



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Project Management for Managers

Lec – 10

Methods of Project Selection (MCDM – I)

Dr. M.K. Barua

Department of Management
Indian Institute of Technology Roorkee



MCDM

- 1) Multi-Attribute Utility Theory,
- 2) Analytic Hierarchy Process,
- 3) Fuzzy Set Theory,
- 4) Case-based Reasoning,
- 5) Data Envelopment Analysis,
- 6) SMART (Simple Multi-Attribute Rating Technique),
- 7) Goal Programming,
- 8) ELECTRE (ELimination and Choice Translating Reality),
- 9) PROMETHEE (Preference ranking organization method for enrichment evaluation)
- 10) Simple Additive Weighting, and
- 11) TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution)



Goal programming??:

In LP, only one dimension of (Z), organizations often have several objectives (conflicting) and not measured in same units.

GP asks mgt to rank objectives.

GP tries to **minimize deviations** from the targets .



Ex:

| | Mobile | Laptop (lt) | |
|-----------|--------|-------------|-------|
| Assembly | 6 | 3 | 90hrs |
| Finishing | 3 | 6 | 72hrs |
| Profit | 120 | 90 | |

If profit is Rs120 per mobile and Rs 90 per laptop, **determine the best combination of mobile and laptop to realize a profit of Rs 2100.**



This is a single goal (profit) problem. Let

D_u = Amount by which profit goal is underachieved.

D_o = Amount by which profit goal is overachieved.

Minimize $Z = D_u$ (underachievement of profit goal)

ST $120x_1 + 90x_2 + D_u - D_o = 2100$

$$6x_1 + 3x_2 \leq 90$$

$$3x_1 + 6x_2 \leq 72$$

$$x_1, x_2, D_u, D_o \geq 0$$

$$6x_1 + 3x_2 + s_1 = 90$$

$$3x_1 + 6x_2 + s_2 = 72$$

$$x_1, x_2, s_1, s_2 \geq 0$$



This is a single goal (profit) problem. Let

Du = Amount by which profit goal is underachieved.

Do = Amount by which profit goal is overachieved.

Minimize $Z = Du$ (underachievement of profit goal)

ST $120x_1 + 90x_2 + Du - Do = 2100$

$$6x_1 + 3x_2 \leq 90$$

$$3x_1 + 6x_2 \leq 72$$

$$x_1, x_2, Du, Do \geq 0$$

$$6x_1 + 3x_2 + s_1 = 90$$

$$3x_1 + 6x_2 + s_2 = 72$$

$$x_1, x_2, s_1, s_2 \geq 0$$

| | | | | | | | | |
|-------|-------------|-------|-------|-----------------|-----------------|-------|-------|-----|
| c_j | 0 | 0 | 0 | 0 | 1 | 0 | | |
| c_B | Basis | x_1 | x_2 | s_1 | s_2 | D_u | D_o | b |
| 1 | D_u | 0 | 0 | $-\frac{50}{3}$ | $-\frac{20}{3}$ | 1 | -1 | 120 |
| 0 | x_1 | 1 | 0 | $\frac{2}{9}$ | $-\frac{1}{9}$ | 0 | 0 | 12 |
| 0 | x_2 | 0 | 1 | $-\frac{1}{9}$ | $\frac{2}{9}$ | 0 | 0 | 6 |
| | Z_j | 0 | 0 | $-\frac{50}{3}$ | $-\frac{20}{3}$ | 1 | -1 | |
| | \bar{c}_j | 0 | 0 | $\frac{50}{3}$ | $\frac{20}{3}$ | 0 | 1 | |

Optimal basic feasible solution

After using simplex method, $x_1 = 12$, $x_2 = 6$, $D_u = 120$, Means actual profit earned is Rs= 2100-120=1980, underachieved by Rs 120.



Ex: If the company sets two equally ranked goals, one to reach a profit of Rs 1500 and the other to meet the mobile goal of 10 units, find the optimal solution.????????



