

# Representation and Analysis of Dynamical Systems

## Test – 40min – without documentation

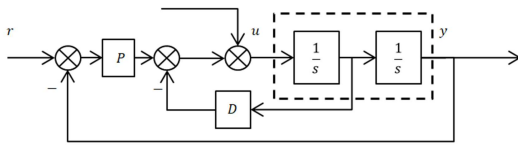
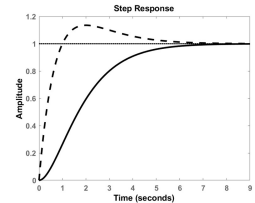
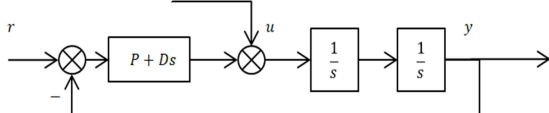
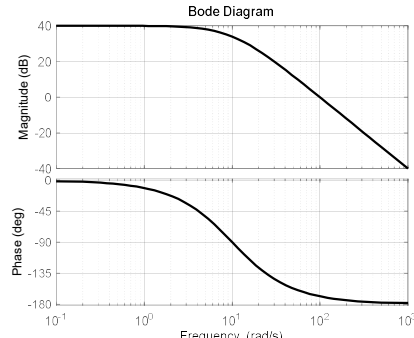
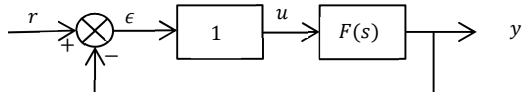
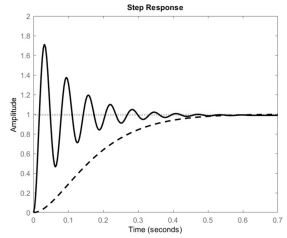
Questions may have more than one positive answer

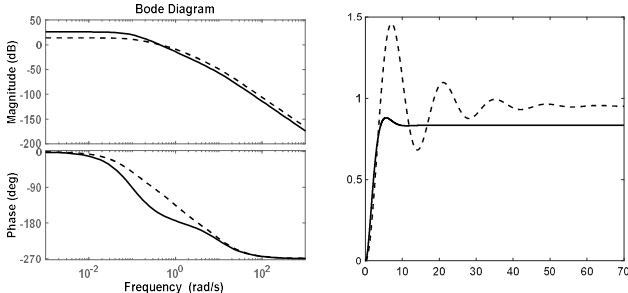
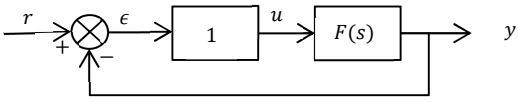
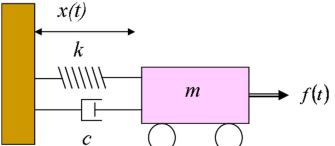
true: 1 points (correct answer is A and you checked A)

partially true: 0,5 points (correct answer is AB and you checked A or B)

false: -0.1 points (correct answer is A and you checked B, or correct answer is AB and you checked AC)

1	We consider a ramp signal: $u(t) = t$	The Laplace transform $U(s)$ of $u(t)$ is: a : $U(s) = 1$ b : $U(s) = 1/s$ c : $U(s) = 1/s^2$
2	The Laplace transform of a signal $u(t)$ is: $U(s) = \frac{1}{10 + s}$	The final value (at $t = \infty$ ) of $u(t)$ is: a : 10 b : 1 c : 0.1 d: none (checking nothing or d:+1)
3	The transfer function of a system is: $U(s) = \frac{1}{10 + s}$	The static gain is: a : 10 b : 1 c : <b>0.1</b>
4	The transfer function of a system is: $U(s) = \frac{1}{10 + s}$	The time constant is: a : 10 s b : 1 s c : <b>0.1 s</b>
5	The transfer function of a system is: $U(s) = \frac{1}{10 + s}$	The bandwidth is: a : <b>10 rad/s</b> b : 1 rad/s c : 0.1 rad/s
6	A linear system (input $u(t)$ , output $y(t)$ ) is driven by the differential equation : $y''(t) - 2y'(t) - 3y(t) = u'(t) + u(t)$ (Initial condition $y(0) = u(0) = 0$ )	The transfer function is: a : $F(s) = \frac{s+1}{s^2-2s-3}$ b : $F(s) = \frac{1}{s-3}$ b : $F(s) = \frac{1}{s^2-2s-3}$
7	A linear system (input $u(t)$ , output $y(t)$ ) is driven by the differential equation : $y''(t) - 2y'(t) - 3y(t) = u'(t) - 3u(t)$ (Initial condition $y(0) = u(0) = 0$ )	The transfer function is $F(s) = \frac{1}{s+1}$ a : <b>True</b> b : False
8	A linear system (input $u(t)$ , output $y(t)$ ) is driven by the differential equation : $y''(t) - 2y'(t) - 3y(t) = u'(t) - 3u(t)$	The system is stable: a: True b : <b>False. The system has an unstable pole at <math>s=3</math>. This pole is cancelled in the transfer function (loss of observability or controllability) but the system remains unstable.</b>

