MA 226 - Assignment Report 10

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Q 1. Generate 10 sample paths for the standard Brownian Motion in the time interval [0, 5] using the recursion

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i} \cdot Z_{i+1}$$

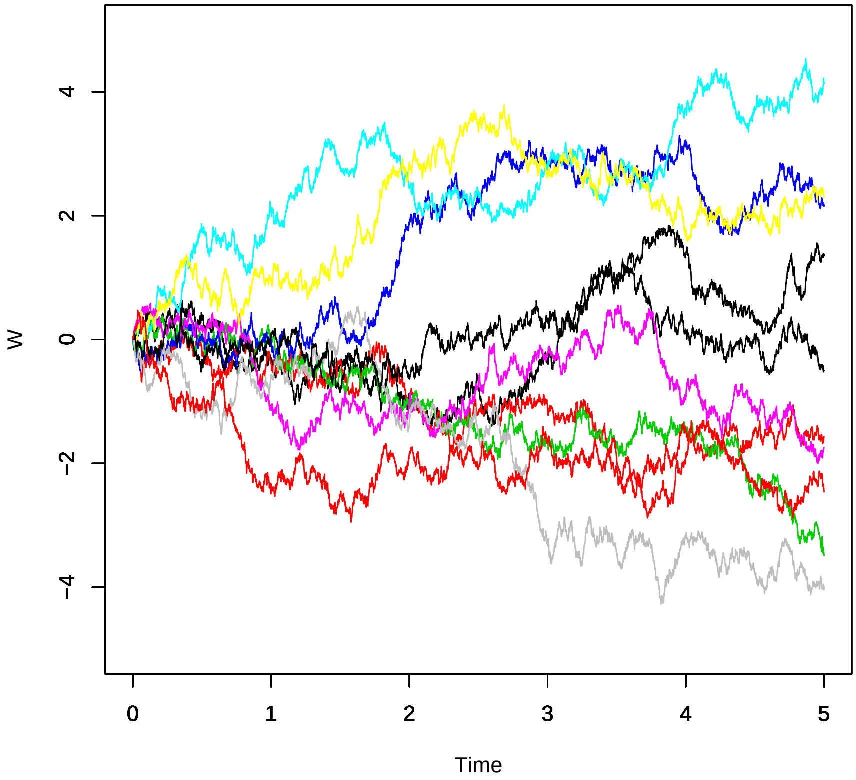
with 5000 generated values for each of the paths where $Z_i \sim \mathcal{N}(0,1)$. Plot all the sample paths in a single figure. Also estimate E[W(2)] and E[W(5)] from the 10 paths that you have generated.

Code for R

```
set.seed(1);

size = 5000;
n = 10;
range_time  = 5;
dt = range_time/size;
sdt = sqrt(dt);

#No matrix assigment, because size will be too large.
W <- vector(length = size+1);
W[1] = 0;
w2 = 0; w5 = 0;

pdf("1.pdf");
for (i in 1:n) {
    for (j in 2:(size + 1)) {
        W[j] = W[j-1] + (sdt * rnorm(1, mean = 0, sd = 1));
    }
    w2 = w2 + (W[(2/dt) + 1]/n);
    w5 = w5 + (W[(5/dt) + 1]/n);
    plot(seq(0, 5, dt), W, type = 'l', xlim = c(0,5), ylim = c(-5,5), col = i, verticals = FALSE, do.points = FALSE, main = "", xlab = "", ylab = "")
    par(new = TRUE)
}
title(ylab = 'W', xlab = 'Time');

cat("\nE[W(2)] =", w2, "\nE[W(5)] =", w5, '\n');
```

Results:

The plot of the sample paths generated for the standard Brownian Motion ::



The values of E[W(2)] and E[W(5)] are estimated to be -0.06122638 and -0.3858699, respectively, from the sample paths generated.

Q 2. Repeat the above exercise with the following Brownian motion ($BM(\mu, \sigma^2)$) discretization

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}.$$

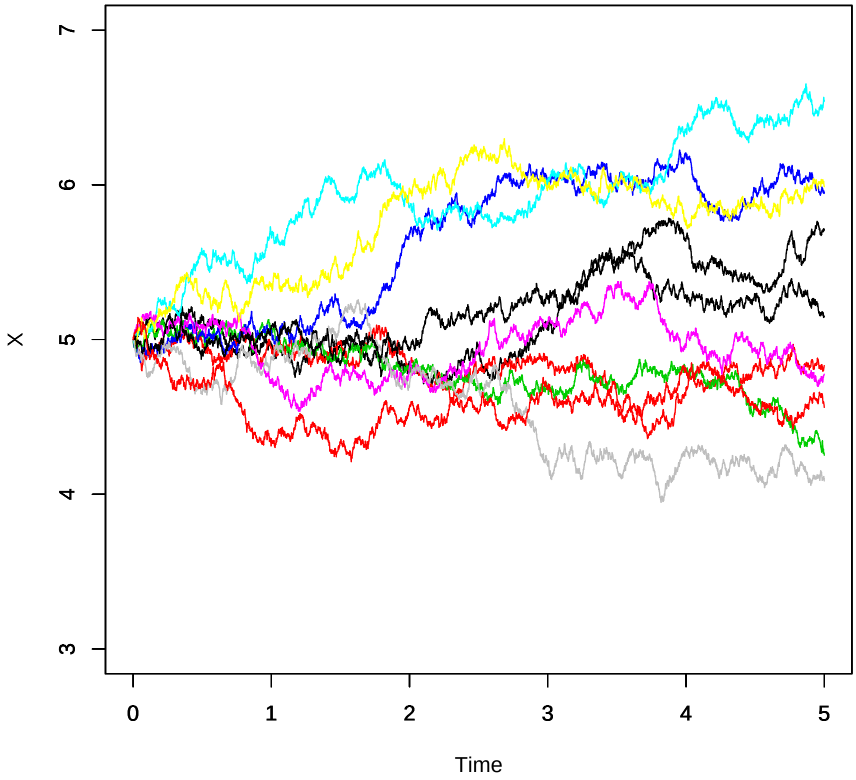
Take $X(0) = 5$, $\mu = 0.06$ and $\sigma = 0.3$.

Code for R

```
set.seed(1);

size = 5000;
n = 10;
range_time  = 5;
dt = range_time/size;
sdt = sqrt(dt);

mu = 0.06;
sigma = 0.3;

#No matrix assigment, because size will be too large.
X <- vector(length = size+1);
X[1] = 5;
x2 = 0; x5 = 0;

pdf("2.pdf");
for (i in 1:n) {
    for (j in 2:(size + 1)) {
       X[j] = X[j-1] + (mu * dt) + (sigma * sdt * rnorm(1, mean = 0, sd = 1));
    }
    x2 = x2 + (X[(2/dt) + 1]/n);
    x5 = x5 + (X[(5/dt) + 1]/n);
    plot(seq(0, 5, dt), X, type = 'l', xlim = c(0,5), ylim = c(3,7), col = i,
        verticals = FALSE, do.points = FALSE, main = "", xlab = "", ylab = "")
    par(new = TRUE)
}
title(ylab = 'X', xlab = 'Time');

cat("\nE[X(2)] =", x2, "\nE[X(5)] =", x5, '\n');
```

Results:

The plot of the sample paths generated for the Brownian Motion with drift μ and diffusion coefficient σ^2 ::



The values of E[X(2)] and E[X(5)] are estimated to be 5.101632 and 5.184239, respectively, from the sample paths generated.

