Aalto University
School of Electrical
Engineering

Module 1
State Feedback Current Control: Continuous-Time Design

Marko Hinkkanen

Politecnico di Torino, February 2017

Introduction

- Model-based controllers are preferred, since they can be automatically tuned based on the known (identified) model parameters
- Various control methods exist: state feedback control is considered here
- Control systems are implemented digitally, but they are often designed in the continuous-time domain

Simple Example System

- Generic LR load with the disturbance voltage is first considered for simplicity
- It could represent, for example, one axis of a 3-phase motor in the dq frame
- Simple system is chosen in order to be able to focus on the control challenges
- Later, magnetic saturation of L as well as LCL filters will be considered
- Controllers and their tuning principles can be extended to 3-phase AC motor drives and grid converters in a straightforward manner
- Observers can be designed and tuned using the same principles

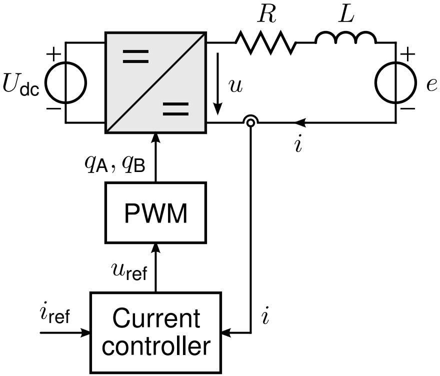
Current Control System

Closed-loop current control enables:

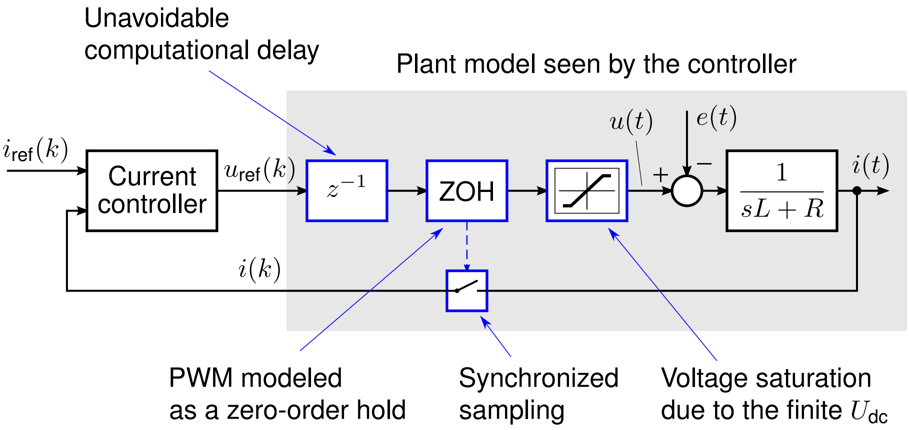
1. Current limitation
2. Precise and fast current control

Example system: LR circuit and the disturbace voltage

$$u = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + e$$



Nonidealities due to Digital Implementation and Actuator Saturation



Outline

Preliminaries

PI Current Control

State Feedback Control

Voltage Saturation and Anti-Windup

1st-Order System

- Transfer function from the input $u(s)$ to the output $y(s)$

$$\frac{y(s)}{u(s)} = G(s) = \frac{K}{1 + s\tau}$$

  where K is the DC gain and τ is the time constant
- Alternative commonly used form

