MA 226 - Assignment Report 4

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Q 1 Simulate 5000 sample of exponential with mean 5. Draw the histogram and the calculate the mean, maximum and minimum. (Use R and C/C++)

Solution: We can write the cumulative density function of Exponential distribution as

$$F_{Exp}(x, \lambda) = (1 - e^{-\lambda x}).$$

To apply inverse transform method we equate $u = (1 - e^{-\lambda x}) \Rightarrow x = -\frac{1}{\lambda}\log(1-u)$ where u is an observation from Uniform(0,1).

Algorithm 1 Generating Random number from Exponential distribution

1: Generate U from $\mathcal{U}[0,1]$.

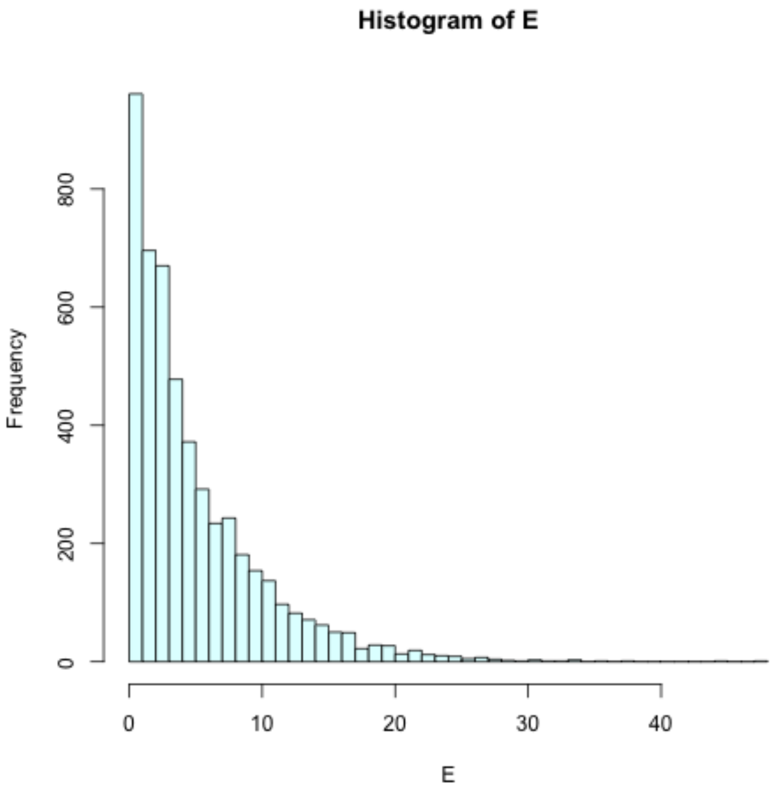
2: Generate X from the following relation $X = -\frac{1}{\lambda}\log(1-U)$.

Code for R

```
genExp<-function(sample, mean) {
  E<-vector(length = sample);

  set.seed(1);
  u<-runif(sample, 0, 1);

  lambda = 1 / mean
  E = ((-1 / lambda) * log(1 - u));

  png("1.png");
  hist(E, breaks = 50, col = "light cyan", plot = TRUE);

  return(c(mean(E), max(E), min(E)));
}

output<-genExp(5000, 5);

cat("The mean, maximum, and minimum are calculated to be", output[1], ",", output
    [2], ", and", output[3], "respectively.\n");
```

Results:

The histogram can be shown as :



(a) ξ_1

Figure 1: *Histogram for n = 5000*

The mean, maximum, and minimum are calculated to be 5.035687 , 47.87766 , and 0.001002004 respectively.

