Speed Neutral Point And Phugoid

Phugoïd Mode : general approach

For takle correctly the Phugoïd Mode, we must consider the general case where the thrust $\vec{F}$ is not applied at the centre of gravity G.

The pitch momentum equation becomes :

$$B \cdot \dot{q} = \frac{1}{2}\rho V^2 SL \cdot Cm_G + F \cdot (z_P - z_G)$$

By differentiation :

$$\delta\dot{q} = \frac{\rho VSLCm_G}{B} \cdot \delta V + \frac{\rho V^2 SL}{2B} \cdot \delta Cm_G + F_V \cdot \frac{z_P - z_G}{B} \cdot \delta V + F_0 \cdot \frac{z_P - z_G}{B} \cdot \Delta\delta x$$

But at trim, $\frac{1}{2}\rho V^2 SL \cdot Cm_G + F \cdot (z_P - z_G) = 0$ $\Rightarrow$ $\rho VSLCm_G = -\frac{2F}{V} \cdot (z_P - z_G)$

So,

$$\delta\dot{q} = \frac{\rho V^2 SL}{2B} \cdot \delta Cm_G + \left(F_V - \frac{2F}{V}\right) \cdot \frac{z_P - z_G}{B} \cdot \delta V + F_0 \cdot \frac{z_P - z_G}{B} \cdot \Delta\delta x$$

Phugoïd Mode : general approach

We obtain the state matrix A and the command matrix B

$$\begin{bmatrix} \delta\dot{V} \\ \delta\dot{\gamma} \\ \delta\dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} x_V & x_\gamma & x_\alpha & x_q \\ z_V & 0 & z_\alpha & z_q \\ -z_V & 0 & -z_\alpha & 1-z_q \\ m_V & 0 & m_\alpha^G & m_q \end{bmatrix} \cdot \begin{bmatrix} \delta V \\ \delta\gamma \\ \delta\alpha \\ q \end{bmatrix} + \begin{bmatrix} x_{\delta x} & 0 \\ 0 & z_{\delta m} \\ 0 & -z_{\delta m} \\ m_{\delta x} & m_{\delta m} \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta x \\ \Delta\delta m \end{bmatrix}$$

$$m_V = \left(F_V - \frac{2F}{V}\right) \cdot \frac{z_P - z_G}{B} \qquad m_\alpha^G = \frac{\rho V^2 SL}{2B} \cdot Cm_\alpha^G = \frac{mV \cdot (X_G - X_F)}{B} \cdot z_\alpha \qquad m_{\delta x} = F_0 \cdot \frac{z_P - z_G}{B}$$

We assume that the Short Period mode is finished : $\delta\dot{\alpha} = \dot{q} = 0$

We want to solve the differential equations set :

$$\begin{bmatrix} \delta\dot{V} \\ \delta\dot{\gamma} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_V & x_\gamma & x_\alpha & x_q \\ z_V & 0 & z_\alpha & z_q \\ -z_V & 0 & -z_\alpha & 1-z_q \\ m_V & 0 & m_\alpha^G & m_q \end{bmatrix} \cdot \begin{bmatrix} \delta V \\ \delta\gamma \\ \delta\alpha \\ q \end{bmatrix}$$

Phugoïd mode : general approach

$$\delta X = \delta X_0 \cdot e^{st} \rightarrow \delta \dot{X} = s \cdot \delta X$$

We find a particular solution such that :

$$\begin{bmatrix} \delta \dot{V} \\ \delta \dot{\gamma} \\ 0 \\ 0 \end{bmatrix} = s \cdot \begin{bmatrix} \delta V \\ \delta \gamma \\ \delta \alpha \\ q \end{bmatrix} = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta V \\ \delta \gamma \\ \delta \alpha \\ q \end{bmatrix}$$

Which leads to :

$$\begin{bmatrix} x_V - s & x_\gamma & x_\alpha & x_q \\ z_V & -s & z_\alpha & z_q \\ -z_V & 0 & -z_\alpha & 1 - z_q \\ m_V & 0 & m_\alpha^G & m_q \end{bmatrix} \cdot \begin{bmatrix} \delta V \\ \delta \gamma \\ \delta \alpha \\ q \end{bmatrix} = 0$$

In order not to get the unique null solution, the determinant shall be equal to zero …

We developp the determinant and get the characteristic equation :

$$s^2 + 2\lambda \cdot s + \omega_0^2 = 0$$

with

$$\omega_0^2 = \frac{g \cdot z_V}{1 - z_q} \cdot \frac{m_\alpha^G - m_V \cdot z_\alpha / z_V}{m_\alpha^G + m_q \cdot z_\alpha / (1 - z_q)}$$

Speed Neutral Point , F_V

ω_0 is a function of m_α^G , so a function of X_G , it can cancel for a certain point resulting

in a s-root equal to 0 and a divergent mode for the phugoïd…

This defines a new point, called the Speed neutral point F_V, given by

$$\boxed{m_\alpha^{F_V} - m_V \cdot z_\alpha / z_V = 0}$$

$$z_V = \frac{2g}{V^2} \qquad \boxed{\rightarrow \frac{X_{F_V}}{L} = \frac{X_F}{L} + \frac{B}{mVL} \cdot \frac{m_V}{z_V}} \qquad m_V = \left(F_V - \frac{2F}{V}\right) \cdot \frac{z_P - z_G}{B}$$

$$\boxed{\rightarrow \frac{X_{F_V}}{L} = \frac{X_F}{L} + \frac{V}{mg} \cdot \left(\frac{F_V}{2} - \frac{F}{V}\right) \cdot \frac{z_P - z_G}{L}}$$

this new point F_V drives the divergence of the Phugoïd

Stability Points Summary

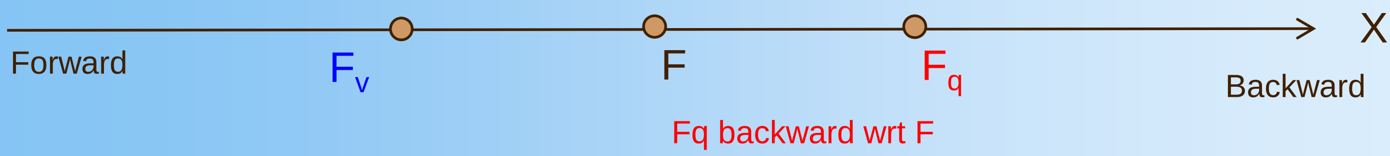
In reality, the Phugoïd can become a-periodic and divergent, a bit like the Short Period mode. It exists a point F_V, the Speed Neutral point, from which the mode becomes divergent.

F_V	$\frac{X_{F_V}}{L} = \frac{X_F}{L} + \frac{B}{mVL} \cdot \frac{m_V}{z_V}$	$\frac{X_{F_V}}{L} = \frac{X_F}{L} + \frac{V}{mg} \cdot \left(\frac{F_V}{2} - \frac{F}{V}\right) \cdot \frac{z_P - z_G}{L}$	→ Phugoîd
F		$\frac{X_F}{L} = 0{,}25 - \frac{Cm_\alpha^{25\%}}{Cz_\alpha}$	
F_q	$\frac{X_{F_q}}{L} = \frac{X_F}{L} - \frac{B}{mVL} \cdot \frac{m_q}{1 - z_q}$	$\frac{X_{F_q}}{L} \approx \frac{X_F}{L} - \frac{\rho SL}{2m} \cdot Cm_q$	→ Short Period Mode

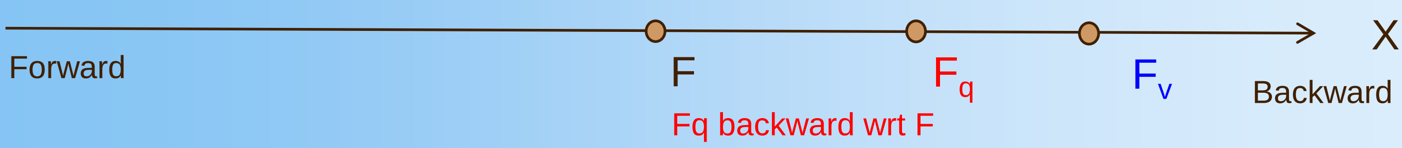
Speed Neutral Point, F_v versus F

The relative X-position of the Speed neutral point F_V with respect to the Aero centre F is function of the relative z-position of the engines with respect to G.

