# Bayesian Dictionary Learning for EEG Source Identification

Trine Nyholm Kragh & Laura Nyrup Mogensen

Mathematical Engineering, MATTEK

Master's Thesis

STUDENT REPORT

AALBORG UNIVERSITY

# AALBORG UNIVERSITY

## STUDENT REPORT

**Title:**

Bayesian Dictionary Learning for EEG
Source Identification

**Theme:**

**Project Period:**

Fall Semester 2019

**Project Group:**

Mattek9

**Participant(s):**

Trine Nyholm Kragh

Laura Nyrup Mogensen

**Supervisor(s):**

Jan Østergaard

**Copies:** 1

**Page Numbers:** 13

**Date of Completion:**

September 17, 2019

**Abstract:**

Here is the abstract

**AALBORG UNIVERSITET**

STUDENTERRAPPORT

**Titel:**
Bayesian Bibliotek Læring for EEG Kilde
Identifikation

**Tema:**

**Projektperiode:**
Efterårssemestret 2019

**Projektgruppe:**
Mattek9

**Deltager(e):**
Trine Nyholm Kragh
Laura Nyrup Mogensen

**Vejleder(e):**
Jan Østergaard

**Oplagstal:** 1

**Sidetal:** 13

**Afleveringsdato:**
17. september 2019

**Abstract:**

Her er resuméet

# Preface

Here is the preface. You should put your signatures at the end of the preface.

Aalborg University, September 17, 2019

| | |
|---|---|
| Trine Nyholm Kragh | Laura Nyrup Mogensen |
| <trijen15@student.aau.dk> | <lmogen15@student.aau.dk> |

# Danish Summary

Dansk resume ?

# Contents

# Introduction

Introduktion til hele projektet, skal kunne læses som en appetitvækker til resten af rapporten, det vi skriver her skal så uddybes senere. Brug dog stadigvæk kilder.

- kort intro a EEG og den brede anveldelse, anvendelse indenfor høreapperat.

- intro af model, problem med overbestemt system

- Seneste forslag til at løse dette

- vi vil efterviser dette og udvide til realtime tracking

- opbygningen af rapporten

# Chapter 1

# Problem Analysis

This chapter examines existing literature concerning source localisation from EEG measurements. At first a motivation for the problem is given, considering especially the application within the hearing aid industry. Further, the state of the art methods are presented follow by a description of the desired contribution.

## 1.1 Motivation

(Hvad er EEG)
EEG recordings or measurements are used within medicine as an imaging technique measuring electric signals on the scalp, caused by brain activity.

The brain consist of a enormous amounts of cells, called neurons. These neurons are mutually connected in neural nets and when a neuron is activated, for instance by some physical stimuli, local current flows are produced[2]. As such the neurons are somehow communicating(?).

The EEG measurements are provided by a varies number of metal electrodes referred to as sensors, places on the head surface reading the electrical signals which are masively amplified and displayed on the computer as a sum of sinusoidal waves relative to time. It takes a large amount of active neurons to generate an electrical activity that is recordable on the scalp as the current then have to penetrate the skull, skin and several other layers.
From this it is clear that the measurements from a single sensor do not correspond to the activity of a single neuron in the brain, but rather a collection of many activities. Here the same neuron activities can be measured by two or more sensors. Furthermore, interfering signals can occur resulting from physical movement of e.g. eyes and jawbone[2].

different frequencies
(Hvad bruges det til)

# Chapter 2

# Sparse Signal Recovery

Through this chapter an introduction to the concept compressive sensing is described with associated theory which later on will be used in the development of the algorithm with used methods known from compressive sensing to estimate the mixing matrix $A$ and the sparse source matrix $X$.

## 2.1 Compressive Sensing

Compressive sensing is the theory of efficient recover/reconstruct a signal from minimal measurements. This recovery is often described as a linear model/system

$$\mathbf{y} = \mathbf{A}\mathbf{x},$$

which consist of observed data $\mathbf{y} \in \mathbb{R}^M$, a matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$ which models the linear measurements and signal $\mathbf{x} \in \mathbb{R}^N$. In compressive sensing terminology, $\mathbf{y}$ is the signal of interest that is wish recovered from minimal measurements meaning that the signal $\mathbf{x}$ must be sparse.

