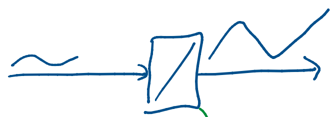
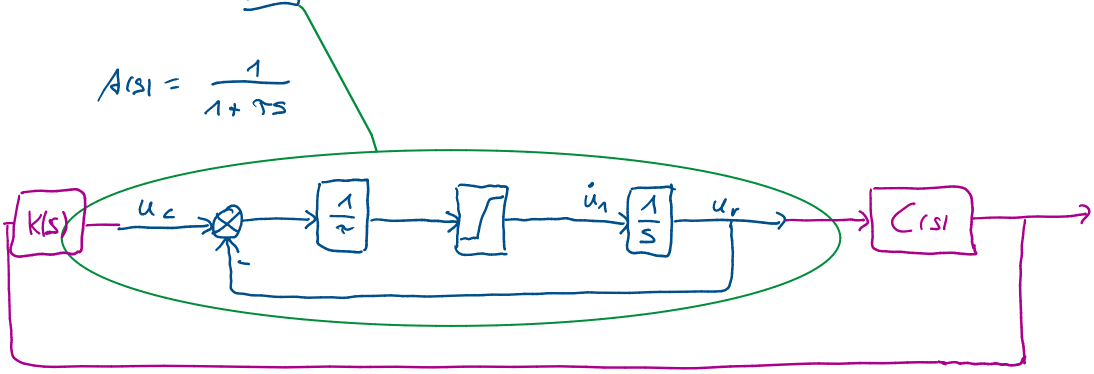
Topic_2

Dienstag, 31. Januar 2023 09:33



$$A(s) = \frac{1}{1+\tau s}$$



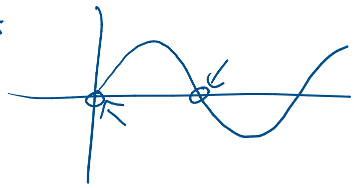
$\dot{x} = \sin(x)$

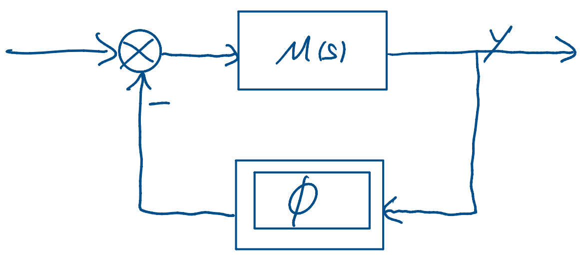
$x_0 = k\pi$ equilibrium points

$x_0 = 0$ $\quad$ $x_\pi = \pi$

$\sin(x) \approx x$ $\quad$ $\xi = x - x_\pi$

$\dot{x} = x$ $\quad$ $\dot{\xi} = -\xi$



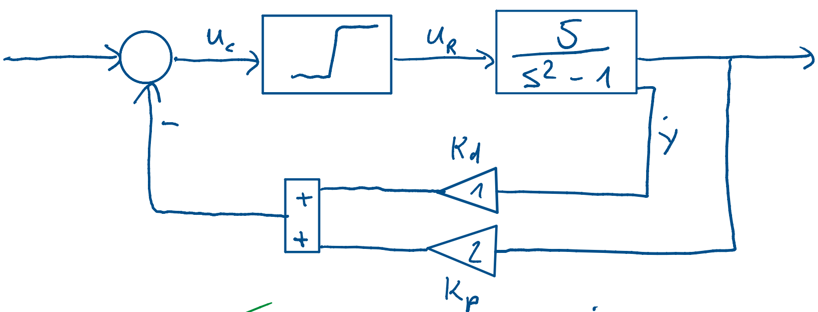


$$M(s) = C(sI - A)^{-1}B + \cancel{D}$$

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$w = -\phi(y) = -\phi(Cx)$$



$$u_c = \cancel{K_c} \cdot \theta_c - K_p\,\theta - K_d \cdot \dot{\theta}$$

$$\begin{cases} x_1 = y \\ x_2 = \dot{y} \end{cases} \qquad \boxed{\dot{x}_1 = x_2}$$

