Institute Supérieur de l'Aéronautique et de l'Espace

Master in Aerospace Engineering

Control of Flexible Structures

Modelling and Analysis of a Flexible Structure

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1 Introduction

The objectives to be achieved in this report are as follows.

- Design a flexible beam in ANSYS.
- Perform modal analysis to find frequencies of different bending and twisting modes.
- Using the ANSYS data, create a simplified dynamic model and check the consistency of the ANSYS results through Bode plot analysis.
- Design a controller to damp these modes.

2 Modelling and Modal Analysis of the beam

2.1 Modelling and Meshing

The flexible beam consists of a plate with a tank that goes through it. The tank contains water that allows vibrations. A pair of piezoelectric actuators are used to dampen the different bending and twisting modes. The structure is modelled in ANSYS as per the dimensions provided in the assignment. The actuators are modelled as ceramics while the plate and the tank are assigned properties of steel and PVC respectively.

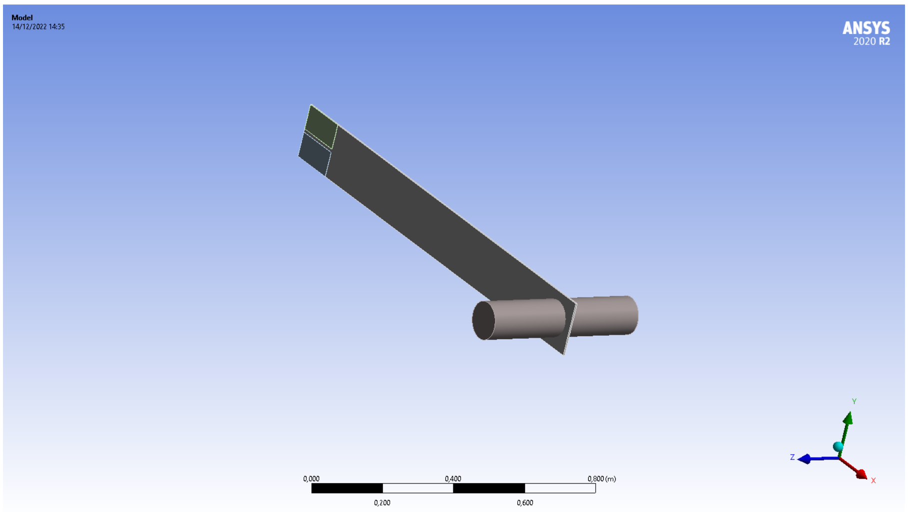

Figure 1: Model of the flexible beam

This was followed by the meshing step which is shown below.

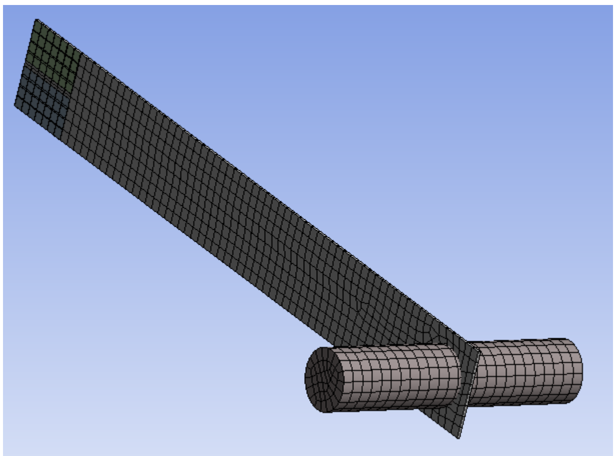

Figure 2: The mesh obtained

2.2 Modal Analysis

The modal analysis is performed after clamping the end of the beam where the actuators are present and establishing bonded connections between the components. The obtained frequencies from this analysis.

Mode	Frequency [Hz]
1,	0,97367
2,	6,3848
3,	8,2913
4,	22,266
5,	26,933
6,	52,224

Figure 3: Modal frequencies

2.2.1 Results

The following images show the 6 different modal shapes corresponding to the different frequencies obtained earlier.

