscillation of G , usually called the wave motion

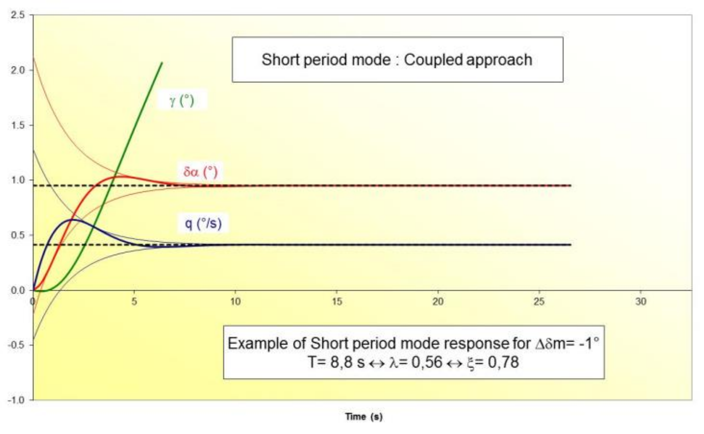
The Damping process is coming from

- the Cm_q (rotation around G)
- the Cz_α (wave motion)

$$\xi = \frac{\lambda}{\omega_0}$$ = relative damping

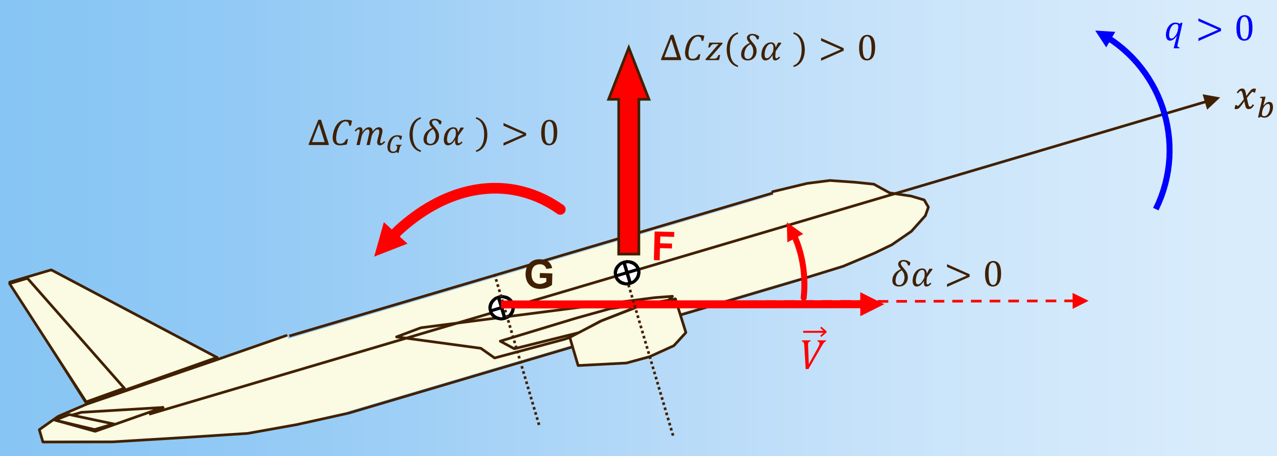


Only Cm_q : $\xi = 0{,}49$



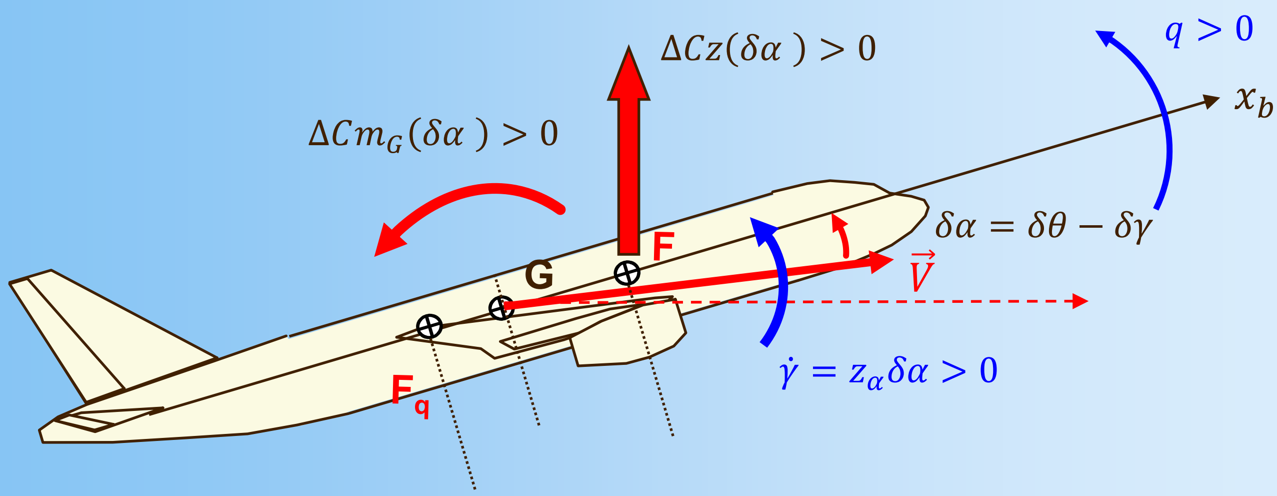
Both $Cm_q + Cz_\alpha$: $\xi = 0{,}78$

Dynamic Stability principle



If G is located after the Aerodynamic center F,
with a positive increase of α, the aircraft will be submitted to
a de-stabilizing pitch moment $\Delta Cm_G(\delta\alpha) > 0$ with a positive pitch rate q
but what happens to the velocity vector $\vec{V}$?

Dynamic Stability principle



Due to the increase of lift $\Delta Cz(\delta\alpha\ ) > 0$, the velocity vector will rotate up resulting in an angle of attack $\delta\alpha = \delta\theta - \delta\gamma$ which will decrease (if the vector $\vec{V}$ is turning faster than the body axis x_b)

This defines a new limiting point, the Manoeuver point, Fq, before which the aircraft is still « dynamically » stable