Solution:

Dup= Amount by which profit goal is underachieved

Dop= Amount by which profit goal is overachieved

Dum= Amount by which mobile goal is underachieved

Dom= Amount by which mobile goal is overachieved

Minimize $Z = \text{Dup} + \text{Dum}$

ST $120x_1 + 90x_2 + \text{Dup} - \text{Dop} = 1500$

$x_1 + \text{Dum} - \text{Dom} = 10$

$6x_1 + 3x_2 \leq 90$

$3x_1 + 6x_2 \leq 72$

$x_1, x_2, \text{Dup}, \text{Dop}, \text{Dum}, \text{Dom} \geq 0$



Solution:

Dup= Amount by which profit goal is underachieved

Dop= Amount by which profit goal is overachieved

Dum= Amount by which mobile goal is underachieved

Dom= Amount by which mobile goal is overachieved

Minimize $Z = \text{Dup} + \text{Dum}$

ST $120x_1 + 90x_2 + \text{Dup} - \text{Dop} = 1500$

$x_1 + \text{Dum} - \text{Dom} = 10$

$6x_1 + 3x_2 \leq 90$

$3x_1 + 6x_2 \leq 72$

$x_1, x_2, \text{Dup}, \text{Dop}, \text{Dum}, \text{Dom} \geq 0$

| c_j | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | |
|-------------|-------|----------------|-------|-------|-----------------|------------------|-----|-----|----------------|
| Basis | x_1 | x_2 | s_1 | s_2 | Dup | Dop | Dum | Dom | b |
| D_{or} | 0 | $\frac{3}{4}$ | 0 | 0 | $\frac{1}{120}$ | $-\frac{1}{120}$ | -1 | 1 | $\frac{5}{2}$ |
| x_1 | 1 | $\frac{3}{4}$ | 0 | 0 | $\frac{1}{120}$ | $-\frac{1}{120}$ | 0 | 0 | $\frac{25}{2}$ |
| s_1 | 0 | $-\frac{3}{2}$ | 1 | 0 | $-\frac{1}{20}$ | $\frac{1}{20}$ | 0 | 0 | 15 |
| s_2 | 0 | $\frac{15}{4}$ | 0 | 1 | $-\frac{1}{40}$ | $\frac{1}{40}$ | 0 | 0 | $\frac{69}{2}$ |
| Z_j | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| \bar{c}_j | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | |

Optimal b.f.s.

After using simplex method, $x_1 = 25/2$, $\text{Dom} = 5/2$, Means profit goal of Rs= 1500 achieved, since both Dup and Dop do not appear in final table.



Analytic Hierarchy Process (AHP)

- Analytic Hierarchy Process is a multi-criteria decision making (MCDM) technique was developed by **Saaty** in 2000 year.
- The analytic hierarchy process (AHP) is also a structured technique for helping people deal with organizing and analyzing **complex decisions**.
- AHP is also a measurement theory that priorities the hierarchy and **consistency of judgmental data** provided by a group of decision makers.
- The AHP provides a **comprehensive and rational** framework for **structuring a problem**, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions



i. **Establish the hierarchy structure**

ii. **Various hierarchies' elements weight computation**

✓ Establishment of pair-wise comparison matrix

✓ The relative importance of two elements is rated using a scale with the values 1, 3, 5, 7, and 9.

| Equally Preferred | Moderately Preferred | Strongly Preferred | Extremely Preferred | Absolutely Preferred |
|-------------------|----------------------|--------------------|---------------------|----------------------|
| 1 | 3 | 5 | 7 | 9 |

✓ 2, 4, 6, and 8 indicate intermediate value.



$$A = [a_{ij}] = \begin{matrix} & \begin{matrix} C_1 & C_2 & & C_n \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} & \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{bmatrix} \end{matrix}$$

Where $a_{ij} = 1$ and $a_{ij} = 1/a_{ji} = 1, 2, \dots, n$.

$$A = [a_{ij}] = \begin{matrix} & \begin{matrix} C_1 & C_2 & & C_n \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} & \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix} \end{matrix}$$

Where $W_i / W_j = a_{ij}$

- Eigen value and eigen vector calculation

$$\lambda_{max} = \sum_{j=1}^n a_{ij} \frac{W_j}{W_i}$$

- Consistency test

$$CI \text{ (Consistency Index)} = \frac{\lambda_{max} - n}{n - 1}$$

$$CR \text{ (Consistency Ratio)} = \frac{CI}{RI}$$

✓ Random index values were already given by Saaty where

| n=1 | n=2 | n=3 | n=4 | n=5 | n=6 | n=7 | n=8 |
|------|------|---------|---------|---------|---------|---------|--------|
| RI=0 | RI=0 | RI=0.52 | RI=0.89 | RI=1.11 | RI=1.25 | RI=1.35 | RI=1.4 |

iii. Overall hierarchy weight computation



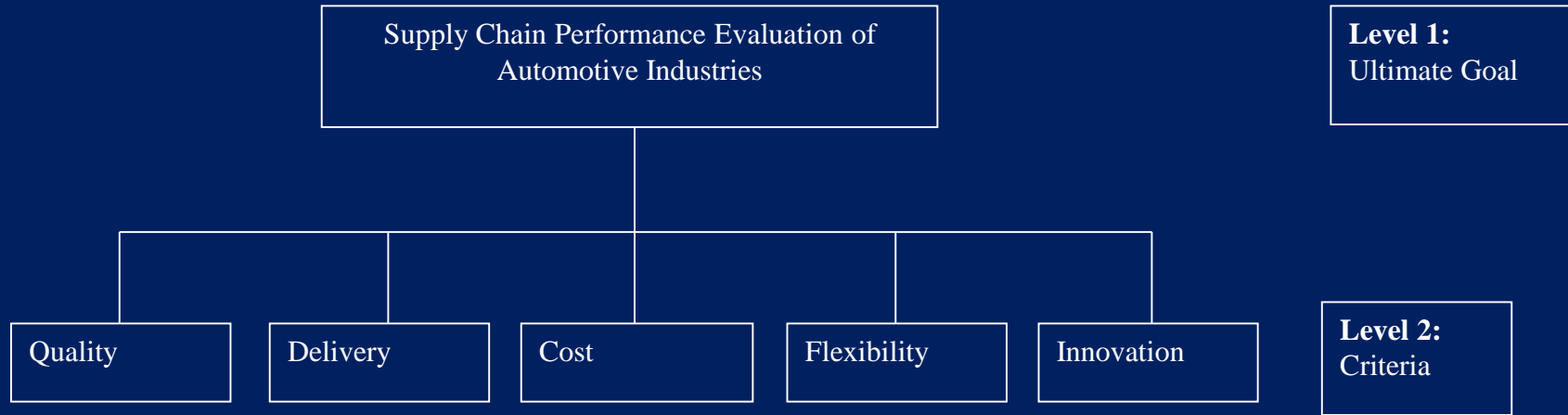


Figure : Hierarchical structure to evaluate the supply chain performance evaluation of automotive industries

| | Quality | Delivery | Cost | Flexibility | Innovation |
|-------------|---------|----------|------|-------------|------------|
| Quality | 1 | 3 | 1 | 1/3 | 9 |
| Delivery | | 1 | 1/3 | 1/5 | 6 |
| Cost | | | 1 | 1/4 | 7 |
| Flexibility | | | | 1 | 8 |
| Innovation | | | | | 1 |

Pair-wise comparison matrix of respondents (Either consensus or highest frequency) for all criteria.

| Respondents | CI | CR | Respondents | CI | CR |
|----------------|---------|---------|-----------------|---------|---------|
| R ₁ | 0.07749 | 0.06918 | R ₉ | 0.08364 | 0.07467 |
| R ₂ | 0.07986 | 0.07131 | R ₁₀ | 0.06853 | 0.06119 |
| R ₃ | 0.06862 | 0.06127 | R ₁₁ | 0.03277 | 0.02926 |
| R ₄ | 0.09214 | 0.08227 | R ₁₂ | 0.08052 | 0.07189 |
| R ₅ | 0.08884 | 0.07933 | R ₁₃ | 0.10024 | 0.08950 |
| R ₆ | 0.09095 | 0.08120 | R ₁₄ | 0.08556 | 0.07640 |
| R ₇ | 0.11118 | 0.09927 | R ₁₅ | 0.04887 | 0.04363 |
| R ₈ | 0.10366 | 0.09255 | R ₁₆ | 0.09692 | 0.08653 |

