Extended Kalman Filter for a Miniature Strapdown Inertial Measurement Unit Attitude estimation

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2D Orientation Estimation

1 State Variable

As the first step, we only estimate the roll angle using the accelerometer sensor along y and z direction which come from d(6) and d(7). The state is one dimensional and is X = θ_x.
For the Kalman Filter Modelling we get the following equations:

$\theta_{k+1} = \theta_k + w$ where w is assume to be of normal distribution

$y = \begin{matrix} a_y \\ a_z \end{matrix} = \begin{matrix} -sin\theta \\ cos\theta \end{matrix} + v$ where the units are g for the accelerometer readings

From, the above equations the Kalman Filter is programmed as follows

F = 1
H = $\begin{matrix} -cos\theta \\ -sin\theta \end{matrix}$ obtained by differentiating the above equation for the output to linearize

The filter is programmed on Matlab as follows and real and simulated data are used to tune the values of Q and R.

```
t=d(1);
% Predict
dt = 0.01; % Taking a sample of 0.01s as readings are available every 0.01s
F = 1; % Unlike for 1 state X = theta, when the value of F will be just 1,
% on including theta_derivative,
% The equation for theta_k+1 = theta_k + delta_t*theta_derivative
X = F*X;
P = F*P*F'+Q;
% Update
if ~isnan(t)
    Y = [d(6)
        d(7)];
    Yhat = [-sin(X) % From dynamic equations
        cos(X)];
    H = [-cos(X) % On differentiating to linearize
        -sin(X)]; % To convert to degrees
    S = H*P*H'+R;
    K = P*H'*inv(S);
    X = X+K*(Y-Yhat);
    P = P-K*S*K';
end
```

For this case, only the accelerometers are used and R is assigned a high value because the accelerometers do not give accurate readings specially when the spirit level is moved horizontally, which is why the gyroscope will be implemented in the next section. High level of confidence exist in the model and hence, the value assigned to Q is less. On observing the model performance for these tunings, a satisfactory agreement of the filter with the simulated data was observed and the graph in the next page summarizes it.

```
%% estimation attitude
qa = 1e-3; %Bruit d'etat orientation
ra = 20*ACCVAR(2:3); %Bruit de mesure accelero.
% The accelerometer gives inaccurate readings when moved horizontally fast, assuming it as a
% rotation. Hence, we have less confidence in Accelerometer than the Gyrometer

rg = GYRVAR(1); %Bruit de mesure gyro
% The above values are obtained on tuning and the Filter performs
% satisfactorily well
Q = diag(qa);
R = diag(ra);
X= 0; %Etat : rotation selon x (rad)
P = deg2rad(10)^2*ones(1);
```

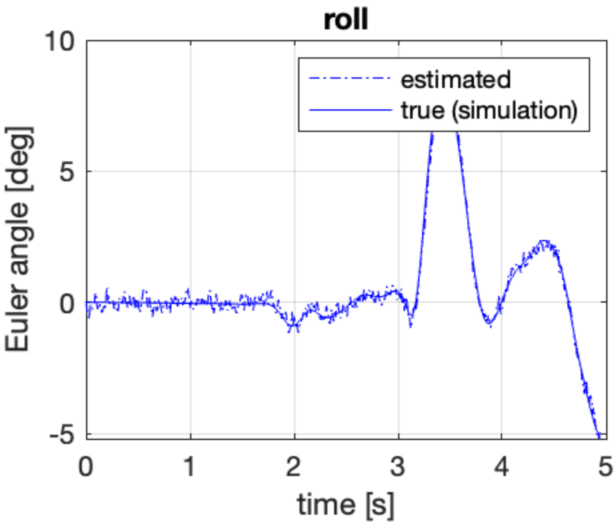

Figure 1. 2D estimation of roll angle for a single state

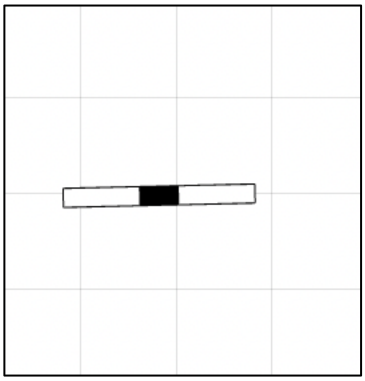

Figure 2. Animation of the Spirit Level

2 State Variable

As discussed above for better and more accurate sensor results, a gyroscope is needed. For enhancing the model further a two dimensional state vector is defined, $X = [\theta_x \quad \dot{\theta_x}]^T$.