Convergence of the Short Period mode

The convergence of the Short Period mode is managed by a new point called the Manœuvre Point F_q located backward to F

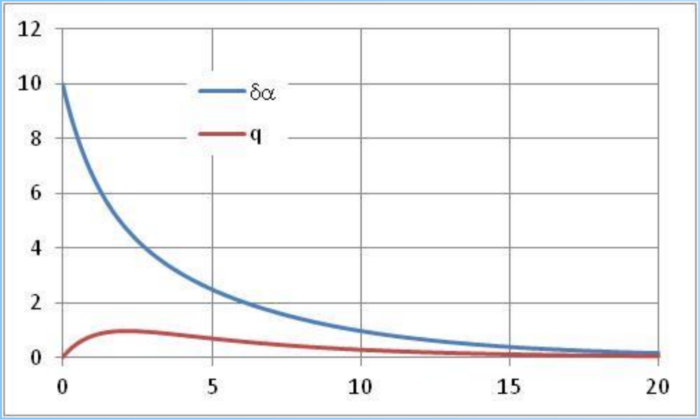
If G is located between F and F_q, the Short Period Mode is a real one but still convergent …

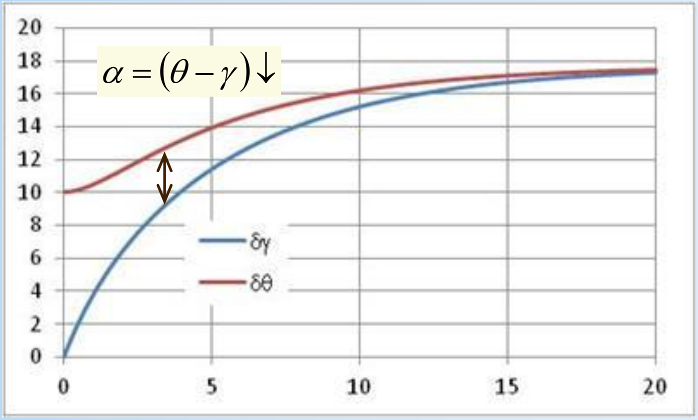
$$\rightarrow \frac{X_{Fq}}{L} \approx \frac{X_F}{L} - \frac{\rho S L}{2m} \cdot Cm_q$$

G between F and Fq : convergent aperiodic

as G backward to F : $m_\alpha^G > 0 \rightarrow q > 0 \rightarrow \theta \uparrow$

however, α decreases, because γ increases more than θ does

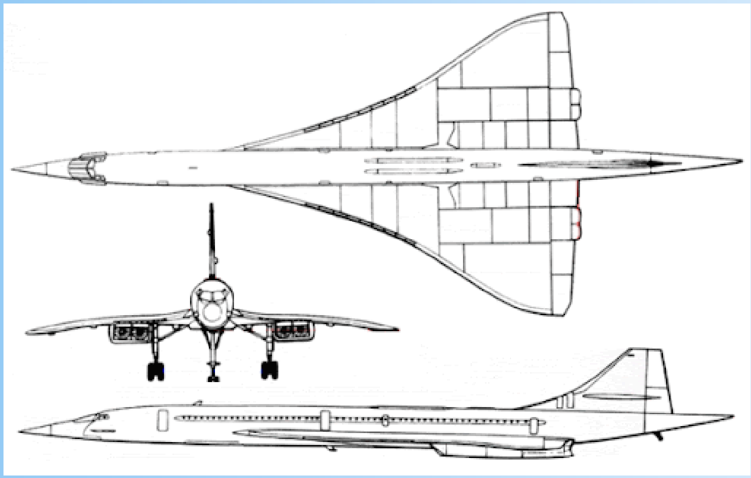




Some remark on the Manoeuver point, Fq

In the most critical cases (high mass / high altitude)
For classical aircraft, the distance between F and Fq is about 5% of the MAC

$$\rightarrow \frac{X_{Fq}}{L} - \frac{X_F}{L} \approx 5\%$$



Concorde is an exception : the Cm_q is very low due to the absence of Horizontal Tail Plane

