The Turning Manoeuvre

Grumman X-29

The dynamics of Turning (with only δl)

$$
\begin{cases}
r_a = \dfrac{\rho V S}{2m} C y_\beta \cdot \beta + \dfrac{g}{V} \cos\theta \cdot \phi \\
\dot{r} = \dfrac{\rho V^2 S L}{2C}\left[C n_\beta \cdot \beta + C n_r \dfrac{rL}{V} + C n_p \dfrac{pL}{V} + C n_{\delta n} \delta n\right] \\
\dot{p} = \dfrac{\rho V^2 S L}{2A}\left[C l_\beta \cdot \beta + C l_r \dfrac{rL}{V} + C l_p \dfrac{pL}{V} + C l_{\delta l} \boldsymbol{\delta l}\right] \\
\dot{\phi} = p
\end{cases}
$$

We want to turn on the right by using only the aileron , δl

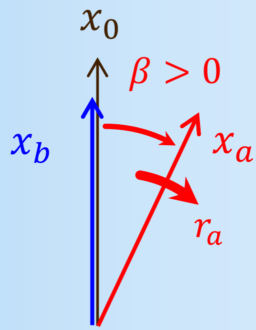
1) The pilot applies $\delta l < 0$ and maintains it.
The first equation which is excited is the roll equation :

$$\dot{p} = \frac{\rho V^2 S L}{2A} \cdot C l_{\delta l} \cdot \delta l \rightarrow p \nearrow$$

2) Then, the kinematic equation : $\dot{\phi} = p \rightarrow \phi \nearrow$

3) Then, the lateral equation :

$$r_a = \frac{g}{V} \cos\theta \cdot \sin\phi \rightarrow r_a \nearrow$$



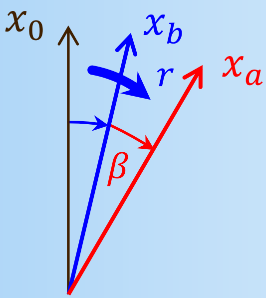
The vector $\vec{V}$ is turning to the right … and the side slip β appears ….

The dynamics of Turning (with only δl)

Up to now, the aircraft has not already turned ; the apparition of the side slip $\beta > 0$ will make it turning to the right

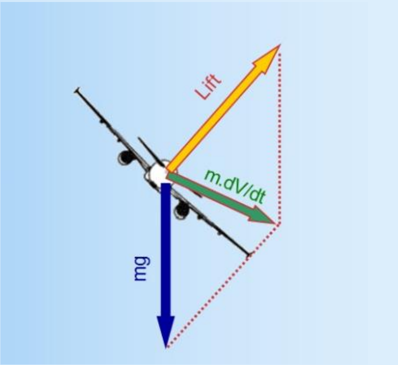
4) Then, the yaw equation : $$\dot{r} = \frac{\rho V^2 SL}{2C} \cdot Cn_\beta \cdot \beta \rightarrow r \nearrow$$

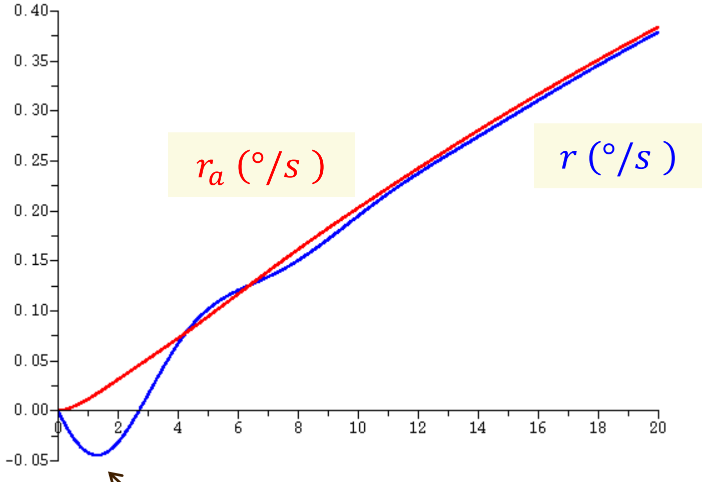
The aircraft is turning to the right….

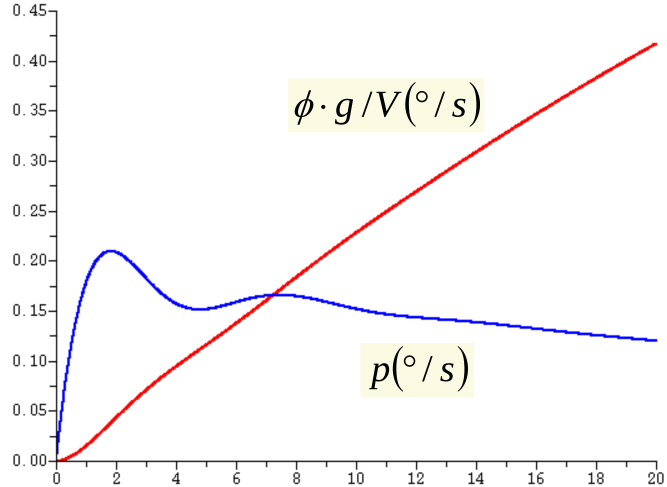


Conclusion : turning with the ailerons proceeds with different steps,

1) The ailerons deflection makes the aircraft banking
2) The Lift Force (within the a/c plane of symmetry) follows and banks
3) Which makes the Velocity vector rotating
4) The good β appears and makes the aircraft rotates …

