Lateral Statics

Northrop B-2

Expression of the Lateral Force equation

We want to compute $\left(\frac{d\vec{V}}{dt}\right)_{R_0}$ with respect to R_a

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \left(\frac{d\vec{V}}{dt}\right)_{R_a} + \vec{\Omega}_{a/0} \wedge \vec{V}$$

with

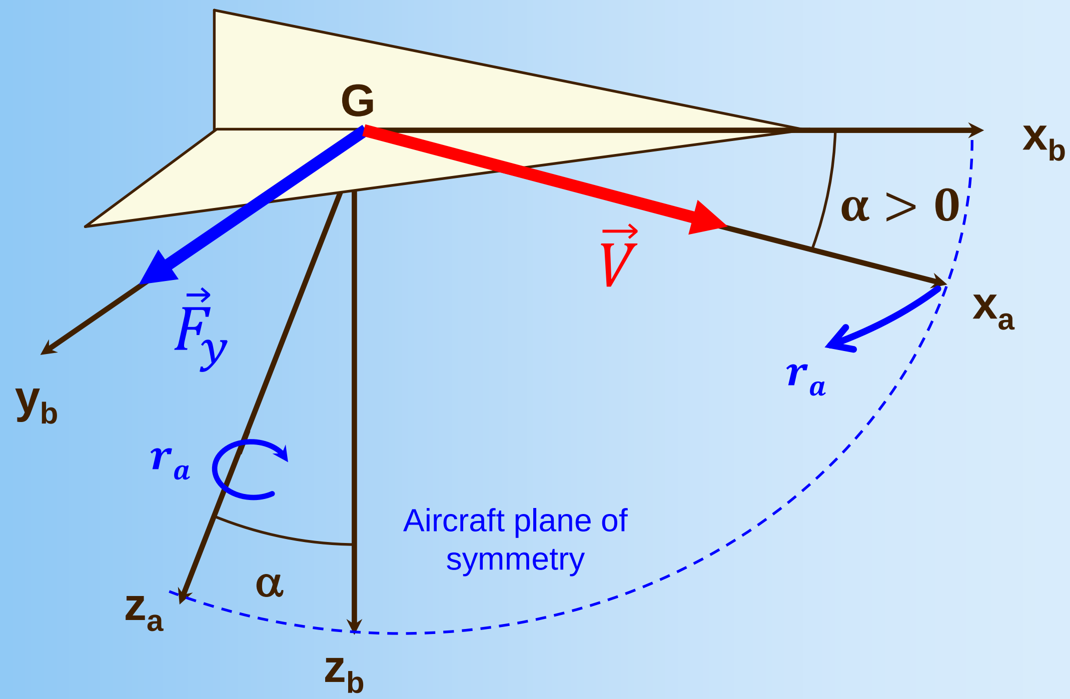
$$\vec{\Omega}_{a/0} = {}_{R_a}\left| \begin{matrix} p_a \\ q_a \\ r_a \end{matrix} \right.$$

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \left| \begin{matrix} \dot{V} \\ 0 \\ 0 \end{matrix} \right. + V \cdot \left| \begin{matrix} p_a \\ q_a \\ r_a \end{matrix} \right. \wedge \left| \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \right. = {}_{R_a}\left| \begin{matrix} \dot{V} \\ V \cdot r_a \\ -V \cdot q_a \end{matrix} \right.$$

The projection of the 1st Newton law with respect to y_a leads to the following relation (where r_a is the rotation rate of $\vec{V}$ with respect to z_a)

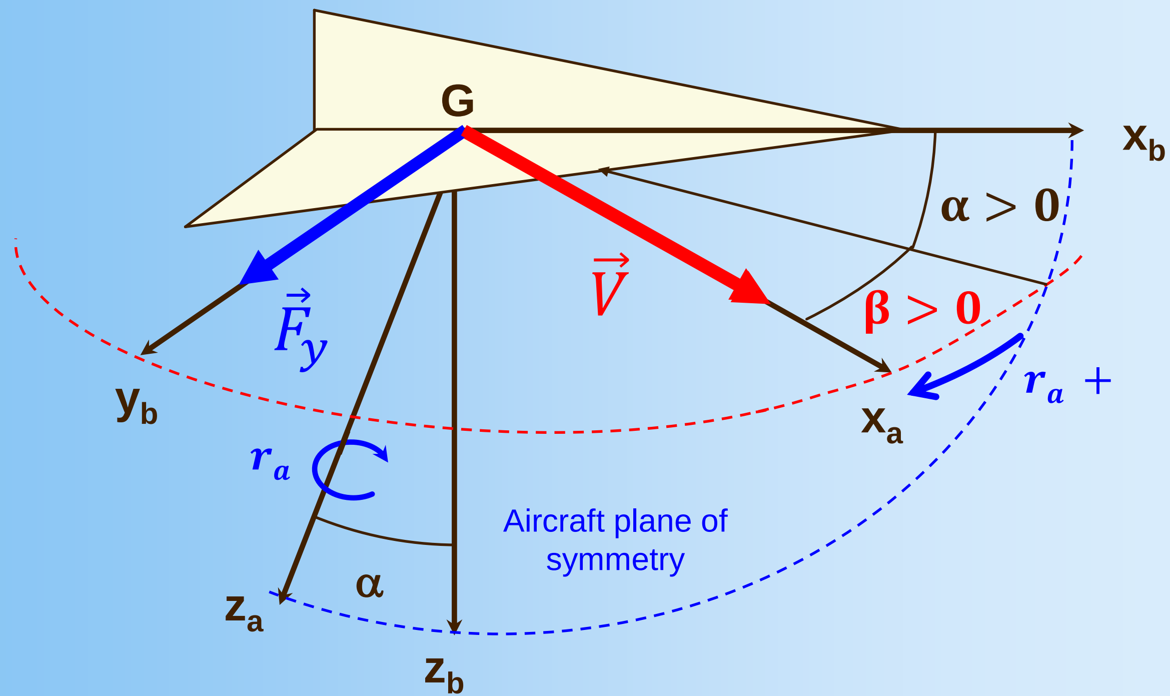
$$\vec{F} = m \cdot \left(\frac{d\vec{V}}{dt}\right)_{R_0} \Rightarrow F_y = mV \cdot r_a$$

Physical Interpretation



Any lateral force $\vec{F}_y$ acting makes rotate the velocity vector $\vec{V}$ around the axis z_a by a rotation r_a : $F_y = mV \cdot r_a$

Physical Interpretation



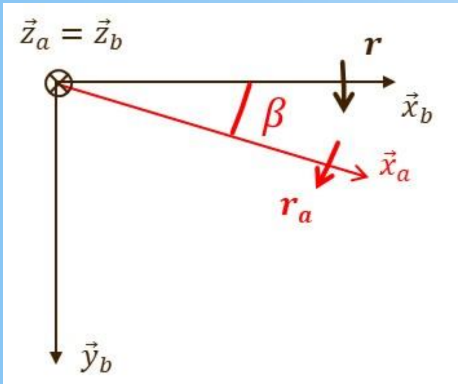
The Velocity vector $\vec{V}$ leaves the aircraft plane of symmetry

