General Flight equations

de Havilland DH.110 Sea Vixen

General Flight equations

The idea is to generalise the 4 differential equations that we have already obtained for a pure longitudinal flight to a general situation (including lateral forces)

We assume that the lift force is banked by a roll angle μ

There is no modification for the Drag equation (projection of the forces on x_a)

$$(x_a) : \; m\dot{V} = F - \frac{1}{2}\rho V^2 SCx - mg \cdot \sin\gamma$$

There is no modification for the Pitch equation (projection of the moments on y_b)

$$(y_b) : B\dot{q} = \frac{1}{2}\rho V^2 SL\left[Cm_0 + Cm_\alpha^G(\alpha - \alpha_0) + Cm_{\delta m}\delta m + Cm_q\frac{qL}{V}\right]$$

For simplicity, we continue to assume that :

- The thrust $\vec{F}$ is aligned with the velocity $\vec{V}$
- The thrust $\vec{F}$ is applied at the centre of gravity G

General Expression of $d\vec{V}/dt$

We compute the projection of $\left(\frac{d\vec{V}}{dt}\right)_{R_0}$ with respect to R_a

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \left(\frac{d\vec{V}}{dt}\right)_{R_a} + \vec{\Omega}_{a/0} \wedge \vec{V}$$

with

$$\vec{\Omega}_{a/0} = {}_{R_a}\left|\begin{matrix} p_a \\ q_a \\ r_a \end{matrix}\right.$$

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \left|\begin{matrix} \dot{V} \\ 0 \\ 0 \end{matrix}\right. + V \cdot \left|\begin{matrix} p_a \\ q_a \\ r_a \end{matrix}\right. \wedge \left|\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}\right. = {}_{R_a}\left|\begin{matrix} \dot{V} \\ V \cdot r_a \\ -V \cdot q_a \end{matrix}\right.$$

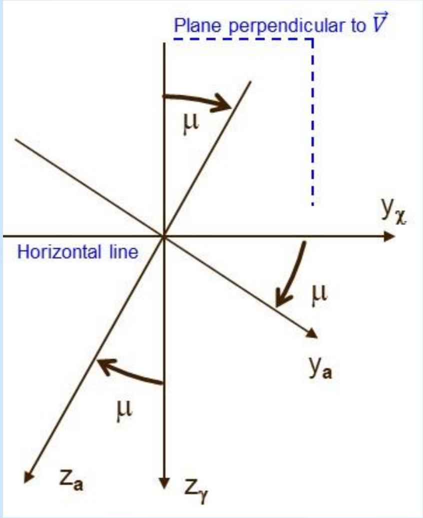
General Expression of $d\vec{V}/dt$

We can use also the general expression obtained in the chapter 1.2

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \dot{V}\,\vec{x}_a + V\cos\gamma\,\dot{\chi}\,\vec{y}_\chi - V\dot{\gamma}\,\vec{z}_\gamma$$

$$\begin{cases} \vec{y}_\chi = \vec{y}_a \cos\mu - \vec{z}_a \sin\mu \\ \vec{z}_\gamma = \vec{y}_a \sin\mu + \vec{z}_a \cos\mu \end{cases}$$

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \left|\begin{array}{c} \dot{V} \\ V\cdot(\dot{\chi}\cos\gamma\cos\mu - \dot{\gamma}\sin\mu) \\ -V\cdot(\dot{\chi}\cos\gamma\sin\mu + \dot{\gamma}\cos\mu) \end{array}\right._{R_a}$$



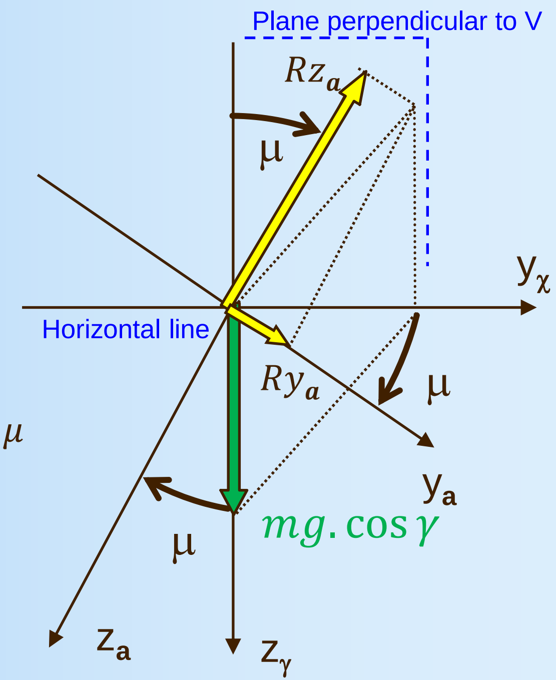
General Flight equations

We obtain the $(\dot{\gamma}, \dot{\chi})$ equations by using the general expression of :

