MA 226 - Assignment Report 11

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A financial asset, the process $\{S(t)\}$ is a GBM with drift parameter μ, volatility parameter σ, and initial value $S(0)$ if

$$S(t) = S(0)exp([\mu - \frac{\sigma^2}{2}]t + \sigma W(t)),$$

where $\{W(t)\}$ a standard BM.

As with the case of a BM, we have a simple recursive procedure to simulate a GBM at $0 = t_0 < t_1 < \cdots t_n$ as

$$S(t_i + 1) = S(t_i)exp([\mu - \frac{\sigma^2}{2}](t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}Z_{i+1}),$$

where $Z_1, Z_2, \cdots, Z_n$ are independent $\mathcal{N}(0,1)$ variates.

In the interval [0, 5], taking both positive and negative values for μ and for at least two different values of σ^2, simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points for each path).

Also, by generating a large number of sample paths, compare the actual and simulated distributions of $S(5)$. Calculate expectation and variance of $S(5)$ and match it with the theoretical values.

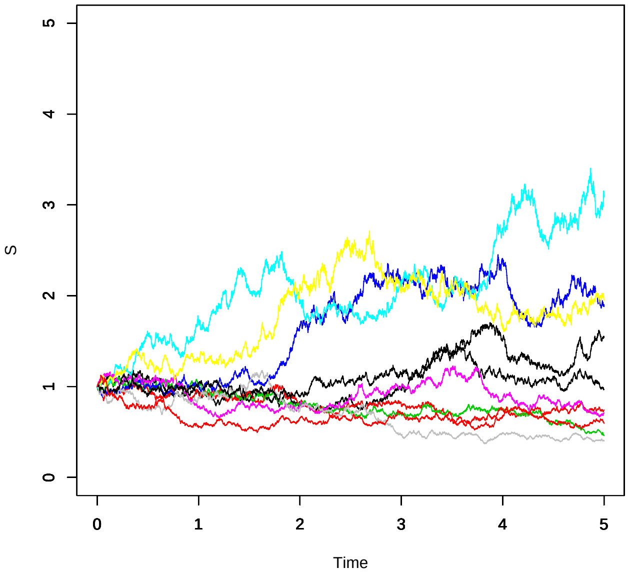
Code for R

```
sink("output.txt");

set.seed(1);

size = 5000;
n = 10;
m = 1000;
range_time  = 5;
dt = range_time/size;
sdt = sqrt(dt);

mu <- c(0.05, -0.05);
sigma <- c(0.25, 0.3);

S0 = 1;

for (p in 1:2) {
  for (q in 1:2) {
    S <- matrix(1, nrow = (size + 1), ncol = 10);

    for (i in 1:n) {
      for (j in 2:(size + 1)) {
        S[j,i] = S[j - 1,i] * exp(((mu[p] - (sigma[q]^2)/2) * dt) + (sigma[q] *
          sdt * rnorm(1, mean = 0, sd = 1)));
      }
    }

    pdf(paste("plot",p,q,".pdf"));
    for (i in 1:n) {
      plot(seq(0, 5, dt), S[,i], type = 'l', xlim = c(0,5), ylim = c(-1,5), col =
          i, verticals = FALSE, do.points = FALSE, main = "", xlab = "", ylab =
        "")
      par(new = TRUE)
    }
    title(ylab = 'S', xlab = 'Time');
#    legend('topright', legend = c(paste("mu =", mu[p]), paste("sigma =", sigma[q])
  ), lty = 0, col = "white", bty = 'n');

#    cat("\n\nTaking mu", mu[p], "and sigma", sigma[q],", and sample size", n,
  "::") ;
```