Side Slip appears : $\beta > 0$ if $F_y > 0$

Any lateral force $\vec{F}_y$ acting creates a side slip β

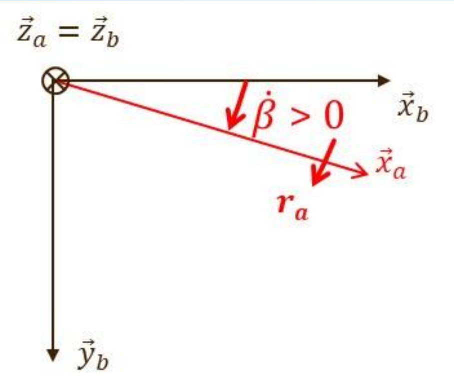
Relation between r_a , β , r

For simplification , $\alpha = 0$



β is constant , any rotation around z_b is equivalent to a rotation around z_a

$r_a = r$



$r = 0$, any β increase corresponds to a rotation around z_a

$r_a = \dot{\beta}$

We generalise the formula with : $\boxed{r_a = \dot{\beta} + r}$

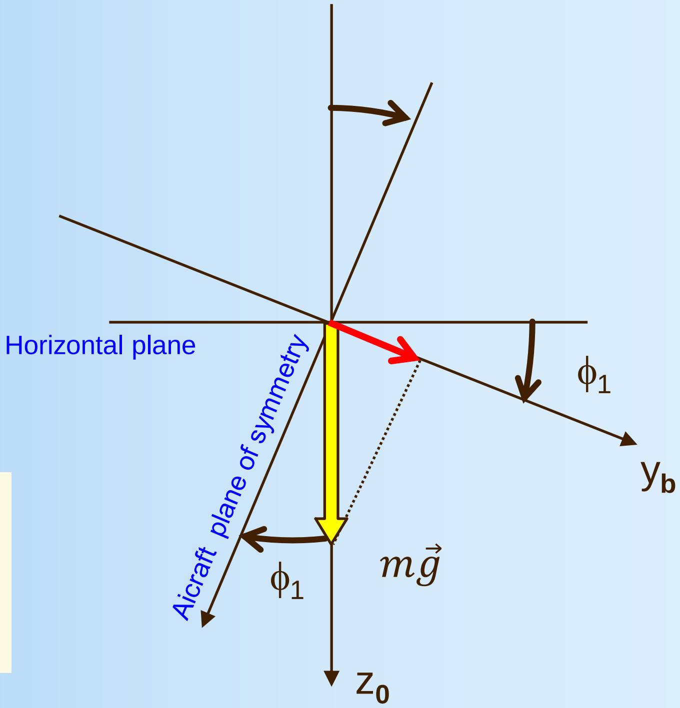
The complete formula (see annex) is given by : $r_a = \dot{\beta} + r\cos\alpha - p\sin\alpha$

Lateral Force equation

$$m\vec{g} + \vec{F}_a + \vec{F} = m \cdot \left(\frac{d\vec{V}}{dt}\right)_{R_0}$$

$$y_a \rightarrow mV \cdot r_a = mg \cdot \sin\phi_1 + \frac{1}{2}\rho V^2\, S \cdot Cy$$

$$mV \cdot (\dot{\beta} + r) = mg \cdot \sin\phi_1 + \frac{1}{2}\rho V^2\, S \cdot Cy$$

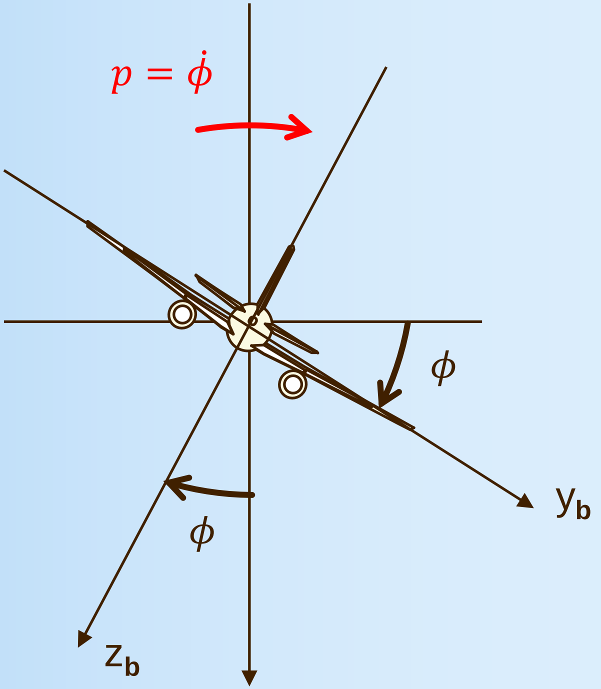


Moment & Kinematic equations

$$\vec{M}_G^a = [I_G] \cdot \left(\frac{d\vec{\Omega}_{ac}}{dt}\right)_{R_0}$$

$$x_b \rightarrow A \cdot \dot{p} = \frac{1}{2}\rho V^2\, SL \cdot Cl_G$$
$$z_b \rightarrow C \cdot \dot{r} = \frac{1}{2}\rho V^2\, SL \cdot Cn_G$$

$$p = \dot{\phi}$$



Lateral Flight equations

$$
\begin{cases}
mV \cdot (\dot{\beta} + r) = mg \cdot \sin\phi_1 + \dfrac{1}{2}\rho V^2\, S \cdot Cy \\
C \cdot \dot{r} = \dfrac{1}{2}\rho V^2\, SL \cdot Cn_G \\
A \cdot \dot{p} = \dfrac{1}{2}\rho V^2\, SL \cdot Cl_G \\
\dot{\phi} = p
\end{cases}
$$

4 differential equations for 4 state variables (β, r, p, ϕ)

I develop the aerodynamic coefficients with neglecting small terms

$$\begin{cases} \dot{\beta} = -r + \dfrac{\rho V S}{2m} \cdot \left(Cy_{\beta} \cdot \beta + \cancel{Cy_p} \cdot \dfrac{pL}{V} + \cancel{Cy_r} \cdot \dfrac{rL}{V} + \cancel{Cy_{\delta l}} \cdot \delta l + \cancel{Cy_{\delta n}} \cdot \delta n \right) + \dfrac{g}{V} \cdot sin\, \phi_1 \\ C \cdot \dot{r} = \dfrac{\rho V^2 S L}{2} \cdot \left(Cn_{\beta} \cdot \beta + Cn_p \cdot \dfrac{pL}{V} + Cn_r \cdot \dfrac{rL}{V} + \cancel{Cn_{\delta l}} \cdot \delta l + Cn_{\delta n} \cdot \delta n \right) \\ A \cdot \dot{p} = \dfrac{\rho V^2 S L}{2} \cdot \left(Cl_{\beta} \cdot \beta + Cl_p \cdot \dfrac{pL}{V} + Cl_r \cdot \dfrac{rL}{V} + Cl_{\delta l} \cdot \delta l + \cancel{Cl_{\delta n}} \cdot \delta n \right) \\ \dot{\phi} = p \end{cases}$$

Lateral flight equations

$$\sin\phi_1 = \cos\theta \cdot \sin\phi$$

$$\begin{cases} \dot{\beta} = -r + \dfrac{\rho V S}{2m} \cdot Cy_\beta \cdot \beta + \dfrac{g}{V} \cdot \overbrace{\sin\phi_1} \\ C \cdot \dot{r} = \dfrac{\rho V^2 S L}{2} \cdot \left(Cn_\beta \cdot \beta + Cn_p \cdot \dfrac{pL}{V} + Cn_r \cdot \dfrac{rL}{V} + Cn_{\delta n} \cdot \delta n \right) \\ A \cdot \dot{p} = \dfrac{\rho V^2 S L}{2} \cdot \left(Cl_\beta \cdot \beta + Cl_p \cdot \dfrac{pL}{V} + Cl_r \cdot \dfrac{rL}{V} + Cl_{\delta l} \cdot \delta l \right) \\ \dot{\phi} = p \end{cases}$$

4 state variables : (β, r, p, ϕ)

