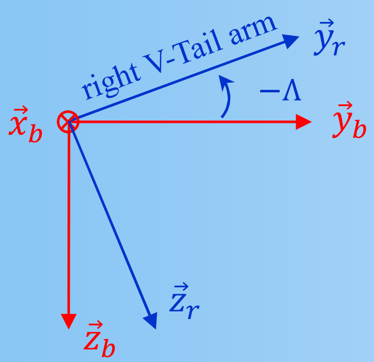
Application to the V-Tail

General Atomics MQ-9 Reaper

Application for the V-Tail concept

The V-Tail is a configuration where both the HTP/VTP are grouped within a same entity



Cirrus Vision SF50

We define a "local" referential associated to the right arm of the V-Tail : $R_r = (x_b, y_r, z_r)$ and so we can define "local" angle of attack / side slip with respect to this referential

Question : what is the « local » α_r, β_r as seen by the right arm of the V-Tail ?

Expression of local aero angles

We need 1 rotation for moving from R_b to R_r : $\quad R_b \underset{(x,-\Lambda)}{\rightarrow} R_r$

$$T_{r/b} = T_x(-\Lambda) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Lambda & -\sin\Lambda \\ 0 & \sin\Lambda & \cos\Lambda \end{bmatrix}$$

We compute the coordinate of the vector $\vec{x}_a$ with respect to R_r

$$\vec{x}_a = \left| \begin{matrix} \cos\alpha\cos\beta \\ \sin\beta \\ \sin\alpha\cos\beta \end{matrix} \right._{R_b} = T_{r/b} \cdot \left| \begin{matrix} \cos\alpha\cos\beta \\ \sin\beta \\ \sin\alpha\cos\beta \end{matrix} \right._{R_b} = \left| \begin{matrix} \cos\alpha\cos\beta \\ \sin\beta\cos\Lambda - \sin\alpha\cos\beta\sin\Lambda \\ \sin\beta\sin\Lambda + \sin\alpha\cos\beta\cos\Lambda \end{matrix} \right._{R_r}$$

Expression of local aero angles

We apply the general relations

$$\begin{cases} \sin\beta_r = \vec{x}_a \cdot \vec{y}_r = \sin\beta\cos\Lambda - \sin\alpha\cos\beta\sin\Lambda \\ \sin\alpha_r\cos\beta_r = \vec{x}_a \cdot \vec{z}_r = \sin\beta\sin\Lambda + \sin\alpha\cos\beta\cos\Lambda \end{cases}$$

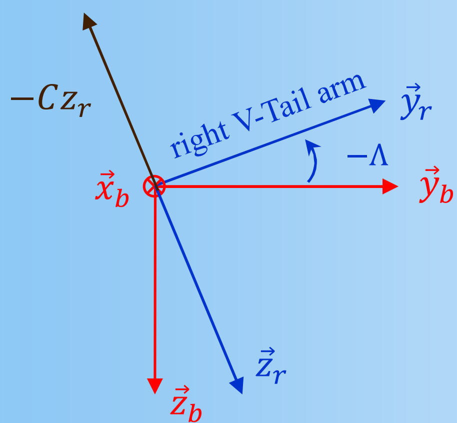
By using the small angles approximations :

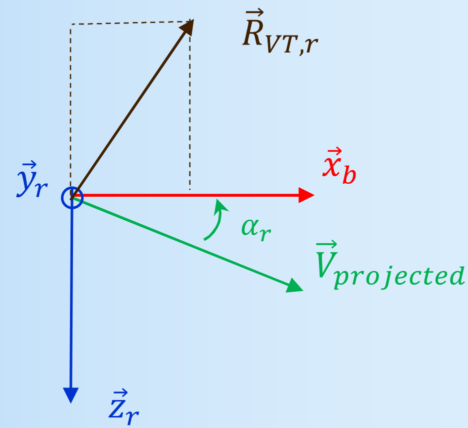
$$\begin{cases} \alpha_l \approx \alpha\cos\Lambda - \beta\sin\Lambda \\ \beta_l \approx \beta\cos\Lambda + \alpha\sin\Lambda \end{cases} \qquad \begin{cases} \alpha_r \approx \alpha\cos\Lambda + \beta\sin\Lambda \\ \beta_r \approx \beta\cos\Lambda - \alpha\sin\Lambda \end{cases}$$

