Aerodynamic Model

Beechcraft Starship

Aerodynamic Model

Aerodynamic Force F (N) and Moment M_G (N.m) are given by

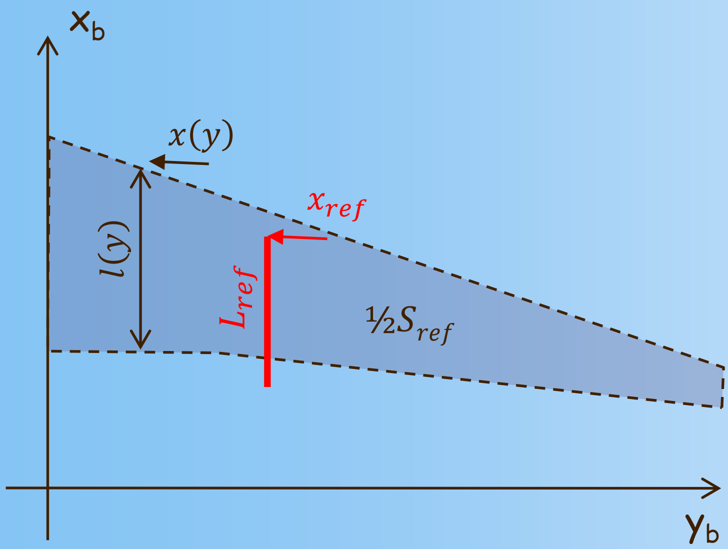
$$F = \frac{1}{2}\rho V^2 S_{ref} \cdot C_F$$

$$\mathrm{M}_A = \frac{1}{2}\rho V^2 S_{ref} L_{ref} \cdot C_M$$

where

- $\frac{1}{2}\rho V^2$ is the dynamic pressure (N/m^2)
- S_{ref} is the reference area (m^2)
- L_{ref} is the reference length (m)
- C_F & C_M are non dimensional aerodynamic coefficients

S_{ref} / L_{ref} Definitions & Calculations



$$\frac{S_{ref}}{2} = \int l(y) \cdot dy$$

$$L_{ref} = \frac{2\int l^2(y) \cdot dy}{S_{ref}}$$

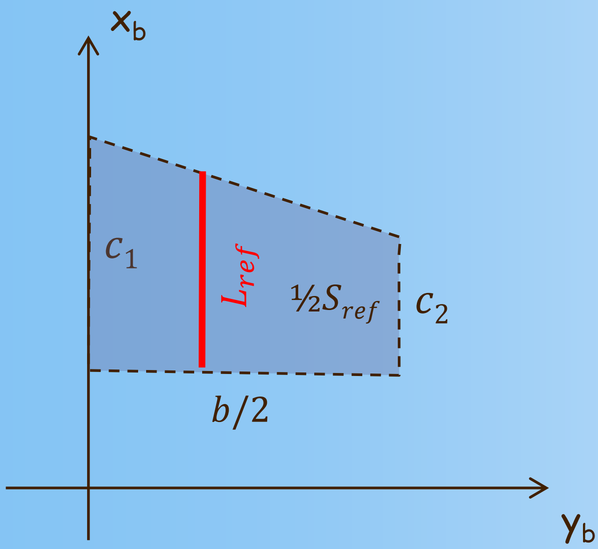
$$x_{ref} = \frac{2\int x(y) \cdot l(y) \cdot dy}{S_{ref}}$$

$$y_{ref} = \frac{2\int y \cdot l(y) \cdot dy}{S_{ref}}$$

The L_{ref} is usually called the Mean Aerodynamic Chord, MAC

This corresponds to a segment of line which is perfectly determined on the 3 views plan, function of the wing chord $l(y)$ variation wrt the y-span

S_{ref} / L_{ref} Definitions & Calculations



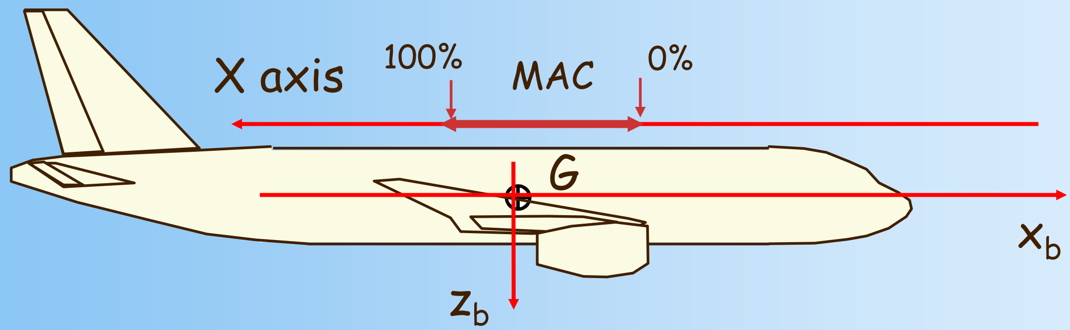
$$S_{ref} = b \cdot \frac{c_1 + c_2}{2}$$

$$L_{ref} = \frac{2c_1}{3} \cdot \frac{1 + \varepsilon + \varepsilon^2}{1 + \varepsilon}$$

$$y_{ref} = \frac{b}{6} \cdot \frac{1 + 2\varepsilon}{1 + \varepsilon}$$

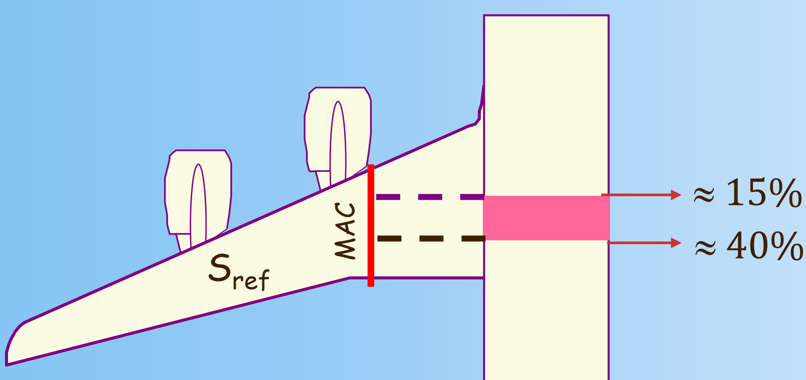
$$\varepsilon = \frac{c_2}{c_1} \quad = \text{taper ratio}$$

If the Lifting surface is a trapezoïdal, the L_{ref} / MAC is a real chord of the wing



For the longitudinal G position, we use the X axis based on MAC (positive aft)

$X/L = 0\% \,/100\%$ corresponds to MAC leading / trailing edge position



CG range is limited from

$X_G/L \approx 15\%$ to 40%

Determination of Aerodynamic Coefficients

SUPAERO

From the real aircraft, a scaled model is built and flown in Wind Tunnel. Then, measurements of Forces (N) and Moments (m.N) at a Reference point A are performed from which the different aerodynamic coefficients are computed.

$$C_F = \frac{F}{\frac{1}{2}\rho V^2 S_{ref}} \qquad C_M = \frac{M_A}{\frac{1}{2}\rho V^2 S_{ref} L_{ref}}$$

Of course, the Reference Surface S and Reference Length L are computed for the scaled model. Then, we can apply these aerodynamic coefficients for the true aircraft using the true associated Reference Surface and Length.