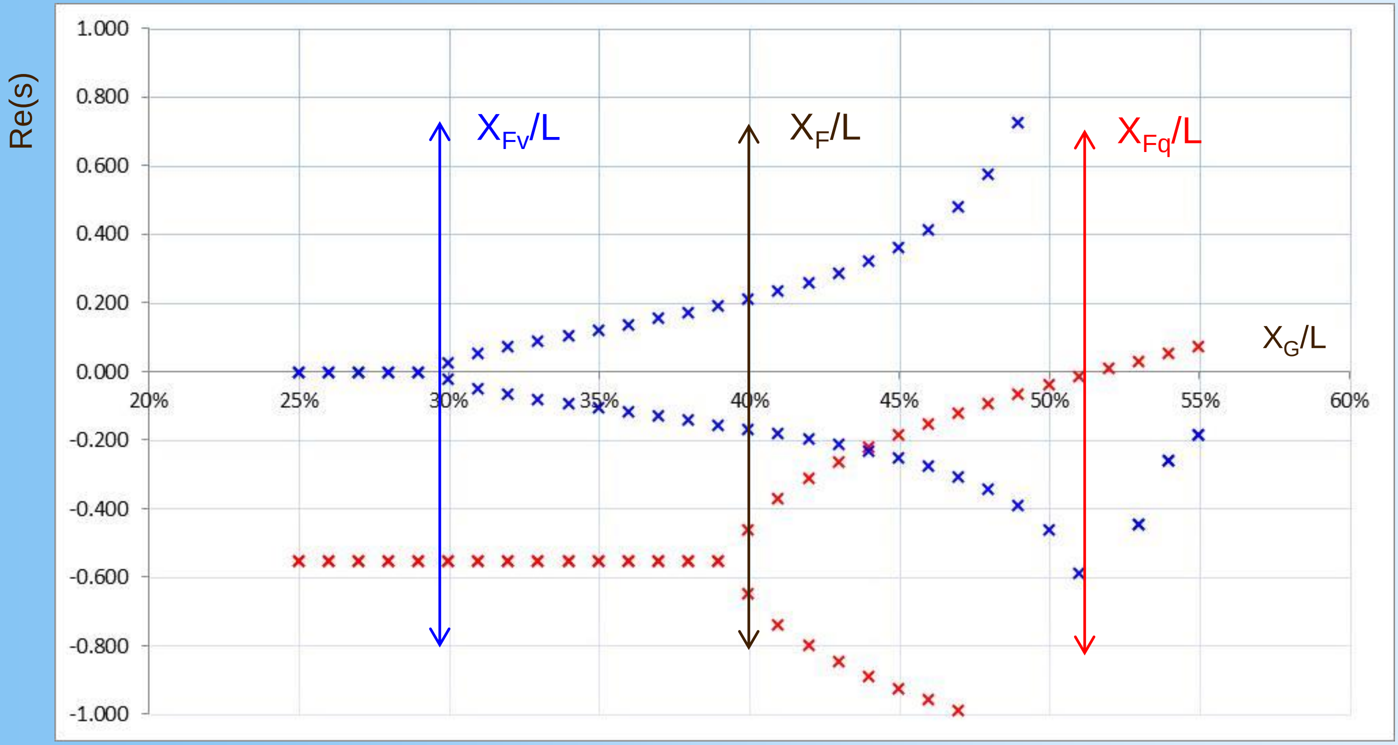
Engines below G : Fv forward wrt F



Engines above G : Fv backward wrt F



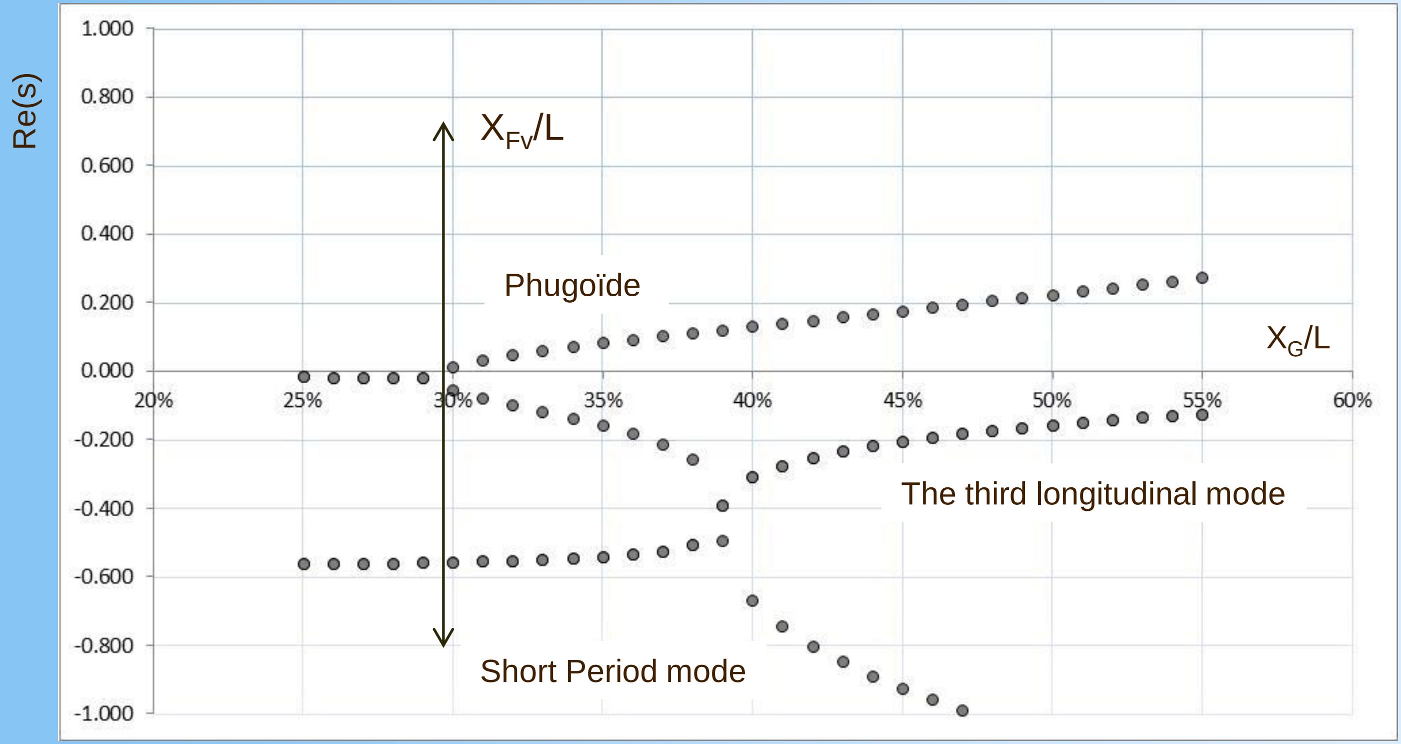
Short Period mode / Phugoïd mode



When treated separately, both longitudinal modes are clearly identified with their associated charateristic points :

- Phugoïd mode in blue with the Speed Neutral point
- Short Period mode in red with the Manoeuvre point

The reality of the Longitudinal modes



In reality, the simplified approach for the 2 separated modes ceases to be valid when interactions occur between them …

The reality of the Longitudinal modes

In reality, the simplified approach for the 2 modes ceases to be valid when interactions occur between the modes

The Phugoïd and the Short Period modes are still easyly identificable ; however, their interaction produces a Third longitudinal mode which is an oscillatory convergent mode

Notice that the aircraft will become unstable when the CG will be located aft to the Speed Neutral Point and that the notion of Manoeuver Point doesn’t exist anymore for the natural aircraft.

However the Manoeuver Point is always present in our modern aircraft because the phugoïd mode is generally strongly damped by an appropriate command law (thrust control law) which makes reveal again the Short Period mode …

Northrop Grumman Firebird

Stability versus Speed

Stability versus speed

From a trimmed situation, we produce a variation of speed
what will be the aircraft initial response in pitch $\dot{q}_{t=0}$?

We compute the initial response in pitch $\dot{q}_{t=0}$, assuming no α variation ($\dot{\alpha} = 0$) and no q variation ($q = 0$)

The set of equations to solve is : $\begin{cases} \delta\dot{\alpha} = -z_V \cdot \delta V - z_\alpha \cdot \delta\alpha = 0 \\ \dot{q}_{t=0} = m_V \cdot \delta V + m_\alpha^G \cdot \delta\alpha \end{cases}$

$$\dot{q}_{t=0} = \left(m_V - \frac{m_\alpha^G}{z_\alpha} \cdot z_V \right) \cdot \delta V = \frac{mVL}{B} \cdot \left(\frac{X_F - X_G}{L} + \frac{B}{mVL} \cdot \frac{m_V}{z_V} \right) \cdot z_V \cdot \delta V$$

