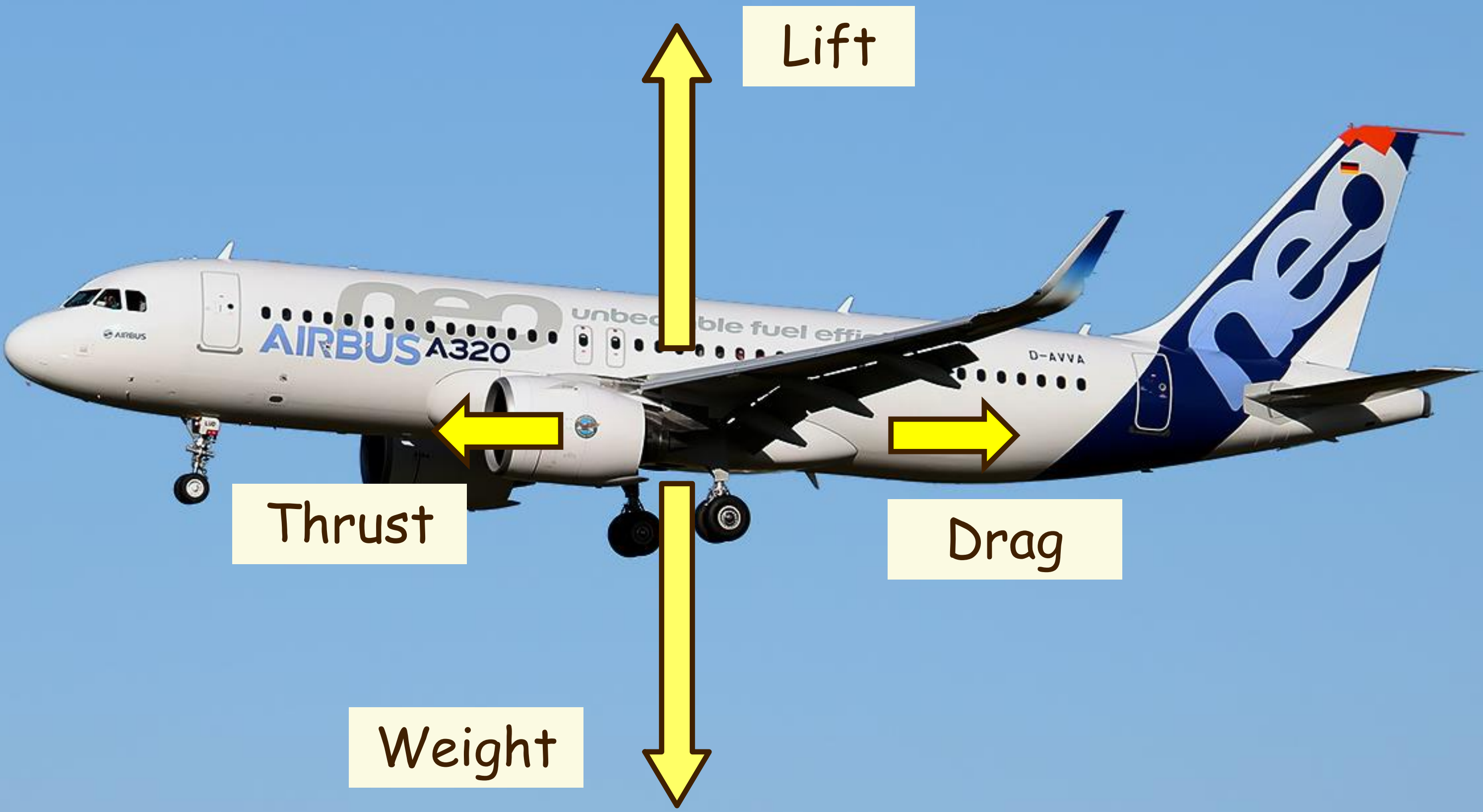
Introduction to Flight Mechanics

Basic Forces acting on aircraft



Basic Forces acting on aircraft

The Lift and Drag are Aerodynamic forces :

- they are created by the motion of the wing (aircraft velocity)
- within a fluid (air)

In cruise flight :

- The lift balances the weight
- The thrust (engine) balances the drag
- The flight variables are constant (velocity, altitude, …)
- We are in steady conditions

