Linear Congruence Generator

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Abstract

A linear congruential generator (LCG) is an algorithm that yields a sequence of pseudo-randomized numbers calculated with a discontinuous piecewise linear equation. The generator is defined by the recurrence relation:

$$x_{i+1} = (ax_i + b) \bmod m$$

$$u_{i+1} = \frac{x_{i+1}}{m}$$

where u is the sequence of pseudorandom values, and

$m, 0 < m$ - the “modulus”

$a, 0 < a < m$ - the “multiplier”

$b, 0 \leq b < m$ - the “increment”

$x_0, 0 \leq x_0 < m$ - the “seed” or “start value”

are integer constants that specify the generator.

If $b = 0$, the generator is often called a multiplicative congruential generator (MCG), or Lehmer RNG. If $b \neq 0$, the method is called a mixed congruential generator.

Problem Statement 1

Generate the sequence of numbers x_i for a = 6, b = 0, m = 11 and x_0 ranging from 0 to10. Also, generate the sequence of numbers x_i for a = 3, b = 0, m = 11, and x_0 ranging from 0 to 10.

Observe the sequence of numbers generated and observe the repetition of values. Tabulate these for each group of values.

How many distinct values are appearing before repetitions? Which, in your view, are the best choices and why?

(Use only C or C++ code)

Program

Code in C

```
#include<stdio.h>
void gen(int a, int b, int m, int x0, int * value, int v){
	int x = x0, i = 0;
	printf("[");
	do{
		printf(" %f",(x/(float)m));
		x = (((a * x) + b) % m);
		i += 1;
	}while(x != x0);
	printf(" ]\tRepetition after %d terms.\n", i);
	value[v] = i;
}
void main(){
	int x0, value[22] = {0}, v = 0;
	for(x0 = 0; x0 <= 10; x0++){
		printf("For a = 6, b = 0, m = 11, x0 = %d :\n\t",x0);
		gen(6, 0, 11, x0, value, v);
		v++;
	}
	for(x0 = 0; x0 <= 10; x0++){
		printf("For a = 3, b = 0, m = 11, x0 = %d :\n\t",x0);
		gen(3, 0, 11, x0, value, v);
		v++;
	}

	int max = 0;
	for(v = 0; v < 22; v++){
		if (max < value[v]){
			max = value[v];
		}
	}

	printf("The best choice/choices are (generating maximum, i.e. %d,terms without repetition):\n", max);

	for(v = 0; v < 22; v++){
		if (max == value[v]){
			if (v < 11){
				printf("\ta = 6, b = 0, m = 11, x0 = %d\n",(v));
			}
			else{
				printf("\ta = 3, b = 0, m = 11, x0 = %d\n",(v-11));
			}
		}
	}
}
```

Output

For a = 6, b = 0, m = 11, x0 = 0 :
[0.000000] Repetition after 1 terms.
For a = 6, b = 0, m = 11, x0 = 1 :
[0.090909 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 2 :
[0.181818 0.090909 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 3 :
[0.272727 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818 0.090909 0.545455]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 4 :
[0.363636 0.181818 0.090909 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545 0.727273]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 5 :
[0.454545 0.727273 0.363636 0.181818 0.090909 0.545455 0.272727 0.636364 0.818182 0.909091]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 6 :
[0.545455 0.272727 0.636364 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818 0.090909]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 7 :
[0.636364 0.818182 0.909091 0.454545 0.727273 0.363636 0.181818 0.090909 0.545455 0.272727]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 8 :
[0.727273 0.363636 0.181818 0.090909 0.545455 0.272727 0.636364 0.818182 0.909091 0.454545]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 9 :
[0.818182 0.909091 0.454545 0.727273 0.363636 0.181818 0.090909 0.545455 0.272727 0.636364]
Repetition after 10 terms.
For a = 6, b = 0, m = 11, x0 = 10 :
[0.909091 0.454545 0.727273 0.363636 0.181818 0.090909 0.545455 0.272727 0.636364 0.818182]
Repetition after 10 terms.
For a = 3, b = 0, m = 11, x0 = 0 :
[0.000000] Repetition after 1 terms.
For a = 3, b = 0, m = 11, x0 = 1 :
[0.090909 0.272727 0.818182 0.454545 0.363636] Repetition after 5 terms.
For a = 3, b = 0, m = 11, x0 = 2 :
[0.181818 0.545455 0.636364 0.909091 0.727273] Repetition after 5 terms.
For a = 3, b = 0, m = 11, x0 = 3 :
[0.272727 0.818182 0.454545 0.363636 0.090909] Repetition after 5 terms.
For a = 3, b = 0, m = 11, x0 = 4 :
[0.363636 0.090909 0.272727 0.818182 0.454545] Repetition after 5 terms.
For a = 3, b = 0, m = 11, x0 = 5 :
[0.454545 0.363636 0.090909 0.272727 0.818182] Repetition after 5 terms.
For a = 3, b = 0, m = 11, x0 = 6 :
[0.545455 0.636364 0.909091 0.727273 0.181818] Repetition after 5 terms.
For a = 3, b = 0, m = 11, x0 = 7 :
[0.636364 0.909091 0.727273 0.181818 0.545455] Repetition after 5 terms.
For a = 3, b = 0, m = 11, x0 = 8 :