Q 3. The Euler approximated recursion with time dependent μ and σ is given by

$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}.$$

Repeat the above exercise by taking

$$Y(0) = 5, \mu(t) = 0.0325 - 0.05t, \sigma(t) = 0.012 + 0.0138t + 0.00125t^2.$$

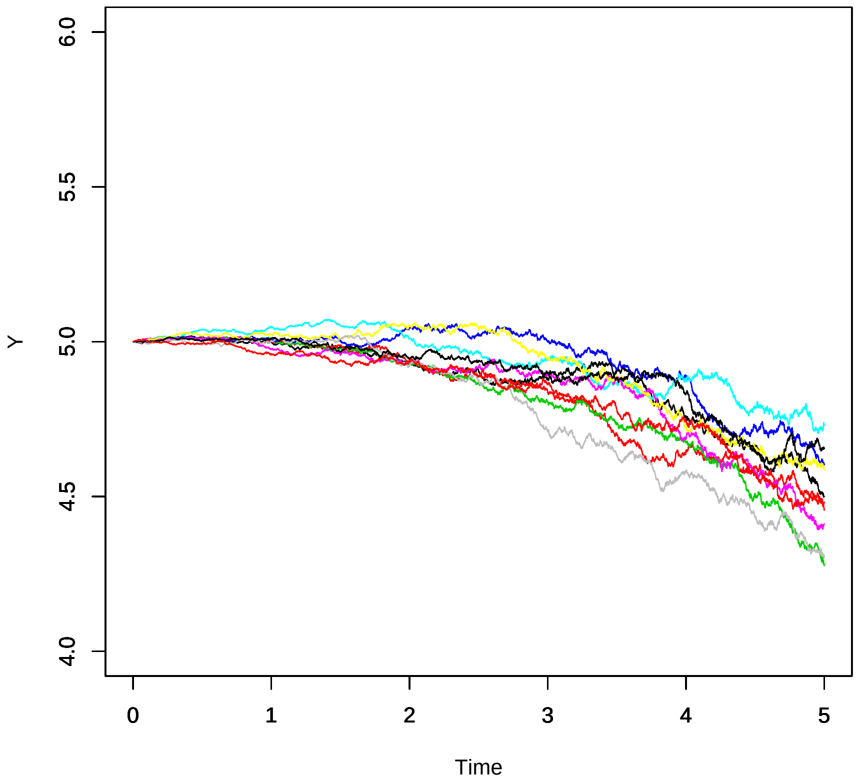
Code for R

```
mu <- function(j){
    return (0.0325 - (0.05*t));
}
sigma <- function(j) {
    return (0.012 + (0.0138*t) + (0.00125*(t^2)));
}

set.seed(1);

size = 5000;
n = 10;
range_time  = 5;
dt = range_time/size;
sdt = sqrt(dt);

#No matrix assigment, because size will be too large.
Y <- vector(length = size+1);
Y[1] = 5;
y2 = 0; y5 = 0;

pdf("3.pdf");
for (i in 1:n) {
    for (j in 2:(size + 1)) {
        t = ((j - 1) * dt);
        Y[j] = Y[j-1] + (mu(t) * dt) + (sigma(t) * sdt * rnorm(1, mean = 0, sd = 1));
    }
    y2 = y2 + (Y[(2/dt) + 1]/n);
    y5 = y5 + (Y[(5/dt) + 1]/n);
    plot(seq(0, 5, dt), Y, type = 'l', xlim = c(0,5), ylim = c(4,6), col = i,
        verticals = FALSE, do.points = FALSE, main = "", xlab = "", ylab = "")
    par(new = TRUE)
```

```
}
title(ylab = 'Y', xlab = 'Time');

cat("\nE[Y(2)] =", y2, "\nE[Y(5)] =", y5, '\n');
```

Results:

The plot of the sample paths generated for the Brownian Motion with time dependent drift $\mu(t)$ and diffusion coefficient $\sigma^2(t)$ ::



The values of E[Y(2)] and E[Y(5)] are estimated to be 4.96492 and 4.499891, respectively, from the sample paths generated.