Remark : Aerodynamic force and moment are produced by the aircraft Velocity with respect to the surrounding air, so V is actually $V_{ac/air}$ and C_F and C_M are function of the orientation of $\vec{V}_{ac/air}$ wrt the aircraft , so function of α and β

Remark : The Reference Aerodynamic Point A is usually taken at 25% of the Reference Length

Principal Aerodynamic coefficients

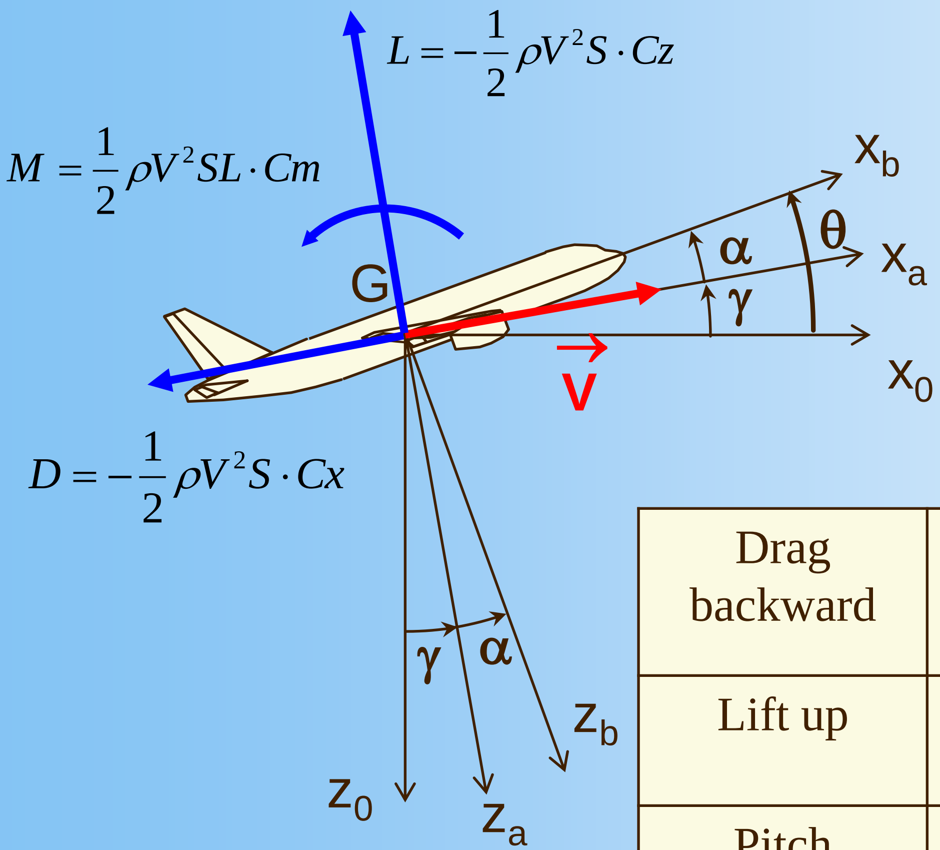
Aerodynamic forces & moments are defined relative to the Aerodynamic reference system $Gx_ay_az_a$ such that :

$$\vec{F} = \frac{1}{2}\rho V^2 \cdot S \cdot \left.\begin{matrix} \\ \\ \\ R_a \end{matrix}\right| \begin{matrix} -Cx \\ Cy_a \\ -Cz \end{matrix}$$

$$\overrightarrow{M_G} = \frac{1}{2}\rho V^2 \cdot SL \cdot \left.\begin{matrix} \\ \\ \\ R_a \end{matrix}\right| \begin{matrix} Cl_G^a \\ Cm_G^a \\ Cn_G^a \end{matrix}$$

The principal aerodynamic coefficients used are Cx & Cz which are the drag & lift coefficients, respectively

Pure Longitudinal Flight



Drag backward	D < 0	Cx > 0
Lift up	L < 0	Cz > 0
Pitch moment up	$M_G > 0$	$Cm_G > 0$

Principal Aerodynamic coefficients

Aerodynamic forces & moments are defined relative to the aircraft reference system $Gx_by_bz_b$ such that :

$$\vec{F} = \frac{1}{2}\rho V^2 \cdot S \cdot \left|_{R_b} \begin{matrix} -Cx_b \\ Cy \\ -Cz_b \end{matrix} \right.$$

$$\overrightarrow{M_G} = \frac{1}{2}\rho V^2 \cdot SL \cdot \left|_{R_b} \begin{matrix} Cl_G \\ Cm_G \\ Cn_G \end{matrix} \right.$$

The principal aerodynamic coefficients used are Cy as lateral aerodynamic force coefficient and Cl_G,Cm_G,Cn_G as aerodynamic moment coefficients

Aerodynamic Coefficients

The Aerodynamic coefficients (Cx, Cy, Cz) and (Cl, Cm, Cn) depend on :

- Mach and Reynolds number
- Aircraft orientation with respect to $\vec{V}_{ac/air}$: α , β
- Aircraft elementary rotations : p , q , r
- Control surfaces deflections : δl , δm , δn

We assume that the Aerodynamic coefficients can be linearized (except for the drag coefficient Cx)

This assumption is valid for a certain range of (α, β), which covers already a wide range of the Flight Domain

Aerodynamic Coefficients : Cz , Cm

For pure Longitudinal Flight, (Cz, Cm_G) function of $(\alpha, q, \delta m)$

$$Cz = Cz_\alpha \cdot (\alpha - \alpha_0) + Cz_q \cdot \frac{qL}{V} + Cz_{\delta m} \cdot \delta m$$

$$Cm_G = Cm_0 + Cm_\alpha^G \cdot (\alpha - \alpha_0) + Cm_q \cdot \frac{qL}{V} + Cm_{\delta m} \cdot \delta m$$