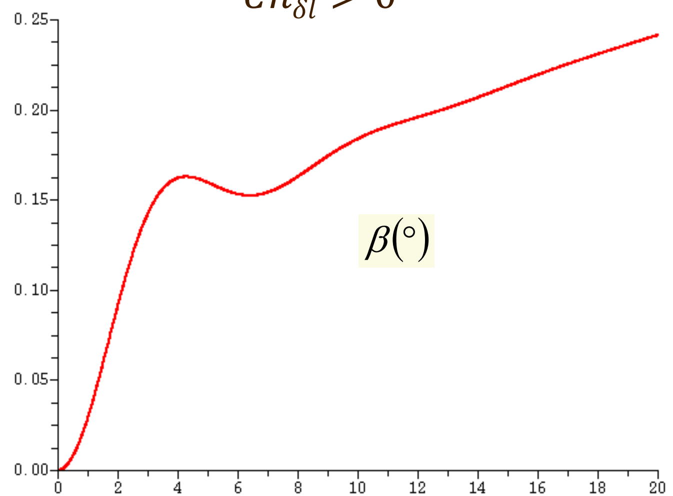


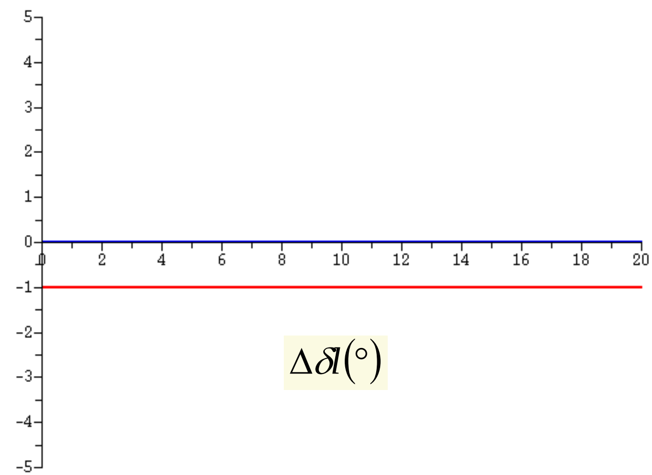


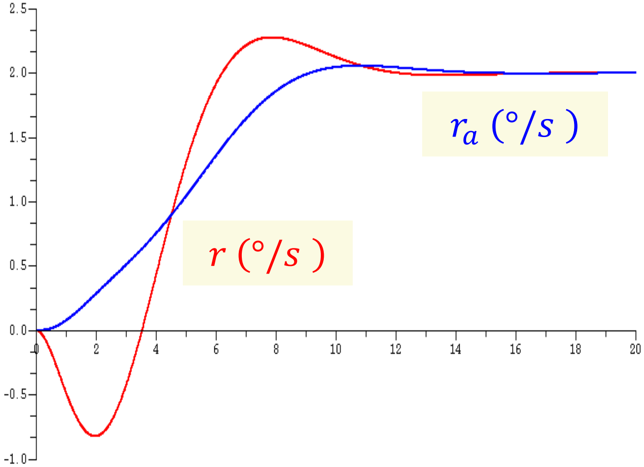


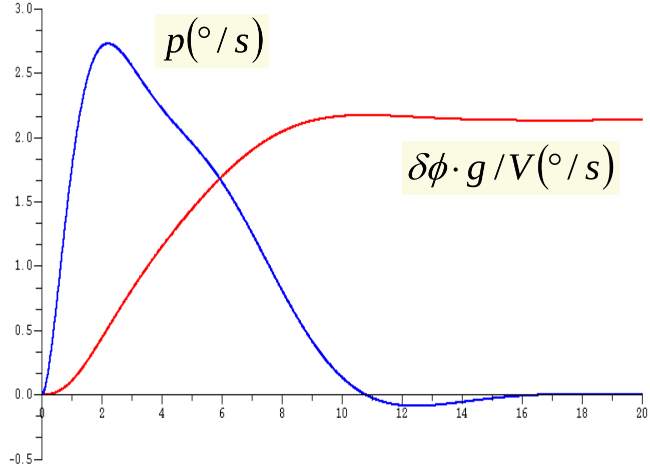
Inverse Yaw effect
$Cn_{\delta l} > 0$

Δδl aileron step input

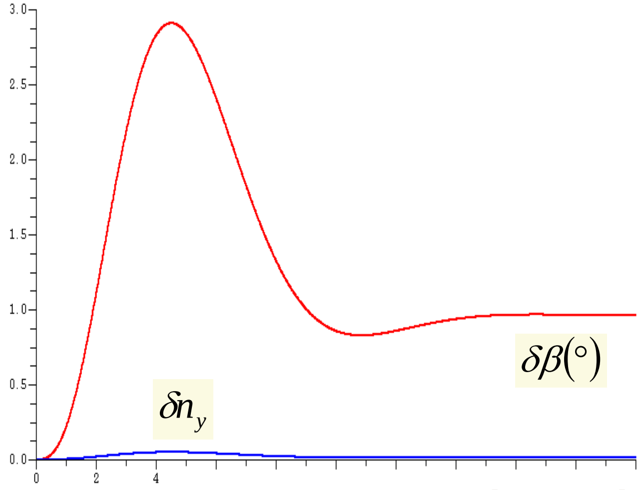


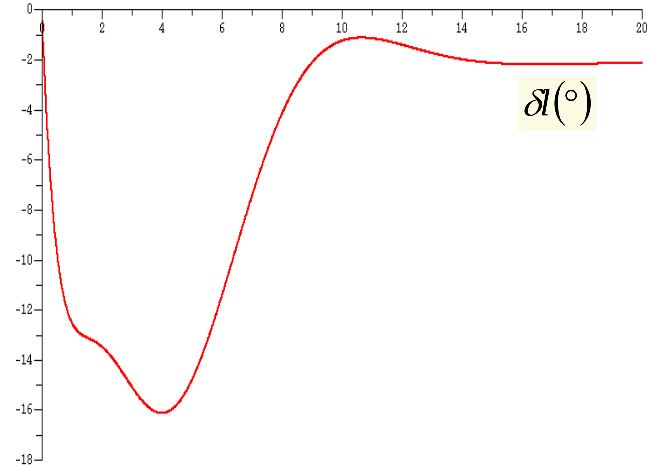




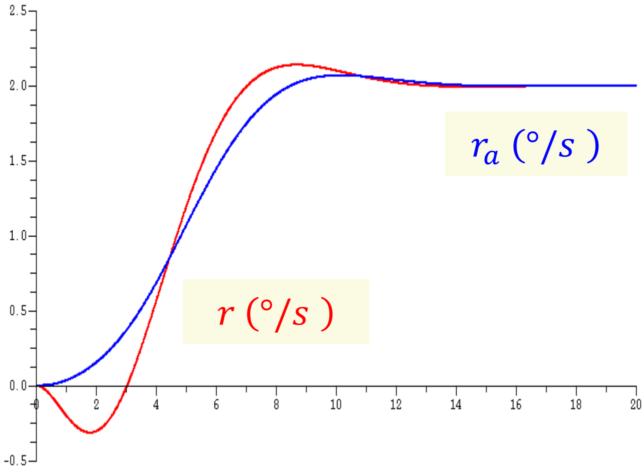


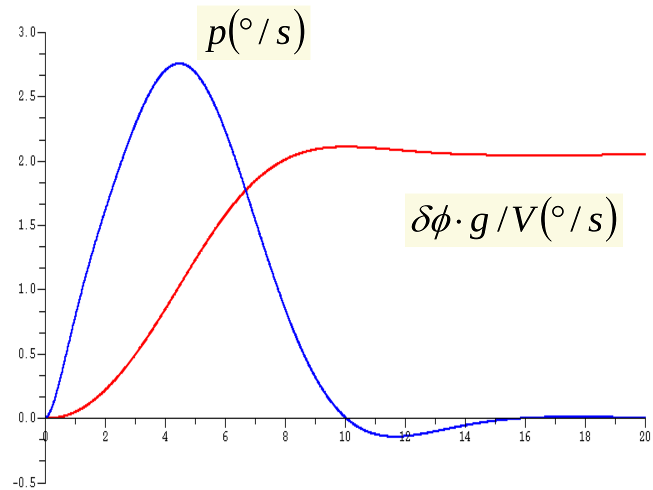
δl < 0 for initiating / good β > 0 / final δl < 0
Spiral Mode Convergent / $Cn_r Cl_\beta - Cn_\beta Cl_r > 0$



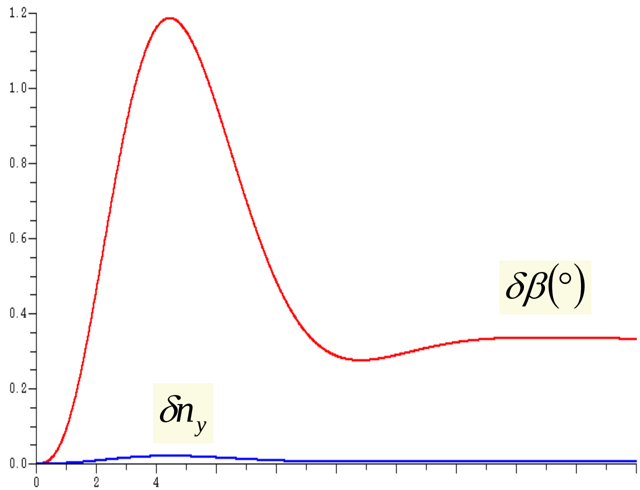


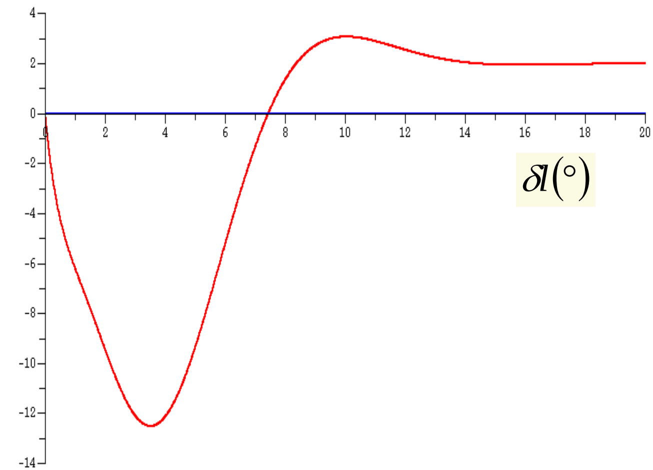
Yaw r objective by δl command / Controlled Aircraft





δl < 0 for initiating / good β > 0 / final δl > 0
Spiral Mode Divergent / $Cn_r Cl_\beta - Cn_\beta Cl_r < 0$





Yaw r objective by δl command / Controlled Aircraft

The dynamics of Turning (with only δn)

$$\begin{cases} r_a = \dfrac{\varrho V S}{2m} C y_\beta \cdot \beta + \dfrac{g}{V}\cos\theta \cdot \phi \\ \dot{r} = \dfrac{\rho V^2 S L}{2C}\left[C n_\beta \cdot \beta + C n_r \dfrac{rL}{V} + C n_p \dfrac{pL}{V} + C n_{\delta n} \boldsymbol{\delta n} \right] \\ \dot{p} = \dfrac{\rho V^2 S L}{2A}\left[C l_\beta \cdot \beta + C l_r \dfrac{rL}{V} + C l_p \dfrac{pL}{V} + C l_{\delta l} \delta l \right] \\ \dot{\phi} = p \end{cases}$$

We want to turn on the right by using only the rudder , δn

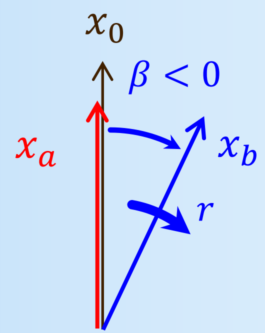
1) The pilot applies $\delta n < 0$ and maintains it.
The first equation which is excited is the yaw equation :

$$\dot{r} = \frac{\rho V^2 S L}{2A} \cdot C l_{\delta n} \cdot \delta n \rightarrow r \nearrow$$

The aircraft is turning to the right

and the side slip β appears ….

$$\beta < 0$$



3) Then, the roll equation :

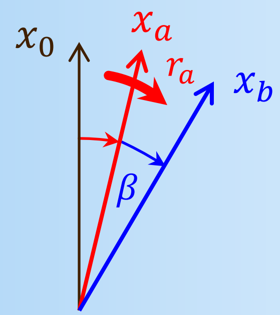
$$\dot{p} = \frac{\rho V^2 S L}{2A} \cdot C l_\beta \cdot \beta \rightarrow p \nearrow$$

The dynamics of Turning (with only δn)

3) Then, the kinematic equation : $\dot{\phi} = p \rightarrow \phi \nearrow$

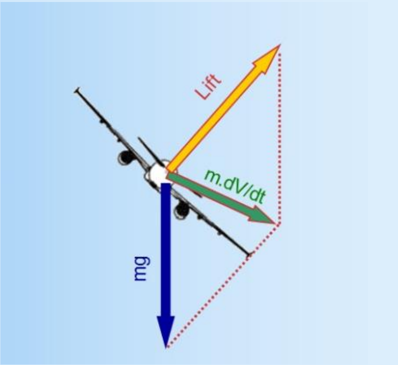
4) Then, the lateral equation : $r_a = \frac{g}{V} \cos\theta \cdot \sin\phi \rightarrow r_a \nearrow$

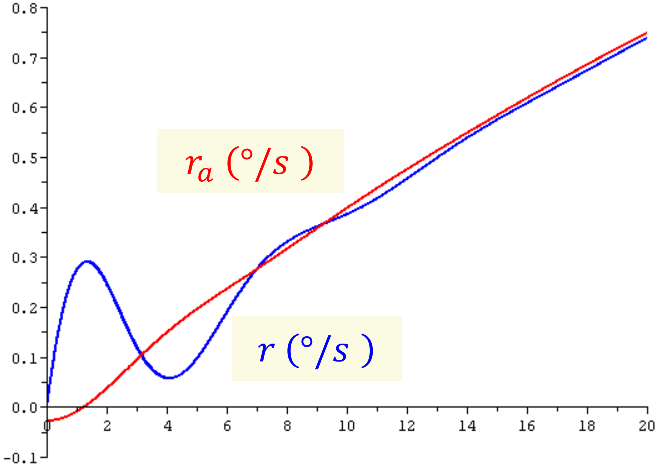
And the vector $\vec{V}$ is turning to the right ….

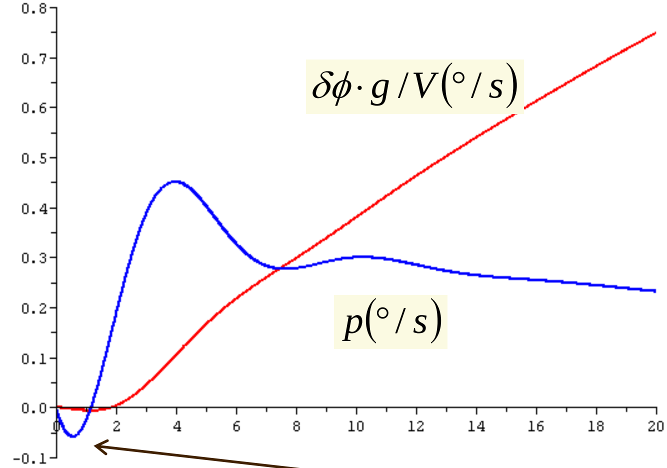


<u>Conclusion</u> : turning with the rudder proceeds with different steps,

1) The rudder deflection makes the aircraft rotating
2) the good β appears and makes the aircraft banks …
3) The Lift Force (within the a/c plane of symmetry) follows and banks
4) And the Velocity vector rotates …

