In-Class Exercise VII: Robustness Disk Margins

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1 Introduction

The purpose of the Flying-Chardonnay project is to design the control law for an automatic drink delivery service. The project key is to maintain the drink glass stable as the vehicle moves around the air.

2 Exercise 1

In this section, some examples of dynamic systems will be studied in order to verify their stability. This will be done by deciding if a certain statement is true or false. The previous statements are the following:

1. A system with a negative gain margin is closed-loop unstable.

False. A system with a negative gain margin does not imply that it is unstable. Having a negative gain margin tells that there is a value of δ less than one which is multiplied by the controller K and makes the closed-loop unstable.

2. A system with a negative phase margins is closed-loop unstable.

False. The same concept developed in the previous explanation can be applied here. A system with a negative phase margin does not imply that it is unstable. Having a negative phase margin tells that there is a value of ϕ less than zero which is added to the system and makes the closed-loop unstable.

3. A system in which MATLAB's allmargin() returns positive gain margins is closed-loop stable.

False. If MATLAB allmargin() returns a positive gain margin does not imply that the system is closed-loop stable, it only tells that there is a critical point when the system's gain is increased. A critical point tells that there is a transition between stable and unstable or even any change in stability.

3 Exercise 2

In this section the gain and phase margins will be computed a plant whose open-loop transfer function is the following:

$$L(s) = \frac{s^2 + 2s + 50}{s^3 + 6s^2 + 11s + 6} \tag{1}$$

Using the MATLAB function *allmargin()* it is possible to compute both the gain and phase margins:

$$GainMargin : [2.812110.6934]$$

$$PhaseMargin : 19.3948$$

As shown previously both gain margins are higher than 1 which means that there are two critical points which can lead to a change in stability or not. To see if the is a region of unstability or stability it is required to compute the root locus of the system (see figure 3.1).

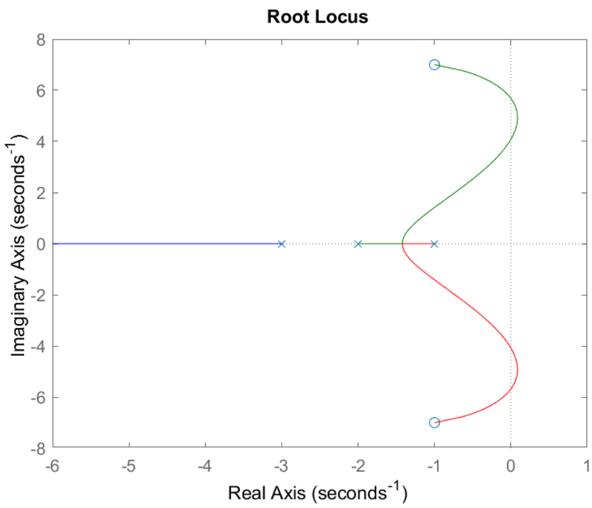

Figure 3.1: Root locus of the plant.

As seen on figure 3.1, there is small region of unstability, therefore, the first critial point changes the stability of the system to unstable and then the second critical point makes the system stable.

The previous margins are computed independently, which means that the gain margins are computed without any disturbance in phase. In order to take into account both the gain and phase disturbances at the same time, it is necessary to adress to disk margins. The first disk margin is the T-based disk margin whose plot is presented on figure 3.2. The gain and phase variations as a disk will also be presented on figure 3.3.

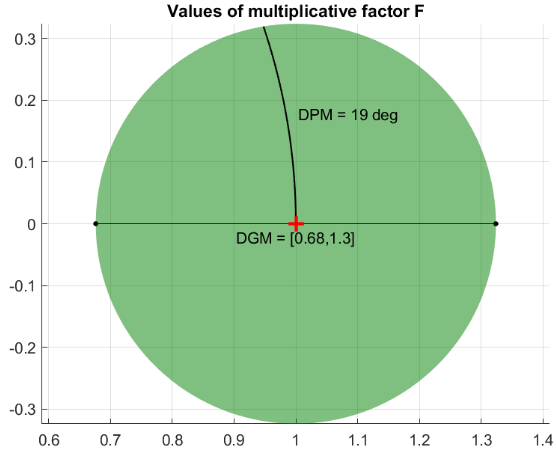

Figure 3.2: T-based disk margin of the plant.

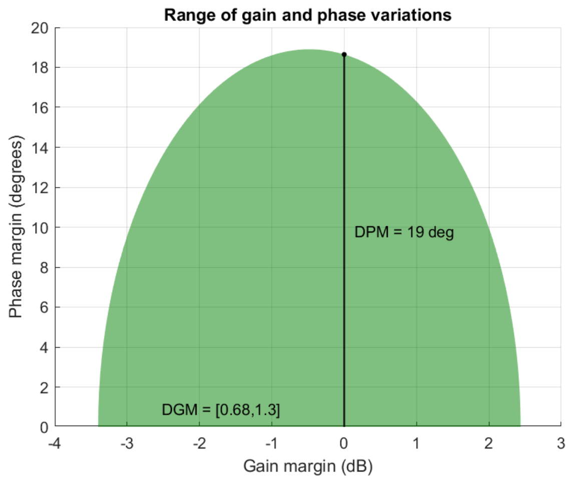

Figure 3.3: Range of phase and gain variations using the T-based disk margin of the plant.