You obtain the expression for the left arm of the V-tail by changing Λ in $-\Lambda$

Expression of V-Tail aero forces

Submitted to a local angle of attack α_r and to an elevator deflection δr,
the right half-tail creates a lift force within its own plane of symmetry $(\vec{x}_b\,,\vec{z}_r)$





$$\vec{R}_{VT,r} = q_\infty \frac{S_h}{2}\left(Cz_\alpha \alpha_r + Cz_{\delta m}\delta r\right) \left|\begin{array}{c} \sin\alpha_r \approx \alpha_r \\ 0 \\ -\cos\alpha_r \approx -1 \end{array}\right._{R_r}$$

q_∞ = dynamic pressure / S_h = ref. surface empennage

Expression of V-Tail aero forces

We have to express $\vec{R}_{VT,r}$ within the aerodynamic referential : $R_r \;\rightarrow\; R_b \;\rightarrow\; R_a$

By using the small angles approximations :

$$T_{a/r} = T_{a/b} \otimes T_{b/r} = \begin{bmatrix} 1 & \beta & \alpha \\ -\beta & 1 & 0 \\ -\alpha & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Lambda & \sin\Lambda \\ 0 & -\sin\Lambda & \cos\Lambda \end{bmatrix}$$

$$\boxed{T_{a/r} = \begin{bmatrix} 1 & \beta\cos\Lambda - \alpha\sin\Lambda & \beta\sin\Lambda + \alpha\cos\Lambda \\ -\beta & \cos\Lambda & \sin\Lambda \\ -\alpha & -\sin\Lambda & \cos\Lambda \end{bmatrix}}$$

$$\vec{R}_{VT,r} = q_\infty \frac{S_h}{2}\left(Cz_\alpha \alpha_r + Cz_{\delta m}\delta r\right) \left.\begin{vmatrix} \alpha_r \\ 0 \\ -1 \end{vmatrix}\right._{R_r} = q_\infty \frac{S_h}{2}\left(Cz_\alpha \alpha_r + Cz_{\delta m}\delta r\right) \cdot T_{a/r} \left.\begin{vmatrix} \alpha_r \\ 0 \\ -1 \end{vmatrix}\right._{R_r}$$

$$\boxed{\vec{R}_{VT,r} = q_\infty \frac{S_h}{2}\left(Cz_\alpha \alpha_r + Cz_{\delta m}\delta r\right) \left.\begin{vmatrix} \alpha_r - (\beta\sin\Lambda + \alpha\cos\Lambda) \approx 0 \\ -\sin\Lambda \\ -\cos\Lambda \end{vmatrix}\right._{R_a}}$$

Expression of aero forces

We sum both contributions of the Vtail arms

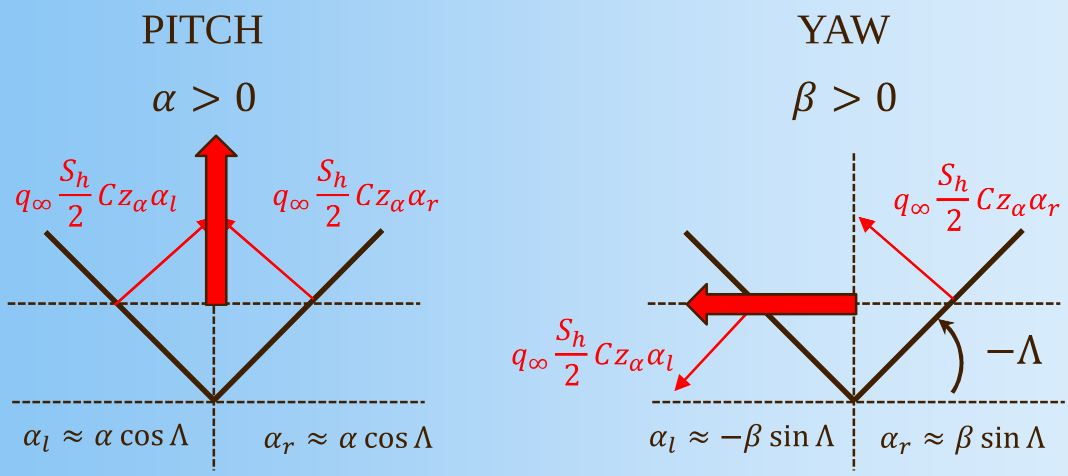
$$\vec{R}_{VT} = q_\infty \frac{S_h}{2}\left((Cz_\alpha \alpha_l + Cz_{\delta m}\delta l) \;{}_{R_a}\!\left|\begin{matrix} 0 \\ \sin\Lambda \\ -\cos\Lambda \end{matrix}\right. + (Cz_\alpha \alpha_r + Cz_{\delta m}\delta r) \;{}_{R_a}\!\left|\begin{matrix} 0 \\ -\sin\Lambda \\ -\cos\Lambda \end{matrix}\right. \right)$$

