MAE1 - Electromagnetism

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Part IV

ELECTROMAGNETIC ENERGY

Energy transfer

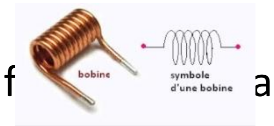
Electromagnetic field acts on charges and can transfer energy.

Inverse phenomen can be true: moving charges can create an electromagnetic field, that conveys energy. (class on antennas)

Conservation of energy, but a radiating body losts energy.

Continuous wave ➔ electrostatic energy : $\frac{1}{2}Cv^2$

magnetostatic energy : $\frac{1}{2}Li^2$

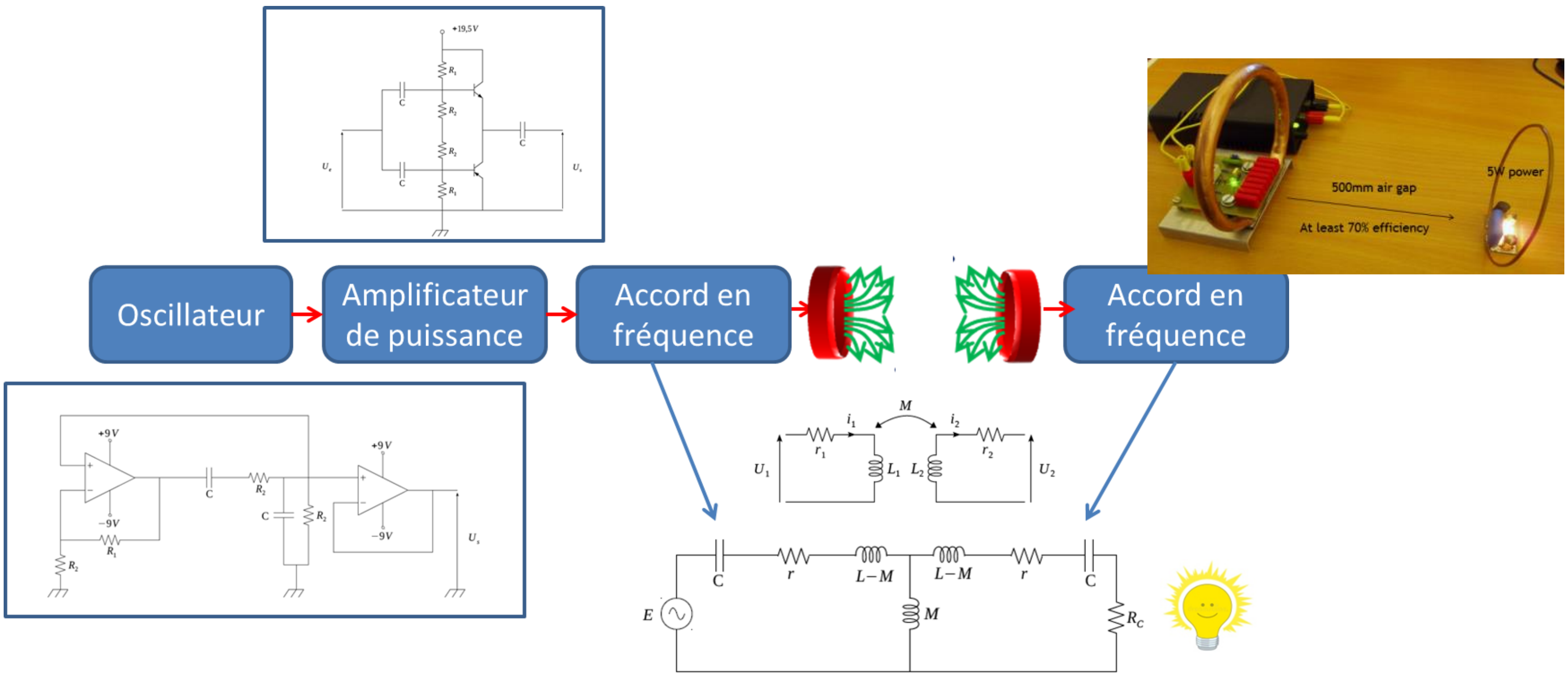


Time-varying ➔ coupling between electric and magnetic f appens to these densities?

