Representation and Analysis of Dynamical Systems
Lab 6
Identification, Simulation & Control of a flexible structure

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Skills to develop:
- modeling of flexible structures
- simulation with state-space representation
- implementation of controllers in a closed-loop system

Objectives of the lab:
- perform tests on a flexible structure
- identify the main parameters of a flexible structure
- study different laws for vibration control

Please deposit a report on the LMS no more than 3 days the lab session.
One report per student.

Description of the context: Active vibration control

In many cases, vibrations are nuisances. They are sources of noise and damage of the devices. Vibration control consists in reducing the amplitude of the vibrations. Two families of solutions are possible for vibration control:
1. passive control of vibrations with materials which damp the vibrations (for example viscoelastic materials)
2. active control with actuators which are controlled to counteract these vibrations. This solution requires an energy source for the actuation of the actuators, but it may prove to be more effective in certain cases (more specifically in low frequencies).

The device you are going to use is a simple example of active vibration control. It is a simple free-clamped beam with:
♣ a piezoelectric actuator which can generate vibrations to counteract the disturbing vibrations and to obtain an overall reduction of the mechanical vibrations on the structure.
♣ a piezoelectric sensor to measure the vibrations of the structure.

The actuator and the sensor are positioned at the clamped extremity of the beam, where they are the most efficient.

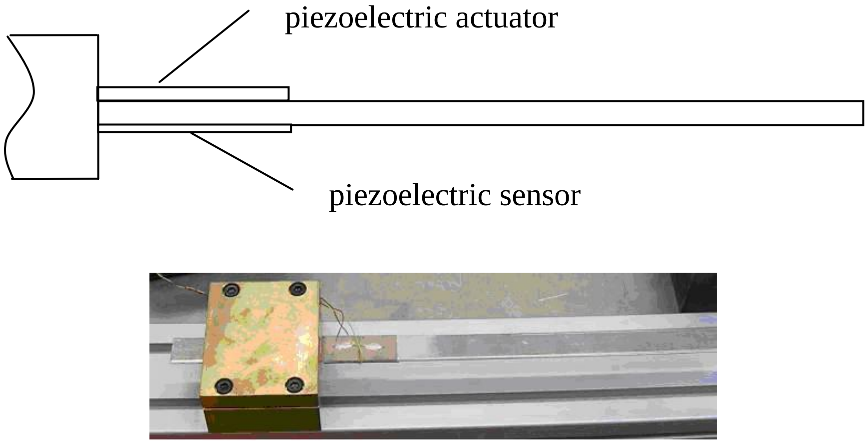


Figure 1 – Cantilever beam: simple example of active vibration control

The piezoelectric actuator is powered up to ±70 V via a voltage amplifier. The piezoelectric sensor sensitive to mechanical deformations generates a variation in electrical charges. In order to convert this signal into a voltage, which is a quantity more easily exploitable, we use a charge amplifier. The signal from the amplifier is an image of the vibrations in the beam. This signal can be processed in order to compute the control voltage of the actuator that can damp the vibrations of the structure.

