
Convair B-36 Peacemaker
Relation between $\vec{V}_{ac/air}$ & $\vec{V}_{ac/grd}$

Relations between $V_{/air}$ & $V_{/grd}$

The angles (γ, χ) define the (absolute) position of the velocity $\vec{V}$ with respect to the Earth Referential R_0. If we are talking about the velocity of the aircraft with respect to the air :

$$\vec{V}_{ac/air} \rightarrow (\gamma_{air}, \chi_{air})$$

If we are talking about the velocity of the aircraft with respect to the ground :

$$\vec{V}_{ac/grd} \rightarrow (\gamma_{grd}, \chi_{grd})$$

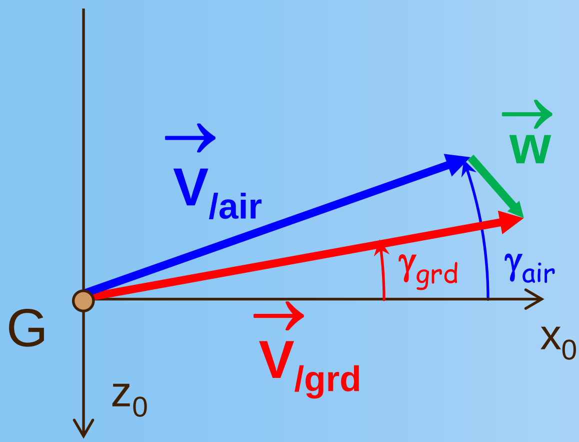
By expressing the velocity $\vec{V}$ wrt the Earth Referential R_0, we can obtain relations between the velocity angles : $(\gamma_{grd}, \chi_{grd})$ & $(\gamma_{air}, \chi_{air})$

$$\vec{V}_{ac/grd} = \vec{V}_{ac/air} + \vec{V}_{air/grd} \Rightarrow \begin{cases} V_{grd} \approx V_{air} + w_x \\ \chi_{grd} \approx \chi_{air} \cdot (1 - w_x/V_{grd}) + w_y/V_{grd} \\ \gamma_{grd} \approx \gamma_{air} \cdot \left(1 - \frac{w_x}{V_{grd}}\right) - w_z/V_{grd} \end{cases}$$

with

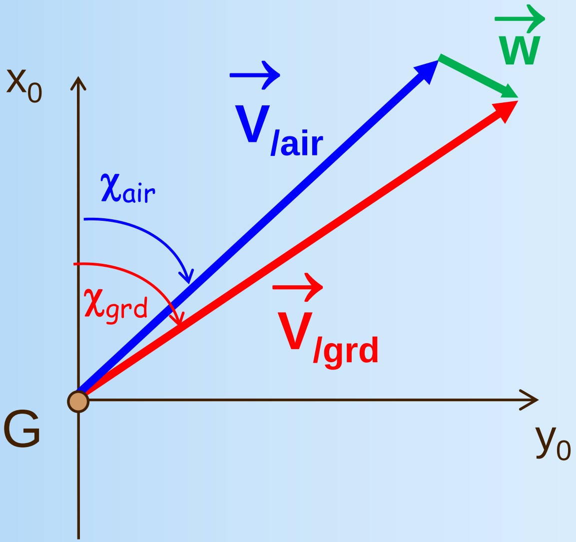
$$\vec{V}_{air/grd} = \left.\begin{matrix} w_x \\ w_y \\ w_z \end{matrix}\right|_{R_0}$$

Relations between $V_{/air}$ & $V_{/grd}$



$w_z > 0$ for descending wind

$$\gamma_{grd} \approx \gamma_{air} \cdot \left(1 - \frac{w_x}{V_{grd}}\right) - \frac{w_z}{V_{grd}}$$



