WORKSHEETS

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Master's Thesis

0.1 Toy Test Example, M-SDL

General Setup

Consider the linear multiple measurement vector model

$$\mathbf{Y} = \mathbf{A}\mathbf{X},$$

with $\mathbf{Y} \in \mathbb{R}^{m \times L}$ being a known measurement matrix, $\mathbf{A} \in \mathbb{R}^{m \times n}$ a known mixing matrix and $\mathbf{X} \in \mathbb{R}^{n \times L}$ being the source matrix we wish to recover in this case.

For this toy example consider a signal $\mathbf{X}$ that is constructed as a merge of k independent signals. As such one column of X is one sample containing k active signals(sources) and n-k zero entries.
A random mixing matrix $\mathbf{A}$ is generated as a random Normal distributed hence it has normalised columns:
`A = np.random.randn(m, n)`
The measurements $\mathbf{Y}$ is generated as the product of $\mathbf{A}$ and $\mathbf{X}$:
`Y = np.dot(A, X)`
The error between all the elements of the true $\mathbf{X}$ and the recovered $\hat{\mathbf{X}}$ by using the mean square error (MSE):

$$\text{MSE} = \frac{1}{L} \sum_{i=1}^{L} (\mathbf{X} - \hat{\mathbf{X}}_i)^2$$

Consider now $\mathbf{Y}$ and $\mathbf{A}$ known then by use of the M-SDL algorithm $\mathbf{X}$ is sought recovered as $hat\mathbf{X}$. The true $\mathbf{X}$ is then to be used for comparison.

Case 1 - $k > m$

The following variables are used:

- $m = 3$ (number of sensors)
- $n = 8$ (number of sources)
- $L = 100$ (number of samples)
- No segmentations – $\mathbf{Y} = \mathbf{A}\mathbf{X}$
- Iterations = 1000
- $k = 4$ is active sources (row-wise)

Results

The MSE of case 1 was found to be 0.141 when rounded to the nearest three decimal. For visual comparison each active source are plotted against the reconstructed source in figure **??**.

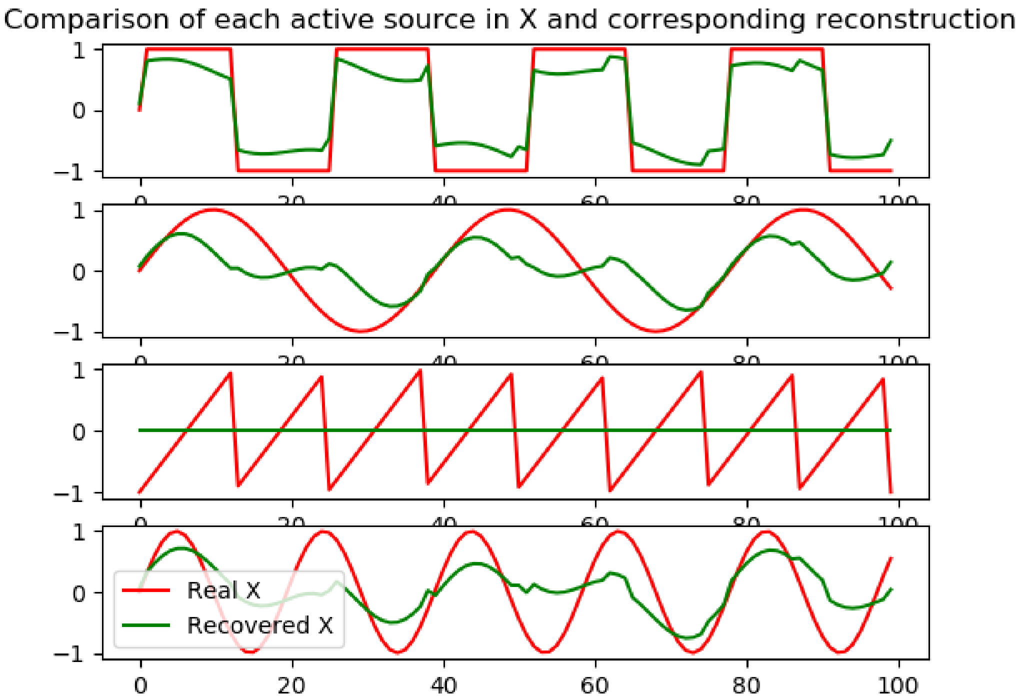

Figure 1: Comparison of each source for k = 4

It is seen that one source was reconstructed as zero, that is the source was not reconstructed in the right location but in another. figure 2 show the comparison for one random chosen sample(row of X),

