MAE1 - Electromagnetism applied to avionics

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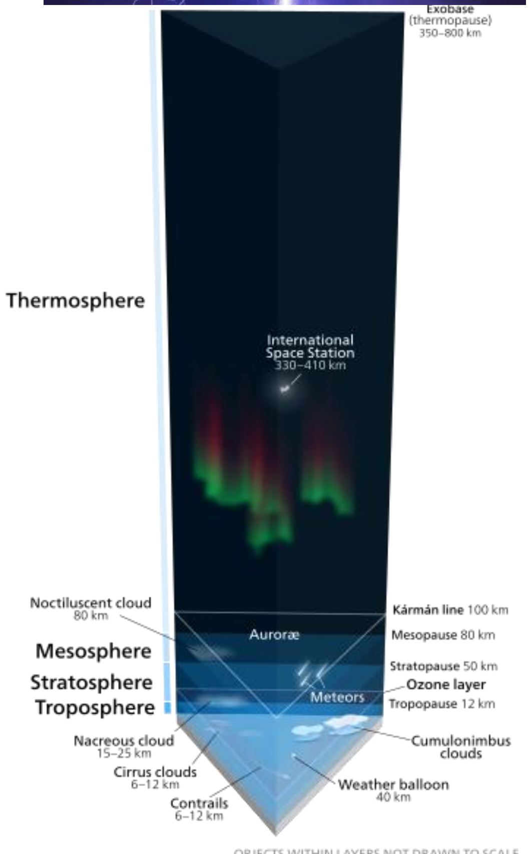
Part VI

- Wave propagation in Vacuum
- Wave propagation in a medium
- What happen at the interface?



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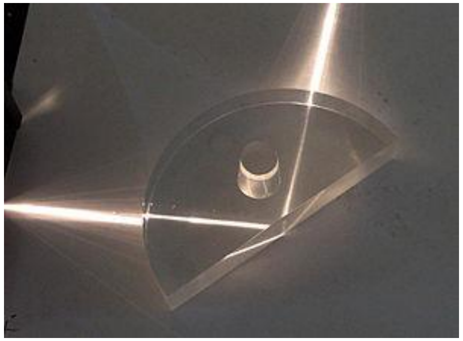


Field in matter

The charge and current densities modified by the matter → 4 field vectors:

- Electrical Field $\vec{E}$
- Magnetic Field $\vec{B}$
- Electric Displacement or induction Field $\vec{D} = \varepsilon\vec{E}$
- Magnetic Intensity or Excitation Field $\vec{H} = \frac{\vec{B}}{\mu}$

$$\boxed{\begin{aligned} &\vec{\nabla}\cdot\vec{D} = \rho \\ &\vec{\nabla}\cdot\vec{B} = 0 \\ &\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ &\vec{\nabla}\times\vec{H} = \left(\vec{j} + \frac{\partial\vec{D}}{\partial t}\right) \end{aligned}}$$

In Vacuum $\varepsilon = \varepsilon_0$, $\mu = \mu_0$

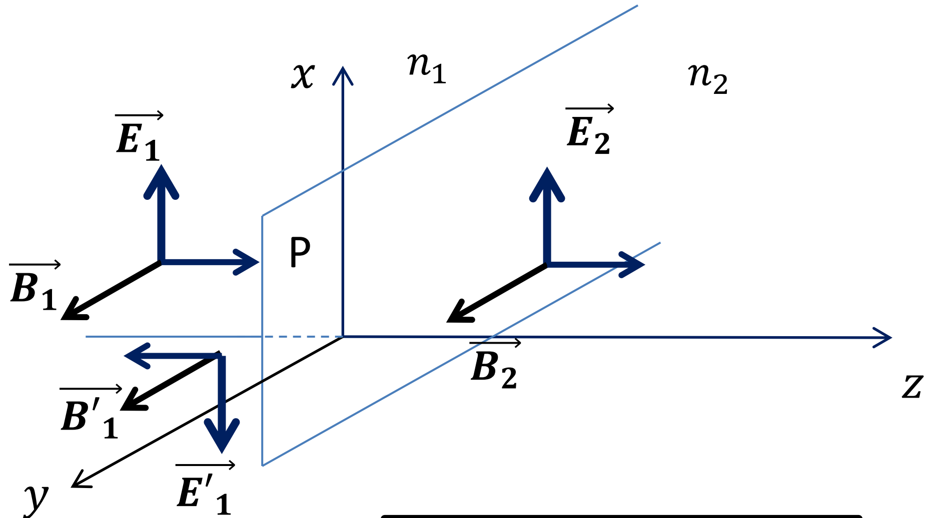
In matter ε (permittivity) , μ (permeability) $\epsilon \mathbb{R}$ or $\mathbb{C}$