Each aero coefficient has no dimension, but also each gradient

This explains the reason of the quantity qL/V :

- The gradient Cz_q has no dimension
- The quantity qL/V has no dimension
- The product $Cz_q \cdot qL/V$ has no dimension

Aerodynamic Coefficient : Cx

Friction / Form drag — Induced drag — Wave drag

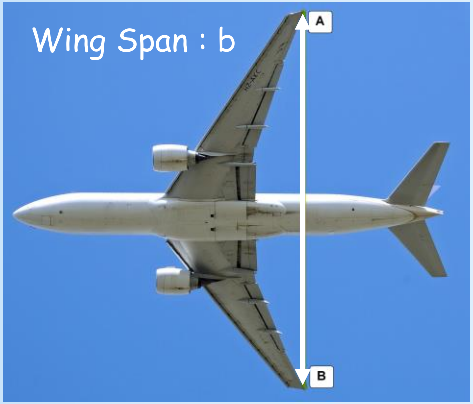
$$Cx = Cx_0 + k_i \cdot Cz^2 + Cx_w$$

$$k_i = \frac{1}{\pi \cdot \lambda \cdot e}$$

Oswald coefficient : $0 \leq e \leq 1$

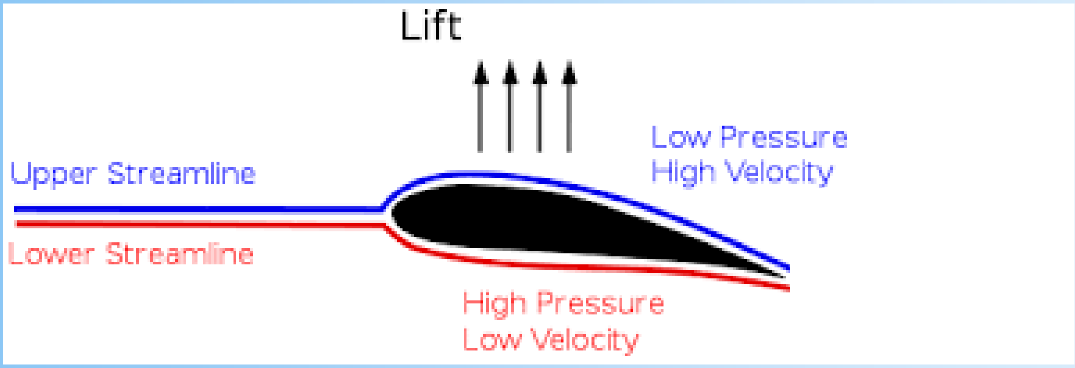
$$\lambda = \frac{b^2}{S}$$

aspect ratio



The coefficient e is function of the lift distribution along the span : $e < 1$ except for the elliptical spanwise lift distribution (Prandtl Linearised Theory) where $e = 1$

Why the wing produces lift ?

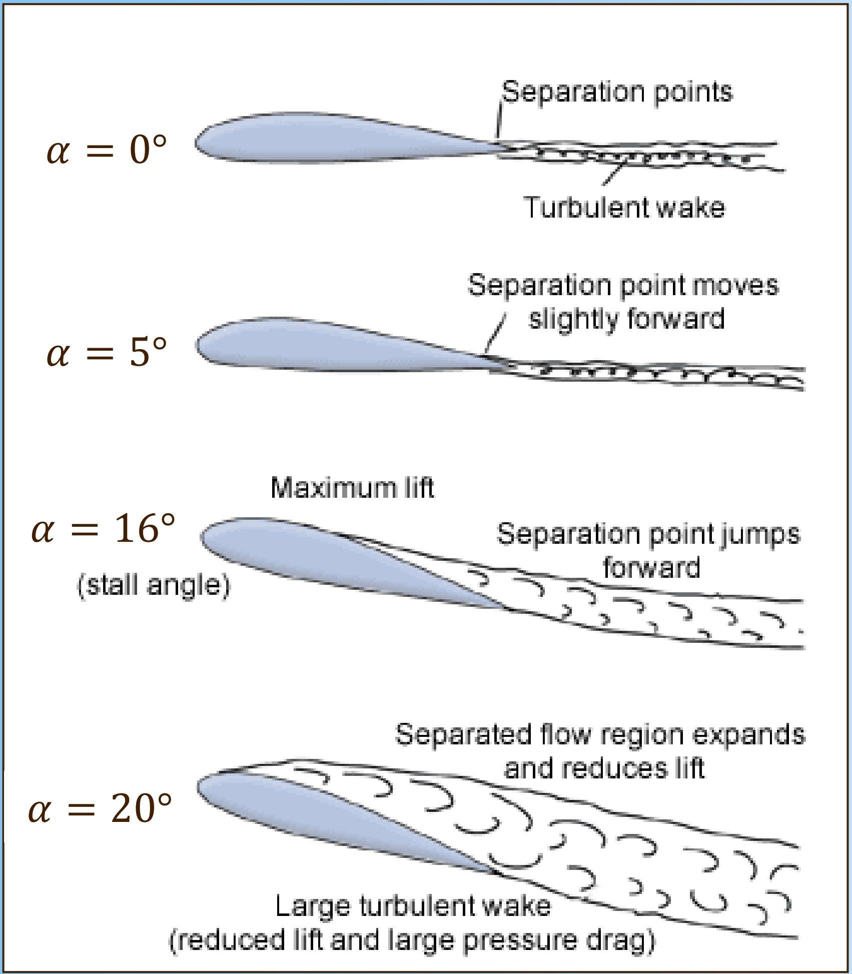


When the wing is animated with a velocity $\vec{V}_{ac/air}$, both upper/lower streamlines splits at the leading edge. Due to the profile shape of the wing, they have a different behaviour :

- Below the wing, the velocity decreases along the streamline resulting in a air pressure increase
- Over the wing, the velocity increases along the streamline resulting in a air pressure decrease

This pressure differential across both sides of the wing produces the Lift force

Why the wing produces lift ?