2 command variables : $(\delta l, \delta n)$

Available command variables : δl / δn

The Ailerons

$$Cl_{\delta l} \cdot \delta l$$

- produces an important roll moment
- δl can vary from -30° to 30°

The Rudder

$$Cn_{\delta n} \cdot \delta n$$

- produces an important yaw moment
- δn can vary from -30° to 30°

Lateral flight Steady State

Steady State ⇔ $\dot{\beta} = \dot{r} = \dot{p} = \dot{\phi} = 0$

$$\begin{cases} -r + \dfrac{\rho V S}{2m} \cdot Cy_\beta \cdot \beta + \dfrac{g}{V} \sin \phi_1 = 0 \\ Cn_\beta \cdot \beta + Cn_r \cdot \dfrac{rL}{V} + Cn_{\delta n} \cdot \delta n = 0 \\ Cl_\beta \cdot \beta + Cl_r \cdot \dfrac{rL}{V} + Cl_{\delta l} \cdot \delta l = 0 \\ p = 0 \end{cases}$$

5 unknowns $(\beta, r, \phi_1, \delta l, \delta n)$ ⇔ 3 equations

•either:

Lateral flight Steady State

$\boxed{r = r_a = 0 \;\Rightarrow\; \text{straight flight}}$

4 unknowns $(\beta, \phi_1, \delta l, \delta n)$ ⇔ 3 equations

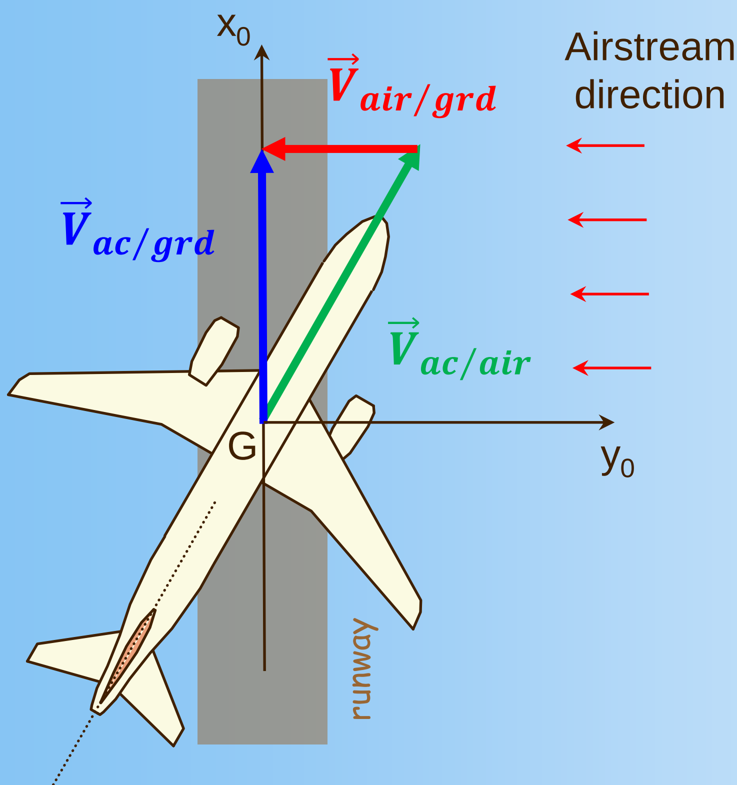
⇨ β imposed ⇨ landing with cross wind

⇨ ϕ_1 imposed ⇨ One Engine Inoperating

$$\begin{cases} Cy_\beta \cdot \beta + Cz \cdot \sin\phi_1 = 0 \\ Cn_\beta \cdot \beta + Cn_{\delta n} \cdot \delta n = 0 \\ Cl_\beta \cdot \beta + Cl_{\delta l} \cdot \delta l = 0 \\ p = 0 \end{cases}$$

Landing with cross wind

$$\vec{V}_{ac/grd} = \vec{V}_{ac/air} + \vec{V}_{air/grd}$$



The pilot is landing with a cross wind coming from the right , $\vec{V}_{air/grd}$

The pilot aligns $\vec{V}_{ac/grd}$ with the runway

The pilot makes no action on the rudder
The aircraft is stable laterally
The aircraft aligns by itself with $\vec{V}_{ac/air}$

The side slip $\beta = 0$

$$\delta n = 0 \rightarrow \beta = 0$$

Landing with cross wind

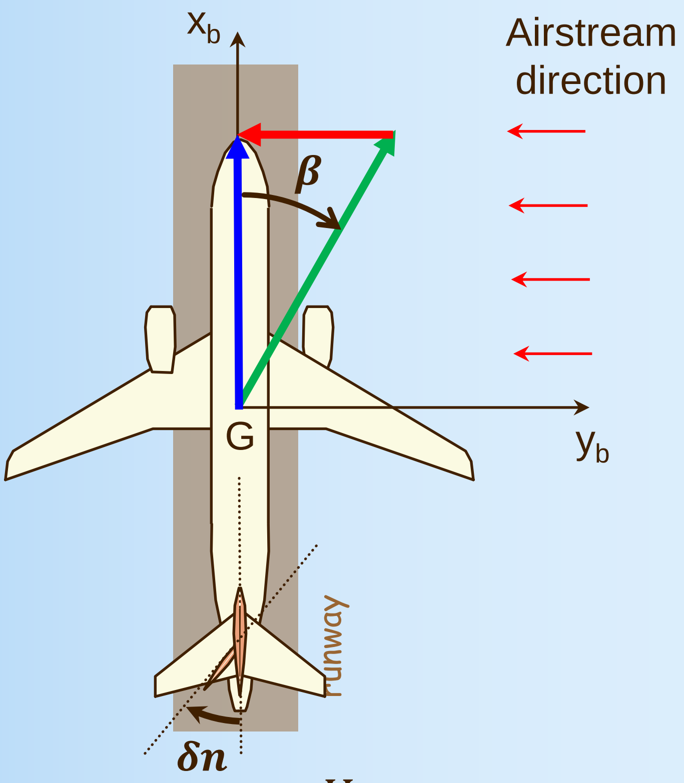
$$\vec{V}_{ac/grd} = \vec{V}_{ac/air} + \vec{V}_{air/grd}$$

The pilot makes the de-crab manoeuvre

With the rudder, he makes turning the aircraft to align the aircraft with the runway

The velocities stay unchanged

The aircraft is now submitted to a side slip



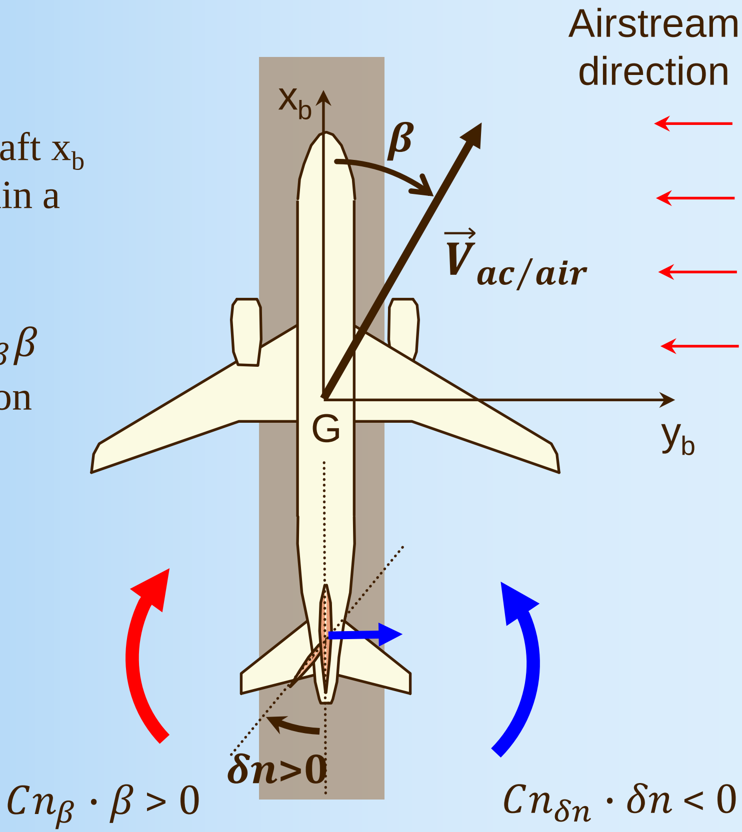
$$\delta n > 0 \rightarrow \beta = \frac{V_{wind}}{V} > 0$$

Landing with cross wind

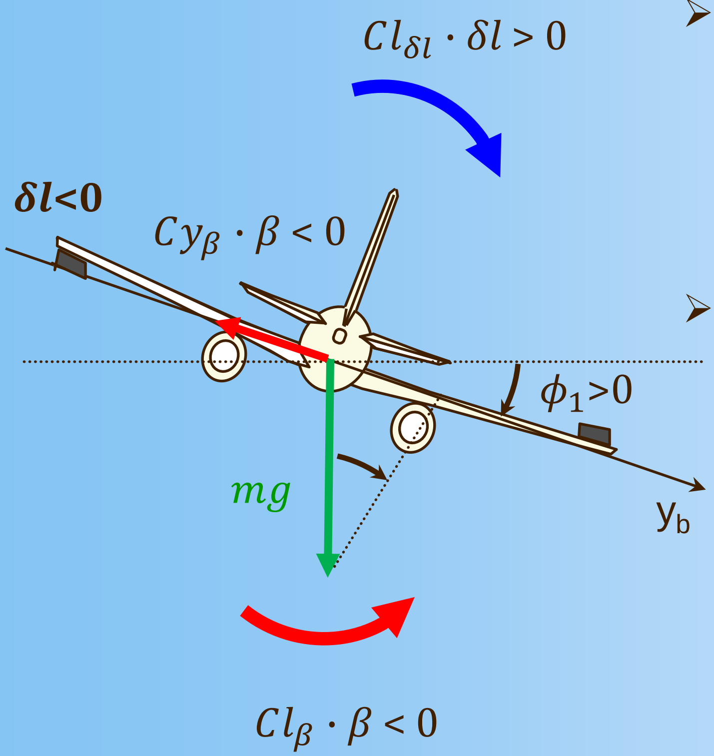
After the decrab, in order to maintain the aircraft x_b aligned with the runway, the pilot shall maintain a positive side slip $\beta = V_w/V$ constant :

- The side slip β creates a yaw moment $Cn_\beta \beta$ which is counteracted by a rudder deflection

$$\underset{+}{Cn_\beta} \cdot \underset{+}{\beta} + \underset{-}{Cn_{\delta n}} \cdot \delta n = 0 \rightarrow \underset{+}{\delta n}$$



Landing with cross wind



- The side slip β creates a roll moment $Cl_\beta \beta$ which is counteracted by an aileron deflection

$$\underset{-}{Cl_\beta} \cdot \underset{+}{\beta} + \underset{-}{Cl_{\delta l}} \cdot \delta l = 0 \rightarrow \underset{-}{\delta l}$$

- The side slip β creates also a lateral force $Cy_\beta \beta$: if the pilot does nothing, this force will make drift the aircraft towards the left of the runway. For maintaining the x_b alignment with the runway, the pilot must bank the aircraft by an angle ϕ_1 in order to counterbalance this lateral force.

$$\frac{1}{2}\rho V^2 S \cdot \underset{-}{Cy_\beta} \cdot \underset{+}{\beta} + mg \sin\phi_1 = 0 \rightarrow \underset{+}{\phi_1}$$

Landing with cross wind

This manoeuvre is performed during the reglementary Flight Tests in order to demonstrate the maximum Cross Wind which can be operated during the Landing of the Aircraft.

During this test, the pilot waits until the last moment for making the aircraft rotation in order to minimise the decrab phase : on recent aircraft, a special Roll law maintains the aircraft “flat” during the decrab, the pilot can put all his attention on the Yaw control

Most of the time, at touch down, the aircraft is not perfectly aligned with the runway, this misalignment is just absorbed by the deformation of the tyres.

One Engine Inoperative equilibrium

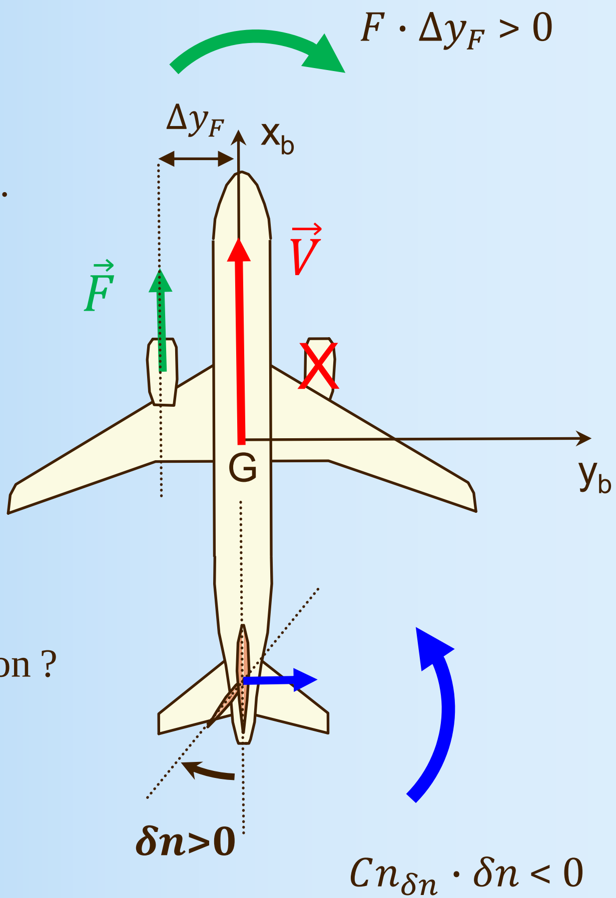
The pilot stops the Right engine.

The Yaw unbalance is solved with a rudder deflection.

$$\underset{+}{F} \cdot \Delta y_F + \frac{1}{2}\rho V^2 S \cdot \underset{-}{Cn_{\delta n}} \cdot \delta n = 0 \rightarrow \underset{+}{\delta n}$$

The pilot has just used the rudder command.

How can he use the ailerons for improving the situation ?