Q 2 Generate 5000 sample from Gamma with parameter $n = 5$ and $\lambda = 5$. Draw the histogram and the calculate the mean, maximum and minimum. (Use R and C/C++)

Solution: For a Gamma(n, λ) random variable, note that because we cannot write an explicit form for the expression of the F^{-1} , it is diffcult to directly apply inverse transform method. However, recall that the sum of independent Exponentials leads us to a Gamma distribution. We can generate from a Gamma distribution by generating n randomnumbers $U_1, U_2, ..., U_n$ and then setting

$$X = -\frac{1}{\lambda}\log(U_1) - ... - \frac{1}{\lambda}\log(U_n) = -\frac{1}{\lambda}\log(U_1...U_n)$$

This works provided n is an integer.

Algorithm 2 Generating Random number from Gamma distribution

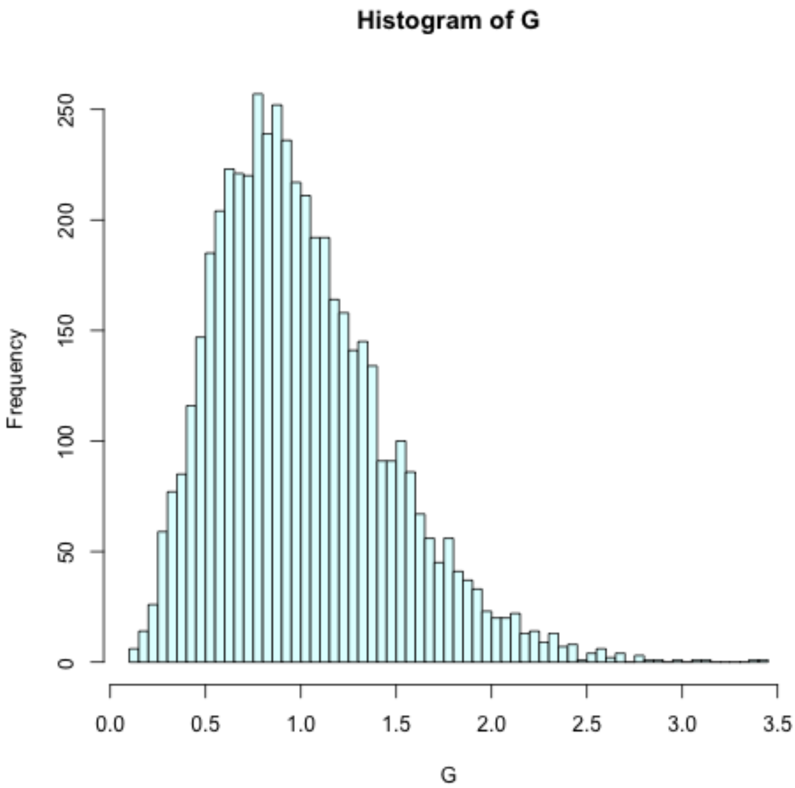
1: Generate $U_1, U_2, ..., U_n$ from $\mathcal{U}[0, 1]$.
2: Generate X from the following relation $X = -\frac{1}{\lambda}\log(U_1...U_n)$.

Code for R

```
genGamma<-function(sample, n, lambda) {
  G<-vector(length=sample);

  set.seed(1);

  for (i in 1:sample) {
    u<-runif(n, 0, 1);
    G[i] = prod(u);
  }

  G = (-1 / lambda) * log(G);

  png("2.png");
  hist(G, breaks = 50, col = "light cyan", plot = TRUE);

  return(c(mean(G), max(G), min(G)));
}

output<-genGamma(5000, 5, 5);

cat("The mean, maximum, and minimum are calculated to be", output[1], ",", output
    [2], ", and", output[3], "respectively.\n");
```

Results:

The histogram can be shown as :



(a) ξ_1

Figure 2: *Histogram for n = 5000*

The mean, maximum, and minimum are calculated to be 1.00498 , 3.447011 , and 0.1211753 respectively.

