Bayesian Dictionary Learning for EEG Source Identification

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Preface

Here is the preface. You should put your signatures at the end of the preface.

Aalborg University, September 17, 2019

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Danish Summary

Dansk resume ?

Contents

Introduction

Introduktion til hele projektet, skal kunne læses som en appetitvækker til resten af rapporten, det vi skriver her skal så uddybes senere. Brug dog stadigvæk kilder.

- kort intro a EEG og den brede anveldelse, anvendelse indenfor høreapperat.
- intro af model, problem med overbestemt system
- Seneste forslag til at løse dette
- vi vil efterviser dette og udvide til realtime tracking
- opbygningen af rapporten

Chapter 1

Problem Analysis?

indhold

Chapter 2

Sparse Signal Recovery

Through this chapter an introduction to the concept compressive sensing is described with associated theory which later on will be used in the development of the algorithm with used methods known from compressive sensing to estimate the mixing matrix A and the sparse source matrix X.

2.1 Compressive Sensing

Compressive sensing is the theory of efficient recover/reconstruct a signal from minimal measurements. This recovery is often described as a linear model/system

$$\mathbf{y} = \mathbf{A}\mathbf{x},$$

which consist of observed data $\mathbf{y} \in \mathbb{R}^M$, a matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$ which models the linear measurements and signal $\mathbf{x} \in \mathbb{R}^N$. In compressive sensing terminology, $\mathbf{y}$ is the signal of interest that is wish recovered from minimal measurements meaning that the signal $\mathbf{x}$ must be sparse.
A signal is said to be k-sparse if the signal has at most k non-zeros coefficient:

$$\|\mathbf{x}\|_0 = \text{card}(\text{supp}(\mathbf{x}) \leq k,$$

where ℓ_0 norm is used. The function card is the cardinality of the support of $\mathbf{x}$. The support of $\mathbf{x}$ is giving as

$$\text{supp}(\mathbf{x}) = \{j \in [N] \; : \; x_j \neq 0\},$$

where $[N]$ a set [1, p. 41]. The set of all k-sparse signal is denoted as

$$\Sigma_k = \{\mathbf{x} \; : \; \|\mathbf{x}\|_0 \leq k\}.$$

We want to recover the signal when there is $M << N$ and $k < M$ [2, p. 8]. This lead to that the matrix $\mathbf{A}$ becomes rank-deficient and therefore have a non-empty null-space [2, p. XX].

The linear model and finding the sparse signal $\mathbf{x}$ can be written as an optimisation problem

$$\min_{\mathbf{z}} \|\mathbf{z}\|_0 \quad \text{s.t} \quad \mathbf{y} = \mathbf{A}\mathbf{z},$$

where $\mathbf{z}$ is all possible candidates to an k-sparse signal $\mathbf{x}$.
Unfortunately, this optimisation problem is non-convex because of ℓ_0 norm and is therefore very difficult to solve leading it to be a NP-hard problem. Instead by replacing the ℓ_0 norm with its convex approximation, the ℓ_1 norm, the optimisation problem become computational feasible [2, p. 27]:

$$\min_{\mathbf{z}} \|\mathbf{z}\|_1 \quad \text{s.t} \quad \mathbf{y} = \mathbf{A}\mathbf{z}.$$

2.1.1 Conditions on the Mixing Matrix

To ensure an exact or an approximate reconstruction of the sparse signal $\mathbf{x}$ some conditions associated on the matrix $\mathbf{A}$ must be satisfied.

Null Space Conditions

The null space property (NSP) is some necessary and sufficient condition for exact recovery. The null space of the matrix A is defined as

$$\mathcal{N}(A) = \{z \ : \ Az = 0\}.$$

ℓ_p norm is given as

$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}, \quad p \in [1, \infty)$$

the norm is used as an measure of the strenght of a signal or as an error.
Then ℓ_0 norm is np hard to calculate and therefore we seek for an approximation within the ℓ_1 norm. Therefore, we instead find the best k-term approximation of the signal

Restricted Isometry Conditions

NSP do not take account for noise and we must therefore look at some stronger conditions which incoperate noise, the following restricted isometry property (RIP) [**\IeC {\textbullet }**].

Definition 2.1 (Restricted Isometry Property)
A matrix A satisfies the RIP of order k if there exists a $\delta_k \in (0,1)$ such that

$$(1-\delta_k)\|x\|_2^2 \le \|Ax\|_2^2 \le (1+\delta_k)\|x\|_2^2,$$

holds for all $x \in \Sigma_k$

If a matrix A satisfy RIP then it will also satisfy the NSP as RIP is strictly stronger than NSP.

Theorem 2.1.1
If A satisfies the RIP of order $2k$ with the constant $\delta_{2k} < \sqrt{2}-1$. Then

$$C = \frac{2}{1-(1+\sqrt{2})\delta_{2k}}$$

Coherence

Another measure used for sparsity is coherence [**IeC {\textbullet }**].

Definition 2.2 (Coherence)
Coherence of the matrix A, denoted as $\mu(A)$, is the largest absolute value between two columns a_i and a_j from A:

$$\mu(A) = \max_{1\le i<j\le n} \frac{|\langle a_i, a_j\rangle|}{\|a_i\|_2\|a_j\|_2}$$

2.2 ICA

2.3 Cov-DL

2.4 MSB

Bibliography

[1] Simon Foucart, Hoger Rauhut. *A Mathematical Introduction to Compressive Sensing*. Springer Science+Business Media New York, 2013.

[2] Yonina C. Eldar, Gitta Kutyniok. *Compressed Sensing: Theory and Application*. Cambridge University Presse, New York, 2012.

Appendix A

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