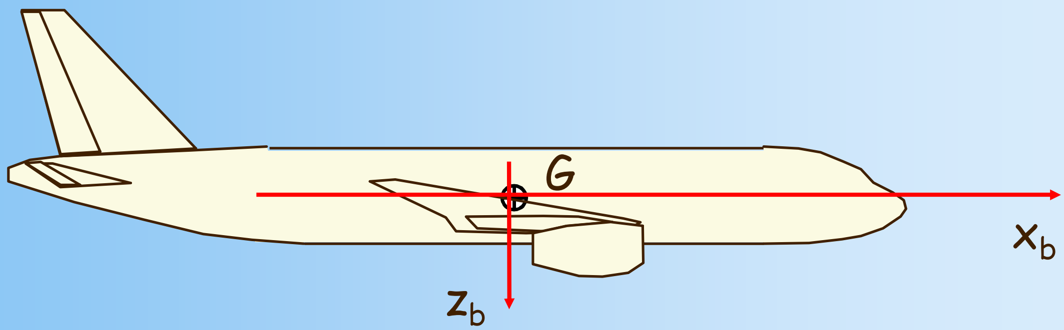
Complements for Inertia

Dornier Do X

Inertia Matrix & Eigenvalues



The Inertia Matrix is a real & symmetrical Matrix
→ its eigenvalues are real

$$I_G = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} = \begin{bmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{bmatrix}$$

$$det[I_G - s \cdot I] = 0$$

$$(B - s) \cdot [(A - s) \cdot (C - s) - E^2] = 0$$

$$\Rightarrow \begin{cases} s_1 = \dfrac{(A - C) - (C - A) \cdot \sqrt{1 + t^2}}{2} \\ s_2 = B \\ s_3 = \dfrac{(A - C) + (C - A) \cdot \sqrt{1 + t^2}}{2} \end{cases}$$

$$t = \frac{2E}{C - A}$$

Inertia Matrix & Eigenvectors

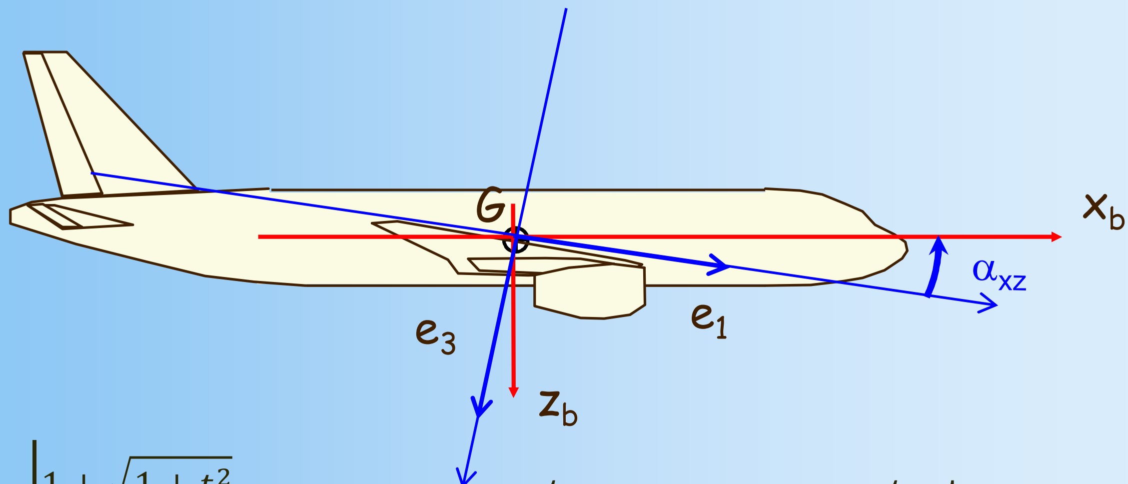
The Inertia Matrix is a real & symmetrical Matrix
→ its eigenvectors are orthogonal each other

$$I_G \cdot \vec{e}_n = s_n \cdot \vec{e}_n \longrightarrow \begin{cases} (A - s_n) \cdot x_n - E \cdot z_n = 0 \\ (B - s_n) \cdot y_n = 0 \\ -E \cdot x_n + (C - s_n) \cdot z_n = 0 \end{cases}$$

For each eigen value, we obtain a solution corresponding to the eigen vector

$$\vec{e}_1 = \left| \begin{array}{c} 1 + \sqrt{1 + t^2} \\ 0 \\ t \end{array} \right. \qquad \vec{e}_2 = \left| \begin{array}{l} 0 \\ 1 \\ 0 \end{array} \right. \qquad \vec{e}_3 = \left| \begin{array}{c} -t \\ 0 \\ 1 + \sqrt{1 + t^2} \end{array} \right.$$

The eigenvector $\vec{e}_1$ gives the direction of the x-main axis of symmetry it makes un angle α_{xz} with respect to x_b



$$\vec{e}_1 = \left| \begin{array}{c} 1+\sqrt{1+t^2} \\ 0 \\ t \end{array} \right. \Longrightarrow \tan\alpha_{xz} = \frac{t}{1+\sqrt{1+t^2}} \Longrightarrow \sqrt{1+t^2} = \frac{t-\tan\alpha_{xz}}{\tan\alpha_{xz}}$$

$$\Longrightarrow t = \frac{2\cdot\tan\alpha_{xz}}{1-\tan^2\alpha_{xz}} = \tan 2\alpha_{xz}$$

$$\boxed{\tan 2\alpha_{xz} = \frac{2E}{C-A}}$$