IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Project Management for Managers

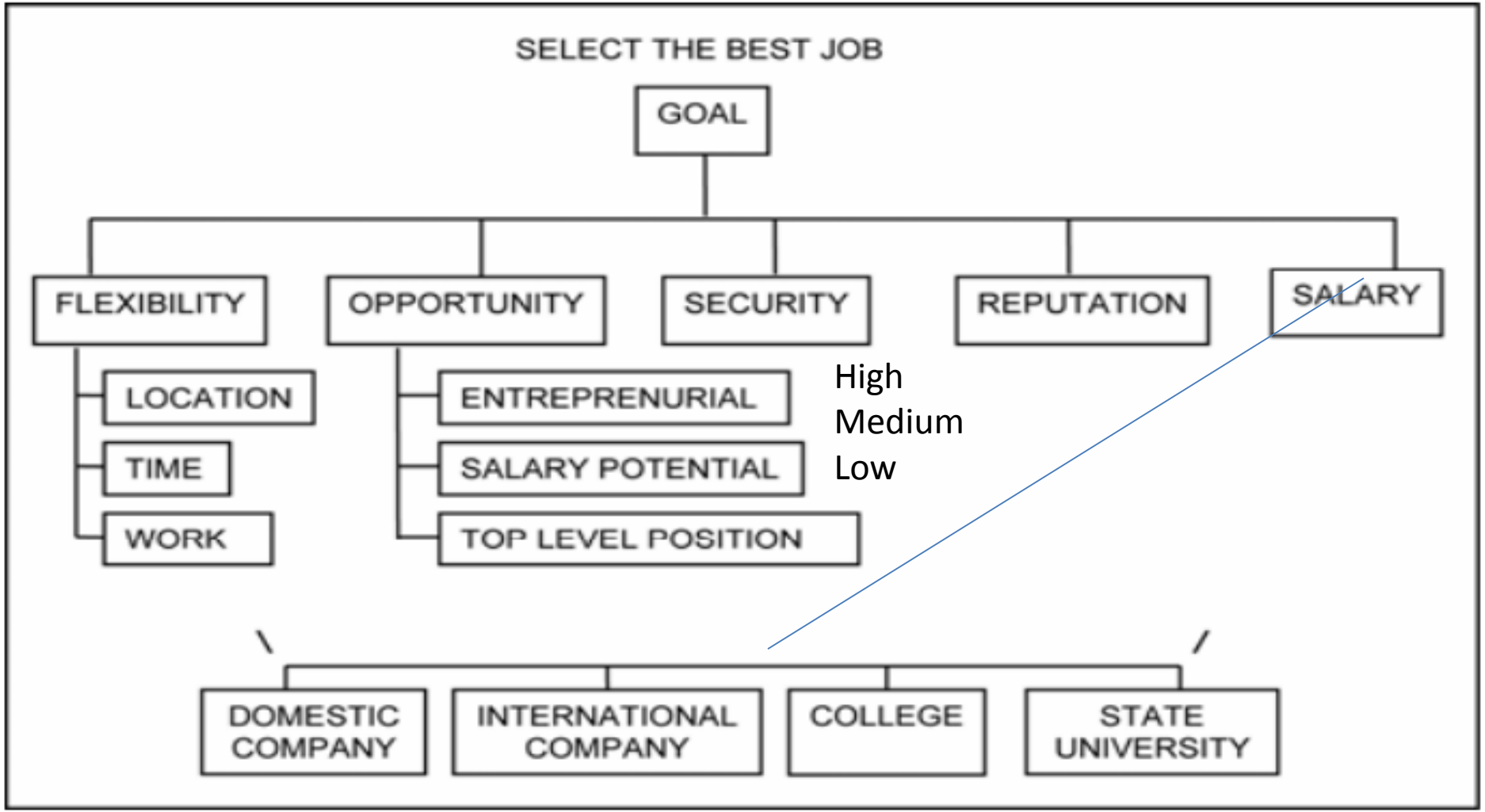
Lec – 11

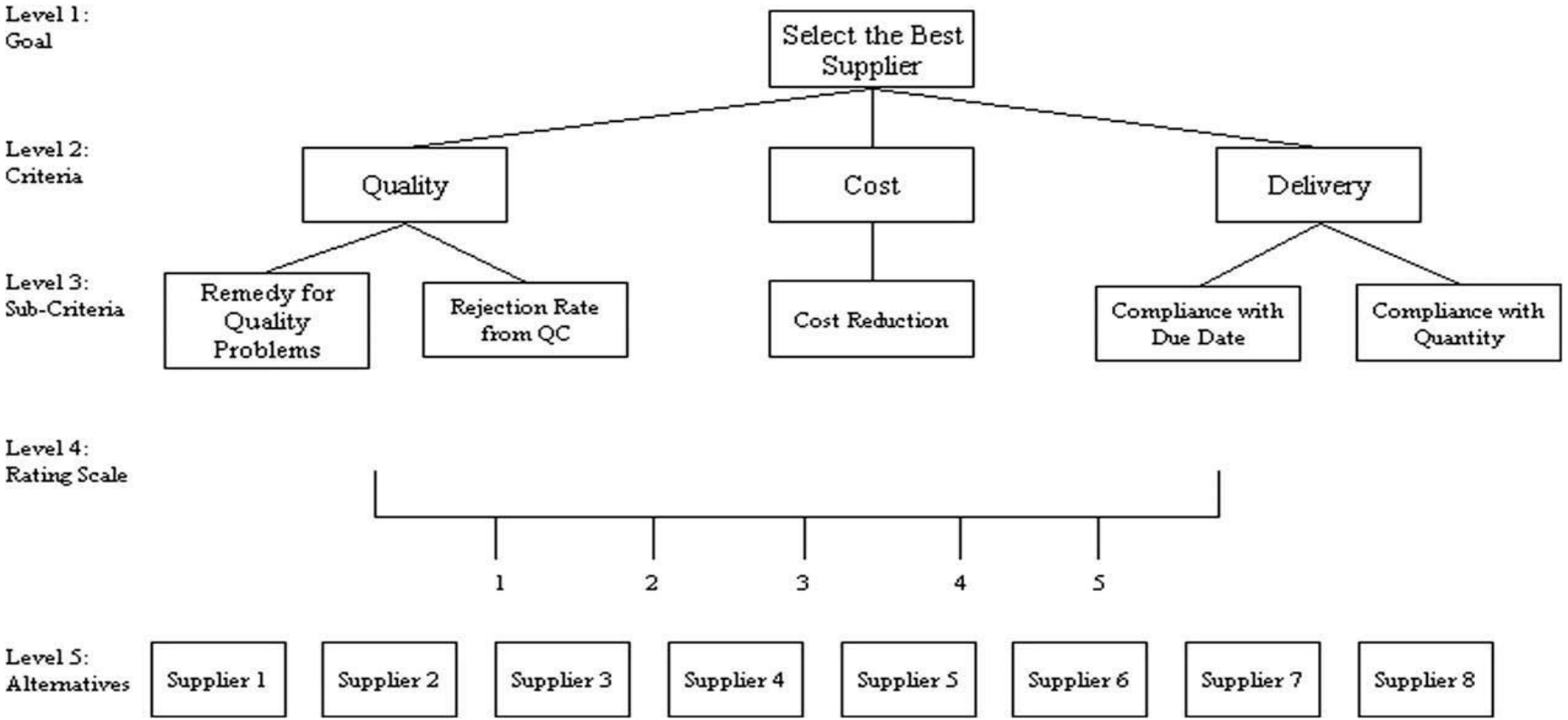
Methods of Project Selection (MCDM – II)