Q 3 Use the rejection method to generate from

$$f(x) = 20x(1-x)^3, \qquad 0 < x < 1.$$

(Use R and C/C++)

Solution: For generating random number from $f(x) = 20x(1-x)^3$ {where $0 < x < 1$} by acceptance-rejection method we choose Uniform(0,1) as our candidate distribution. We write the probability density function of Uniform distribution as $g(x) = 1$.
We choose c maximizing $\frac{f(x)}{g(x)} = \frac{20x(1-x)^3}{1}$.
Maximum of c will attain at $x = \frac{1}{4}$ i.e. $c = \frac{135}{64}$.

Algorithm 3 Generating random number from $f(x) = 20x(1-x)^3$ {where $0 < x < 1$} by acceptance-rejection method

1: Generate U_1, U_2 from $\mathcal{U}[0,1]$.
2: **if** $U_2 \le \frac{f(U_1)}{cg(U_1)} = \frac{20U_1(1-U_1)^3}{c}$ **then**
3: $X = U_1$.
4: **else**
5: return to step 1.
6: **end if**

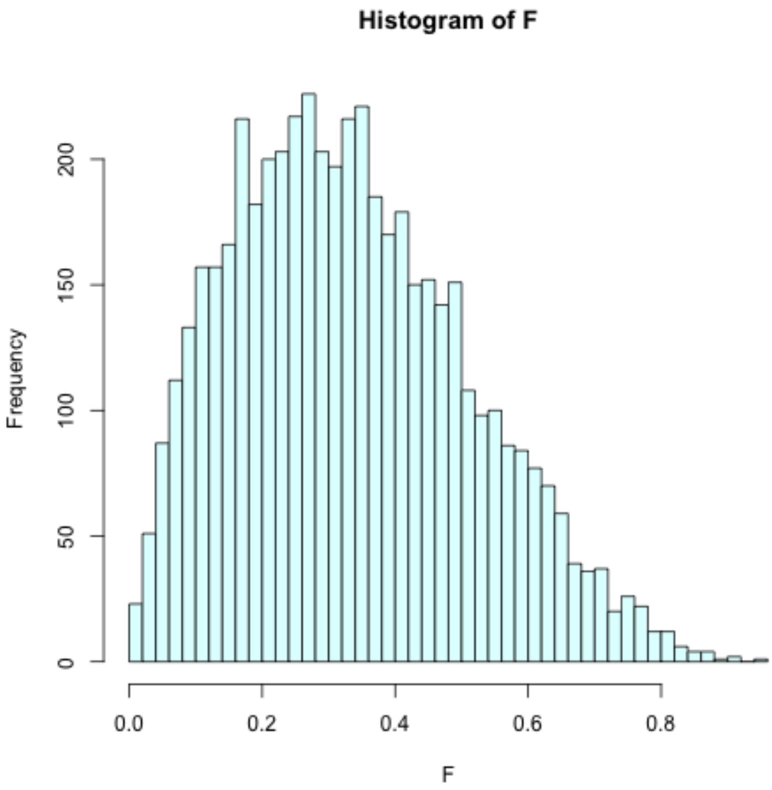
Code for R

```
f<-function(x) {
  return (20 * x * ((1 - x)^3));
}

genF<-function(sample) {

  F<-vector(length = sample);

  c = (135 / 64);

  set.seed(1);

  for (i in 1:sample) {
    repeat {
      u<-runif(2, 0, 1);
      if(u[2] <= (f(u[1])/c)){
```

```
                F[i] = u[1];
                break;
            }
        }
    }

    png("3.png");
    hist(F, breaks = 50, col = "light cyan", plot = TRUE);

    return(c(mean(F), var(F), max(F), min(F)));
}

output<-genF(5000);

cat("The mean, variance, maximum, and minimum are calculated to be", output[1], ",", output[2], ",", output[3], ", and", output[4], "respectively.\n");
```

Results:

The histogram can be shown as :



(a) ξ_1

Figure 3: *Histogram for n = 5000*

The mean, variance, maximum, and minimum are calculated to be 0.333445 , 0.03102842 , 0.9569733 , and 0.00372441 respectively.