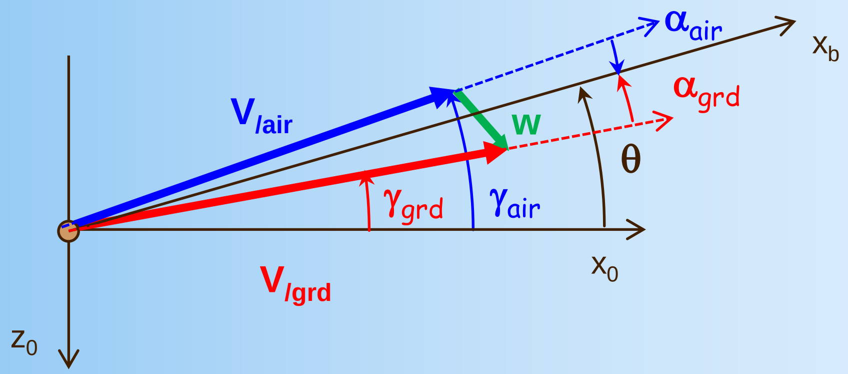
$w_y > 0$ for wind from the left

$$\chi_{grd} \approx \chi_{air} \cdot \left(1 - \frac{w_x}{V_{grd}}\right) + \frac{w_y}{V_{grd}}$$

Relations between $V_{/air}$ & $V_{/grd}$

For a Longitudinal Flight, the relation $\theta = \alpha_{grd} + \gamma_{grd} = \alpha_{air} + \gamma_{air}$ leads to express $\alpha_{grd}/\alpha_{air}$

$$\begin{cases} \alpha_{grd} + \gamma_{grd} = \alpha_{air} + \gamma_{air} \\ \gamma_{grd} \approx \gamma_{air} \cdot \left(1 - \dfrac{w_x}{V_{grd}}\right) - \dfrac{w_z}{V_{grd}} \end{cases} \Rightarrow \alpha_{air} \approx \alpha_{grd} - \gamma_{air} \cdot \frac{w_x}{V_{grd}} - \frac{w_z}{V_{grd}}$$



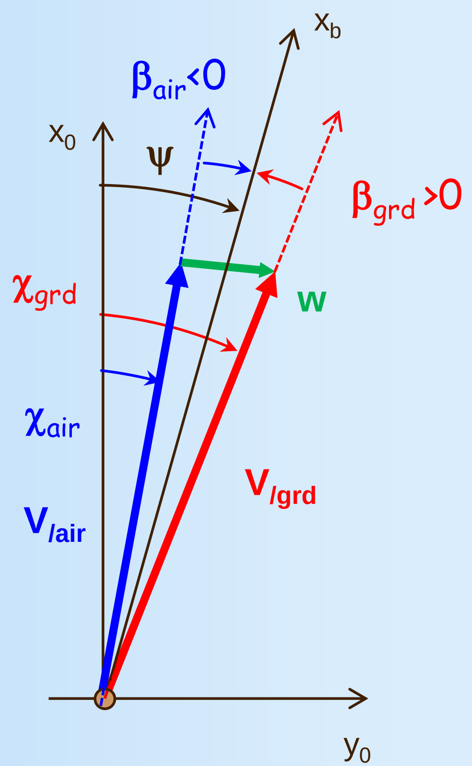
Relations between $V_{/air}$ & $V_{/grd}$

For a Flight in the plane (x_0, y_0), the relation $\psi = \chi_{grd} - \beta_{grd} = \chi_{air} - \beta_{air}$ leads to express β_{grd}/β_{air}

$$\begin{cases} \chi_{grd} - \beta_{grd} = \chi_{air} - \beta_{air} \\ \chi_{grd} \approx \chi_{air} \cdot \left(1 - \dfrac{w_x}{V_{grd}}\right) + \dfrac{w_y}{V_{grd}} \end{cases}$$

$$\Downarrow$$

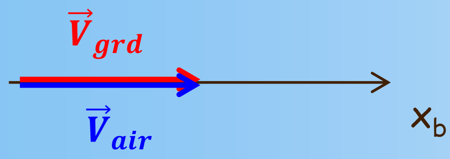
$$\beta_{air} \approx \beta_{grd} + \chi_{air} \cdot \frac{w_x}{V_{grd}} - \frac{w_y}{V_{grd}}$$



Longitudinal Flight with vertical Wind

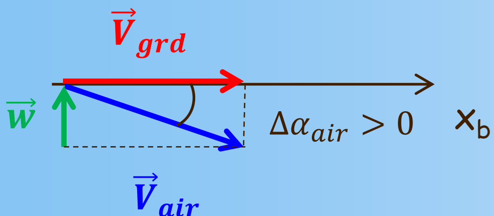
Antonov AN-225

Response from a Wind Ascending Step



Initial trim , no Wind : $\vec{F}_{aero} + \vec{F}_{thrust} + m\vec{g} = \vec{0}$

