Representation and Analysis of Dynamic Systems
Lab 1

Exercise 1: basic properties of the Laplace transform

1.1 Given a signal u(t) and its Laplace transform U(s) write the delay theorem, the final value theorem, the initial value theorem, the derivation theorem and the integration theorem.

1.2 Give the Laplace transform of the three basic signals $u(t)$: impulse, unit step and exponential $u(t) = e^{-\frac{t}{\tau}}$

1.3 Compute the Laplace transform of $u(t) = \cos(\omega t)$ (hint : express cos as the sum of two exponentials)

Exercise 2: basic exercises with the Laplace transform

A system with input $u(t)$ and output $y(t)$ (Laplace transforms $U(s)$ and $Y(s)$) is known by a simple differential equation:

$$a\frac{dy(t)}{dt} + y(t) = u(t)$$

Initial condition: $y(0) = 0$

2.1 What is the unit of a? For the following take $a = 2$

2.3 Compute the transfer function $F(s) = \frac{Y(s)}{U(s)}$ and create the transfer function F with Matlab

2.4 Compute the output $y(t)$ when the input is an impulse at $t = 0$. Compare with the impulse response obtained with Matlab (**impulse**)

2.5 Compute the output $y(t)$ when the input is unit step at $t = 0$. Compare with the step response obtained with Matlab (**step**)

Exercise 3: a bit more complicated with the Laplace transform

3.1 Compute the output $y(t)$ when the input $u(t)$ is an impulse for the following cases:

$$Y_1(s) = \frac{1}{s(1+2s)}U(s), Y_2(s) = \frac{1}{s(1-3s)}U(s), Y_3(s) = \frac{1}{(1+10s)(1+s)}U(s), Y_4(s) = \frac{1+2s}{(1+10s)(1+s)}U(s),$$
$$Y_5(s) = \frac{1-2s}{s(1+10s)}U(s)$$

Hint: decompose the rational fractions as a sum of simple ones

3.2 Plot $y(t)$ using Matlab

3.3 Compute the output $y(t)$ when the input $u(t)$ is unit step for the following case:

$$Y_1(s) = \frac{1}{1+2s}U(s),$$

Exercise 4: bloc-diagram manipulation

A system is described by a bloc diagram as in figure 1. This system as two inputs (reference $r(t)$, and perturbation $p(t)$) and one output $y(t)$.

4.1 Find $F_1(s)$, $F_2(s)$, $F_3(s)$, and $F_4(s)$ such as:

$$Y(s) = F_1(s)R(s) + F_2P(s)$$
$$U(s) = F_3(s)R(s) + F_4P(s)$$

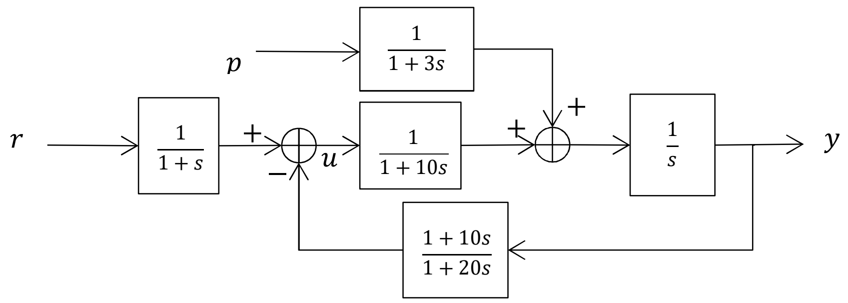

Figure 1

4.2 Is the system stable?

4.2 Plot $u(t)$ and $y(t)$ when the input $r(t)$ is a unitary step. Idem when the perturbation $p(t)$ is a unitary step. Find the final value using Matlab and by formal calculation (final value theorem).

Exercise 5: final and initial value theorem

We want to analyze the time response $y(t)$ of a system to a unitary step impulse. The transfer function is:

$$F(s) = \frac{1 - s}{(1 + 2s)(1 + 10s)}$$

1. What is the final value of $y(t)$?

2. Analyze the time response near $t = 0$

3. Plot the time response