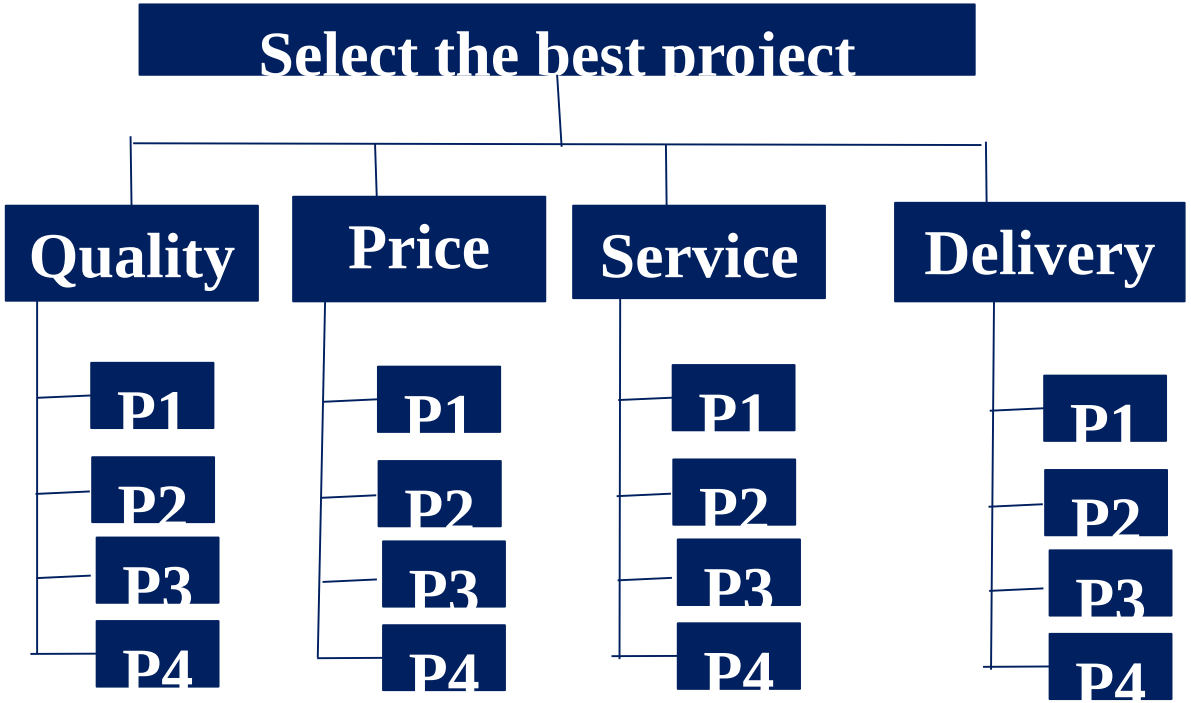
Dr. M.K. Barua

Department of Management

Indian Institute of Technology Roorkee


SELECT THE BEST JOB
GOAL
FLEXIBILITY
OPPORTUNITY
SECURITY
REPUTATION
SALARY
LOCATION
TIME
WORK
ENTREPRENURIAL
SALARY POTENTIAL
TOP LEVEL POSITION
High
Medium
Low
DOMESTIC COMPANY
INTERNATIONAL COMPANY
COLLEGE
STATE UNIVERSITY


Level 1:
Goal
Select the Best Supplier
Level 2:
Criteria
Quality
Cost
Delivery
Level 3:
Sub-Criteria
Remedy for Quality Problems
Rejection Rate from QC
Cost Reduction
Compliance with Due Date
Compliance with Quantity
Level 4:
Rating Scale
1
2
3
4
5
Level 5:
Alternatives
Supplier 1
Supplier 2
Supplier 3
Supplier 4
Supplier 5
Supplier 6
Supplier 7
Supplier 8


Select the best project
Quality
Price
Service
Delivery
P1
P2
P3
P4
P1
P2
P3
P4
P1
P2
P3
P4
P1
P2
P3
P4

Pair wise comparison matrix and computations: evaluation criteria

Original matrix	Quality	Price	Service	Delivery
Quality	1	2	4	3
Price		1	3	3
Service			1	2
Delivery				1

Pair wise comparison matrix and computations: evaluation criteria

Original matrix	**Q**	**P**	**S**	**D**
Q	1	2	4	3
P	½	1	3	3
S	¼	1/3	1	2
D	1/3	1/3	1/2	1
Column total	25/12	11/3	17/2	9

Adjusted matrix	Q	P	S	D	Weights (row avg.)
Q	12/25	6/11	8/17	3/9	0.457
P	6/25	3/11	6/17	3/9	0.300
S	3/25	1/11	2/17	2/9	0.138
D	4/25	1/11	1/17	1/9	0.105
				Total	1.0

Supplier pair wise comparison matrices and priorities

A: wrt quality

	P1	P2	P3	P4	Wts
P1	1	5	6	1/3	?
P2		1	2	1/6	?
P3			1	1/8	?
P4				1	?