As a consequence, we can assume $F \approx Fq$

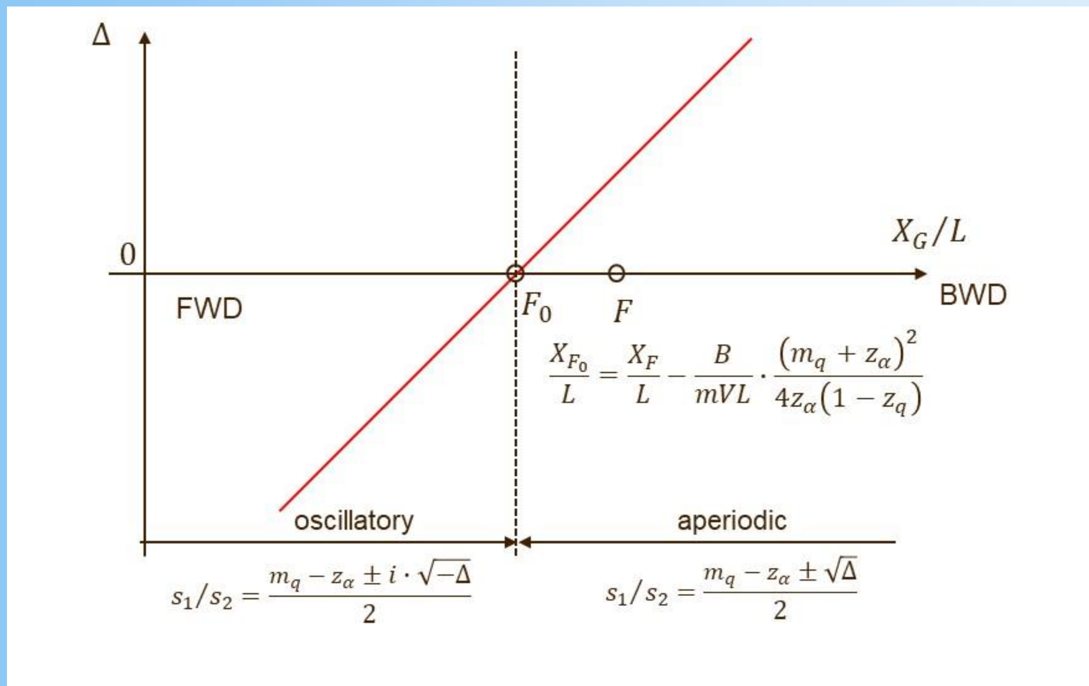
Short Period mode versus CG

NOT in PROGRAM

$$s^2 - (m_q - z_\alpha)\cdot s - (1 - z_q)\cdot m_\alpha^G - m_q z_\alpha = 0$$

$$\Delta = (m_q - z_\alpha)^2 + 4\cdot\left[(1 - z_q)\cdot m_\alpha^G + m_q z_\alpha\right]$$

$$\begin{cases} m_\alpha^G = \dfrac{\rho V^2 S L}{2B}\cdot Cm_\alpha^G = \dfrac{mVL}{B}\cdot\dfrac{X_G - X_F}{L}\cdot z_\alpha \\ z_\alpha = \dfrac{\rho V S}{2m}\cdot Cz_\alpha\ , m_q = \dfrac{\rho V S L^2}{2B}\cdot Cm_q \end{cases}$$

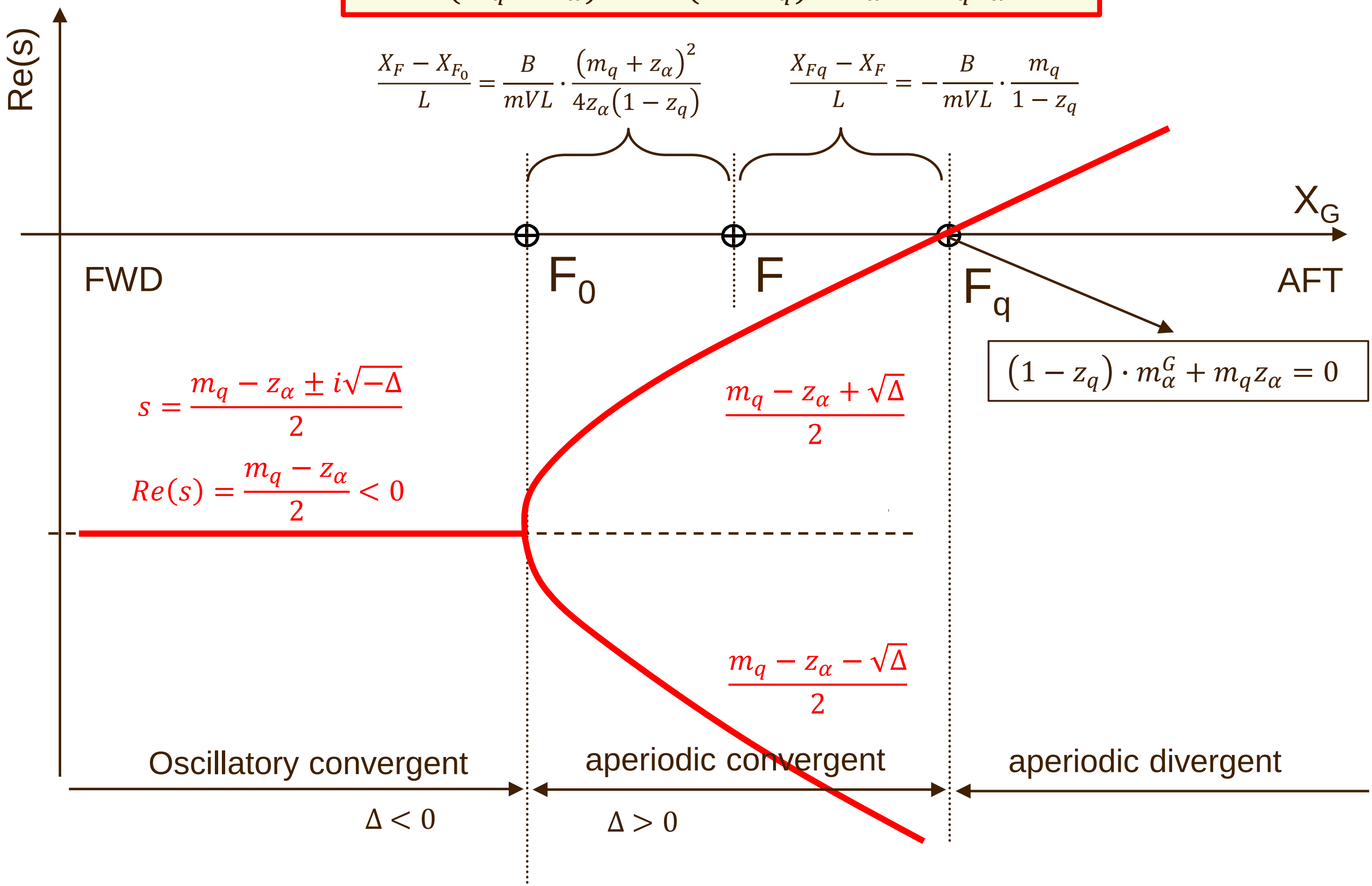


The Short Period mode ceases to be oscillatory for a CG before the F point

Short Period mode versus CG

NOT in PROGRAM

$$\boxed{s^2 - (m_q - z_\alpha)\cdot s - (1 - z_q)\cdot m_\alpha^G - m_q z_\alpha = 0}$$



Short Period mode versus CG

NOT in PROGRAM

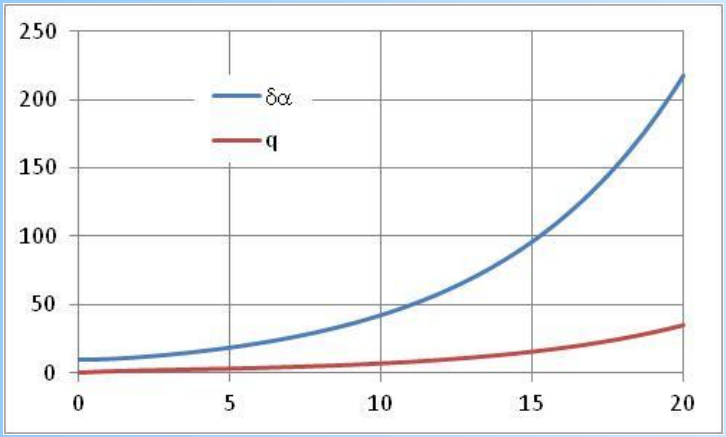
The upper real branch $\frac{m_q - z_\alpha + \sqrt{\Delta}}{2}$ cuts the x-absciss when : $\left(1 - z_q\right) \cdot m_\alpha^G + m_q z_\alpha = 0$

This defined a new point called F_q such that : $\frac{X_{Fq}}{L} = \frac{X_F}{L} - \frac{B}{mVL} \cdot \frac{m_q}{1 - z_q}$

The Manœuvre point F_q is the limit of the convergence for the Short Period mode

The divergence of the Short Period mode is non-periodic

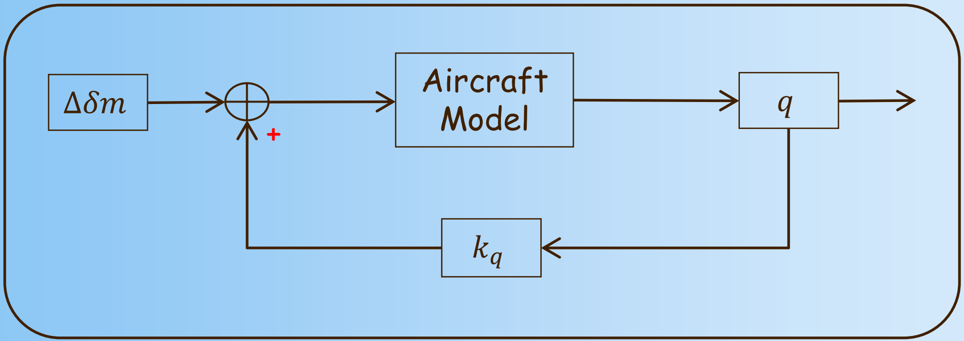
G backward wrt Fq : aircraft unstable



Pitch Damper Principle

The damping of the Short Period mode is driven by : $\lambda = \dfrac{-m_q + z_\alpha}{2}$

If I want to improve this damping, I have to increase either $-m_q$ or z_α



The principle of the pitch damper is to increase $-m_q$ by making a feedback loop with the (measured) variable q on the command δm

$$\Delta\delta m \rightarrow \Delta\delta m + k_q \cdot q$$

Pitch Damper Principle

natural aircraft : variation of pitch equation

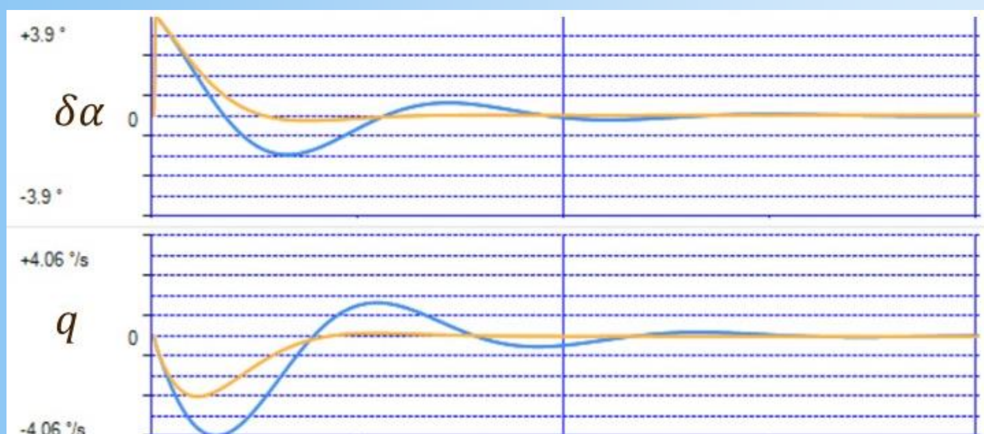
$$\dot{q} = m_\alpha^G \cdot \delta\alpha + \bar{m}_q \cdot q + \bar{m}_{\delta m} \cdot \Delta\delta m$$

$$\Delta\delta m \rightarrow \Delta\delta m + k_q \cdot q$$

$$\dot{q} = m_\alpha^G \cdot \delta\alpha + \left(\bar{m}_q + k_q \bar{m}_{\delta m}\right) \cdot q + m_{\delta m} \cdot \Delta\delta m$$

aircraft with feedback law : variation of pitch equation

The m_q is negatively increased by $k_q m_{\delta m}$ (if $k_q > 0$)



we increase the damping ξ from 0,33 to 0,70 with a gain k_q=0,34

Phugoïd Mode : simplified approach

The Phugoïd Mode is the eigen mode of the Longitudinal Dynamics linked to the variables (V, γ).

$$\begin{bmatrix} \delta\dot{V} \\ \delta\dot{\gamma} \\ \delta\dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} x_V & x_\gamma & x_\alpha & x_q \\ z_V & 0 & z_\alpha & z_q \\ -z_V & 0 & -z_\alpha & 1 - z_q \\ 0 & 0 & m_\alpha^G & m_q \end{bmatrix} \cdot \begin{bmatrix} \delta V \\ \delta\gamma \\ \delta\alpha \\ q \end{bmatrix}$$