$\varepsilon = \varepsilon_0\varepsilon_r$ & $\mu = \mu_0\mu_r$

From the Ohm local Law : $\vec{j} = \gamma\vec{E}$ where γ is the conductivity

Reflection, refraction at the boundary LHI media: NORMAL INCIDENCE

Simplest Case : **Normal incidence** of a plane wave on a dielectric plan (P), **non magnetic media**



Transmitted wave $(\overrightarrow{E_2}, \overrightarrow{B_2})$

Reflected wave $(\overrightarrow{E'_1}, \overrightarrow{B'_1})$

Maxwell equations

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right)$$

Applied to

$(\overrightarrow{E_1}, \overrightarrow{B_1}, \overrightarrow{D_1}, \overrightarrow{H_1})$, $(\overrightarrow{E_2}, \overrightarrow{B_2}, \overrightarrow{D_2}, \overrightarrow{H_2})$,

$(\overrightarrow{E'_1}, \overrightarrow{B'_1}, \overrightarrow{D'_1}, \overrightarrow{H'_1})$

Normal Incidence

@ The interface $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

$(\overrightarrow{E_1}, \overrightarrow{B_1}, \overrightarrow{D_1}, \overrightarrow{H_1})$, incident travelling wave, direction $+z$

$(\overrightarrow{E_2}, \overrightarrow{B_2}, \overrightarrow{D_2}, \overrightarrow{H_2})$, transmitted travelling wave, direction $+z$

$(\overrightarrow{E'_1}, \overrightarrow{B'_1}, \overrightarrow{D'_1}, \overrightarrow{H'_1})$, reflected travelling wave , direction $-z$

$$\overrightarrow{E_1} = \overrightarrow{e_x} E_{1x} e^{i(k_1 z - \omega t)}$$
$$\overrightarrow{E'_1} = -\overrightarrow{e_x} E_{1x} e^{-i(k_1 z + \omega t)}$$
$$\overrightarrow{E_2} = \overrightarrow{e_x} E_{2x} e^{i(k_2 z - \omega t)}$$

$$k_1 = \frac{n_1 \omega}{c} \qquad k_2 = \frac{n_2 \omega}{c}$$

Normal Incidence

From Maxwell Faraday $i\vec{k} \times \vec{E} = i\omega\vec{B}$

$$\Rightarrow \qquad \frac{n}{c}\overrightarrow{e_z} \times \vec{E} = \vec{B}$$

$$c\overrightarrow{B_1} = n_1\overrightarrow{e_y}E_{1x}e^{i(k_1 z-\omega t)}$$

$$c\overrightarrow{B'_1} = n_1\overrightarrow{e_y}E_{1x}e^{-i(k_1 z+\omega t)}$$

$$c\overrightarrow{B_2} = n_2\overrightarrow{e_x}E_{2x}e^{i(k_2 z-\omega t)}$$

Normal Incidence, Non Magnetic Media

At the **interface, z=0**, the pulsation ω is the same for the incident, reflected and transmitted wave.

As considering a **normal incident**, the tangential components of E and B is only considered and are continuous at the boundary.

