Aalto University
School of Electrical
Engineering

Module 4
State Feedback Current Control: Magnetic Saturation and Gain-Scheduling

Marko Hinkkanen

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Introduction

- In the previous examples, the plant was assumed to be linear
- In practice, the magnetic circuit can be highly nonlinear
  (for example, in synchronous reluctance motors)
- Nonlinearities due to the magnetic saturation have to be properly (but not accurately) modelled and taken into account in the control system
- This improves not only the control performance but also the robustness against parameter errors
- Suitable algebraic saturation model may simplify both the self-commissioning and tuning problems

Outline

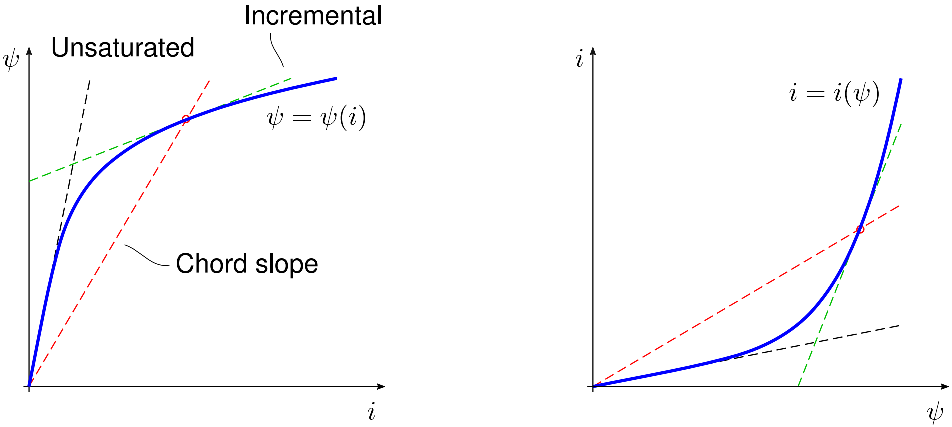
Saturation Characteristics

Algebraic Saturation Models

Gain-Scheduled Current Controller

Improved Gain-Scheduled Current Controller

Example Saturation Characteristics



Two representations of the same saturation characteristics

Inductances

- Chord-slope inductance

$$L(i) = \frac{\psi(i)}{i}$$

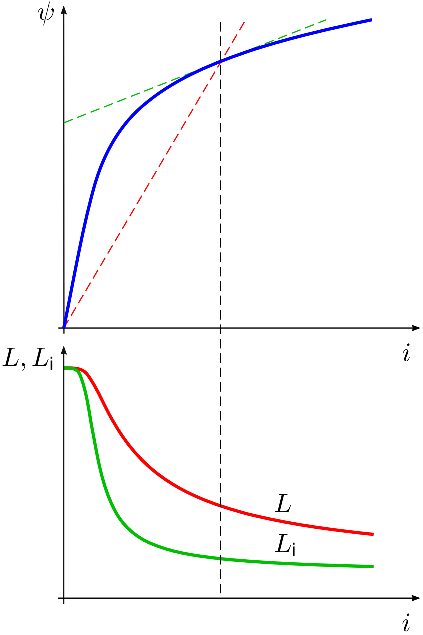
- Incremental inductance

$$L_{\mathsf{i}}(i) = \frac{\partial \psi(i)}{\partial i}$$

- Appears in the voltage equation

$$\frac{\mathrm{d}\psi(i)}{\mathrm{d}t} = \frac{\partial \psi(i)}{\partial i}\frac{\mathrm{d}i}{\mathrm{d}t}$$

if the current is the state variable



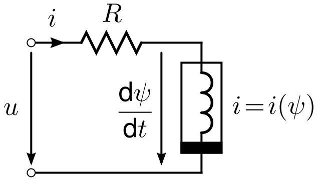
Dynamic Model

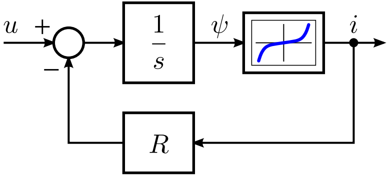
- Nonlinear inductor $i = i(\psi)$ with a series resistance R
- Flux linkage as the state variable

