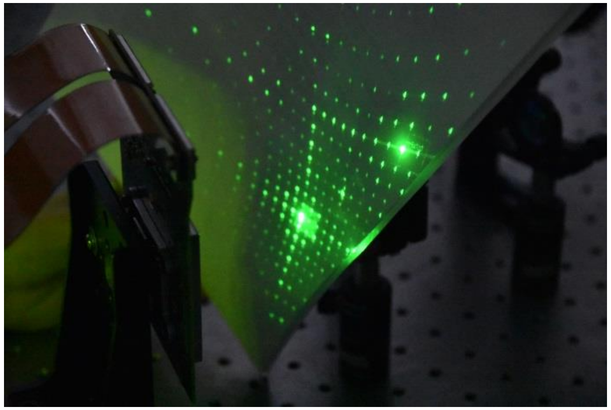
MAE 1 - ELECTROMAGNETISM

Angélique Rissons-Malivert

2020-2021

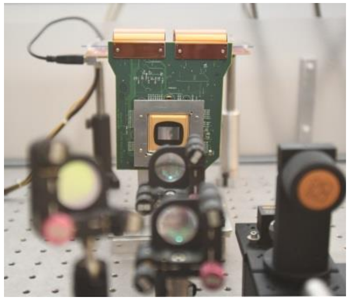
Introduction



Who am I?

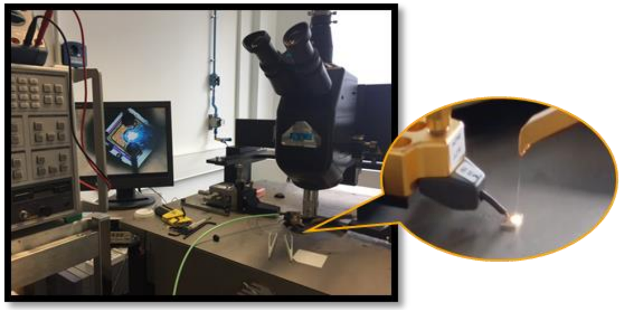
- Angélique Rissons-Malivert
- ISAE-Supaero Professor

What am I doing here?



- Academic: Microwave-Photonics teaching, Supaero 2A responsible
- Research: Optical telecommunication & Microwave-Photonics
  - Research team: PAMPA (Photonic Antenna Microwave PlasmA)
  - Research Department : DEOS (Department Electronic and Signal)

How to contact me:



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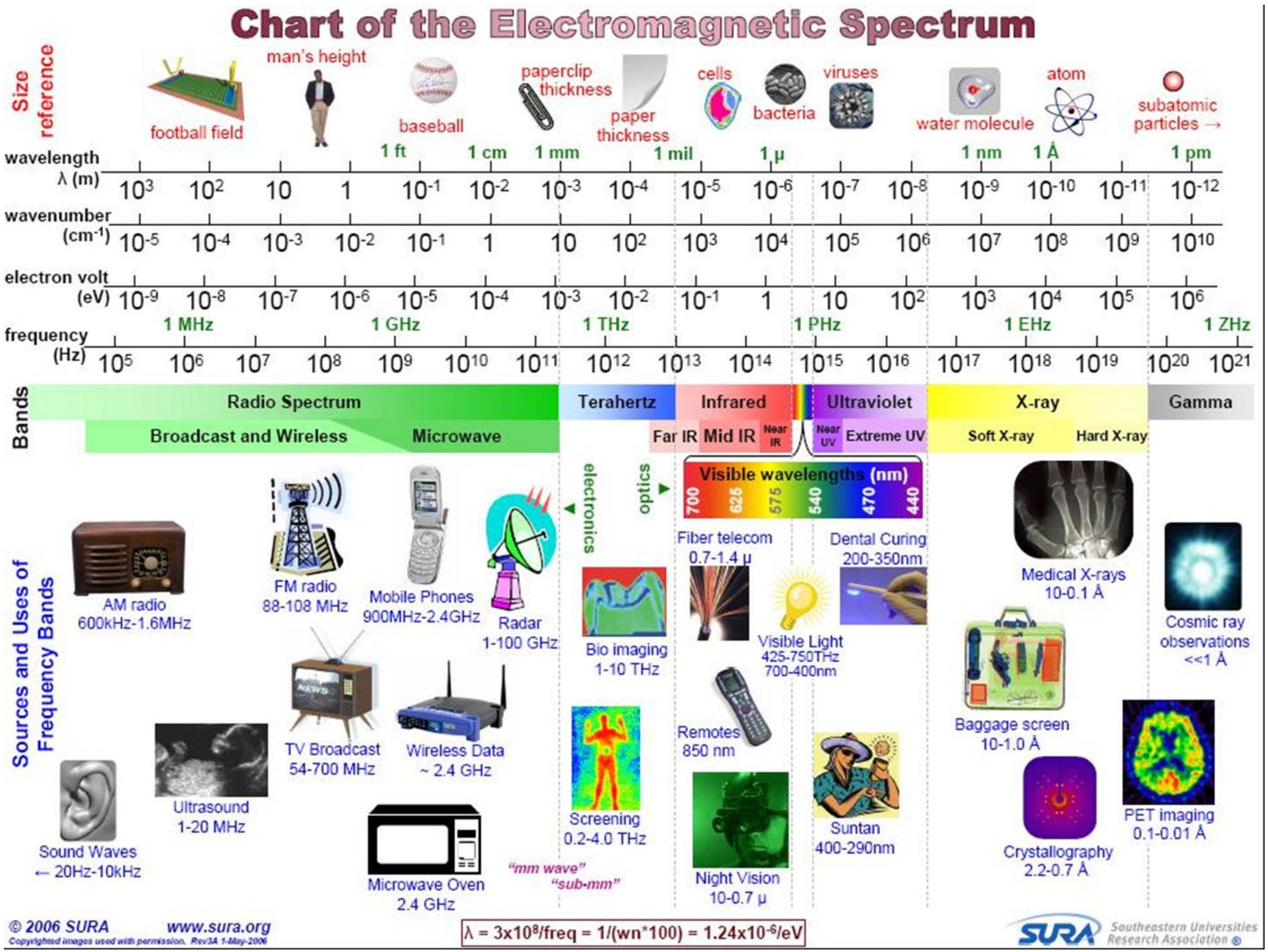
Outlines

Electromagnetism Spectrum



Electromagnetism field Quantities & Keywords

Books :

- **Foundations of Electromagnetism**, Reitz, Milford, Christy.
- **Equations de Maxwell Ondes électromagnetiques**, M. &N. Hulin, Perrin
- **Microondes Volume 1 & 2**, Paul F. Combes

Physical Quantities:

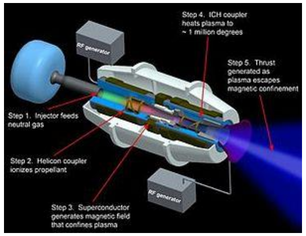
- $\vec{E}$ electric field vector, $\vec{B}$ magnetic Field Vector, $\overrightarrow{H} = \frac{\vec{B}}{\mu_0}$ magnetic intensity vector, $\vec{D} = \varepsilon_0 \vec{E}$ electric displacement vector
- λ Wavelength From Microwave(few meters) to visible optics (500nm)
- f or ν Frequency from MHz to PHz
- ϵ_0 vacuum permittivity ($8.854 \times 10^{-12}\ F/m$)
- μ_0 vacuum permeability ($4\pi \times 10^{-7}\ H/m$)
- c speed of the light ($3 \times 10^{-8}\ m/s$)

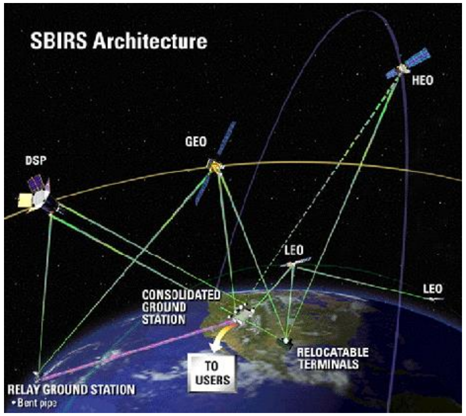
Keywords:

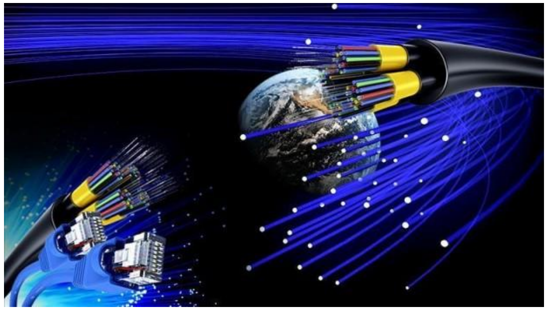
- Electrical & magnetic coupling or interaction
- Microwave
- Radiofrequency
- Induction
- Wave Propagation
- Wave corpuscular duality
- Plasma
- Optics and Photonics

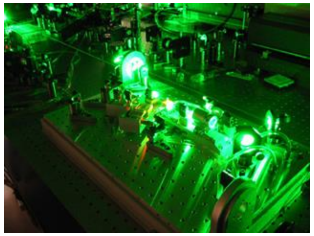
Application Field - Physical layer and Technologies

Instruments (Attitude control), Radar and Lidar, Communications and Navigation, Telemeasurement Astronomy, Structural Health Monitoring, Scientific space mission (Curiosity, PHARAO,…)

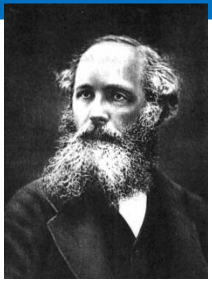








Outlines



From Electrostatic and Magnetostatic laws

Steady state Electromagnetic Field:

$div\,\vec{E} = \frac{\rho}{\varepsilon_0}$ or $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ Gauss Law

$div\,\vec{B} = 0$ or $\vec{\nabla} \cdot \vec{B} = 0$ General Magnetism Law

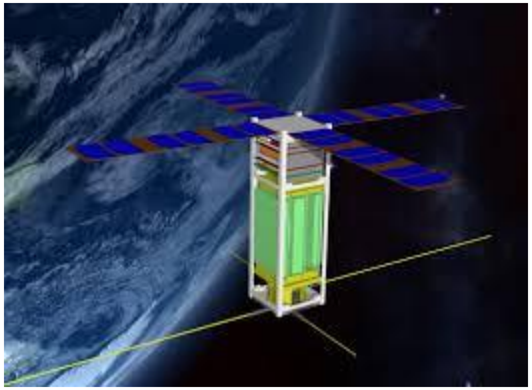
$\overrightarrow{curl}\vec{E} = 0$ or $\vec{\nabla} \times \vec{E} = 0$ Faraday Law

$\overrightarrow{curl}\vec{B} = \mu_0\vec{J}$ or $\vec{\nabla} \times \vec{B} = \mu_0\vec{J}$ Ampere Law

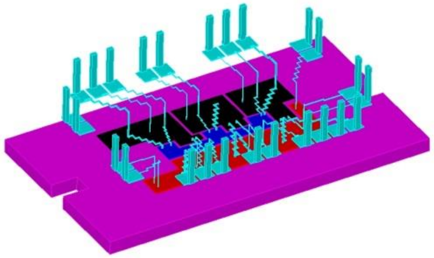
Applications

- Faraday Law Induced Current
  - the cycle dynamo

- Ampere Law Displacement Charge Magnetorqer/ Foucault Current



- EMC, Inductive coupling in an electronic board



Time-Varying Maxwell Equations

(1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ Local Gauss Law (electric flux density)

(2) $\vec{\nabla} \cdot \vec{B} = 0$ General Magnetism Law

(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faraday Law (relationship between Electric and Magnetic Field)

(4) $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$ Ampere law (current flow in a wire creating a magnetic field)

B and E resolution of equations cf. MemOperator

Double cross product of Maxwell-Faraday equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}) - \Delta \vec{E}$$

$$-\vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

Maxwell Ampere $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$

$$\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}) - \Delta \vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\boxed{\Delta \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0} \vec{\nabla} \rho + \mu_0 \frac{\partial \vec{j}}{\partial t}}$$