$$\vec{R}_{VT} = q_\infty \frac{S_h}{2}\left\{ Cz_\alpha \;{}_{R_a}\!\left|\begin{matrix} 0 \\ \sin\Lambda\,(\alpha_l - \alpha_r) \\ -\cos\Lambda\,(\alpha_l + \alpha_r) \end{matrix}\right. + Cz_{\delta m} \;{}_{R_a}\!\left|\begin{matrix} 0 \\ \sin\Lambda\,(\delta_l - \delta_r) \\ -\cos\Lambda\,(\delta_l + \delta_r) \end{matrix}\right. \right\}$$

By using the expression of α_l and α_r

$$\vec{R}_{VT} = q_\infty S_h \left\{ Cz_\alpha \;{}_{R_a}\!\left|\begin{matrix} 0 \\ -\beta\cdot\sin^2\Lambda \\ -\alpha\cdot\cos^2\Lambda \end{matrix}\right. + Cz_{\delta m} \;{}_{R_a}\!\left|\begin{matrix} 0 \\ \sin\Lambda\cdot\dfrac{\delta_l - \delta_r}{2} \\ -\cos\Lambda\cdot\dfrac{\delta_l + \delta_r}{2} \end{matrix}\right. \right\}$$

V-Tail response to α/β



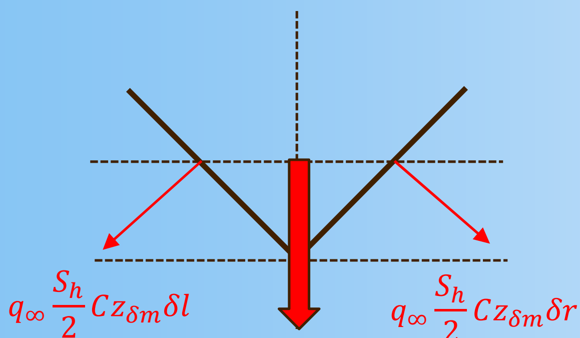
$$\vec{R}_z = -q_\infty S_h Cz_\alpha\, \alpha \cdot \cos^2\Lambda \cdot \vec{z}_a$$

$$\vec{R}_y = -q_\infty S_h Cz_\alpha\, \beta \cdot \sin^2\Lambda \cdot \vec{y}_a$$

V-Tail response to δl/δr

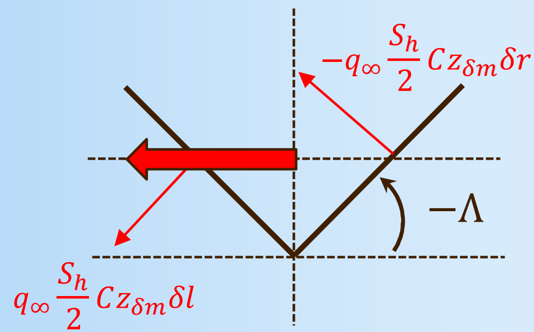
PITCH

$$\delta l = \delta r < 0$$



YAW

$$\delta l = -\delta r < 0$$



$$\vec{R}_z = -q_\infty S_h Cz_{\delta m} \cdot \frac{\delta l + \delta r}{2} \cdot \cos\Lambda \cdot \vec{z}_a$$

$$\vec{R}_y = q_\infty S_h Cz_{\delta m} \cdot \frac{\delta l - \delta r}{2} \cdot \sin\Lambda \cdot \vec{y}_a$$