[0.727273 0.181818 0.545455 0.636364 0.909091] Repetition after 5 terms.

For a = 3, b = 0, m = 11, x0 = 9 :

[0.818182 0.454545 0.363636 0.090909 0.272727] Repetition after 5 terms.

For a = 3, b = 0, m = 11, x0 = 10 :

[0.909091 0.727273 0.181818 0.545455 0.636364] Repetition after 5 terms.

The best choice/choices are (generating maximum, i.e. 10,terms without repetition):

a = 6, b = 0, m = 11, x0 = 1

a = 6, b = 0, m = 11, x0 = 2

a = 6, b = 0, m = 11, x0 = 3

a = 6, b = 0, m = 11, x0 = 4

a = 6, b = 0, m = 11, x0 = 5

a = 6, b = 0, m = 11, x0 = 6

a = 6, b = 0, m = 11, x0 = 7

a = 6, b = 0, m = 11, x0 = 8

a = 6, b = 0, m = 11, x0 = 9

a = 6, b = 0, m = 11, x0 = 10

Problem Statement 2

Generate a sequence u_i with m = 244944, a = 1597, 51749 (take x_0 as per your choice). Try to group the values in the ranges 0 - 0.05, 0.05 - 0.10, 0.10 - 0.15, ... and see their frequencies (i.e. the number of values falling in a group).

For at least 5 different values of the number of values generated, tabulate the frequencies in each case, draw bar diagrams of these data and put in your observations.

(Use both R and C / C ++ code)

*Addition: You already have two sets of a. Either you take b = 0 or suitable choices of b (say prime to m). To generate different sets you can take different seeds.

Program

Code in C

```
#include <stdio.h>
void gen(int a, int x0, int * f){
	long long int x = x0; int i;
	do{
		x = (((a * x) + 0) % 244944);
		i = ((x/(float)244944)/0.05);
		if (i == 20) {i--;}
		*(f + i) += 1;
	}while(x != x0);
	for(i = 0; i < 20; i++){
		*(f + 20) += *(f + i);
	}
}
void main(){
	int x0[5], f[5][21] = {0}, i;
//Generating the frequeny diistributions, along with taking input the value of x0s:
	printf("For a = 1597, b = 0 three values of x0:\n");
	for(i = 0; i < 3; i++){
		printf("x0_%d ? ", (i+1));
		scanf("%d", &x0[i]);
		gen(1597, x0[i], *(f + i));
	}

	printf("For a = 51749, b = 0 two values of x0:\n");
	for(i = 3; i < 5; i++){
		printf("x0_%d ? ", (i+1 - 3));
		scanf("%d", &x0[i]);
		gen(51749, x0[i], *(f + i));
	}
//Printing the frequency distibutions:
	printf("With a = 1597, b = 0, m = 244944 :\n");
	for(i = 0; i < 3; i++){
		printf("And, with the value of x0 = %d, the frequency distribution is:\n", x0[i]);
		for(int j = 0; j < 20; j++){
			printf("\t%.2f-%.2f\t:\t%d\n", (j/(float)20), ((j+1)/(float)20), f[i][j]);
		}
		printf("Total numbers : %d\n\n", f[i][20]);
	}

		printf("\n\nWith a = 51749, b = 0, m = 244944 :\n");
	for(i = 3; i < 5; i++){
		printf("And, with the value of x0 = %d, the frequency distribution is:\n", x0[i]);
		for(int j = 0; j < 20; j++){
			printf("\t%.2f-%.2f\t:\t%d\n", (j/(float)20), ((j+1)/(float)20), f[i][j]);
		}
		printf("Total numbers : %d\n\n", f[i][20]);
	}
}
```