Flying Vehicles

Balloon

Helicopter

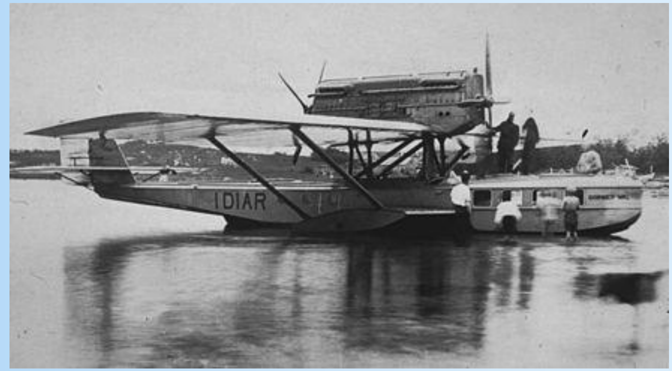
Missile

Rocket

Flying Vehicles

Fokker Dr.I Triplane

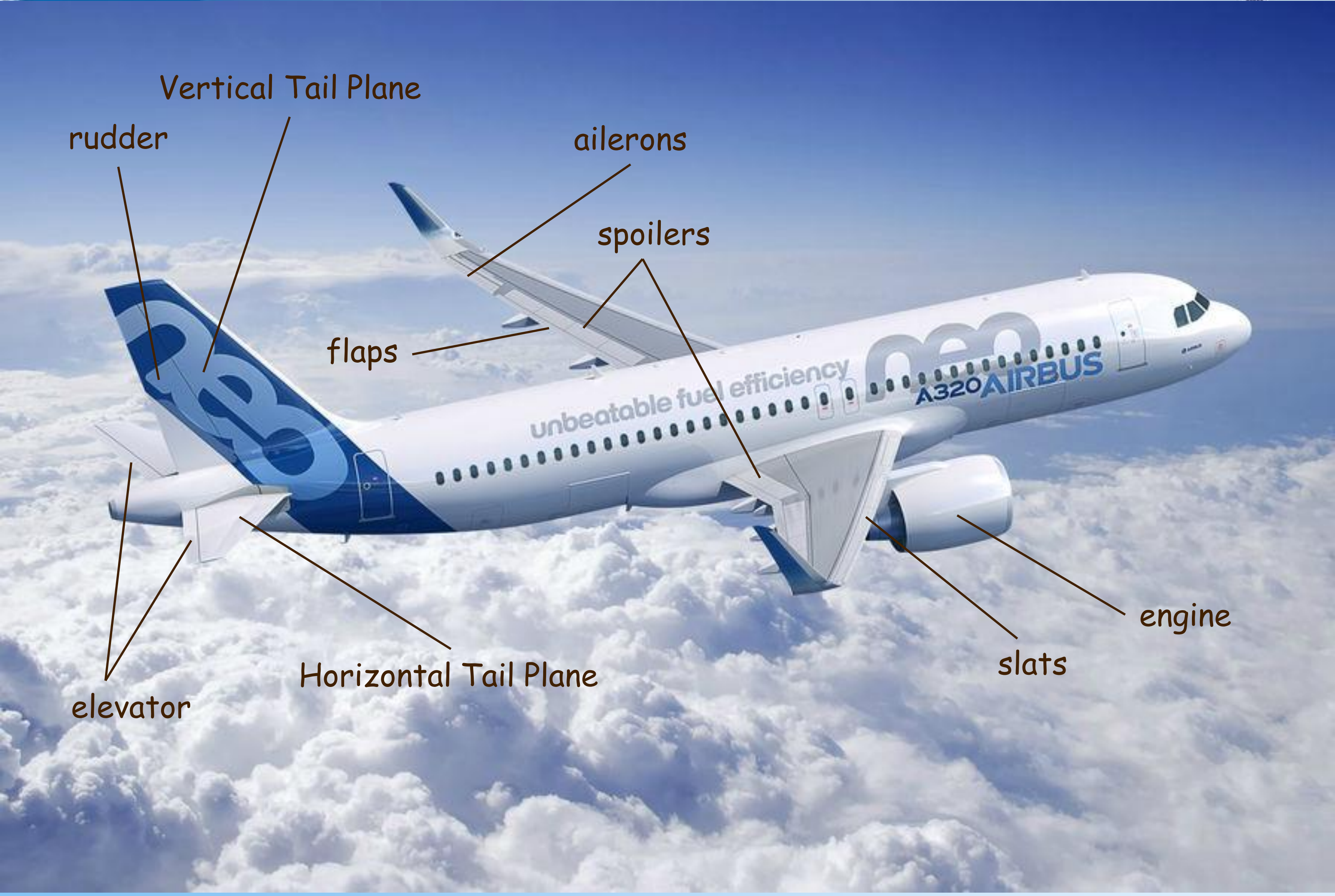
Dornier Wal Seaplane



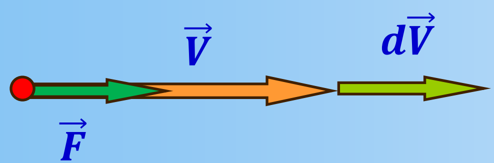
Northrop B2 Flying Wing

Saab 37 Viggen Canard

Aircraft : moveables and components

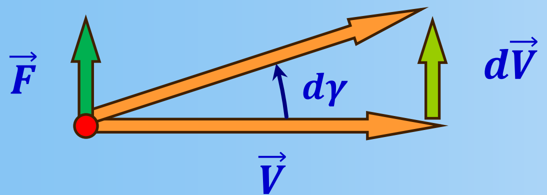


$$\vec{F} = m \cdot \frac{d\vec{V}_G}{dt}$$



Modification of the velocity module (acceleration / deceleration)

$$F = m \cdot \frac{dV}{dt}$$

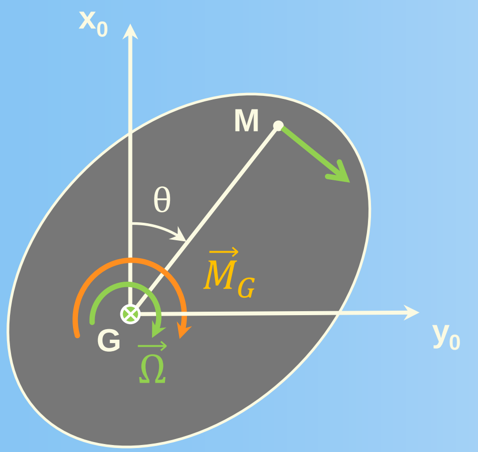


Modification of the velocity orientation (trajectory curvature modification)

$$F = m \cdot V \frac{d\gamma}{dt}$$

Movement around the Centre of Gravity

$$\vec{M}_G = [I_G] \cdot \frac{d\vec{\Omega}}{dt}$$



Modification of the Body orientation
(Body rotation around the CG)

$$\vec{\Omega} = \frac{d\theta}{dt} \cdot \vec{z}_0$$

<u>remark</u> : $\vec{\Omega}$ is a pseudo-vector
its orientation is linked to the Body rotation
(right hand rule)

Movement of / around the Centre of Gravity

The 1^{st} Newton Law says that :

- a Force applied to the CG modifies the velocity vector of the body

The 2^{nd} Newton Law says that :

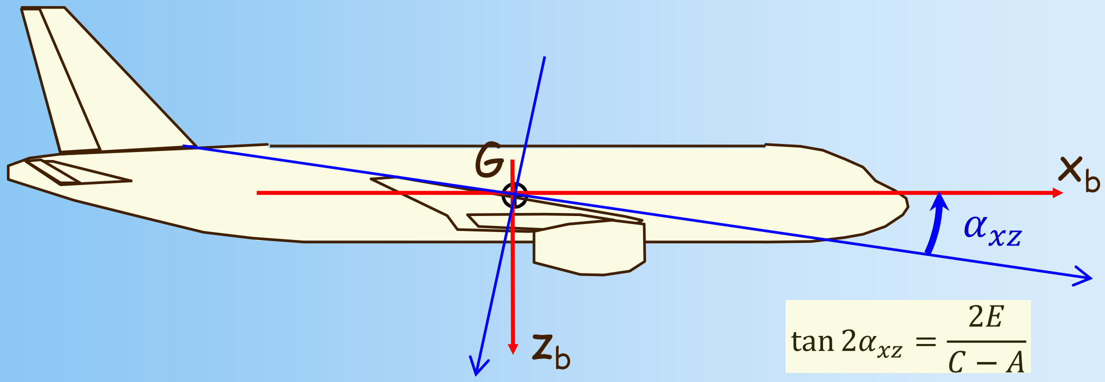
- a Moment applied to the CG modifies the orientation of the body

Try to see it also in the opposite way :

If I want to modify the velocity vector (trajectory control),
$\rightarrow$ I shall modify the Forces acting on the CG

If I want to modify the body orientation (aircraft control),
$\rightarrow$ I shall modify the Moments acting on the CG

Inertia Matrix : $[I_G]$



Assuming a plane of symmetry (x_b, z_b), the coefficients I_{xy} / I_{yz} vanish :

$$[I_G] = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} = \begin{bmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{bmatrix}$$

Inertia Matrix : $[I_G]$

The presence of the fin and engines produces a misalignment of the principal x-axis of inertia with the x_b axis ; this is responsible of the $E = I_{xz}$ product of Inertia

This term is generally small (compared to the main inertia) :
we will neglect it for the rest of the course