Remember that : $\dfrac{X_{F_V}}{L} = \dfrac{X_F}{L} + \dfrac{B}{mVL} \cdot \dfrac{m_V}{z_V}$

Stability versus speed

From a trimmed situation, we produce a variation of speed
what will be the aircraft initial response in pitch ?

$$\Rightarrow \quad \dot{q}_{t=0} = \frac{mVL}{B} \cdot \frac{X_{F_V} - X_G}{L} \cdot z_V \cdot \delta V$$

The answer depends on the relative position of G
with respect to the Speed Neutral poinf F_V

If G is forward wrt F_V $\left(X_G < X_{F_V}\right)$,

a positive δV creates an instantaneous $\dot{q}_{t=0} > 0$,
the aircraft is rotating up, the angle of attack α is increased
the drag $0{,}5\rho V^2 SCx$ is increased and the velocity decreases $\left(\dot{V} < 0\right)$

The aircraft is stable with respect to speed (the aircraft comes back to its initial trim)

The Baghdad A300 Incident

The facts

- November 22nd, 2003. An A300-B4 cargo-transformed, S/N 094, operated by DHL, takes off from Baghdad to Bahrain.

- Trip is short, payload is light (Mail) and TOW : 100t

- A special Take Off procedure is applied to minimize ground proximity time and threat exposure duration :
  - Take off with slats only and maximum thrust
  - Early retraction of slats
  - Climb at optimum climb speed (215kts)

- Passing 8000ft, a strong impact shakes the structure. Immediately, one announces the loss of two hydraulic circuits (green and yellow).

- Twenty seconds later, the captain feels the controls stiffening as the last hydraulic circuit (blue) is lost.

The facts

A/C configuration is then the following :

- All hydraulics lost
  - Ailerons, rudder and elevators are « floating » in the wind (zero hinge moment)
  - THS is frozen
  - Spoilers are inoperative and prevented to deflect (sucked by the airflow) by a non return valve. But one of them is slightly leaking.
  - Slats and flaps configuration are retracted and frozen
- Left wing in fire and associated fuel tank is emptying
- A significative amount of the left wing surface is missing

- **BUT**: BOTH ENGINES ARE STILL RUNNING

- **QUESTION** : is it possible to control the aircraft with only engines ?

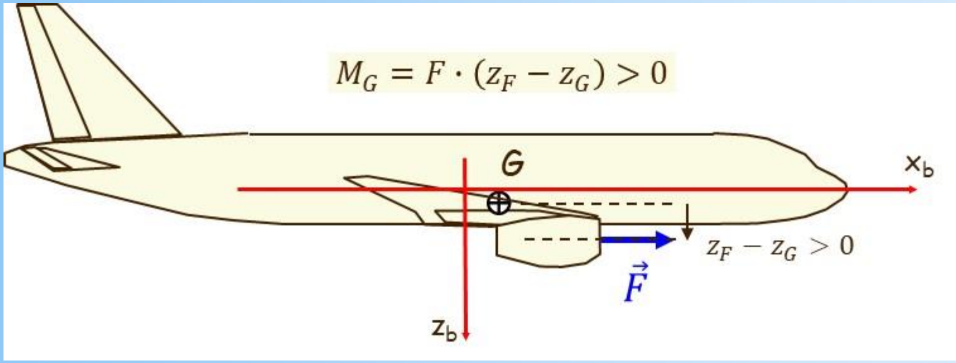
The problem to be solved

You have just the thrust command for controlling the aircraft

The aircraft is trimmed : you increased the thrust . What is the next trim ?

We have already answered to this question during the main course :

- Starting from a trim situation, you increase the thrust
- The next trim is characterised by an increase of the slope (the aircraft is climbing) keeping the same initial velocity
- However, these results were obtained with a strong assumption : the engines are at the same z-level than the CG ; this is obviously not the case of this aircraft