$\rightarrow\ \alpha_{air} = \alpha_{grd}\ /\ \gamma_{air} = \gamma_{grd}$



The Wind Pertubation starts :

- The state variables $\vec{V}_{grd}/\alpha_{grd}/\gamma_{grd}$ don't change (only their derivatives)
- The air variables $\vec{V}_{air}/\alpha_{air}/\gamma_{air}$ see the change

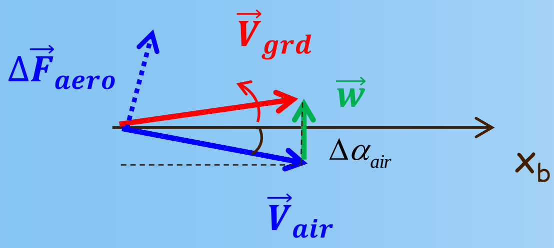
$\rightarrow\ \Delta\alpha_{air} > 0\ /\ \Delta\gamma_{air} < 0$

$\rightarrow\ \Delta\alpha_{grd} = 0\ /\ \Delta\gamma_{grd} = 0$

The aerodynamic force $\vec{F}_{aero}$
is modified

$$\vec{F}_{aero} + \vec{F}_{thrust} + m\vec{g} = m\frac{d\vec{V}_{grd}}{dt}$$

When the perturbation starts,
the state vector $\vec{V}_{grd}$ doesn't change,
only its derivative …

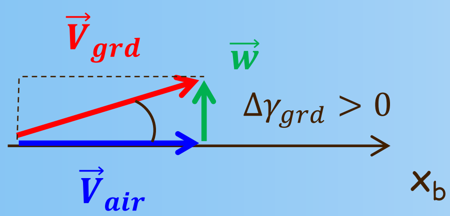


During the transients :

- Due to the α_{air} change, a change of lift force appears …
- Which makes turning up $\vec{V}_{grd}$ … and mechanically $\vec{V}_{air}$
- Which makes decrease α_{air}

$\rightarrow\ \Delta\alpha_{air} > 0\ /\ \Delta\vec{F}_{aero}$

$$\vec{F}_{aero} + \vec{F}_{thrust} + m\vec{g} = m\frac{d\vec{V}_{grd}}{dt}$$



Final Trim :

- We recover the initial α_{air}, we recover the initial lift force and the initial trim situation …
- The ground variables $\vec{V}_{grd}/\alpha_{grd}/\gamma_{grd}$ see the change

