Theoretical modeling of erbium-doped fiber amplifiers with excited-state absorption

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Numerical solution of a general rate-equation model of an erbium-doped fiber amplifier highlights several important features of the device. A difference in strong-signal gain between copropagating and counterpropagating signal pump schemes in the presence of pump excited-state absorption is predicted, and this is confirmed by experiment. The detriment in the noise figure previously associated with counterpropagating schemes for only small-signal conditions is shown to be similar for strong signals, corresponding to a power-amplifier operating regime. In addition, a correction to the noise figure is given for the limit of a low-gain amplifier.

The erbium-doped silica fiber amplifier[1,2] is receiving much interest as an optical amplifier in the third optical telecommunications window near 1.55 μm. Advantages of erbium-doped fiber amplifiers include high gain that is polarization independent, the absence of Fresnel reflections, and quantum-limited noise behavior.[3] In addition, the recent demonstrations of excellent performance and near-100% quantum efficiency when pumped at wavelengths free from pump excited-state absorption[4] (ESA) indicate that compact and practical devices will soon become a reality.

In this Letter we present results of a full strong-signal numerical analysis of the amplifier to describe the noise and gain characteristics for both copropagating and counterpropagating signal pump schemes. Previous research has analyzed the problem on the basis of negligible amplified spontaneous emission (ASE) and ESA[5] or for small signals only in the counterpropagating scheme.[6] A comprehensive treatment of the effect of modal overlaps and ESA on small-signal gain has been given by Armitage,[7] and the spectral evolution of ASE has been given by Desurvire and Simpson.[8] In this research we make approximations with regard to modal overlaps and ASE in order to simplify the approach, although without sacrificing the limiting effects of either. We show that under strong-signal conditions the noise detriment associated with the counterpropagating scheme is similar to that of the small-signal case, and that for amplifiers operating in the low-gain limit the noise figure goes to unity, as expected from physical considerations. However, we also show that a difference in strong-signal gain is expected between the two possible pump schemes in the presence of pump ESA, with the counterpropagating scheme showing the greatest gain. This result is shown to agree with experimental measurements.

The behavior of the erbium amplifier, in common with other rare-earth-doped fiber devices, can be described in terms of rate equations for the population-inversion density, the pump field, the signal field, and ASE. If we assume rapid relaxation from pump bands into the metastable level and uniform dopant and field distributions across the fiber core, at a given axial position z in the fiber these can be written as

$$\frac{dN_2(z)}{dt} = W_p(z)[N_{\text{tot}} - N_2(z)] - W_s(z)[(1+\alpha)N_2(z) - \alpha N_{\text{tot}}] - \frac{N_2(z)}{\tau_{21}}, \quad (1)$$

$$\frac{dP_p{}^{+}(z)}{dz} = -P_p{}^{+}(z)\sigma_{\text{abs}}[N_{\text{tot}} - N_2(z)] - P_p{}^{+}(z)\sigma_{\text{ESA}}N_2(z), \quad (2)$$

$$\frac{dP_s{}^{+,-}(z)}{dz} = P_s{}^{+,-}(z)\gamma(z), \quad (3)$$

$$\frac{dP_f{}^{+,-}(z)}{dz} = \mu(z)h\nu\Delta\nu\gamma(z) + P_f{}^{+,-}(z)\gamma(z), \quad (4)$$

$$\text{local gain} \qquad \gamma(z) = \eta_s\sigma_{21}[(1+\alpha)N_2(z) - \alpha N_{\text{tot}}]. \quad (5)$$

Here N_2 and N_{tot} are the metastable-level population density and the total erbium dopant concentration, respectively, W_p is the pump rate, W_s is the stimulated emission rate, and τ_f is the metastable-level lifetime, where τ_f has been measured as (12 ± 0.2) msec for the two germanosilicate fibers used here. In equilibrium, i.e., $dN_2(z)/dt = 0$, Eq. (1) can be explicitly written as

$$N_2(z) = \frac{N_{\text{tot}}[W_p(z) + \alpha W_s(z)]}{W_p(z) + (1+\alpha)W_s(z) + 1/\tau_f}. \quad (6)$$