For what the energy transfer?

Energy transfer: where is the energy ? ➔ need to draw up local energy balance

- Energy transfer by **near-field harvesting** (Short-range, field spreads in all the directions)



https://www.eleceng.adelaide.edu.au/students/wiki/projects/index.php/Projects:2015s1-71_Inductive_Power_Transfers

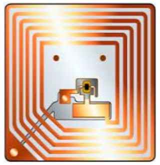
For what the energy transfer ?

Energy transfer: where is the energy ? ➔ need to draw up local energy balance

- Which applications for **near-field harvesting** ?

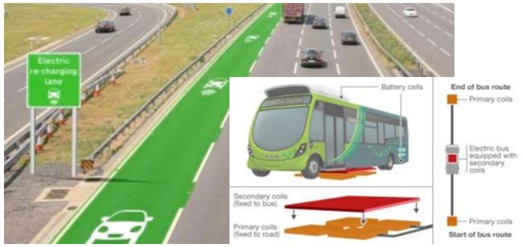
RFID: Radio Frequency IDentification

125 kHz



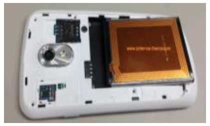
WPT: Wireless Power Transfer

Electric recharging lane



NFC: Near-Field Contact

Mobile phone

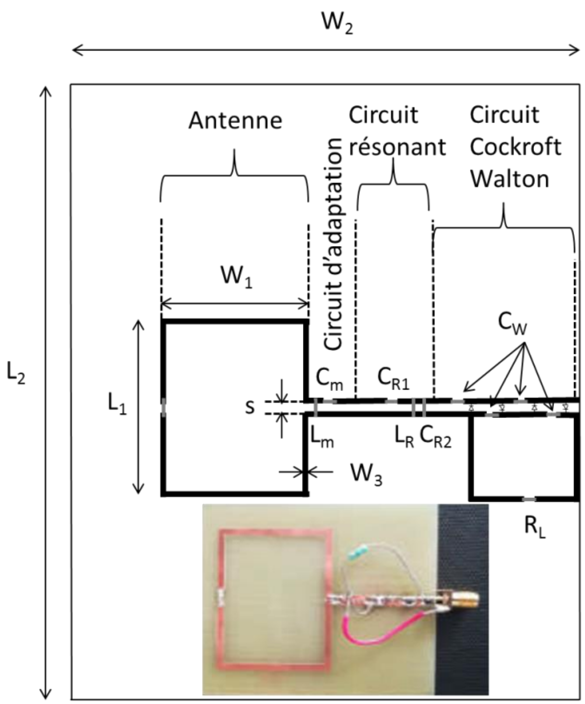


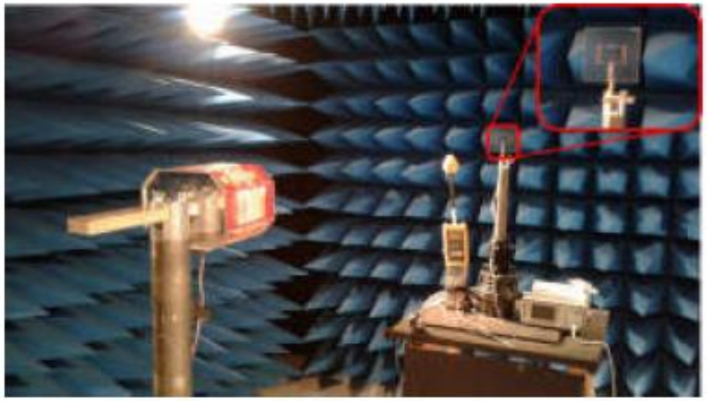
https://www.leaseprotect.fr/portique/
https://mashable.com/2015/08/17/electric-car-charging-uk/?europe=true

For what the energy transfer?