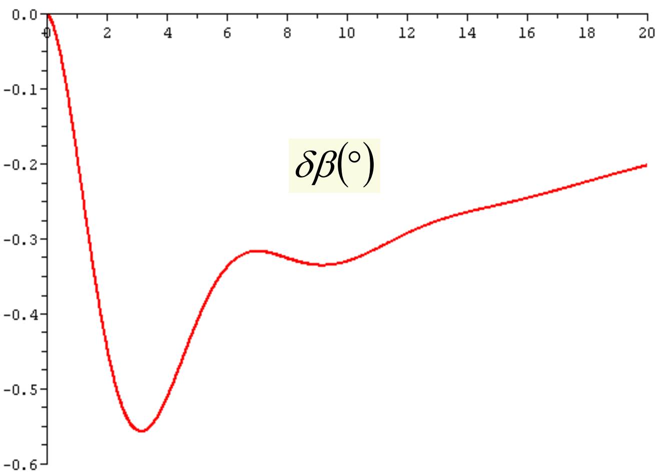


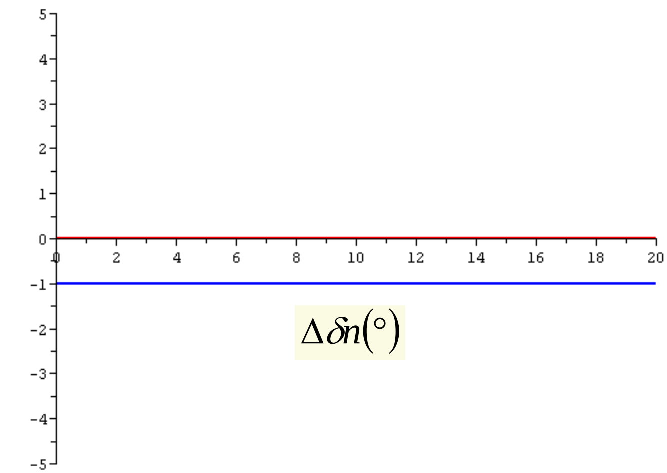


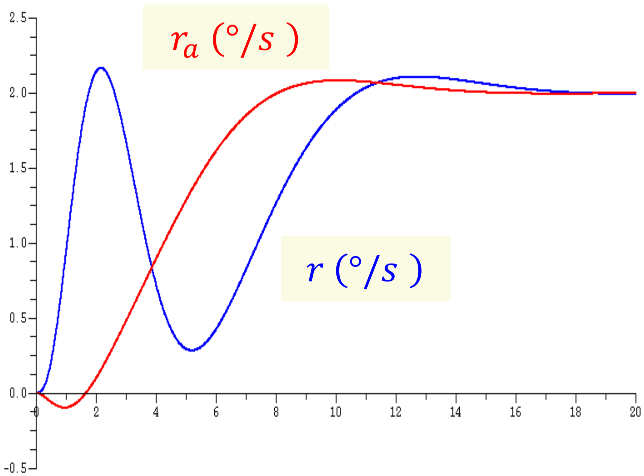


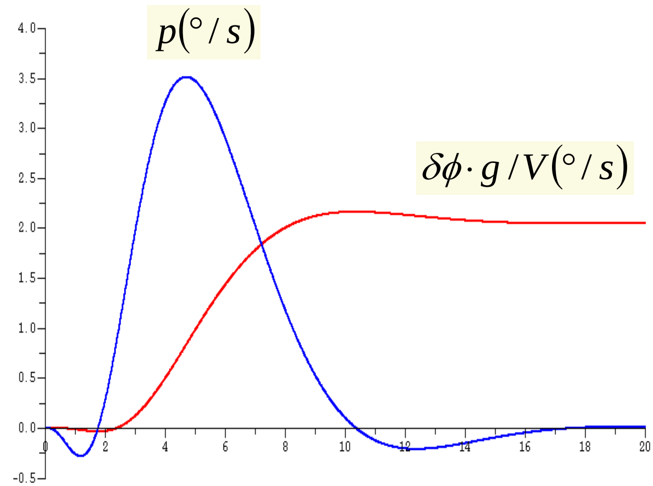
Initial effect
$Cl_{\delta n} > 0$

Δδn rudder step input



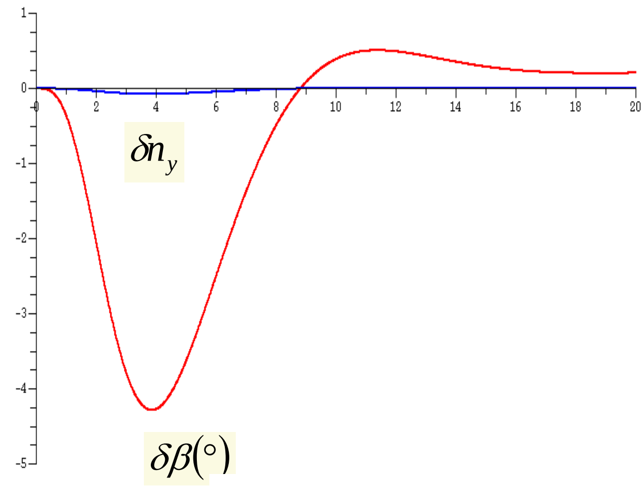


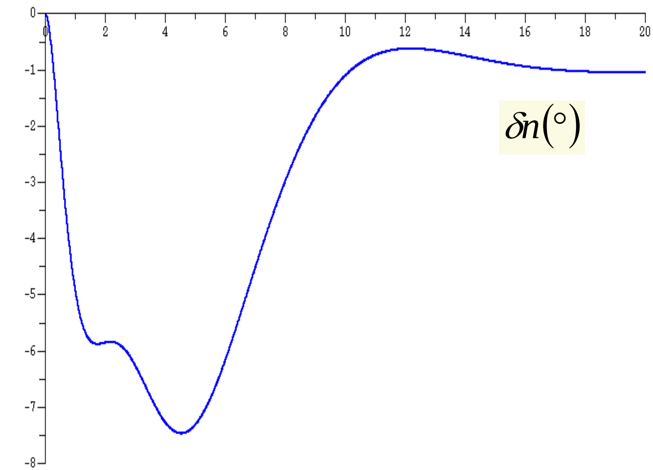




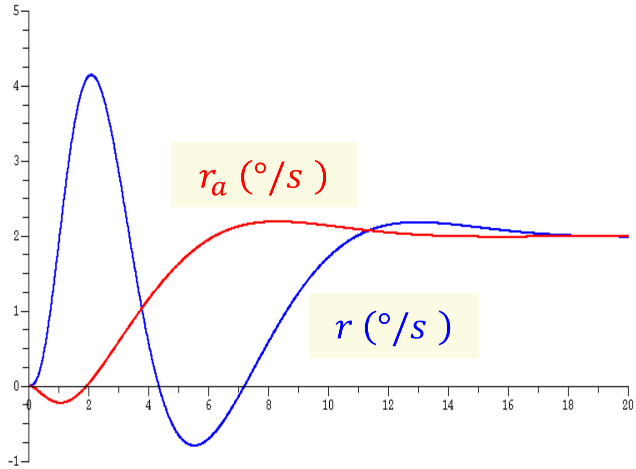
δn < 0 for initiating / good β < 0 / final δn < 0

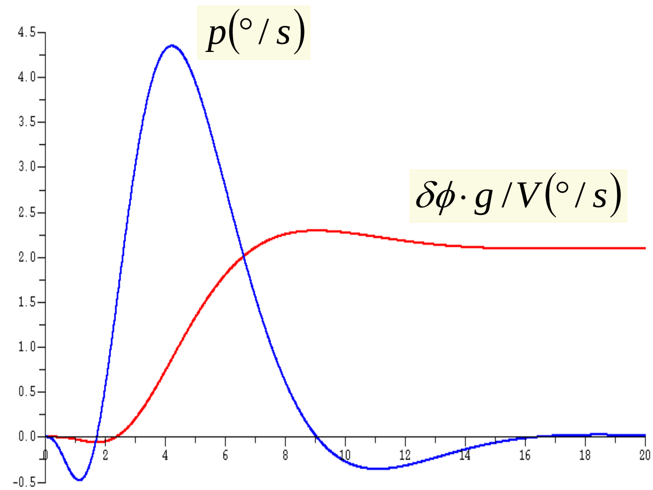
Spiral Mode Convergent / $Cn_r Cl_\beta - Cn_\beta Cl_r > 0$



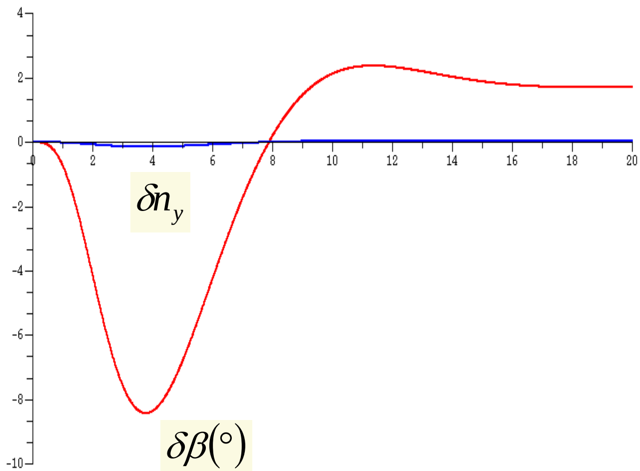


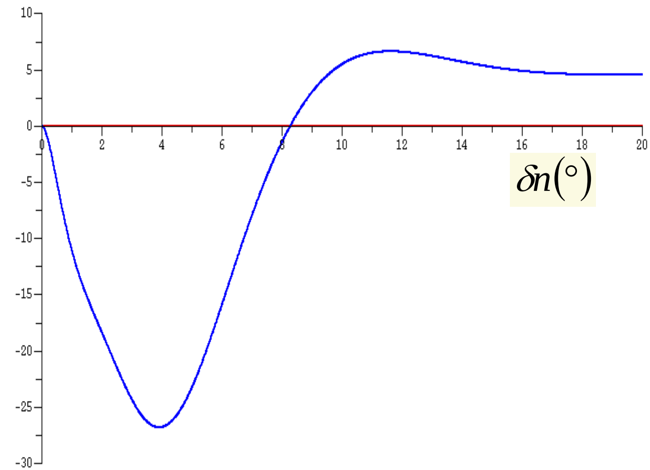
Yaw r objective by δn command / Controlled Aircraft





δn < 0 for initiating / good β < 0 / final δn > 0
Spiral Mode Divergent / $Cn_r Cl_\beta - Cn_\beta Cl_r < 0$





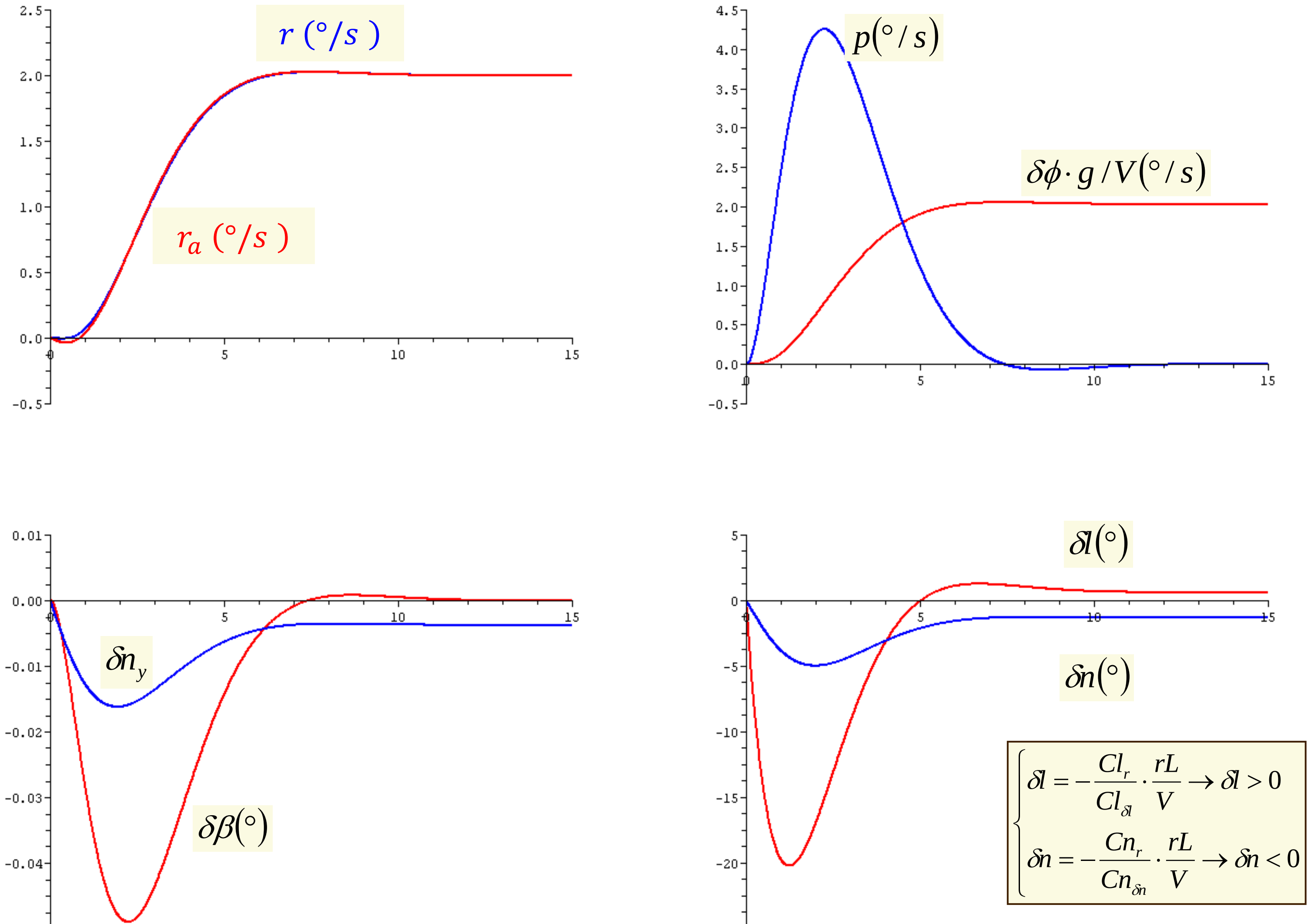
Yaw r objective by δn command / Controlled Aircraft