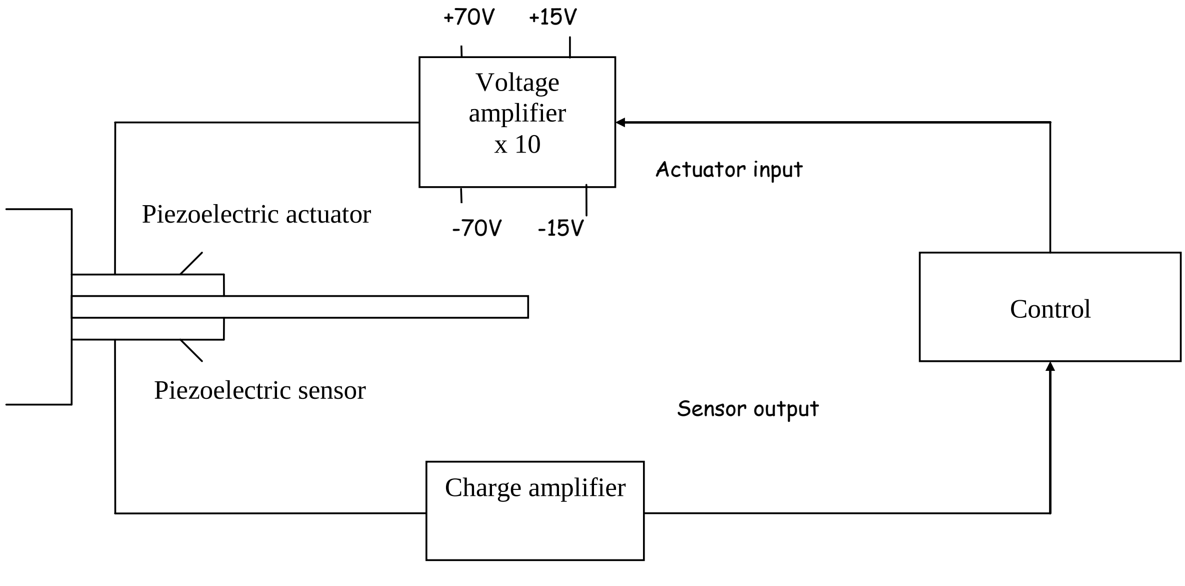


Figure 2 – Scheme of the active vibration control device

In this lab, we are using the device to model the flexible beam and the control is performed in simulation with Simulink.

Part 1: Identification of the flexible beam model (Measure)

Question 1: Method 1 - Frequential analysis

The objective of this question is to plot and study the Bode diagrams of the system around the **first mode of resonance**.

We are using the beam in this configuration:

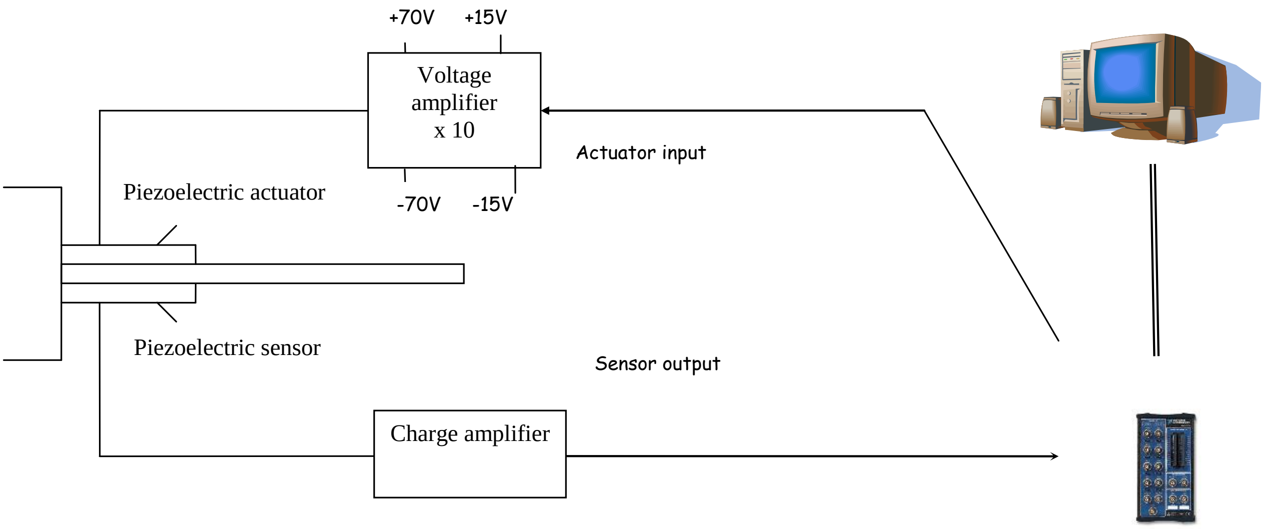


Figure 3 – Scheme for the frequential analysis

Note :

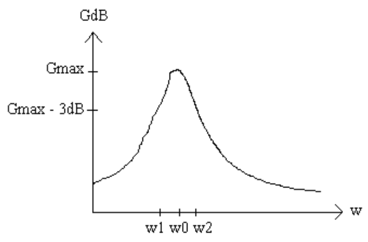
- *The charge amplifier is supplied by an external battery. Please switch it off between 2 measurements.*
- *Set-up of the charge amplifier:*
  - *Sensibility: 1.00*
  - *mV/unit out : 0.1*
  - *acc : 1m/s²*
  - *upper frequency limit: 1 KHz*

Load the Matlab code named « Frequencyanalysis» (available on the LMS).
Switch on the power supply and the charge amplifier.

Run the program to plot the Bode diagrams of the beam between 20Hz and 30Hz with a step of 0.5 Hz. Use the Bode diagrams to measure the first resonance mode.

Run the program again to plot the Bode diagrams of the beam around the first resonance mode ± 0.5Hz with a step of 0.05 Hz.