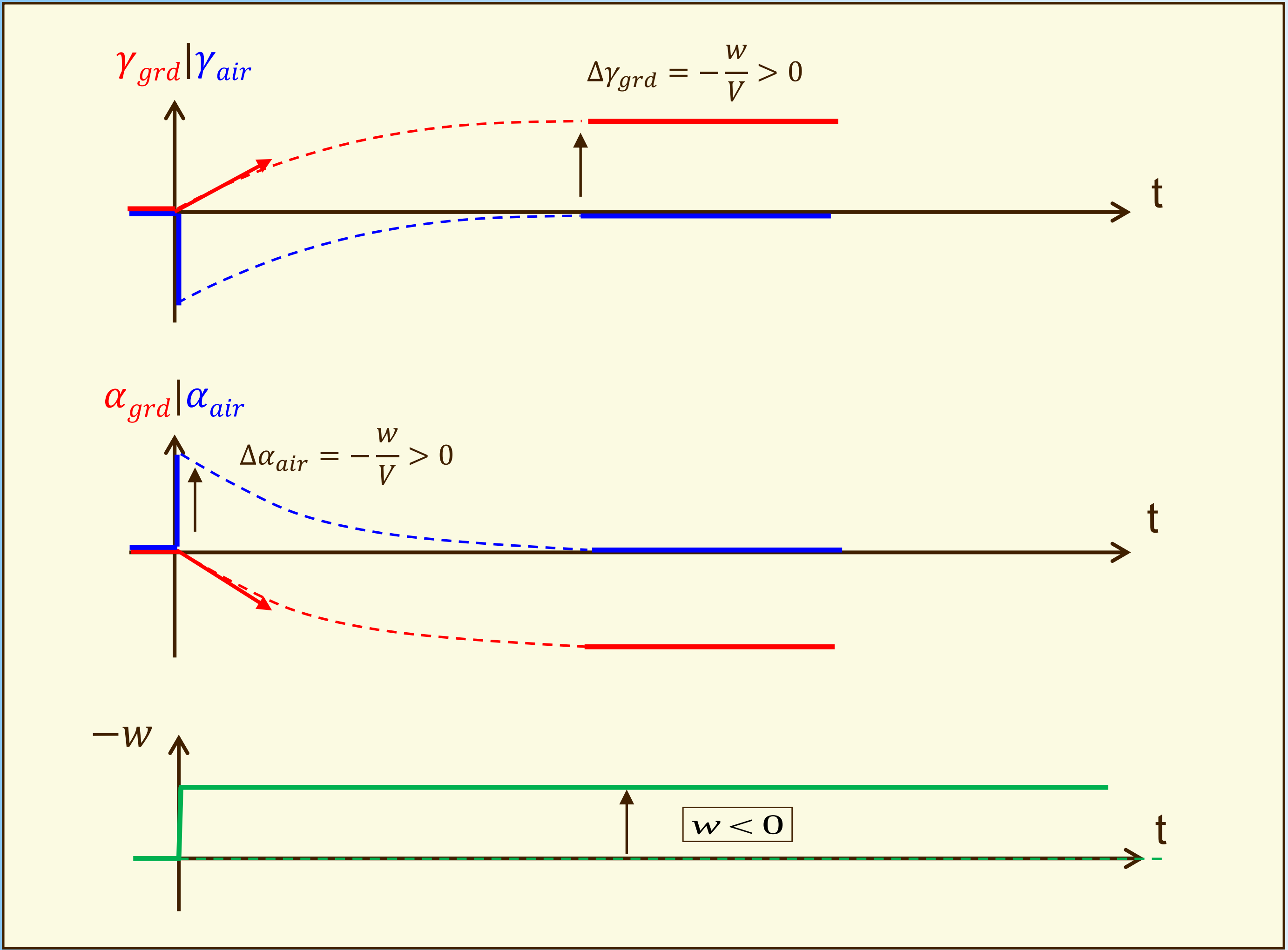
$\rightarrow\ \Delta\alpha_{air} = 0\ /\ \Delta\gamma_{air} = 0$

$\rightarrow\ \Delta\alpha_{grd} < 0\ /\ \Delta\gamma_{grd} > 0$

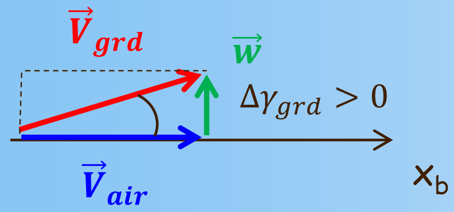
$$\underbrace{\vec{F}_{aero} + \vec{F}_{thrust}}_{\substack{\text{Only function of air variables} \\ \vec{V}_{air}/\alpha_{air}/\beta_{air}}} + m\vec{g} = \vec{0}$$

The trim is only function of air variables so $\vec{V}_{air}/\alpha_{air}/\beta_{air}$ do not see "the wind" when the trim is set …

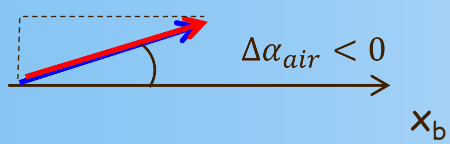
Response from a Wind Ascending Step



Response from a Wind Ascending Gust



$\vec{V}_{grd} = \vec{V}_{air}$



At the moment when the gust is finished $(\vec{w} = \vec{0})$:

- We recover instantaneously $\vec{V}_{air} = \vec{V}_{grd}$
- The air variables $\vec{V}_{air}/\alpha_{air}/\gamma_{air}$ see the change