A: wrt price

	P1	P2	P3	P4	Wts
P1	1	1/3	5	8	?
P2		1	7	9	?
P3			1	2	?
P4				1	?

Supplier pair wise comparison matrices and priorities

A: wrt service

	P1	P2	P3	P4	Wts
P1	1	5	4	8	?
P2		1	1/2	4	?
P3			1	5	?
P4				1	?

A: wrt delivery

	P1	P2	P3	P4	Wts
P1	1	3	1/5	1	?
P2		1	1/8	1/3	?
P3			1	5	?
P4				1	?

Calculation of the weights A: wrt quality

	P1	P2	P3	P4
P1	1.00	5.00	6.00	0.33
P2	0.20	1.00	2.00	0.17
P3	0.17	0.50	1.00	0.13
P4	3.00	6.00	8.00	1.00
Column total	4.37	12.50	17.00	1.63

	P1	P2	P3	P4
P1	1/4.37	5.00/12.50	6.00/17	0.33/1.63
P2	0.20/4.37	1.00/12.50	2.00//17	0.17/1.63
P3	0.17/4.37	0.50/12.50	1.00//17	0.13/1.63
P4	3.00/4.37	6.00/12.50	8.00//17	1.00/1.63

	P1	P2	P3	P4	Row Sum	Normalized
P1	0.229008	0.4	0.352941	0.205128	1.187077	0.296769
P2	0.045802	0.08	0.117647	0.102564	0.346013	0.086503
P3	0.038168	0.04	0.058824	0.076923	0.213915	0.053479
P4	0.687023	0.48	0.470588	0.615385	2.252996	0.563249

A wrt Price				
	P1	P2	P3	P4
P1	1.00	0.33	5.00	8.00
P2	3	1	7	9
P3	0.166667	0.142857	1	2
P4	0.125	0.111111	0.5	1
Column Sum	4.29	1.59	13.50	20.00

	P1	P2	P3	P4	Row Sum	Normailzed
P1	0.23301	0.21	0.37037	0.4	1.213380079	0.30334502
P2	0.699029	0.63	0.518519	0.45	2.297547645	0.574386911
P3	0.038835	0.09	0.074074	0.1	0.302909026	0.075727256
P4	0.029126	0.07	0.037037	0.05	0.186163251	0.046540813

Supplier pair wise comparison matrices and priorities

A: wrt quality

	P1	P2	P3	P4	Wts
P1	1	5	6	1/3	0.297
P2		1	2	1/6	0.08
P3			1	1/8	0.53
P4				1	0.563

A: wrt service

	P1	P2	P3	P4	Wts
P1	1	5	4	8	0.597
P2		1	1/2	4	0.140
P3			1	5	0.214
P4				1	0.050

A: wrt price

	P1	P2	P3	P4	Wts
P1	1	1/3	5	8	0.303
P2		1	7	9	0.573
P3			1	2	0.078
P4				1	0.046

A: wrt delivery

	P1	P2	P3	P4	Wts
P1	1	3	1/5	1	0.151
P2		1	1/8	1/3	0.060
P3			1	5	0.638
P4				1	0.151

Computation of weights: supplier alternatives

Adjusted matrix	Weights (row avg.)
Quality	0.457
Price	0.300
Service	0.138
Delivery	0.105

	Quality	Price	Service	Delivery	Wts (Q*P*S*D)
P1	.457*.297	.300*.303	.138*.597	.105*.151	.325
P2	.457*.087	.300*.573	.138*.140	.105*.060	.237
P3	.457*.053	.300*.078	.138*.214	.105*.638	.144
P4	.457*.563	.300*.046	.138*.050	.105*.151	.294

TOPSIS METHOD

TOPSIS METHOD

Technique of Order Preference by Similarity to Ideal Solution

This method considers three types of attributes or criteria

- Qualitative **benefit** attributes/criteria
- Quantitative **benefit** attributes/criteria
- **Cost** attributes or criteria

- In this method **two artificial alternatives** are hypothesized:

- **Ideal** alternative: the one which has the **best level** for all attributes considered.