$$A = I_{xx} = \sum m \cdot (y^2 + z^2)$$

$$B = I_{yy} = \sum m \cdot (x^2 + z^2)$$

$$C = I_{zz} = \sum m \cdot (x^2 + y^2)$$

$$D = I_{xy} = \sum m \cdot xy = 0$$

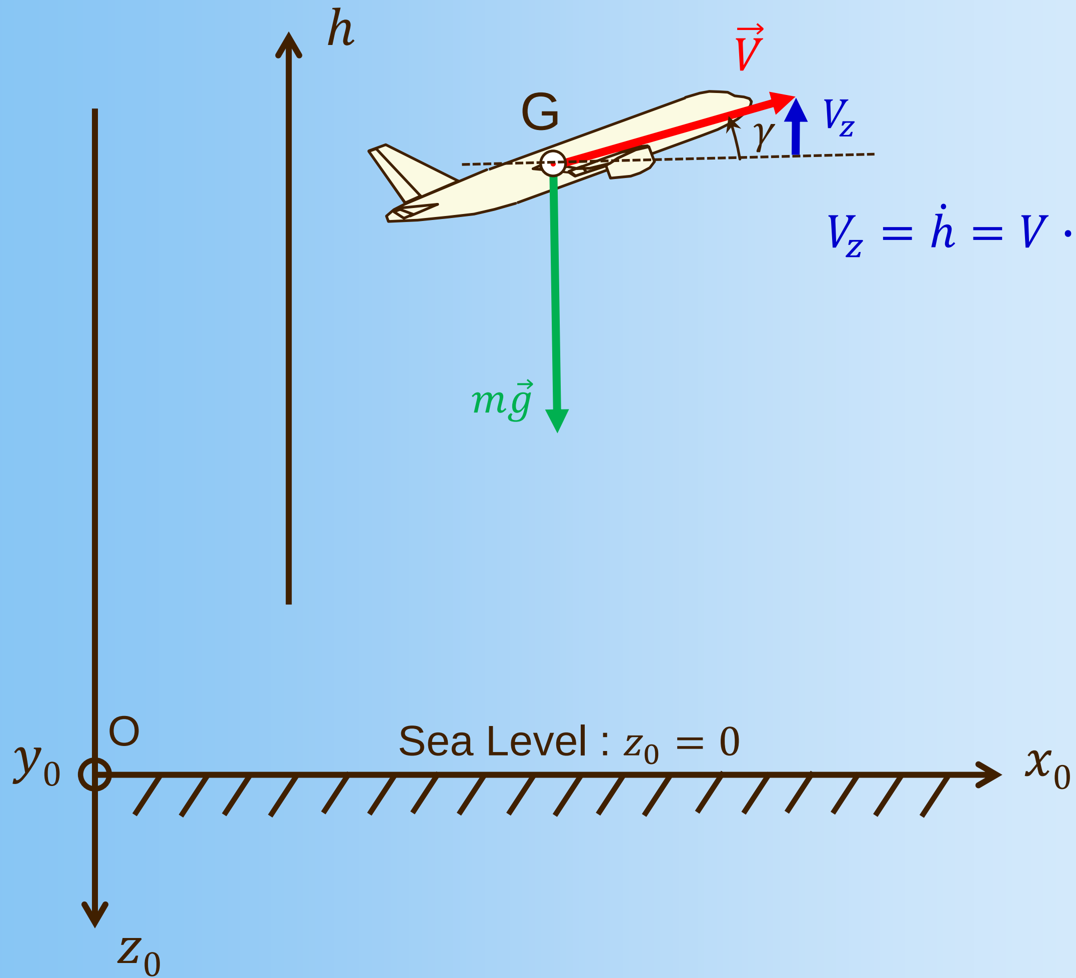
$$E = I_{xz} = \sum m \cdot xz$$

$$F = I_{yz} = \sum m \cdot yz = 0$$

Flight mechanics reference systems

Fairchild A-10 Thunderbolt

Earth reference system $Ox_0y_0z_0$



$$V_z = \dot{h} = V \cdot \sin\gamma$$

Earth reference system $Ox_0y_0z_0$

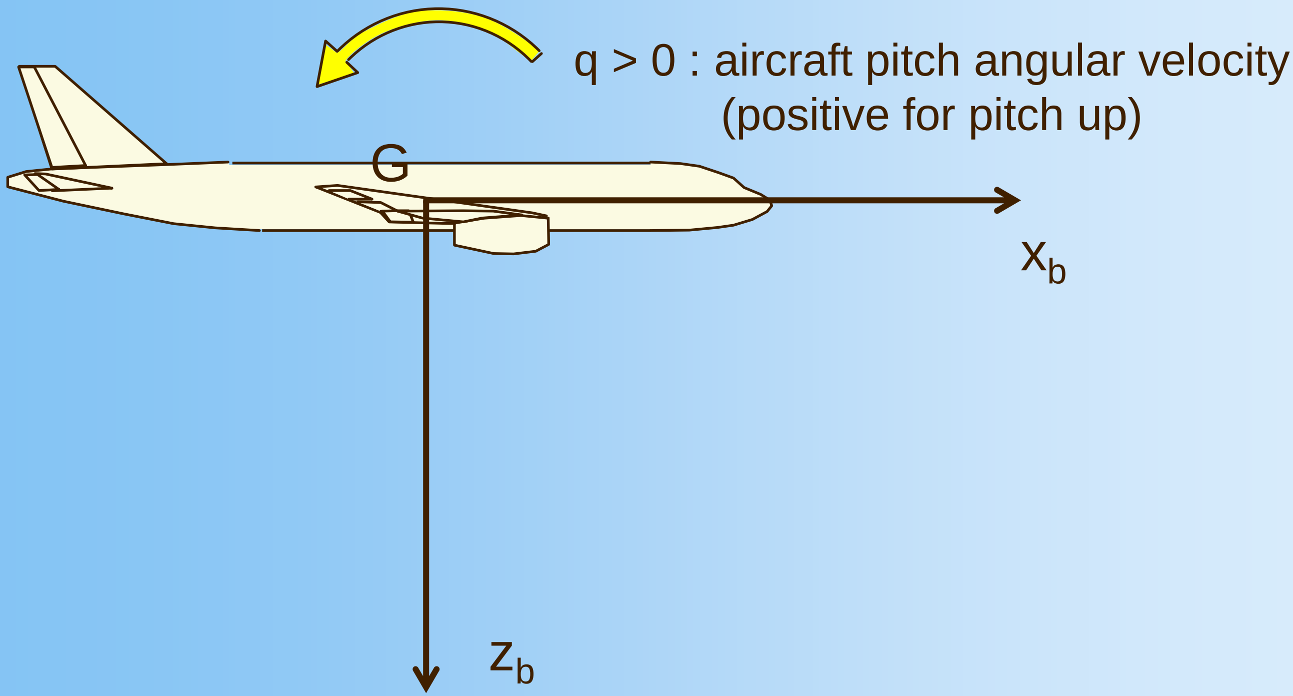
- The Earth is assumed flat and still (no rotation about itself) (*)
- z_0 is vertical , parallel to the gravity vector , positive downwards
- Ox_0y_0 is the Sea Level Earth horizontal plane
- R_0 is assumed to be an Inertial Referential : we do not take into account the additional inertial forces (centrifugal and Coriolis forces)

- We define the altitude h as $h = -z_0$

- We define the vertical speed : $V_z = \dot{h} = V \cdot \sin\gamma$

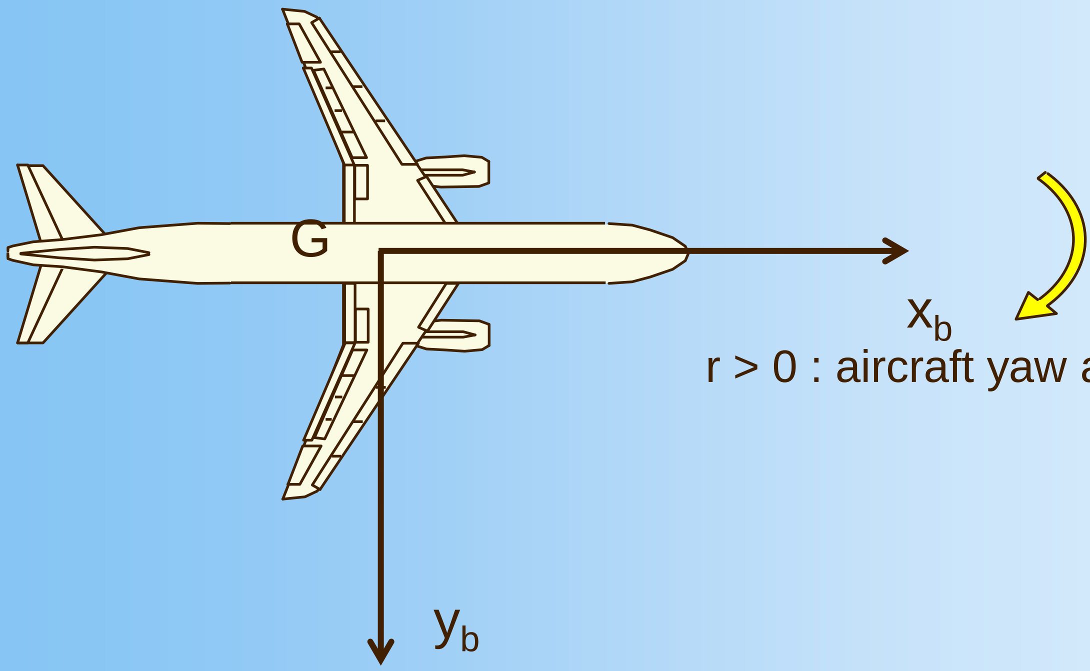
(*) : these assumptions are completely OK for flight mechanics purposes where the times and distances stay small. For performance purposes, it becomes sometimes difficult to neglect the additional inertial forces …