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = u - Ri$$
$$i = i(\psi)$$

- Current as the state variable

$$L_\mathrm{i}(i)\frac{\mathrm{d}i}{\mathrm{d}t} = u - Ri$$





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Current as Independent Variable

- Simple algebraic functions for $\psi = \psi(i)$ or for $L = L(i)$ are hard to find
- Rational functions generally fit well to the measured data, but they may also show vertical asymptotes or nonmonotonic behaviour
- Nonmonotonic behaviour makes the function noninvertible, which limits its applicability
- Various piecewise functions could also be used to model $\psi = \psi(i)$

Flux as Independent Variable: Polynomial Function

- Polynomial function

$$i(\psi) = (c_0 + c_1|\psi| + \ldots + c_S|\psi|^S)\psi$$

- Coefficient $c_0 = 1/L_0$ is the inverse of the unsaturated inductance
- Fits well to measured data if S is high enough (typically $S = 4\ldots7$ suffices)
- Typically, some coefficients tend to be negative, which may result in a nonmonotonic function outside the measured data range

Flux as Independent Variable: Simplified Polynomial Function

- Simplified polynomial function

$$i(\psi) = (c_0 + c_S|\psi|^S)\psi$$

- S is a positive exponent and c_0 and c_S are positive coefficients
- Function is monotonic and (numerically) invertible
- Equivalent inductance function

$$L(\psi) = \frac{\psi}{i(\psi)} = \frac{1}{c_0 + c_S|\psi|^S}$$

LLS Fitting

- Assume positive current and flux samples

$$i = c_0\psi + c_S\psi^{S+1}$$

- Model is linear with respect to c_0 and c_S
- LLS problem in a vector form

$$\underbrace{\begin{bmatrix} i(1) \\ i(2) \\ \vdots \\ i(N) \end{bmatrix}}_{\boldsymbol{y}} = \underbrace{\begin{bmatrix} \psi(1) & \psi(1)^{S+1} \\ \psi(2) & \psi(2)^{S+1} \\ \vdots & \vdots \\ \psi(N) & \psi(N)^{S+1} \end{bmatrix}}_{\boldsymbol{X}} \underbrace{\begin{bmatrix} c_0 \\ c_S \end{bmatrix}}_{\boldsymbol{\beta}} + \underbrace{\begin{bmatrix} \varepsilon(1) \\ \varepsilon(2) \\ \vdots \\ \varepsilon(N) \end{bmatrix}}_{\boldsymbol{\varepsilon}}$$

where $\boldsymbol{y}$ is the vector of the current samples, $\boldsymbol{X}$ is regressor matrix, $\boldsymbol{\beta}$ is the parameter vector, $\boldsymbol{\varepsilon}$ is the residual vector, and N is the number of samples

- Sum of the squared residuals

$$J(\boldsymbol{\beta}) = \boldsymbol{\varepsilon}^{\mathsf{T}}\boldsymbol{\varepsilon} = \sum_{n=1}^{N} \varepsilon(n)^2$$

- Parameter vector minimizing J

$$\boldsymbol{\beta} = (\boldsymbol{X}^{\mathsf{T}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathsf{T}}\boldsymbol{y}$$

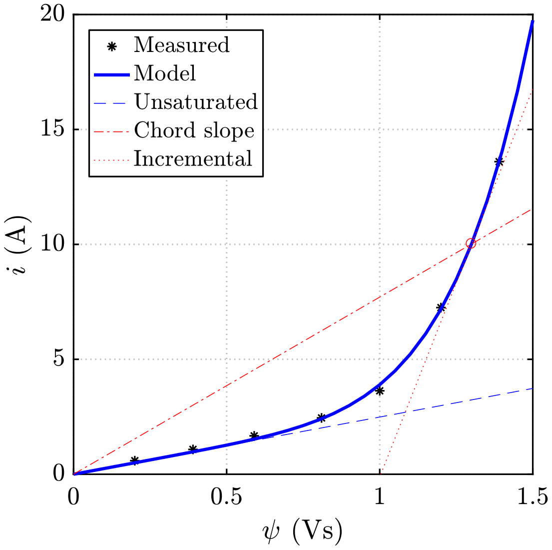
  or in MATLAB simply `b = X\y`
- Fitting can be done for pre-selected values of S and the best fit can be chosen