Use the last Bode diagrams to compute the damping ratio with the formula given in Figure 4.



Quality factor: Q = $\omega_0/(\omega_2-\omega_1)$
Damping ratio: $\varepsilon = 1/(2Q)$

Figure 4 – Measurement of the damping ratio using the frequency response

Question 2: Method 2 - Temporal analysis

The objective of this question is to plot the response of the system to a pull-up solicitation to excite the first resonance mode and to measure the damping ratio and the period of the first resonance mode.

We are using the beam in this configuration:

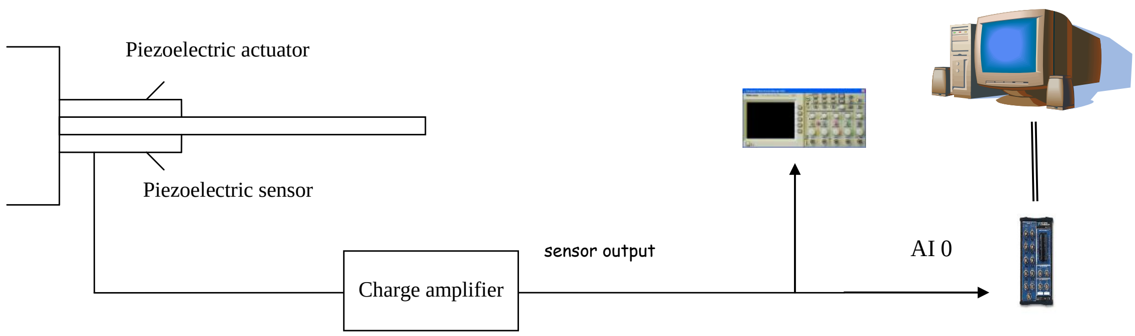


Figure 5 – Scheme for the temporal analysis

Load the Matlab code named « Timeanalysis» (available on the LMS).
Switch on the charge amplifier.
Run the program and straight after lauching the code, pull-up **gently** the beam.
After 10s, you will get the time response.
Use the time response to measure the first resonance frequency and the damping ratio using this formula:

$$\varepsilon = \frac{\ln\left(\frac{O_{k-1}}{O_k}\right)}{\sqrt{4\pi^2 + \ln^2\left(\frac{O_{k-1}}{O_k}\right)}}$$

where O_k is the overshoot of the k^{th} oscillation.

Question 3: analyze the previous results.

Part 2: Simulation of the flexible beam (Matlab-Simulink)

Open a Matlab script and write the model of the beam under the form of a second order system:

$$F(s) = \frac{X(s)}{U(s)} = \frac{K}{\frac{s^2}{\omega^2{}_{n1}} + \frac{2\xi_1}{\omega_{n1}}s + 1}$$

Consider K = 1e^{-4}.
For ω_{n1} and ξ_1, use the values we have computed in the part 1 of the lab.

Compute the state-space representation (A,B,C,D) of the flexible beam with $[x \quad \dot{x}]^T$ as state vector and x as output.

Open a Simulink file and do the diagram of Figure 6 with the state-space system that represents the flexible beam, K_a the amplifier gain and K_s the sensor gain, with K_a = 10 and K_s = 100.

Choose $[5e-3 \quad 0]$ for the initial conditions to simulate the pull-up test (initial position non null $x_0 = 5e-3$ and initial velocity null) and observe the output of the system. Compare the settling time of the simulated response with the measured settling time of measurements.