$$\left(\frac{d\vec{V}}{dt}\right)_{R_0} = \left|\begin{matrix} \dot{V} \\ V\cdot(\dot{\chi}\cos\gamma\cos\mu - \dot{\gamma}\sin\mu) = V\cdot r_a \\ -V\cdot(\dot{\chi}\cos\gamma\sin\mu + \dot{\gamma}\cos\mu) = -V\cdot q_a \end{matrix}\right._{R_a}$$

We obtain the equations with respect to y_a and z_a :

$$\begin{cases} mV(\dot{\chi}\cos\gamma\cos\mu - \dot{\gamma}\sin\mu) = \frac{1}{2}\rho V^2 SCy + mg\cos\gamma\sin\mu \\ -mV\underbrace{(\dot{\chi}\cos\gamma\sin\mu + \dot{\gamma}\cos\mu)}_{q_a} = -\frac{1}{2}\rho V^2 SCz + mg\cos\gamma\cos\mu \end{cases}$$



The $\dot{\alpha}$ equation is obtained with (see annex 1.1) :

$$q_a \approx q - \dot{\alpha} \rightarrow \dot{\alpha} = q - q_a = q - \left(\frac{\rho V S Cz}{2m} - \frac{g}{V}\cos\gamma\cos\mu\right)$$

General Flight equations

The general Flight equations are listed below ; of course, we recover the Pure Longitudinal Flight equations by assuming Cy=0 and μ=0°

$$
\begin{cases}
(x_a) : \; m\dot{V} = F - \dfrac{1}{2}\rho V^2 SCx - mg \cdot \sin\gamma \\
(y_a) : \dot{\chi}\cos\gamma\cos\mu - \dot{\gamma}\sin\mu = \dfrac{\rho VSCy}{2m} + \dfrac{g}{V}\cos\gamma\sin\mu \\
(z_a) : \dot{\gamma}\cos\mu + \dot{\chi}\cos\gamma\sin\mu = \dfrac{\rho VSCz}{2m} - \dfrac{g}{V}\cos\gamma\cos\mu \\
(y_b) : B\dot{q} = \dfrac{1}{2}\rho V^2 SL\left[Cm_0 + Cm_\alpha^G(\alpha - \alpha_0) + Cm_{\delta m}\delta m + Cm_q\dfrac{qL}{V}\right] \\
(z_a) : \dot{\alpha} \approx q - q_a = q - \dfrac{\rho VSCz}{2m} + \dfrac{g}{V} \cdot \cos\gamma\cos\mu
\end{cases}
$$