The pump and stimulated emission rates, $W_p(z)$ and $W_s(z)$, are given by

$$W_s(z) = \frac{P_s{}^{+,-}(z) + P_f{}^{+,-}(z)}{h\nu_s a}\sigma_{21}\eta_s, \quad (7)$$

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$$W_p(z) = \frac{P_p^{+}(z)}{h\nu_p a}\sigma_{abs}\eta_p, \qquad (8)$$

where a is the core area of the fiber and η_p and η_s are the proportion of the pump and signal powers propagated within the fiber core. P_p, P_s, and P_{f1} are the pump, signal, and ASE powers, respectively (the plus sign indicates copropagating and the minus sign indicates counterpropagating relative to the pump direction), and σ_{abs}, σ_{ESA}, and σ_{21} are the pump absorption, pump ESA, and stimulated emission cross sections, respectively. The term α is given by the ratio σ_{12}/σ_{21} at the signal wavelength. Although we are aware of an analysis that gives $\sigma_{12}/\sigma_{21} < 1$ for aluminosilicate fibers using a Fuchtbauer–Ladenburg relation,[8] for this fiber type by comparison of the small-signal umpumped attenuation with the small-signal gain under conditions of heavy pumping at 980 nm we find that $\alpha \approx 1.25 \pm 0.1$. In the expression describing ASE, $h\nu$ and $\Delta\nu$ are the photon energy and optical bandwidth of the ASE, respectively, and the term μ is defined as $N_2(z)/[(1+\alpha)N_2(z) - \alpha N_{tot}]$. The ASE bandwidth is taken to be 2 nm on the basis that ASE-induced saturation is experimentally observed to occur when the ASE spectrum has narrowed to approximately 2 nm. Hence, although this value will give ASE powers that are too low at low gains, the important limit of ASE-induced gain saturation is accounted for. These equations can be integrated numerically for arbitrary signal powers in the copropagating and counterpropagating configurations, and the solutions can be iterated to give a self-consistent condition for the signal, the ASE, and the pump fields.

Numerical solution of the above equations was performed for 665-nm pumping of a 0.21-N.A., 1050-nm cutoff germanosilicate fiber doped with $\approx 8.8 \times 10^{18}$ cm^{-3} Er^{3+} ions (400 parts in 10^6). A value of 6.5×10^{-21} cm^2 (Ref. 9) was used for σ_{21}, and σ_{abs} was inferred from spectral attenuation data of a highly multimode fiber to be approximately half this value. At a 665-nm pump wavelength the ratio $\sigma_{ESA}/\sigma_{abs}$ has been measured to be ≈ 0.2.[10] The overlap of pump radiation with the core was assumed to be complete owing to the multimode nature of the pump, and the signal overlap with the core (η_s) was taken to be 0.4. Figure 1 shows the predicted variation of gain with the fiber length for 0.1-μW and 100-μW signal inputs at 1.536 μm, for 100 mW of pump power, in the copropagating and counterpropagating pump schemes. Superimposed onto the theoretical curves are experimental values previously reported,[3] which were obtained with a fiber characterized by the above parameters. One interesting feature of the theoretical curves is the predicated difference in gain in the saturated regime between the copropagating and counterpropagating pump schemes. At the position of maximum gain the counterpropagating scheme is expected to show more gain, and this is confirmed in the experimental data for the 100-μW signal input. This feature is believed to result from a reduction of pump ESA owing to reduced inversion in the pump input end at high signal powers with the counterpropagating scheme. Using the overlap of $\eta_s = 0.4$ (slightly lower than the proportion of the power propagated in the core, 0.62, at this fiber V value[11]) the maximum gain for the two cases is predicted. However, the saturation region is experimentally observed to be more spread out than that predicted by the simple model. We believe this may be due to the interplay of the field and dopant distributions in the fiber, which have been generalized in this treatment.

In a similar way, numerical calculations were performed for the 980-nm pump wavelength that has recently[4] been shown to be highly efficient owing to the absence of pump ESA.[10] Experimental data were obtained with a different fiber, although it had nominally the same core composition as the previous fiber. The parameters that were different from the 665-nm pump case were N.A. = 0.16, a cutoff of 975 nm, an Er^{3+} concentration of 5.1×10^{18} cm^{-3}, $\sigma_{abs} = 4.9 \times 10^{-21}$ cm^2, $\sigma_{ESA}/\sigma_{abs} = 0$, and a pump power of 15 mW. The pump overlap with the dopant distribution was assumed to be 83%, which corresponds to the proportion of the fundamental mode propagated in the core[11] and the signal overlap of 0.37. Note that this signal overlap is in the same ratio (0.65) to the signal power propagated in the core[11] as with previous fiber. Figure 2 shows the experimental and theoretical data for the 980-nm pump wavelength obtained with the above fiber. A good match is again observed between theory and experiment. Note that in this case the theoretical

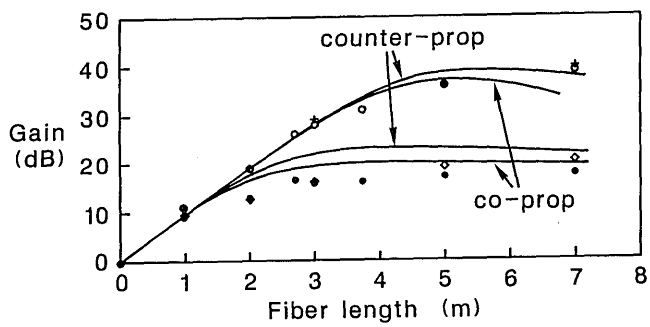

Fig. 1. Theoretical (curves) and experimental (points) gain versus the fiber length for 100 mW of pump power at 665 nm. Copropagating signal inputs were 0.1 μW (open circles) and 100 μW (filled circles) at 1.536 μm; counterpropagating signal inputs were 0.1 μW (crosses) and 100 μW (diamonds) at 1.536 μm.

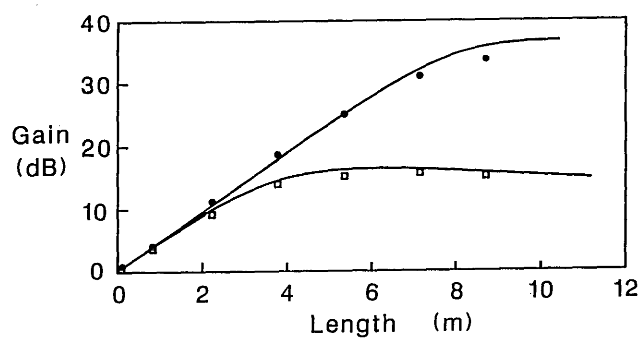

Fig. 2. Theoretical (curves) and experimental (points) gain versus the fiber length for 15 mW of pump power at 980 nm for copropagating and counterpropagating 0.1-μW (circles) and 100-μW (squares) signal inputs at 1.536 μm.

