Bayesian Dictionary Learning for EEG Source Identification

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Master's Thesis

Mathematical Engineering
Aalborg University
http://www.aau.dk

AALBORG UNIVERSITY
STUDENT REPORT

Title:
Bayesian Dictionary Learning for EEG Source Identification

Theme:

Project Period:
Fall Semester 2019

Project Group:
Mattek9

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Copies: 1

Page Numbers: 9

Date of Completion:
September 17, 2019

Abstract:

Here is the abstract

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STUDENTERRAPPORT

Matematik-Teknologi
Aalborg Universitet
http://www.aau.dk

Titel:
Bayesian Bibliotek Læring for EEG Kilde Identifikation

Tema:

Projektperiode:
Efterårssemestret 2019

Projektgruppe:
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Deltager(e):
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Oplagstal: 1

Sidetal: 9

Afleveringsdato:
17. september 2019

Abstract:

Her er resuméet

Preface

Here is the preface. You should put your signatures at the end of the preface.

Aalborg University, September 17, 2019

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Danish Summary

Dansk resume ?

Contents

Introduction

Introduktion til hele projektet, skal kunne læses som en appetitvækker til resten af rapporten, det vi skriver her skal så uddybes senere. Brug dog stadigvæk kilder.

- kort intro a EEG og den brede anveldelse, anvendelse indenfor høreapperat.
- intro af model, problem med overbestemt system
- Seneste forslag til at løse dette
- vi vil efterviser dette og udvide til realtime tracking
- opbygningen af rapporten

Chapter 1

Problem Analysis?

indhold

Chapter 2

Theory

2.1 Compressive Sensing

Compressive sensing is used to recover high-dimensional signal from incomplete measurements using efficient algorithm. In compressive sensing a linear model is used to describe the relationship:

$$y = Ax,$$

where x is a $1 \times N$ vector and A is a matrix of size $M \times N$.
As we want $M << N$ then A becomes rank-deficieny and therefore have a nonempty nullspace.
We want x to be a sparse representation, meaning that we have a signal of lenght N we want to represent it with k « N nonzero coefficient
signal can be well-approximated from a linearly combination of few elements extracted from a known basis or dictionary. If the representation is exact then the signal is sparse. A signal x said to be k-sparse when it has at most k nonzeros in x:

$$\|x\|_0 \leq k,$$

where

$$\Sigma_k = \{x \;:\; \|x\|_0 \leq k\},$$

denote the set of all k-sparse signals [**\IeC {\textbullet }**].
ℓ_p norm is given as

$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}, \quad p \in [1, \infty)$$

the norm is used as an measure of the strenght of a signal or as an error.

Then ℓ_0 norm is np hard to calculate and therefore we seek for an approximation within the ℓ_1 norm. Therefore, we instead find the best k-term approximation of the signal
some conditions must be satisfied to insure that we recover all sparse representation of a signal. Some of the most known conditions explore the null space: Null Space Conditions.
The null space of the matrix A is defined as

$$\mathcal{N}(A) = \{z \ : \ Az = 0\}.$$

NSP do not take account for noise and we must therefore look at some stronger conditions which incoperate noise, the following restricted isometry property (RIP) [**IeC {\textbullet }**].

Definition 2.1 (Restricted Isometry Property)
A matrix A satisfies the RIP of order k if there exists a $\delta_k \in (0,1)$ such that

$$(1-\delta_k)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1+\delta_k)\|x\|_2^2,$$

holds for all $x \in \Sigma_k$

If a matrix A satisfy RIP then it will also satisfy the NSP as RIP is strictly stronger than NSP.

Theorem 2.1.1
If A satisfies the RIP of order $2k$ with the constant $\delta_{2k} < \sqrt{2}-1$. Then

$$C = \frac{2}{1-(1+\sqrt{2})\delta_{2k}}$$

Another measure used for sparsity is coherence [**IeC {\textbullet }**].

Definition 2.2 (Coherence)
Coherence of the matrix A, denoted as $\mu(A)$, is the largest absolute value between two columns a_i and a_j from A:

$$\mu(A) = \max_{1\leq i<j\leq n} \frac{|\langle a_i, a_j\rangle|}{\|a_i\|_2\|a_j\|_2}$$

Appendix A

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