The second disk margin is the S-T balanced (symmetric) disk margin whose plot is presented on figure 3.4. The gain and phase variations as a disk will also be presented on figure 3.4.

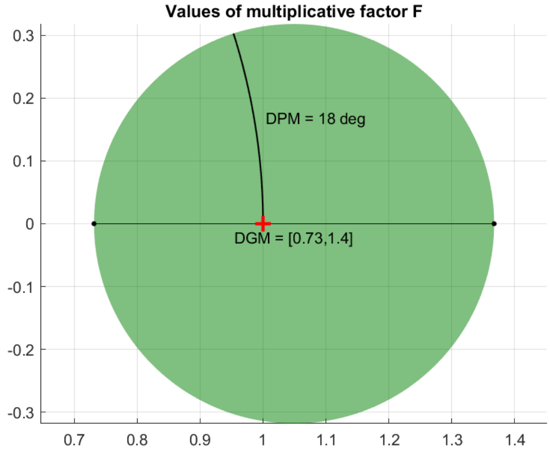

Figure 3.4: S-T balanced (symmetric) disk margin of the plant.

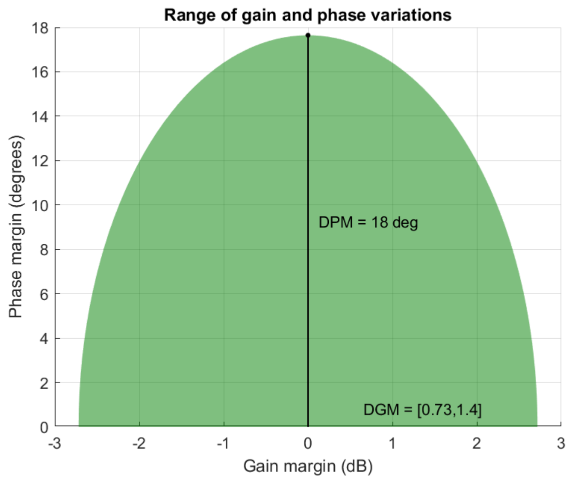

Figure 3.5: Range of phase and gain variations using the S-T balanced (symmetric) disk margin of the plant.

As seen on figures 3.2 and 3.4, the requirement of 6dB as disk margins is not reached in the case of this plant. Therefore, it would be required to increase the gain margins modifying the poles of the closed loop system.

Finally, with a set of experimental data of real-life frequency uncertainties the stabilty of the system will be studied. That will be done by comparision of the experimental points and the S-based disk margin. This comparision is presented on figure 3.6.

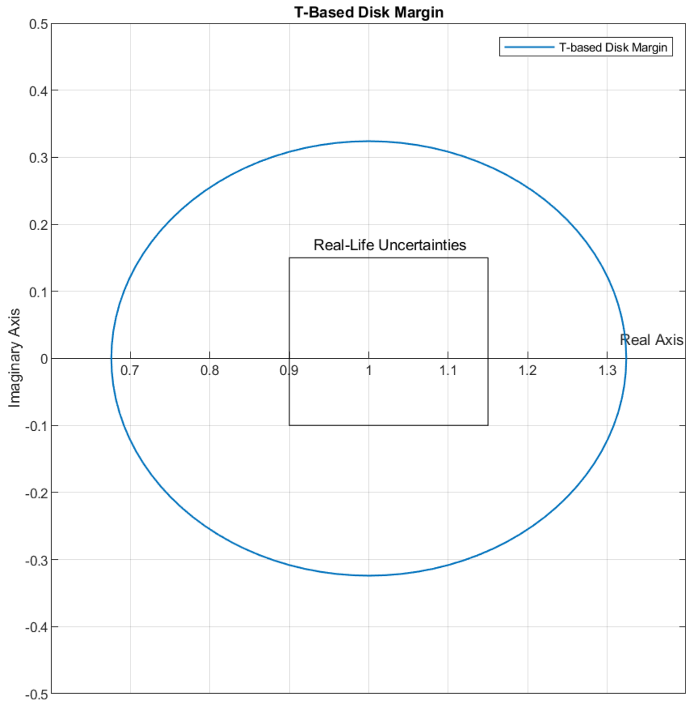

Figure 3.6: Comparision between the region of uncertainty and the S-based disk margin.

As seen on figure 3.6, the region of uncertainty is within the region of stability of the system, therefore, the plant robustle stable enough.

4 Exercise 3

In this section, the relation between phase margins ($\bar{\phi}$ and $\underline{\phi}$), gain margins ($\bar{g}$ and $\underline{g}$) and the S-T balanced (symmetric) disk margin ($\bar{\alpha}_{ST}$) is computed:

$$\begin{cases} \bar{g} \geq \frac{1+\frac{\bar{\alpha}_{ST}}{2}}{1-\frac{\bar{\alpha}_{ST}}{2}} \\ \underline{g} \leq \frac{1-\frac{\bar{\alpha}_{ST}}{2}}{1+\frac{\bar{\alpha}_{ST}}{2}} \\ \bar{\phi} \geq \cos^{-1}\left(\frac{1-(\frac{\bar{\alpha}_{ST}}{2})^2}{1+(\frac{\bar{\alpha}_{ST}}{2})^2}\right) \\ \underline{\phi} \leq -\cos^{-1}\left(\frac{1-(\frac{\bar{\alpha}_{ST}}{2})^2}{1+(\frac{\bar{\alpha}_{ST}}{2})^2}\right) \end{cases} \tag{2}$$