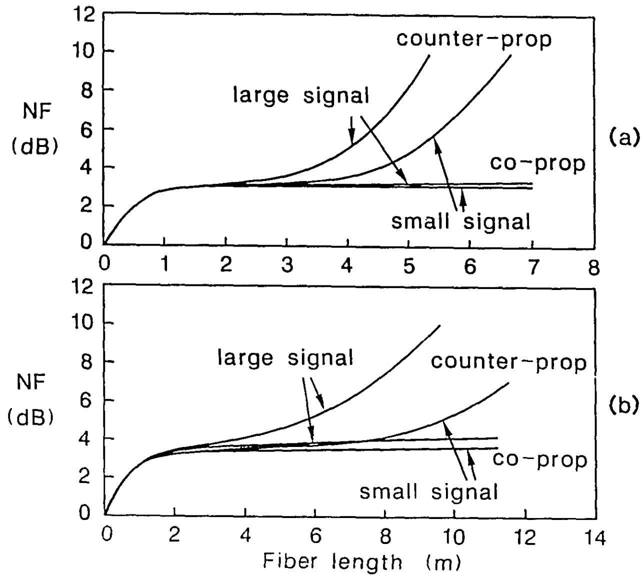

Fig. 3. Theoretical noise figure (NF) versus the fiber length for (a) 100 mW of pump power at 665 nm and (b) 15 mW of pump power at 980 nm. The signal inputs are 0.1 μW (small) and 100 μW (large) at 1.536 μm.

data for copropagating and counterpropagating signal pump schemes were derived to be the same within the errors of the numerical model and are indistinguishable in Fig. 2. Hence we conclude that the presence of pump ESA in the case of 665-nm pumping is responsible for a difference in the strong-signal gain.

It has been shown that the noise induced into an optical signal by the erbium amplifier is dominated by a combination of signal–spontaneous beat noise and spontaneous–spontaneous beat noise.[3] Since optical filtering can potentially be used to reduce the spontaneous–spontaneous contribution, the limiting noise source is likely to be signal–spontaneous beat noise. A convenient way of modeling the signal–spontaneous beat noise detriment of the amplifier is to plot the noise figure (NF) of the amplifier defined in a manner similar to that in Ref. 6, $\mathrm{NF} = [2\mu_{\mathrm{eff}}(G-1)/G + 1/G]$, against the amplifier length, where G is the single-pass gain of the amplifier (defined as the exponential of the integral of the local gain over the fiber length) and μ_{eff} is defined as

$$\mu_{\mathrm{eff}} = \frac{P_{f1}{}^{+,-}}{2h\nu\Delta\nu(G-1)}. \qquad (9)$$

In physical terms μ_{eff} relates to $\mu(z)$, which is defined above, in the region of the amplifier where $(G-1)/G$ is considerably less than unity. In practice, this corresponds to the signal input end of the fiber. Note that $\mu_{\mathrm{eff}}(G-1)$ is equivalent to N_{sp} in Ref. 6. Figure 3 shows the predicted noise figure for 0.1-μW and 100-μW signals pumping at (a) 665 nm and (b) 980 nm. The small-signal results in both cases agree closely with the data of Olshansky,[6] indicating a degradation in the signal-to-noise ratio of $\approx$6 dB at the point of maximum gain in the counterpropagating case and just over the 3-dB quantum limit for the copropagating case. The copropagating noise figure for 980-nm pumping is slightly higher than that of 665-nm pumping because of the lower initial inversion that results from a substantially lower pump power in this case. The higher signal level is also seen to increase the copropagating noise figure owing to a reduction of inversion at the signal input end of the fiber. At the higher signal level the degradation in the signal-to-noise ratio is again seen to be $\approx$6 dB for the counterpropagating case at the optimum length both with and without ESA. Note, however, that the noise figure is shown to go to unity at vanishingly small amplifier lengths. This low-gain limit was not treated by Olshansky,[6] and it follows from simple physical reasoning that the noise figure should go to unity under such conditions.

We have shown the implementation of a numerical model to describe the behavior of the erbium amplifier in the presence of ESA and for arbitrary signal inputs. The results show that in the presence of ESA there exists a difference in strong-signal gain between copropagating and counterpropagating pump schemes, with the counterpropagating scheme being favorable. This result is validated by experimental data. In addition, we have shown that in the strong-signal limit the degradation in the signal-to-noise ratio at the fiber length giving maximum gain is expected to be similar to the small-signal case, both with and without ASE. A correction to the noise figure for low-gain (<10-dB) amplifiers has also been given.

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