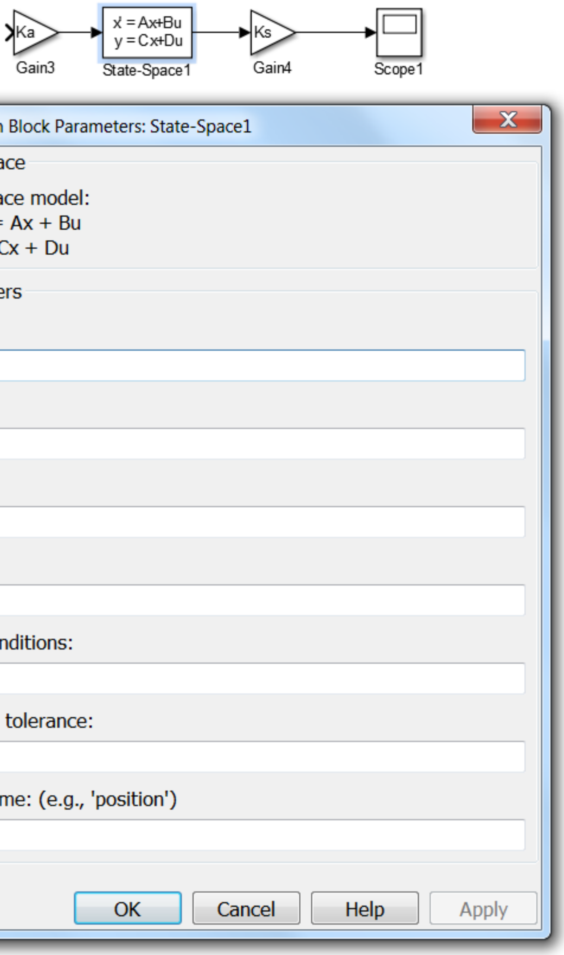


Figure 6 – Scheme for the open-loop analysis and simulation of the pull-up test

Choose the good parameters for the simulation:

- simulation ⇒ model configuration parameters (fixed-step and fixed step-size = 1e-4)

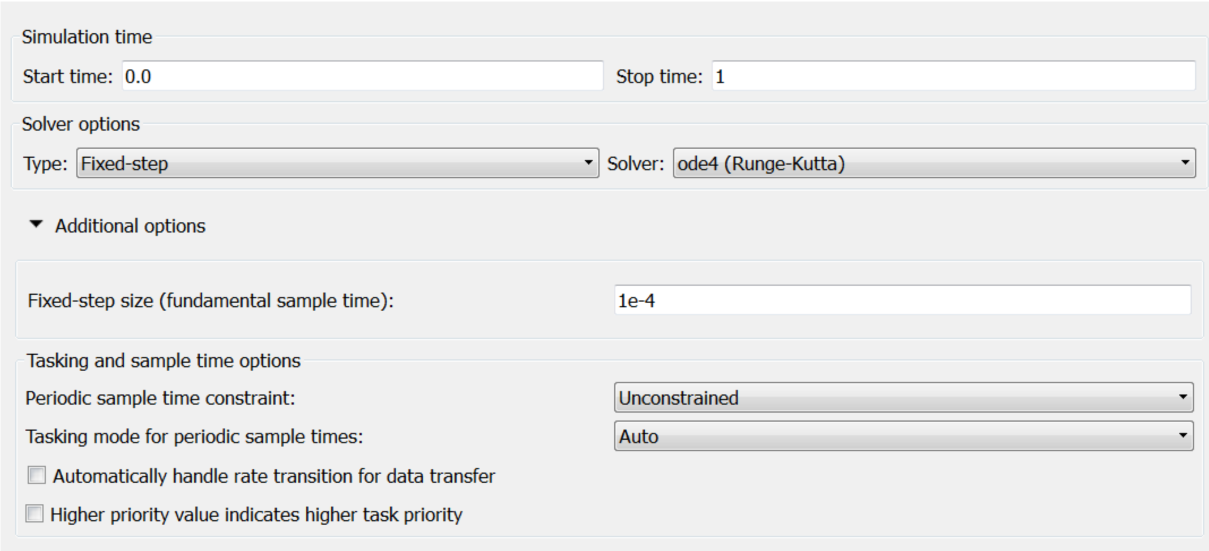


Figure 7 – Model configuration parameters

If you do not observe the whole signal, open the scope properties with the button and unselect the option 'Limit data points to last: 5000'.

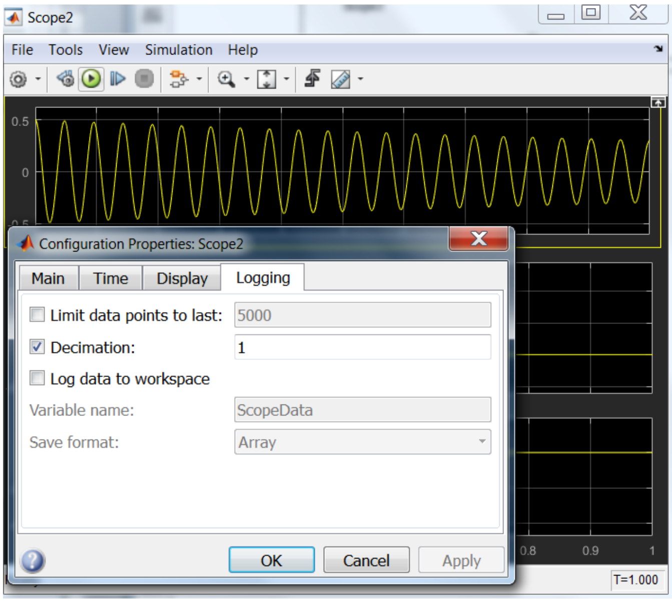


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Figure 8 – Scope properties

Part 3: Active vibration control (Simulation)

Case 1: Model with one mode

To reduce the vibrations, we are choosing a very simple control law called damping injection. This control law can be applied here because the actuator and the sensor are co-located (at the same position on the beam). It consists on a feedback on the velocity gain:

$$u = -K_a K_c K_s v$$

that will increase the damping coefficient of the closed loop system.