Body reference system $Gx_by_bz_b$



- Gx_b is the fuselage horizontal reference, pointed forwards
- Gx_bz_b is the aircraft plane of symmetry
- q measures the pitch or the rotation of R_b around y_b

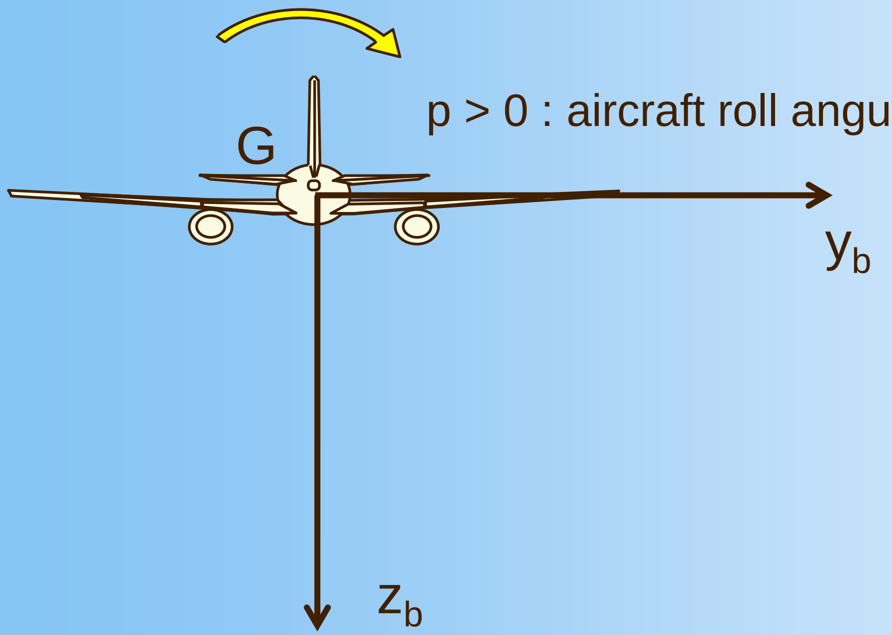
Body reference system $Gx_by_bz_b$



r > 0 : aircraft yaw angular velocity

- Gy_b is perpendicular w.r.t. the aircraft plane of symmetry
- Gy_b is pointed towards the right of the pilot
- r measures the yaw or the rotation of R_b around z_b

Body reference system $Gx_by_bz_b$



- Gz_b is pointed downwards
- p measures the roll or the rotation of R_b around x_b

Aircraft Elementary Rotation : (p,q,r)

The elementary rotations (p,q,r) define the aircraft rotation with respect to the Earth reference system R_0.
They define the absolute rotation of the aircraft : $\vec{\Omega}_{ac/R_0}$

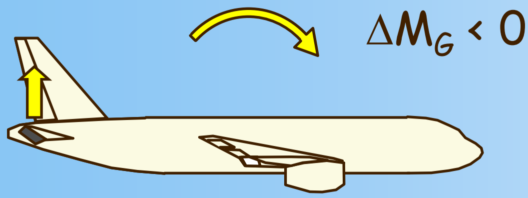
As the Body reference system R_b is « embedded » within the aircraft, the absolute rotation of R_b is the same as for the aircraft :

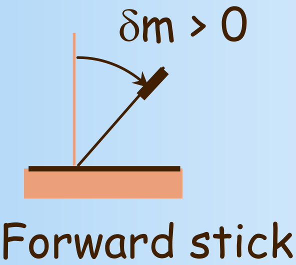
$$\vec{\Omega}_{R_b/R_0} = \vec{\Omega}_{ac/R_0}$$

Furthermore, we must understand that the absolute elementary rotations (p,q,r) are measured by 3 gyrometers located near the aircraft Center of Gravity, so (p,q,r) are the components of the Aircraft (absolute) rotation but projected with respect to the Body Reference system R_b :

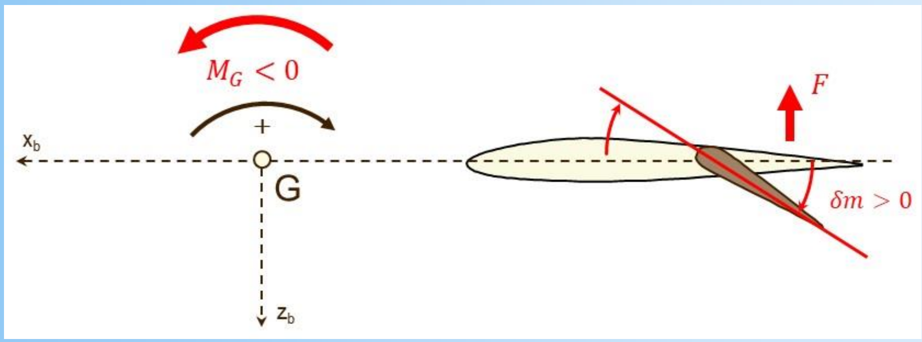
$$\vec{\Omega}_{R_b/R_0} = \vec{\Omega}_{ac/R_0} = {}_{R_b}\left|\begin{matrix} p \\ q \\ r \end{matrix}\right.$$

Momentum around the Gy_b axis or pitch axis





Pitch

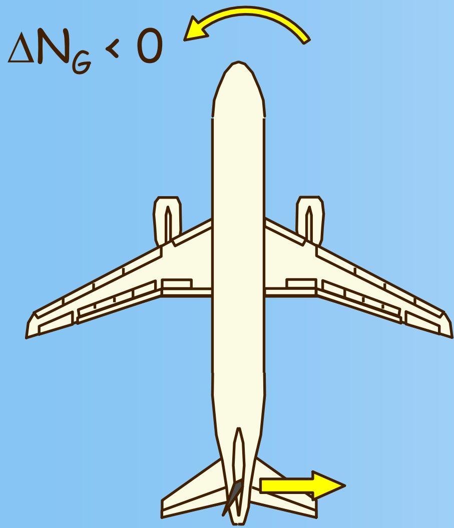


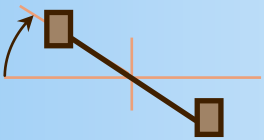
A positive $\delta m > 0$ elevator deflection creates
a negative pitch momentum $\Delta M_G < 0$
(around the aircraft Gy_b axis)

$\delta m>0$ corresponds to a downward motion of the elevator

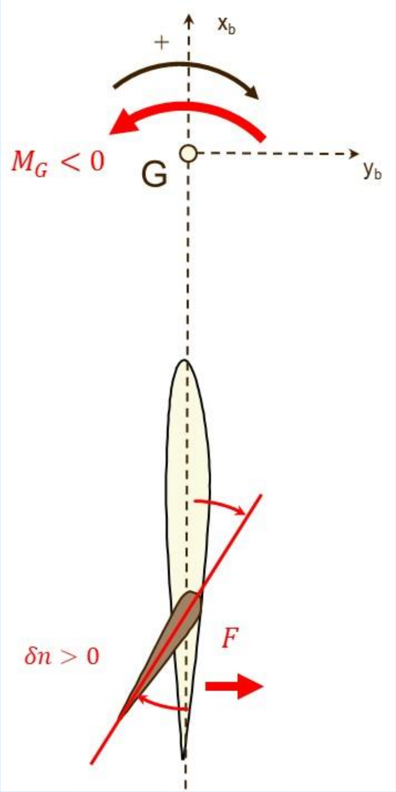
Momentum around the Gz_b axis or yaw axis

Yaw





Rudder pedals
to the left

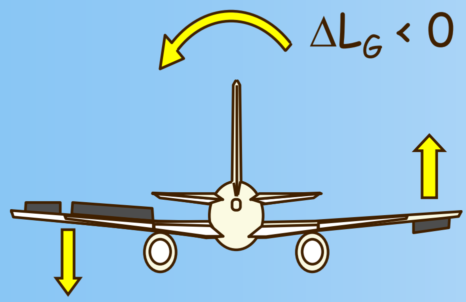


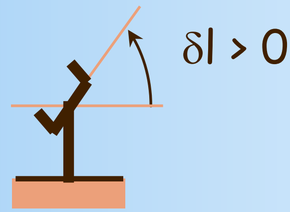
A positive $\delta n > 0$ rudder deflection creates a negative yaw momentum $\Delta N_G < 0$ (around the aircraft Gz_b axis)

$\delta n>0$ corresponds to a rudder motion to the left

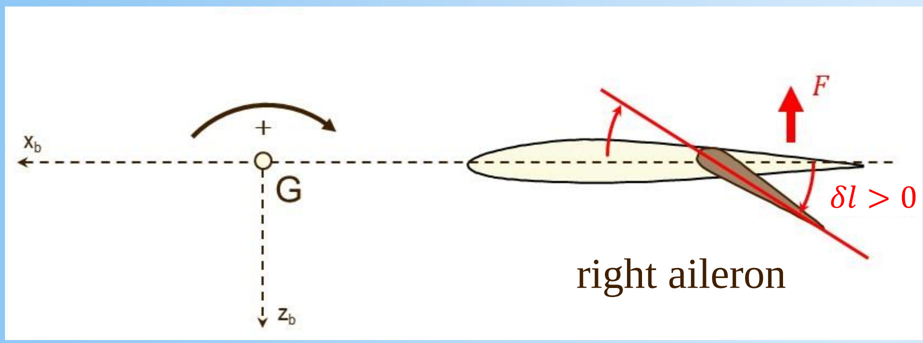
Momentum around the Gx_b axis or roll axis

Roll





Left stick (or control wheel)



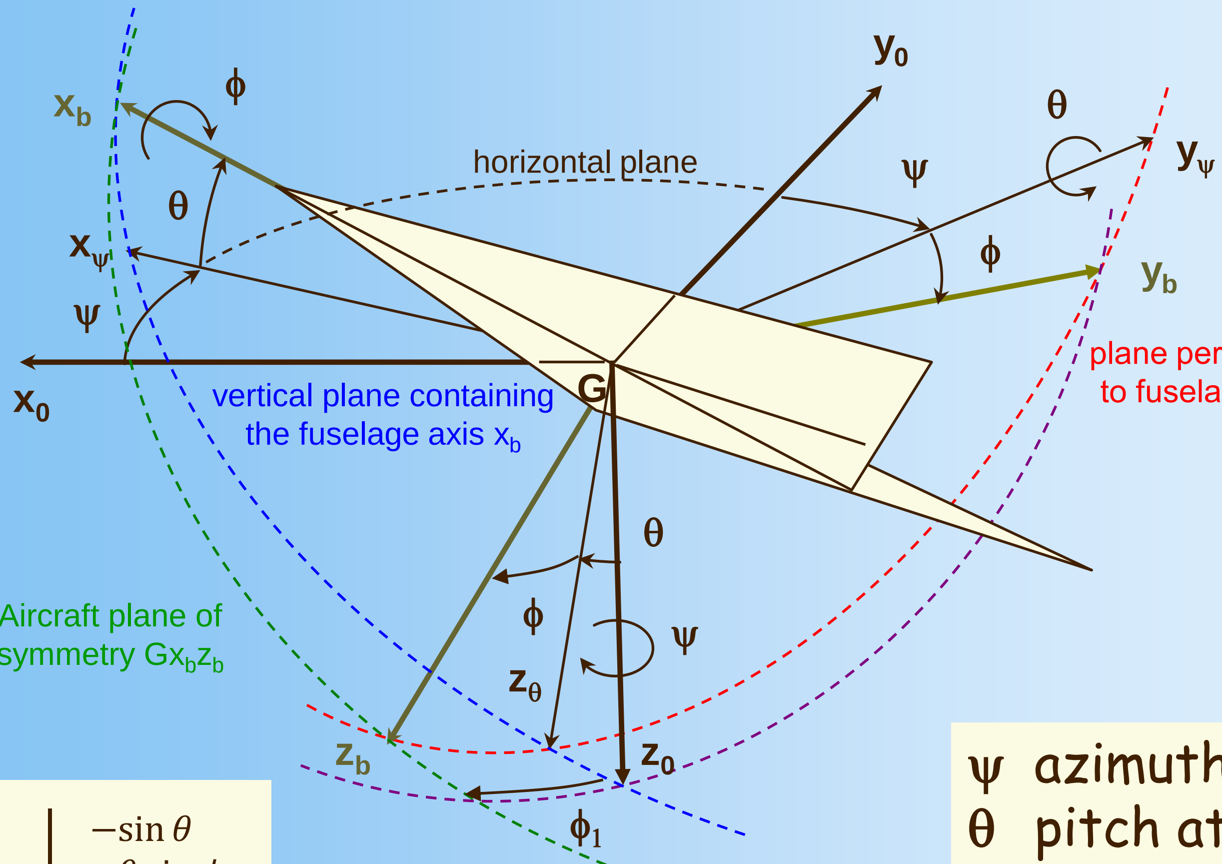
A positive $\delta l > 0$ aileron deflection creates a negative roll momentum $\Delta L_G < 0$ (around the aircraft Gx_b axis)

*$\delta l > 0$ corresponds to a downward motion of the **right** aileron (same convention as the elevator δm)*

Synthesis

Axis	Momentum (N.m)	Controls (°)	Angular velocity (rd/s)
around x_b or roll axis	L	δl	p
around y_b or pitch axis	M	δm	q
around z_b or yaw axis	N	δn	r

Rotation Angles from R_0 to R_b



$$\vec{z}_0 = \left| \begin{array}{c} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{array} \right._{R_b}$$

ψ azimuth angle
θ pitch attitude
ϕ roll angle

Rotation Angles from R_0 to R_b

We need 2 rotations for moving the axis x_0 to x_b :

- ψ = rotation within the local horizontal plane until the x-axis is merging within the vertical plane containing x_b
- θ = rotation within the local vertical plane until the x-axis is merging with the x_b axis