By considering the associated sub matrix :

$$A = \begin{bmatrix} x_V & -g \\ 2g/V^2 & 0 \end{bmatrix}$$

$$x_V = \frac{1}{m} \cdot (F_V - \rho V S C x)$$

$$det[A - s \cdot I] = 0$$

$$det \begin{bmatrix} x_V - s & -g \\ 2g/V^2 & -s \end{bmatrix} = 0 \Rightarrow s^2 - x_V \cdot s + 2g^2/V^2 = 0$$

$$s^2 + 2\lambda \cdot s + \omega_0^2 = 0$$

$$\Delta' = \lambda^2 - \omega_0^2 = -\omega_n^2 < 0$$

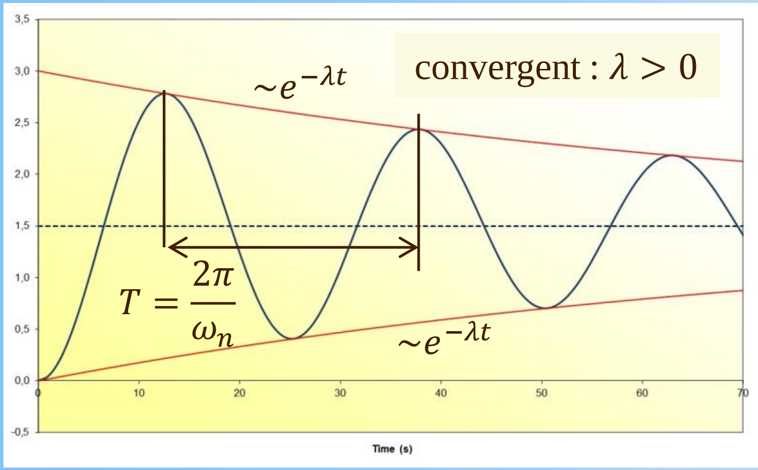
$$\Delta' = -2g^2/V^2 + \frac{x_V^2}{4} < 0$$

the eigenvalues are complex and conjugate : $s \,/\, \bar{s} = -\lambda \pm i \cdot \sqrt{-\Delta'} = -\lambda \pm i \cdot \omega_n$

Short Period Mode : general approach

The s-eigenvalues are complex, the solution on the form $\delta X = \delta X_0 \cdot e^{st}$ is periodic function of the time

$$s \,/\, \bar{s} = -\lambda \pm i \cdot \omega_n \quad \Rightarrow \quad \delta X \sim e^{-\lambda t} \cdot (\delta X_1 \cos \omega_n t + \delta X_2 \sin \omega_n t)$$



The damping term is small

$$\lambda = -\frac{x_V}{2} = \frac{\rho V S C x - F_V}{2m}$$

The period is linked to the velocity

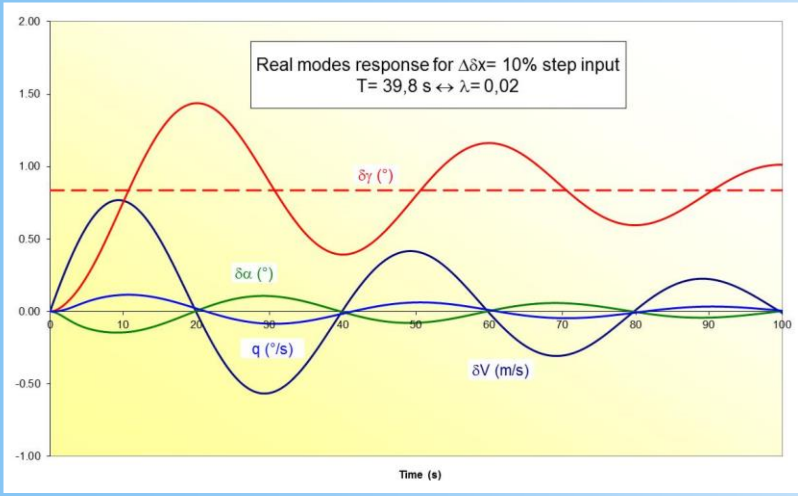
$$T = \frac{2\pi}{\omega_n} \approx \sqrt{2}\pi \cdot \frac{V}{g} \approx 0{,}45 \cdot V$$

Phugoïd mode : simplified approach

When the Short Period Mode is finished ($\alpha = Cte$ & $q = 0$), there is the activation of a new oscillation. This oscillation, called the Phugoïd, is driven by the 2 State variable (V, γ) ; it is characterised by a Long Period time and is weakly damped.

In reality, it is nearly a motion at Constant Energy : the oscillations are linked to the energy exchange between the velocity (kinematic) and altitude (potential). The damping processes are vey low, mainly driven by the Drag Force and the variations of the Thrust versus the velocity V, which explains the fair assumption of Constant Energy principle.

This simplified approach of the Phugoïd leads to an oscillatory motion which is always convergent, whatever the CG position.