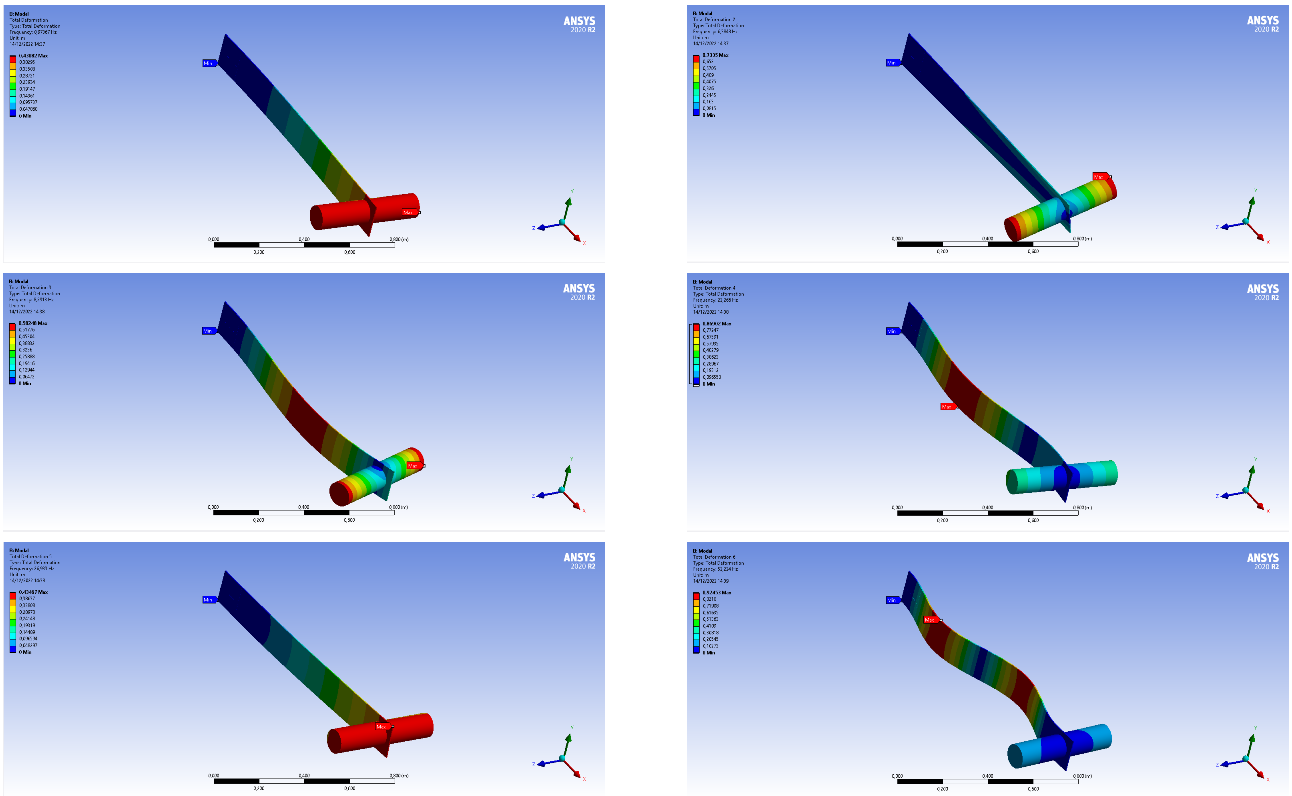

Figure 4: Different Modal Shapes

3 Reduced Order Model in MATLAB

The reduced-model of a structure with k piezoelectric actuators can be modeled for n modes in the modal base with a mechanical equation and an electrical equation.

$$M\ddot{q} + D_s\dot{q} + Kq = \theta V + F^e \tag{1}$$

$$q_c = \theta^T q + C_0 V \tag{2}$$

In ANSYS, the modal base is normalized to the mass matrix. Thus, we have:

$$M = I_n \tag{3}$$

Therefore, the stiffness matrix becomes through the modal formalization of the problem:

$$K = diag[\omega_i^2] \tag{4}$$

In general, the damping ratios is difficult to calculate for each mode. However, it can be measured through experiments on the structure. For our analysis, the damping matrix is obtained through the Bastile assumption:

$$D_s = diag(2\zeta_i)\sqrt{KM} \tag{5}$$

Using all this information along with the values of the frequency modes and physical displacements provided in the assignment, the state-space representation of the flexible beam can be calculated. The following MATLAB code creates this state space model. The C matrix taken is for both displacements and velocities.

```
n = 3;
k = 2;

M = eye(n);

w = [1.3164 ; 11.243 ; 13.944]*2*pi;
epsilon = [0.0067 ; 0.01 ; 0.002];

theta = [-0.387e-3 -0.387e-3
         -0.317e-2 -0.317e-2
         -0.776e-3 0.785e-3];

phi = [0.645  -0.4056   0.5205
       0.645  -0.4064  -0.5278
       0.207   0.769   -0.0045];

K = diag(w.^2);
D = diag(2*epsilon)*sqrt(K*M);

A = [zeros(n) eye(n)
     -inv(M)*K -inv(M)*D];
B = [zeros(n,k) ; inv(M)*theta];
C = [phi zeros(n)]; %C = [phi zeros(n) ; zeros(n) phi];
D = zeros(n,k);   %D = zeros(2*n,k);

sys = ss(A,B,C,D)
```

3.1 Results and Discussion

The matrices as well as the bodes plots for all modes are shown below.

```
sys =

  A =
              x1         x2         x3         x4         x5         x6
   x1          0          0          0          1          0          0
   x2          0          0          0          0          1          0
   x3          0          0          0          0          0          1
   x4     -68.41          0          0    -0.1108          0          0
   x5          0      -4990          0          0     -1.413          0
   x6          0          0      -7676          0          0    -0.3505

  B =
                u1           u2
   x1            0            0
   x2            0            0
   x3            0            0
   x4    -0.000387    -0.000387
   x5     -0.00317     -0.00317
   x6    -0.000776     0.000785

  C =
              x1         x2         x3         x4         x5         x6
   y1      0.645    -0.4056     0.5205          0          0          0
   y2      0.645    -0.4064    -0.5278          0          0          0
   y3      0.207      0.769    -0.0045          0          0          0

  D =
       u1   u2
   y1   0    0
   y2   0    0
   y3   0    0
```

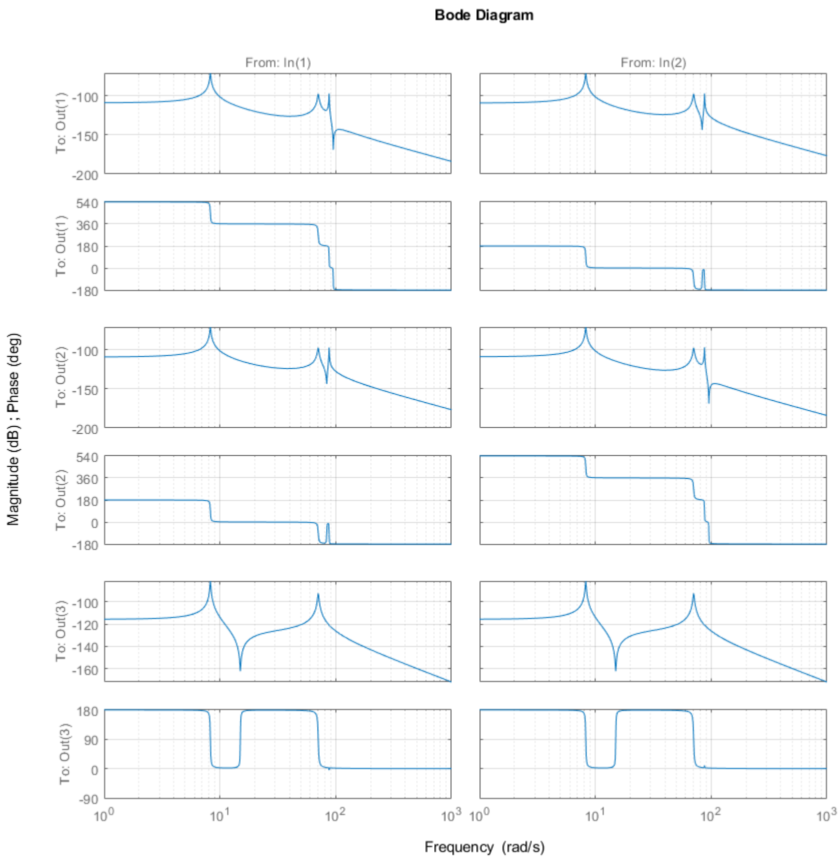

Figure 5: Bode Plots (Open Loop)

The following points are noteworthy from these bode plots.