Energy transfer: where is the energy ? ➔ need to draw up local energy balance

- Energy transfer by radiating harvesting (Long-range, transmitter can focused on the receiver) ➔ **antenna = electromagnetic transceiver**



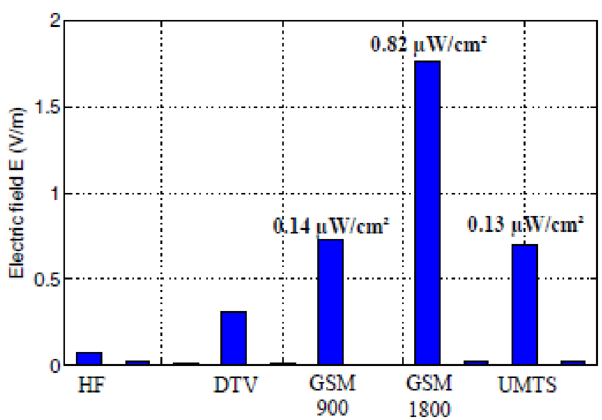


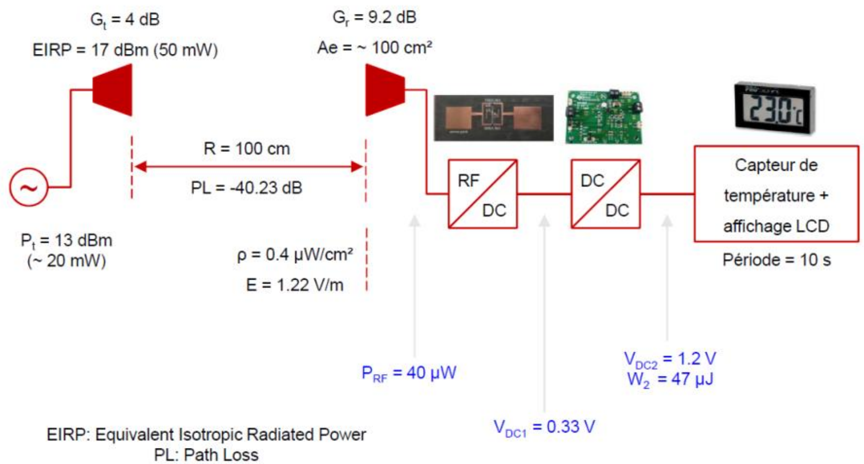
For what the energy transfer ?

Energy transfer: where is the energy ? ➔ need to draw up local energy balance

- Which applications for radiating harvesting ➔ **sensor measurement**

By opportune energy harvesting or energy transfer ➔ required electromagnetic energy





ANFR: 190m far away from a radio transmitter

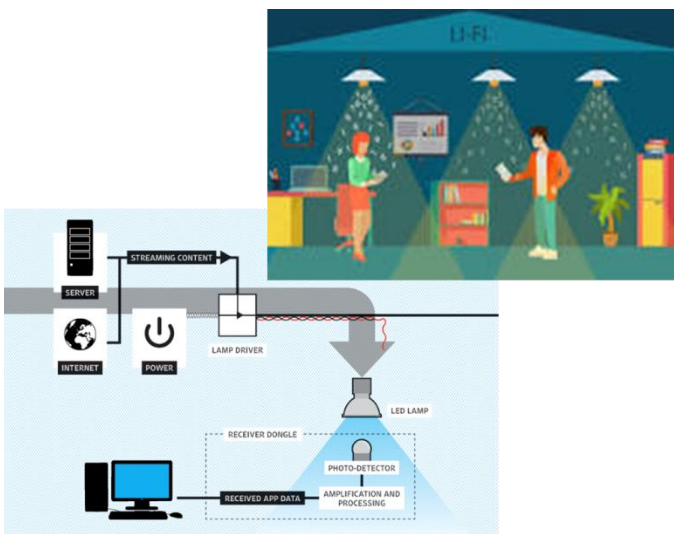
ANFR, French Agency for Frequencies management. (Agence Nationale des Fréquences). http://www.cartoradio.fr/cartoradio/web/.

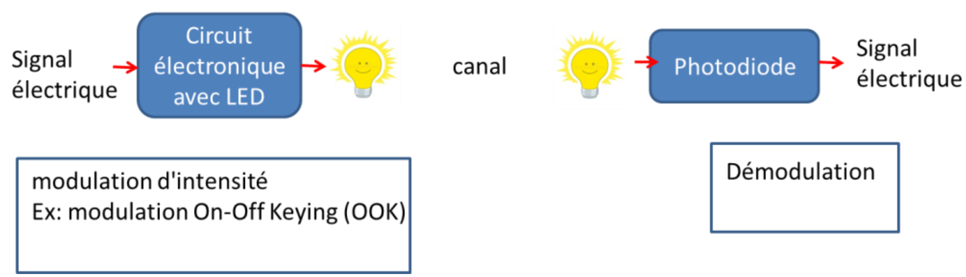
For what the energy transfer ?