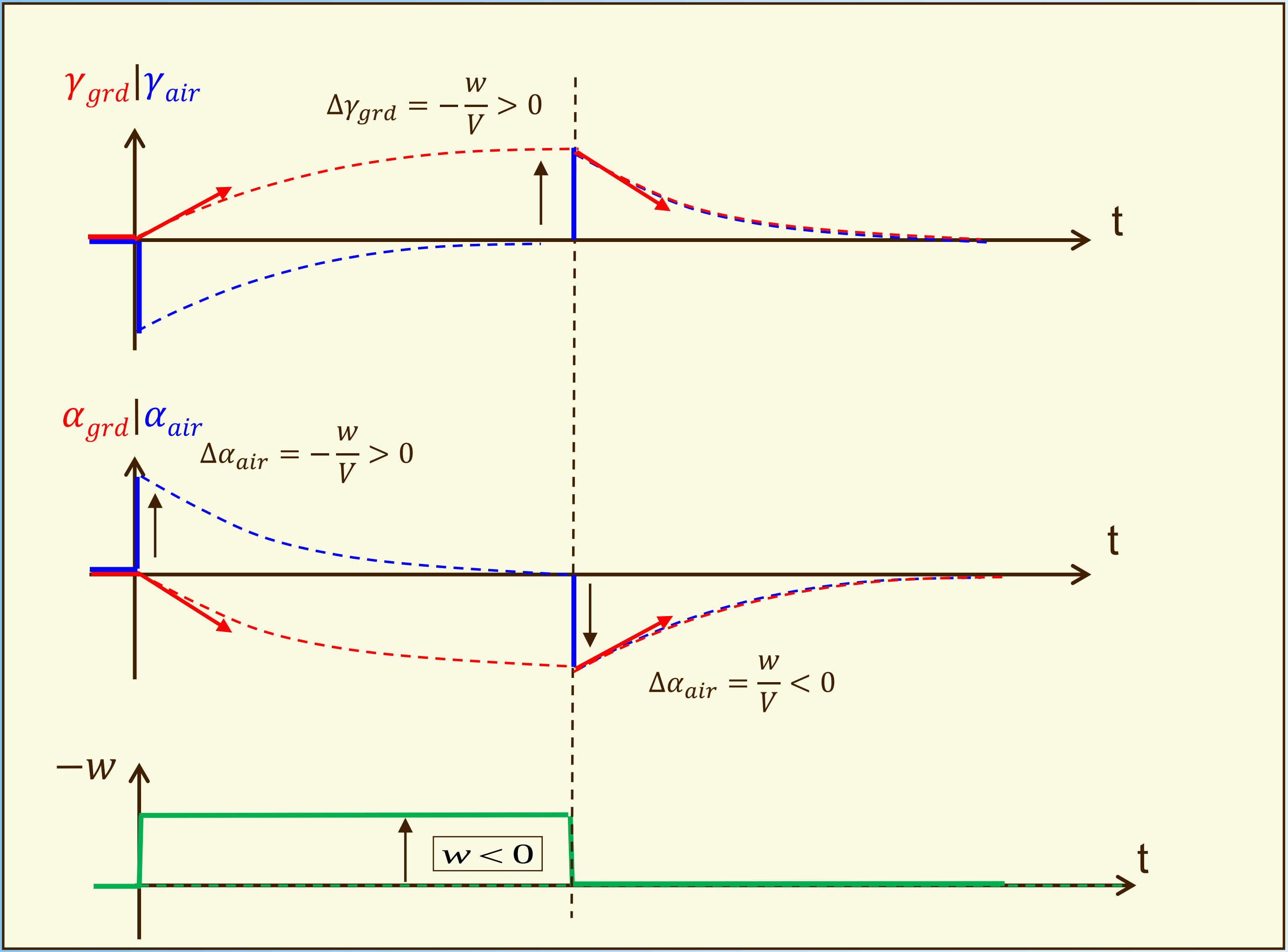
$\rightarrow \ \Delta\alpha_{air} = \Delta\gamma_{grd} < 0$

$\rightarrow \ \Delta\gamma_{air} = \Delta\gamma_{grd} > 0$

During the transients :

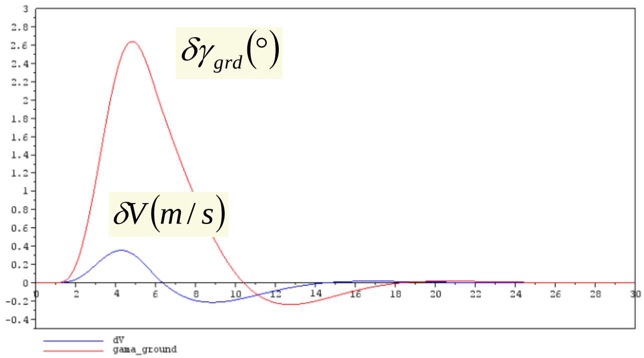
- The negative α_{air} creates a down-lift force …
- Which makes turning down $\vec{V}_{grd}$ … and mechanically $\vec{V}_{air}$
- Until the final trim

