final

December 23, 2019

[2]:
```
import numpy as np
from scipy.stats import norm
from scipy.optimize import fsolve
```

[3]:
```
t0 = np.arange(0, 2, 0.25)
t1 = np.arange(0.25, 2.25, 0.25)
```

[4]:
```
# F(0, t0, t1)
F = np.array([6, 8, 9, 10, 10, 10, 9, 9])/100.
```

[5]:
```
# P(0, t1)
P = np.zeros(8)
P[0] = 1/( 1 + ( t1[0] - t0[0] ) * F[0] )
for i in range(1,8):
    P[i] = P[i-1] / ( 1 + ( t1[i] - t0[i] ) * F[i] )
```

[6]:
```
P
```

[6]:
```
array([ 0.98522167,  0.9659036 ,  0.944649  ,  0.92160878,  0.89913052,
        0.87720051,  0.8578978 ,  0.83901986])
```

[7]:
```
# R_swap(first_reset_time, maturity)
# first_reset_time = 1
# maturity = 2
# quarterly fixed payments i.e. delta = 0.25
delta = 0.25
R_swap = ( P[3] - P[-1] ) / ( delta * P[4:].sum() )
```

[8]:
```
R_swap
```

[8]:
```
0.095114321443452166
```

[9]:
```
# ATM
K = R_swap
```

[10]:
```
N = 1
```

[ ]:

[11]:
```
black_d1 = lambda sigma: ( np.log(R_swap/K) + 0.5 * sigma**2 * (t0[4] - t0[0])␣
↪) / ( sigma * np.sqrt(t0[4] - t0[0]) )
black_d2 = lambda sigma: ( np.log(R_swap/K) - 0.5 * sigma**2 * (t0[4] - t0[0])␣
↪) / ( sigma * np.sqrt(t0[4] - t0[0]) )
```

[12]:
```
black_swaption_payer = lambda sigma : N * delta * P[4:].sum() * ( R_swap*norm.
↪cdf(black_d1(sigma)) - K*norm.cdf(black_d2(sigma)) )
```

[13]:
```
black_implied_sigma = fsolve(lambda sigma: black_swaption_payer(sigma) - 0.01,␣
↪0.1)
```

[14]:
```
black_implied_sigma, black_implied_sigma*10**2
```

[14]:
```
(array([ 0.30468099]), array([ 30.46809903]))
```

[ ]:

[15]:
```
bachelier_D = lambda sigma: ( R_swap - K ) / ( sigma * np.sqrt(t0[4] - t0[0]) )
```

[16]:
```
bachelier_swaption_payer = lambda sigma : N * delta * P[4:].sum() * sigma * np.
↪sqrt(t0[4] - t0[0]) * ( bachelier_D(sigma)*norm.cdf(bachelier_D(sigma)) +␣
↪norm.pdf(bachelier_D(sigma)) )
```

[17]:
```
bachelier_implied_sigma = fsolve(lambda sigma: bachelier_swaption_payer(sigma)␣
↪- 0.01, 0.1)
```

[18]:
```
bachelier_implied_sigma, bachelier_implied_sigma*10**4
```

[18]:
```
(array([ 0.02886782]), array([ 288.67823777]))
```

[ ]:

[19]:
```
black_swaption_payer(0.5), black_swaption_payer(0.5)*10**2
```

[19]:
```
(0.016304098259164831, 1.6304098259164832)
```

[ ]: