MA 226 - Assignment Report 7

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Q 1. Generate 50 randam numbers from Geometric distribution of the form :

$$f(x;p) = pq^{i-1} \qquad i = 1,2,... \qquad 0 < p < 1.$$

Draw the probability mass function.

Solution: In Geometric distribution with parameter p, we have $P(X = i) = pq^{i-1}, i \leq 1$.
Note that the cumulative probability $P(X \leq j) = \sum_{i=1}^{j} P(X = i) = 1 - q^j$.
Generate $U \sim \mathcal{U}(0,1)$ and set $X = j$ if $1 - q^{j-1} \leq U < 1 - q^j$.
$U \sim \mathcal{U}(0,1) \Rightarrow (1-U) \sim \mathcal{U}(0,1)$.

$$\begin{aligned} X &= min\{j | q^j < 1 - U\} \\ &= min\{j | j \times log(q) < log(1-U)\} \\ &= min\{j | j > \frac{log(1-U)}{log(q)}\} \\ &= min\{j | j > \frac{log(1-U)}{log(q)}\} \\ &= \left\lceil \frac{log(1-U)}{log(q)} \right\rceil \end{aligned} \tag{1}$$

Code for R

```
genGeometric <- function(sample, p) {
   u <- runif(sample,0,1);
   G <- ceiling(log(u)/log(1-p));
   return (G);
}

set.seed(1);
p = runif(1,0,1); #Taking value of 'p', i.e. probability for success in a trial .

sample = 50;

G <- genGeometric(sample, p); #Generating Geometric Random Numbers.
Density <- p * ((1 - p)^(G - 1));

cat("The value of p taken is",p,".\n");
cat("The sample mean and variance, for the", sample, "random numbers generated from
    Geometric distribution, with parameter p =", p, ", are calculated to be", mean(G
    ),"and",var(G),"respectively.\n");

```

```
pdf("1.pdf");
#plot(G, Density, xlab = "x", ylab = "p(x)", main = paste("Probability mass function
    \nGeometric distribution , n =", sample), col = "red");
plot(G, Density, xlab = "x", ylab = "p(x)", main = "", col = "red");
legend("topright", legend = paste("parameter", p), lty = 0, col = "red", bty = 'n');
```

Results:

The plots can be shown as :

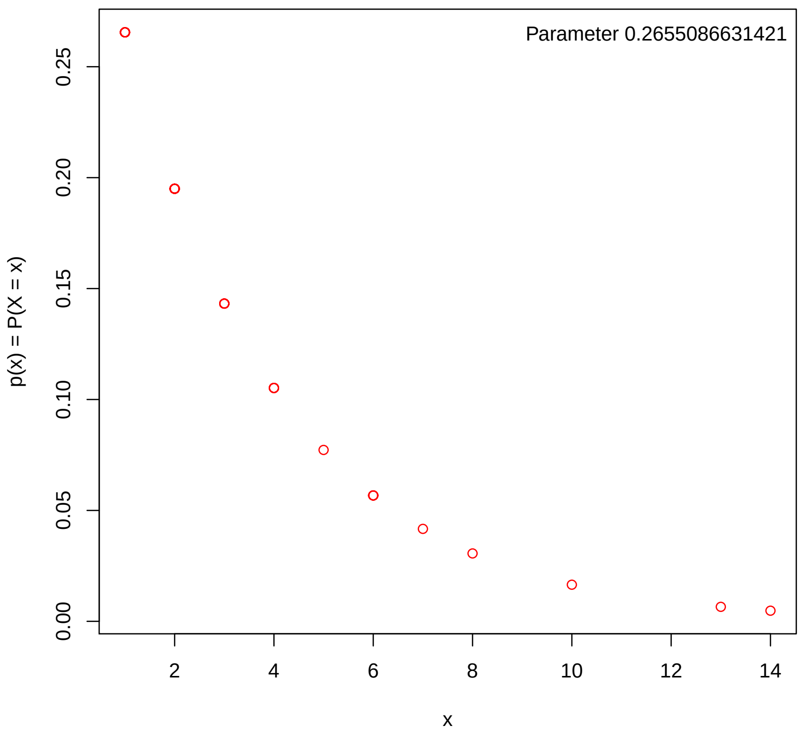


Figure 1: *Probability mass function for 50 samples from Geometric distribution*

The value of p taken is 0.2655087.

The sample mean and variance, for the 50 random numbers generated from Geometric distribution, with parameter p = 0.2655087, are calculated to be 3.42 ,and 8.615918 respectively.

These values are close to the theoretical ones, $\frac{1}{p} = \frac{1}{0.2655087} = 3.766355$, and $\frac{1-p}{p^2} = \frac{1-0.2655087}{0.2655087^2} = 10.41907$.

Q 2. Generate 50 random numbers from Poisson distribution with mean 2. Draw the probability mass function and the cumulative distribution function.

Solution: In the case of Poisson, we exploit the recursion property $p_{i+1} = \frac{\lambda}{i+1} p_i$, for $i \geq 0$.

Algorithm 1 Generating Random number from Poisson distribution with parameter λ.

1: Generate U from $\mathcal{U}(0,1)$.
2: Set $i = 0$, $p = e^{-l}$ and $F = p$.
3: **if** $U < F$ **then**
4: $X = i$.
5: **else**
6: Set $p = \frac{\lambda}{i+1} p$, $F = F + p$ and $i = i + 1$.
7: Return to step 3.
8: **end if**

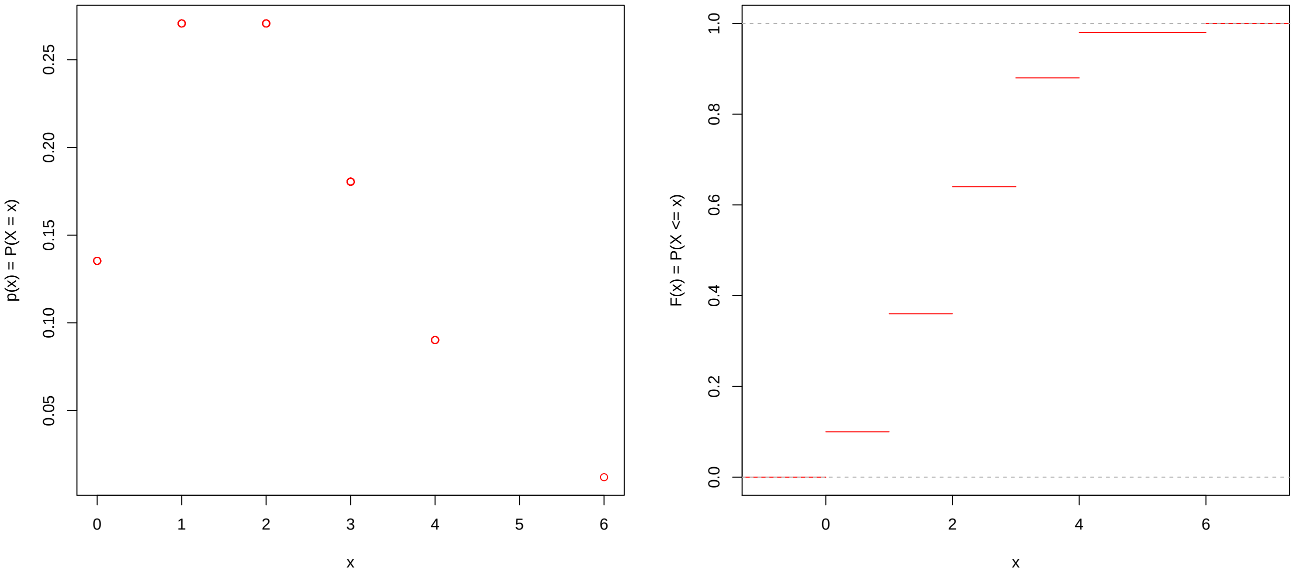
Code for R

```
genPoisson <- function(sample, l) {
   P <- vector(length = sample);
   for (j in 1:sample) {
      u = runif(1,0,1);
      i = 0;
      p = exp(-l);
      F = p;
      repeat {
         if (u < F) {
            P[j] = i;
            break;
         }
         else {
            p = ((l * p) / (i + 1));
            F = F + p;
            i = i + 1;
         }
      }
   }
   return (P);
}

set.seed(1);
```

```

sample = 50;
l = 2;

P <- genPoisson(sample, l);
s_P <- sort(P);

Density <- ((exp(-l) * (l^(P))) / factorial(P));

cat("The sample mean and variance, for the", sample, "Poisson random numbers generated, are calculated to be", mean(P), ",and", var(P), "respectively.\n");

pdf("2_pmf.pdf");
#plot(P, Density, xlab = "x", ylab = "p(x) = P(X = x)", main = paste("Probability mass function\nPoisson distribution (with mean 2), n =", sample), col = "red");
plot(P, Density, xlab = "x", ylab = "p(x) = P(X = x)", main = "", col = "red");

pdf("2_CDF.pdf");
#plot(ecdf(s_P), xlab = "x", ylab = "F(x) = P(X <= x)", do.points = FALSE, main = paste("Cumulative distribution function\nPoisson distribution (with mean 2), n =", sample), col = "red");
plot(ecdf(s_P), xlab = "x", ylab = "F(x) = P(X <= x)", do.points = FALSE, main = "", col = "red");
```

Results:

The plots can be shown as :



(a) *Probability mass function for 50 samples*

(b) *Cumulative Distribution Function for 50 samples*

Figure 2: *Poisson distribution with* $\lambda = 2$

The sample mean and variance, for the 50 Poisson random numbers generated, are calculated to be 2.06 ,and 1.649388 respectively. These values are close to the theoretical ones, $\lambda = 2$.

