Machine Learning for Econometrics

Debiased/Double Machine Learning

Anthony Strittmatter

Reference

Chernozhukov, Chetverikov, Demirer, Duflo, Hansen, and Newey (2017): "Double/Debiased/Neyman Machine Learning of Treatment Effects", American Economic Review, 107 (5), pp. 261-265, download.

Overview

Potential Outcome Framework

Notation:

- D_i binary treatment dummy (e.g., assignment to training program)
- $Y_i(1)$ potential outcome under treatment (e.g., earnings under participation in training)
- $Y_i(0)$ potential outcome under non-treatment (e.g., earnings under non-participation in training)

Infeasible parameter:

- Individual causal effect: $\delta_i = Y_i(1) - Y_i(0)$

Feasible parameters:

- Average Treatment Effect (ATE): $\delta = E[Y_i(1) - Y_i(0)] = E[\delta_i]$
- Average Treatment Effect on the Treated (ATET): $\rho = E[\delta_i | D_i = 1]$

Identifying Assumptions for ATE

- **Stable Unit Treatment Value Assumption (SUTVA):**

$$Y_i = Y_i(1)D_i + Y_i(0)(1 - D_i)$$

- **No Support Problems:**

$$\epsilon < Pr(D_i = 1|X_i = x) = p(x) < 1 - \epsilon$$

for some small $\epsilon > 0$ and all x in the support of X_i

- **Conditional Independence Assumption (CIA):**

$$Y_i(1), Y_i(0) \perp\!\!\!\perp D_i | X_i = x$$

for all x in the support of X_i

Modified Outcome Method for ATE

Inverse Probability Weighting:

$$Y^*_{i,IPW} = \frac{D_i}{p(X_i)} Y_i - \frac{1 - D_i}{1 - p(X_i)} Y_i = \frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i$$

with the propensity score $p(x) = Pr(D_i = 1 | X_i = x)$.

ATE: $\delta = E[Y^*_{i,IPW}]$ and $\hat{\delta} = \dfrac{1}{N} \sum_{i=1}^{N} \hat{Y}^*_{i,IPW}$

Proof of Identification

$$
\begin{aligned}
\delta &= E\left[Y_i(1)\right] - E\left[Y_i(0)\right] \overset{LIE}{=} \int E\left[Y_i(1)|X_i = x\right] - E\left[Y_i(0)|X_i = x\right] f_X(x)dx \\
&\overset{CIA}{=} \int E\left[Y_i(1)|D_i = 1, X_i = x\right] - E\left[Y_i(0)|D_i = 0, X_i = x\right] f_X(x)dx \\
&= \int E\left[Y_i|D_i = 1, X_i = x\right] - E\left[Y_i|D_i = 0, X_i = x\right] f_X(x)dx \\
&= \int E\left[D_i Y_i|D_i = 1, X_i = x\right] - E\left[(1 - D_i)Y_i|D_i = 0, X_i = x\right] f_X(x)dx \\
&\overset{LIE}{=} \int E\left[\frac{D_i Y_i}{p(X_i)}\middle| X_i = x\right] - E\left[\frac{(1 - D_i)Y_i}{1 - p(X_i)}\middle| X_i = x\right] f_X(x)dx \\
&= \int E\left[\frac{D_i Y_i}{p(X_i)} - \frac{(1 - D_i)Y_i}{1 - p(X_i)}\middle| X_i = x\right] f_X(x)dx \\
&= \int E\left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i\middle| X_i = x\right] f_X(x)dx \overset{LIE}{=} E\left[\frac{D_i - p(X_i)}{p(X_i)(1 - p(X_i))} Y_i\right]
\end{aligned}
$$

Reference: Horvitz and Thompson (1952)

Modified Outcome Method with IPW

Advantages:

- Generic approach
- Sparsity assumptions can be avoided by appropriate choice of estimator for propensity score
- Heterogeneous treatment effects

Disadvantages:

- Potentially omitting "weak outcome confounders"
- Shows weak performance in simulations (Knaus, Lechner, and Strittmatter, 2018)
- Not $\sqrt{N}$-consistent in high-dimensional setting

Overview

Conditional Mean Differences

Identification:

$$
\begin{aligned}
\delta &= E[Y_i(1)] - E[_iY(0)] \\
&\overset{LIE}{=} \int E[Y_i(1)|X_i = x] - E[Y_i(0)|X_i = x] f_X(x)dx \\
&\overset{CIA}{=} \int E[Y_i(1)|D_i = 1, X_i = x] - E[Y_i(0)|D_i = 0, X_i = x] f_X(x)dx \\
&= \int \underbrace{E[Y_i|D_i = 1, X_i = x]}_{=\mu_1(x)} - \underbrace{E[Y_i|D_i = 0, X_i = x]}_{=\mu_0(x)} f_X(x)dx
\end{aligned}
$$