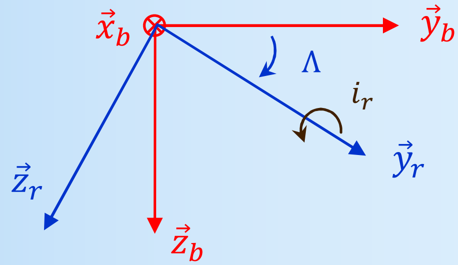
Inversed V-Tail

General Atomics MQ-1 Predator

The Predator drone characteristics

The Predator drone is equipped with an Inversed Vtail :

- There is no control surface on the Vtail
- But each arm can rotate around their y-axis



We need 2 rotations for moving from R_b to R_r : $R_b \underset{(x,\Lambda)}{\rightarrow} R_i \underset{(y,i_r)}{\rightarrow} R_r$

$$T_{r/b} = T_{r/i} \otimes T_{i/b} = T_y(i_r) \otimes T_x(\Lambda) = \begin{bmatrix} \cos i_r & 0 & -\sin i_r \\ 0 & 1 & 0 \\ \sin i_r & 0 & \cos i_r \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Lambda & \sin\Lambda \\ 0 & -\sin\Lambda & \cos\Lambda \end{bmatrix}$$

$$T_{r/b} = \begin{bmatrix} \cos i_r & \sin i_r \sin\Lambda & -\sin i_r \cos\Lambda \\ 0 & \cos\Lambda & \sin\Lambda \\ \sin i_r & -\cos i_r \sin\Lambda & \cos i_r \cos\Lambda \end{bmatrix}$$

Expression of local angle of attack

We compute the coordinate of the vector $\vec{x}_a$ with respect to R_r

By using the small angles approximations :

$$\vec{x}_a = T_{r/b} \cdot \left|\begin{matrix}\cos\alpha\cos\beta \\ \sin\beta \\ \sin\alpha\cos\beta\end{matrix}\right._{R_b} \approx \left|\begin{matrix}1 \\ \alpha\sin\Lambda + \beta\cos\Lambda \\ \alpha\cos\Lambda - \beta\sin\Lambda + i_r\end{matrix}\right._{R_r}$$

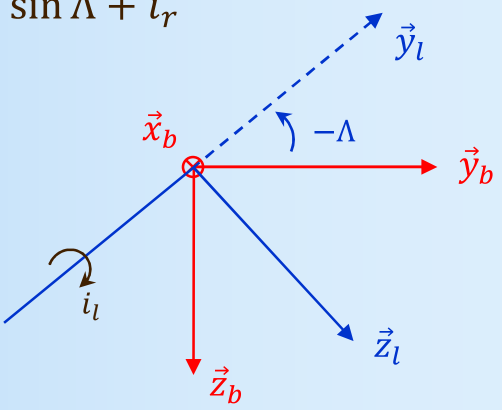
We apply the general relations

$$\begin{cases} \sin\beta_r = \vec{x}_a \cdot \vec{y}_r = \alpha\sin\Lambda + \beta\cos\Lambda \\ \sin\alpha_r\cos\beta_r = \vec{x}_a \cdot \vec{z}_r = \alpha\cos\Lambda - \beta\sin\Lambda + i_r \end{cases}$$

$$\begin{cases} \alpha_r \approx \alpha\cos\Lambda - \beta\sin\Lambda + i_r \\ \alpha_l \approx \alpha\cos\Lambda + \beta\sin\Lambda + i_l \end{cases}$$

right Vtail nose up : $i_r > 0$

left Vtail nose up : $i_l > 0$



Expression of aero forces

$$T_{a/r} = T_{a/b} \otimes T_{b/r} = \begin{bmatrix} 1 & \beta & \alpha \\ -\beta & 1 & 0 \\ -\alpha & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & i_r \\ i_r \sin\Lambda & \cos\Lambda & -\sin\Lambda \\ -i_r \cos\Lambda & \sin\Lambda & \cos\Lambda \end{bmatrix}$$

$$T_{a/r} = \begin{bmatrix} 1 & \beta\cos\Lambda + \alpha\sin\Lambda & i_r + \alpha\cos\Lambda - \beta\sin\Lambda \\ -\beta + i_r\sin\Lambda & \cos\Lambda & -\sin\Lambda \\ -\alpha - i_r\cos\Lambda & \sin\Lambda & \cos\Lambda \end{bmatrix}$$