This phenomenon is amplified with the increase of the angle of attack, α

→ Cz increases when α increases

up to a certain point …

The upper streamline has more and more difficulties to follow the wing profile : a flow separation occurs which expands all over the wing

- The pressure recovers the p_∞ value, the pressure differential process ceases

- Stall is achieved with a Lift decrease

Wave Drag : Cx_w

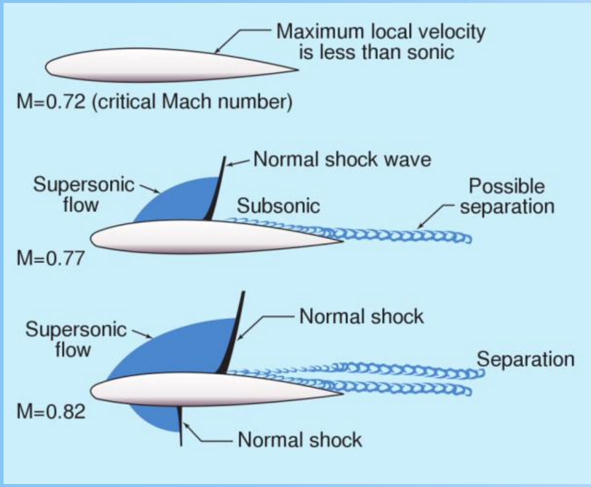
When the Mach increases, the acceleration of the flowfield (particularly at extrados) creates a supersonic region terminated by a re-compression shock.

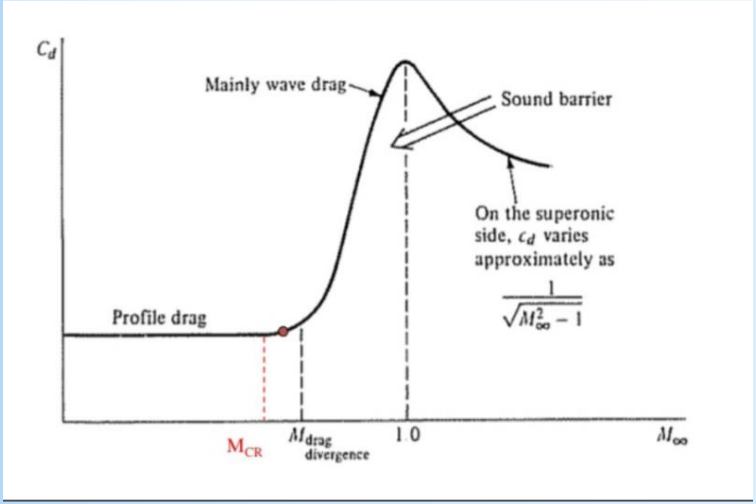
This creates a separation flowfield behind associated to a strong drag increase.

We define the Mach M_{div} when the drag increase achieves 20 drag count (*)

Obviously, the Cruise Mach will be before M_{div}

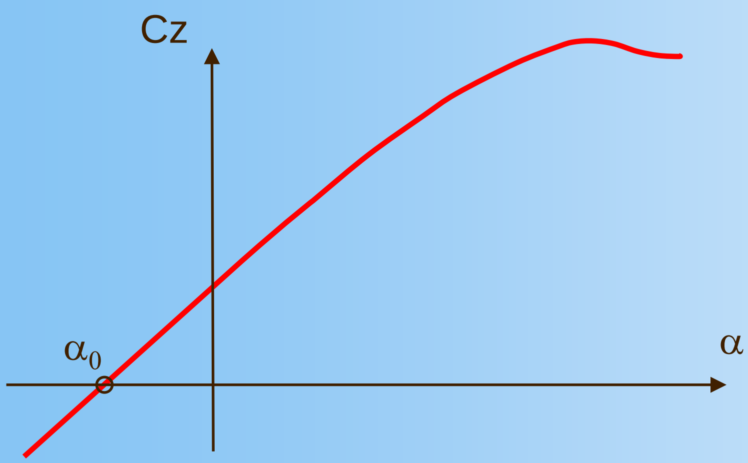
For the present course, we will assume the Cx_w is negligible





$(*)\ \Delta Cx = 20\ dc = 0{,}0020$

Physics of Lift surface : α effect



$$Cz = Cz_\alpha \cdot (\alpha - \alpha_0)$$

$\boxed{Cz_\alpha \text{ is positive}}$

The Lift coefficient Cz can be considered as a linear function versus α until Stall phenomenon appears . The Lift gradient Cz_α is mainly a function of the Wing aspect ratio $\lambda = b^2/S$

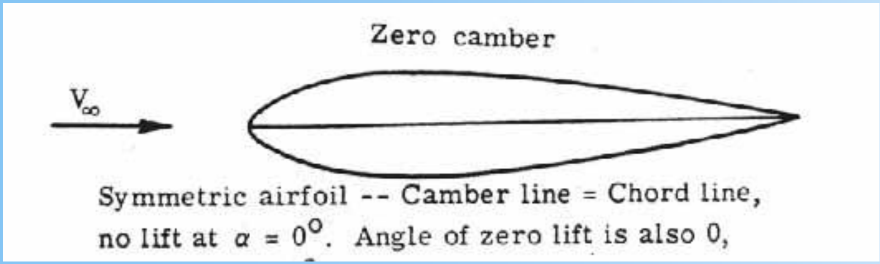
$$Cz_\alpha = \frac{2\pi\lambda}{2 + \sqrt{4 + (1 + \tan^2\varphi_{50} - M^2)\lambda^2}}$$

Diederich formula

$$Cz_\alpha \rightarrow \frac{2\pi}{\sqrt{(1 + \tan^2\varphi_{50} - M^2)}} \approx 2\pi$$

when $\lambda \rightarrow \infty$

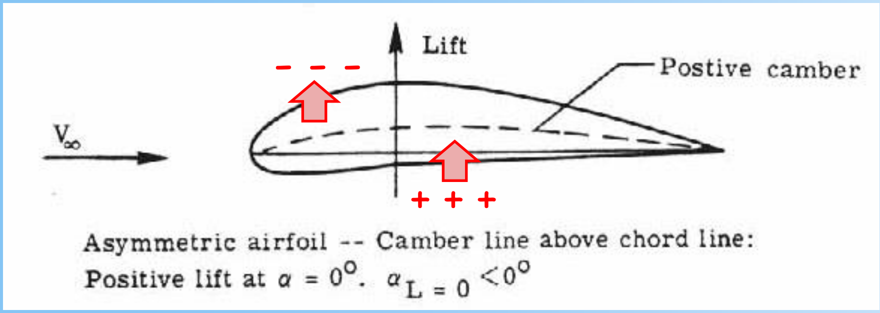
Physics of Lift surface : camber effect



no camber

$Cz(\alpha = 0°) = 0$

$\Rightarrow \alpha_0 = 0°$



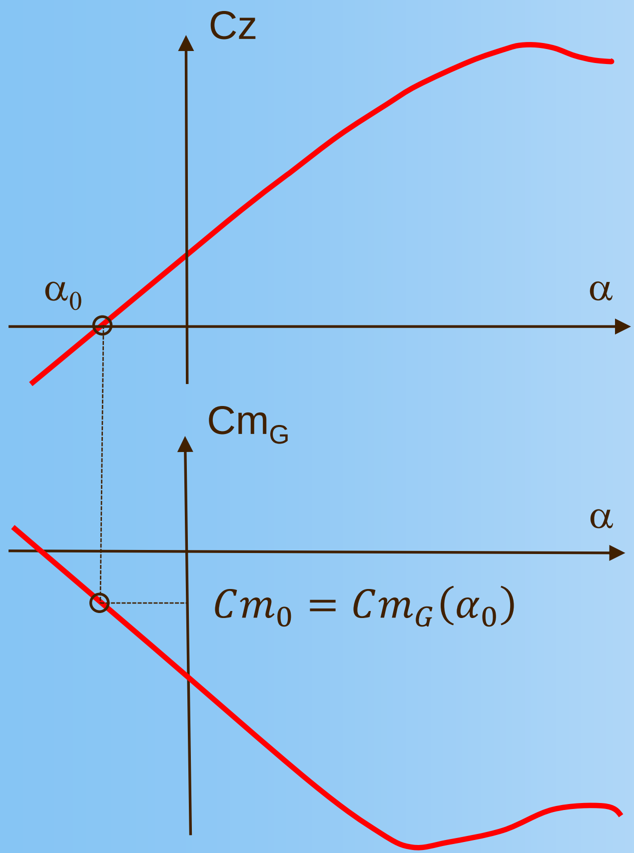
positive camber

$Cz(\alpha = 0°) > 0$

$\Rightarrow \alpha_0 < 0°$

The zero lift angle α_0 is driven by the camber of the profile : the more the wing is (positive) cambered, the more α_0 is negative

Physics of Lift surface : α effect



$$Cz = Cz_\alpha \cdot (\alpha - \alpha_0)$$

Cz_α is positive

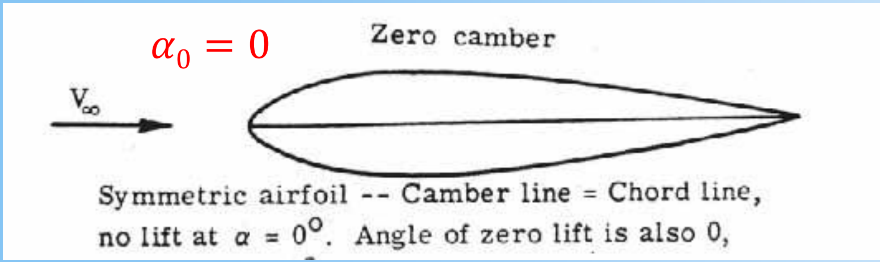
$$Cm_G = Cm_0 + Cm_\alpha^G \cdot (\alpha - \alpha_0)$$

Cm_α^G is negative

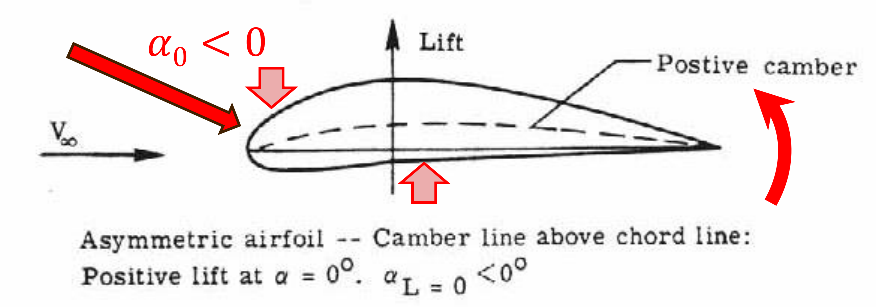
The pitch coefficient Cm can be also considered as linear function versus α

The Lift gradient Cm_α is assumed to be negative (refers to Stability)

Physics of Lift surface : camber effect



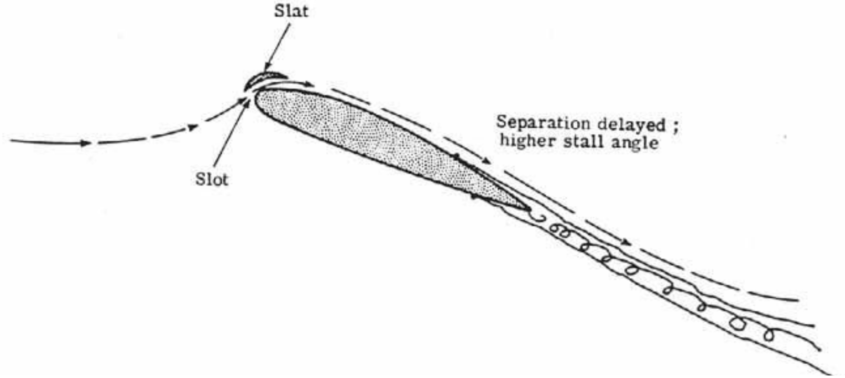
no camber
$Cm(\alpha_0) = Cm_0 = 0$

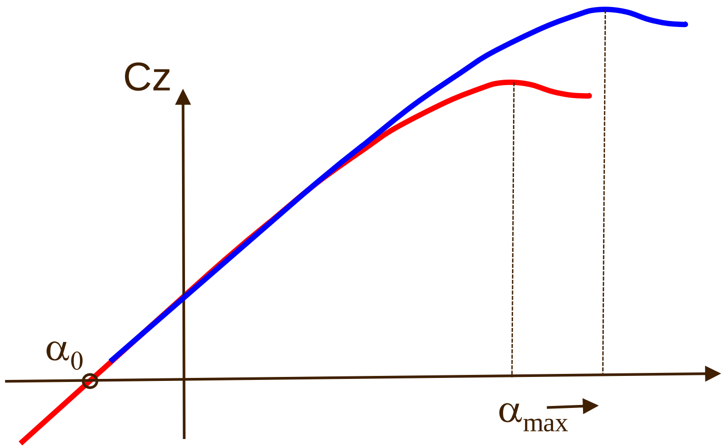


positive camber
$Cm(\alpha_0) = Cm_0 < 0$

The $Cm_0 = Cm_G(\alpha_0)$ pitch coefficient is also driven by the camber of the profile : the more the wing is (positive) cambered, the more Cm_0 is negative

