HW2: TRAJECTORY GENERATION FOR NONLINEAR SYSTEMS

Exercise 1

This exercise is part of a series of activities leading to the implementation of an interactive MIMO control law for the `Flying-Chardonnay`, an automatic drink delivery device. This exercise exploits the MATLAB model implemented previously, and follows the following configuration its parameters (in S.I. units):

$m_d = 1$	$m_c = 1$	$l = 1$	$l_d = 1$	$J = 1$	$C_D = 0.01$	$g = 10$

1. **(10pts)** Find a trajectory $\left(\boldsymbol{x}(t), \boldsymbol{u}(t)\right)$ that **shifts the drone horizontally in** 4**m** while keeping the level of water reasonably high. Additionally, please follow the following design constraints:

   (a) Avionics (sensors/actuators) sample frequency: 50Hz;

   (b) Maneuver maximum time: 2s.

   To measure the water level and animate your trajectory, please use the MATLAB files `[Exercise Support] MATLAB: Flying-Chardonnay Animation` posted on the LMS. You can also use the `[Lecture Example] MATLAB: MPC for Automatic Rocket Landing` entry to look for inspiration.

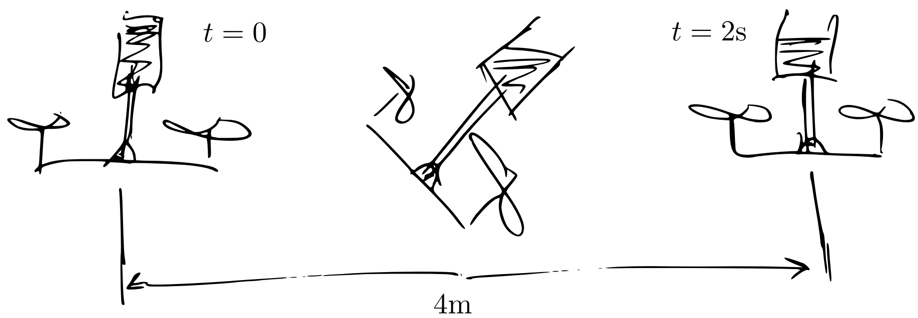

Figure 1: An trajectory example.

Exercise 2

Consider the nonlinear discrete-time system represented by the state equations:

$$
\begin{aligned}
\boldsymbol{x}_{k+1} &= \begin{pmatrix} \cos u_k & -\sin u_k \\ \sin u_k & \cos u_k \end{pmatrix} \boldsymbol{x}_k \\
\boldsymbol{y}_k &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \boldsymbol{x}_k
\end{aligned}
$$

where $\boldsymbol{x}_k \in \mathbb{R}^2$, $\boldsymbol{y}_k \in \mathbb{R}^2$, and $u_k \in \mathbb{R}$. Assume the system is initially at $\boldsymbol{x}_0 = (1, 0)$, and that its desired final position at $k = N$ is $\boldsymbol{x}_N = (0, 1)$. The final objective of this exercise will be to find **by hand** the optimal trajectory $\{(\boldsymbol{x}_0, u_0), (\boldsymbol{x}_1, u_1), \cdots, (\boldsymbol{x}_N, u_N)\}$ that minimizes the following cost function:

$$
J = \sum_{k=0}^{N} \alpha |\boldsymbol{y}_k|^2 + \beta u_k^2 \tag{1}
$$

where $\alpha > 0$ and $\beta > 0$.

1. **(7pts)** Find the optimal trajectory for a prediction horizon $N = 2$.
2. **(3pts)** Find the optimal trajectory for a prediction horizon $N = 2021$.