- As expected, resonance is found at 3 frequencies, namely, 8.3 rad/s, 70.56 rad/s and 87.61 rad/s. The twisting mode corresponds to the highest frequency, 87.61 rad/s.
- Points A and B are located at the top and bottom end of the beam while point C is located at the center-line of the beam. Therefore, point C will not experience any influence due to the twisting mode.
- This is confirmed in the bode plots corresponding to point C where only the bending modes are observed.
- There is anti-resonance observed in some bode plots as well. According the plots, the presence of these zeros seems to depend on the location of the actuators.
- The twisting mode has the lowest damping (at 87.61 rad/s). This is followed by the first bending mode (at 8.3 rad/s) and second bending mode (at 70.56 rad/s).
- A lower damping corresponds to a sharp resonance peak, and this phenomenon can be clearly observed from the bode plot.

4 Controller Design

This section explains the design of an LQR controller to damp the previously seen modes. Before designing the controller, the temporal response is checked and the controllability of the system is verified. It is found that the controllability matrix is full-rank which confirms that this system is fully controllable.

4.1 Open Loop Temporal Response

The structure is excited with an initial condition provided in the assignment. The following graphs show the temporal response to those excitations.

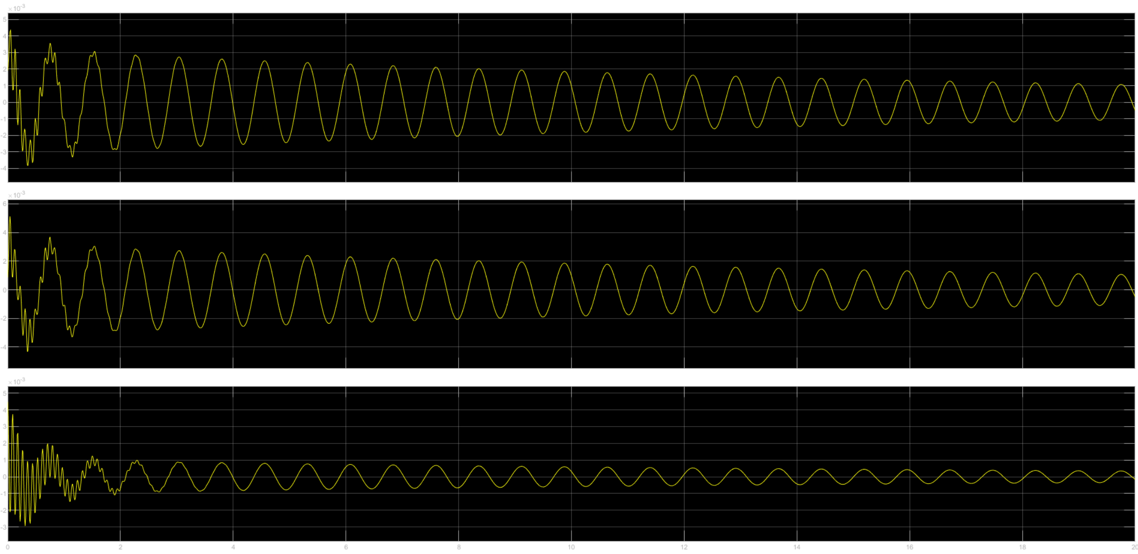

Figure 6: Temporal Response (Open Loop)

- It is observed that the displacements are mildly damped and continue to oscillate over time.
- In the third modal displacement plot, certain high frequency oscillations are observed in the firs few seconds. This is an effect of the second and third modes.

4.2 Closed Loop Temporal Response

It is also important to note that the C matrix taken for this analysis takes both displacements and velocities. The implementation of the controller comes with a saturation limit of 120V on the piezoelectric actuators, without any limit on the number of sensors.
The system is controlled via an LQR regulator along with a P-gain to amplify the error. This LQR controller is given on the state feedback and involved tuning the Q and R matrices. These matrices represent the error tolerances in states and inputs respectively.
The Q matrix should have large values in order to penalize errors in states (composed of displacements and velocities) since the objective is to dampen out all the modes. The Q matrix provides more weight to the first and third modes, since they are less damped.
The R matrix, however, is kept small for the actuators to be able to properly provide control input. Finally, a proportional gain of 1000 is added to amplify the error.
The Q and R matrices are as follows:

```
Q =
       1000           0           0           0           0           0
          0          10           0           0           0           0
          0           0        1000           0           0           0
          0           0           0        1000           0           0
          0           0           0           0          10           0
          0           0           0           0           0        1000

R =
  1      0
  0      1
```

The obtained gain matrix is as shown.

```
Gain =

 -0.0028    -0.0000    -0.0000    -1.7606    -0.0112    -1.1046
 -0.0028    -0.0000     0.0001    -1.7606    -0.0112     1.1174
```

The simulink model for the controlled system is shown below.