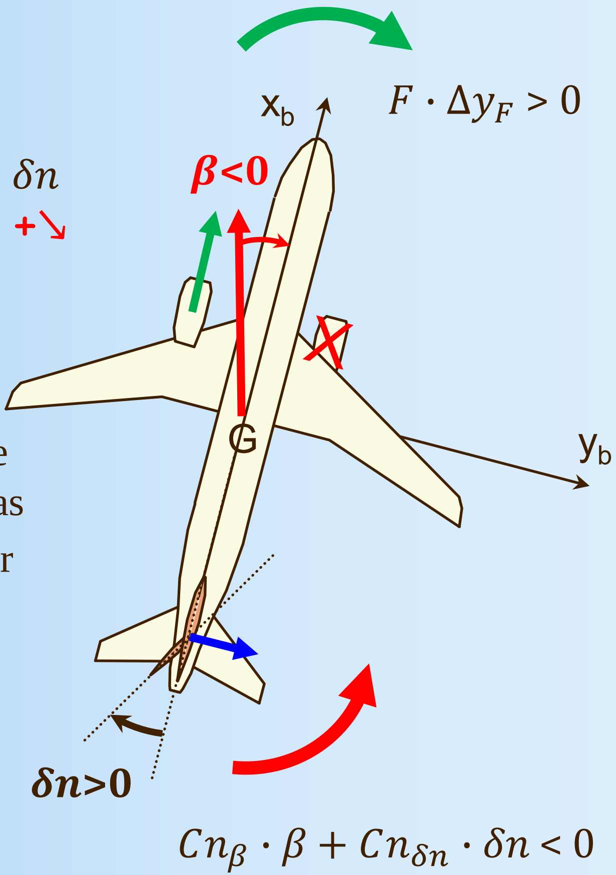
One Engine Inoperative equilibrium

The pilot is going to create the "good" side slip β in order to improve the Yaw balance :

$$F \cdot \underset{+}{\Delta y_F} + \frac{1}{2}\rho V^2 S \cdot \left[\underset{-}{Cn_\beta} \cdot \underset{+}{\beta} + \underset{-}{Cn_{\delta n}} \cdot \underset{+}{\delta n}\right] = 0 \rightarrow \underset{+\searrow}{\delta n}$$

If the pilot choose negative side slip β, it will help the Yaw balance by creating a $Cn_\beta \beta$ with the same sign as $Cn_{\delta n}\delta n$ resulting in a reduction of the original rudder deflection.

Now, how the pilot creates the "good" side slip β ?



One Engine Inoperative equilibrium

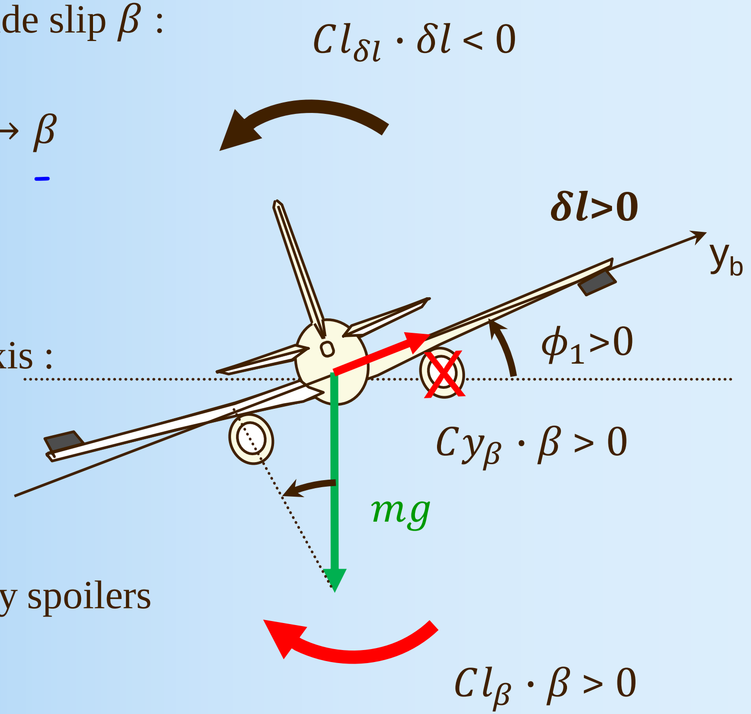
The pilot banks the aircraft on the good side (the good side is the one with the engine working !) ; by doing this, a lateral force $mg\sin\phi_1$ appears which creates the good side slip β :

$$\frac{1}{2}\rho V^2 S \cdot \underset{-}{Cy_\beta} \cdot \beta + mg \cdot \sin \underset{-}{\phi_1} = 0 \rightarrow \underset{-}{\beta}$$

Then, the ailerons are used for balancing the roll axis :

$$\underset{-}{Cl_\beta} \cdot \underset{-}{\beta} + \underset{-}{Cl_{\delta l}} \cdot \delta l = 0 \rightarrow \underset{+}{\delta l}$$

Remark : during this situation, we prevent to deploy spoilers because producing too much drag



Engine failure (OEI) = VMC / Controlability

This manoeuvre is performed during the reglementary Flight Tests in order to demonstrate the Minimum Control Speed, VMC.

The aircraft is trimmed and the pilot cuts One Engine ; the rudder is moving for balancing the yaw axis. Then, the pilot draws slowly on the stick, the angle of attack increases and the velocity decreases.
Hence, the rudder moment $\frac{1}{2}\rho V^2 S \cdot Cn_{\delta n}\delta n$ decreases and the pilot is obliged to put more rudder deflection for keeping the yaw balance.
When the rudder stop δn_{max} is achieved, it is no more possible to trim the aircraft with One Engine Inoperative and the pilot notes the achieved velocity which is called the VMC.

Of course, the pilot is clever : he knows that if he banks the aircraft on the good side, it eases the yaw balance and then he can continue to decrease the speed and improve the VMC !
Of course, the regulator knows it and he imposes that the bank angle ϕ_1 shall not exceed +/-5°

VMC = Minimum air Control speed for OEI (One Engine Inoperative)

VMCA at Take-Off / VMCL at Landing

VMC's shall be as small as possible in order to improve operationnal performances

Lateral Statics
Steady Turn

Breguet 763 Deux Ponts Provence

Lateral flight Steady State

Steady State $\Leftrightarrow$ $\dot{\beta} = \dot{r} = \dot{p} = \dot{\phi} = 0$

$$\begin{cases} -r + \dfrac{\rho V S}{2m} \cdot Cy_\beta \cdot \beta + \dfrac{g}{V} \sin \phi_1 = 0 \\ Cn_\beta \cdot \beta + Cn_r \cdot \dfrac{rL}{V} + Cn_{\delta n} \cdot \delta n = 0 \\ Cl_\beta \cdot \beta + Cl_r \cdot \dfrac{rL}{V} + Cl_{\delta l} \cdot \delta l = 0 \\ p = 0 \end{cases}$$

5 unknowns $(\beta, r, \phi_1, \delta l, \delta n)$ $\Leftrightarrow$ 3 equations

•either:

Lateral flight Steady State

$$\boxed{r = r_a \neq 0 \Rightarrow \text{steady turn}}$$

4 unknowns $(\beta, \phi_1, \delta l, \delta n)$ ⇔ 3 equations

⇨ δl or δn imposed ⇨ steady turn

⇨ $\beta = 0$ ⇨ coordinate steady turn

⇨ $\phi_1 = 0$ ⇨ flat steady turn

$$\begin{cases} r = \dfrac{\rho V S}{2m} \cdot Cy_\beta \cdot \beta + \dfrac{g}{V} \cdot \sin \phi_1 \\ Cl_\beta \cdot \beta + Cl_r \cdot \dfrac{rL}{V} = -Cl_{\delta l} \cdot \delta l \\ Cn_\beta \cdot \beta + Cn_r \cdot \dfrac{rL}{V} = -Cn_{\delta n} \cdot \delta n \\ p = 0 \end{cases}$$

Steady Turn Solution with δl or δn

steady turn with δl : $\delta n = 0$

$$\frac{rL}{V} = \frac{\overset{+}{Cn_\beta}\,\overset{-}{Cl_{\delta l}}}{Cl_\beta Cn_r - Cn_\beta Cl_r} \cdot \delta l$$

steady turn with δn : $\delta l = 0$

$$\frac{rL}{V} = \frac{-\overset{-}{Cl_\beta}\,\overset{-}{Cn_{\delta n}}}{Cl_\beta Cn_r - Cn_\beta Cl_r} \cdot \delta n$$

$$\begin{cases} r = \dfrac{\rho V S}{2m} \cdot Cy_\beta \cdot \beta + \dfrac{g}{V} \cdot \sin\phi_1 \\ Cl_\beta \cdot \beta + Cl_r \cdot \dfrac{rL}{V} = -Cl_{\delta l} \cdot \delta l \\ Cn_\beta \cdot \beta + Cn_r \cdot \dfrac{rL}{V} = -Cn_{\delta n} \cdot \delta n \\ p = 0 \end{cases}$$