For the next visualisation we look at the first 4 sources of $\mathbf{X}$ and $\hat{\mathbf{X}}$ to see how well the estimation is.

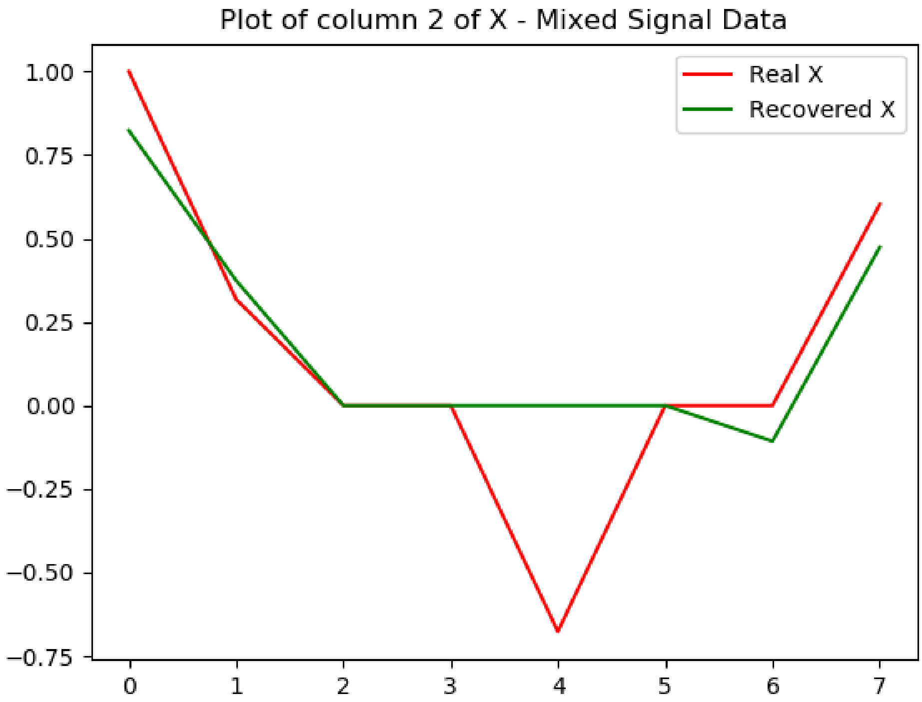

Figure 2: comparison of a single sample

Case 2 - $k < m$

The following variables are used:

- $m = 6$ (number of sensors)
- $n = 8$ (number of sources)
- $L = 100$ (number of samples)
- No segmentations – $\mathbf{Y} = \mathbf{AX}$
- Iterations = 1000
- $k = 4$ is active sources (row-wise)

Results

The MSE of case 1 was found to be 0.010 when rounded to the nearest three decimal. For visual comparison each active source are plotted against the reconstructed source in figure **??**.