Output

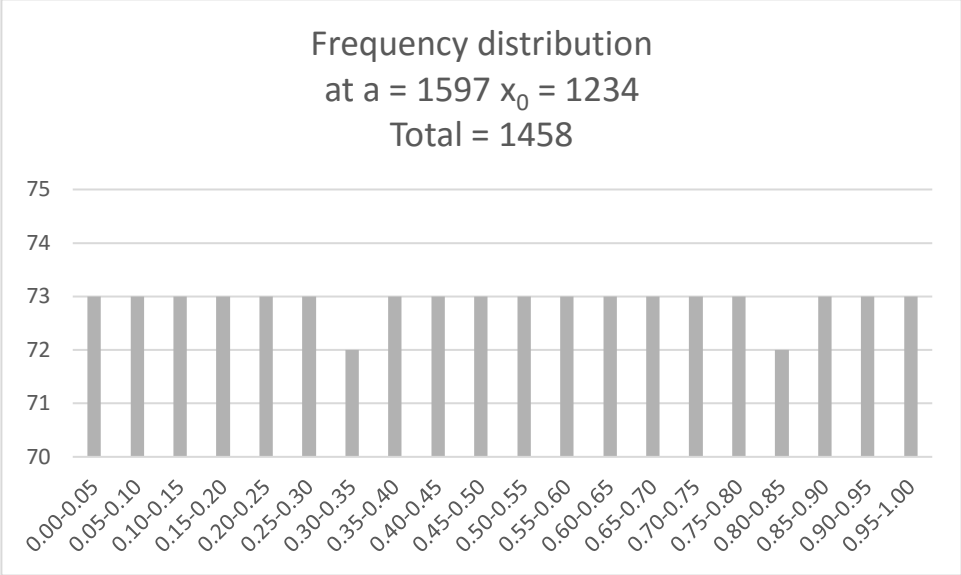
Table

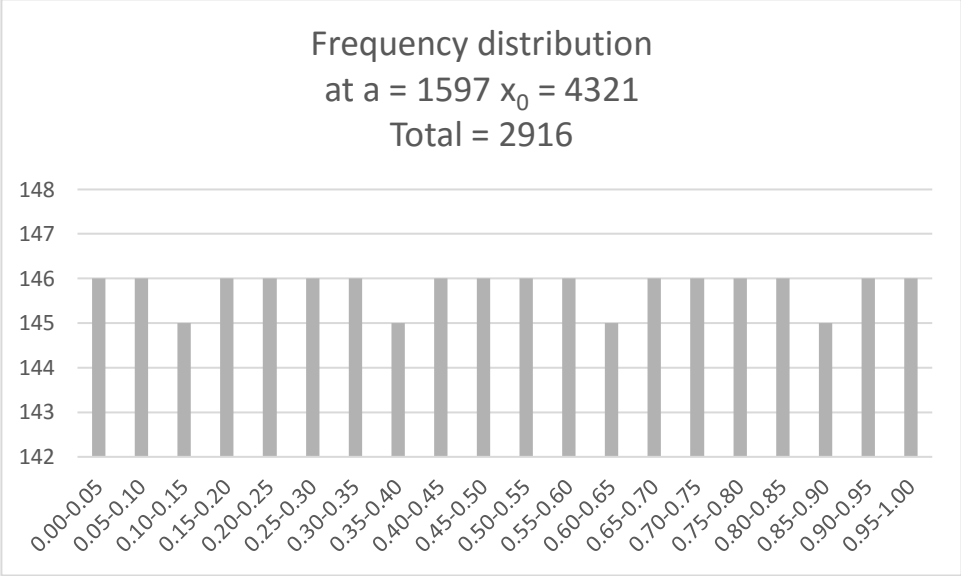
With the values of a = 1597, 51749, b = 0 and m = 244944, the frequency distributions at different values of x_0 are:

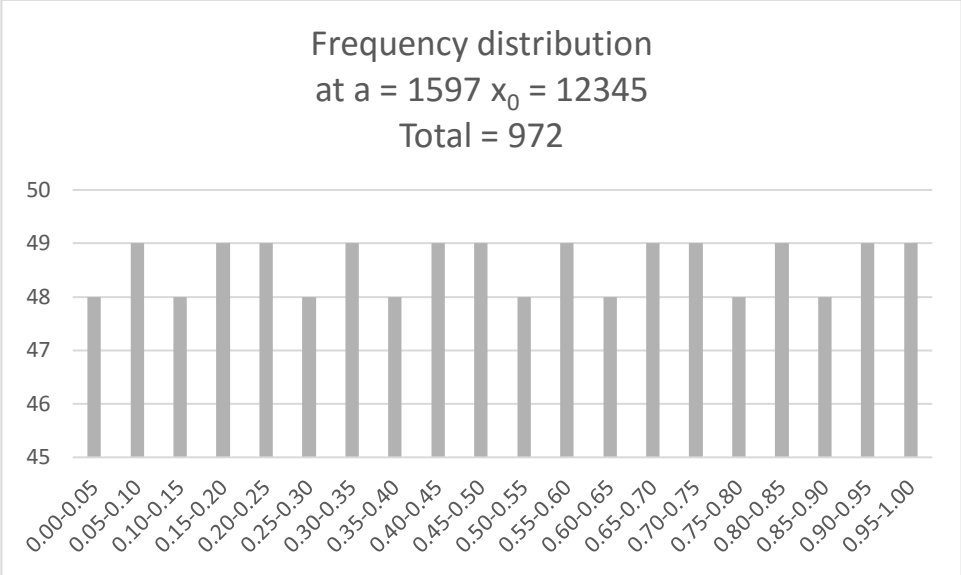
a	1597	1597	1597	51749	51749
$x_0 \rightarrow$ / **Range ↓**	1234	4321	12345	54321	123456
0.00 - 0.05	73	146	48	16	7
0.05 - 0.10	73	146	49	15	10
0.10 - 0.15	73	145	48	17	8
0.15 - 0.20	73	146	49	17	7
0.20 - 0.25	73	146	49	15	9
0.25 - 0.30	73	146	48	16	8
0.30 - 0.35	72	146	49	17	7
0.35 - 0.40	73	145	48	16	9
0.40 - 0.45	73	146	49	16	8
0.45 - 0.50	73	146	49	17	8
0.50 - 0.55	73	146	48	16	8
0.55 - 0.60	73	146	49	15	8
0.60 - 0.65	73	145	48	17	9
0.65 - 0.70	73	146	49	17	7
0.70 - 0.75	73	146	49	15	8
0.75 - 0.80	73	146	48	16	9
0.80 - 0.85	72	146	49	17	7
0.85 - 0.90	73	145	48	16	8
0.90 - 0.95	73	146	49	16	10
0.95 - 1.00	73	146	49	17	7
Total	1458	2916	972	324	162