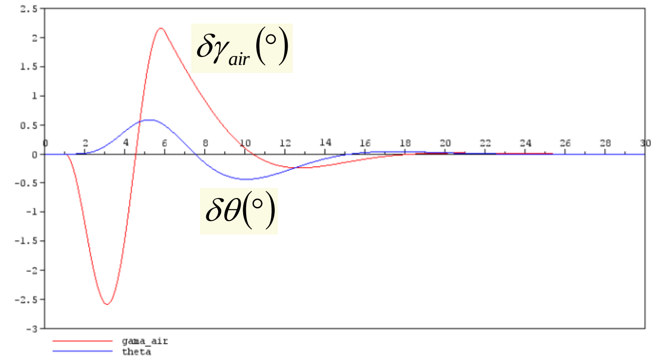
Response from a Wind Ascending Gust

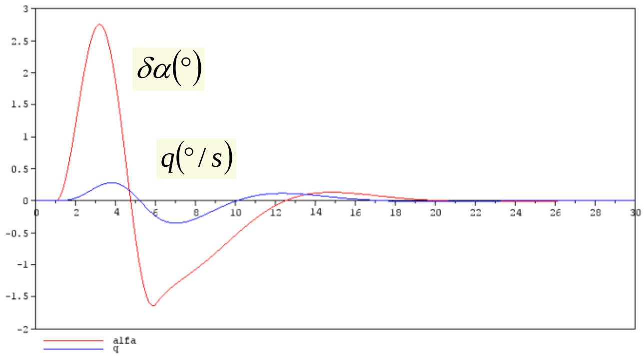


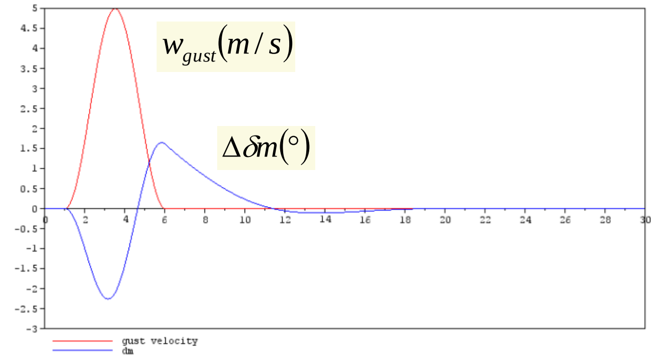
Response from a Wind Ascending Gust

Pure Longitudinal Flight with a ascendant wind gust









Notice the evolution of $\Delta\delta m$: the simulation is performed with a normal law for preventing the phugoid

Formation Flight : Tip Vortex application

Formation Flight Principle

An aircraft submitted to a constant ascending wind $(-w > 0)$ will steadily climb

$$\Delta\gamma_{grd} = -\frac{w}{V} > 0$$

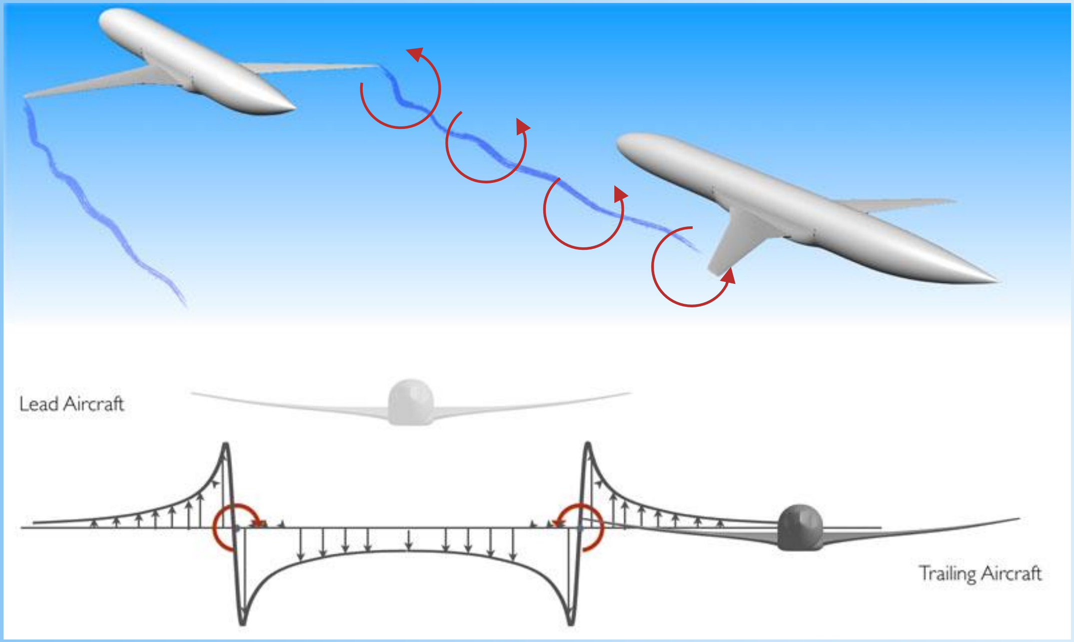
Imagine an aircraft in cruise : the flight is at a given level $(\gamma_{grd} = 0)$ associated to a given thrust, F

Imagine the same aircraft but submitted to a constant ascending wind
If we want to maintain the flight at a given level $(\gamma_{grd} = 0)$, we must decrease the thrust

$$\Delta F = -mg \cdot \Delta\gamma_{grd} = mg \cdot \frac{w}{V} < 0$$

We save thrust, we save fuel, we save cost …

Formation Flight : Tip Vortex application



The left wing of the trailing aircraft interferes with the tip vortices produced by the lead aircraft

This creates a constant ascending wind on the left wing of the trailing aircraft
The trailing aircraft will save thrust, fuel and cost …

Wind at Take-off and Landing

SAAB 35 Draken

Take-Off / Landing with Head Wind

The question is : why head wind is favourable for take-off and landing ?

Take-Off and Landing Velocities are associated with Cz capability :

- A certain Cz for take-off , associated to a velocity V_R (at rotation) and V_{LOF} (at lift-off)
- A certain Cz for landing, associated to a certain velocity V_{LS} (on the glide)

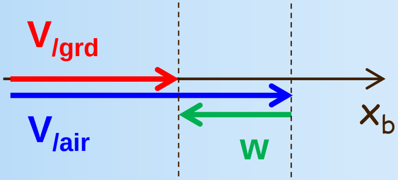
Associated to these Cz capability, these velocities are **air speed** and the aircraft shall demonstrate these air speed whatever the wind.

However, the (landing / take-off) ground distance covered by the aircraft is associated to **ground speed**, so the lower the ground speed will be, the shorter the ground distance will be …

As the air speed velocity (in steady condition) are the same whatever the wind, head wind is favourable because the associated ground speed will be lower as well as the take-off / landing distance shorter.

$$V_{gr} = V_{air} + w_x$$

with $w_x < 0$



Steady Head Wind

Approach Speed : V_{ref} / V_{app}

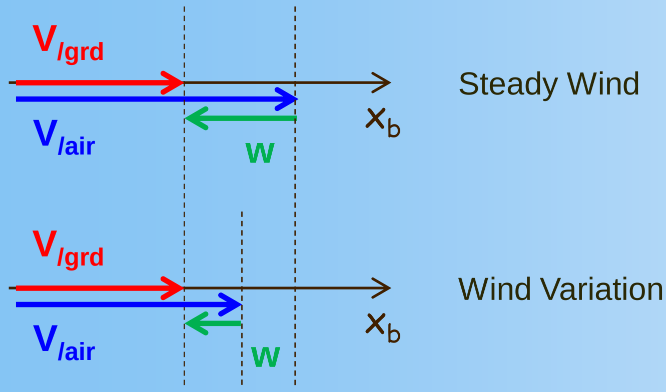
The V_{ref} is the reference speed for landing performance calculation ; the Regulations impose the following relation :

$$V_{ref} \geq 1.23 \cdot V_{S1g}$$

with V_{S1g} given by :

$$mg = \frac{1}{2}\rho_0 V_{S1g}^2 \cdot SCz_{\max}$$

In case of Head Wind, as $V_{/air}$ is given by V_{ref}, $V_{/grd}$ can decrease accordingly.
Now, imagine a transitory loss of Head Wind, we know that $V_{/air}$ will be immediately impacted and, in that case, will decrease. So, there is a potential issue of stall / security …



Hence, in case of landing with Head Wind, the Regulations recommend to increase the landing speed by adopting the V_{app} given by :

$$V_{app} = V_{ref} + \frac{w}{2}$$

with w/2 limited by 20 kts
JAR-OPS 1.515 (b)