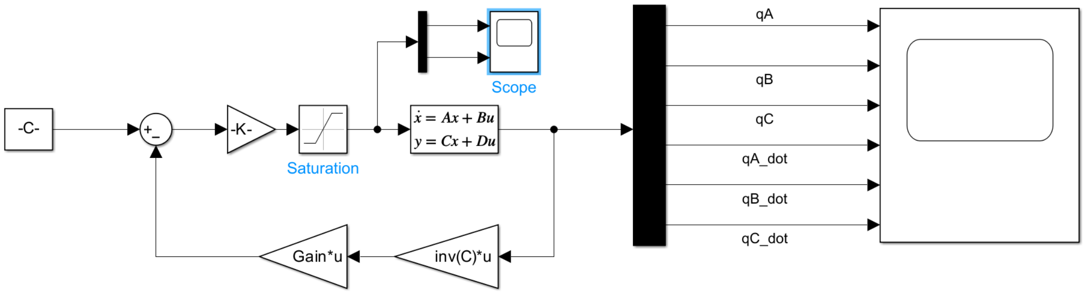

Figure 7: Simulink Model of the Controlled System

The system behaviour with this feedback loop is shown below.

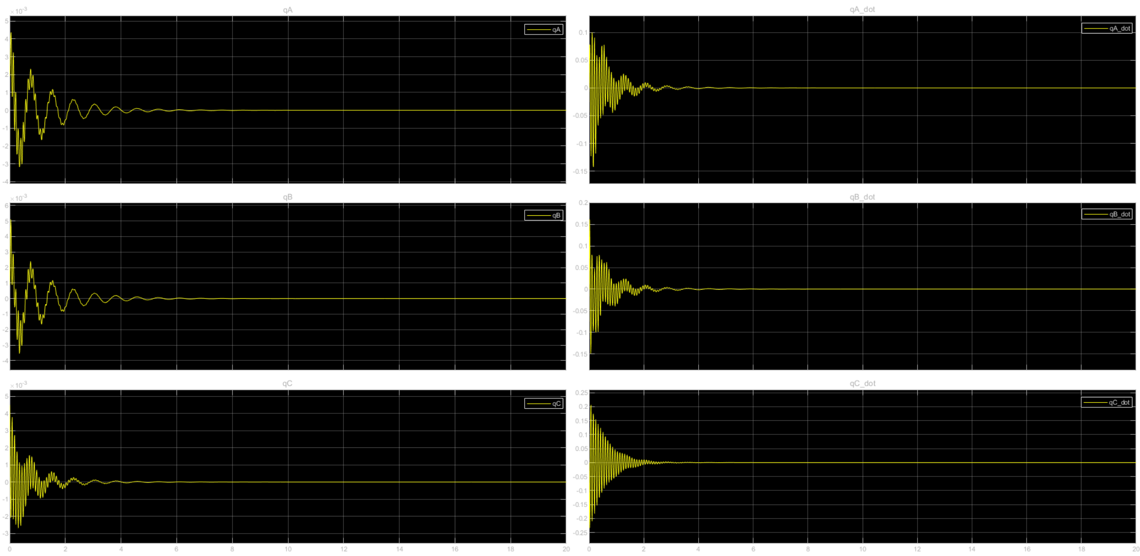

Figure 8: System Behaviour in state feedback

We see that this controller is able to quickly dampen out all vibrations in about 6 seconds. The following plot shows how the control input changes with time.

