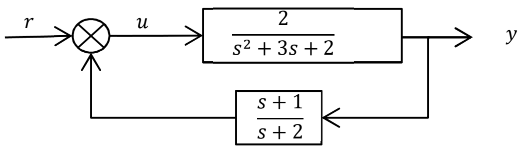
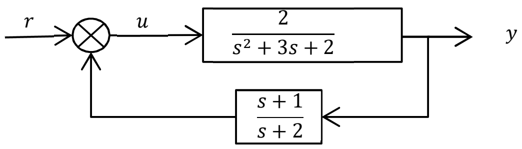
Representation and Analysis of Dynamical Systems

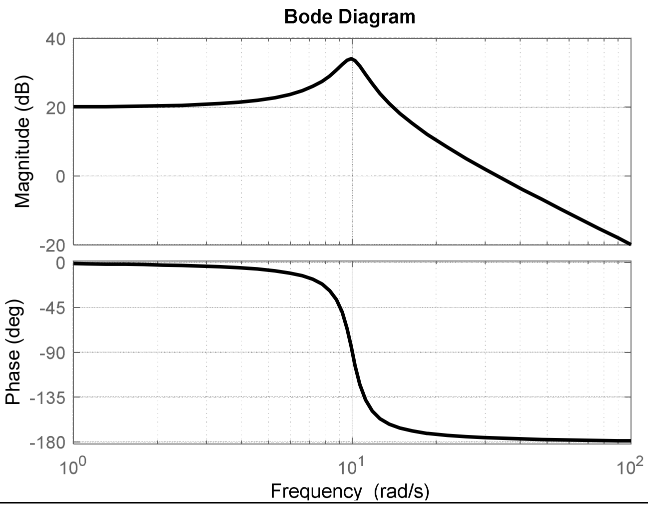
Test – 40min – without documentation

$u(t)$ is unitary step signal	1. The Laplace transform $U(s)$ of $u(t)$ is: a : $U(s) = 1$ b : $U(s) = 1/s$	
The Laplace transform of a signal $u(t)$ is: $$U(s) = \frac{10}{10+s}$$	2. The final value (at $t = \infty$) of $u(t)$ is: a : 10 b : 1 c : 0	
The transfer function of a system is: $$U(s) = \frac{10}{10+s}$$	3. The static gain is: a: 10 b : 1 c : 0	
A linear system (input $u(t)$, output $y(t)$) is driven by the differential equation:$y''(t) - 3\,y'(t) + 2y(t) = u(t)$	4. The transfer function is: a: $F(s) = \frac{1}{s^2-3s+2}$ b : $F(s) = \frac{1}{2s^2-3s+1}$	
The system (input $u(t)$, output $y(t)$) driven by the following equation $$y''(t) + 3\,y'(t) - 2y(t) = u(t)$$ Is stable	5. The system is stable: a: True b : False	
A linear system (input $u(t)$, output $y(t)$) is driven by the differential equation: $$y''(t) + 3\,y'(t) + 2y(t) = u(t) + u'(t)$$	6. The transfer function is: $$F(s) = \frac{1}{s+2}$$ a: True b : False	
Consider the system: 	7. The transfer function between r and y is: $$\frac{y}{r} = \frac{2s+2}{s^3+5s^2+10s+6}$$ a: True b : False	
Consider the system: 	8. The transfer function between r and u is: $$\frac{u}{r} = \frac{s^2+4s+4}{s^2+4s+6}$$ a: True b : False	
Consider the transfer function of a system: $$F(s) = \frac{1}{s}$$	9. The system can be stabilized with a pure proportional controller: a: Yes b: No	

The transfer function of a system is:

$$F(s) = \frac{A}{\omega_0^2 + 2\sigma\omega_0 s + s^2}$$

The Bode plot is given below:



10. The correct set of coefficients is:

a: $A = 1\,; \sigma = 0.1\,;\ \omega_0 = 10$

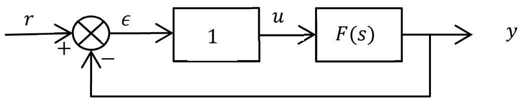
b : $A = 1000\,; \sigma = 0.1\,;\ \omega_0 = 1$

c : $A = 10\,; \sigma = 10\,;\ \omega_0 = 1$

d : $A = 1000\,; \sigma = 0.1\,;\ \omega_0 = 10$

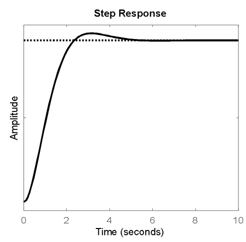
e : none

The system given by the last question is included in a closed loop such as:

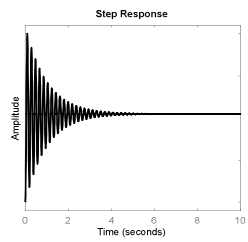


11. The correct step response (input $r(t)$ is a step) is:

a:



b:



Consider the transfer function of a system:

$$F(s) = \frac{1}{1 - s}$$

12. The system is stable

a: Yes

b: No

Consider the transfer function of a system:

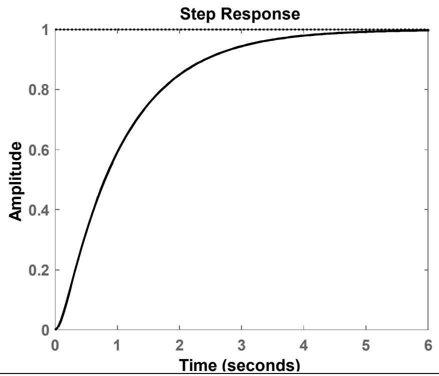
$$F(s) = \frac{1}{1 - s}$$

13. The system can be stabilized with a pure proportional controller:

a: Yes

b : No

The step response of a system is given below:



14. This step response correspond to the transfer function:

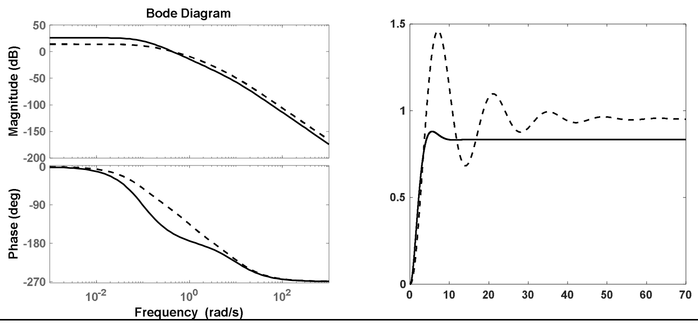
a: $F_1 = \frac{1}{1+0.1s}$

b: $F_1 = \frac{10}{1+s}$

c: $F_1 = \frac{1}{1+1.1s+0.1s^2}$

d: $F_1 = \frac{1}{1+0.1s+0.1s^2}$

We give the Bode diagram (open loop) and step response (in closed loop) of two systems (solid line and dashed line):



15. The system with dashed line (resp. solid) of the Bode diagram corresponds to the system with dashed line (resp. solid) of the step response:

a: Yes

b: No

The open loop transfer function H is:

$$H(s) = \frac{100}{s^2 + 4s}$$

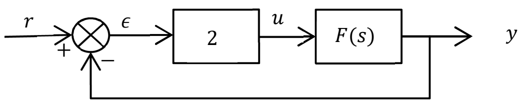
16. This system in closed loop has a zero static error:

a: Yes

b: No

A system is given by its transfer function:

$$F(s) = \frac{1 - s}{1 + s + s^2}$$



17. This system is stable **in open loop** (input u(t) output y(t)):

a: Yes

b: No

18. This system is stable **in closed loop** (input r(t) output y(t)):

a: Yes

b: No

A system (input $u(t)$ output $y(t)$) is driven by the differential equation:

$$y''(t) + 2y'(t) + y^2(t) = u'(t) + u(t)$$

The Laplace transform of $u(t)$ (resp$y(t)$) is $U(s)$ (resp $Y(s)$)