Grumman Mohawk

Load Factor expression for Turning Manoeuvre

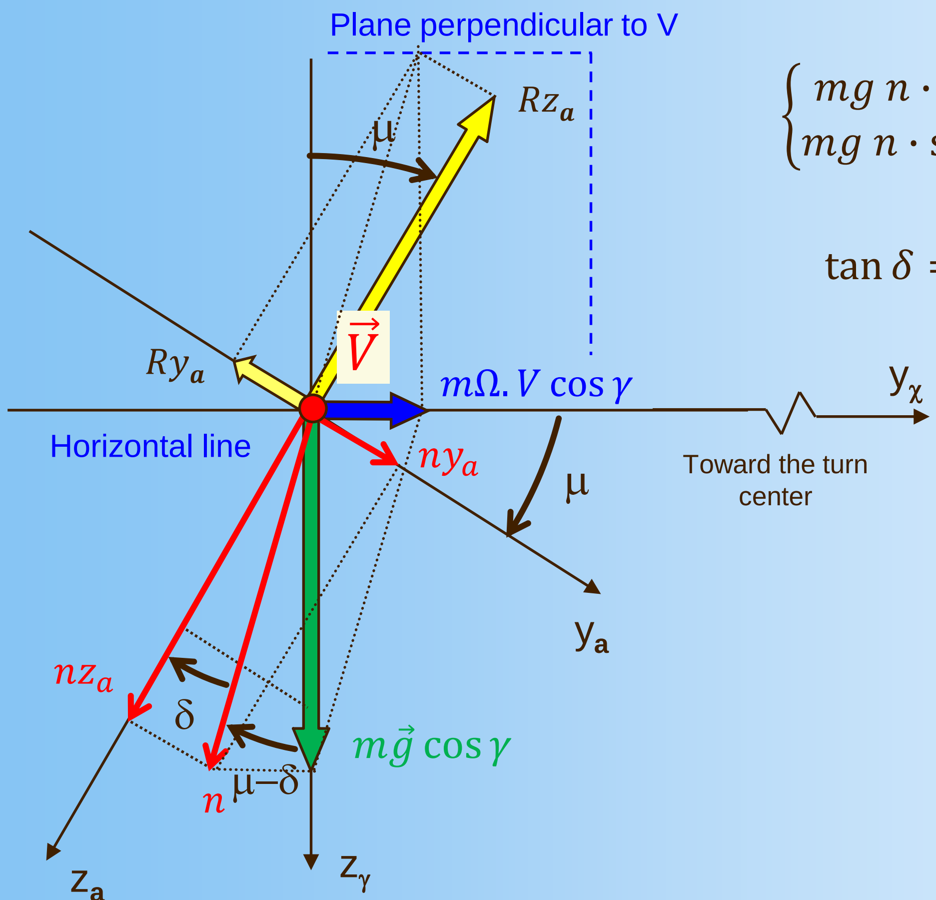
Load factor for Turn Manoeuvre

The following slides gives general expression of load factor during a steady turning manoeuvre

- General case with lateral forces
- General cases with no lateral forces
    - Expressed with respect to the Aerodynamic Referential System
    - Expressed with respect to the Body Referential System

Of course, the load factors which are measured by the sensors within the aircraft correspond to the formula expressed with respect to the Body Referential System

