Correction to exercise 03:

Question 1:

For this code $k_1 = 3$, $n_1 = 6$

Hence $R = \frac{k_1}{n_1} = \frac{1}{2}$.

Question 2:

The rate of a code belongs to the interval $[0;1]$.

- A rate of 0 means that for every information bit, an infinite number of redundancy bits is transmitted $\rightarrow$ this has no practical interest.
- A rate of 1 means that no error correction was implemented.

Question 3:

Decreasing the code rate entails an increase in redundancy, hence, a better error probability (lower).

Question 4:

For this code, since $k = 3$, there are $2^3 = 8$ possible codewords.

Question 5:

To check if a codeword is valid or not, one has to compute $c \cdot H^t$ and check if it is equal to $\underline{0}$.

1. $c_1 = [1\,0\,1\,0\,1\,0]$.

$$c \cdot H^t = (1\,0\,1\,0\,1\,0)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad [\text{mod } 2]$$

$$= [0\;1\;1] \neq [0\;0\;0] \rightarrow \text{not a codeword}$$

- $c_2 = [0\ 0\ 0\ 0\ 0\ 0]$ $\rightarrow$ all 0 codeword is always a valid codeword.

$$c_2 \cdot H^t = [0\ 0\ 0].$$

- $c_3 = [0\ 0\ 0\ 1\ 1\ 1]$

$$c_3\, H^t = [0\ 0\ 0] \rightarrow \text{valid codeword}$$

- $c_4 = [1\ 1\ 1\ 0\ 1\ 0]$

$$c_4 \cdot H^t = [0\ 0\ 1] \neq [0\ 0\ 0] \rightarrow \text{not a valid codeword}$$

Question 6:

- Code 3 is a repetition Code.
- Code 2 is a permutation of the generator matrix of code 1.

→ Repetition coding and uncoded communication have the same BER and BLER curves. This is because the gain induced on the probability of error by repetition coding ($R = 1/3$) is destructed by the fact that a repetition code uses 3 times more energy. Since the BER & BLER curves are equal, one should not use repetition coding because it divides the bitrate by 3, without improving the BER & BLER.

→ The codes $\mathcal{C}_1$ and $\mathcal{C}_2$ have the same performance. This is due the fact that since the generator matrix G_2 is a permutation of G_1 (swap between 1st and last row) the obtained codebooks are equivalent, and hence the performances are the same.

→ Uncoded / repetition coding out performs coding for low $\frac{E_b}{N_0}$, while coding with $\mathcal{C}_1$ or $\mathcal{C}_2$ is better at high $\frac{E_b}{N_0}$

Question 7:

For a BER close to 10 dB, I would suggest using either code $\mathcal{C}_1$ or $\mathcal{C}_2$.
Now, code $\mathcal{C}_1$ has the advantage of being a systematic code, which allows a quick encoding, hence, $\mathcal{C}_1$ would be faster to implement than $\mathcal{C}_2$.