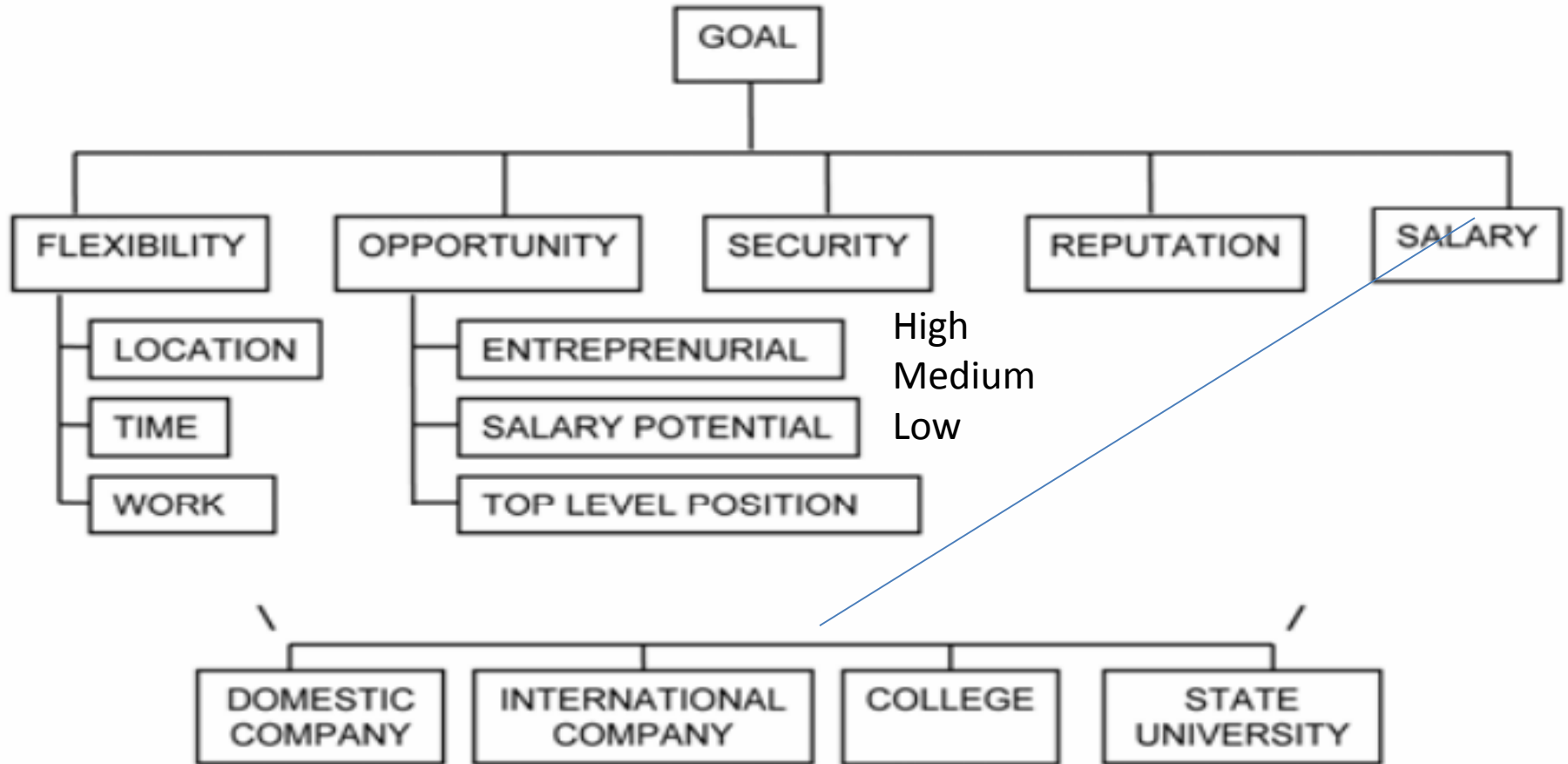
Table : Consistency index (CI) and consistency ratio (CR) of all respondents

| <i>Criteria</i> | <i>Weights of Criteria</i> | <i>Rank</i> |
|------------------------------|----------------------------|-------------|
| Quality = C ₁ | 0.2937 | 2 |
| Delivery = C ₂ | 0.1995 | 3 |
| Cost = C ₃ | 0.1533 | 4 |
| Flexibility = C ₄ | 0.3036 | 1 |
| Innovation = C ₅ | 0.0499 | 5 |

Table : Relative weights of the criteria and global priority



SELECT THE BEST JOB



Level 1:
Goal

Select the Best
Supplier

Level 2:
Criteria

Quality

Cost

Delivery

Level 3:
Sub-Criteria

Remedy for
Quality
Problems

Rejection Rate
from QC

Cost Reduction

Compliance with
Due Date

Compliance with
Quantity

Level 4:
Rating Scale

1

2

3

4

5

Level 5:
Alternatives

Supplier 1

Supplier 2

Supplier 3

Supplier 4

Supplier 5

Supplier 6

Supplier 7

Supplier 8