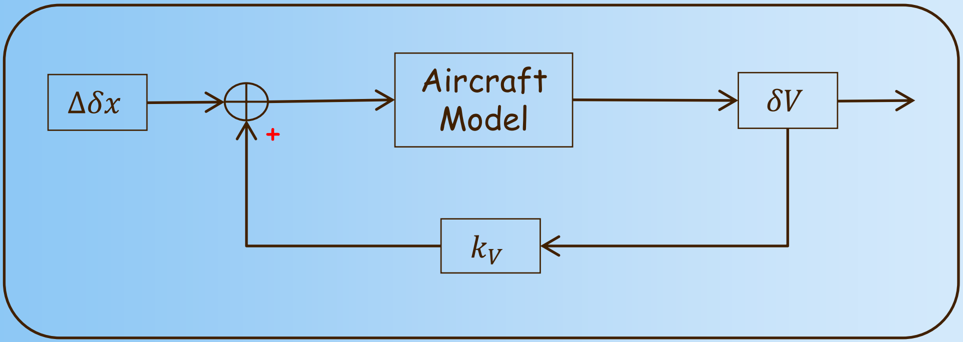


Typical Phugoïd oscillations excited by a Thrust step.
Notice the (α,q) variables oscillate also in line with the Phugoïd.

Auto-Throttle Principle

The damping of the Phugoïd mode is driven by : $\lambda = \dfrac{\rho V S C x - F_V}{2m}$

If I want to improve this damping, I have to increase either Cx or $-F_V$



The principle of the auto-throttle is to increase $-F_V$ by making a feedback loop with the (measured) variable V on the command δx

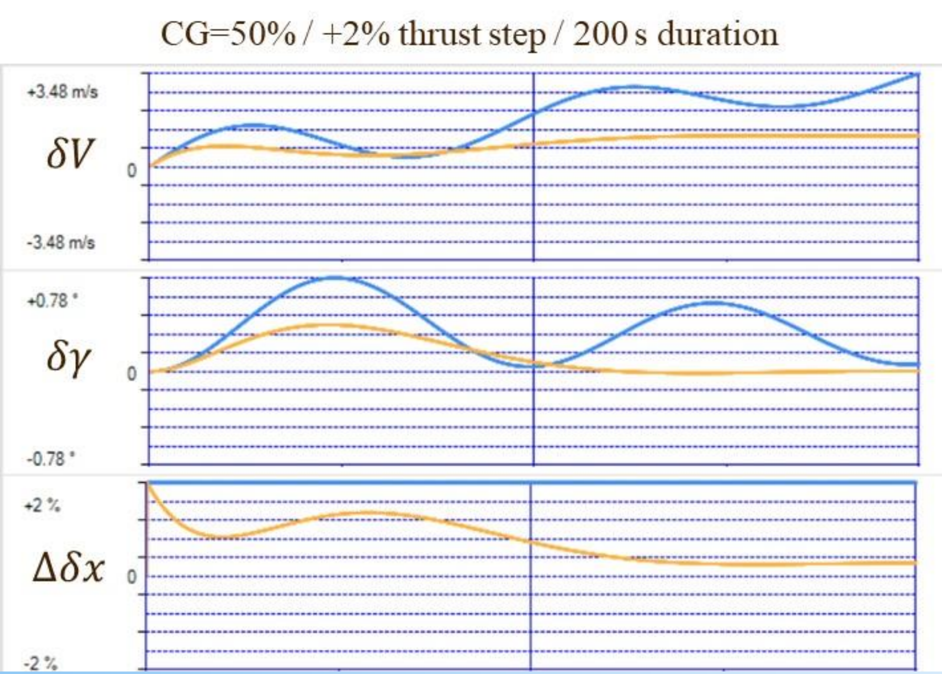
$$\Delta\delta x \rightarrow \Delta\delta x + k_V \cdot \delta V$$

Auto-Throttle Principle

$$\Delta\delta x \to \Delta\delta x + k_V \cdot \delta V$$

$$\delta F = F_V^{-} \cdot \delta V + F_0 \cdot \Delta\delta x \quad \Rightarrow \quad \delta F = (F_V^{-} + k_V \cdot F_0^{+}) \cdot \delta V + F_0 \cdot \Delta\delta x$$

The F_V is negatively increased by $k_V F_0$ (if $k_V < 0$)