The problem to be solved

Starting from a trim situation, you increase the thrust.
The general lift/pitch variation equations are :

$$\begin{cases} \delta\dot{\alpha} = -z_V \cdot \delta V - z_\alpha \cdot \delta\alpha \\ \delta\dot{q} = m_V \cdot \delta V + m_\alpha^G \cdot \delta\alpha + m_{\delta x} \cdot \Delta\delta x \end{cases}$$

$$z_V = \frac{2g}{V^2} \quad m_V = \left(F_V - \frac{2F}{V}\right) \cdot \frac{z_P - z_G}{B} \qquad m_\alpha^G = \frac{\rho V^2 S L}{2B} \cdot Cm_\alpha^G = \frac{mV \cdot (X_G - X_F)}{B} \cdot z_\alpha \quad m_{\delta x} = F_0 \cdot \frac{z_P - z_G}{B}$$

The next trim is given by :

$$\begin{cases} -z_V \cdot \Delta V - z_\alpha \cdot \Delta\alpha = 0 \\ m_V \cdot \Delta V + m_\alpha^G \cdot \Delta\alpha = -m_{\delta x} \cdot \Delta\delta x \end{cases}$$

$$\Delta\alpha = \frac{m_{\delta x}/z_\alpha}{m_V/z_V - m_\alpha^G/z_\alpha} \cdot \Delta\delta x \qquad \Delta V = \frac{-m_{\delta x}/z_V}{m_V/z_V - m_\alpha^G/z_\alpha} \cdot \Delta\delta x$$

with :

$$\frac{m_V}{z_V} - \frac{m_\alpha^G}{z_\alpha} = \frac{mVL}{B} \cdot \frac{X_{F_V} - X_G}{L}$$

The problem resolution

The next trim depends on the relative position of G wrt the Speed Neutral poinf F_V

If G is forward wrt F_V : $X_G < X_{F_V} \rightarrow \frac{m_V}{z_V} - \frac{m_\alpha^G}{z_\alpha} > 0$

$$\Delta\delta x > 0 \rightarrow \Delta\alpha > 0 \;\; \& \;\; \Delta V < 0$$

a thrust increase $\Delta\delta x > 0$ creates a new trim with an increase of α and a decrease of V

If G is aft wrt F_V : $X_G > X_{F_V} \quad \rightarrow \frac{m_V}{z_V} - \frac{m_\alpha^G}{z_\alpha} < 0$

$$\Delta\delta x > 0 \rightarrow \Delta\alpha < 0 \;\; \& \;\; \Delta V > 0$$

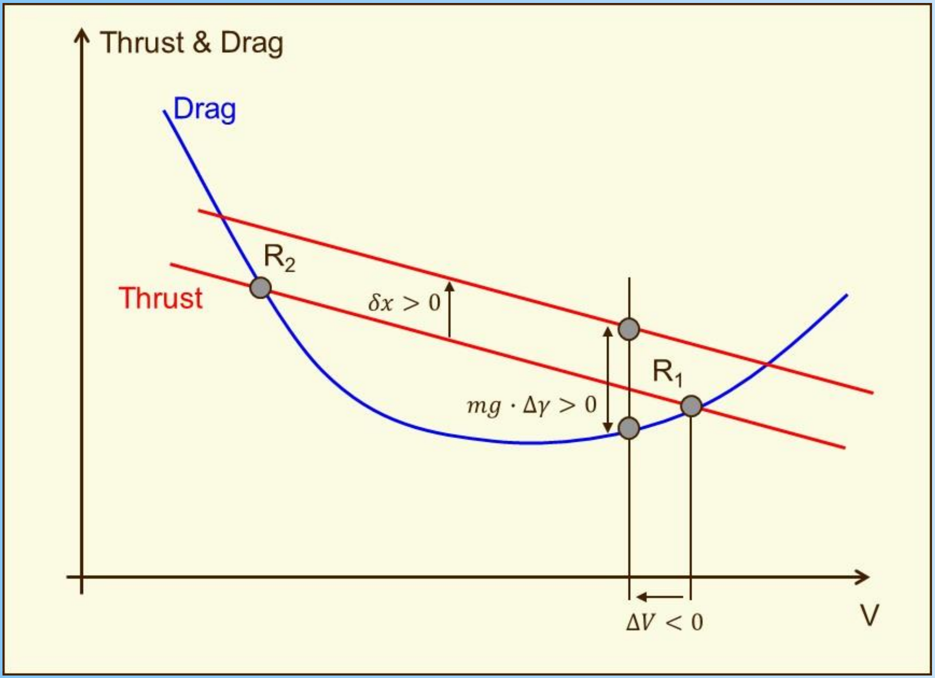
a thrust increase $\Delta\delta x > 0$ creates a new trim with a decrease of α and an increase of V