The dynamics of Turning …

Turning with only δl or only δn is possible by creating the good side slip β for completing the manoeuvre …

Turning with only δl :
$\rightarrow$ $\delta l < 0$ makes banking the aircraft and rotating the velocity vector
$\rightarrow$ the good side slip $\beta > 0$ is produced and the aircraft turns, thanks to Cn_β
Turning with only δn :
$\rightarrow$ $\delta n < 0$ makes producing the good side slip $\beta < 0$
$\rightarrow$ The aircraft banks and the velocity turns, thanks to Cl_β

By coordinating both controls δl and δn, you can perform the turning manoeuvre without side slip (keeping $\beta=0$). This is the so called Coordinate Steady Turn



Yaw r / β=0° objectives by δl / δn commands / Controlled Aircraft

Lateral Dynamics

Piaggio Avanti

Small state variables variation principle

$$\begin{cases} mV\cdot(\dot\beta + r) = \dfrac{1}{2}\rho V^2 S C y_\beta \beta + mg \overbrace{\cos\theta\sin\phi}^{\sin\phi_1 = \cos\theta\sin\phi} \\ C\dot r = \dfrac{1}{2}\rho V^2 SL\left[Cn_\beta\beta + Cn_r\dfrac{rL}{V} + Cn_p\dfrac{pL}{V} + Cn_{\delta n}\delta n\right] \\ A\dot p = \dfrac{1}{2}\rho V^2 SL\left[Cl_\beta\beta + Cln_r\dfrac{rL}{V} + Cl_p\dfrac{pL}{V} + Cl_{\delta l}\delta l\right] \\ \dot\phi = p \end{cases}$$

Starting from an equilibrium, we consider small variations $(\delta\beta, \delta r = r, \delta p = p, \delta\phi)$ obtained by small command variations $(\Delta\delta l, \Delta\delta n)$; this is obtained by differentiating the flight equations :

$$\delta\dot\beta \approx \delta\left[\frac{\varrho V S}{2m} C y_\beta \beta - r + \frac{g}{V}\cos\theta\cdot\sin\phi\right] \approx \frac{\varrho V S}{2m} C y_\beta\cdot\delta\beta - \delta r + \frac{g}{V}\cos\theta\cdot\delta\phi$$

Small state variables variation principle

$$
\begin{cases}
\delta\dot{\beta} = \dfrac{\varrho V S}{2m} C y_\beta \cdot \delta\beta - r + \dfrac{g}{V}\cos\theta \cdot \delta\phi \\
\dot{r} = \dfrac{\rho V^2 S L}{2C}\left[C n_\beta \cdot \delta\beta + C n_r \cdot \dfrac{rL}{V} + C n_p \cdot \dfrac{pL}{V} + C n_{\delta n} \cdot \Delta\delta n\right] \\
\dot{p} = \dfrac{\rho V^2 S L}{2A}\left[C l_\beta \cdot \delta\beta + C l_r \cdot \dfrac{rL}{V} + C l_p \cdot \dfrac{pL}{V} + C l_{\delta l} \cdot \Delta\delta l\right] \\
\delta\dot{\phi} = p
\end{cases}
$$

We obtain the state matrix A and the command matrix B

$$
\begin{bmatrix} \delta\dot{\beta} \\ \dot{r} \\ \dot{p} \\ \delta\dot{\phi} \end{bmatrix}
=
\begin{bmatrix} y_\beta & -1 & 0 & \dfrac{g}{V}\cos\theta \\ n_\beta & n_r & n_p & 0 \\ l_\beta & l_r & l_p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\cdot
\begin{bmatrix} \delta\beta \\ r \\ p \\ \delta\phi \end{bmatrix}
+
\begin{bmatrix} 0 & 0 \\ 0 & n_{\delta n} \\ l_{\delta l} & 0 \\ 0 & 0 \end{bmatrix}
\cdot
\begin{bmatrix} \Delta\delta l \\ \Delta\delta n \end{bmatrix}
$$

$$
\delta\dot{X} = A \cdot \delta X + B \cdot \delta U
$$

Pure Roll mode : general solution

The first mode of the Lateral dynamics is called the Pure Roll mode.
It is a first order motion driven by the roll speed variable, p.

$$\dot{p} = l_p \cdot p$$

$$p = p_0 \cdot e^{st} \rightarrow \dot{p} = s \cdot p$$

$$s = l_p = -\frac{1}{\tau} \qquad l_p = \frac{\rho V S L^2}{2A} \cdot Cl_p$$

s is a real eigen value → a-periodic mode ; convergent because $s < 0$

The a-periodic mode is convergent : the characteristic time $\tau = -1/l_p > 0$

$$p = p_0 \cdot e^{-t/\tau}$$

Pure Roll mode : general solution

From the initial equilibrium, I apply a roll command variation $\Delta\delta l$

$$\dot{p} = l_p \cdot p + l_{\delta l} \cdot \Delta\delta l \quad \Rightarrow \quad p = p_0 \cdot e^{-t/\tau} + p_\infty$$

I have to add to the general solution a particular solution, p_∞

This particular solution obtained for : $\dot{p} = 0 \rightarrow p_\infty = -l_{\delta l} \cdot \Delta\delta l / l_p$

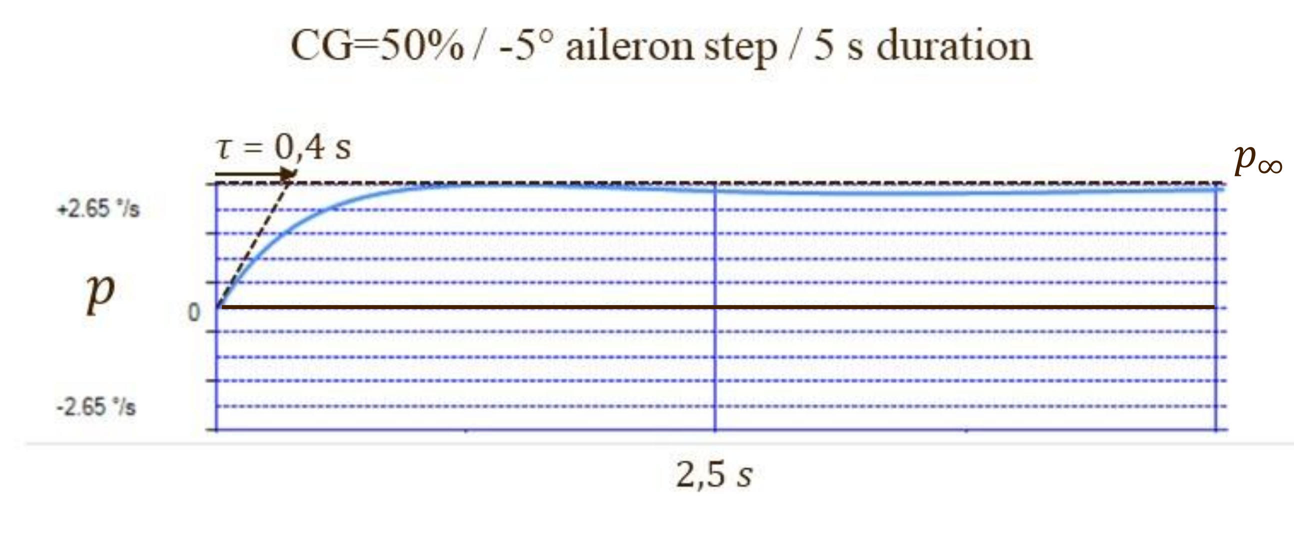
Initial condition : $p_{t=0} = 0 \rightarrow p_0 = -p_\infty$

$$\Rightarrow \quad p = -p_\infty \cdot e^{-t/\tau} + p_\infty = p_\infty \cdot \left(1 - e^{-t/\tau}\right)$$

$$\tau = -\frac{1}{l_p} > 0$$

$$p_\infty = -\frac{l_{\delta l} \cdot \Delta\delta l}{l_p}$$

DC8 / 0 ft / 125 m/s / -5° aileron step



The pure Roll mode is a short convergent real mode :

- the a-periodic response is fast , characterised by a short time constant, τ
- Notice that we can guess the value of τ graphically : by drawing the slope at t=0, the intersection with the asymptote gives τ

$$p' = \frac{p_\infty}{\tau} \cdot e^{-\frac{t}{\tau}} \rightarrow p'_{t=0} = \frac{p_\infty}{\tau}$$

Side Slip Oscillation Mode

The Side Slip Oscillation Mode is an eigen mode linked to the variables (β, r)

By considering the associated sub matrix :

$$\begin{vmatrix} \delta\dot{\beta} \\ \dot{r} \end{vmatrix} = \begin{bmatrix} y_\beta & -1 \\ n_\beta & n_r \end{bmatrix} \cdot \begin{vmatrix} \delta\beta \\ r \end{vmatrix}$$

$$\begin{cases} y_\beta = \dfrac{\rho V S}{2m} \cdot C y_\beta \\ n_\beta = \dfrac{\rho V^2 S L}{2C} \cdot C n_\beta \qquad n_r = \dfrac{\rho V S L^2}{2C} \cdot C n_r \end{cases}$$

$$det[A - s \cdot I] = 0$$

$$det \begin{bmatrix} y_\beta - s & -1 \\ n_\beta & n_r - s \end{bmatrix} = 0 \quad \Rightarrow \quad s^2 - (n_r + y_\beta) \cdot s + n_\beta + n_r y_\beta = 0$$

$$s^2 + 2\lambda \cdot s + \omega_0^2 = 0$$

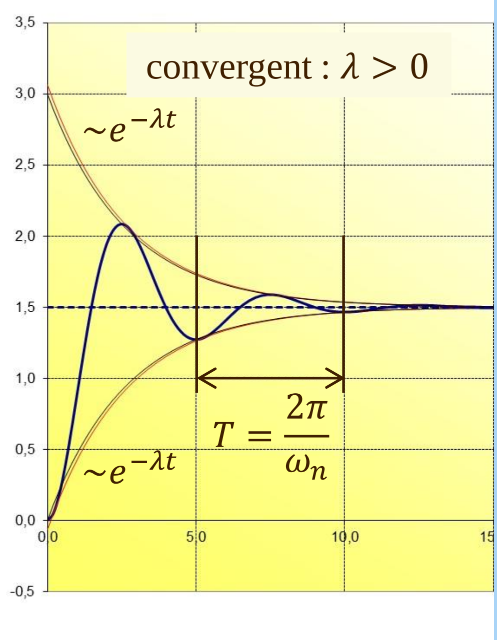
$$\Delta' = \lambda^2 - \omega_0^2 = -\omega_n^2 < 0 \qquad \Delta' \approx -(n_\beta + n_r y_\beta) + \frac{(n_r + y_\beta)^2}{4} < 0$$

the eigenvalues are complex and conjugate : $s \,/\, \bar{s} = -\lambda \pm i \cdot \sqrt{-\Delta'} = -\lambda \pm i \cdot \omega_n$