we increase the damping ξ from 0,06 to 0,54 with a gain k_V=-0,015

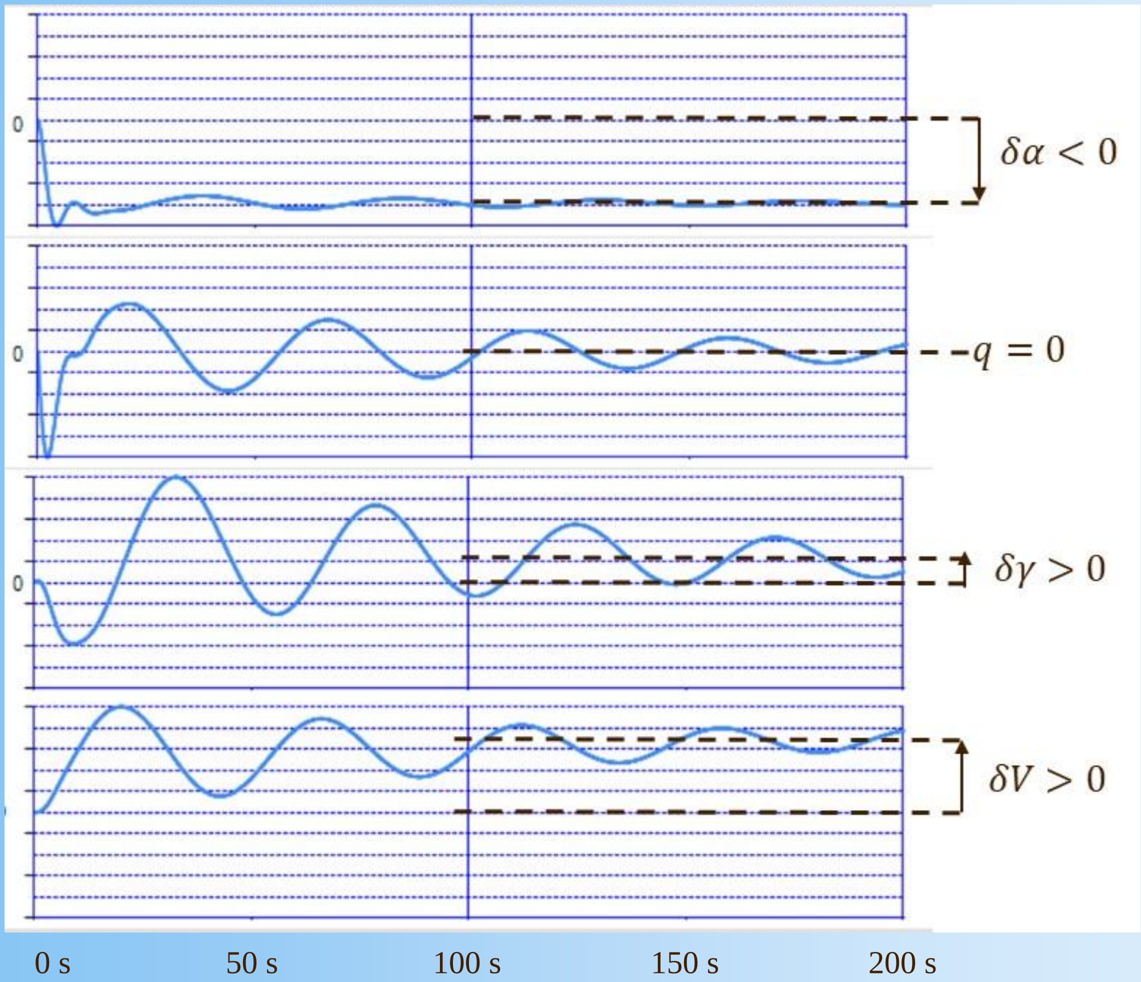
Longitudinal dynamics : Conclusions

Any Longitudinal Dynamics can be considered as
a combination of both eigen modes :

	Short Period Mode	Phugoïd
Variables	(α, q)	(V, γ)
Period	function of Cm_α Short Period a-periodic divergent for aft CG	$T \approx 0.45 \cdot V$ Long Period Periodic Convergent
Damping	function of Cm_q & Cz_α Strong Damping	function of Cx & F_V Weak Damping

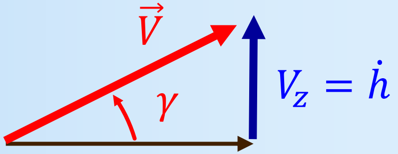
General Response from a Δδm variation

Time simulation 200s **Concorde / 5000 ft / 100 m/s / 5° elevator step**



The Altitude Convergence mode

This (last) mode is linked to the variation of the state variable, h (altitude)
It assumes that the Short Period and the Phugoïd modes are finished
It is a motion performed at constant Dynamic Pressure



$$\begin{cases} \dot{\alpha} = 0 \\ \dot{\gamma} = 0 \end{cases} \rightarrow \begin{cases} \alpha = Cte \\ mg = \frac{1}{2}\rho V^2 S \cdot Cz \end{cases} \rightarrow \rho V^2 = Cte$$

The dynamics of the altitude is given by the 1st order differential equation : $\delta\dot{h} = V \cdot \delta\gamma$

$$\delta h = \delta h_0 \cdot e^{st} = \delta h_0 \cdot e^{-\frac{t}{\tau}}$$

The eigenvalue is a real number. The solution is an aperiodic function of time

τ is called the characteristic time : τ is positive, the mode is convergent

General Response from a $\Delta\delta x$ variation

Concorde / 5000 ft / 100 m/s / 5% thrust step