The problem resolution

What happens to the slope ? The slope variation is given by : $mg \cdot \Delta\gamma = \Delta\left\{F - \frac{1}{2}\rho V^2 SCx\right\}$

The resolution can be performed graphically

We restrict the problem : CG is fwd $\left(X_G < X_{F_V}\right)$ and V is large (first regime)



$\Delta\delta x > 0$

→ The thrust level increases

If G is forward wrt F_V

$\Delta\delta x > 0 \rightarrow \Delta V < 0$

→ the drag level decreases

Graphically, we obtain

$\rightarrow \Delta\gamma > 0$

→ as a result, the aircraft reacts « normally » , we increase the thrust and the slope increases

The Baghdad A300 incident

We come back to the Baghdad A300 incident.

The CG of the aircraft is forward : so, the aircraft is stable (versus α and V)
and the velocity is sufficiently high (first regime)
We have demonstrated that you can use the thrust for controlling your slope
(as usual, you want to land so you decrease the trust for starting your descent)

Of course,
- You cannot control anymore the speed which is now a result :

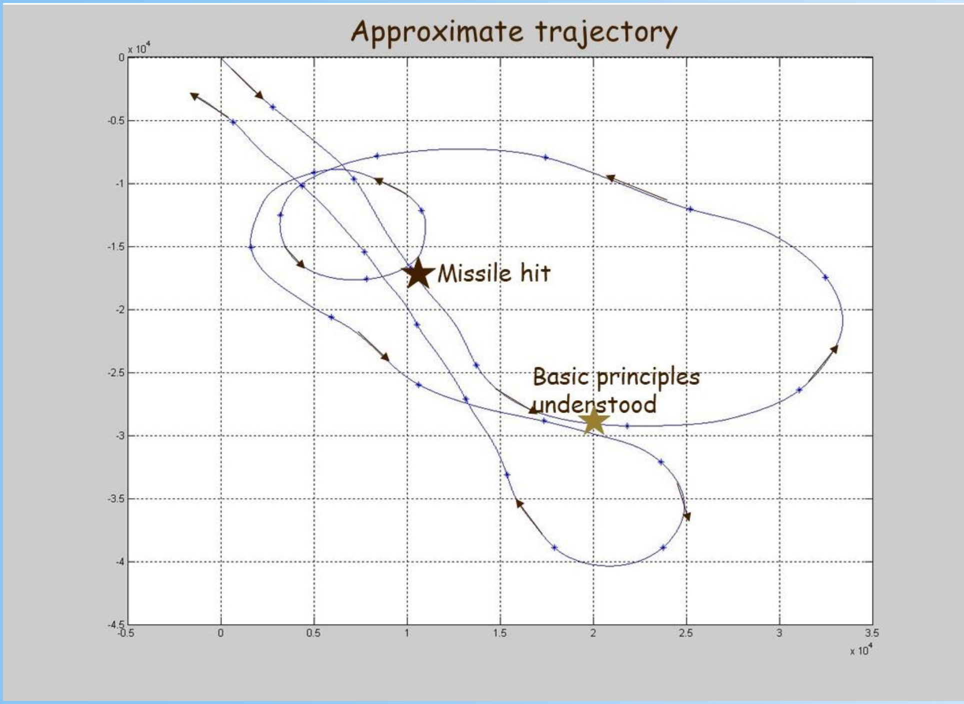
note that against intuition (and initial reaction), a thrust reduction will finally induce a speed increase and vice versa.
- You have also to live with the phugoïd which is no more articificially damped

due to the inoperant systems

But there is a chance for landing …

Take the time for understand

The behaviour of the crew was perfect : no panic, they took time to learn the new behaviour of the aircraft : they managed to control the pitch using symmetrical thrust order, extending the landing gear to limit the speed increase. Then they learned to control the roll using assymmetrical thrust order, going through some roll excursions beyond 30°.



Having learned to manage the flight path, the crew decided to attempt the landing.

Additionally the left wing was on fire and there was a fuel leak!… It was time to come back and land.

Bring the A/C to the ground

Disregarding some light damage on the landing gear (deflated tyres)

The aircraft was intact (but for the wing !)

No injuries for the crew or people on ground

Conclusion

Well done Mario, Eric, Steeve ; remarkable job !