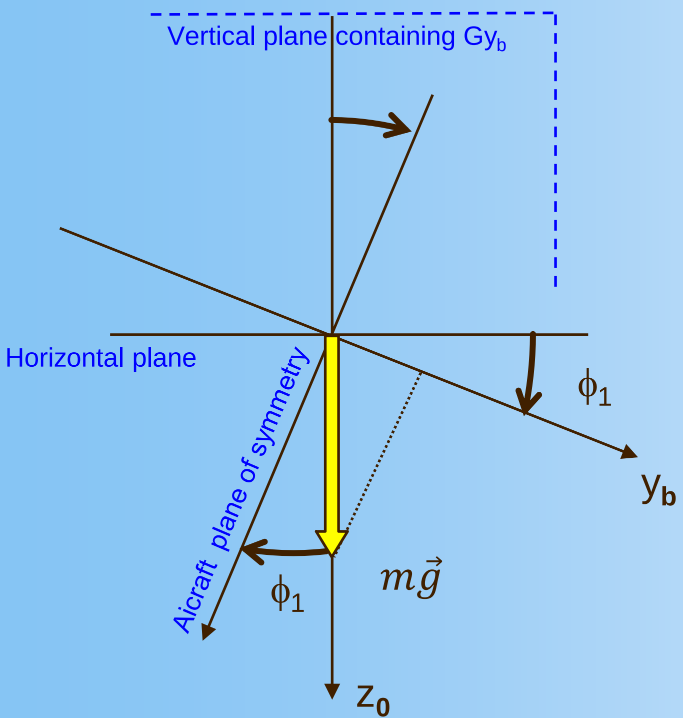
And a last rotation for moving the z axis within the aircraft plane of symmetry ($x_b z_b$) :

- ϕ = rotation around the Aircraft x-axis (x_b)

$$\vec{\Omega}_{b/0} = \dot{\psi}\ \vec{z}_0 + \dot{\theta}\ \vec{y}_\psi + \dot{\phi}\ \vec{x}_b$$

If $\phi = 0$, then the aircraft plane of symmetry ($x_b z_b$) is vertical

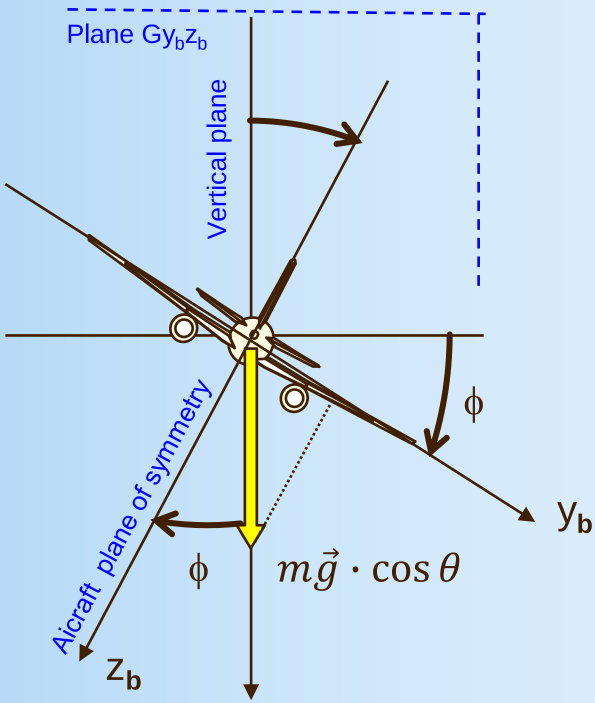
Roll Angle ϕ / Bank Angle ϕ_1



ϕ_1 : bank angle

angle between y_b axis and the horizontal plane

angle between (z_0) and the a/c plane of symmetry



ϕ : roll angle

rotation angle around Gx_b

rotation angle of the a/c plane of symmetry around the x_b axis

$m\vec{g}$ component on y_b :

$$\sin\phi_1 = \sin\phi\cos\theta$$

Differences between ϕ and ϕ_1

We consider the vertical plane containing the wings : Gy_bz_0

- ϕ_1 = angle between z_0 and the a/c plane of symetry
- ϕ_1 = angle between the wings Gy_b and the horizontal plane
- The component of mg w.r.t. to $Gy_b = m\vec{g}\sin\phi_1$

We consider the plane : Gy_bz_b

- ϕ = « roll » rotation angle around Gx_b
- The projection of mg w.r.t. to the plane $Gy_bz_b = m\vec{g}\cos\theta$
- The component of mg w.r.t. to $Gy_b = m\vec{g}\sin\phi\cos\theta$

$m\vec{g}$ component on y_b : $\boxed{\sin\phi_1 = \sin\phi\cos\theta}$

Roll angle ϕ and Bank angle ϕ_1 are the same
when the x_b axis is within the horizontal plane (θ=0°)

Aerodynamic reference system $Gx_ay_az_a$

It is the air surrounding the aircraft which produces the Aerodynamic Forces & Moments acting on the Aircraft (by producing a certain pressure distribution along the different Aerodynamic surfaces)

So, it is Mandatory to know the velocity vector of the surrounding air mass flow with respect to the aircraft ; we call it $\vec{V}_{ac/air}$; a little Observer sit on the Aircraft will see the air mass flow arriving at the aircraft animated with this velocity : $\vec{V}_{air/ac} = -\vec{V}_{ac/air}$

By definition, the Aerodynamic referential is defined such as :

- Gx_a = along the velocity vector $\vec{V}_{ac/air}$
- Gz_a = perpendicular to Gx_a and within the a/c plane of symmetry
- Gy_a = for getting the final direct triedal

Some remarks …

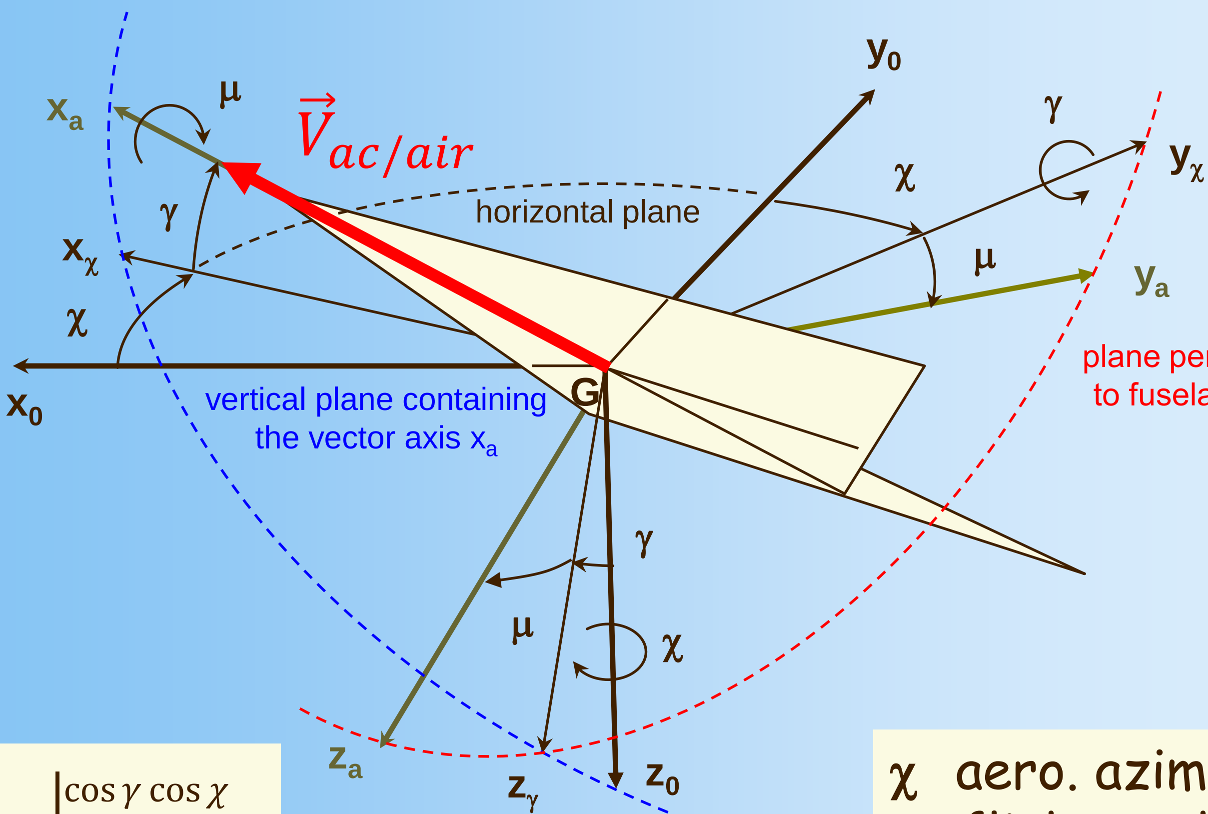
Remark 1 : be careful, the velocity $\vec{V}_{ac/air}$ is not necessary the velocity of the aircraft with respect to the ground $\vec{V}_{ac/grd}$, if there is some wind $\vec{V}_{air/grd}$, then

