Beta dis

$$f(x) = \frac{1}{B(\alpha_1, \alpha_2)} x^{\alpha_1 - 1} (1-x)^{\alpha_2 - 1}$$

PAGE:

DATE:

$0 \le x \le 1$

$$B(\alpha_1, \alpha_2) = \frac{\Gamma\alpha_1 \, \Gamma\alpha_2}{\Gamma(\alpha_1 + \alpha_2)}$$

To maximize $f(x)$,

$$\ln f = \ln C + (\alpha_1 - 1)\ln x + (\alpha_2 - 1)\ln(1-x)$$

$$x^{\circ} = \frac{\alpha_1 - 1}{\alpha_1 + \alpha_2 - 2}$$

Alg

1. Generate U_1, U_2 $\sim$ uniform $(0,1)$

2. ~~Step~~, Accept U_1 if
$cU_2 \le f(U_1)$
or goto 1

Normal dis

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad -\infty < x < \infty$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$X_1 \sim \phi(x)$

$X = \sigma X_1 + \mu$

$X_1 \sim N(0,1)$

Scanned by CamScanner

Laplace dis

so, $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

$g(x) = \frac{1}{2} e^{-|x|}$

cdf of X: $G(x) = \int_{-\infty}^{x} \frac{1}{2} e^{-|x|} dx$

$$= \begin{cases} \frac{1}{2} e^{x} & x < 0 \\ 1 - \frac{1}{2} e^{-x} & x > 0 \end{cases}$$

$u \sim$ unif $(0,1)$

$u < \frac{1}{2} \Rightarrow u = \frac{1}{2} e^{x} \Rightarrow x = \ln 2u$

$u > \frac{1}{2} \Rightarrow u = 1 - \frac{1}{2} e^{-x}$

$\Rightarrow x = -\ln\{2(1-u)\}$

Use laplace for Normal.

$$\frac{\phi(x)}{g(x)} = \frac{2}{\sqrt{2\pi}} e^{-\frac{x^2}{2} + |x|}$$

$$= \sqrt{\frac{2e}{\pi}} e^{-(|x|-1)^2} \leq \sqrt{\frac{2e}{\pi}} = c$$

Scanned by CamScanner

PAGE:

DATE:

Generate $X_1 \sim g(\ )$ & $U \sim unif(0,1)$

Accept X_1 if $cU\, g(X_1) \le f(X_1)$

X_i	1	2	3	4	5	
	p_1	p_2	p_3	p_4	p_5	$\Sigma p = 1$
$u = \cdot 2$	$\parallel$	$\parallel$				
	$1/N$	$1/N$	$1/N$			

$$P[X \le x] = F_n(x) = \begin{cases} 0 & x < 1 \\ 1/N & 1 \le x < 2 \\ 2/N & 2 \le x < 3 \\ \vdots & \\ 1 & \end{cases}$$

$u = 0.25$

1.25

for $u \in [0,1)$

~~inf {x : F(x) ≥ u}~~

X inf $\{x : F(x) \ge u\}$

$X = \lfloor Nu \rfloor + 1$ if $Nu \notin Int$

if

X_i	x_1	x_2	x_3
	p_1	p_2	p_3

$X = X_{(Int(Nu)\, +1)}$

if $Nu \notin Z$

else

$X = X_{(Nu)}$

Scanned by CamScanner

PAGE:

DATE:

~~Generate~~

Step 1 Generate U_1 , U_2

2 Set $X = \text{Int}(U_1 N) + 1$

3 Accept X if

if $\cancel{C U_2}$ $C \cdot U_2 \cdot \frac{1}{N} \leq p_x$

$$C = \max_i \frac{p_i}{1/N}$$

Half Normal

$$f(x) = 2\phi(x)$$
$$= \frac{2}{\sqrt{2\pi}} e^{-x^2/2}$$

$g(x) = e^{-x}$ $X = -\ln(1-u)$

$$\frac{f(x)}{g(x)} = \sqrt{\frac{2}{\pi}} e^{-x^2/2 + x}$$

$$c = \sqrt{\frac{2e}{\pi}}$$

~~A~~ Accept X , u

$$cu\,g(x) \leq f(x)$$

Theoretical acceptance probability = $\frac{1}{c}$