Assignment - 4: Linear Systems Descriptions

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```
clear all
clc
close all

% Questions
%   If wind is a perturbation, how can we hover trim with zero angles?
%   If the chardonnay dynamics already takes into account??
%   x_dot = Ax+Bu+Ew  or   x_dot = Ax+Bu where u is a 2*2 matrix
```

Exercise 1.1 and 1.2

```
% Initially, we assume to disturbance to be present. Therefore, the linear
% model is delta_x_dot = A*delta_x + B*delta_u

% Creating a structure for the system's physical properties
drone = struct('m_d',1,'m_c',1,'l',1,'l_d',1,'J',1,'C_D',0.01,'g',10);

syms x u w pn pd T1 T2 vn vd theta thetad gamma gammad wn wd

x = [pn ; pd ; vn ; vd ; theta ; thetad ; gamma ; gammad];
u = [T1 ; T2];
w = [wn ; wd];
x_dot = chardonnay_dynamics(x,u,w,drone);
x_dot = x_dot(3:8);
x = x(3:8);

xt = zeros(6,1);
ut = 10*ones(2,1);
wt = ones(2,1);

A = jacobian(x_dot,x);
B = jacobian(x_dot,u);
E = jacobian(x_dot,w);

At = double(subs(A,[x ; u ; w],[xt ; ut ; wt]));
Bt = double(subs(B,[x ; u ; w],[xt ; ut ; wt]));
Et = double(subs(E,[x ; u ; w],[xt ; ut ; wt]));

delta_x_dot = At*(x - xt) + Bt*(u - ut) + Et*(w - wt);

Ct = eye(6);
Dt = zeros(6,2);

sys = ss(At,Bt,Ct,Dt);
Gs = tf(sys) %Transfer matrix for G(s)
```

```
Gs =
```

```
  From input 1 to output...
             20 s^2 - 2.665e-14 s - 199.9
   1:  ----------------------------------------
       s^5 + 0.01 s^4 - 19.99 s^3 - 0.09995 s^2

       -0.5 s^5 - 0.005 s^4 + 9.995 s^3 + 0.04998 s^2 - 0.1 s + 1.108e-15
   2:  -----------------------------------------------------------------
            s^6 + 0.015 s^5 - 19.99 s^4 - 0.1999 s^3 - 0.0004997 s^2

       -1
   3:  ---
       s^2
```

```
sys_1 = ss(At,Et,Ct,Dt);
Gd = tf(sys_1) %Transfer matrix for G(d)
```

```
Gd =

  From input 1 to output...
        0.01 s^2 + 4.441e-18 s - 0.09995
   1:  ---------------------------------
       s^3 + 0.01 s^2 - 19.99 s - 0.09995

                      -5e-05 s - 6.133e-23
   2:  --------------------------------------------------
       s^4 + 0.015 s^3 - 19.99 s^2 - 0.1999 s - 0.0004997

   3:  0

   4:  0
```

Exercise 2

Ex 2.1 (Impulse matrix (H(t)))

H(t) = $(C * e^{\mathrm{At}} * B) + D\delta(t)$

Since D=0,

H(t) = $C * e^{\mathrm{At}} * B$

A = $\begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$e^{\mathrm{At}} = \sum_{n=0}^{\infty} \frac{t^n A^n}{n!}$$

$$e^{\mathrm{At}} = A^0 + \mathrm{tA} + 0 + \ldots\ldots$$

$$e^{\mathrm{At}} = I + \mathrm{At} = \begin{bmatrix} 1+5t & -3t & 2t \\ 15t & 1-9t & 6t \\ 10t & -6t & 1+4t \end{bmatrix}$$

Therfore:

$$H(t) = \begin{bmatrix} 1+3t & -3t \\ 9t & 1-9t \end{bmatrix}$$

Ex 2.2 (Markov parameters (H(k)))

$$H(t) = \sum_{k=0}^{\infty} \frac{H_k * t^k}{k!} = H_0 + H_1 * t + H_2 * t^2 + \ldots\ldots\ldots$$

Therefore,

$$H_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad H_1 = \begin{bmatrix} 3 & -3 \\ 9 & -9 \end{bmatrix}$$ (Markov Parameters)

Ex 2.3 (Transfer Matrix (H(s)))

$$H(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix} + \begin{bmatrix} \frac{3}{s^2} & -\frac{3}{s^2} \\ \frac{9}{s^2} & -\frac{9}{s^2} \end{bmatrix}$$

$$H(s) = \begin{bmatrix} \frac{s+3}{s^2} & -\frac{3}{s^2} \\ \frac{9}{s^2} & \frac{s-9}{s^2} \end{bmatrix}$$

Ex 2.4

```
clear all
clc
close all

A = [5 -3 2 ; 15 -9 6 ; 10 -6 4];
B = [1 0 ; 0 1 ; -1 0];
C = [1 0 0 ; 0 1 0];
D = [0 0 ; 0 0];

H = tf(ss(A,B,C,D))
```

```
H =

  From input 1 to output...
                   s^2 + 3 s - 1.228e-14
   1:  ---------------------------------------------
       s^3 + 4.125e-15 s^2 - 6.084e-14 s + 6.652e-29

                      9 s - 3.118e-14
   2:  ---------------------------------------------
       s^3 + 4.125e-15 s^2 - 6.084e-14 s + 6.652e-29

  From input 2 to output...
```

```
                    -3 s + 1.469e-15
   1:  ---------------------------------------------
       s^3 + 4.125e-15 s^2 - 6.084e-14 s + 6.652e-29

                        s^2 - 9 s
   2:  ---------------------------------------------
```

We say that a system has zeros at infinity if the limit as s->infinity or the value of the transfer function is equal to zero; this happens whenever you have more poles than zeros. You will see this in the root locus plot as asymptotes which go to infinity (the number of asymptotes is equal to the number of zeros at infinity). MATLAB sometimes computes these zeros at infinity as being large finite numbers.When this happens, some of the coefficients in the numerator that are supposed to be zero end up being very small numbers. This is the reason why the MATLAB command yields a slightly different transfer function matrix than the one computed by hand.

Exercise 3

Ex 3.1

```
s = tf("s");
G1 = [0 -1/s; 0 2/s];
G2 = [3/s 5/s; 0 0];
config1 = G1*G2
```

```
config1 =

  From input 1 to output...
   1:  0

   2:  0

  From input 2 to output...
   1:  0

   2:  0

Static gain.
```

```
config2 = G2*G1
```

```
config2 =

  From input 1 to output...
   1:  0

   2:  0

  From input 2 to output...
       7 s^2
   1:  -----
        s^4

   2:  0

Continuous-time transfer function.
```

Therefore, it can be proved that $\mathrm{G1}(s) * \mathrm{G2}(s) \neq \mathrm{G2}(s) * \mathrm{G1}(s)$.

Ex 3.2

Similarly it can be said that in a MIMO system one could not interchange the arrangement of blocks in a series connection block diagram because in this case the matrixes are being multiplies but one could interchange the arrangement of bloks in a parallel connection since it is just an addition of Matrixes.