$$\frac{Y}{u_R} = \frac{5}{s^2-1} \Rightarrow \ddot{y} - y = 5u_R$$

$$\boxed{\ddot{y} = y + 5u_R}$$

$$\dot{x}_2 = x_1 + 5u_R$$

$$u_1 = \mathrm{sat}(u_c) = -\mathrm{sat}(2x_1 + x_2)$$

$|2x_1 + x_2| \leq 1$ sat. not active

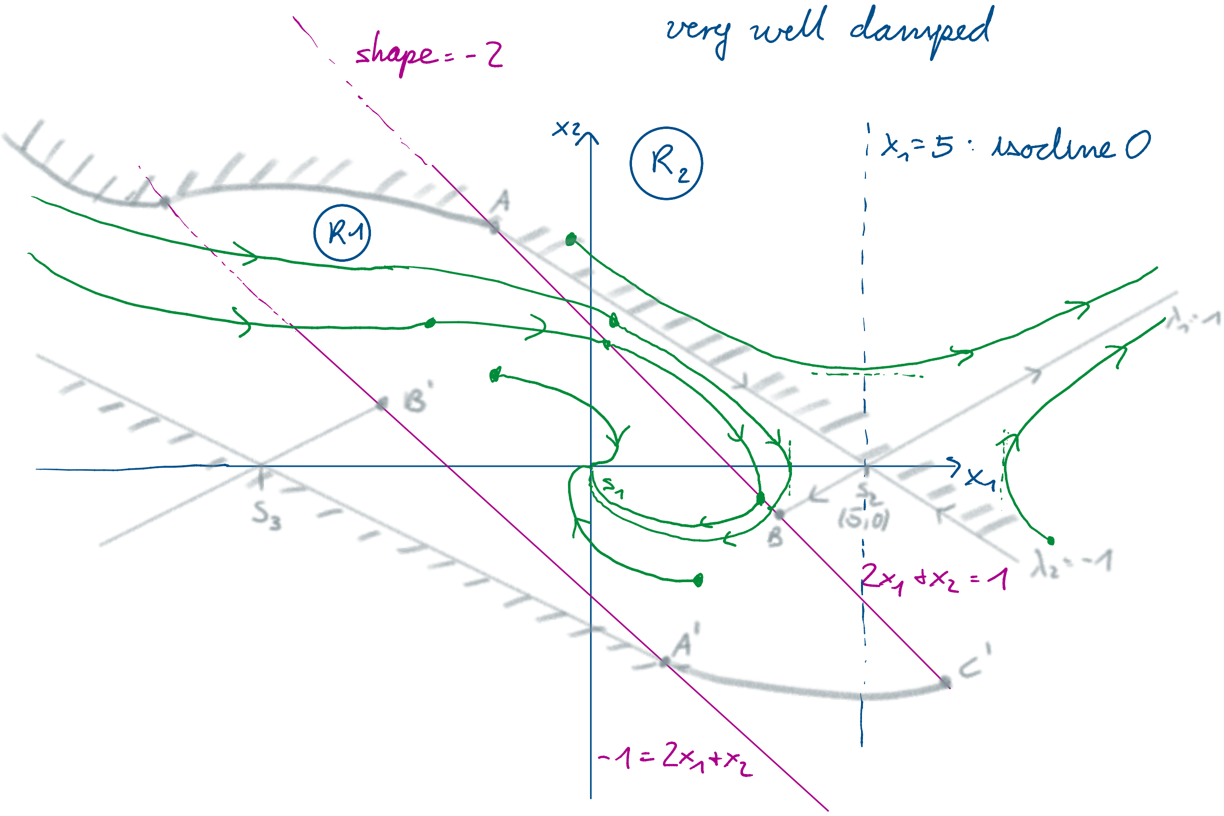
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -9x_1 - 5x_2 \end{cases} \qquad A_1 = \begin{bmatrix} 0 & 1 \\ -9 & -5 \end{bmatrix}$$

$$\det(sI - A) = \begin{vmatrix} s & -1 \\ 9 & s+5 \end{vmatrix} = s^2 + 5s + 9$$