Side Slip Oscillation Mode

The s-eigenvalues are complex, the solution on the form $\delta X = \delta X_0 \cdot e^{st}$ is periodic function of the time

$$s \,/\, \bar{s} = -\lambda \pm i \cdot \omega_n \quad \Rightarrow \quad \delta X \sim e^{-\lambda t} \cdot (\delta X_1 \cos \omega_n t + \delta X_2 \sin \omega_n t)$$



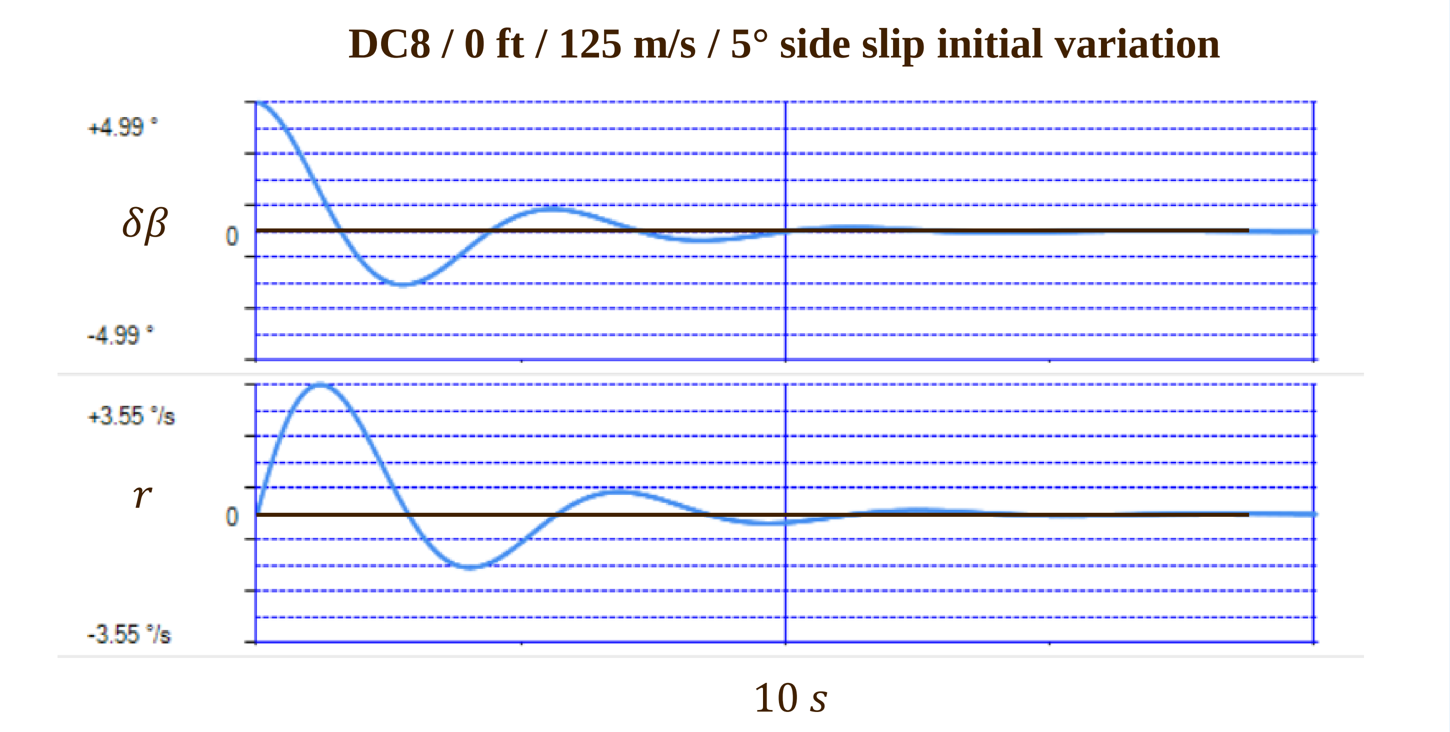
The damping process is linked to $(Cn_r \,, Cy_\beta)$

$$-\lambda = \frac{n_r + y_\beta}{2} < 0$$

The period is linked to the aircraft lateral stability

$$T = \frac{2\pi}{\omega_n} \approx \frac{2\pi}{\sqrt{n_\beta}}$$

General Response from a δβ variation



The Side Slip mode is a short convergent oscillating mode :

- The time period is about 5 to 10 s
- The oscillations ceases at about twice the time-period

Sideslip oscillations mode

The side slip oscillation mode is a periodic mode between the state variables (β, r).
It presents numerous analogies with respect to the Longitudinal Short Period mode

	Short Period Mode	Sideslip Mode
Variables	(α, q)	(β, r)
Period	function of Cm_α Short Period	function of Cn_β Short Period
Damping	function of Cm_q & Cz_α Strong Damping	function of Cn_r & Cy_β Strong Damping

The reality : the Dutch Roll mode

The side slip oscillation mode as presented before is a pure theoretical exercise. A pure yaw motion doesn't exist because there is a strong coupling with the roll motion.

This coupling can be identify with the coefficients l_β / n_p of the A matrix :

- Any pure side motion induces a roll perturbation because of l_β
- Which induces a perturbation in yaw because of n_p

$$\begin{bmatrix} \delta\dot{\beta} \\ \dot{r} \\ \dot{p} \\ \delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} y_\beta & -1 & 0 & \frac{g}{V}\cos\theta \\ n_\beta & n_r & \boxed{n_p} & 0 \\ \boxed{l_\beta} & l_r & l_p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta\beta \\ r \\ p \\ \delta\phi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & n_{\delta n} \\ l_{\delta l} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta l \\ \Delta\delta n \end{bmatrix}$$

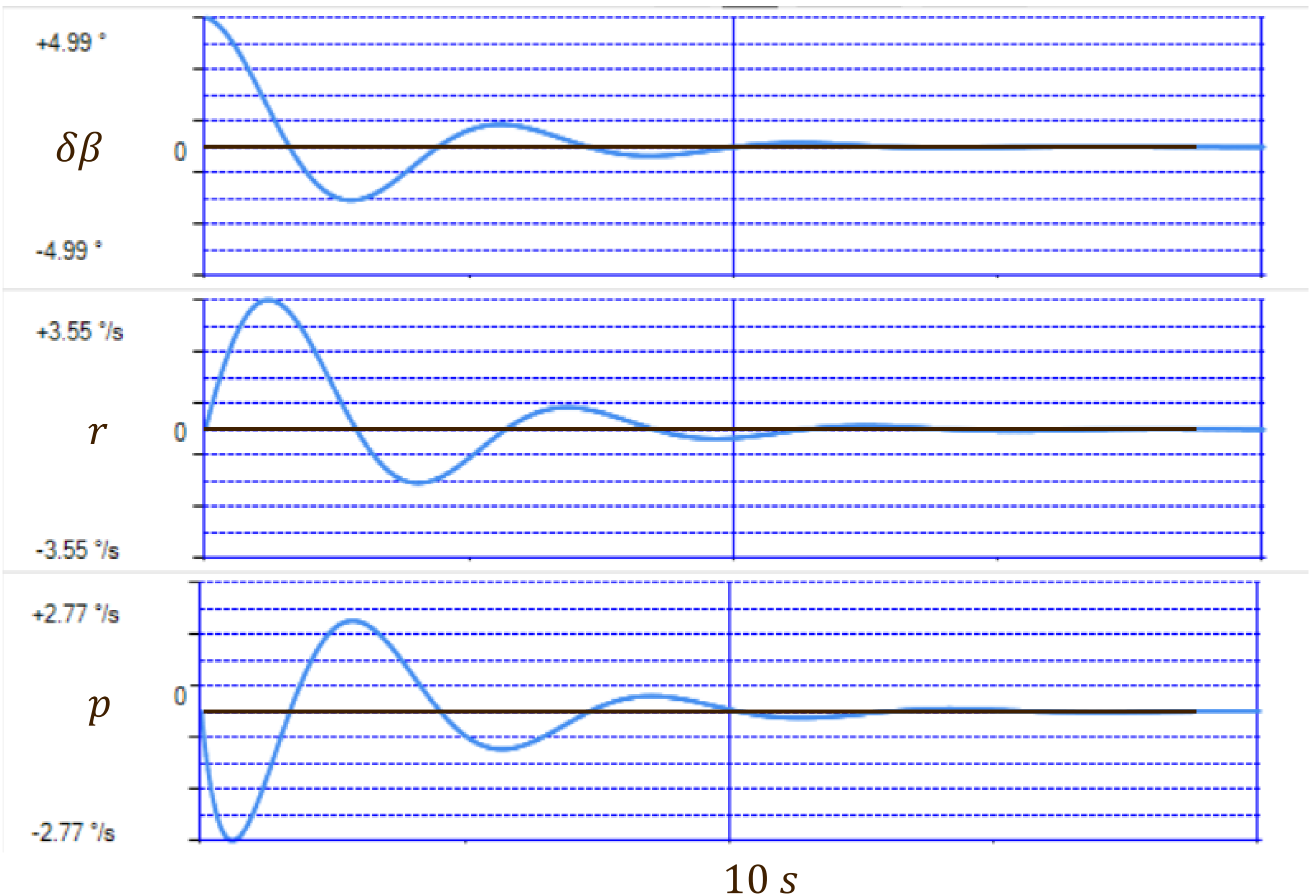
This complex yaw/roll motion is identify as the Dutch Roll mode. It is a periodic mode (even for aft CG). Its damping can be assess by the approximate formula :

$$\lambda \approx -\frac{1}{2}\left(y_\beta + n_r + \frac{n_p - g/V}{n_\beta + l_p^2} \cdot l_\beta \right)$$