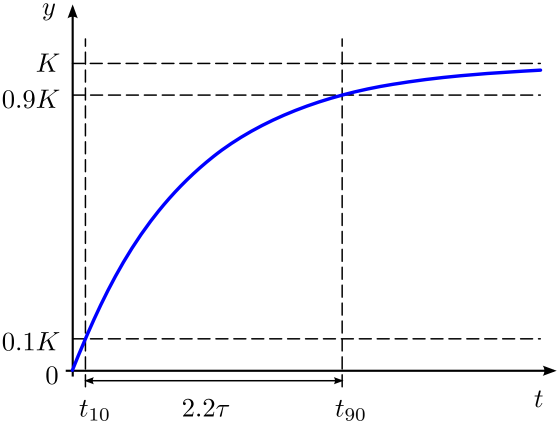
$$G(s) = \frac{K\alpha}{s + \alpha}$$

  where the 3-dB bandwidth is $\alpha = 1/\tau$

1st-Order System: Step Response

- Unit-step response

$$y(t) = K\left(1 - \mathrm{e}^{-t/\tau}\right)$$

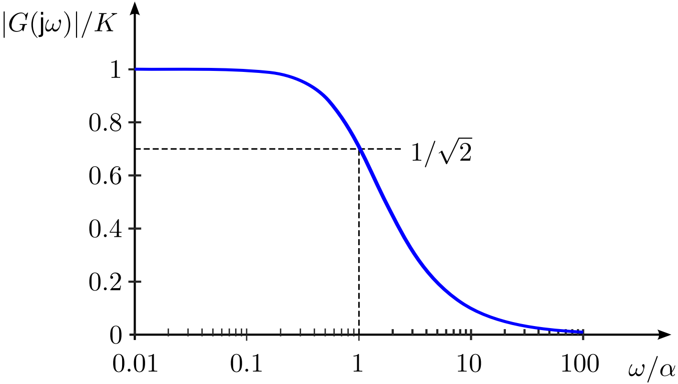


1st-Order System: Magnitude of the Frequency Response

- Magnitude of the frequency response

$$|G(\mathrm{j}\omega)| = \frac{K\alpha}{\sqrt{\alpha^2 + \omega^2}}$$

- Amplification at $\omega = \alpha$ is $|G(\mathrm{j}\alpha)| = K/\sqrt{2} \approx 0.71K$

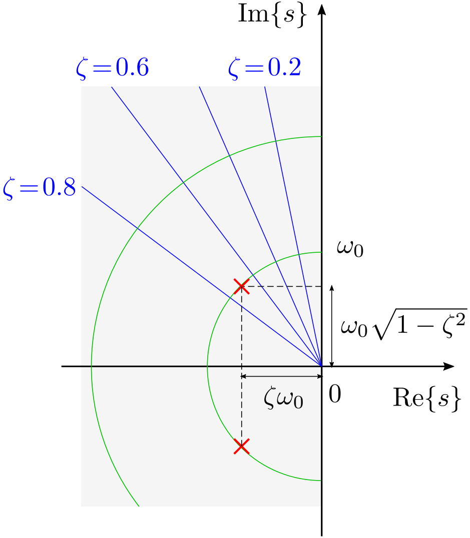


2nd-Order System

- Example transfer function from the input u to the output y

$$\frac{y(s)}{u(s)} = G(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

- Characteristic polynomial is often parametrized using
  - ω_0 = undamped angular frequency
  - ζ = damping ratio
- No zero in this example system

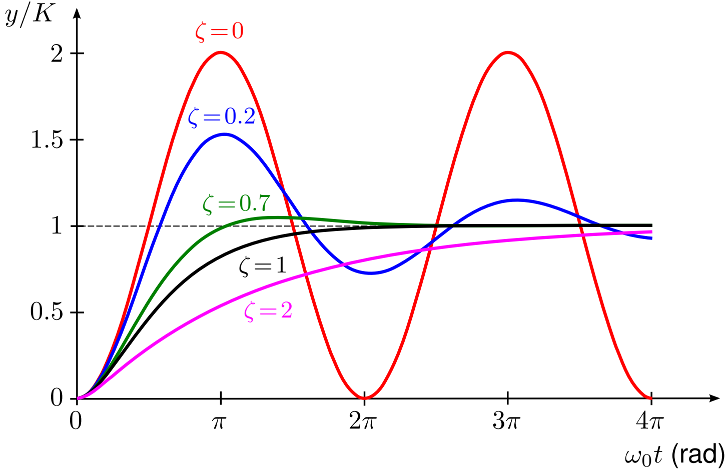


2nd-Order System: Step Response

- 2nd-order system

$$G(s) = \frac{y(s)}{u(s)} = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

- Response $y(t)$ to the step input $u(t)$ is shown
- No overshoot if $\zeta \geq 1$



Step responses can be easily plotted using numerical simulation tools. If needed, an analytical solution could be obtained using the inverse Laplace transformation.

2nd-Order System: Magnitude of the Frequency Response

- Consider a sinusoidal input

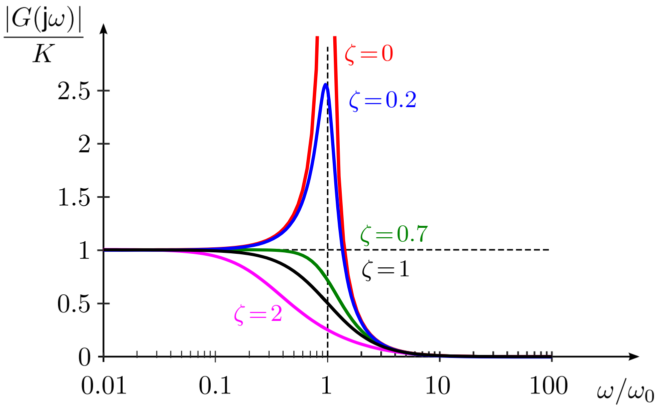
$$u(t) = U \sin(\omega t)$$

- For $\zeta > 0$, the output in steady state is

$$y(t) = AU \sin(\omega t + \phi)$$

where

$$A = |G(\mathrm{j}\omega)| \qquad \phi = \angle G(\mathrm{j}\omega)$$



State-Space Form

- State-space model consists of coupled 1st-order differential equations
- Derivatives $\mathrm{d}\boldsymbol{x}/\mathrm{d}t$ depend on the states $\boldsymbol{x}$ and the system input u

$$
\begin{aligned}
\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u \\
y &= \boldsymbol{C}\boldsymbol{x}
\end{aligned}
$$