Exercise 4.1

- Consider the following measured data ($N = 7$)

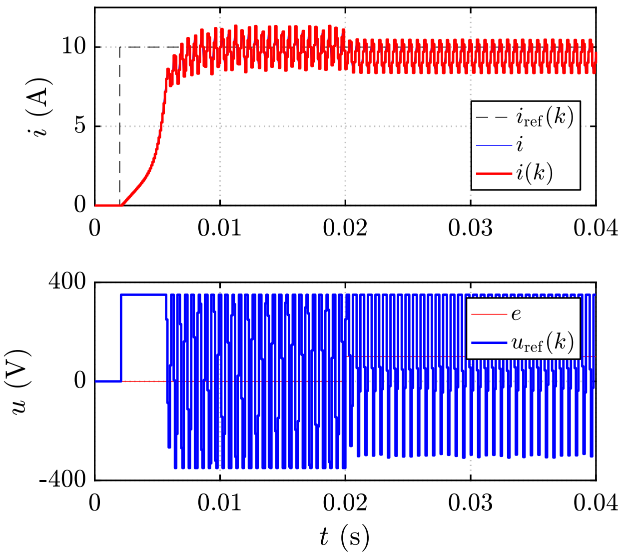
$$\psi = \begin{bmatrix} 0.20 \\ 0.39 \\ 0.59 \\ 0.81 \\ 1.00 \\ 1.20 \\ 1.39 \end{bmatrix} \text{ Vs} \qquad i = \begin{bmatrix} 0.59 \\ 1.08 \\ 1.67 \\ 2.45 \\ 3.63 \\ 7.25 \\ 13.6 \end{bmatrix} \text{ A}$$

- Fit the simplified polynomial function to the data (use $S = 4 \dots 6$)



Exercise 4.2

- In your simulation model, modify the plant by replacing constant L with a nonlinear inductor: $S = 5$, $c_0 = 2.5$ H^{-1}, $c_S = 1.4$ (Vs)$^{-S}$
- Decrease T_s down to 100 μs
- Leave the controller as it is ($\hat{L} =$ 170 mH corresponds to the chord-slope inductance at 10 A)
- Simulate the model
- Fixing this chattering problem is not easy (without significantly decreasing the bandwidth or increasing the sampling frequency)



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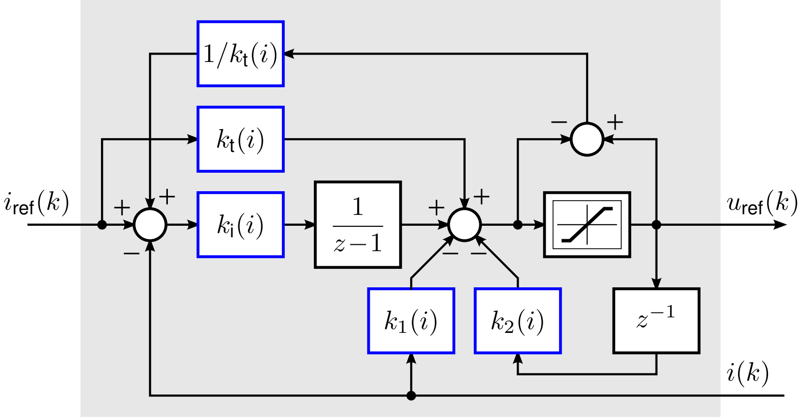
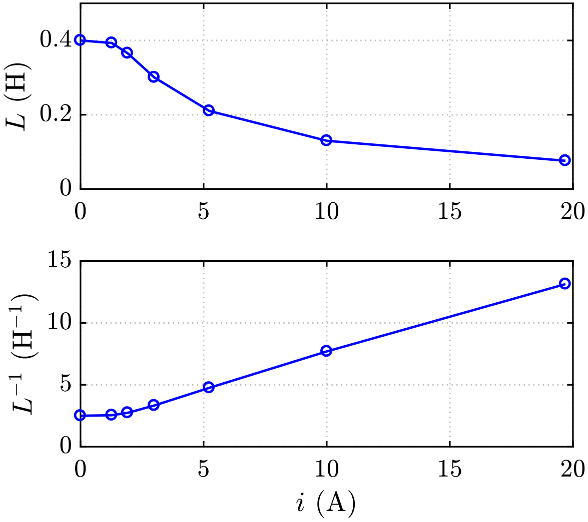
Gain-Scheduled Current Controller