$$= s^2 + 2\xi\omega s + \omega^2$$

$$\begin{cases} \omega = 3 \\ \xi = \frac{5}{6} \in [0{,}8 \quad 1[\end{cases}$$

very well damped



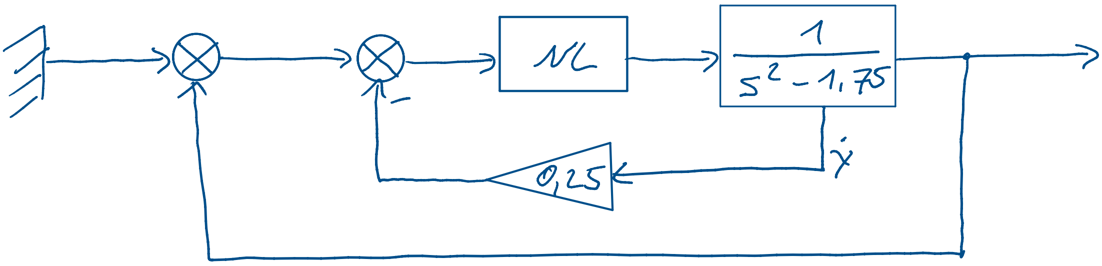
$\underline{2x_1 + x_2 > 1}$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1 - 5 \end{cases} \qquad x_{s2} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \det(sI - A_2) = s^2 - 1$$

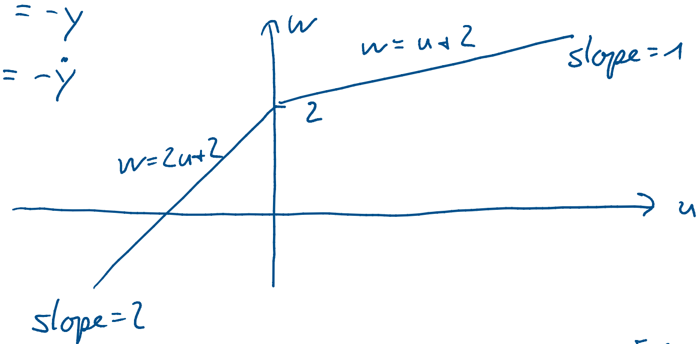
$$\lambda_1 = 1$$

$$\lambda_2 = -1$$



$x_1 = -y$

$x_2 = -\dot{y}$



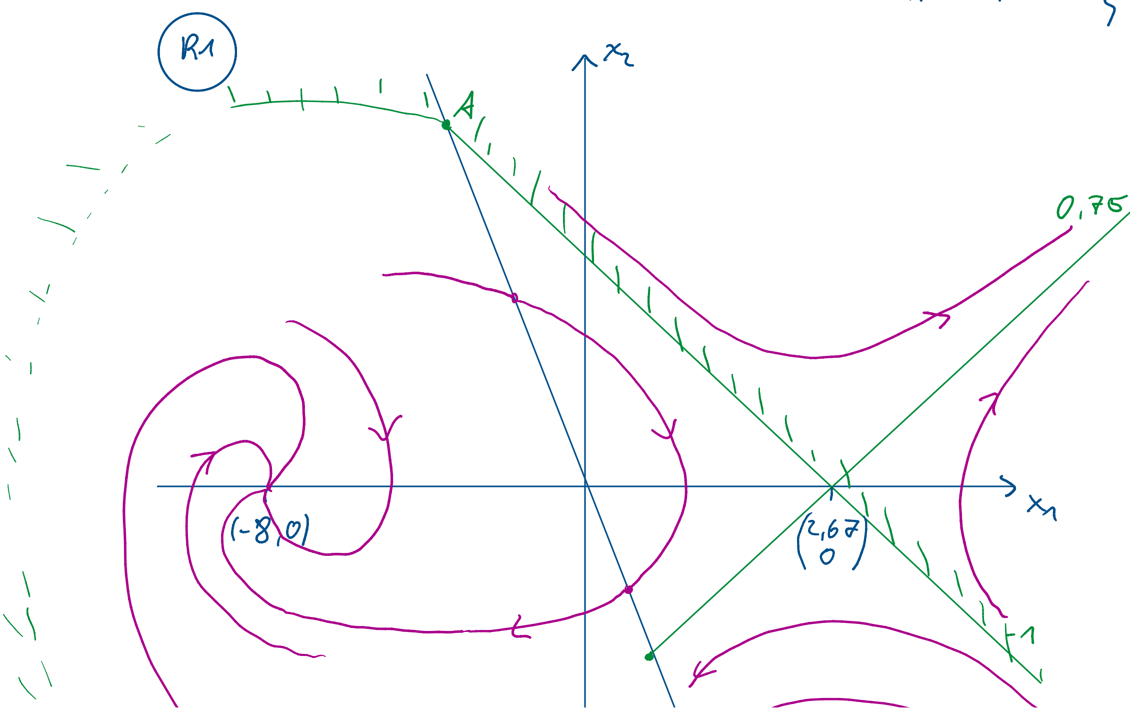
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 1{,}75\, x_1 - w \end{cases}$$

