# Representation and Analysis of Dynamic Systems
# Lab 1

## Exercise 1: basic properties of the Laplace transform

1.1 Given a signal u(t) and its Laplace transform U(s) write the delay theorem, the final value theorem, the initial value theorem, the derivation theorem and the integration theorem.

*Just write the equations:*

$$u(t-t_0) \Rightarrow e^{-t_0 s}U(s)$$

$$u(\infty) = \lim_{s\to 0} sU(s)$$

$$u(0) = \lim_{s\to\infty} sU(s)$$

$$\frac{du(t)}{dt} \Rightarrow sU(s) - u(0^+)$$

$$\int_0^t u(t)dt \Rightarrow \frac{1}{s}U(s)$$

1.2 Give the Laplace transform of the three basic signals $u(t)$: impulse, unit step and exponential $u(t) = e^{-\frac{t}{\tau}}$

*Just write the equations:*

| | |
|---|---|
| $\delta(t)$ | 1 |
| 1 | $\frac{1}{s}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |

1.3 Compute the Laplace transform of $u(t) = \cos(\omega t)$ (hint : express $cos$ as the sum of two exponentials)

$$\cos(\omega t) = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$U(s) = \frac{1}{2}\left(\frac{1}{s+i\omega} + \frac{1}{s-i\omega}\right) = \frac{s}{(s+i\omega)(s-i\omega)} = \frac{s}{s^2+\omega^2}$$

Solution: it's not really an exercise, just copying formulas from the course (or any other source!) It must be stressed that formulas of 1.1 and 1.2 are very basic and much better to be well known

## Exercise 2: basic exercises with the Laplace transform

A system with input $u(t)$ and output $y(t)$ (Laplace transforms $U(s)$ and $Y(s)$) is known by a simple differential equation:

$$a\frac{dy(t)}{dt} + y(t) = u(t)$$

Initial condition: $y(0) = 0$

2.1 What is the unit of a? For the following take $a = 2$

Solution: dimension of a: time

2.3 Compute the transfer function $F(s) = \frac{Y(s)}{U(s)}$ and create the transfer function F with Matlab

*When considering initial condition:*

$$Y(s) = \frac{1}{1+as}U(s) + \frac{ay(0)}{1+as}$$

*Remark: both terms have the same denominator*

*Transfer function is by definition $F(s) = \frac{1}{1+as}$ ("transfer between u and y") the other term is for initial condition*

Solution: $F(s) = \frac{1}{1+a\,s}$

>> F = tf(…)

Hint :

>> s = tf('s'); % Create the transfer function s, can be used just like a symbolic operator

>> F = 1/(1+a*s) ;

Hint : explore F (look into the structure, especially F.num and F.den)

2.4 Compute the output $y(t)$ when the input is an impulse at $t = 0$. Compare with the impulse response obtained with Matlab (**impulse**)

$$U(s) = 1$$

$$Y(s) = \frac{1}{1+as}$$

$$= \frac{\frac{1}{a}}{\frac{1}{a}+s}$$

$$y(t) = 1/ae(-\frac{t}{a})$$

Solution: compare result obtained with the formula and the one obtained with Matlab.

2.5 Compute the output $y(t)$ when the input is unit step at $t = 0$. Compare with the step response obtained with Matlab (**step**)

$$U(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{1+as}\frac{1}{s} = \frac{Y1}{1+as} + \frac{Y2}{s}$$

$$Y1 = \lim((1+as)Y(s)\ quand\ s \to -\frac{1}{a} = -a$$

$$Y(s) = -\frac{a}{1+as} + \frac{1}{s}$$

$$= -\frac{1}{\frac{1}{a}+s} + \frac{1}{s}$$

$$y(t) = 1 - e(-t/a)$$

## Exercise 3: a bit more complicated with the Laplace transform

3.1 Compute the output $y(t)$ when the input $u(t)$ is an impulse for the following cases:

$$Y_1(s) = \frac{1}{s(1+2s)}U(s), Y_2(s) = \frac{1}{s(1-3s)}U(s), Y_3(s) = \frac{1}{(1+10s)(1+s)}U(s), Y_4(s) = \frac{1+2s}{(1+10s)(1+s)}U(s),$$
$$Y_5(s) = \frac{1-2s}{s(1+10s)}U(s)$$