- If saturation characteristics $\hat{\psi} = \hat{\psi}(i)$ are known or identified, we can try to apply simple gain-scheduling to compensate for the saturation effects
- Gains depend on the measured current $i(k)$ via the inductance estimate $\hat{L}(i)$

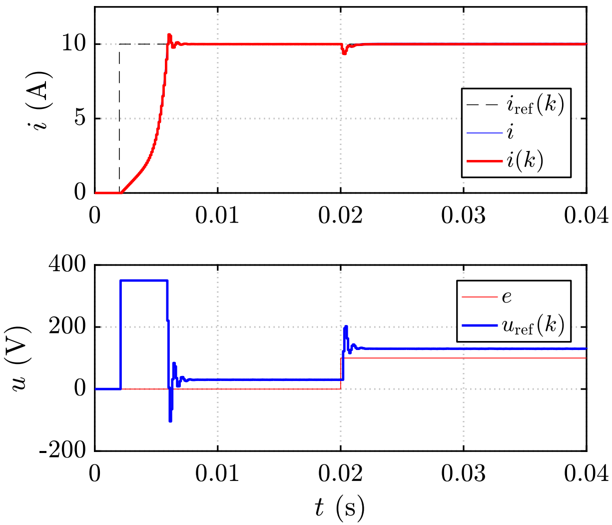
Exercise 4.3

- For gain-scheduling, inductance values $\hat{L}(i)$ could be stored in the look-up table
- Calculate and plot $\hat{L}(i)$ using the simplified polynomial saturation model
- Calculate and plot also $1/\hat{L}(i)$
- Inverse inductance is better for interpolation and extrapolation (less data points needed)



Exercise 4.4

- From now on, it is more convenient to implement the controller as a MATLAB Function
- Modify the controller so that the gains depend on $\hat{L}(i)$ or $1/\hat{L}(i)$
- Use the data points from previous exercise together with linear interpolation and extrapolation (`interp1.m`)
- Simulate the model
- Still some problems in the control performance



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Nonlinear Hold-Equivalent Sampled-Data Model

- Let us assume that the inductance depends on time but stays constant between sampling instants
- This assumption leads to a nonlinear sampled-data model

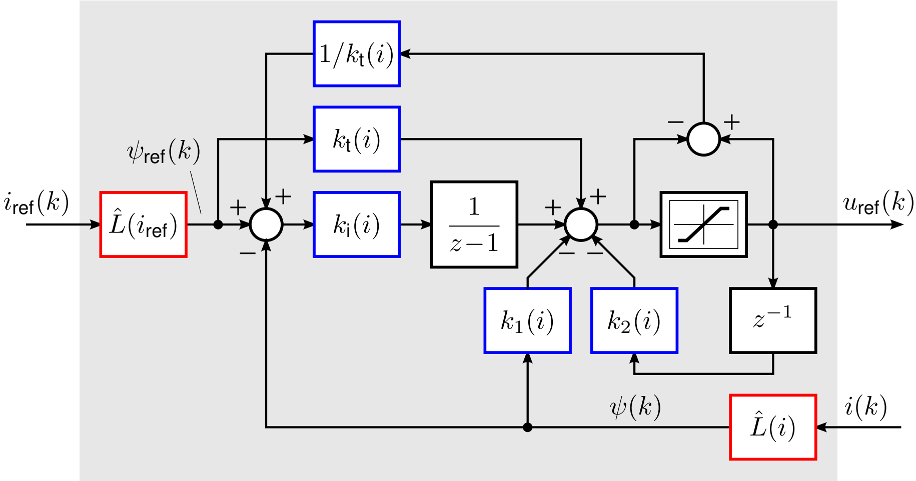
$$\begin{aligned} \psi(k+1) &= \phi(k)\psi(k) + \gamma(k)[u(k) - e(k)] \\ i(k) &= \psi(k)/L(k) \end{aligned}$$