$$u = x_1 + 0{,}25\, x_2$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0{,}25 & -0{,}5 \end{bmatrix}$$

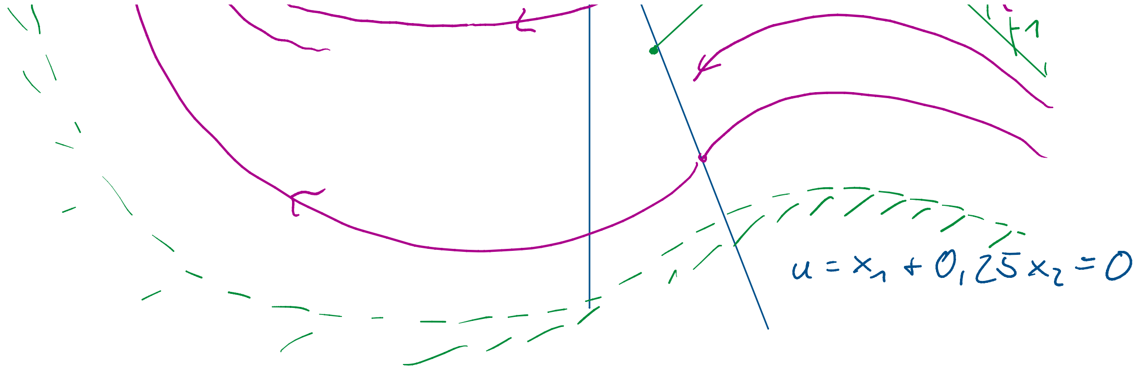
$$B_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

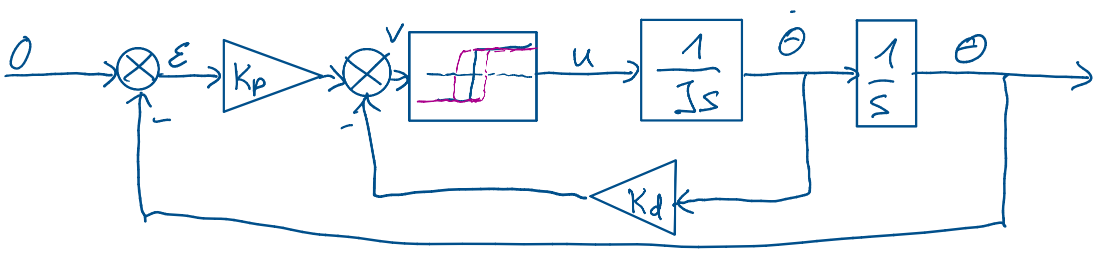
$$\omega_n = 0{,}5 = \xi$$



$$A_2 = \begin{bmatrix} 0 & 1 \\ 0{,}75 & -0{,}25 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$





$$x_1 = \varepsilon = -\theta$$

$$x_2 = \dot\varepsilon = -\dot\theta \qquad \dot x_1 = x_2$$

$$J\ddot\theta = u \Rightarrow \dot x_2 = -\frac{1}{J}u \qquad \dot x_2 = -\frac{M}{J}\,\mathrm{sign}(v)$$

