HW3: Linearization, Gain Scheduling and Regions of Attraction

Exercise 1 (Linearization)

This in-class exercise is part of a series of activities leading to the implementation of an interactive MIMO control law for the `Flying-Chardonnay`, an automatic drink delivery device. This exercise exploits the MATLAB model implemented previously, and follows the following configuration its parameters (in S.I. units):

$m_d = 1$	$m_c = 1$	$l = 1$	$l_d = 1$	$J = 1$	$C_D = 0.01$	$g = 10$

1. **(10pts)** For a trimmed hovering condition, find a linearized state-space model $\Delta\dot{\boldsymbol{x}} = A\Delta\boldsymbol{x} + B\Delta\boldsymbol{u} + E\Delta\boldsymbol{w}$ using any linearization technique seen in class (by hand, Finite Differences or Complex-Step), where $\boldsymbol{u} = (T_1, T_2)$, $\boldsymbol{x} = (v_n, v_d, \theta, \dot{\theta}, \gamma, \dot{\gamma})$, and $\boldsymbol{w} = (w_n, w_d)$. Justify the use of your chosen technique and write down the respective values of (A, B, E).

Exercise 2 (Phase Planes and Linearization)

Assume the simple pendulum seen in class which dynamics is given by

$$\frac{d}{dt}\begin{pmatrix}\theta \\ \dot{\theta}\end{pmatrix} = \begin{pmatrix}\dot{\theta} \\ -\frac{g}{l}\sin\theta - \frac{b}{ml^2}\dot{\theta} + \frac{1}{ml^2}u(t)\end{pmatrix} \tag{1}$$

1. **(5pts)** Using a numerical tool (e.g., MATLAB or any alternative software of your choice), plot its Phase Portrait for $u(t) = 0$ around the trim point $(\theta, \dot{\theta}) = (0, 0)$ for the following choice of parameters (in S.I. units):

$g = 10$	$l = 1$	$b = 1$	$m = 1$

   Secondly, plot the Phase Portrait of its associated linearized model around the same trim point (*Hint: Take a look at the* `quiver`*(x,y,u,v) function in MATLAB*). How do the two plots compare?

2. **(2pts)** Plot the same exercise (i.e., nonlinear and linearized Phase Portraits) for $u(t) = 0$ around the trim point $(\theta, \dot{\theta}) = (\pi, 0)$. How do they compare?

Exercise 3 (Phase Portraits and Regions of Attraction)

Assume the following **nonlinear** system in 1D:

$$\dot{x}(t) = \alpha x^3(t) + u(t) \tag{2}$$

where $\alpha > 0$. To stabilize this system at the trim condition $x = 0$, we design the following **linear** PD controller:

$$u(t) = -k_p x(t) - k_d \dot{x}(t) \tag{3}$$

with $k_p > 0$ and $k_d > 0$.

1. **(3pts)** Is this controller locally stable? If so, what is its region of attraction (give this result in function of k_p, k_d and α)?