Data transfer: Internet by radio waves (WiFi: Wireless Fidelity), and by light (LiFi: Light Fidelity)

WiFi
- Microwave
- Bi-directionnal communication
- Need modem

LiFi
- Optic (modulating of LED by digital signal)
- Use the existing network
- Unidirectionnal communication (downward)
- Less saturated
- Faster (10 times Wifi?)
- More secure (light crossing not though the walls)
- More adapted to environment (hospital, underground parking)





Correspondence between field and circuit

POWER AND ENERGY DENSITIES

($\vec{E}$ et$\vec{H}$ represent peak values)

$\frac{1}{2}\oiint(\vec{E} \times \vec{H}^{*}) \cdot d\boldsymbol{s}$

$\frac{1}{2}\int_V \ \rho\left|\vec{E}\right|^2 dv$ (dissipated real power)

$\frac{1}{4}\int_V \ \epsilon\left|\vec{E}\right|^2 dv$ (time-average electric stored energy)

$\frac{1}{4}\int_V \ \mu\left|\vec{H}\right|^2 dv$ (time-average magnetic stored energy)

POWER AND ENERGY

(v et i represent peak values)

$\mathrm{P} = \frac{1}{2} vi$ (power voltage current relation)

$P_d = \frac{1}{2}\frac{v^2}{R}$ (power dissipated in a resistor)

$\frac{1}{4} C v^2$ (energy stored in a capacitor)

$\frac{1}{4} L i^2$ (energy stored in an inductor)

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2$$

$$U_L = L\int_0^I I dI = \frac{1}{2}LI^2$$

ELECTROMAGNETIC ENERGY

From Electrostatic & Magnetostatic

Electrostatic potential energy:

$$U_E = \frac{1}{2}\iiint_v \vec{E}\cdot\vec{D}dv = \frac{1}{2}\iiint_v \varepsilon_0\, E^2 dv$$

Energy of a system of charge producing an E field

Magnetostatic potential energy:

$$U_M = \frac{1}{2}\iiint\limits_v \vec{H}\cdot\vec{B}dv = \frac{1}{2}\iiint\limits_v \frac{B^2}{\mu_0}\, dv$$

Energy stored by the B Field

CONSERVATION of ELECTROMAGNETIC ENERGY

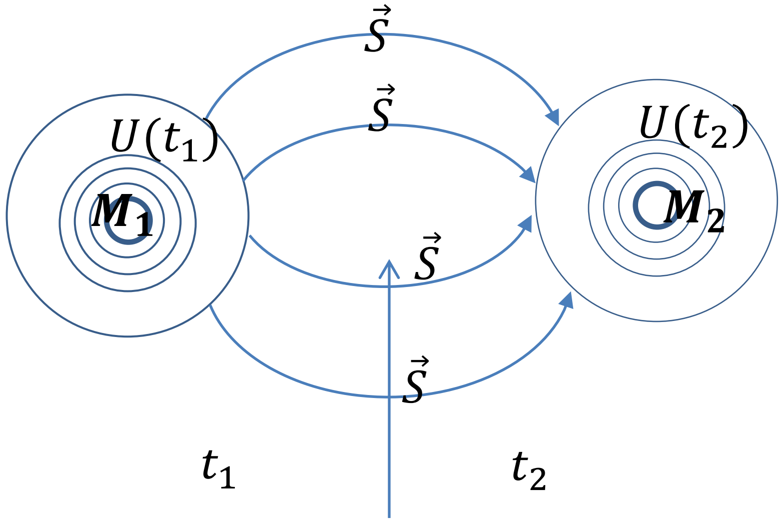
$$\vec{\nabla} \cdot \vec{S} + \frac{\partial U}{\partial t} = -\vec{j} \cdot \vec{E}$$

U, the density of energy, contribution of E and B field (connected) .