$$u = M\,\mathrm{sign}(v)$$

$$v = K_p\varepsilon - K_d\dot\theta$$

$$= -(K_p\theta + K_d\dot\theta)$$

$$= K_p x_1 + K_d x_2$$

$$\begin{cases} \dot x_1 = x_2 \Rightarrow \dot x_1 = x_{20} - \frac{M}{J}t \\ \dot x_2 = -\frac{M}{J} \Rightarrow x_2 = x_{20} - \frac{M}{J}t \end{cases}$$

$$x_1 = x_{10} + x_{20}t - \frac{M}{2J}t^2$$

$$x_1 = L(x_2, x_2^2)$$

$$x_1 = x_{10} + \frac{J}{2M}(x_{20}^2 - x_2^2)$$

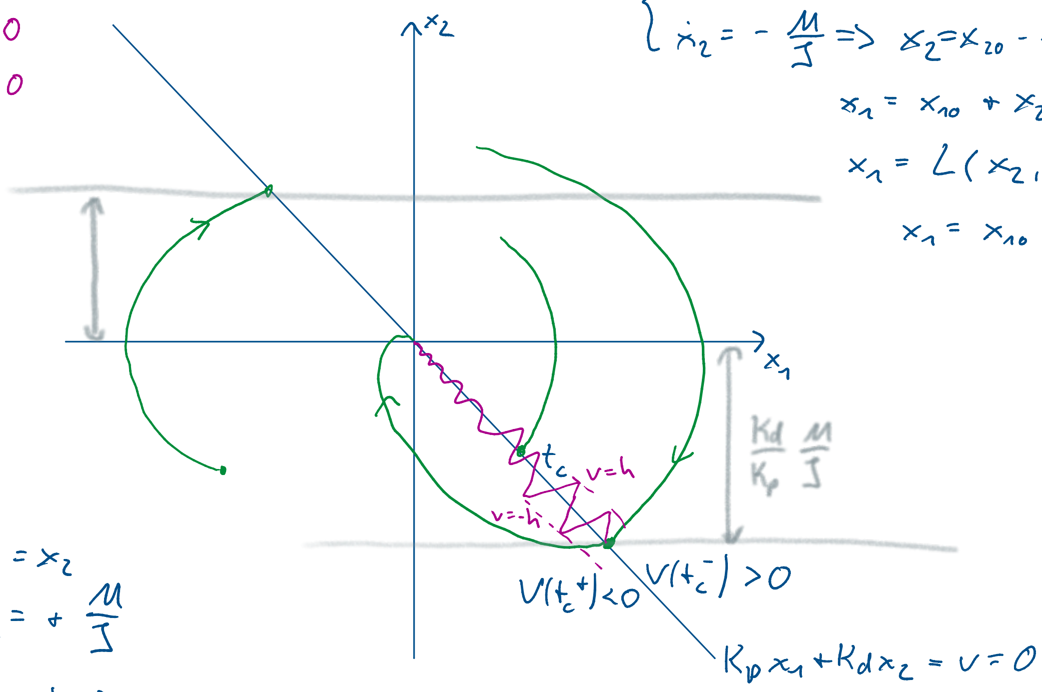
$$K_p x_1 + K_d x_2 \approx 0$$

$$K_p x_1 + K_d\dot x_1 = 0$$

$$\dot x_1 = -\frac{K_p}{K_d}x_1$$

$$\boxed{\dot x_1 = -\frac{1}{\tau}x_1}$$

$$\boxed{\tau = \frac{K_d}{K_p}}$$



$$\begin{cases} \dot x_1 = x_2 \\ \dot x_2 = +\frac{M}{J} \end{cases}$$

$$(V(t_c^-)) > 0$$

$$(V(t_c^+)) < 0$$

$$V(t) = K_p x_1(t) + K_d x_2(t)$$

$$\dot{V}(t_c^-) = K_p x_2(t_c^-) + K_d \cdot \left(-\frac{M}{J}\right) > 0$$

$$\boxed{\dot{V}(t_c^+) = K_p x_2(t_c^+) + K_d \left(\frac{M}{J}\right)} < 0 \ ?$$

$$K_p x_2 + K_d \frac{M}{J} < 0$$

$$\boxed{x_2 < -\frac{K_d}{K_p} \frac{M}{J}}$$