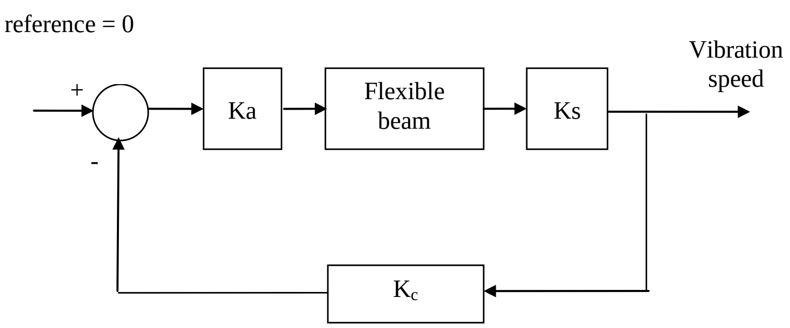

Figure 7 – Scheme for the vibration control: damping injection

Simulate the damping injection with Simulink. Be careful to the matrices C and D to observe the variables x and v. Choose the best value of the gain controller $\mathrm{K_c}$ to reduce vibrations without saturating the actuator (±70V max).

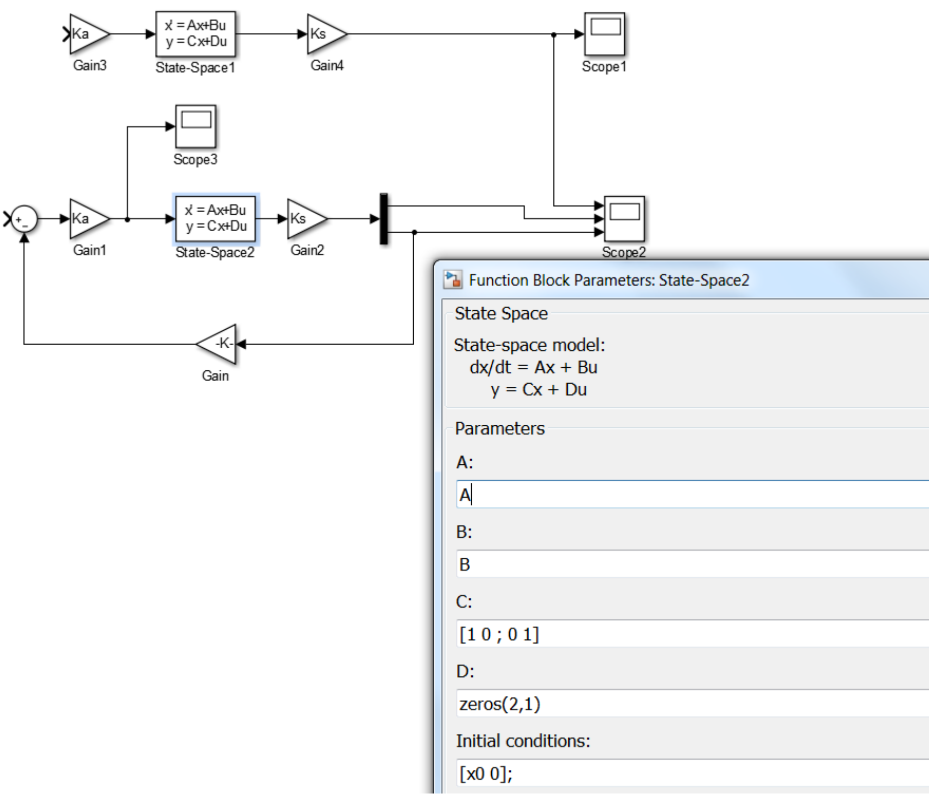

Figure 8 – Simulink diagram for the control by damping injection

<u>Case 2</u>: Model with two modes

We have only considered the first resonance mode of the beam but the beam has indeed an infinite number of modes.