@z=0, E-Field follows: $E_{1x} - E'_{1x} = E_{2x}$

@z=0 & for a non magnetic medium,
B-Field follows: $n_1(E_{1x}+E'_{1x}) \quad = n_2E_{2x}$

Normal Incidence, Non Magnetic Media

By knowing the magnitude E_{1x}, we find:

$$E'_{1x} = \frac{n_2 - n_1}{n_2 + n_1} E_{1x}$$

$$E_{2x} = \frac{2n_1}{n_2 + n_1} E_{1x}$$

For given refractive index of each medium

$$\frac{E'_{1x}}{E_{1x}} \qquad \frac{E_{2x}}{E_{1x}}$$

Fresnel coefficients for normal incidence

Reflection Fresnel coefficient

$$\frac{E'_{1x}}{E_{1x}} = r_{12}$$

Transmission Fresnel coefficient

$$\frac{E_{2x}}{E_{1x}} = t_{12}$$

“12” index give the direction from medium 1 to medium 2

$$r_{12} = \frac{n_2 - n_1}{n_2 + n_1}$$

$$t_{12} = \frac{2n_1}{n_2 + n_1}$$

The reflected and transmitted E field is not directly measurable but we measure the Poynting Vector (or intensity of the wave)

$$\vec{S} = \vec{E} \times \vec{B} = \vec{k}\frac{E^2}{\omega\mu_0} = \frac{nE^2}{\mu_0 c}\overrightarrow{e_z}$$

Poynting Vector & Transmittance and Reflectance

$$\overrightarrow{S_1} = \frac{n_1 {E_{1x}}^2}{c}\overrightarrow{e_z} \qquad \overrightarrow{S'_1} = \frac{n_1 {E'_{1x}}^2}{c}\overrightarrow{e_z} \qquad \overrightarrow{S_2} = \frac{n_2 {E_{2x}}^2}{c}\overrightarrow{e_z}$$

We denote the Reflectance or Reflection coefficient R_n:

$$\frac{\overrightarrow{S'_1}}{\overrightarrow{S_1}} = R_n = {r_{12}}^2$$

We denote the Transmittance or transmission coefficient T_n:

$$\frac{\overrightarrow{S_2}}{\overrightarrow{S_1}} = T_n = \frac{n_2}{n_1}{t_{12}}^2$$

Energy conservation at the interface

$$R_n + T_n = 1$$

For any interface of non-conducting media, this result expresses the conservation of Energy.

In a case of circular or elliptic polarization, the equations could be hold separately for each polarization and for the total intensity. Only the power (which is the flux of intensity) integrate all polarization on the surface of the detector.

As the energy of the incident wave is either reflected or transmitted, no energy is stored at this kind of interface.
To store energy (Solar Panel or detector), the material has to be active (charge, current).

Applications

Air-Glass interface: $n_{air} = 1, \quad n_{glass} = 1.5$

Value of Reflectance and Transmittance:

$$R_n = \quad 0.04$$

$$T_n = \quad 0.96$$

If $n_2 > n_1$, in normal incidence, 96% of energy is transmitted, the Glass is transparent.

If $n_2 < n_1$, in normal incidence, the energy is mainly reflected, this is the case of a mirror

Air-Water application

Air-Water interface: $n_{air} = 1$

- for visible light $n_{water} = 1.33$

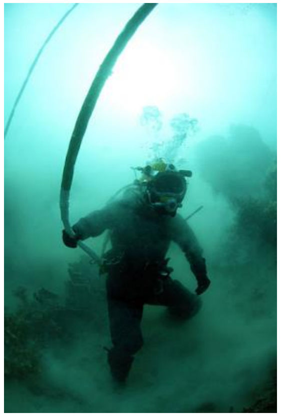
$$\Rightarrow R_n = \quad 0.02$$

- For Radio wave below $\omega = 10^{11} rad \cdot s^{-1} \Leftrightarrow f \approx 16GHz$, $\varepsilon_r = 81$

(NB: Radar frequencies are 3, 5, 9 GHz bands)

$$n_{water} = \sqrt{\varepsilon_r} = 9$$

$$R_n = \qquad 0.64$$



Geolocation under the water?

