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Olivia Judson

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Guest Column: Loves Me, Loves Me Not (Do the Math)

By Steven Strogatz

"In the spring," wrote Tennyson, "a young man's fancy lightly turns to thoughts of love." And so in keeping with the spirit of the season, this week's column looks at love affairs — mathematically. The analysis is offered tongue in cheek, but it does touch on a serious point: that the laws of nature are written as differential equations. It also helps explain why, in the words of another poet, "the course of true love never did run smooth."

To illustrate the approach, suppose Romeo is in love with Juliet, but in our version of the story, Juliet is a fickle lover. The more Romeo loves her, the more she wants to run away and hide. But when he takes the hint and backs off, she begins to find him strangely attractive. He, on the other hand, tends to echo her: he warms up when she loves him and cools down when she hates him.

What happens to our star-crossed lovers? How does their love ebb and flow over time? That's where the math comes in. By writing equations that summarize how Romeo and Juliet respond to each other's affections and then solving those equations with calculus, we can predict the course of their affair.

The resulting forecast for this couple is, tragically, a never-ending cycle of love and hate. At least they manage to achieve simultaneous love a quarter of the time.

The model can be made more realistic in various ways. For instance, Romeo might react to his own feelings as well as to Juliet's. He might be the type of guy who is so worried about throwing himself at her that he slows himself down as his love for her grows. Or he might be the other type, one who loves feeling in love so much that he loves her all the more for it.

Add to those possibilities the two ways Romeo could react to Juliet's affections — either increasing or decreasing his own — and you see that there are four personality types, each corresponding to a different romantic style.

My students and those in Peter Christopher's class at Worcester Polytechnic Institute have suggested such descriptive names as Hermit and Malevolent Misanthrope for the particular kind of Romeo who damps out his own love and also recoils from Juliet's. Whereas the sort of Romeo who gets pumped by his own ardor but turned off by Juliet's has been called a Narcissistic Nerd, Better Latent Than Never, and a Flirting Fink. (Feel free to post your own suggested names for these two types and the other two possibilities.)

Although these examples are whimsical, the equations that arise in them are of the far-reaching kind known as differential equations. They represent the most powerful tool humanity has ever created for making sense of the material world. Sir Isaac

Newton used them to solve the ancient mystery of planetary motion. In so doing, he unified the heavens and the earth, showing that the same laws of motion applied to both.

In the 300 years since Newton, mankind has come to realize that the laws of physics are always expressed in the language of differential equations. This is true for the equations governing the flow of heat, air and water; for the laws of electricity and magnetism; even for the unfamiliar and often counterintuitive atomic realm where quantum mechanics reigns.

In all cases, the business of theoretical physics boils down to finding the right differential equations and solving them. When Newton discovered this key to the secrets of the universe, he felt it was so precious that he published it only as an anagram in Latin. Loosely translated, it reads: “It is useful to solve differential equations.”

The silly idea that love affairs might progress in a similar way occurred to me when I was in love for the first time, trying to understand my girlfriend’s baffling behavior. It was a summer romance at the end of my sophomore year in college. I was a lot like the first Romeo above, and she was even more like the first Juliet. The cycling of our relationship was driving me crazy until I realized that we were both acting mechanically, following simple rules of push and pull. But by the end of the summer my equations started to break down, and I was even more mystified than ever. As it turned out, the explanation was simple. There was an important variable that I’d left out of the equations — her old boyfriend wanted her back.

In mathematics we call this a three-body problem. It's notoriously intractable, especially in the astronomical context where it first arose. After Newton solved the differential equations for the two-body problem (thus explaining why the planets move in elliptical orbits around the sun), he turned his attention to the three-body problem for the sun, earth and moon. He couldn't solve it, and neither could anyone else. It later turned out that the three-body problem contains the seeds of chaos, rendering its behavior unpredictable in the long run.

Newton knew nothing about chaotic dynamics, but he did tell his friend Edmund Halley that the three-body problem had "made his head ache, and kept him awake so often, that he would think of it no more."

I'm with you there, Sir Isaac.

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NOTES:

For models of love affairs based on differential equations, see Section 5.3 in Strogatz, S. H. (1994) "Nonlinear Dynamics and Chaos." Perseus, Cambridge, MA.

For Newton's anagram, see page vii in Arnol'd, V. I. (1988) "Geometrical Methods in the Theory of Ordinary Differential Equations." Springer, New York.

Chaos in the three-body problem is discussed in Peterson, I. (1993) "Newton's Clock: Chaos in the Solar System." W.H.

Freeman, San Francisco.

For the quote about how the three-body problem made Newton's head ache, see page 158 in Volume II of Brewster, D. (1855) "Memoirs of the Life, Writings, and Discoveries of Sir Isaac Newton." Thomas Constable and Company, Edinburgh.

For readers who are curious about the math used here:
In the first story above, Romeo's behavior was modeled by the differential equation $dR/dt = aJ$. This equation describes how Romeo's love (represented by R) changes in the next instant (represented by dt). The amount of change (dR) is just a multiple (a) of Juliet's current love (J) for him. This equation idealizes what we already know – that Romeo's love goes up when Juliet loves him – by assuming something much stronger. It says that Romeo's love increases in direct linear proportion to how much Juliet loves him. This assumption of linearity is not emotionally realistic, but it makes the subsequent analysis much easier. Juliet's behavior, on the other hand, was modeled by the equation $dJ/dt = -bR$. The negative sign in front of the constant b reflects her tendency to cool off when Romeo is hot for her. Given these equations and an assumption about how the lovers felt about each other initially (R and J at time $t = 0$), one can use calculus to inch R and J forward, instant by instant. In this way, we can figure out how much Romeo and Juliet love (or hate) each other at any future time. For this elementary model, the equations should be familiar to students of math and physics: Romeo and Juliet behave like simple harmonic oscillators.