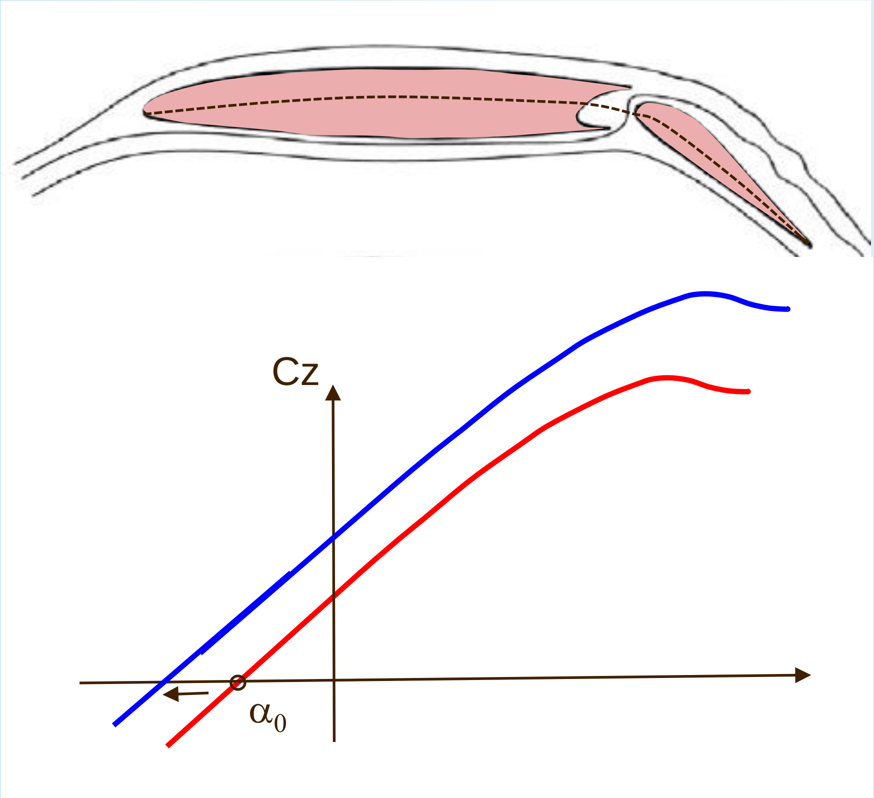
Physics of Lift surface : slat effect





The slot behind the extended slat lets pass more fast "fresh" moving air, speeding up the flow over the wing : the streamline can re-attach to the wing profile.
The stall process is delayed

Physics of Lift surface : flap effect



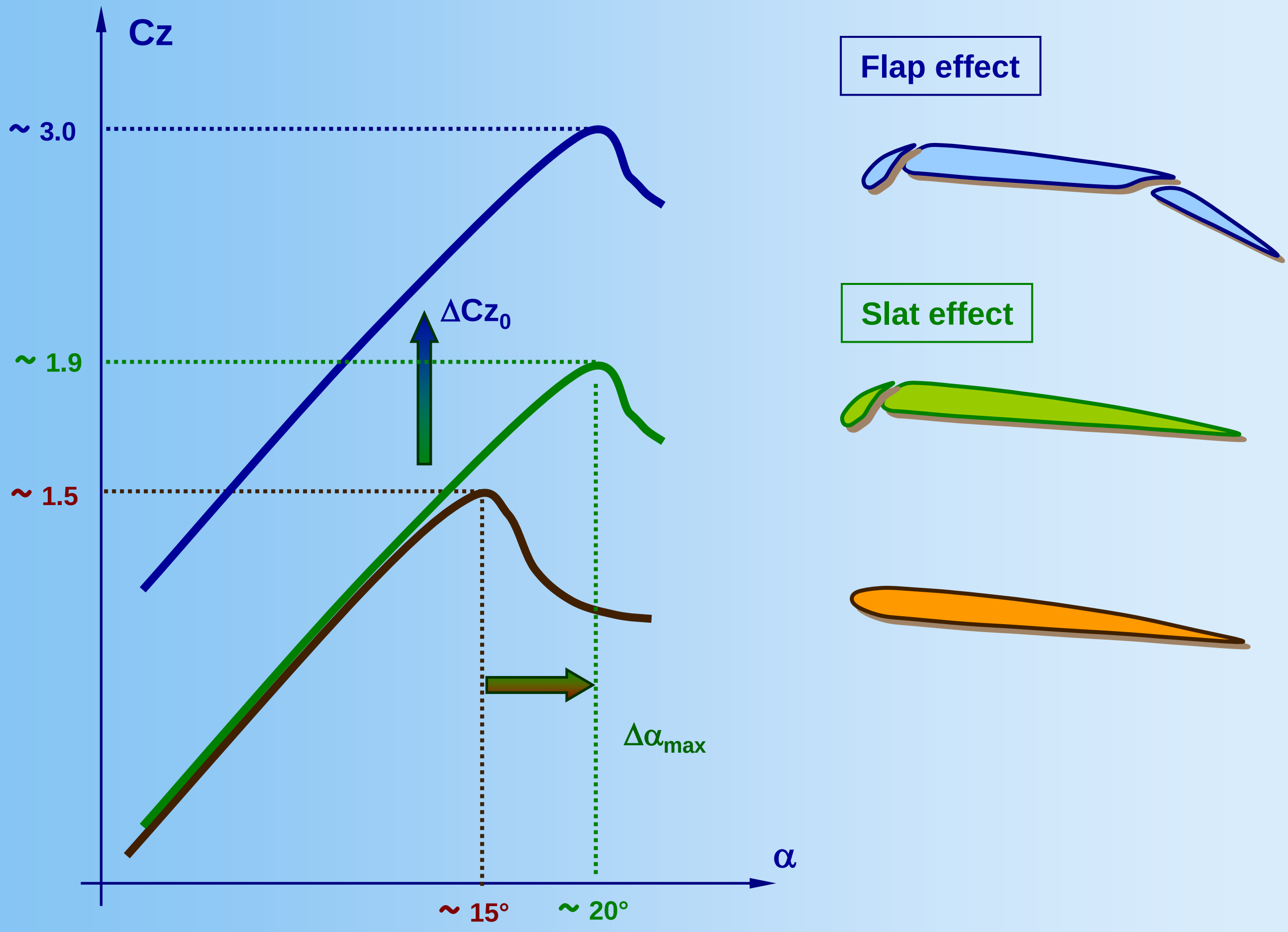
The flap deflection increases the (positive) camber of the profile : α_0 becomes more negative , the lift curve is translated up.
Notice that there is also a slot effect with a (small) delay of the stall