- **Negative** ideal alternative: the one which has the worst attribute values.

- TOPSIS selects the alternative that is **closest** to the **ideal** solution and **farthest** from **negative** ideal alternative.

Input to TOPSIS

- TOPSIS assumes that we have m **alternatives** (options) and n **criteria** and we have the **score** of each option with respect to each criterion.
- Let x_{ij} score of option i with respect to criterion j

  We have a matrix $X = (x_{ij})$ $m \times n$ matrix.
- Let J be the set of **benefit attributes** or criteria (more is better???)
- Let J' be the set of **negative attributes** or criteria (less is better???)

Steps of TOPSIS

- Step 1: Construct **normalized** decision matrix.
- This step transforms various attribute dimensions **into non-dimensional attributes**, which allows comparisons across criteria.
- Normalize scores or data as follows:

$$r_{ij} = x_{ij} / (\Sigma x^2_{ij}) \quad \text{for } i = 1, \ldots, m; j = 1, \ldots, n$$

- Step 2: Construct the **weighted normalized** decision matrix.
- Assume we have a set of weights for each criteria w_j for j = 1,…n.
- Multiply each column of the **normalized** decision **matrix** by its associated **weight**.
- An element of the new matrix is:

$$v_{ij} = w_j r_{ij}$$

- Step 3: Determine the **ideal and negative** ideal solutions.

- **Ideal** solution.

  $A^* = \{ v_1^*, \ldots, v_n^* \}$, where

  $v_j^* = \{ \max_i (v_{ij}) \text{ if } j \in J ; \min_i (v_{ij}) \text{ if } j \in J' \}$

- **Negative** ideal solution.

  $A' = \{ v_1', \ldots, v_n' \}$, where

  $v' = \{ \min_i (v_{ij}) \text{ if } j \in J ; \max_i (v_{ij}) \text{ if } j \in J' \}$

- Step 4: Calculate the **separation** measures for each alternative.

- The separation from the **ideal** alternative is:

$$S_i^* = \left[\sum_j (v_j^* - v_{ij})^2 \right]^{1/2} \qquad i = 1, \ldots, m$$

- Similarly, the separation from the **negative ideal** alternative is:

$$S'_i = \left[\sum_j (v_j' - v_{ij})^2 \right]^{1/2} \qquad i = 1, \ldots, m$$

- Step 5: Calculate the **relative closeness** to the ideal solution C_i^*

$$C_i^* = S'_i / (S_i^* + S'_i) \ , \qquad 0 < C_i^* < 1$$

Select the option with C_i^* closest to 1.

Applying TOPSIS Method to Example

Weight	0.1	0.4	0.3	0.2
	Style	Reliability	Fuel Eco.	Cost
Civic	7	9	9	8
Saturn	8	7	8	7
Ford	9	6	8	9
Mazda	6	7	8	6

Applying TOPSIS to Example

- m = 4 alternatives (car models)
- n = 4 attributes/criteria

- x_{ij} = score of option i with respect to criterion j
  $X = \{x_{ij}\}$ 4×4 score matrix.
- J = set of **benefit** attributes: style, reliability, fuel economy (more is better)
- J' = set of **negative** attributes: cost (less is better)

Steps of TOPSIS

- Step 1(a): calculate $(\Sigma x_{ij})^{1/2}$ for each column

	Style	Rel.	Fuel	Cost
Civic	49	81	81	64
Saturn	64	49	64	49
Ford	81	36	64	81
Mazda	36	49	64	36
Σx_{ij}	230	215	273	230
$(\Sigma x)^{1/2}$	15.17	14.66	16.52	15.17

Steps of TOPSIS

- Step 1 (b): divide each column by $(\Sigma x^2_{ij})^{1/2}$ to get $\mathbf{r_{ij}}$

	Style	Rel.	Fuel	Cost
Civic	0.46	0.61	0.54	0.53
Saturn	0.53	0.48	0.48	0.46
Ford	0.59	0.41	0.48	0.59
Mazda	0.40	0.48	0.48	0.40

????