$\vec{\boldsymbol{S}}$**,** the Poynting Vector, a flux or an energy current density (displacement of energy)

$\vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{E}}$ is the work done by the local field on charged particles per volume unit.

See Showme https://www.showme.com/sh/?h=CbEnpdA



CONSERVATION of ELECTROMAGNETIC ENERGY

If $\vec{\nabla} \cdot \vec{S}$=0 : local conservation of energy

If $\vec{\nabla} \cdot \vec{S} \neq 0 \; and \; \vec{j} \cdot \vec{E} = 0$ (non conducting medium)

$\vec{\nabla} \cdot \vec{S} + \frac{\partial U}{\partial t} = 0 \Rightarrow$equation of continuity for charge and U replace the charge density , verify $\vec{\nabla} \cdot \vec{S}$ is the rate of energy flow per unit of area.

U and S are quadratic ⇔ Real Anyway ⇒ physically measurable

Density of Electromagnetic Energy U and Electromagnetic Energy Flux S are measurable anyway!!

Plane Wave in Vacuum

$$\vec{B} = \frac{\vec{k}}{\omega} \times \vec{E}$$

Poynting vector: $\vec{S} = \frac{\vec{E}\times(\vec{k}\times\vec{E})}{\omega\mu_0} = \vec{k}\frac{E^2}{\omega\mu_0}$

While $\vec{k} \perp \vec{E}$, $\vec{k} = k \cdot \overrightarrow{u} = \frac{\omega}{c}\overrightarrow{u}$

Unit propagation vector

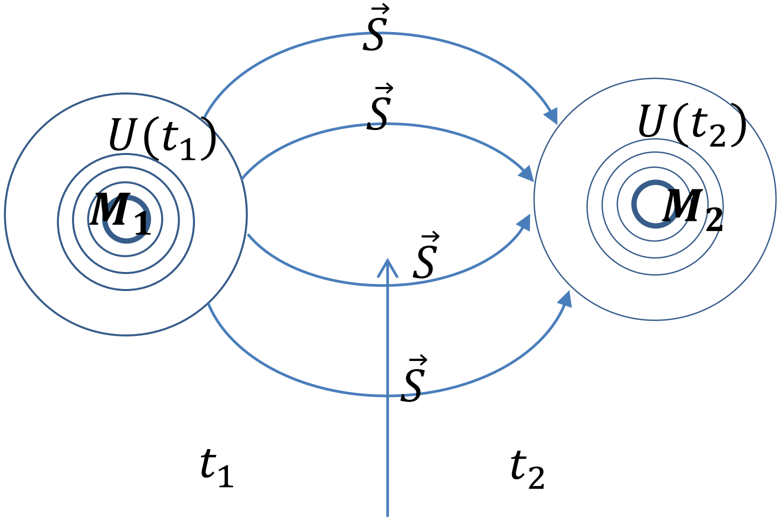
$$\vec{S} = \frac{E^2}{c\mu_0}\vec{u} = c\varepsilon_0 E^2 \vec{u}$$

Density of energy : $U = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} = \varepsilon_0 E^2$

Plane Wave in Vaccum

- Density of Energy : $U = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$

- Poynting Vector : $\vec{S} = \mathrm{cU}\vec{u}$



Physically $(U;\vec{S})$ - Density of energy & current of energy density – could be compared to $(\rho,\vec{j})$

Application in Optics

I is the Optical intensity, of the electromagnetic plane wave received by time unit by a surface unit ⊥ to $\vec{k}$. I is thus the mean of the Poynting Vector $\vec{S}$.

$$I = \langle S \rangle = c\varepsilon_0 |E|^2$$

For a laser source, we consider 2 components of the Electrical Field E_s and E_p with the same magnitude such as $\langle S \rangle = \langle S_s \rangle + \langle S_p \rangle = \frac{c\varepsilon_0}{2}\left(|E_s|^2 + \left|E_p\right|^2\right) = c\varepsilon_0 |E|^2$

The photodetected power, will be converted in an electrical current.

$$P = I.A_{phot}$$

The power P is measured in W, A_{phot} is the active area of the photodiode.

I is expressed in $\boldsymbol{W.m^{-2}}$ or in $\boldsymbol{J \cdot s^{-1} \cdot m^{-2}}$

Example Of energy transfered by a radio antenna

We assume a plane wave emitted by a Radio Source S. The Power emitted by S is equal to 1MW and is uniformly distributed into half space above the Earth. A detector measured the field at the R=1000km from the emitter.