We have to express $\vec{R}_{z,r}$ within the aerodynamic referential : $R_r \rightarrow R_b \rightarrow R_a$

$$\vec{R}_{VT,r} = q_\infty \frac{S_h}{2} Cz_\alpha \alpha_r \; {}_{R_r}\left| \begin{matrix} \alpha_r \\ 0 \\ -1 \end{matrix} \right. = q_\infty \frac{S_h}{2} Cz_\alpha \alpha_r \; {}_{R_a}\left| \begin{matrix} \alpha_r - (i_r + \alpha\cos\Lambda - \beta\sin\Lambda) \approx 0 \\ \sin\Lambda \\ -\cos\Lambda \end{matrix} \right.$$

Expression of Aero forces

We add the left arm contribution (do not forget to change Λ to $-\Lambda$)

$$\vec{R}_{VT} = q_\infty \frac{S_h}{2} Cz_\alpha \left|\begin{matrix} 0 \\ \sin\Lambda\,(\alpha_r - \alpha_l) \\ -\cos\Lambda\,(\alpha_l + \alpha_r) \end{matrix}\right._{R_a} = q_\infty \frac{S_h}{2} Cz_\alpha \left|\begin{matrix} 0 \\ -\sin\Lambda\,(2\beta\sin\Lambda + i_l - i_r) \\ -\cos\Lambda\,(2\alpha\cos\Lambda + i_l + i_r) \end{matrix}\right._{R_a}$$

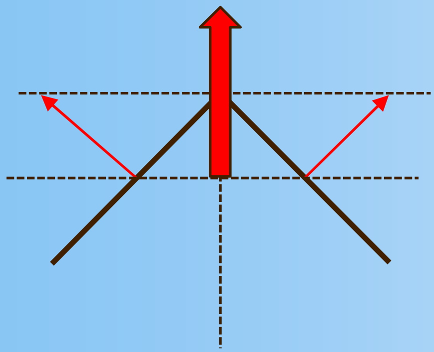
$$\boxed{\vec{R}_{VT} = q_\infty S_h Cz_\alpha \left(\left|\begin{matrix} 0 \\ -\beta\cdot\sin^2\Lambda \\ -\alpha\cdot\cos^2\Lambda \end{matrix}\right._{R_a} + \left|\begin{matrix} 0 \\ -\dfrac{i_l - i_r}{2}\cdot\sin\Lambda \\ -\dfrac{i_l + i_r}{2}\cdot\cos\Lambda \end{matrix}\right._{R_a} \right)}$$

The (i_l, i_r) command variables give the possibility to control the aircraft in pitch and yaw