The steady state solution is function of the quantity :

$$Cl_\beta Cn_r - Cn_\beta Cl_r = \begin{vmatrix} Cl_\beta & Cl_r \\ Cn_\beta & Cn_r \end{vmatrix}$$

meaning of : $Cl_\beta Cn_r - Cn_\beta Cl_r$

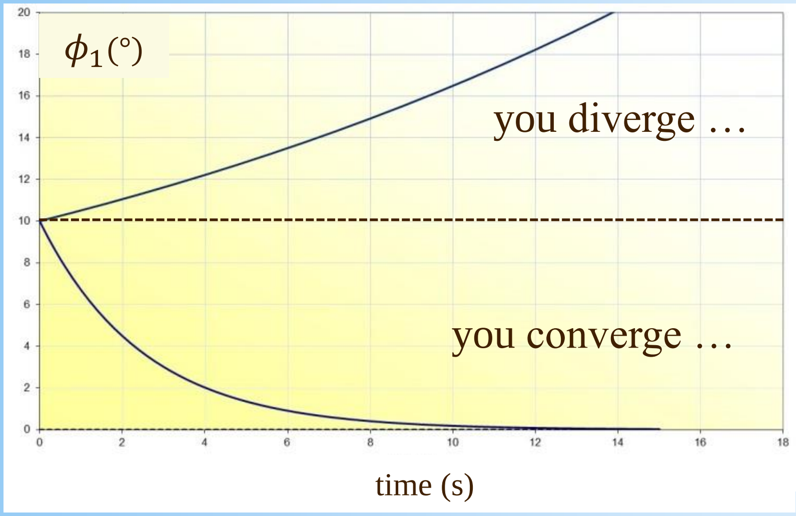
The aircraft is banked at $\phi_1 > 0$ and the pilot puts the stick at neutral : $\delta l = 0$

What happens ? Does the aircraft come back to $\phi_1 = 0$ or continue banking ?

In that situation, there are :

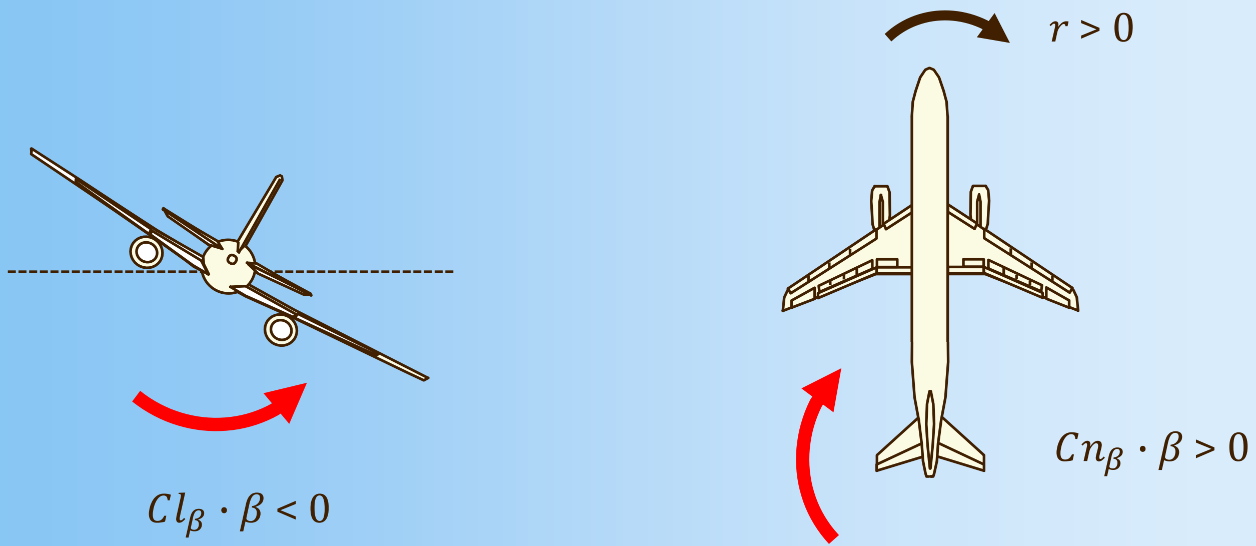
- aerodynamic moments which are opposing to bank (the aircraft will come back)
- aerodynamic moments which are making turn (the aircraft will continue turning)

Depending on the magnitude of these moments, they are 2 possibilities …



meaning of : $Cl_\beta Cn_r - Cn_\beta Cl_r$

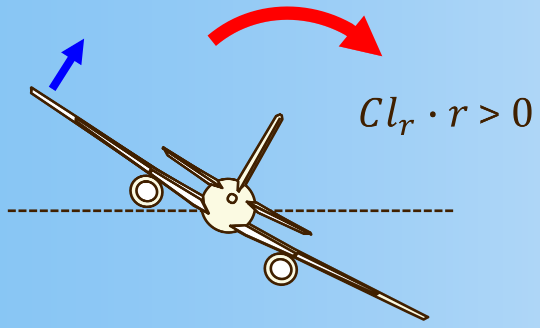
Banking $\phi_1 > 0$ on the right creates side slip on the right : $\beta > 0$

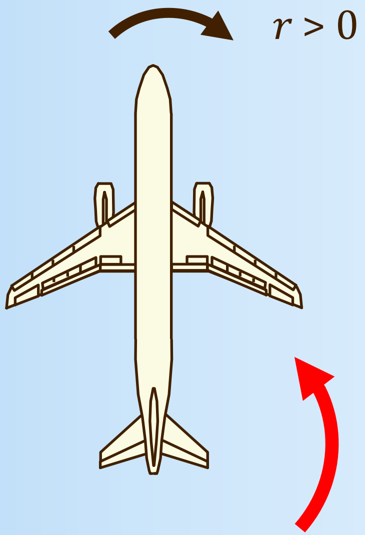


The Cl_β (dihedral effect) is opposing and reducing the bank

The Cn_β (wheathercock effect) makes turning by producing yaw rate $r > 0$

Banking $\phi_1 > 0$ on the right creates positive yaw rate : $r > 0$





The Cl_r makes banking more and turning more consequently

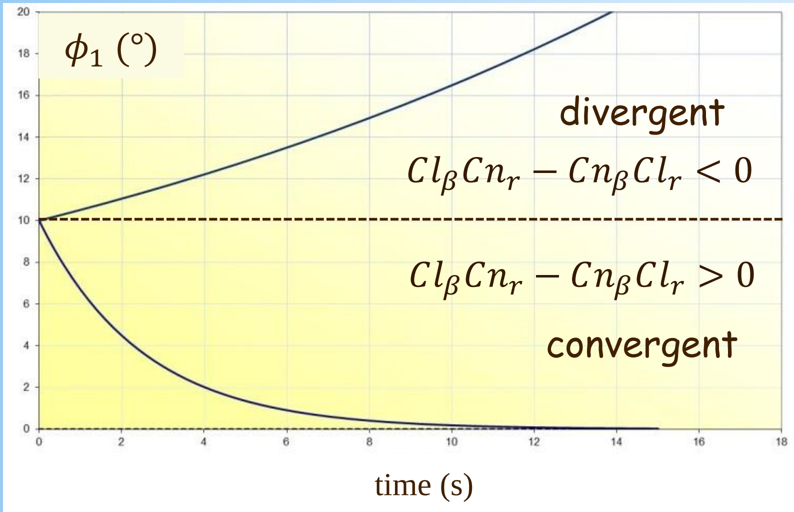
The Cn_r (damping effect) naturally decreases the yaw rate r

meaning of : $Cl_\beta Cn_r - Cn_\beta Cl_r$

The aircraft is banked at $\phi_1 > 0$ and the pilot does nothing : $\delta l = \delta n = 0$

Against Turn	Pro Turn
$Cl_\beta < 0$	$Cn_\beta > 0$
$Cn_r < 0$	$Cl_r > 0$

The sign of $\boldsymbol{Cl_\beta Cn_r - Cn_\beta Cl_r}$ is associated to the aircraft characteristics for turning by itself or not. It is linked to the convergence / divergence of the Spiral Mode…



Steady Turn Solution with δl or δn

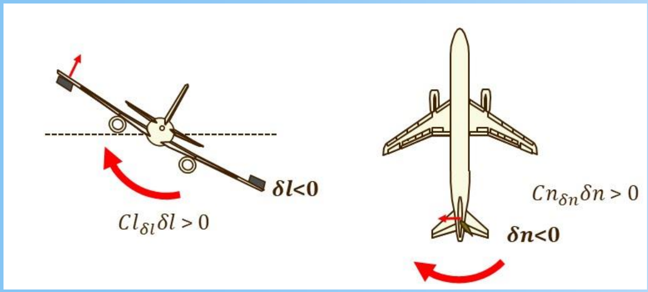
Steady Turn solution to the right r > 0, what is the sign of δl / δn ?

$$\frac{rL}{V} = \frac{\overset{+}{Cn_\beta}\overset{-}{Cl_{\delta l}}}{Cl_\beta Cn_r - Cn_\beta Cl_r} \cdot \delta l \qquad\qquad \frac{rL}{V} = \frac{-\overset{-}{Cl_\beta}\overset{-}{Cn_{\delta n}}}{Cl_\beta Cn_r - Cn_\beta Cl_r} \cdot \delta n$$

<u>Spiral Mode Convergent</u>

$$Cl_\beta Cn_r - Cn_\beta Cl_r > 0$$

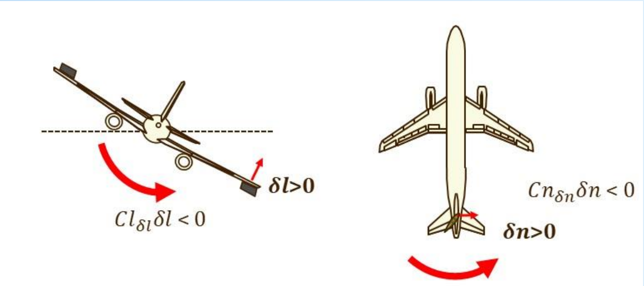
→ the aircraft doesn't turn naturally by itself
→ you must put command to force the aircraft to turn steady
→ $\delta l < 0$ or $\delta n < 0$



<u>Spiral Mode Divergent</u>

$$Cl_\beta Cn_r - Cn_\beta Cl_r < 0$$

→ the aircraft turns naturally by itself
→ you must put command against the turning, to prevent the aircraft to diverge
→ $\delta l > 0$ or $\delta n > 0$



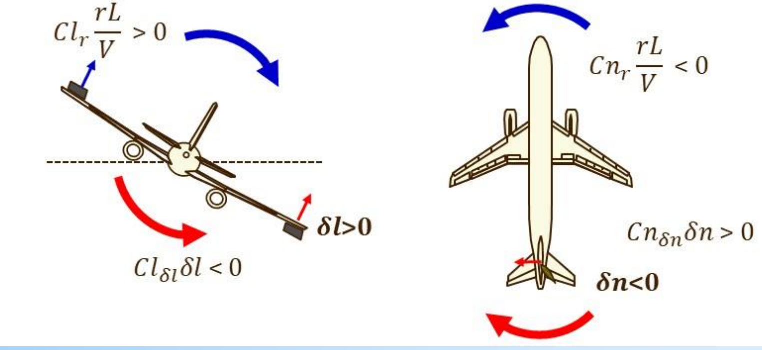
The Coordinate Steady Turn (β = 0°)

$$\begin{cases} r = \dfrac{\rho V S}{2m} \cdot Cy_\beta \cdot \beta + \dfrac{g}{V} \cdot \sin\phi_1 \\ Cl_\beta \cdot \beta + Cl_r \cdot \dfrac{rL}{V} = -Cl_{\delta l} \cdot \delta l \\ Cn_\beta \cdot \beta + Cn_r \cdot \dfrac{rL}{V} = -Cn_{\delta n} \cdot \delta n \\ p = 0 \end{cases}$$

Coordinate Steady Turn : $\beta = 0$

$$r = r_a = \frac{g}{V} \cdot \sin\phi_1$$

$$\begin{cases} \delta l = -\dfrac{Cl_r^{+}}{Cl_{\delta l}^{-}} \cdot \dfrac{rL}{V} \\ \delta n = -\dfrac{Cn_r^{-}}{Cn_{\delta n}^{-}} \cdot \dfrac{rL}{V} \end{cases}$$



This is the typical Operationnal Turning manoeuvre :
the control law manages the manoeuvre by keeping β= 0°

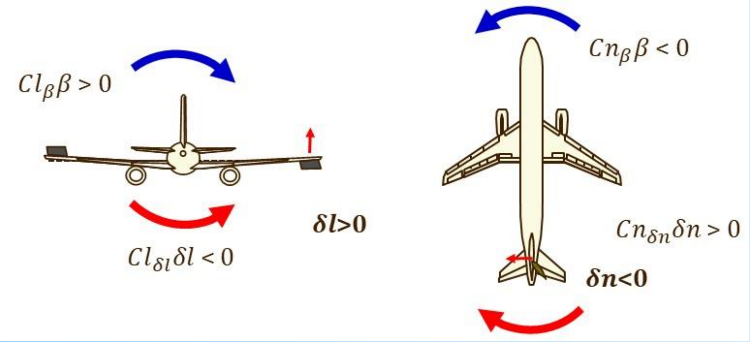
The Flat Steady Turn

$$\begin{cases} r = \dfrac{\rho V S}{2m} \cdot Cy_\beta \cdot \beta + \dfrac{g}{V} \cdot \sin\phi_1 \\ Cl_\beta \cdot \beta + Cl_r \cdot \dfrac{rL}{V} = -Cl_{\delta l} \cdot \delta l \\ Cn_\beta \cdot \beta + Cn_r \cdot \dfrac{rL}{V} = -Cn_{\delta n} \cdot \delta n \\ p = 0 \end{cases}$$

Flat Steady Turn : $\phi_1 = 0$

$$r = r_a = \frac{\rho V S}{2m} \overset{-}{Cy_\beta} \cdot \beta$$

$$\begin{cases} \delta l \approx -\dfrac{\overset{-}{Cl_\beta}}{\overset{-}{Cl_{\delta l}}} \cdot \beta \\ \delta n \approx -\dfrac{\overset{+}{Cn_\beta}}{\overset{-}{Cn_{\delta n}}} \cdot \beta \end{cases}$$



Purely academic ; can be related to the decrab manoeuvre at landing with cross wind. Difficult situation because the pilot has to control both yaw and roll.