Hint: decompose the rational fractions as a sum of simple ones

3.2 Plot $y(t)$ using Matlab

3.3 Compute the output $y(t)$ when the input $u(t)$ is unit step for the following case:

$$Y_1(s) = \frac{1}{1+2s}U(s),$$

Hint: the step response of $\frac{1}{1+2s}$ is the impulse response of $\frac{1}{s(1+2s)}$

## Exercise 4: bloc-diagram manipulation

A system is described by a bloc diagram as in figure 1. This system as two inputs (reference $r(t)$, and perturbation $p(t)$) and one output y$(t)$.

4.1 Find $F_1(s)$, $F_2(s)$, $F_3(s)$, and $F_4(s)$ such as:

$$Y(s) = F_1(s)R(s) + F_2P(s)$$
$$U(s) = F_3(s)R(s) + F_4P(s)$$

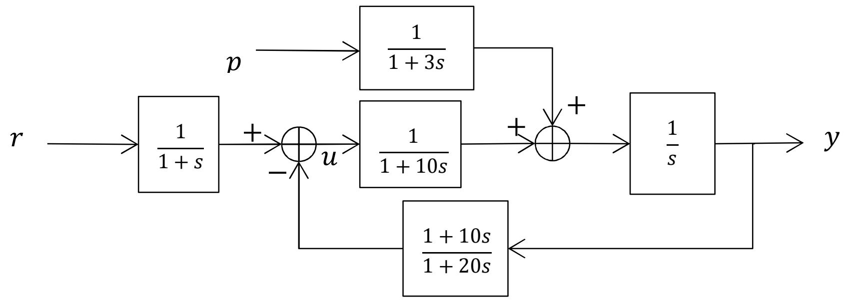


Figure 1

solution: no systematic method, just writing the equations. For transfer from $r \rightarrow y$ set $p = 0$

*Matlab result:*

*F1 =*

*0.1 s + 0.005*

*--------------------------------------------*

*s^4 + 1.15 s^3 + 0.205 s^2 + 0.06 s + 0.005*

*F2 =*

*0.3333 s + 0.01667*

*--------------------------------------*

*s^3 + 0.3833 s^2 + 0.06667 s + 0.01667*

*F3 =*

*s^2 + 0.05 s*

*-----------------------------*

*s^3 + 1.05 s^2 + 0.1 s + 0.05*

*F4 =*

*-0.1111 s - 0.005556*

*-----------------------------------------------------*

*s^4 + 0.7167 s^3 + 0.1944 s^2 + 0.03889 s + 0.005556*

4.2 Is the system stable?

*Matlab: pole(F)*

*System stable because each pole in the left hand plane*

*Remark: the system is made by interconnection of stables systems, but results in something unstable…*

4.2 Plot $u(t)$ and $y(t)$ when the input $r(t)$ is a unitary step. Idem when the perturbation $p(t)$ is a unitary step. Find the final value using Matlab and by formal calculation (final value theorem).

## Exercise 5: final and initial value theorem

We want to analyze the time response $y(t)$ of a system to a unitary step impulse. The transfer function is:

$$F(s) = \frac{1-s}{(1+2s)(1+10s)}$$

1. What is the final value of $y(t)$?

$y(\infty) = \lim_{s\to 0}\left(s\frac{1}{s}F(s)\right) = 1$

2. Analyze the time response near $t = 0$

initial value => $y(0)$

$y(0) = lim_{s\to\infty}\left(s\frac{1}{s}F(s)\right)$ =0

tangent => initial value of $y'(0)$

$y'(0) = lim_{s\to\infty}\left(s\frac{1}{s}F(s)s\right) = -1/20$ *tangent is negative, this is due to negative zero (non minimum phase system)*

negative transient, due to positive zero (root of the denominator). Makes control difficult.

3. Plot the time response

Matlab, compare with the step response of $\frac{1}{1+10s}$