High Lift Systems example

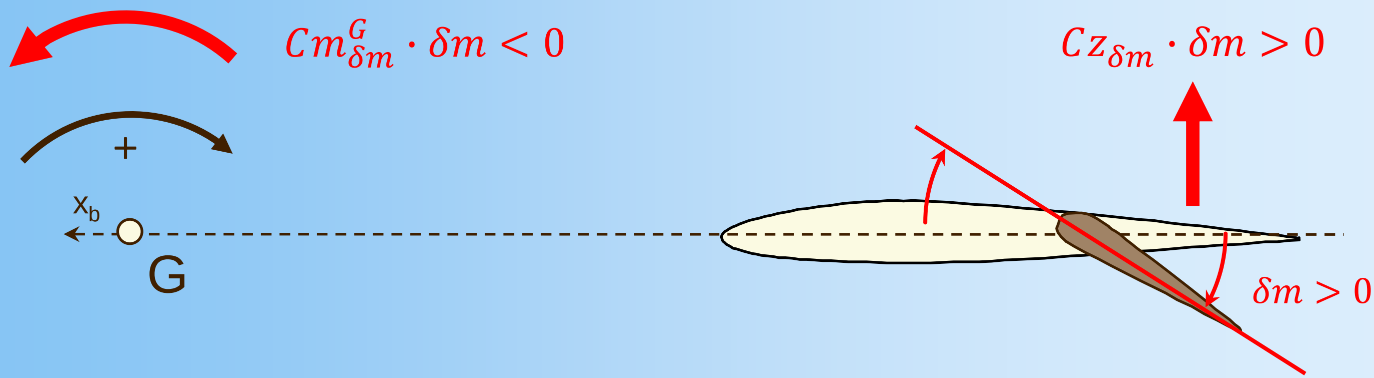
Single Slotted Flap

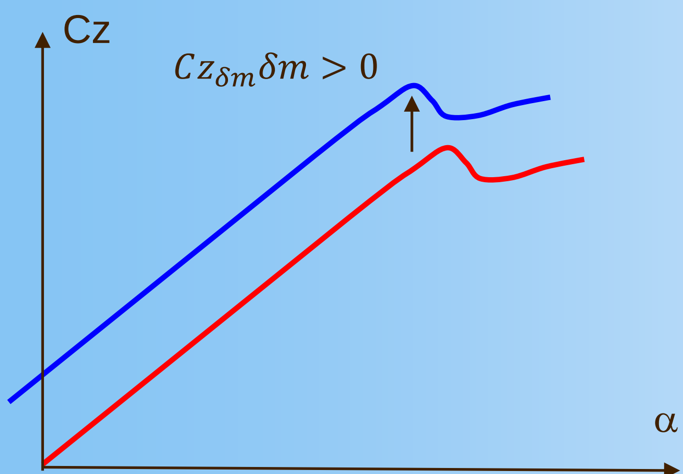
Double Slotted Flap

High Lift Physics synthesis



Physics of Lift surface : δm effect





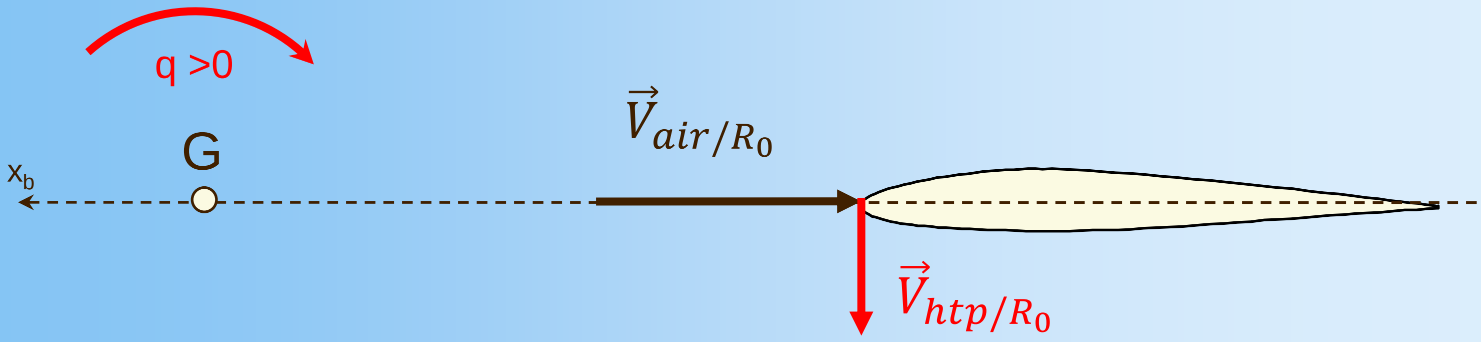
$$Cz = Cz_{\alpha} \cdot (\alpha - \alpha_0) + Cz_{\delta m} \cdot \delta m$$

$Cz_{\delta m}$ is positive

The elevator deflection produces the same result as a flap deflection with a global translation of the lift curve

$Cm_{\delta m}$ is negative

Pitch rate on empennage : Cz_q & Cm_q

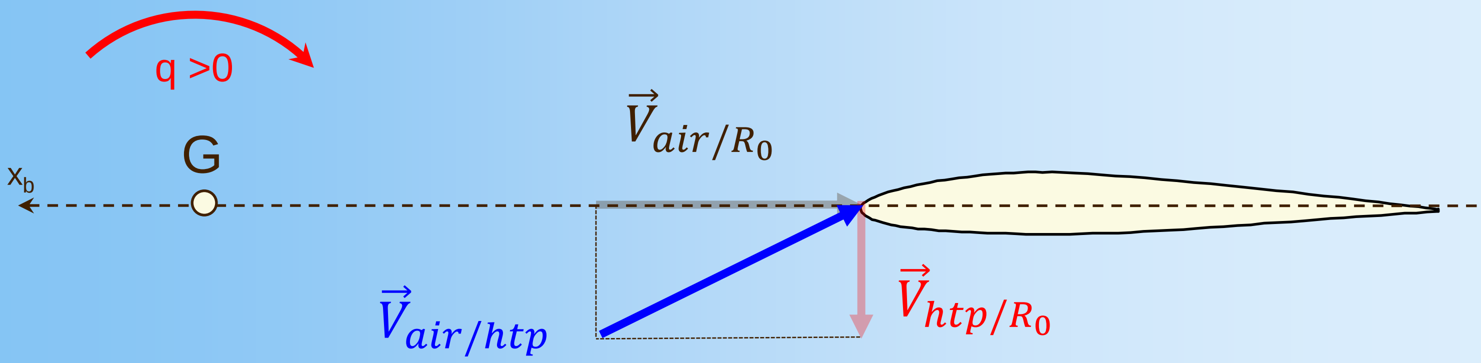


The Aircraft is rotating around G : an observer (fixed on R_0) sees :