$$\vec{V}_{ac/grd} = \vec{V}_{ac/air} + \vec{V}_{air/grd}$$

Remark 2 : within the 1st Newton Law, the velocity used is the velocity of the aircraft with respect to the ground (with respect to a Galilean Referential)

$$m \cdot \frac{d\vec{V}_{ac/grd}}{dt} = \vec{F}$$

Remark 3 : it is the velocity $\vec{V}_{ac/air}$ which is responsible of the aerodynamic forces/moments, so its knowledge (intensity, orientation wrt the aircraft) is essential : this is the function of the numerous anemometric probes …



$$\vec{x}_a = \left|\begin{matrix} \cos\gamma\cos\chi \\ \cos\gamma\sin\chi \\ -\sin\gamma \end{matrix}\right._{R_0}$$

χ aero. azimuth
γ flight path angle
μ aero. roll angle

Rotation Angles from R_0 to R_a

We need 2 rotations for moving the axis x_0 to x_a :

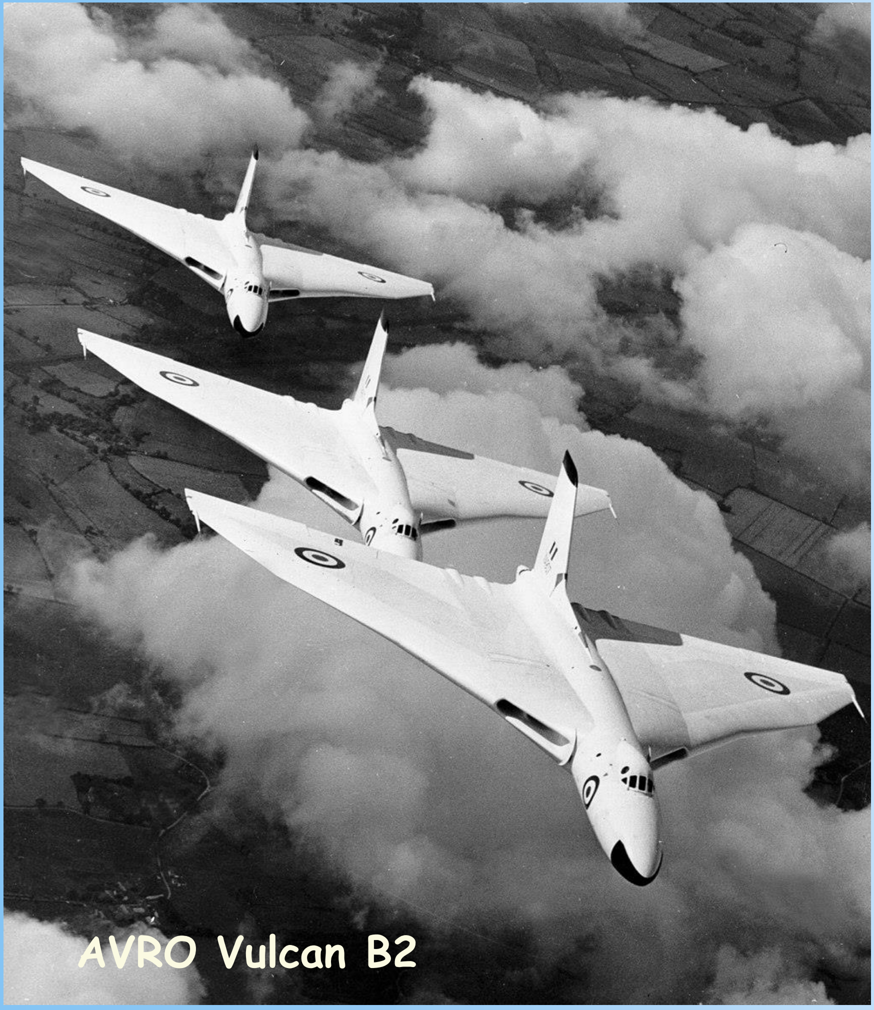
- χ = rotation within the local horizontal plane until the x-axis is merging within the vertical plane containing x_a
- γ = rotation within the local vertical plane until the x-axis is merging with the x_a axis

And a last rotation for moving the z axis within the aircraft plane of symmetry ($x_b z_b$) :

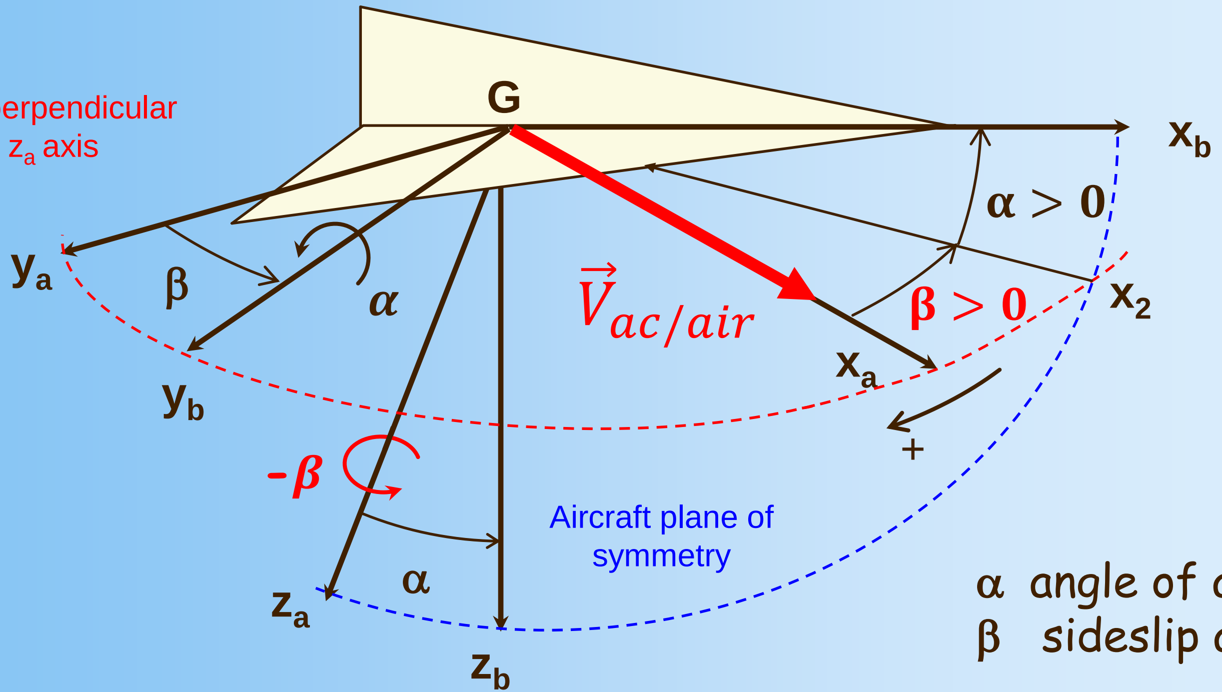
- μ = rotation around the x_a-axis

$$\vec{\Omega}_{a/0} = \dot{\chi}\ \vec{z}_0 + \dot{\gamma}\ \vec{y}_\chi + \dot{\mu}\ \vec{x}_a$$