Q 3. Draw the histogram based 50 generated random numbers from the mixture of two Weibull distributions :

$$f(x;\beta_1,\theta_1,\beta_2,\theta_2,p) = pf_1(x;\beta_1,\theta_1) + (1-p)f_2(x;\beta_2,\theta_2)$$

where $f_1(.)$ and $f_2(.)$ are two Weibull distributions of the form : $f(x;\beta,\theta) = \beta\theta^\beta x^{\beta-1}e^{-(\theta x)^\beta}$ where, $\beta_1 = 2, \theta_1 = 1, \beta_2 = 1.5, \theta_2 = 1, p = 0.4$.

Solution: Consider simulating from a distribution with mass function $P(X = j) = \alpha p_j^{(1)} + (1-\alpha)p_j^{(2)}$, $j \leq 0$, $0 < \alpha < 1$.
If X_1 and X_2 are the random variables with respective mass functions $p_j^{(1)}$ and $p_j^{(2)}$, then

$$X = \begin{cases} X_1 & \text{with probability } \alpha \\ X_2 & \text{with probability } (1-\alpha) \end{cases}$$

Generating random number from Weibull distributions of the form $f(x;\beta,\theta) = \beta\theta^\beta x^{\beta-1}e^{-(\theta x)^\beta}$: Then $F(x;\beta,\theta) = 1 - e^{-(\theta x)^\beta}$. And, using Inverse Transform Method, $X = \frac{1}{\theta}(-log(1-U))^{\frac{1}{\beta}}$ where, $U \sim \mathcal{U}(0,1)$.

Algorithm 2 Generating random number from the mixture of given two Weibull distributions.

1: Generate U_1, U_2 from $\mathcal{U}(0,1)$.
2: **if** $U_1 < p$ **then**
3: Generate X_1 from the relation $X_1 = \frac{1}{\theta_1}(-log(1-U_2))^{\frac{1}{\beta_1}}$.
4: $X = X_1$.
5: **else**
6: Generate X_2 from the relation $X_2 = \frac{1}{\theta_2}(-log(1-U_2))^{\frac{1}{\beta_2}}$.
7: $X = X_2$.
8: **end if**

Code for R

```
genWeibull <- function(val, beta, theta) {
    return ((1/theta) * ((-log(1 - val))^(1/beta)));
}

genMix <- function(sample, beta_1, theta_1, beta_2, theta_2, p) {
    M <- vector(length = sample);
    for (i in 1:sample) {
        u <- runif(2,0,1);
        if (u[1] < p) {
            M[i] = genWeibull(u[2], beta_1, theta_1);
        }
        else {
            M[i] = genWeibull(u[2], beta_2, theta_2);
        }
    }
    return (M);
}

set.seed(1);

sample = 50;

M <- genMix(sample, 2, 1, 1.5, 1, 0.4);

cat("The sample mean and variance, for the", sample, "random numbers generated from mixture of the two given Weibull distributions, are calculated to be", mean(M), ",and", var(M),"respectively.\n");

pdf("3.pdf");
#hist(M, xlab = "x", breaks = 50, main = paste("Histogram of Mixed Distribution\nn =", sample), col = "red");
hist(M, xlab = "x", main = "", col = "red");
```

Results:

The histogram can be shown as :

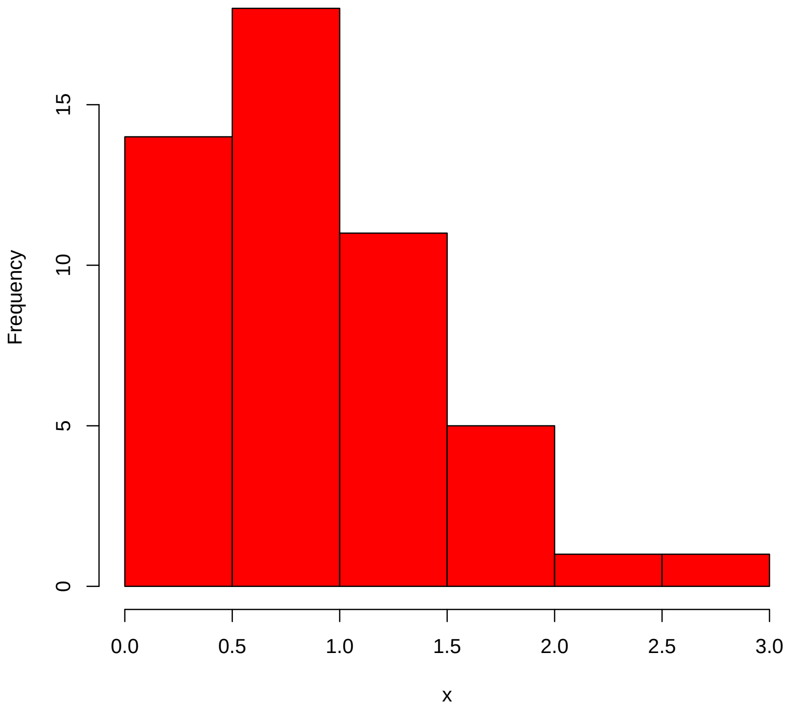


Figure 3: *Histogram of Mixed Distribution for 50 samples*

The sample mean and variance, for the 50 random numbers generated from mixture of the two given Weibull distributions, are calculated to be 0.8864908 ,and 0.305633 respectively.