Estimator:

$$\hat{\delta} = \frac{1}{N} \sum_{i=1}^{N} (\hat{\mu}_1(X_i) - \hat{\mu}_0(X_i))$$

with $\hat{\mu}_1(x) = \hat{E}[Y_i|D_i = 1, X_i = x]$ and $\hat{\mu}_0(x) = \hat{E}[Y_i(0)|D_i = 0, X_i = x]$ being the estimated conditional expectations of the potential outcomes.

Overview

Inverse Probability Weighting

T-Learner

Double/Debiased Machine Learning

Double/Debiased Machine Learning (DML)

Efficient Score:

$$
\begin{aligned}
Y^*_{i,DML} &= \mu_1(X_i) - \mu_0(X_i) + \frac{D_i - p(X_i)}{p(X_i)(1-p(X_i))} Y_i - \frac{D_i}{p(X_i)} \mu_1(X_i) + \frac{1-D_i}{1-p(X_i)} \mu_0(X_i) \\
&= \mu_1(X_i) - \mu_0(X_i) + \frac{D_i(Y_i - \mu_1(X_i))}{p(X_i)} - \frac{(1-D_i)(Y_i - \mu_0(X_i))}{1-p(X_i)}
\end{aligned}
$$

ATE: $\delta = E[Y^*_{i,DML}]$ and $\hat{\delta} = \dfrac{1}{N}\sum_{i=1}^{N} \hat{Y}^*_{i,DML}$

We can use standard ML methods to estimate $\hat{\mu}_1(x)$, $\hat{\mu}_0(x)$, and $\hat{p}(x)$.

Reference: Chernozhukov et al., 2017

Proof of Identification

$$
\begin{aligned}
\delta =& E\left[\mu_1(X_i) - \mu_0(X_i) + \frac{D_i(Y_i - \mu_1(X_i))}{p(X_i)} - \frac{(1-D_i)(Y_i - \mu_0(X_i))}{1-p(X_i)}\right] \\
=& E\left[\frac{D_i - p(X_i)}{p(X_i)(1-p(X_i))} Y_i + \frac{(p(X_i) - D_i)\mu_1(X_i)}{p(X_i)} - \frac{(D_i - p(X_i))\mu_0(X_i)}{1-p(X_i)}\right] \\
=& \int E\left[\frac{D_i - p(X_i)}{p(X_i)(1-p(X_i))} Y_i + \frac{(p(X_i) - D_i)\mu_1(X_i)}{p(X_i)}\right. \\
& \left. - \frac{(D_i - p(X_i))\mu_0(X_i)}{1-p(X_i)} \right| X_i = x\Bigg] f_X(x)dx \\
=& \int \left(E\left[\left. \frac{D_i - p(X_i)}{p(X_i)(1-p(X_i))} Y_i \right| X_i = x\right] + \frac{E[p(X_i) - D_i | X_i = x]}{p(x)} \mu_1(x) \right. \\
& \left. - \frac{E[D_i - p(X_i) | X_i = x]}{1-p(x)} \mu_0(x) \right) f_X(x)dx \\
=& \int E\left[\left. \frac{D_i - p(X_i)}{p(X_i)(1-p(X_i))} Y_i \right| X_i = x\right] f_X(x)dx = E\left[Y_i(1) - Y_i(0)\right]
\end{aligned}
$$

Reference: Robins and Rotnitzki (1995)

DML Cross-Fitting Algorithm

1. Partition the data randomly in samples S^A and S^B
2. Estimate the nuisance parameters $\hat{\mu}_1^A(x)$,$\hat{\mu}_0^A(x)$, and $\hat{p}^A(x)$ in S^A; and $\hat{\mu}_1^B(x)$,$\hat{\mu}_0^B(x)$, and $\hat{p}^B(x)$ in S^B with ML
3. Calculate the efficient scores in samples S^A and S^B, respectively:

$$\hat{Y}_{i,DML}^{A*} = \hat{\mu}_1^B(X_i^A) - \hat{\mu}_0^B(X_i^A) + \frac{D_i^A(Y_i^A - \hat{\mu}_1^B(X_i^A))}{\hat{p}^B(X_i^A)} - \frac{(1 - D_i^A)(Y_i^A - \hat{\mu}_0^B(X_i^A))}{1 - \hat{p}^B(X_i^A)}$$

$$\hat{Y}_{i,DML}^{B*} = \hat{\mu}_1^A(X_i^B) - \hat{\mu}_0^A(X_i^B) + \frac{D_i^B(Y_i^B - \hat{\mu}_1^A(X_i^B))}{\hat{p}^A(X_i^B)} - \frac{(1 - D_i^B)(Y_i^B - \hat{\mu}_0^A(X_i^B))}{1 - \hat{p}^A(X_i^B)}$$