$$\theta_{k+1} = \theta_k + \dot{\theta}\,\Delta t$$

$$\dot{\theta_{k+1}} = \dot{\theta_k}$$

$$y = \begin{matrix} a_y \\ a_z \\ \dot{\theta} \end{matrix} = \begin{matrix} -sin\theta \\ cos\theta \\ \dot{\theta} \end{matrix}$$

For the above set of equations:

$$F = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

$$H = \begin{matrix} -cos\theta & 0 \\ -sin\theta & 0 \\ 0 & 180/\pi \end{matrix}$$

For this system the component of R matrix corresponding to the gyroscope is assigned a lower value to insinuate more confidence on its readings when compared to the accelerometer.

```
t=d(1);
% Predict
dt = 0.01; % Taking a sample of 0.01s as readings are available every 0.01s
F = [1 dt ; 0 1]; % Unlike for 1 state X = theta, when the value of F will be just 1, on includi
% The equation for theta_k+1 = theta_k + delta_t*theta_derivative
X = F*X;
P = F*P*F'+Q;
% Update
if ~isnan(t)
    Y = [d(6)
        d(7)
        d(2)];
    Yhat = [-sin(X(1)) % From dynamic equations
        cos(X(1))
        180/pi*(X(2))];
    H = [-cos(X(1)) 0 % On differentiating to linearize
        -sin(X(1)) 0
        0  180/pi]; % To convert to degrees
    S = H*P*H'+R;
    K = P*H'*inv(S);
    X = X+K*(Y-Yhat);
    P = P-K*S*K';
end
```

```
%% estimation attitude
qa = 1e-3; %Bruit d'etat orientation
ra = 100*ACCVAR(2:3); %Bruit de mesure accelero. The accelerometer gives inaccurate readingd when mo
% Hence, we have less confidence in Accelrometer than the Gyrometer
rg = GYRVAR(1); %Bruit de mesure gyro
% The above values are obtained on tuning and the Filter performs
% satisfactorily well
Q = diag([qa qa]);
R = diag([ra(1) ra(2) rg]);
X=[0 0]'; %Etat : rotation selon x (rad)
P = deg2rad(10)^2*ones(2);
```

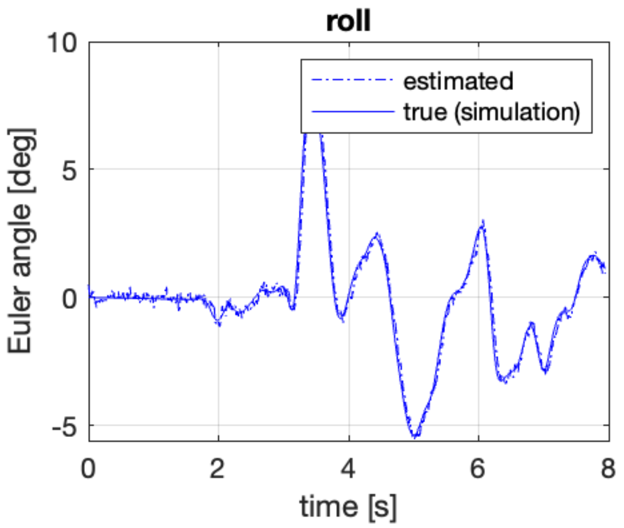

Figure 3. 2D estimation of roll angle for two state variables

Validation with Real System

The same code was used with the given IMU (with both 1 state and 2 state) and the results was satisfactory. The produced model was closely following the actual system.

Discussion

- For an aircraft moving with constant acceleration such as in the case of coordinated turn: One notable property of the EFK is that is implements a linear model for the non-linear system which may lead to estimation errors. In such scenarios it is advised to use non-linear Kalman Filters.

- If the sensor is designed for a specific purpose: The EFK can be modelled keeping the system properties and characteristics in mind, tuning the Q and R matrices based on our confidence level on the system model and the sensor used.

- If the gyrometer bias is not constant: When the gyrometer bias needs to be estimated, it can be added as an additional parameter in the state vector and the F and H matrices can be accordingly modified.