Turn Manœuvre / General Case

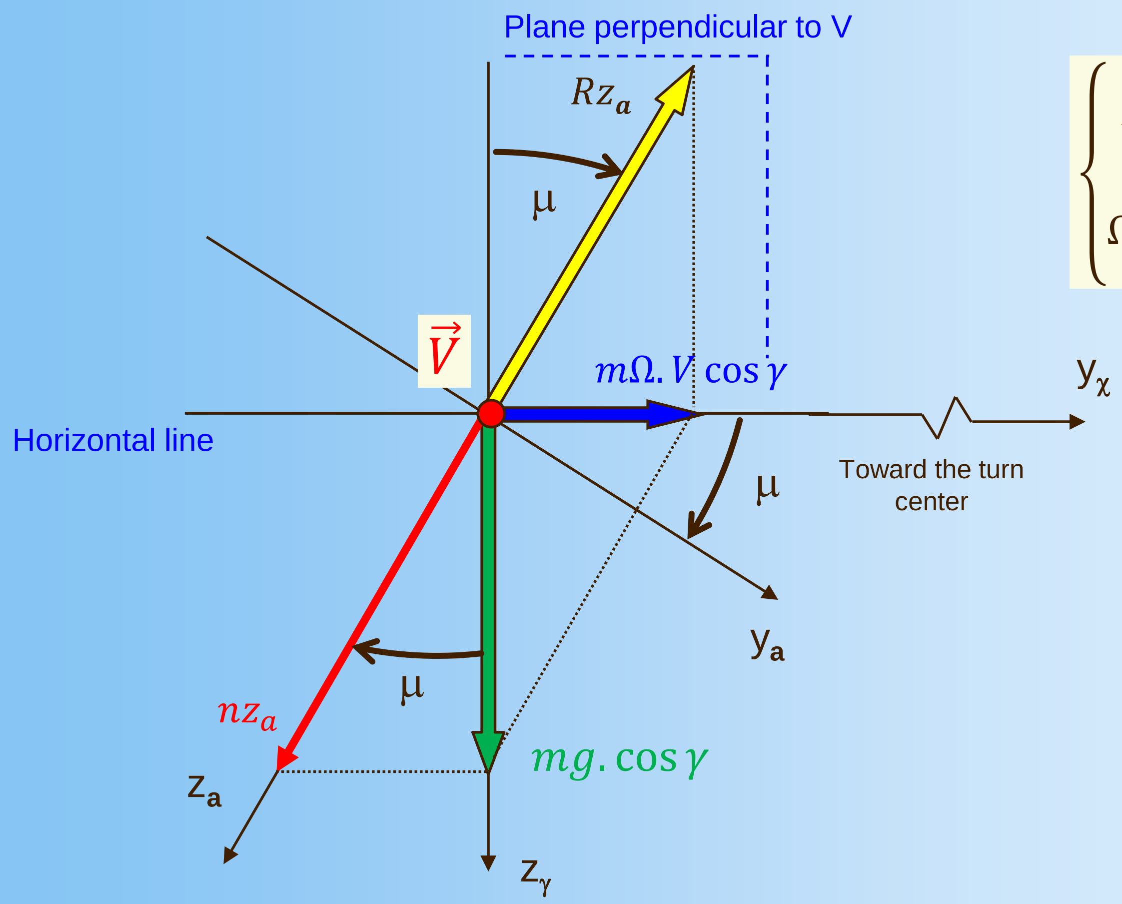


$$\begin{cases} mg\, n \cdot \cos(\mu - \delta) = mg \cdot \cos\gamma \\ mg\, n \cdot \sin(\mu - \delta) = m\Omega V \cdot \cos\gamma \end{cases}$$

$$\tan\delta = \frac{Ry_a}{Rz_a} = \frac{ny_a}{nz_a}$$

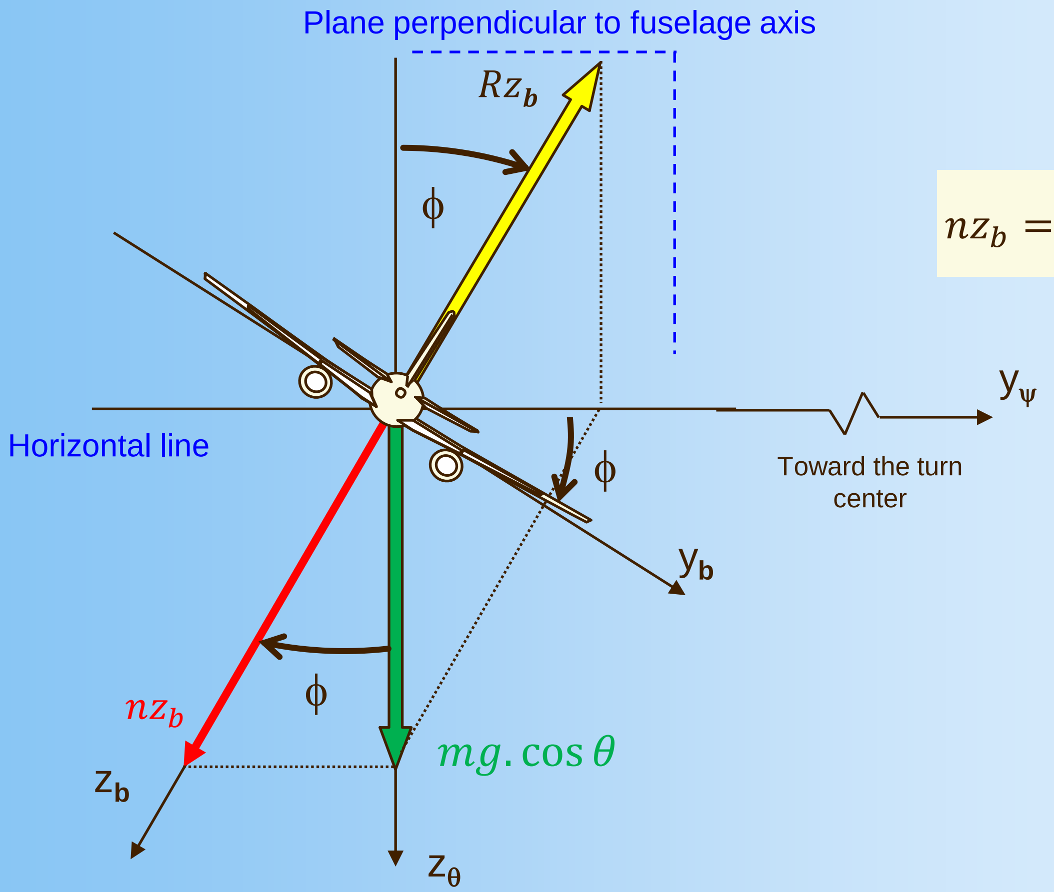
$$\begin{cases} ny_a = \dfrac{\cos\gamma \sin\delta}{\cos(\mu - \delta)} \\ nz_a = \dfrac{\cos\gamma \cos\delta}{\cos(\mu - \delta)} \\ \Omega = \dfrac{g}{V} \cdot \tan(\mu - \delta) \end{cases}$$

Turn Manœuvre / $ny_a = 0$



$$\begin{cases} nz_a = \dfrac{\cos\gamma}{\cos\mu} \\ \Omega = \dfrac{g}{V}\cdot\tan\mu \end{cases}$$

Turn Manœuvre / $ny_b = 0$



$$nz_b = \frac{\cos\theta}{\cos\phi}$$