- Using $\psi(k) = L(k)i(k)$, we get a mathematically equivalent form

$$i(k+1) = \frac{L(k)\phi(k)}{L(k+1)}i(k) + \frac{\gamma(k)}{L(k+1)}[u(k) - e(k)]$$

- We should know the future inductance in order to be able to use the last form as the basis for the controller (or observer)!
- In the previous controller, we inherently assumed that $L(k) = L(k+1)$

Gain-Scheduled Current Controller With Flux as a State Variable



- State variable changed from i to ψ and gains dependent on $\hat{L}(i)$
- For constant $\hat{L}$, the controller is equivalent to the previous ones

Gains for the Modified Controller

- Gains for the new state variable (just compare the block diagrams)

$$k_{\mathsf{t}} = \frac{b_1}{\gamma} \qquad k_2 = a_2 + \phi + 1 \qquad k_1 = \frac{a_1 - \phi(1 - k_2) + k_2}{\gamma} \qquad k_{\mathsf{i}} = k_1 + \frac{a_0 - k_2\phi}{\gamma}$$

- Model parameters depend on the current

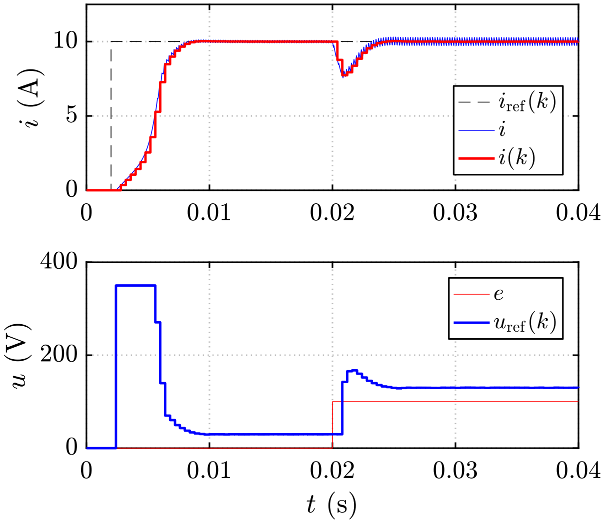
$$\phi = \mathsf{e}^{-[R/L(i)]T_{\mathsf{s}}} \qquad \gamma = \frac{1 - \mathsf{e}^{-[R/L(i)]T_{\mathsf{s}}}}{R/L(i)}$$

- Example selection $a_0 = 0$, $a_1 = \beta^2$, $a_2 = -2\beta$, $b_1 = 1 - \beta$ gives

$$\frac{\psi(z)}{\psi_{\mathsf{ref}}(z)} = \frac{1 - \beta}{z(z - \beta)}$$

Exercise 4.5

- Change the state variable of the controller from i to ψ
- For illustration purposes, increase T_s up to 400 μs
- Simulate the model
- Also test the robustness against measurement noise and saturation model errors
- Note that this scheme is still very easy to tune for a user



Further Reading

- H. A. A. Awan, T. Tuovinen, S. E. Saarakkala, and M. Hinkkanen, “Discrete-time observer design for sensorless synchronous motor drives,” *IEEE Trans. Ind. Applicat.*, 2016.
- S. E. Saarakkala, M. Sokolov, M. Hinkkanen, J. Kataja, and K. Tammi, “State-space flux-linkage control of bearingless synchronous reluctance motors,” in *Proc. IEEE-ECCE*, Milwaukee, WI, 2016.
- M. Hinkkanen, P. Pescetto, E. Mölsä, S. E. Saarakkala, G. Pellegrino, and R. Bojoi, “Sensorless self-commissioning of synchronous reluctance motors at standstill without rotor locking,” *IEEE Trans. Ind. Applicat.*, 2016, early access.