HW4: Linear Systems Descriptions

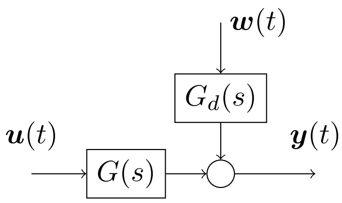
Exercise 1 (Frequency Domain Models)

This exercise is part of a series of activities leading to the implementation of an interactive MIMO control law for the `Flying-Chardonnay`, an automatic drink delivery device. This exercise exploits the MATLAB model implemented previously, and follows the following configuration its parameters (in S.I. units):

$m_d = 1$	$m_c = 1$	$l = 1$	$l_d = 1$	$J = 1$	$C_D = 0.01$	$g = 10$

1. **(5pts)** Using the hovering linear state-space model approximation computed in the previous exercise, find the system transfer matrix $G(s)$, assuming full-state output, i.e., $\boldsymbol{y}(t) = \boldsymbol{x}(t)$.

2. **(5pts)** Compute the disturbance transfer matrix $G_d(s)$, assuming the disturbance architecture below.



Exercise 2 (Linear Model Representations)

Consider the following state-space system:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} \boldsymbol{u} \tag{1}$$

$$\boldsymbol{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{u} \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^3$, $\boldsymbol{u} \in \mathbb{R}^2$ and $\boldsymbol{y} \in \mathbb{R}^2$. Unless otherwise stated, answer the following questions **by hand**.

1. **(2pts)** Find the equivalent Impulse Matrix $H(t)$ description.

2. **(2pts)** Find the equivalent Markov Parameters H_k description.

3. **(2pts)** Find the equivalent Transfer Matrix $H(s)$ description.

4. **(2pts)** Using the MATLAB command `tf(ss(A,B,C,D))`, compute the equivalent Transfer Matrix $H(s)$ description. How does this compare with the one computed by hand?

Exercise 3 (Noncommutative Property of MIMO Systems)

Consider the following transfer matrices:

$$G_1(s) = \begin{bmatrix} 0 & \frac{-1}{s} \\ 0 & \frac{2}{s} \end{bmatrix} \quad G_2(s) = \begin{bmatrix} \frac{3}{s} & \frac{5}{s} \\ 0 & 0 \end{bmatrix} \tag{3}$$

1. **(1pts)** Compute $G_1(s)G_2(s)$ and $G_2(s)G_1(s)$. Is $G_1(s)G_2(s) = G_2(s)G_1(s)$?

2. **(1pts)** Based on that, could one interchange the arrangement of the blocks in a series connection block diagram? What about in a parallel connection diagram?