Interface with conducting medium

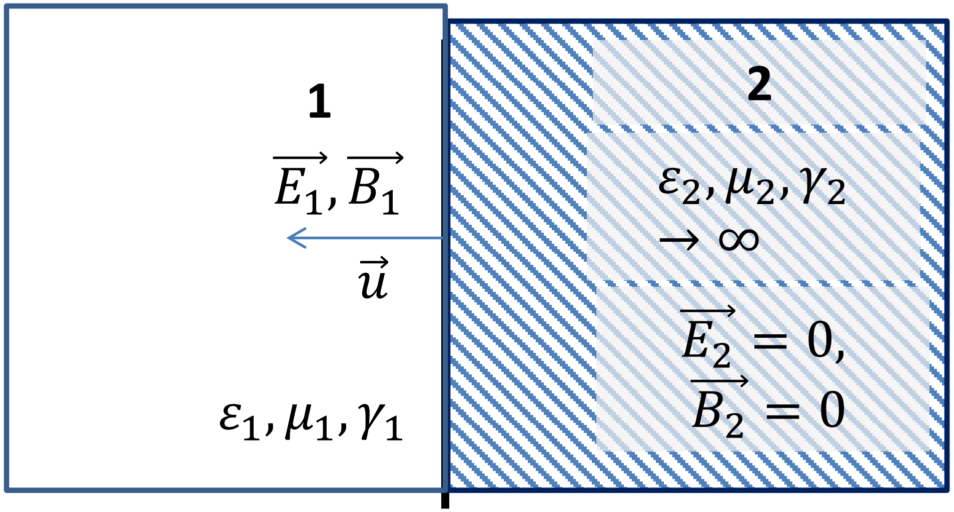
- Medium 2 : perfect conductor $\Rightarrow \gamma \to \infty$, $\overrightarrow{E_2}$ & $\overrightarrow{H_2} \Rightarrow$ Null while skin depth $\delta \;=\sqrt{\dfrac{2}{\mu_0\gamma \;\; \omega}} = 0$
- At the interface, surface current $\overrightarrow{I_s}$ $(A \cdot m^{-1})$ and surface charges $Q_s(C \cdot m^{-1})$ appears
- If we consider a vector $\vec{u}$, a normal unity vector, oriented from medium 2 to the medium 1

$$\vec{u} \times \overrightarrow{E_1} = 0$$

$$\vec{u} \times \overrightarrow{H_1} = \overrightarrow{I_s}$$

$$\vec{u} \cdot \overrightarrow{D_1} = Q_s$$

$$\vec{u} \cdot \overrightarrow{B_1} = 0$$



Example

For $f = 10GHz$,we could verify $\delta = \sqrt{\frac{2}{\mu_0 \gamma \omega}} < \frac{\lambda}{100}$ and $\gamma > \frac{100}{12\lambda}$

With $\mu_0 = 4\pi \cdot 10^{-7} kg.m.A^{-2}.s^{-2}$

$$\lambda = \frac{c}{f} = 0.03\text{m}$$

$$\delta\ (f) = \sqrt{\frac{1}{\pi \mu_0 \gamma\ f}}$$

if $\delta < \frac{\lambda}{100}$, we could verify

$$\gamma > \frac{100}{12\lambda}$$

Interface between 2 medium separated by a thin layer conducting medium

- At the interface: thin layer of conducting medium with a null thickness for perfect conductor if $\gamma \rightarrow \infty$, or a thickness < $\frac{\delta}{10}$ for a finite value of γ
- At the interface, surface current $\overrightarrow{I_s}$ $(A \cdot m^{-1})$ and surface charges $Q_s (C \cdot m^{-1})$ appears
- If we consider a vector $\vec{u}$, a normal unity vector, oriented from medium 2 to the medium 1

$$\vec{u} \times \left(\overrightarrow{E_1} - \overrightarrow{E_2}\right) = 0$$
$$\vec{u} \times \left(\overrightarrow{H_1} - \overrightarrow{H_2}\right) = \overrightarrow{I_s}$$
$$\vec{u} \cdot \left(\overrightarrow{D_1} - \overrightarrow{D_2}\right) = \overrightarrow{Q_s}$$
$$\vec{u} \cdot \left(\overrightarrow{B_1} - \overrightarrow{B_2}\right) = 0$$