Steps of TOPSIS

- Step 1 (b): divide each column by $(\Sigma x^2_{ij})^{1/2}$ to get $\mathbf{r}_{ij}$

	Style	Rel.	Fuel	Cost
Civic	0.46	0.61	0.54	0.53
Saturn	0.53	0.48	0.48	0.46
Ford	0.59	0.41	0.48	0.59
Mazda	0.40	0.48	0.48	0.40

7/15/17

- Step 2 : multiply each column by w_j to get v_{ij}.

Weight	0.1	0.4	0.3	0.2
	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192	0.144	0.080

- Step 3 (a): determine ideal solution A*.

A* = {0.059, 0.244, 0.162, 0.080}

	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192 ↑	0.144	0.080 ↓

- Step 3 (a): find negative ideal solution A'.

A' = {0.040, 0.164, 0.144, 0.118}

	Style	Rel.	Fuel	Cost
Civic	0.046	0.244	0.162	0.106
Saturn	0.053	0.192	0.144	0.092
Ford	0.059	0.164	0.144	0.118
Mazda	0.040	0.192 ↓	0.144	0.080 ↑

- Step 4 (a): determine separation from ideal solution

$A^* = \{0.059, 0.244, 0.162, 0.080\}$

$S_i^* = [\Sigma (v_j^* - v_{ij})^2]^{1/2}$ for each row

	Style	Rel.	Fuel	Cost
Civic	$(.046-.059)^2$	$(.244-.244)^2$	$(.162-.162)^2$	$(.106-.080)^2$
Saturn	$(.053-.059)^2$	$(.192-.244)^2$	$(.144-.162)^2$	$(.092-.080)^2$
Ford	$(.059-.059)^2$	$(.164-.244)^2$	$(.144-.162)^2$	$(.118-.080)^2$
Mazda	$(.040-.059)^2$	$(.192-.244)^2$	$(.144-.162)^2$	$(.080-.080)^2$

- Step 4 (a): determine separation from ideal solution S_i^*

	$\Sigma(v_j^* - v_{ij})^2$	$S_i^* = [\Sigma (v_i^* - v_{ij})^2]^{1/2}$
Civic	**0.000845**	**0.029**
Saturn	**0.003208**	**0.057**
Ford	**0.008186**	**0.090**
Mazda	**0.003389**	**0.058**

- Step 4 (b): find separation from negative ideal solution $A' = \{0.040, 0.164, 0.144, 0.118\}$

$S_j' = [\sum (v_i' - v_{ij})^2]^{1/2}$ for each row

	Style	Rel.	Fuel	Cost
Civic	$(.046-.040)^2$	$(.244-.164)^2$	$(.162-.144)^2$	$(.016-.0118)^2$
Saturn	$(.053-.040)^2$	$(.192-.164)^2$	$(0)^2$	$(.092-.118)^2$
Ford	$(.059-.040)^2$	$(.164-.164)^2$	$(0)^2$	$(0)^2$
Mazda	$(.040-.040)^2$	$(.192-.164)^2$	$(0)^2$	$(.080-.118)^2$

- Step 4 (b): determine separation from negative ideal solution S_i'

	$\Sigma(v_j' - v_{ij})^2$	$S_i' = [\Sigma (v_j' - v_{ij})^2]^{1/2}$
Civic	**0.006904**	**0.083**
Saturn	**0.001629**	**0.040**
Ford	**0.000361**	**0.019**
Mazda	**0.002228**	**0.047**

- Step 5: Calculate the relative closeness to the ideal solution $C_i^* = S'_i / (S_i^* + S'_i)$

	$S'_i / (S_i^* + S'_i)$	C_i^*		
Civic	**0.083/0.112**	**0.74**	←	**BEST**
Saturn	**0.040/0.097**	**0.41**		
Ford	**0.019/0.109**	**0.17**		
Mazda	**0.047/0.105**	**0.45**		