$$l_\beta = \frac{\rho V^2 S L}{2A} \cdot Cl_\beta$$

Dutch Roll from a δβ variation

DC8 / 0 ft / 125 m/s / 5° side slip initial variation



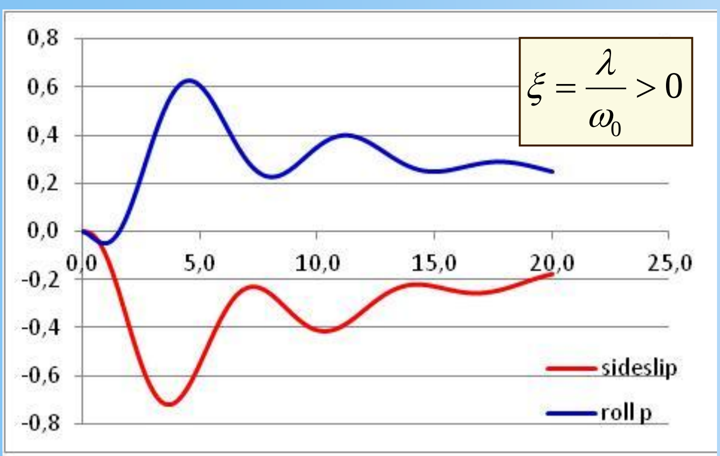
•either

The reality : the Dutch Roll mode

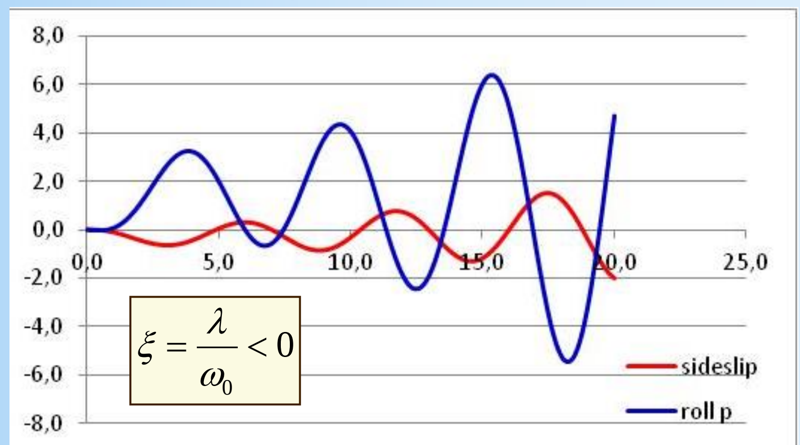
$$\lambda = -\frac{1}{2}\left(\underbrace{y_{\overset{-}{\beta}} + n_{\overset{-}{r}}}_{-} + \underbrace{\frac{n_{\overset{-}{p}} - g/V}{n_\beta + l_p^2} \cdot l_{\overset{-}{\beta}}}_{+} \right)$$

The corrective / additive term is a de-stabilizing term as the product of 2 negative terms.
Hence, the Dutch Roll mode may become divergent but staying always periodic …

Convergent Dutch Roll



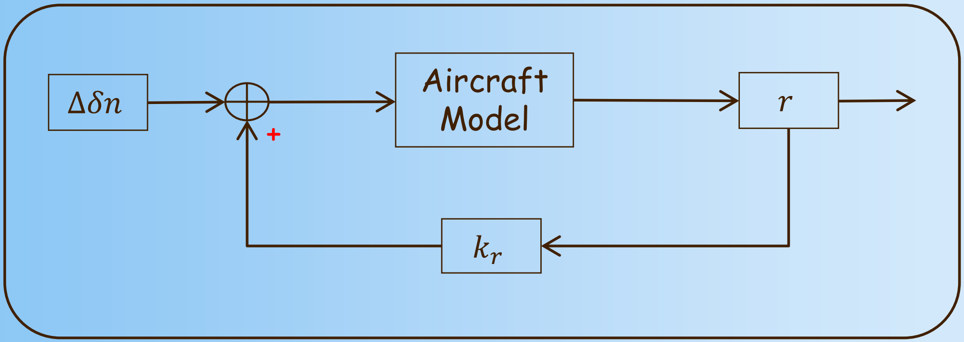
Divergent Dutch Roll
Cl_β more negative



Yaw Damper Principle

The damping of the Dutch Roll mode is driven by : $$\lambda = -\frac{y_\beta + n_r}{2}$$

If I want to improve this damping, I have to increase either $-n_r$ or $-y_\beta$



The principle of the yaw damper is to increase $-n_r$ by making a feedback loop with the (measured) variable r on the command δn

$$\Delta\delta n \rightarrow \Delta\delta n + k_r \cdot r$$

Yaw Damper Principle

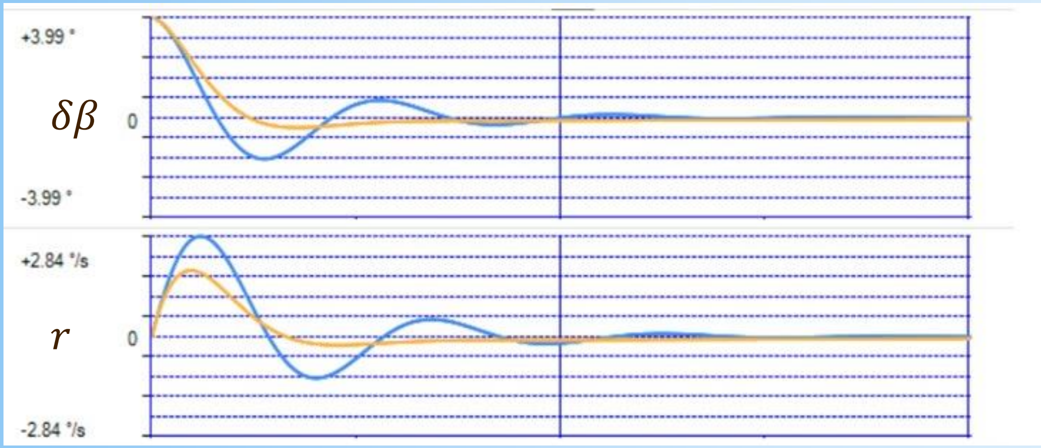
$$\dot{r} = n_\beta \cdot \delta\beta + \bar{n}_r \cdot r + \bar{n}_{\delta n} \cdot \Delta\delta n$$

natural aircraft : variation of yaw equation

$$\dot{r} = n_\beta \cdot \delta\beta + (\bar{n}_r + k_r \bar{n}_{\delta n}) \cdot r + n_{\delta n} \cdot \Delta\delta n$$

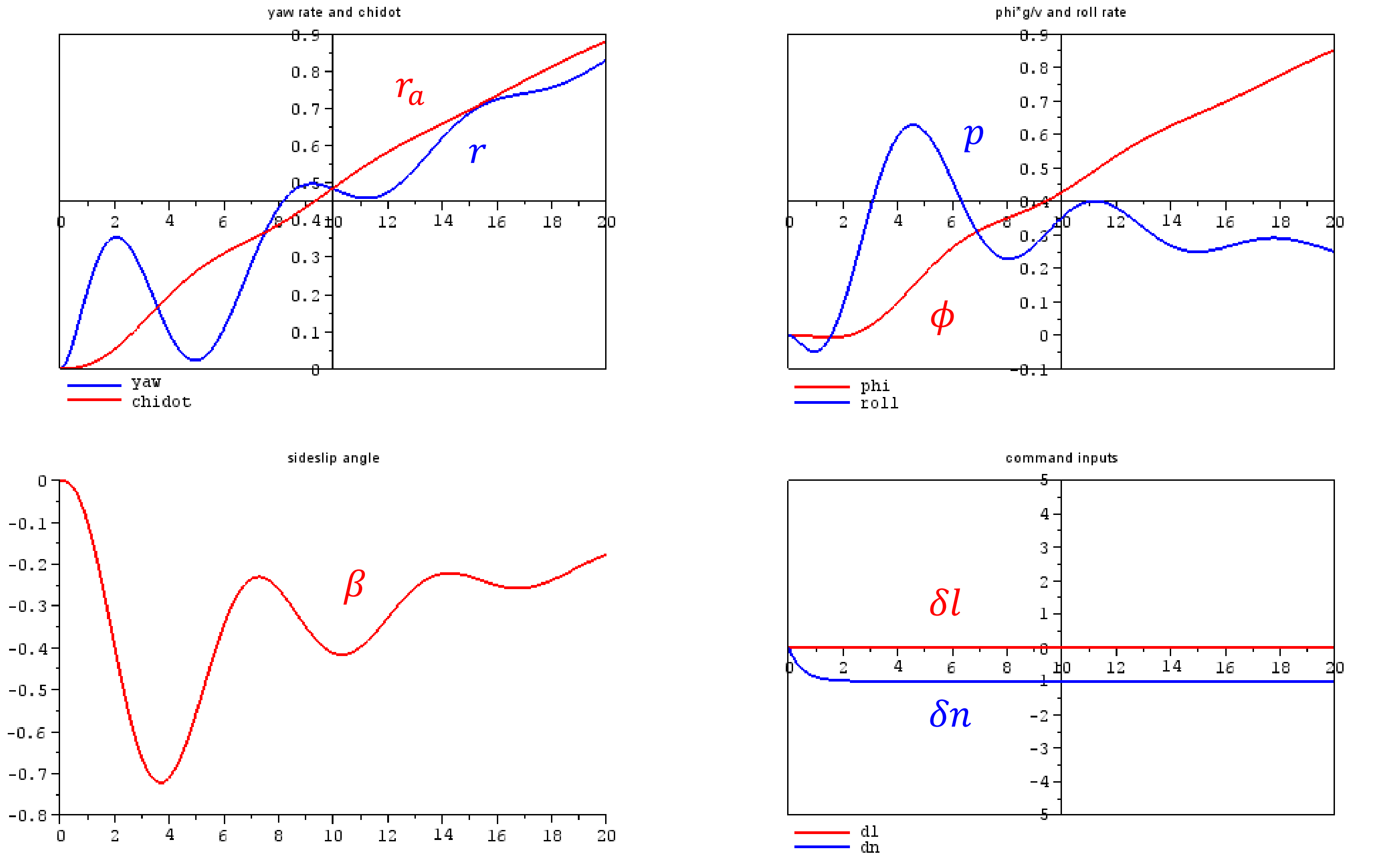
aircraft with feedback law : variation of yaw equation

The n_r is negatively increased by $k_r n_{\delta n}$ (if $k_r > 0$)



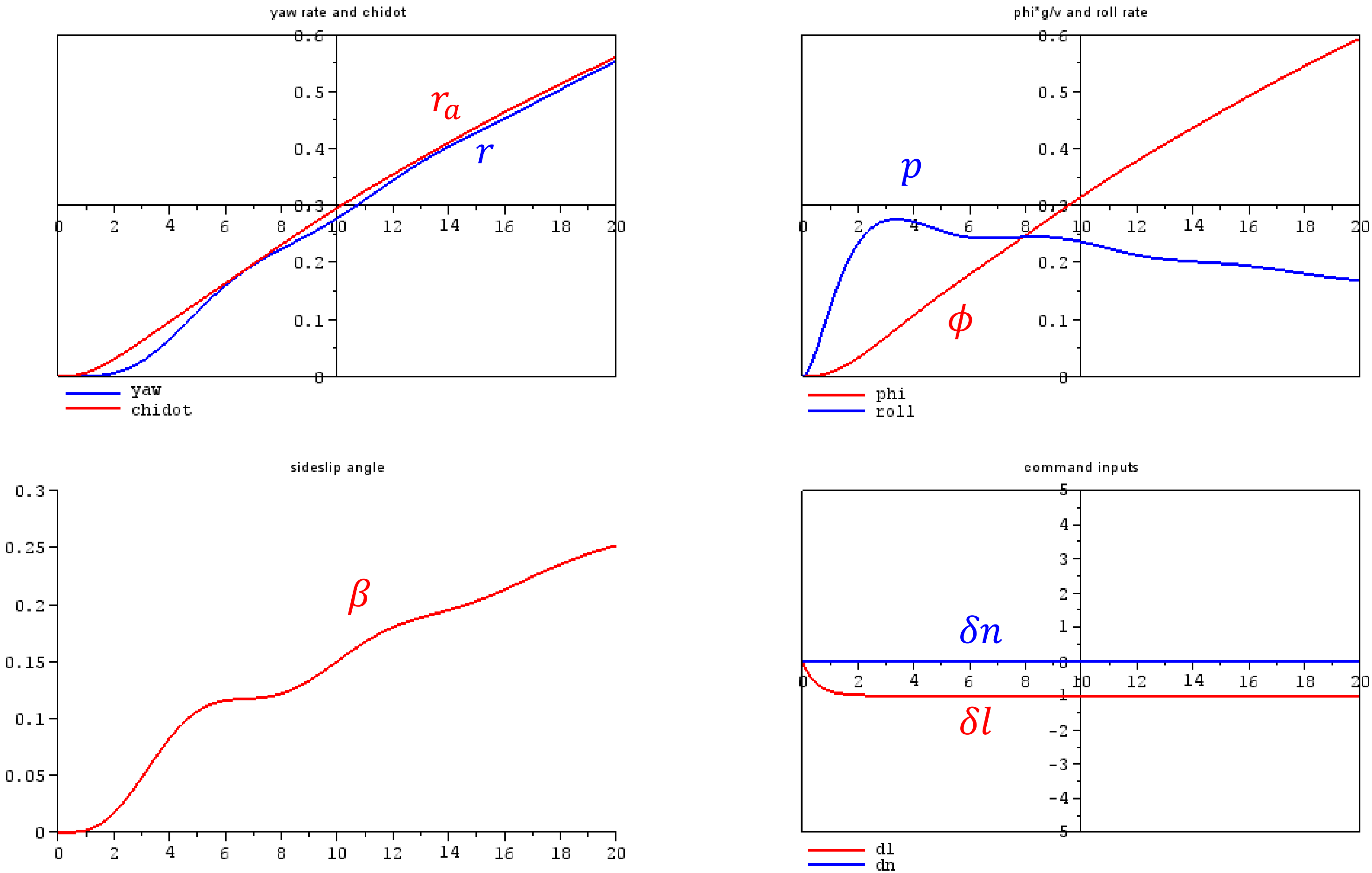
we increase the damping ξ from 0,27 to 0,67 with a gain k_r=1,27

Dutch Roll mode by yaw command



Strong Dutch Roll excitation with rudder control

Dutch Roll mode by roll command



Weak Dutch Roll excitation with aileron control / roll mode is identified

The Spiral mode

This (last) mode is linked to the variation of the state variable, ϕ Roll angle
It assumes that the Pure Roll and the Dutch Roll modes are finished

The dynamics of the Roll angle is given by the 1st order differential equation : $\dot{\phi} = p$

$$\delta\phi = \delta\phi_0 \cdot e^{-\frac{t}{\tau}}$$

The eigenvalue is a real number. The solution is an aperiodic function of time

τ is called the characteristic time : assuming an initial Roll angle variation $\delta\phi_0$

- if τ is positive, the spiral mode is convergent : after a certain time, the Roll angle perturbation $\delta\phi$ tends to zero
- if τ is negative, the spiral mode is divergent : the Roll angle perturbation $\delta\phi$ tends to infinity

The Spiral mode

The spiral mode is an a-periodic motion with a time constant τ given by :

$$\tau \approx \frac{V}{g}\cdot\frac{\overset{-}{n_p}\,\overset{-}{l_\beta} - \overset{+}{n_\beta}\,\overset{-}{l_p}}{n_r l_\beta - n_\beta l_r}$$

$$\begin{cases} l_\beta = \dfrac{\rho V^2 SL}{2A}\cdot Cl_\beta < 0 \\ n_\beta = \dfrac{\rho V^2 SL}{2C}\cdot Cn_\beta > 0 \end{cases} \quad \begin{cases} l_r = \dfrac{\rho VSL^2}{2A}\cdot Cl_r > 0 \\ n_r = \dfrac{\rho VSL^2}{2C}\cdot Cn_r < 0 \end{cases} \quad \begin{cases} l_p = \dfrac{\rho VSL^2}{2A}\cdot Cl_p < 0 \\ n_p = \dfrac{\rho VSL^2}{2C}\cdot Cn_p < 0 \end{cases}$$

The sign of τ is given by the sign of the quantity : $n_r l_\beta - n_\beta l_r$

We have already seen this quantity which is associated to the aircraft characteristics for turning by itself.

If $n_r l_\beta - n_\beta l_r$ is positive : the moments opposing to turn overcome,
→ the aircraft doesn't turn naturally by itself
→ $\tau > 0$ and the Spiral mode is convergent

If $n_r l_\beta - n_\beta l_r$ is negative : the moments helping to turn overcome,
→ the aircraft turns naturally by itself
→ $\tau < 0$ and the Spiral mode is divergent

Spiral mode : calculation of s

NOT in PROGRAM

When the Roll Mode / Dutch Roll are finished ($\dot{p} = \dot{\beta} = \dot{r} = 0$), the last mode called the Spiral mode activates. It is a first order motion driven by the roll angle variable, ϕ.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} y_\beta & -1 & 0 & g/V \\ n_\beta & n_r & n_p & 0 \\ l_\beta & l_r & l_p & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta\beta \\ r \\ p \\ \delta\phi \end{bmatrix} \quad \Rightarrow \quad \delta\dot{\phi} = s \cdot \delta\phi \quad \begin{bmatrix} y_\beta & -1 & 0 & g/V \\ n_\beta & n_r & n_p & 0 \\ l_\beta & l_r & l_p & 0 \\ 0 & 0 & 1 & -s \end{bmatrix} \cdot \begin{bmatrix} \delta\beta \\ r \\ p \\ \delta\phi \end{bmatrix} = 0$$

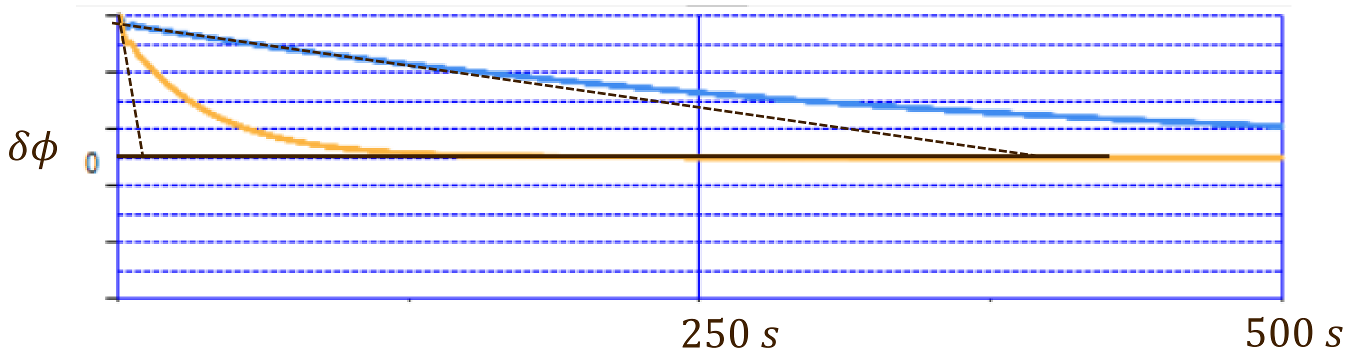
$$det \begin{bmatrix} y_\beta & -1 & 0 & g/V \\ n_\beta & n_r & n_p & 0 \\ l_\beta & l_r & n_p & 0 \\ 0 & 0 & 1 & -s \end{bmatrix} = 0 \quad \Rightarrow \quad -g/V \cdot \begin{bmatrix} n_\beta & n_r & n_p \\ l_\beta & l_r & l_p \\ 0 & 0 & 1 \end{bmatrix} - s \cdot \begin{bmatrix} y_\beta & -1 & 0 \\ n_\beta & n_r & n_p \\ l_\beta & l_r & l_p \end{bmatrix} = 0$$

$$-g/V \cdot \begin{bmatrix} n_\beta & n_r \\ l_\beta & l_r \end{bmatrix} - s \cdot \left\{ y_\beta \cdot \begin{bmatrix} n_r & n_p \\ l_r & l_p \end{bmatrix} + \begin{bmatrix} n_\beta & n_p \\ l_\beta & l_p \end{bmatrix} \right\} = 0$$

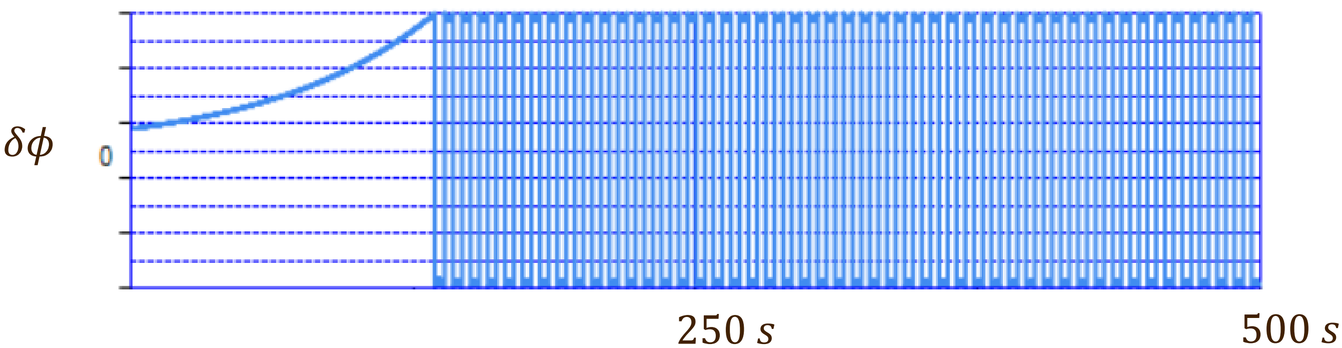
$$\Rightarrow \quad s = -\frac{1}{\tau} = -\frac{g}{V} \cdot \frac{\begin{bmatrix} n_\beta & n_r \\ l_\beta & l_r \end{bmatrix}}{\begin{bmatrix} n_\beta & n_p \\ l_\beta & l_p \end{bmatrix}}$$