19. The relationship between $U(s)$ and $Y(s)$ is:

$$s^2Y(s) + 2sY(s) + Y(s)^2 = sU(s) + U(s)$$

a: True

b: False

A system (input $u(t)$ output $y(t)$ internal state $x(t)$ is driven by the state space equation:

$$\begin{cases} \dot{x}(t) = -x(t)^2 + u(t) \\ y(t) = x(t)^2 \end{cases}$$

The state and output variation near the equilibrium point (U_0, X_0, Y_0) are $(\delta u(t), \delta x(t), \delta y(t))$ such as:

$$u(t) = U_0 + \delta u(t)$$
$$x(t) = X_0 + \delta x(t)$$
$$y(t) = Y_0 + \delta y(t)$$

The system is trimmed at the equilibrium point corresponding to $u(t) = U_0 = 4$.

20. The linearized state space equation near the equilibrium point is:

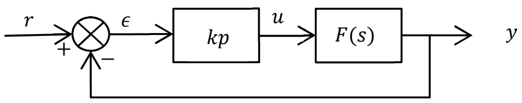
a: $\begin{cases} \dot{\delta x}(t) = -4\delta x(t) + \delta u(t) \\ \delta y(t) = 4\delta x(t) \end{cases}$

b: $\begin{cases} \dot{\delta x}(t) = -2\delta x(t) + \delta u(t) \\ \delta y(t) = 2\delta x(t) \end{cases}$

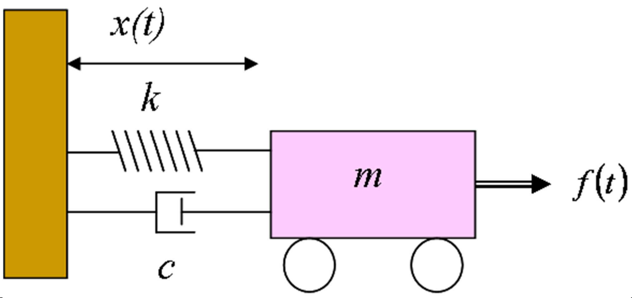
c: $\begin{cases} \dot{\delta x}(t) = 4\delta x(t) + \delta u(t) \\ \delta y(t) = -4\delta x(t) \end{cases}$

A system is given by its transfer function:

$$F(s) = \frac{2 - s}{-1 + s + s^2}$$



21. Which one of the following is true:
a: closed loop stable if $kp < 0$
b: closed loop stable if $kp > 1$
c: closed loop stable if $kp = 0.7$
d: unstable for any value of kp

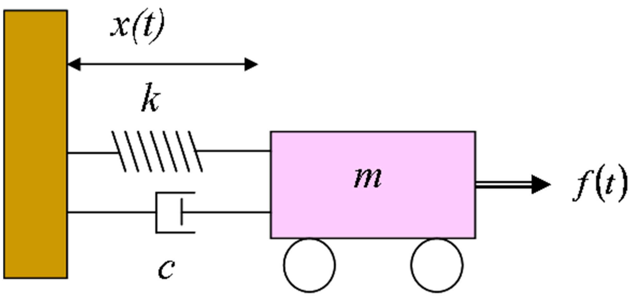


22. The potential energy is
a: $E_p = \frac{1}{2}kx^2$

b: $E_p = \frac{1}{2}m\dot{x}^2$

c: $E_p = 0$

State Space representation
Consider the differential equation that characterizes a mechanical system with a mass M, a damper of constant C and a stiffness K:
$m\ddot{x} + c\dot{x} + kx = F$,
F being the force applied to the system.
The output of the system is the displacement x.



23. A possible representation of the mechanical system is:
a:

$$X = [x \quad \dot{x}]^t$$
$$\dot{X} = \begin{bmatrix} 0 & 1 \\ \frac{k}{m} & \frac{c}{m} \end{bmatrix} X + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$
$$Y = [1 \quad 0]X$$

b:

$$X = [x \quad \dot{x}]^t$$
$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} X + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$
$$Y = [1 \quad 0]X$$

b:

$$X = [x \quad \dot{x}]^t$$
$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{k}{m} \end{bmatrix} X + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$
$$Y = [1 \quad 0]X$$