- States $\boldsymbol{x}$ depend on the history, but not on the present values of the inputs
- Output y depends only on the states (in physical systems)
- State variables are typically associated with the energy storage
  - Current i of an inductor (or its flux linkage $\psi = Li$)
  - Voltage u of a capacitor (or its charge $q = Cu$)
  - Speed v of a mass (or its momentum $p = mv$)
- Choice of state variables is not unique (as shown in the parenthesis above)

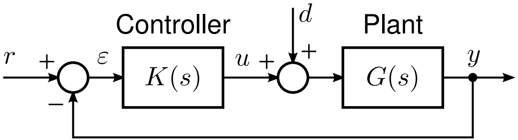
Closed-Loop Control

- Closed-loop transfer function

$$\frac{y(s)}{r(s)} = H(s) = \frac{L(s)}{1 + L(s)}$$

where $L(s) = K(s)G(s)$ is the loop transfer function

- Typical control objectives
  - Zero control error in steady state
  - Well-damped and fast transient response



$r =$ reference
$\varepsilon =$ control error
$u =$ control output
$d =$ load disturbance
$y =$ output

The stability of the closed-loop system $H(s)$ is often evaluated indirectly via the loop transfer function $L(s)$. For example, the gain and phase margins can be read from a Bode plot or a Nyquist plot of $L(\mathrm{j}\omega)$. In these lectures, we mainly analyse the closed-loop transfer function $H(s)$ directly.

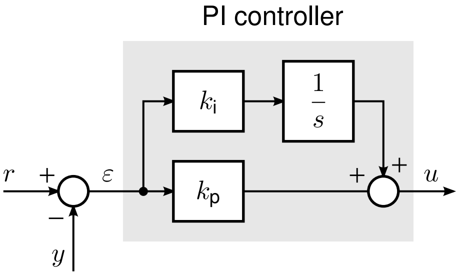
PI Controller

- Most common controller type
- Time domain

$$u = k_\mathsf{p}\varepsilon + \int k_\mathsf{i}\varepsilon \, \mathsf{d}t$$

- Transfer function

$$\frac{\varepsilon(s)}{u(s)} = K(s) = k_\mathsf{p} + \frac{k_\mathsf{i}}{s}$$



Outline

Preliminaries

PI Current Control

State Feedback Control

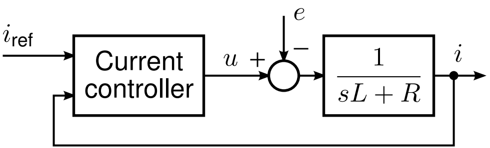
Voltage Saturation and Anti-Windup

Simplified System Model

- System model

$$L\frac{\mathrm{d}i}{\mathrm{d}t} = u - Ri - e$$

- Switching-cycle averaged quantities
- Ideal voltage production: $u = u_{\mathrm{ref}}$
- Computational delay is omitted
- Voltage e is a load disturbance
- P controller cannot drive the steady-state error to zero



PI Current Controller

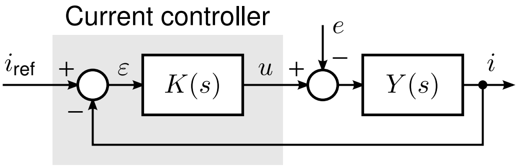
- Time domain

$$u = k_\mathsf{p}\varepsilon + \int k_\mathsf{i}\varepsilon \, \mathsf{d}t$$

- Transfer function

$$\frac{\varepsilon(s)}{u(s)} = K(s) = k_\mathsf{p} + \frac{k_\mathsf{i}}{s}$$

- How to tune the gains k_p and k_i?



$$\frac{i(s)}{u(s)} = Y(s) = \frac{1}{sL + R}$$

Closed-Loop Transfer Function

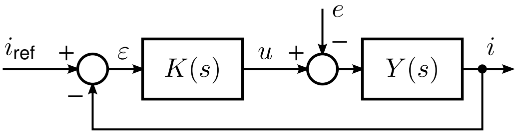
- Closed-loop transfer function

$$\frac{i(s)}{i_{\text{ref}}(s)} = H(s) = \frac{K(s)Y(s)}{1 + K(s)Y(s)}$$

- Desired closed-loop system

$$H(s) = \frac{\alpha_{\text{c}}}{s + \alpha_{\text{c}}}$$

  where α_{c} is the bandwidth
- Time constant of the closed-loop system is $\tau_{\text{c}} = 1/\alpha_{\text{c}}$