General Response from a δϕ variation

DC8 / 0 ft / 125 m/s / 30° roll angle step

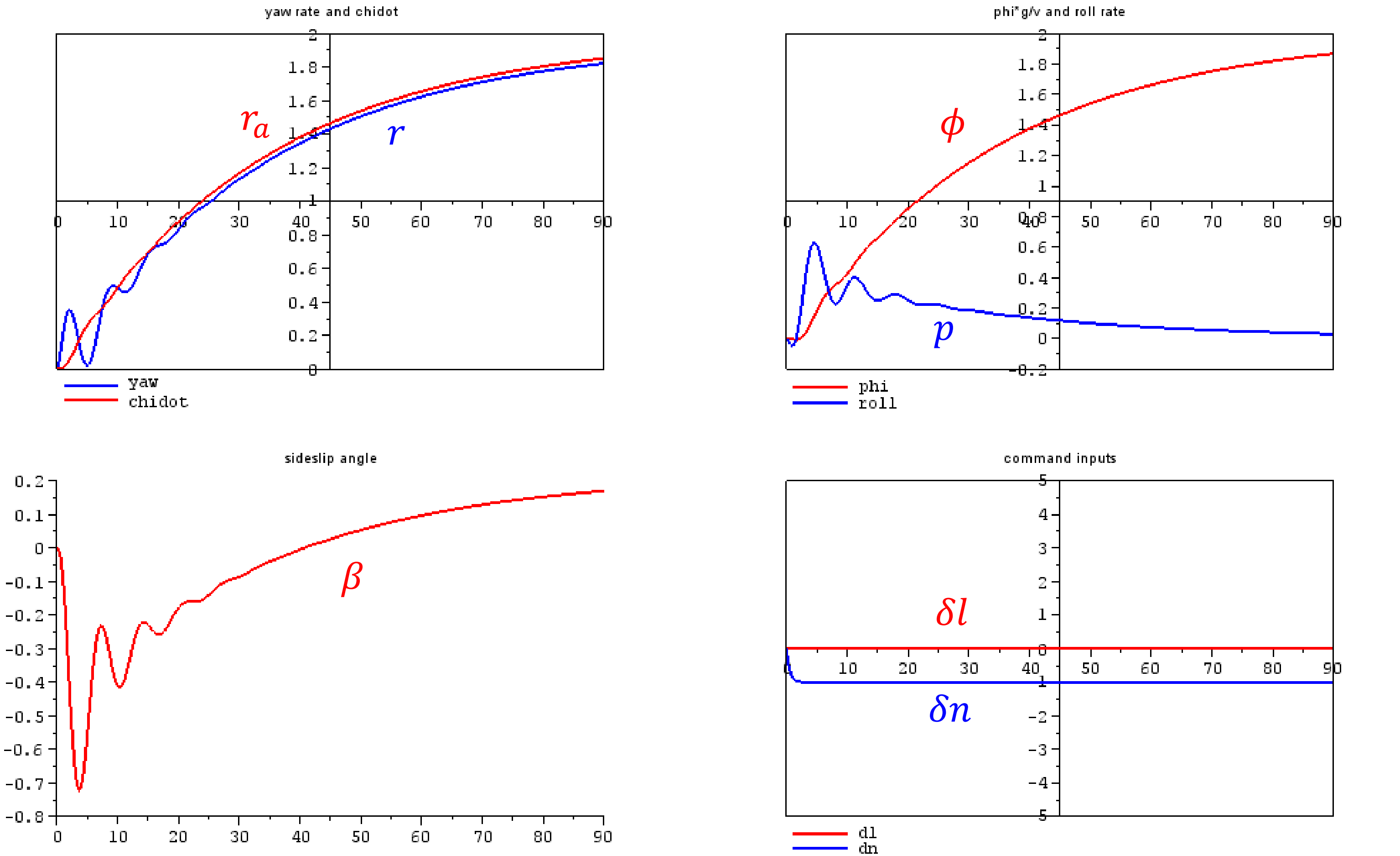


Nominal $Cl_\beta : \tau = 343\ s$ / $2 \times Cl_\beta : \tau = 36{,}5\ s$



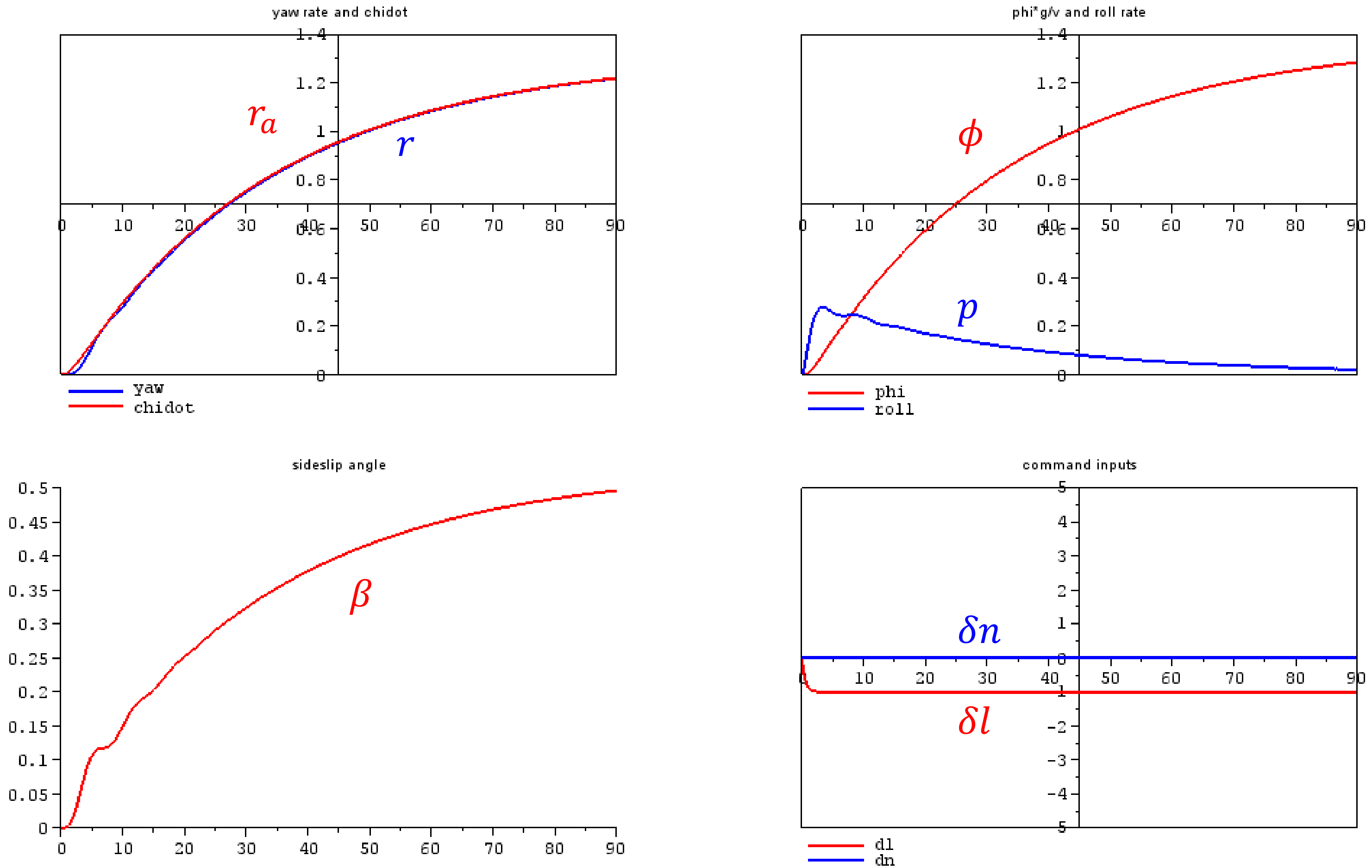
$2 \times Cn_\beta : \tau = -73{,}5\ s$

Spiral mode mode by yaw command



Pure Roll mode & Dutch Roll finished / Spiral mode is identified

Spiral mode mode by roll command



Pure Roll mode & Dutch Roll finished / Spiral mode is identified

Managing the Spiral mode Divergence

The spiral mode convergence / divergence is associated to the sign of $n_r l_\beta - n_\beta l_r$

Imagine that you have an aircraft with a Spiral mode convergent but a poor Dutch Roll ?

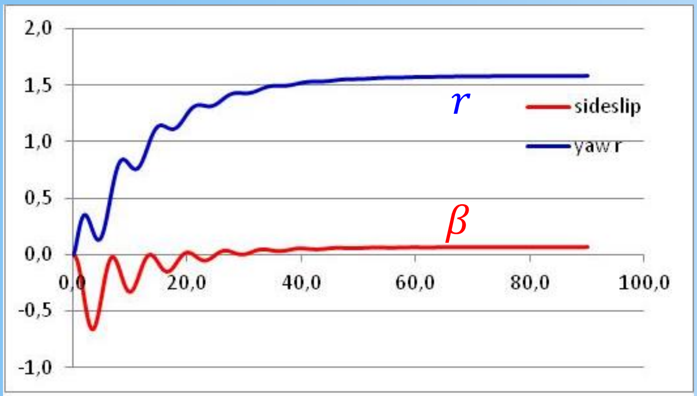
A Spiral mode which is too convergent is not good because you will consume a lot of roll controll for turning and a weakly damped Dutch Roll mode is not good also…
The idea is to manage the Cl_β level …

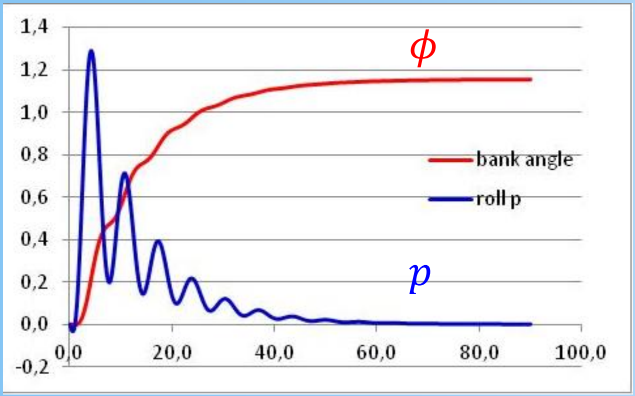
If you make your Cl_β less negative (for instance by putting less dihedral to the wing)
→ The Dutch Roll will become more convergent
→ The Spiral mode will not converge as well or may even risk to be divergent, but you can accept it because the divergence will stay slow and easily controllable

Wing Tip Downlet can mitigate the too negative Cl_β characteristic of the classical Winglet …

Managing the Spiral mode Divergence

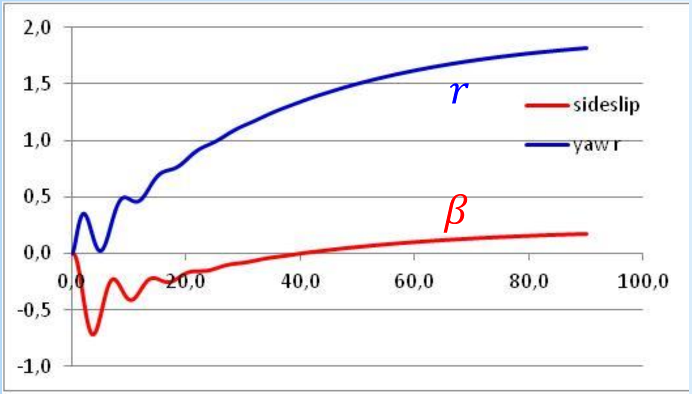
l_β

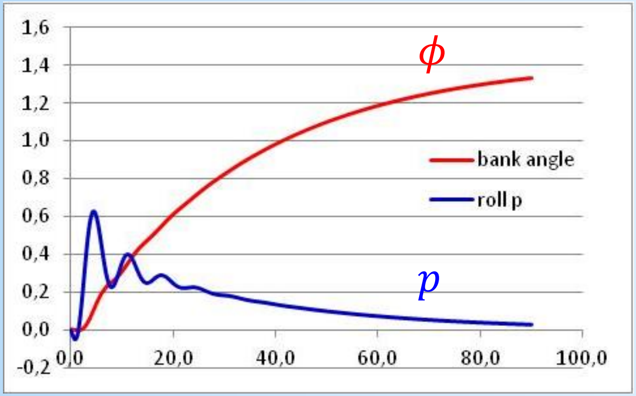




$\xi_{dr} = 11\% \;/\; \tau_{sp} = 12\ s$

$l_\beta/2$





$\xi_{dr} = 18\% \;/\; \tau_{sp} = 33\ s$