9	<p>Consider the system (controller <math>P = 1</math> ; <math>D = 2</math>, system <math>F = 1/s^2</math>):</p> 	<p>The input <math>r(t)</math> is a unitary step. The output <math>y(t)</math> is:</p>  <p><b>a : solid</b> <b>b : dashed</b></p>
10	<p>Consider the system (controller <math>P = 1</math> ; <math>D = 2</math>, system <math>F = 1/s^2</math>):</p> 	<p>The damping of the closed loop system is:</p> <p><b>a : <math>\sigma = 0.5</math></b> <b>b : <math>\sigma = 1</math></b></p> <p>The natural frequency of the closed loop system is :</p> <p><b>c : <math>\omega_0 = 1</math></b> <b>d : <math>\omega_0 = 0.1</math></b></p>
11	<p>The Bode plot of a transfer function <math>F(s)</math> is given:</p> 	<p>The corresponding transfer function is:</p> <p><b>a : <math>F(s) = \frac{100}{1+0.1s}</math></b> <b>b : <math>F(s) = \frac{1}{1+0.1s}</math></b> <b>c : <math>F(s) = \frac{100}{s^2+20s+100}</math></b> <b>d : <math>F(s) = \frac{10000}{s^2+20s+100}</math></b></p>
12	<p>The system given by the last question is included in a closed loop such as:</p>  <p>And the step response is:</p> 	<p>The correct step response (input <math>r(t)</math> is a step) is:</p> <p><b>a : solid</b> <b>b : dash</b></p>
13	<p>Consider the transfer function of a system:</p> $F(s) = \frac{1}{1-s+s^2}$	<p>The system is stable</p> <p><b>a : Yes</b> <b>b : No</b></p>
14	<p>Consider the transfer function of a system:</p> $F(s) = \frac{1}{1-s+s^2}$	<p>The system can be stabilized with a pure proportional controller:</p> <p><b>a : Yes</b> <b>b : No</b></p>

15	<p>We give the Bode diagram (open loop) and step response (in closed loop) of two systems (solid line and dashed line):</p> 	<p>The system with dashed line (resp. solid) of the Bode diagram corresponds to the system with dashed line (resp. solid) of the step response:</p> <p>a : Yes b : No</p>
16	<p>A system is given by its transfer function:</p> $F(s) = \frac{1 - 2s}{1 + s + s^2}$ 	<p>This system is stable <b>in open loop</b> (input <math>u(t)</math> output <math>y(t)</math>):</p> <p>a : Yes b : No</p> <p>This system is stable <b>in closed loop</b> (input <math>r(t)</math> output <math>y(t)</math>):</p> <p>c : Yes d : No</p>
17	<p>A system (input <math>u(t)</math> output <math>y(t)</math> internal state <math>x(t)</math>) is driven by the state space equation:</p> $\begin{cases} \dot{x}(t) = -3x(t) - x(t)^2 + u^2(t) \\ y(t) = x(t)^2 \end{cases}$ <p>The state and output variation near the equilibrium point <math>(U_0, X_0, Y_0)</math> are <math>(\delta u(t), \delta x(t), \delta y(t))</math> such as:</p> $\begin{aligned} u(t) &= U_0 + \delta u(t) \\ x(t) &= X_0 + \delta x(t) \\ y(t) &= Y_0 + \delta y(t) \end{aligned}$ <p>The system is trimmed at the equilibrium point corresponding to <math>u(t) = U_0 = 2</math>.</p>	<p>The equilibrium point corresponds to <math>\begin{cases} U_0 = 2 \\ X_0 = 2 \end{cases}</math></p> <p>a : true b : false</p> <p>The linearized dynamic state space equation near the equilibrium point is</p> $\begin{cases} \delta \dot{x}(t) = -5\delta x(t) + 2\delta u(t) \\ \delta y(t) = -2\delta x(t) \end{cases}$ <p>c : true d : false</p> <p><b>this question could not be answered without knowing the equilibrium point. Suppose we take <math>U_0 = 2, X_0 = 1</math> as an equilibrium point then the linearized state space equation is:</b></p> $\begin{cases} \delta \dot{x}(t) = -5\delta x(t) + 4\delta u(t) \\ \delta y(t) = -2\delta x(t) \end{cases}$ <p><b>(question not rated)</b></p>
18		<p>The kinetic energy is</p> <p>a : <math>E_k = \frac{1}{2} k x^2</math> b : <math>E_k = \frac{1}{2} m \dot{x}^2</math> c : <math>E_k = 0</math></p>

19	<p>We consider an electrical system with input <math>u(t)</math> and output <math>y(t)</math>. The corresponding differential equations are:</p> $\begin{cases} u(t) = Ri(t) + L \frac{di(t)}{dt} + v_c(t) \\ y(t) = v_c(t) + R i(t) \\ C \frac{dv_c(t)}{dt} = i(t) \end{cases}$	<p>A possible representation of the mechanical system is (with <math>X = [x_1(t) \ x_2(t)]^t</math>):</p> <p>a : <math display="block">\begin{cases} \dot{X} = \begin{bmatrix} -\frac{R}{L} &amp; \frac{1}{L} \\ \frac{1}{C} &amp; 0 \end{bmatrix} X + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t) \\ y(t) = [R \ 1]X \end{cases}</math></p> <p>b : <math display="block">\begin{cases} \dot{X} = \begin{bmatrix} -\frac{R}{L} &amp; -1 \\ \frac{1}{L} &amp; 0 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} R &amp; \frac{1}{C} \end{bmatrix} X \end{cases}</math></p> <p>c : <math display="block">\begin{cases} \dot{X} = \begin{bmatrix} R &amp; L \\ 1 &amp; R \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y(t) = [C \ 0]X \end{cases}</math></p> <p><b>d: none</b></p>
20	<p>We consider linear system represented by a state space equation:</p> $\begin{cases} \dot{X} = [-1]X + [1]u(t) \\ y(t) = [1]X \end{cases}$	<p>This system is stable:</p> <p><b>a : yes</b></p> <p><b>b : no</b></p>