4. Calculate ATE,

$$\hat{\delta} = \frac{1}{2}(\underbrace{\hat{E}[\hat{Y}_{i,DML}^{A*}|S^A]}_{=\hat{\delta}_A} + \underbrace{\hat{E}[\hat{Y}_{i,DML}^{B*}|S^B]}_{=\hat{\delta}_B}),$$

Asymptotic Results for ATE

- Main Regularity Condition: Convergence rate of nuisance parameters is at least $\sqrt[4]{N}$.
- ATE can be estimated $\sqrt{N}$-consistently

$$\sqrt{N}(\hat{\delta} - \delta) \xrightarrow{d} N(0, \sigma^2)$$

with $\sigma^2 = Var(Y^*_{i,DML})$ and $Var(\hat{\delta}) = \sigma^2/N$

- Split sample estimator of σ^2

$$\hat{\sigma}^2 = \frac{1}{2}\left(\hat{\sigma}^2_A + (\hat{\delta}_A - \hat{\delta})^2\right) + \frac{1}{2}\left(\hat{\sigma}^2_B + (\hat{\delta}_B - \hat{\delta})^2\right)$$

for $\hat{\delta} = 1/2(\hat{\delta}_A + \hat{\delta}_B)$

Advantages of DML

Advantages compared to IPW and Conditional Mean Differences:

- Treatment and outcome equations are modelled explicitly
- Double robustness property
- $\sqrt{N}$-consistent and asymptotically normal even under high-dimensional confounding

Efficient Score for ATET

$$Y^*_{i,ATET} = \frac{D_i(Y_i - \mu_0(X_i))}{p} - \frac{p(X_i)(1 - D_i)(Y_i - \mu_0(X_i))}{p(1 - p(X_i))}$$

with $p = Pr(D_i = 1)$.

ATET: $\rho = E[Y^*_{i,ATET}]$ and $\hat{\rho} = \frac{1}{N}\sum_{i=1}^{N} \hat{Y}^*_{i,ATET}$

Estimator of Asymptotic Variance:

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{Y}^*_{i,ATET} - \hat{\rho}\right)^2$$

and $\hat{Var}(\hat{\rho}) = \hat{\sigma}^2/N$

References: Chernozhukov et al., 2017, Farrell, 2015

Proof of Identification for ATET

$$
\begin{aligned}
\rho =& E\left[\frac{D_i(Y_i-\mu_0(X_i))}{p} - \frac{p(X_i)(1-D_i)(Y_i-\mu_0(X_i))}{p(1-p(X_i))}\right] \\
=& \int E\left[\frac{D_iY_i}{p} - \frac{p(X_i)(1-D_i)Y_i}{p(1-p(X_i))} - \frac{(D_i-p(X_i))\mu_0(X_i)}{p(1-p(X_i))}\middle| X_i = x\right] f_X(x)dx \\
=& \int \Bigg(\frac{E[D_iY_i|X_i=x]}{p} - \frac{p(x)E[(1-D_i)Y_i|X_i=x]}{p(1-p(x))} \\
& - \frac{E[D_i-p(X_i)|X_i=x]}{p(1-p(x))}\mu_0(x)\Bigg) f_X(x)dx \\
=& \int \left(\frac{E[D_iY_i|X_i=x]}{p} - \frac{p(x)E[(1-D_i)Y_i|X_i=x]}{p(1-p(x))}\right) f_X(x)dx \\
=& \int \frac{p(x)}{p}\left(E[D_iY_i|D_i=1,X_i=x] - E[(1-D_i)Y_i|D_i=0,X_i=x]\right) f_X(x)dx \\
=& \int \left(E[Y_i(1)|D_i=1,X_i=x] - E[Y_i(0)|D_i=0,X_i=x]\right) f_{X|D=1}(x)dx \\
=& \int \left(E[Y_i(1)|D_i=1,X_i=x] - E[Y_i(0)|D_i=1,X_i=x]\right) f_{X|D=1}(x)dx \\
=& E[Y_i(1)-Y_i(0)|D_i=1]
\end{aligned}
$$