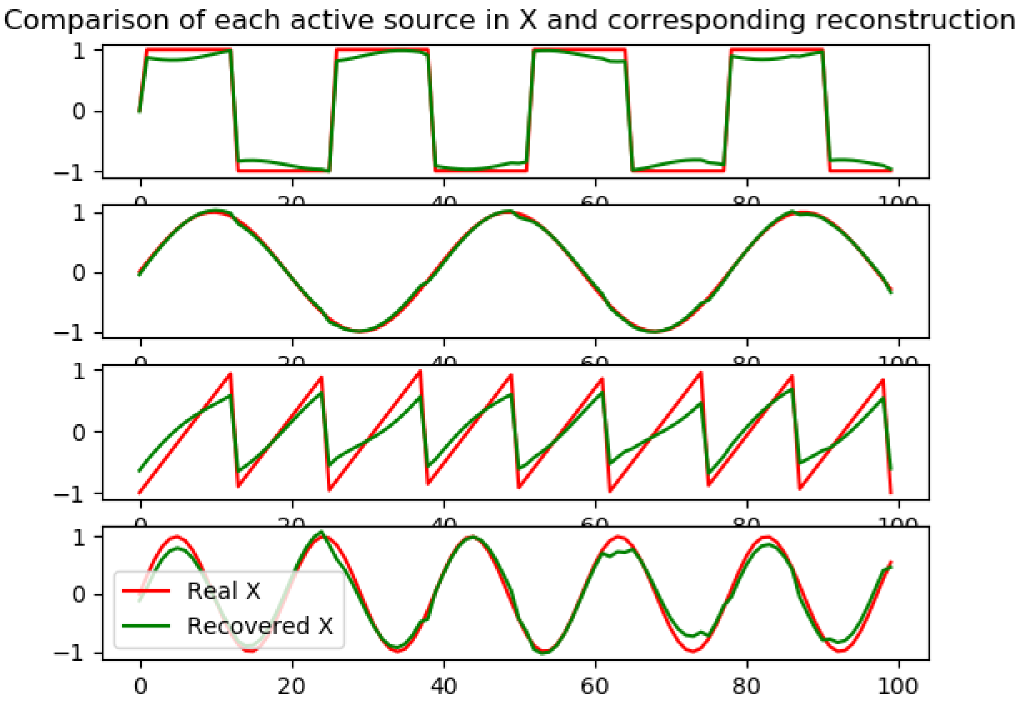

Figure 3: Comparison of each source for k = 4

Figure 4 show the comparison for one random chosen sample(row of X),

For the next visualisation we look at the first 4 sources of $\mathbf{X}$ and $\hat{\mathbf{X}}$ to see how well the estimation is.

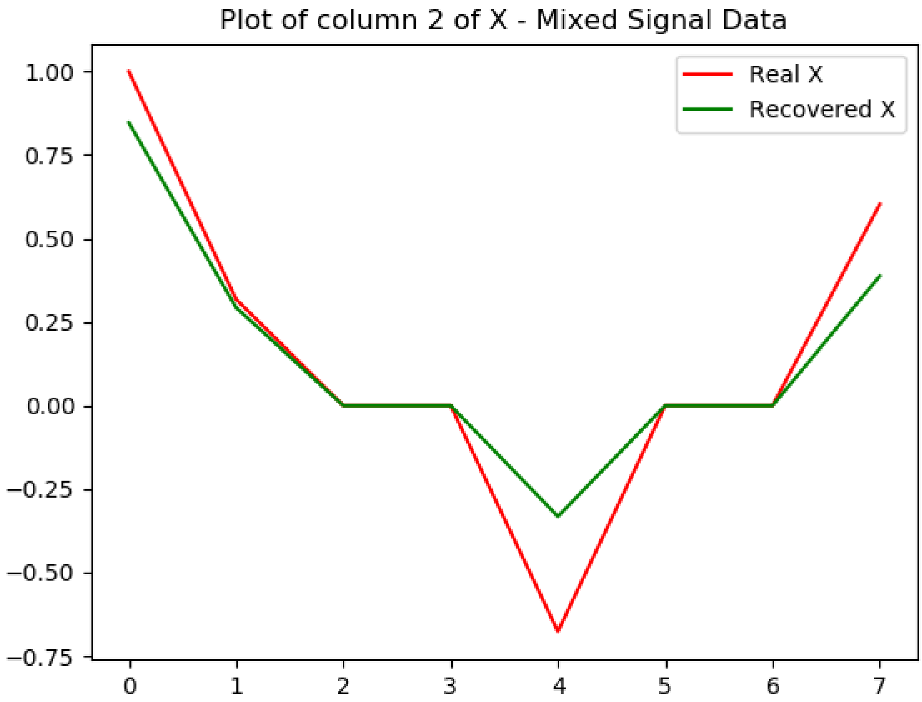

Figure 4: comparison of a single sample