If $\mu = 0$, then the z_a axis is within the vertical plane containing $\vec{V}_{ac/air}$



Angle of Attack
Side Slip
(α,β)



α angle of attack
β sideslip angle

$$\vec{x}_a = \begin{vmatrix} \cos\alpha\cos\beta \\ \sin\beta \\ \sin\alpha\cos\beta \end{vmatrix}_{R_b}$$

Take care ! The convention imposes that β is positive when the wind is coming from the right : the sign of β doesn't follow the positive rotation around the z_a axis

Rotation Angles from R_a to R_b

We need 2 rotations for moving from R_a to R_b :

- β = rotation around the z_a axis until the x-axis is merging within the aircraft plane of symmetry
- α = rotation within the aircraft plane of symmetry (around the y_b axis) until the x-axis is merging with the x_b axis

We need only 2 angles for getting from R_a to R_b because the z_a axis is ,by definition, within the Gx_bz_b plane (aircraft symmetry plane).

$$\vec{\Omega}_{b/a} = -\dot{\beta}\,\vec{z}_a + \dot{\alpha}\,\vec{y}_b$$

Remark : the rotation with respect to the z_a axis is negative whereas β is imposed positive

Some remarks …

Remark 1 :

- β is the angle between $\vec{V}_{ac/air}$ and the aircraft plane of symmetry
- β positive, when the flowstream is coming from the right
- if β = 0°, $\vec{V}_{ac/air}$ is within the a/c plane of symetry ($x_b z_b$) and $y_a = y_b$

$$\boxed{\sin\beta = \vec{x}_a \cdot \vec{y}_b}$$

Remark 2 :

- α is an angle measured within the aircraft plane of symmetry
- α is not the angle between x_b and $\vec{V}_{ac/air}$ but between x_b and the projection of $\vec{V}$ within the aircraft plane of symmetry
- α positive when the flowstream is coming from downwards
- if α = 0°, $\vec{V}_{ac/air}$ is within the plane ($x_b y_b$) and $z_a = z_b$

$$\boxed{\sin\alpha\cos\beta = \vec{x}_a \cdot \vec{z}_b}$$

$$\vec{x}_a \cdot \vec{z}_b = (\vec{x}_2 \cos\beta + \vec{y}_b \sin\beta) \cdot \vec{z}_b = \vec{x}_2 \cdot \vec{z}_b \cos\beta = \sin\alpha \cdot \cos\beta$$

Relation between $\vec{\Omega}_{a/0}$ and $\vec{\Omega}_{b/0}$

NOT in PROGRAM

By composition,

$$\vec{\Omega}_{a/0} = \vec{\Omega}_{a/b} + \vec{\Omega}_{b/0}$$

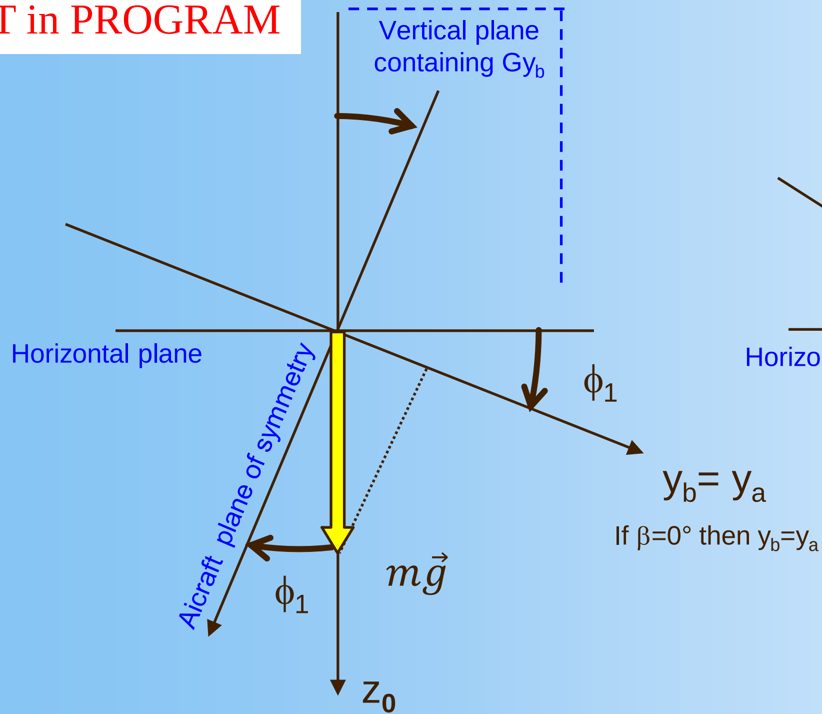
By definition, as for $\vec{\Omega}_{b/0} = {}_{R_b}\left| \begin{matrix} p \\ q \\ r \end{matrix} \right.$, we defines $\vec{\Omega}_{a/0} = {}_{R_a}\left| \begin{matrix} p_a \\ q_a \\ r_a \end{matrix} \right.$

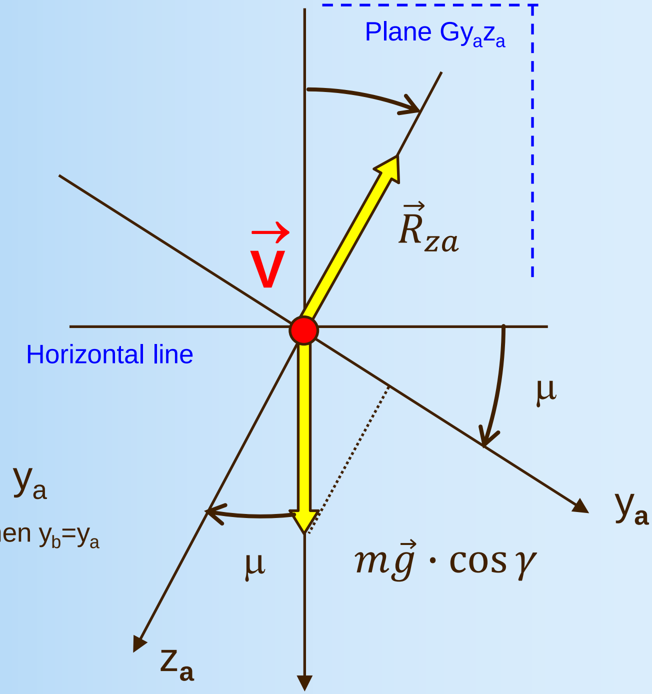
The calculation is performed within the annex, assuming the small angles approximation,

$$p_a \approx p + r \cdot \alpha$$
$$q_a \approx q - \dot{\alpha}$$
$$r_a \approx \dot{\beta} + r - p \cdot \alpha$$

Relation μ , ϕ_1 / simple case with β = 0°

NOT in PROGRAM





ϕ_1 : bank angle

angle between y_b axis and the horizontal plane
angle between (z_0) and the a/c plane of symmetry

μ : aero. roll angle

rotation angle around Gx_a
rotation angle of the Lift Force around the Velocity vector

$m\vec{g}$ component on y_a :

$$\sin\phi_1 = \sin\phi\cos\theta = \sin\mu\cos\gamma$$