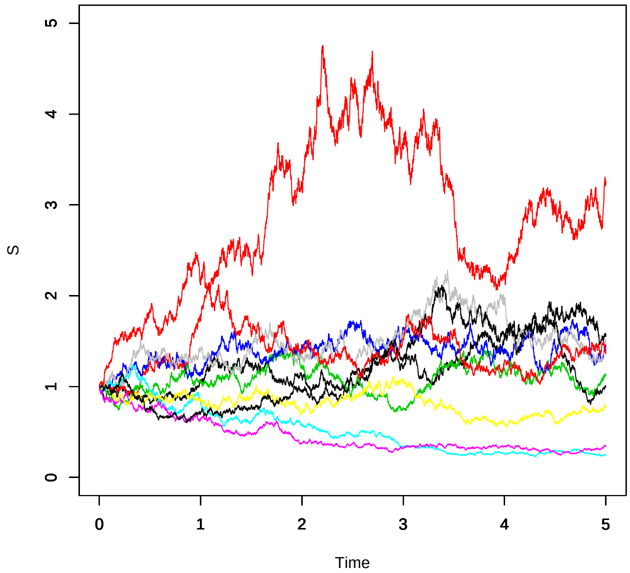
```
#     cat("\nSample expectation and variance of S(5) are estimated to be", mean(S[(5/dt + 1),]), ", and", var(S[(5/dt + 1),]), ", respectively.");
#     cat("\nWhile, theoretical expectation and variance of S(5) are", (S0 * exp(mu[p] * 5)), ", and", (S0 * exp(2 * mu[p] * 5) * (exp((sigma[q]^2) * 5) - 1)), ", respectively.");

      s <- vector(length = m);
      for (i in 1:m) {
         s[i] = 1;
         for (j in 2:(size + 1)) {
            s[i] = s[i] * exp(((mu[p] - (sigma[q]^2)/2) * dt) + (sigma[q] * sdt * rnorm(1, mean = 0, sd = 1)));
         }
      }
      cat("\n\nTaking mu", mu[p], "and sigma", sigma[q],", and sample size", m, "::") ;
      cat("\nSample expectation and variance of S(5) are estimated to be", mean(s), ", and", var(s), ", respectively.");
      cat("\nWhile, theoretical expectation and variance of S(5) are", (S0 * exp(mu[p] * 5)), ", and", (S0 * exp(2 * mu[p] * 5) * (exp((sigma[q]^2) * 5) - 1)), ", respectively.");

   }
}

sink();
```

Results

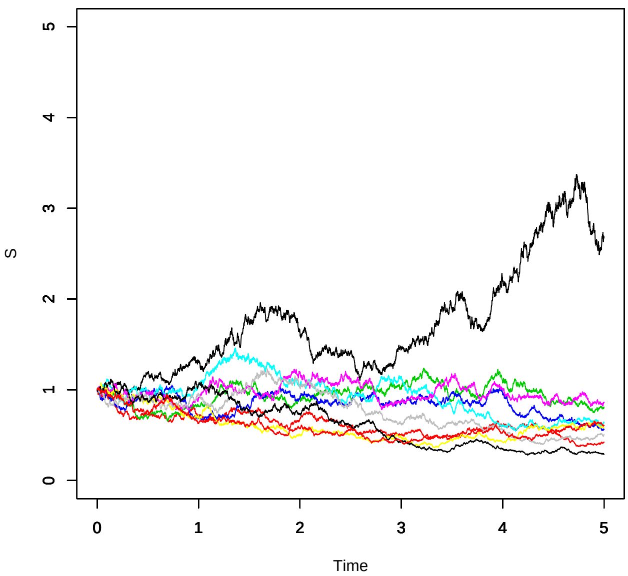
The plots of the sample paths generated for the Geomeric Brownian Motion ::



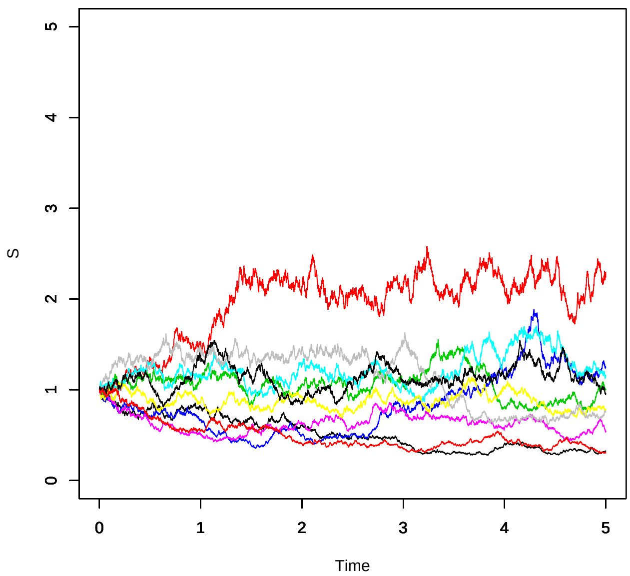
(a) $\mu = 0.05$ *and* $\sigma = 0.25$



(b) $\mu = 0.05$ *and* $\sigma = 0.3$



(c) $\mu = -0.05$ *and* $\sigma = 0.25$



(d) $\mu = -0.05$ *and* $\sigma = 0.3$

Figure 1: *Plots of sample paths generated for the Geomeric Brownian Motion*

Comparision of the theoretical and simulated distributions of $S(5)$, i.e. expectations and variances ::

μ	σ	Expectation		Variance	
		Sample	Theoretical	Sample	Theoretical
0.05	0.25	1.280236	1.284025	0.5402041	0.6048135
	0.3	1.32613	1.284025	1.094235	0.9369884
-0.05	0.25	0.7602637	0.7788008	0.2082394	0.2224985
	0.3	0.7724962	0.7788008	0.3620434	0.3446988

Table 1: *Expectation and Variance of $S(5)$, taking 1000 samples.*