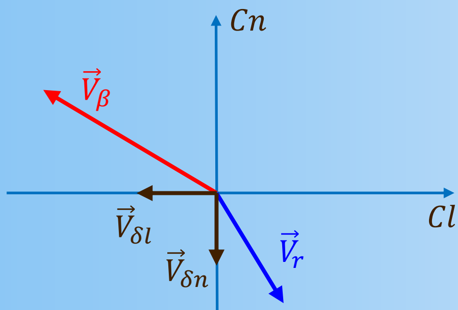
An interesting Remark …

NOT in PROGRAM

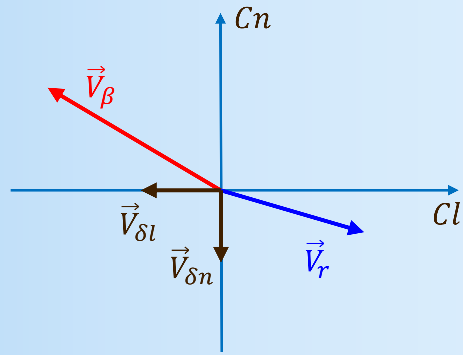
We can interpret the lateral trim by a vectoriel sum of 4 vectors :

$$\begin{cases} Cl_\beta \cdot \beta + Cl_r \cdot \dfrac{rL}{V} + Cl_{\delta l} \cdot \delta l = 0 \\ Cn_\beta \cdot \beta + Cn_r \cdot \dfrac{rL}{V} + Cn_{\delta n} \cdot \delta n = 0 \end{cases} \Rightarrow \left|\begin{matrix} Cl_\beta \\ Cn_\beta \end{matrix}\right. \cdot \beta + \left|\begin{matrix} Cl_r \\ Cn_r \end{matrix}\right. \cdot \frac{rL}{V} + \left|\begin{matrix} Cl_{\delta l} \\ 0 \end{matrix}\right. \cdot \delta l + \left|\begin{matrix} 0 \\ Cn_{\delta n} \end{matrix}\right. \cdot \delta n = 0$$

Then it is possible to solve the lateral trim by a graphical resolution :



Spiral Mode Convergent :
$Cl_\beta Cn_r - Cn_\beta Cl_r > 0$

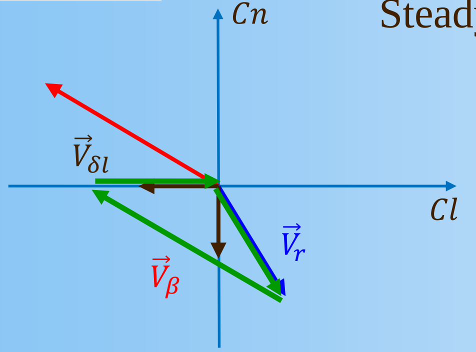


Spiral Mode Divergent :
$Cl_\beta Cn_r - Cn_\beta Cl_r < 0$

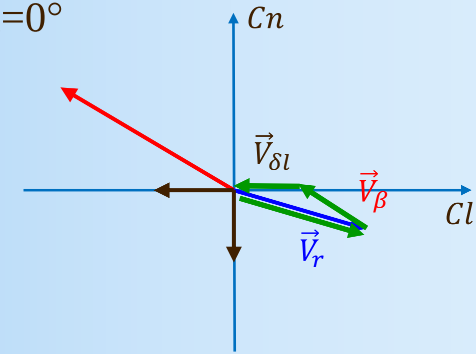
Steady Turning to be solved

NOT in PROGRAM

Steady Turn / δn=0°

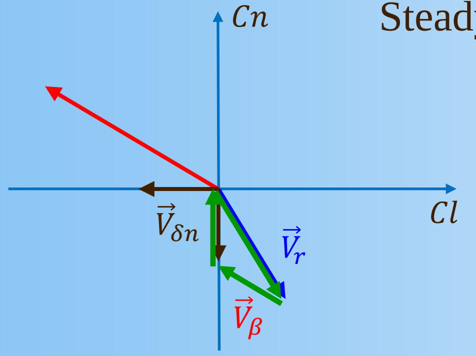


Spiral Mode Convergent :
$\beta > 0 \ / \ \delta l < 0$

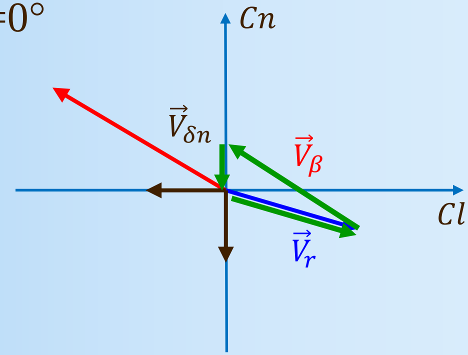


Spiral Mode Divergent :
$\beta > 0 \ / \ \delta l > 0$

Steady Turn / δl=0°



Spiral Mode Convergent :
$\beta > 0 \ / \ \delta n < 0$

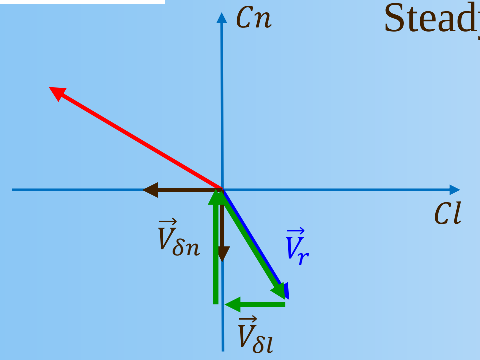


Spiral Mode Divergent :
$\beta > 0 \ / \ \delta n > 0$

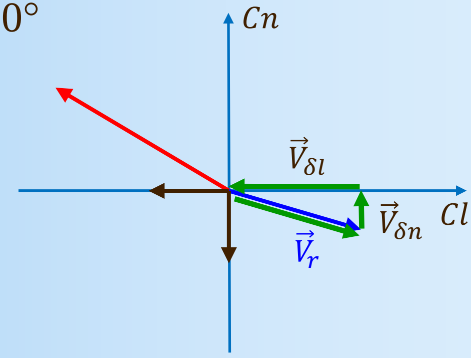
But also Straight Lateral Flight

NOT in PROGRAM

Steady Turn / β=0°

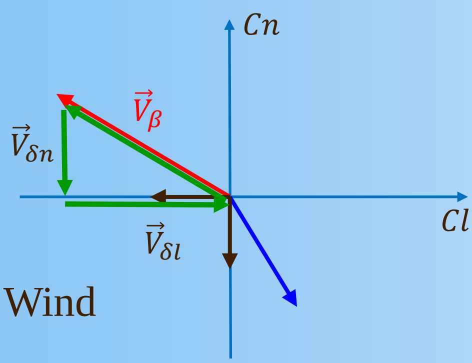


Spiral Mode Convergent :
$\delta l > 0 / \ \delta n < 0$

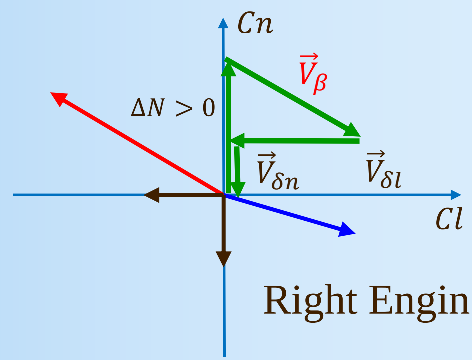


Spiral Mode Divergent :
$\delta l > 0 / \ \delta n < 0$

Cross Wind



Cross Wind / $\beta > 0$:
$\delta l < 0$ / $\delta n > 0$



Right Engine Failure

Right Engine Failure / $\Delta N > 0$:
$\beta < 0$ / $\delta l > 0$ / $\delta n > 0$