A signal is said to be $k$-sparse if the signal has at most $k$ non-zeros coefficient:

$$\|\mathbf{x}\|_0 = \mathrm{card}(\mathrm{supp}(\mathbf{x}) \le k,$$

where $\ell_0$ norm is used. The function card is the cardinality of the support of $\mathbf{x}$. The support of $\mathbf{x}$ is giving as

$$\mathrm{supp}(\mathbf{x}) = \{j \in [N] \ : \ x_j \ne 0\},$$

where $[N]$ a set [1, p. 41]. The set of all $k$-sparse signal is denoted as

$$\Sigma_k = \{\mathbf{x} \ : \ \|\mathbf{x}\|_0 \le k\}.$$

We want to recover the signal when there is $M \ll N$ and $k < M$ [3, p. 8]. This lead to that the matrix $\mathbf{A}$ becomes rank-deficient and therefore have a non-empty

null-space [3, p. XX].

The linear model and finding the sparse signal **x** can be written as an optimisation problem

$$\min_{\mathbf{z}} \|\mathbf{z}\|_0 \quad \text{s.t} \quad \mathbf{y} = \mathbf{Az},$$

where **z** is all possible candidates to an $k$-sparse signal **x**.

Unfortunately, this optimisation problem is non-convex because of $\ell_0$ norm and is therefore very difficult to solve leading it to be a NP-hard problem. Instead by replacing the $\ell_0$ norm with its convex approximation, the $\ell_1$ norm, the optimisation problem become computational feasible [3, p. 27]:

$$\min_{\mathbf{z}} \|\mathbf{z}\|_1 \quad \text{s.t} \quad \mathbf{y} = \mathbf{Az}.$$

### 2.1.1   Conditions on the Mixing Matrix

To ensure an exact or an approximate reconstruction of the sparse signal **x** some conditions associated on the matrix **A** must be satisfied.

**Null Space Conditions**

The null space property (NSP) is some necessary and sufficient condition for exact recovery. The null space of the matrix $A$ is defined as

$$\mathcal{N}(A) = \{z \; : \; Az = 0\}.$$

$\ell_p$ norm is given as

$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}, \quad p \in [1, \infty)$$

the norm is used as an measure of the strenght of a signal or as an error.

Then $\ell_0$ norm is np hard to calculate and therefore we seek for an approximation within the $\ell_1$ norm. Therefore, we instead find the best $k$-term approximation of the signal

**Restricted Isometry Conditions**

NSP do not take account for noise and we must therefore look at some stronger conditions which incoperate noise, the following restricted isometry property (RIP) [**\IeC {\textbullet }**].

**Definition 2.1 (Restricted Isometry Property)**

A matrix $A$ satisfies the RIP of order $k$ if there exists a $\delta_k \in (0, 1)$ such that

$$(1 - \delta_k)\|x\|_2^2 \le \|Ax\|_2^2 \le (1 + \delta_k)\|x\|_2^2,$$

holds for all $x \in \Sigma_k$

If a matrix $A$ satisfy RIP then it will also satisfy the NSP as RIP is strictly stronger than NSP.

**Theorem 2.1.1**

If $A$ satisfies the RIP of order $2k$ with the constant $\delta_{2k} < \sqrt{2} - 1$. Then

$$C = \frac{2}{1 - (1 + \sqrt{2})\delta_{2k}}$$

**Coherence**

Another measure used for sparsity is coherence [\IeC {\textbullet }].

**Definition 2.2 (Coherence)**

Coherence of the matrix $A$, denoted as $\mu(A)$, is the largest absolute value between two columns $a_i$ and $a_j$ from $A$:

$$\mu(A) = \max_{1 \le i < j \le n} \frac{|\langle a_i, a_j \rangle|}{\|a_i\|_2 \|a_j\|_2}$$

## 2.2   ICA

## 2.3   Cov-DL

## 2.4   MSB

# Bibliography

[1] Simon Foucart, Hoger Rauhut. *A Mathematical Introduction to Compressive Sensing.* Springer Science+Business Media New York, 2013.

[2] Teplan, M. "Fundamentals of EEG". In: *Measurement science review* 2 (2002).

[3] Yonina C. Eldar, Gitta Kutyniok. *Compressed Sensing: Theory and Application.* Cambridge University Presse, New York, 2012.

# Appendix A

# Appendix A