Consider now the beam model with two modes such that:

$$F_2(s) = \frac{X(s)}{U(s)} = \frac{K}{\frac{s^2}{\omega^2{}_{n1}}+\frac{2\xi_1}{\omega_{n1}}s+1} \; \frac{1}{\frac{s^2}{\omega^2{}_{n2}}+\frac{2\xi_2}{\omega_{n2}}s+1}$$

with $\omega_{n2} = 2\pi 120$ and $\xi_2 = 0.05$.

Compute the state-space representation (A2,B2,C2,D2) that corresponds to $\frac{1}{\frac{s^2}{\omega^2{}_{n2}}+\frac{2\xi_2}{\omega_{n2}}s+1}$.

Add it in the Simulink (Figure 9).

Test your controller and analyze the results. If necessary, propose a new value of Kc.

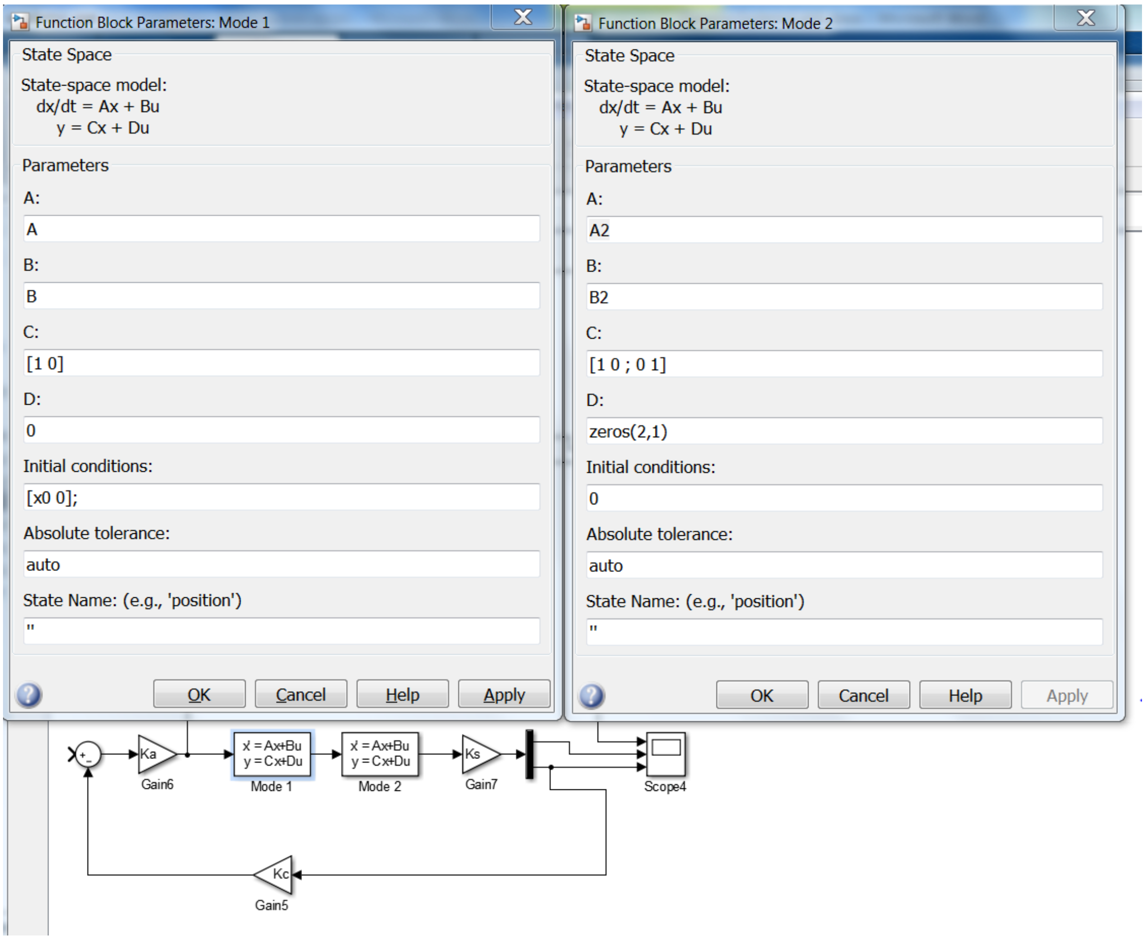

Figure 9 – Simulink diagram for the control by damping injection and beam with 2 modes

<u>Analysis</u>:

Analyze your results with one mode and two modes (using the root locus, Bode diagrams,…)