- Compute the value of the Electrical Field (in Volt / m) at the distance R.
- Discussion on the size of the receiver and the power dissipated in the atmosphere.

Antenna Power

The power, P is the flux of the poynting vector S through a surface surrounding the transmitter (half sphere of a Radius R) centered to the emitter.

P$= \langle S \rangle (2\pi R^2) = c\varepsilon_0 |E|^2 (2\pi R^2)$

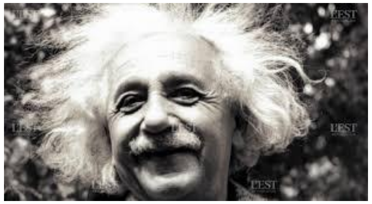
So $|E|^2 = \frac{P}{2c\varepsilon_0(\pi R^2)} = 6.10^{-5}$

Thus $E = 8.10^{-3} V.m^{-1}$

Conclusions: an antenna of few cm could receive the signal of few mV but the emitter is powerfull (WallPlug efficiency < 30%)

The Photon: Wave Particle Duality

Since beginning of 20 century (Einstein, DeBroglie, Planck...)



Particles generating a Wave, through energetic exchanges: quantum of energy (energy packets)

Photon = quantum of energy.

$E = hv = \hbar\omega$ Planck relation, $h = 6.62 \cdot 10^{-34} J \cdot s$

The momentum vector (related to the moving particles)

$\vec{p} = \hbar\vec{k}$ & $|\vec{p}| = \dfrac{hv}{c}$

The Photon: Wave Particle Duality

- Photons velocity = c (Relativity)
- U Density of electromagnetic energy carried by n photons per volume unit:

$$U = \sum_{n} hv$$

The Photon: Wave Particle Duality

Compute the energy value of 1 photon at 1.55µm Wavelength .

1. Relation between Wavelength and Frequency

2. $v = \frac{c}{\lambda} = \frac{3\ 10^{8}}{1.55\ 10^{-6}} = \frac{3\ 10^{8+6}}{1.55}\ v \sim 2 10^{14}\text{Hz}\ v = 192THz$

2. Find the magnitude unit of hv : $10^{-34} * 10^{14} = 10^{-20} = 10^{-5} fJ$

a- mJ

b- MJ

c- nJ

d – fJ

3. Compute the value of $h\nu$

ν=1.92·10^(14) and h=6.62·10^(−34)$J \cdot s$

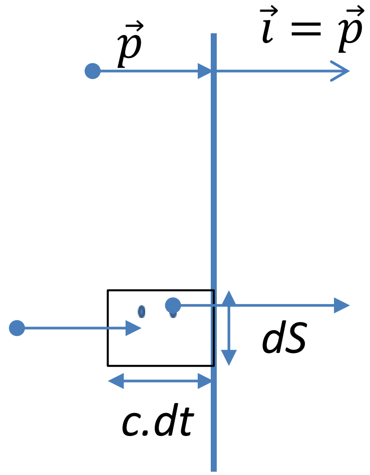
$h\nu$=12.81·10^(−20)J

4. Discussion on the number of Photons and electromagnetic energy

To obtain an energy of 1J , 10^19 Photons are required

The Wave plane interaction with a plane surface

Momentum vector correspond to a translate radiation pressure

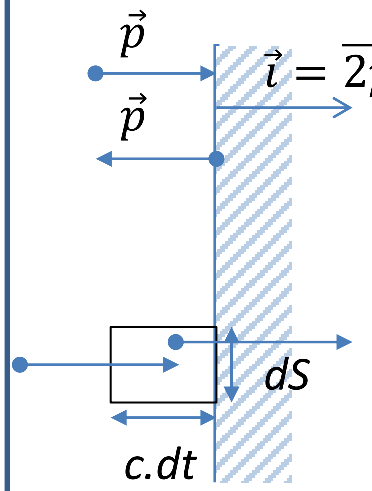


Screen

$\vec{\imath}$ Pulse received by screen:

$$\vec{\imath} = \vec{p}$$

Photon absorbed by the screen
Inelastic collision / absorbed Energy

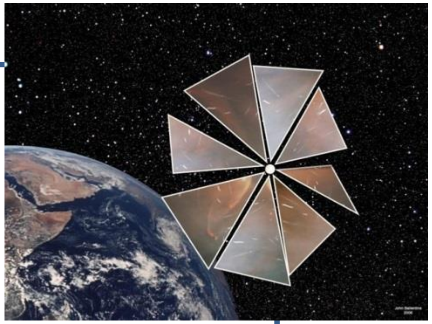


Mirror

$\vec{\imath}$ Pulse received by Mirror:

$$\vec{\imath} = 2\vec{p}$$

Photon reflected by the mirror
Elastic collision /kinetic Energy



Solar Sail

Radiation pressure

During dt , on the elementary area dS, the target receive *n* Photons (contained by the volume, cylinder of radius area dS and length *cdt*).

***U* is the density of energy of the incident wave** $n = \frac{U}{\hbar\omega}$.

d^2N particles in dS received :

$$d^2N = n \cdot c \cdot dt \cdot dS = \cdot \frac{U \cdot c}{\hbar\omega} dt \cdot dS = \frac{U}{\hbar k} \cdot dt \cdot dS$$

d^2I is the total pulse

$$d^2I = d^2N \cdot i \begin{cases} = \hbar k \cdot \dfrac{U}{\hbar k} \cdot dt \cdot dS = \mathrm{U} \cdot dt \cdot dS & \quad Screen \\ = 2\hbar k \cdot \dfrac{U}{\hbar k} \cdot dt \cdot dS = 2\mathrm{U} \cdot dt \cdot dS & \quad Mirror \end{cases}$$

Radiation pressure

dF , Force applied to dS:

$$d^2I = \mathrm{dF\ dt}$$

$$dF = UdS \quad or \quad dF = 2UdS$$

The Electromagnetic radiation due to the light creates a radiative force proportional to the illuminated area ⇒ **Radiation Pressure**

$p = U$ for a perfect screen

$p = 2U$ for a perfect mirror

For Further Practice **Solar Sail: Lesson complement**

Numerical application

- See https://www.showme.com/sh/?h=WL7QBmq

Memory Aid: Plane Wave in Vacuum

From Maxwell Equation in vacuum (without charge and current), **Wave Equation**

$$\Delta\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Delta\vec{B} = \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$$

IN VACUUM

The wave number is $k = \frac{\omega}{c}$

From Maxwell Equation : $\vec{E}\perp\vec{k}$, $\vec{B}\perp\vec{k}$

$\vec{E}\perp\vec{B}$

$c|\vec{B}| = |\vec{E}|$

Wave impedance in vacuum

$$\eta_0 = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$$

Harmonic wave

Wavelength $\lambda = \frac{2\pi}{k} = cT$

Period $T = \frac{2\pi}{\omega}$

Frequency $f = \frac{\omega}{2\pi}$

Velocity

Light speed : $c \approx 3 \cdot 10^8 \mathrm{m/s}$

Phase velocity $v_\varphi = \frac{\omega}{k}$

Group velocity $v_g = \frac{d\omega}{dk}$

Polarization

Linear polarization: $\varphi = 0 \ or \ \pi$

Circular polarization :

$\varphi = \pm\frac{\pi}{2}$ and $E_{ox} = E_{oy}$

Elliptical polarization: each other

Memory Aid: Plane Wave in Vacuum

Polarization

Linear polarization: $\varphi = 0\ or\ \pi$

Circular polarization : $\varphi = \pm\frac{\pi}{2}$ and $E_{ox} \quad = E_{oy}$

Elliptical polarization: each other

Malus Law, Intensity $I \sim {E_0}^2 cos^2\theta$

Energy and Power

Density of Energy : $U = \varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$

Poynting Vector : $\vec{S} = \mathrm{cU}\vec{u}$

Intensity : $I = \langle S \rangle = \frac{c\varepsilon_0}{2}|E|^2$

Power : $P = I.A$ (A is the area of the detector)

Planck relation $E = hv = \hbar\omega$
Planck constant : $h = 6.62 \cdot 10^{-34} J \cdot s$
$\hbar = \frac{h}{2\pi}$