B and E resolution of equations cf. MemOperator

Double cross product of Maxwell-Faraday equation

$$\vec{\nabla} \times \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}) - \Delta \vec{E}$$

$$-\vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

Maxwell Ampere $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$

$$\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}) - \Delta \vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right)$$

Maxwell-Gauss eq.

$$\frac{\rho}{\varepsilon_0}$$

$$\boxed{\Delta \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0} \vec{\nabla} \rho + \mu_0 \frac{\partial \vec{j}}{\partial t}}$$

B and E resolution of equations cf. MemOperator

Double cross product of Maxwell-Faraday equation

$$\vec{\nabla}\times \quad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{E}) - \Delta\vec{E}$$

$$-\vec{\nabla}\times\left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

Maxwell Ampere $\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{j} + \epsilon_0\frac{\partial \vec{E}}{\partial t}\right)$

$$\vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{E}) - \Delta\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\vec{j} + \varepsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}\right)$$

Maxwell-Gauss eq.

$$\frac{\rho}{\varepsilon_0}$$

$$\boxed{\Delta\vec{E} - \varepsilon_0\mu_0\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0}\vec{\nabla}\rho + \mu_0\frac{\partial \vec{j}}{\partial t}}$$

B resolution

Double cross product of Maxwell-Ampere equation

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{B}) - \Delta \vec{B}}$$

$$\boxed{\vec{\nabla} \times \left(\mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right) = \left(\mu_0 \vec{\nabla} \times \vec{j} + \varepsilon_0 \mu_0 \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t}\right)}$$

B field conservation

$$\boxed{\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{E}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial \vec{B}}{\partial t}\right)}$$

Maxwell-Faraday eq.

$$\boxed{-\Delta \vec{B} = \mu_0 \vec{\nabla} \times \vec{j} - \varepsilon_0 \mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{B}}{\partial t}\right)}$$

$$\boxed{\Delta \vec{B} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu_0 \vec{\nabla} \times \vec{j}}$$

B resolution

Double cross product of Maxwell-Ampere equation

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{B}) - \Delta \vec{B}$$

B field conservation

$(\vec{\nabla} \cdot \vec{B}) = 0$

$$\vec{\nabla} \times \left(\mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \right) = \left(\mu_0 \vec{\nabla} \times \vec{j} + \varepsilon_0 \mu_0 \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t} \right)$$

$$\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) = - \frac{\partial}{\partial t} \left(\frac{\partial \vec{B}}{\partial t} \right)$$

Maxwell-Faraday eq.

$$-\Delta \vec{B} = \mu_0 \vec{\nabla} \times \vec{j} - \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \left(\frac{\partial \vec{B}}{\partial t} \right)$$

$$\boxed{\Delta \vec{B} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu_0 \vec{\nabla} \times \vec{j}}$$

B and E resolution of equations cf. MemOperator

$\overrightarrow{\nabla}$ Double Cross Product of $\overrightarrow{(E\ or\ B)}$

$\overrightarrow{\nabla}$ Cross product of the Differentiating $\overrightarrow{(B\ or\ E)}$

Navier Equation form

Introduce $\overrightarrow{\nabla}$ dot product of $\vec{A}$

$$\Delta\vec{A} - \varepsilon_0\mu_0\frac{\partial^2\vec{A}}{\partial t^2}$$

$$\Delta\vec{E} - \varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0}\vec{\nabla}\rho + \mu_0\frac{\partial\vec{j}}{\partial t}$$

$$\Delta\overrightarrow{B} - \varepsilon_0\mu_0\frac{\partial^2\overrightarrow{B}}{\partial t^2} = -\mu_0\vec{\nabla}\times\vec{j}$$

Results I Electromagnetism - Maxwell Equation(Spatial/temporal equations)

(1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ Local Gauss Law (electric flux density)

(2) $\vec{\nabla} \cdot \vec{B} = 0$ General Magnetism Law

(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$ Faraday Law (relationship between E & B Field)

(4) $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$ Ampere law (current flow in a wire creating a B field)

E & B Resolution

$$\Delta\vec{E} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0}\vec{\nabla}\rho + \mu_0 \frac{\partial \vec{j}}{\partial t}$$

$$\Delta\vec{B} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu_0 \vec{\nabla} \times \vec{j}$$

Electromagnetic Wave Equation?

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