Pitch / Yaw control

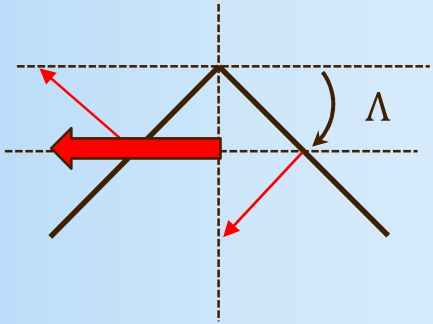
PITCH CONTROL

$$i_l = i_r > 0$$



YAW CONTROL

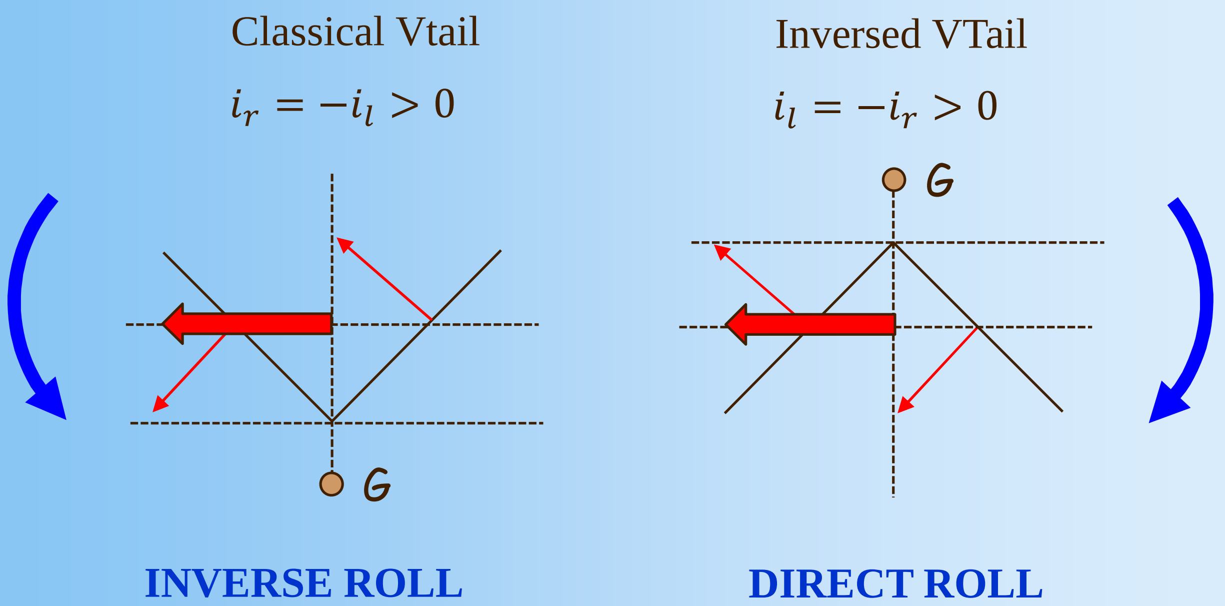
$$i_l = -i_r > 0$$



$$\vec{R}_z = -q_\infty S_h C z_\alpha \frac{i_l + i_r}{2} \cdot \cos\Lambda \cdot \vec{z}_a$$

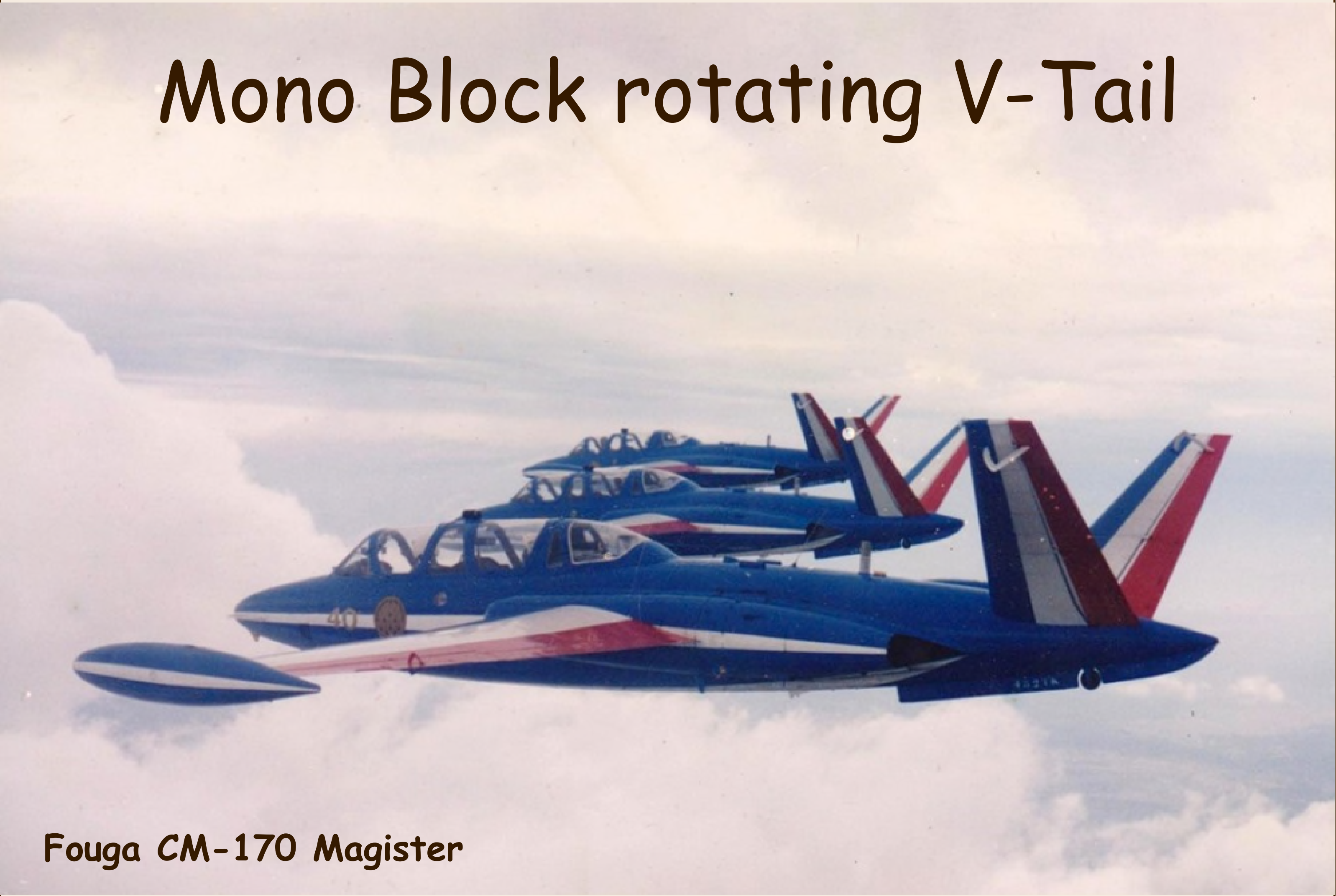
$$\vec{R}_y = -q_\infty S_h C z_\alpha \frac{i_l - i_r}{2} \cdot \sin\Lambda \cdot \vec{y}_a$$