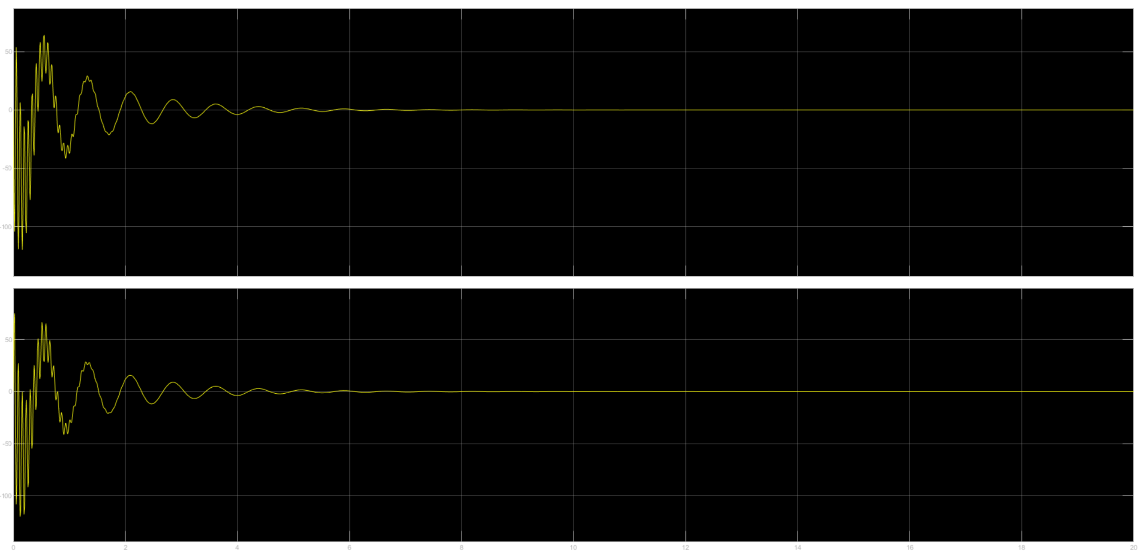

Figure 9: Actuator Input Evolution

It is observed that the designed controller input voltage in both actuators does not reach the saturation voltage of 120V.

The closed-loop system is generated using the following MATLAB code snippet.

```
u1 = 5e-3;
u2 = 4.5e-3;
u3 = 1e-3;

initial_conditions = [u1 ; u2 ; u3 ; zeros(n,1)];

Qw = [1e3 ; 10 ; 1e3 ; 1e3 ; 10 ; 1e3];
Q = diag(Qw)
R = 1*eye(k)
P = 1000;

[Gain,~,~] = lqr(sys,Q,R);

model_sys = P*sys;
feedback_sys = Gain*inv(C);

CL_sys = feedback(model_sys,feedback_sys)
```

The following tables compare the damping ratios of the open-loop and closed-loop systems.

```
    damp(sys)

         Pole                 Damping       Frequency       Time Constant
                                          (rad/seconds)       (seconds)

 -5.54e-02 + 8.27e+00i      6.70e-03        8.27e+00          1.80e+01
 -5.54e-02 - 8.27e+00i      6.70e-03        8.27e+00          1.80e+01
 -7.06e-01 + 7.06e+01i      1.00e-02        7.06e+01          1.42e+00
 -7.06e-01 - 7.06e+01i      1.00e-02        7.06e+01          1.42e+00
 -1.75e-01 + 8.76e+01i      2.00e-03        8.76e+01          5.71e+00
 -1.75e-01 - 8.76e+01i      2.00e-03        8.76e+01          5.71e+00

 damp(CL_sys)

         Pole                 Damping       Frequency       Time Constant
                                          (rad/seconds)       (seconds)

 -7.37e-01 + 8.24e+00i      8.91e-02        8.27e+00          1.36e+00
 -7.37e-01 - 8.24e+00i      8.91e-02        8.27e+00          1.36e+00
 -7.42e-01 + 7.06e+01i      1.05e-02        7.06e+01          1.35e+00
 -7.42e-01 - 7.06e+01i      1.05e-02        7.06e+01          1.35e+00
 -1.04e+00 + 8.76e+01i      1.19e-02        8.76e+01          9.59e-01
 -1.04e+00 - 8.76e+01i      1.19e-02        8.76e+01          9.59e-01
```

These numbers clearly show the increased damping ratios as well as the decreased time constants of the controlled system which confirms the effectiveness of the LQR controller. The least changes are seen in the second mode which is expected as the weightage given for this mode is low in the Q matrix. The analysis is concluded by comparing the bode plot of the 2 systems. The bode plots are plotted only for the displacement states.

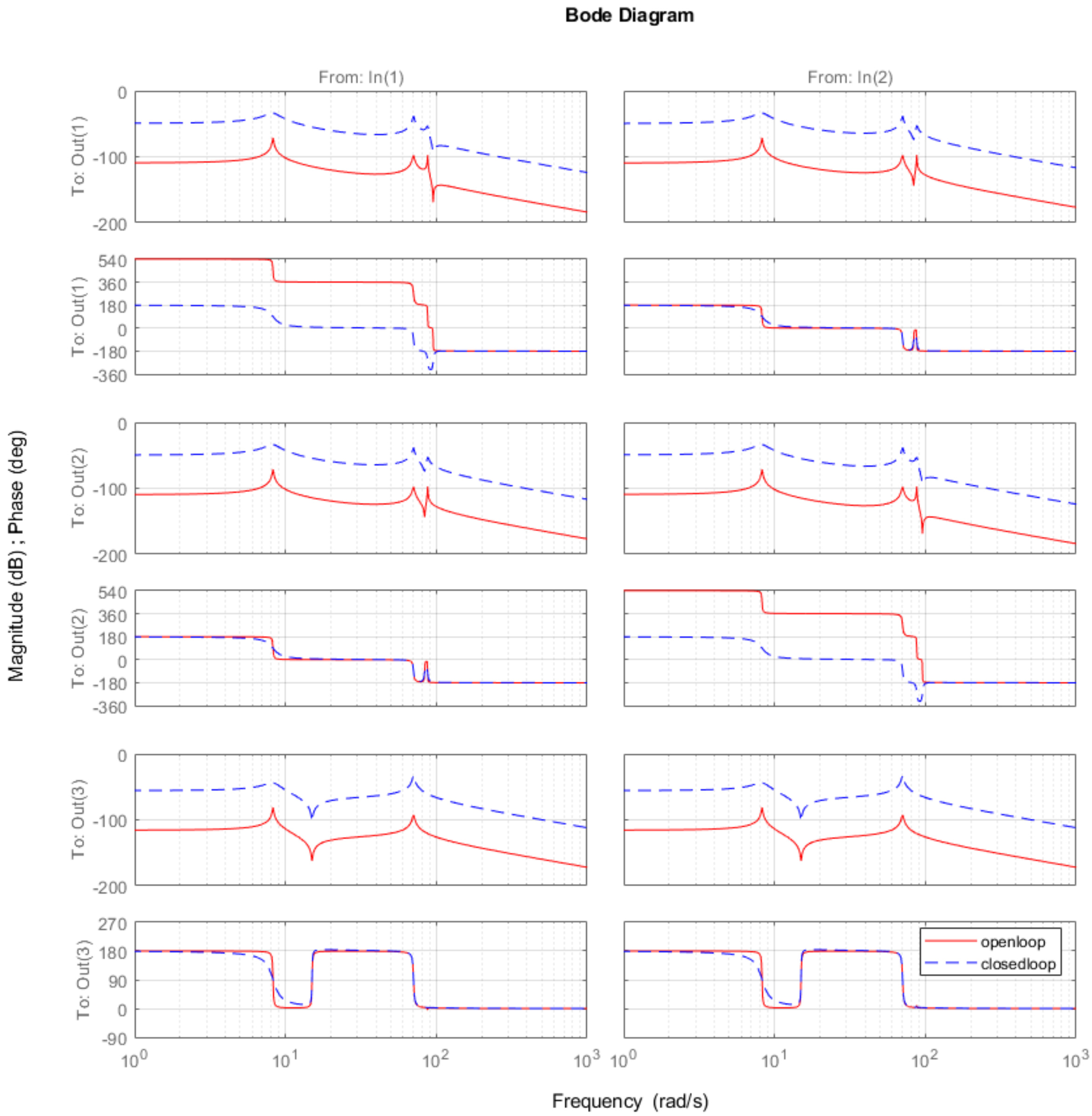

Figure 10: Bode Plots (Open Loop vs Closed Loop)

The main improvements that we see in the closed-loop bode plots are that the curves peaks have flattened which corresponds to increased damping. The phase plots have also become smoother as compared to more or less steps before.