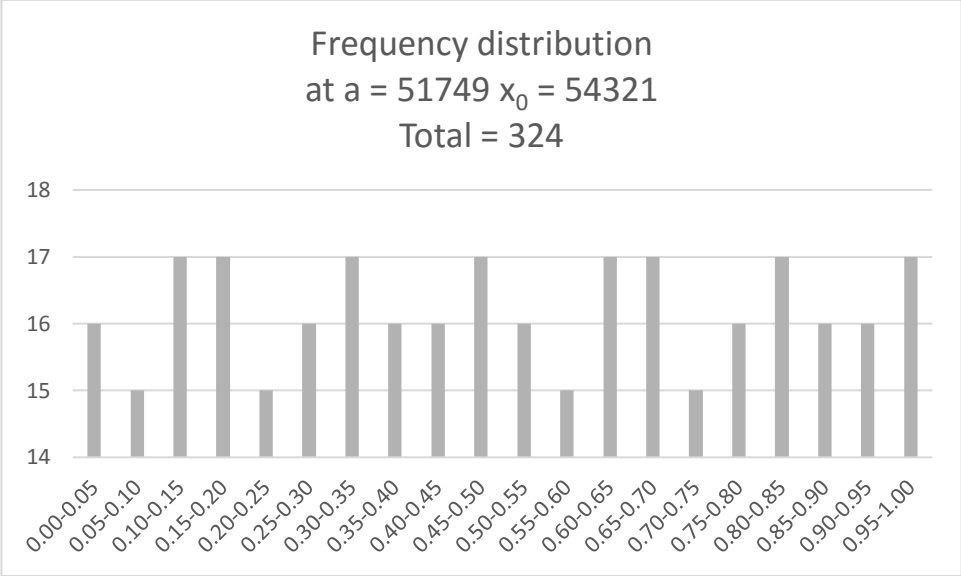
Bar-Graphs

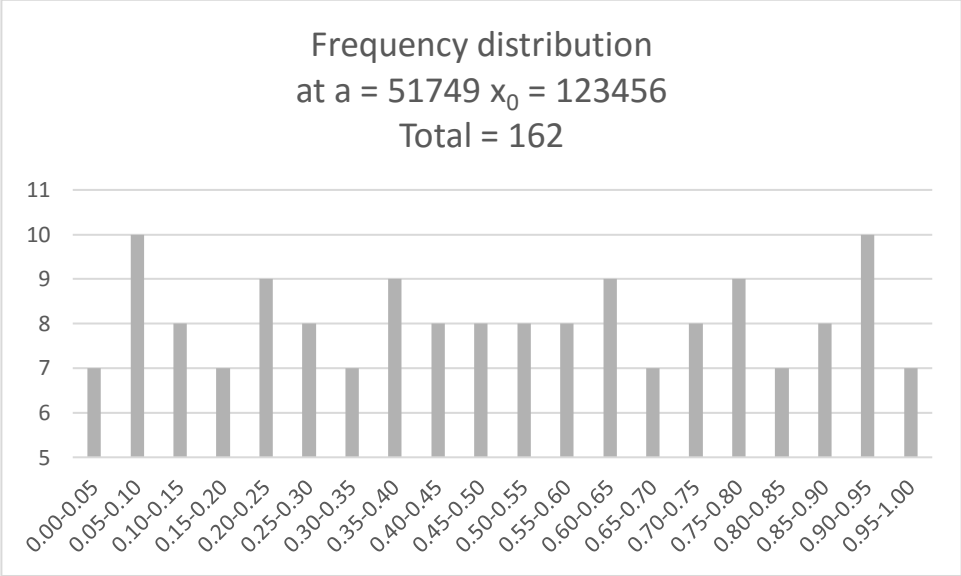
With the values of a = 1597, 51749, b = 0 and m = 244944, the bar-diagrams/bar-graphs of the frequency distributions at different values of x_0 are:






Frequency distribution
at a = 1597 x_0 = 12345
Total = 972
50
49
48
47
46
45
0.00-0.05
0.05-0.10
0.10-0.15
0.15-0.20
0.20-0.25
0.25-0.30
0.30-0.35
0.35-0.40
0.40-0.45
0.45-0.50
0.50-0.55
0.55-0.60
0.60-0.65
0.65-0.70
0.70-0.75
0.75-0.80
0.80-0.85
0.85-0.90
0.90-0.95
0.95-1.00


Frequency distribution
at a = 51749 x_0 = 54321
Total = 324
18
17
16
15
14
0.00-0.05
0.05-0.10
0.10-0.15
0.15-0.20
0.20-0.25
0.25-0.30
0.30-0.35
0.35-0.40
0.40-0.45
0.45-0.50
0.50-0.55
0.55-0.60
0.60-0.65
0.65-0.70
0.70-0.75
0.75-0.80
0.80-0.85
0.85-0.90
0.90-0.95
0.95-1.00