Inversed versus Conventional



You apply a Yaw command for turning right, due to the relative Center of Gravity position, naturally the Inversed Vtail banks the aircraft on the right : hence, with the Inversed Vtail, you can also control the roll. This explains why there is no roll control surfaces (ailerons nor spoilers) on the Predator wing !

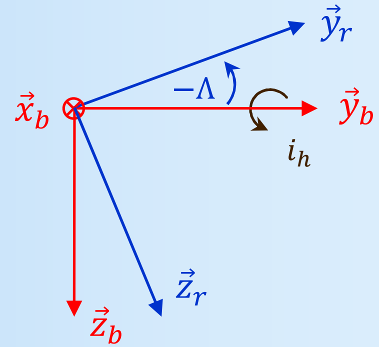
Mono Block rotating V-Tail



Fouga CM-170 Magister

The Predator drone characteristics

We assume that the V-Tail can rotate with respect to the y_b axis



We need 2 rotations for moving from R_b to R_r :

$$R_b \underset{(y,i_h)}{\rightarrow} R_i \underset{(x,-\Lambda)}{\rightarrow} R_r$$

$$T_{r/b} = T_{r/i} \otimes T_{i/b} = T_x(-\Lambda) \otimes T_y(i_h) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Lambda & -\sin\Lambda \\ 0 & \sin\Lambda & \cos\Lambda \end{bmatrix} \otimes \begin{bmatrix} \cos i_h & 0 & -\sin i_h \\ 0 & 1 & 0 \\ \sin i_h & 0 & \cos i_h \end{bmatrix}$$

$$T_{r/b} = \begin{bmatrix} \cos i_h & 0 & -\sin i_h \\ -\sin i_h \sin\Lambda & \cos\Lambda & -\cos i_h \sin\Lambda \\ \sin i_h \cos\Lambda & \sin\Lambda & \cos i_h \cos\Lambda \end{bmatrix}$$

Expression of local angle of attack

We compute the coordinate of the vector $\vec{x}_a$ with respect to R_r

By using the small angles approximations :

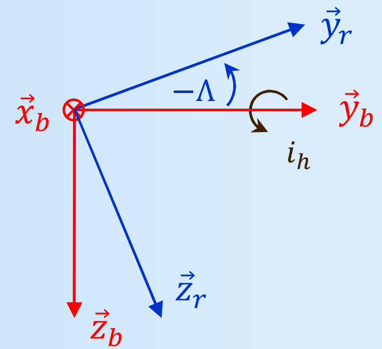
$$\vec{x}_a = T_{r/b} \cdot \left| \begin{matrix} \cos\alpha\cos\beta \\ \sin\beta \\ \sin\alpha\cos\beta \end{matrix} \right._{R_b} \approx \left| \begin{matrix} 1 \\ -\alpha\sin\Lambda + \beta\cos\Lambda - i_h\sin\Lambda \\ \alpha\cos\Lambda + \beta\sin\Lambda + i_h\cos\Lambda \end{matrix} \right._{R_r}$$

We apply the general relations

$$\begin{cases} \sin\beta_r = \vec{x}_a \cdot \vec{y}_r = -\alpha\sin\Lambda + \beta\cos\Lambda - i_h\sin\Lambda \\ \sin\alpha_r\cos\beta_r = \vec{x}_a \cdot \vec{z}_r = \alpha\cos\Lambda + \beta\sin\Lambda + i_h\cos\Lambda \end{cases}$$

$$\begin{cases} \alpha_r \approx \alpha\cos\Lambda + \beta\sin\Lambda + i_h\cos\Lambda \\ \alpha_r \approx \alpha\cos\Lambda - \beta\sin\Lambda + i_h\cos\Lambda \end{cases}$$

Vtail nose up : $i_h > 0$



Expression of aero forces

$$T_{a/r} = T_{a/b} \otimes T_{b/r} \approx \begin{bmatrix} 1 & \beta & \alpha \\ -\beta & 1 & 0 \\ -\alpha & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & -i_h \sin\Lambda & i_h \cos\Lambda \\ 0 & \cos\Lambda & \sin\Lambda \\ -i_h & -\sin\Lambda & \cos\Lambda \end{bmatrix}$$

$$T_{a/r} \approx \begin{bmatrix} 1 & \beta\cos\Lambda - \alpha\sin\Lambda - i_h\sin\Lambda & \alpha\cos\Lambda + \beta\sin\Lambda + i_h\cos\Lambda \\ -\beta - i_h & \cos\Lambda & \sin\Lambda \\ -\alpha - i_h & -\sin\Lambda & \cos\Lambda \end{bmatrix}$$

We have to express $\vec{R}_{z,r}$ within the aerodynamic referential : $R_r \;\rightarrow\; R_b \;\rightarrow\; R_a$

$$\vec{R}_{VT,r} \approx q_\infty \frac{S_h}{2} C z_\alpha \alpha_r \left|\begin{matrix} \alpha_r \\ 0 \\ -1 \end{matrix}\right._{R_r} = q_\infty \frac{S_h}{2} C z_\alpha \alpha_r \left|\begin{matrix} \alpha_r - (\alpha\cos\Lambda + \beta\sin\Lambda + i_h\cos\Lambda) \approx 0 \\ -\sin\Lambda \\ -\cos\Lambda \end{matrix}\right._{R_a}$$

Expression of Aero forces

We add the left arm contribution (do not forget to change Λ to $-\Lambda$)

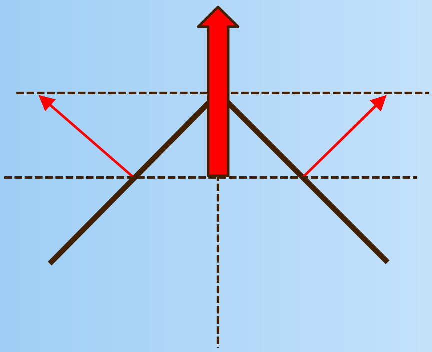
$$\vec{R}_{VT} \approx q_\infty \frac{S_h}{2} Cz_\alpha \;{}_{R_a}\left|\begin{array}{c} 0 \\ -\sin\Lambda\,(\alpha_r - \alpha_l) \\ -\cos\Lambda\,(\alpha_l + \alpha_r) \end{array}\right. \approx q_\infty S_h Cz_\alpha \;{}_{R_a}\left|\begin{array}{c} 0 \\ -\sin\Lambda \cdot \beta \sin\Lambda \\ -\cos\Lambda \cdot (\alpha\cos\Lambda + i_h \cos\Lambda) \end{array}\right.$$

$$\boxed{\vec{R}_{VT} \approx q_\infty S_h Cz_\alpha \left({}_{R_a}\left|\begin{array}{c} 0 \\ -\beta \cdot \sin^2\Lambda \\ -\alpha \cdot \cos^2\Lambda \end{array}\right. + {}_{R_a}\left|\begin{array}{c} 0 \\ 0 \\ -i_h \cdot \cos^2\Lambda \end{array}\right. \right)}$$

The i_h command variables give the possibility to control the aircraft in pitch

PITCH CONTROL

$i_h > 0$



$$\vec{R}_z = -q_\infty S_h C z_\alpha i_h \cdot \cos^2 \Lambda \cdot \vec{z}_a$$