- the air striking the empennage with a velocity $\vec{V}_{air/R_0}$
- the empennage moving down with a velocity $\vec{V}_{htp/R_0}$

htp for Horizontal Tail Plane

Pitch rate on empennage : Cz_q & Cm_q

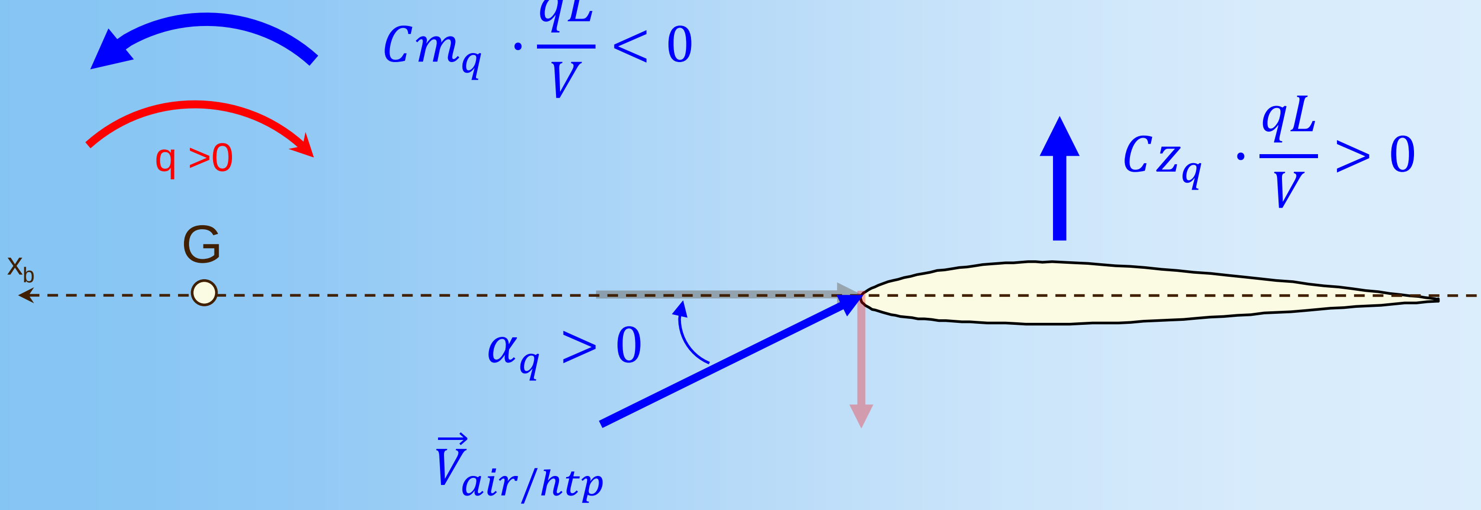


The Aircraft is rotating around G : an observer (fixed on the empennage) sees the air striking the empennage with a velocity $\vec{V}_{air/htp}$ such that

$$\vec{V}_{air/R_0} = \vec{V}_{air/htp} + \vec{V}_{htp/R_0}$$

The observer, fixed on the empennage, sees a relative velocity given by the speed composition

Pitch rate on empennage : Cz_q & Cm_q

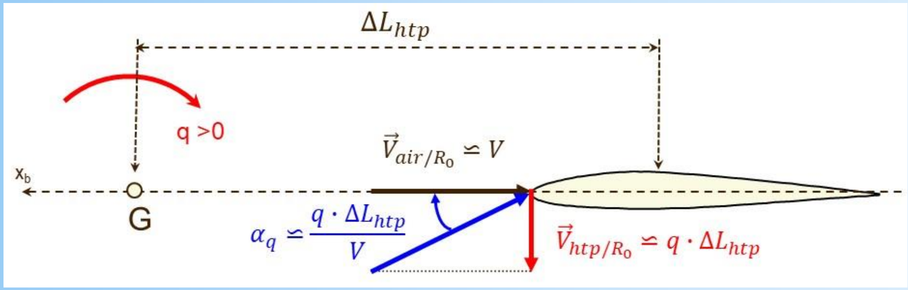


The Aircraft is rotating around G : the observer (fixed on the empennage) sees the air striking the empennage with an angle of attack $\alpha_q > 0$
The empennage, as for any lifting surface, reacts by producing a lift force which gives a pitching moment at G

Notice that the q pitch rotation creates a pitch moment opposite to the q rotation
This phenomenon is called a damping process …

Pitch rate on empennage : Cz_q & Cm_q

NOT in PROGRAM



The HTP reacts with a Lift Force : $S_{htp} \cdot Cz_\alpha^{htp} \alpha_q = S_{htp} \cdot Cz_\alpha^{htp} \cdot \dfrac{q\Delta L_{htp}}{V}$

We identify this moment to : $\boxed{S_{htp} \cdot Cz_\alpha^{htp} \cdot \dfrac{q\Delta L_{htp}}{V} = S \cdot Cz_q \cdot \dfrac{qL}{V} \Rightarrow Cz_q = Cz_\alpha^{htp} \cdot \dfrac{S_{htp}\Delta L_{htp}}{SL}}$

This Lift Force creates a pitch moment at G : $-S_{htp} \cdot Cz_\alpha^{htp} \alpha_q \cdot \Delta L_{htp} = -S_{htp}\Delta L_{htp} \cdot Cz_\alpha^{htp} \cdot \dfrac{q\Delta L_{htp}}{V}$

We identify this moment to :

$$\boxed{-S_{htp}\Delta L_{htp} \cdot Cz_\alpha^{htp} \cdot \frac{q\Delta L_{htp}}{V} = SL \cdot Cm_q \cdot \frac{qL}{V} \Rightarrow Cm_q = -Cz_\alpha^{htp} \cdot \frac{S_{htp}\Delta L_{htp}^2}{SL^2}}$$

Longitudinal aerodynamic coefficients

	Positive when …	Cz	Cm
Angle of attack α (°)	when flow field from downward	$Cz_{\alpha} > 0$	$Cm_{\alpha} < 0$ longitudinal stability
Elevator deflection δm (°)	Elevator trailing edge down	$Cz_{\delta m} > 0$	$Cm_{\delta m} < 0$
Pitch angular rate q (rd/s)	according to pitch-up rotation	$Cz_{q} > 0$	$Cm_{q} < 0$ pitch damping