Frequency distribution
at a = 51749 x_0 = 123456
Total = 162
11
10
9
8
7
6
5
0.00-0.05
0.05-0.10
0.10-0.15
0.15-0.20
0.20-0.25
0.25-0.30
0.30-0.35
0.35-0.40
0.40-0.45
0.45-0.50
0.50-0.55
0.55-0.60
0.60-0.65
0.65-0.70
0.70-0.75
0.75-0.80
0.80-0.85
0.85-0.90
0.90-0.95
0.95-1.00

Observation

The numbers generated are distributed evenly (almost) across the range [0,1), as observed for the cases with a = 1597 (i.e. x_0 = 1234, 4321, 12345), in the tables of frequency distributions and their bar-diagrams/bar-graphs.

While they are a little less evenly distributed (or say a pattern seems to form with regions of higher probability and those of lower probability), as observed for the cases with a = 51749 (i.e. x_0 = 54321, 123456), in similar aforementioned grounds.

Problem Statement 3

Generate a sequence u_i with a = 1229, b = 1, m = 2048. Plot in a two-dimensional graph the points (u_{i-1}, u_i), i.e., the points (u_1, u_2), (u_2, u_3), (u_3, u_4), $\cdots$ What are your observations?

Program

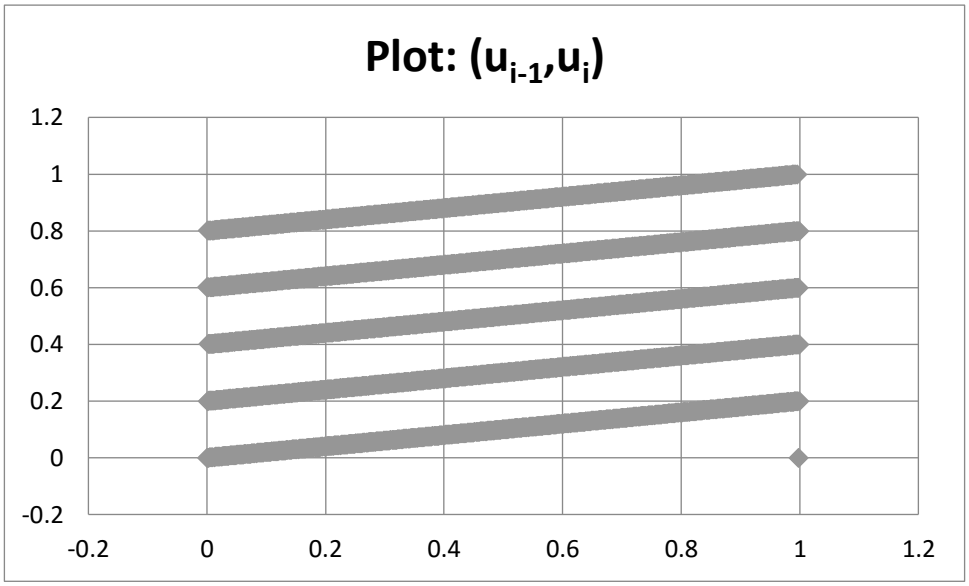
Code in C

```
#include<stdio.h>
void main(){
	int x0, x;
	float u, u1;
	printf("x0 ? ");
	scanf("%d", &x0);
	x = x0;
	u1 = (x/(float)2048);
	do{
		u = u1;
		x = (((1229 * x) + 1) % 2048);
		u1 = (x/(float)2048);
		printf("(%g,%g)\t", u, u1);
	}while(x != x0);
	printf("\n");
}
```

Output

Plot

With the values of a = 1229, b = 1 m = 2048, the plot of the sequence (u_{i-1}, u_i) is:



Observation

The sequence of the numbers u_i generated are pseudorandom instead of random.

If they were truly i.i.d. Uniform(0,1) we'd see points randomly dispersed in the unit square, in the plot of sequence (u_{i-1}, u_i). But instead the points fall entirely on some definite region (lines).