Example Tuning Principle

- Let us equal the closed-loop transfer function with the desirable one

$$H(s) = \frac{K(s)Y(s)}{1 + K(s)Y(s)} = \frac{\alpha_\mathrm{c}}{s + \alpha_\mathrm{c}} \qquad \Rightarrow \qquad K(s)Y(s) = \frac{\alpha_\mathrm{c}}{s}$$

- Controller $K(s)$ can be solved

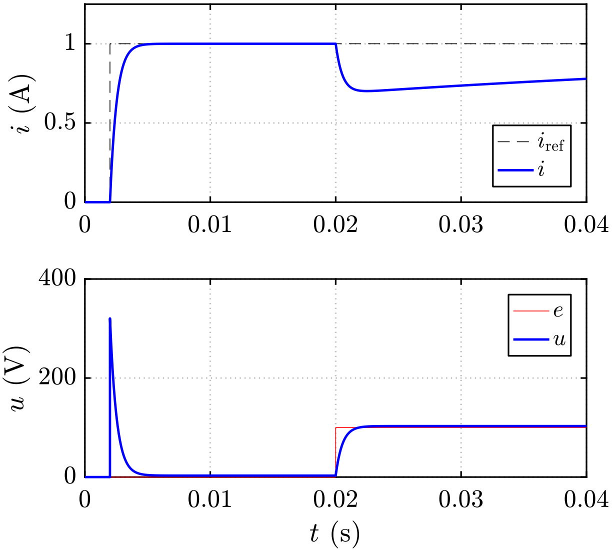
$$K(s) = \frac{\alpha_\mathrm{c}}{sY(s)} = \frac{\alpha_\mathrm{c}}{s}(sL + R) = \alpha_\mathrm{c} L + \frac{\alpha_\mathrm{c} R}{s}$$

- Result: PI controller with the gains $k_\mathrm{p} = \alpha_\mathrm{c}\hat{L}$ and $k_\mathrm{i} = \alpha_\mathrm{c}\hat{R}$
- Discretization: Angular sampling frequency $2\pi/T_\mathrm{s}$ should be at least one decade more than the bandwidth α_c

Exercise 1.1

- Build a Simulink model for the RL load with disturbance voltage
- Use the following parameters: $R = 3\ \Omega$ and $L = 0.17$ H
- Implement the continuous-time PI controller
- Omit PWM, delays, and voltage saturation
- Tune the controller according to the given principle, use $\alpha_\mathsf{c} = 2\pi \cdot 300$ rad/s
- Simulate using similar reference and disturbance as in the following example

- Current rise time agrees with the designed bandwidth
- PI controller tuned this way is sensitive to the load disturbance e
- Load-disturbance rejection can be improved by increasing k_i
- Try to improve the disturbance rejection. What happens to the reference tracking?



Outline

Preliminaries

PI Current Control

State Feedback Control

Voltage Saturation and Anti-Windup

State Feedback With Integral Action and Reference Feedforward

- System model

$$L\frac{\mathrm{d}i}{\mathrm{d}t} = u - Ri - e$$

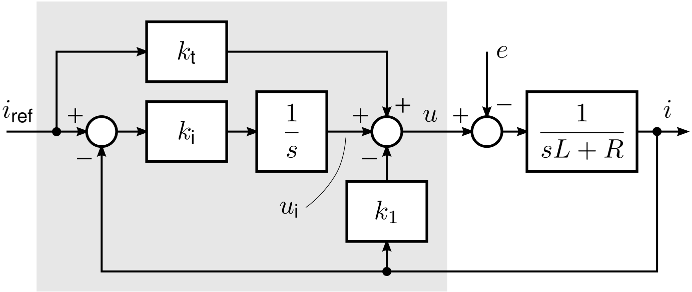
- Control law

$$\frac{\mathrm{d}u_\mathrm{i}}{\mathrm{d}t} = k_\mathrm{i}(i_\mathrm{ref} - i)$$
$$u = k_\mathrm{t} i_\mathrm{ref} - k_1 i + u_\mathrm{i}$$

- PI controller as a special case

$$k_\mathrm{t} = k_1$$

Current controller



- k_1 = state feedback gain
- k_t = reference feedforward gain
- k_i = integral gain

Closed-Loop Response

- Closed-loop current response in the Laplace domain

$$i(s) = \underbrace{\frac{k_{\mathsf{t}} s + k_{\mathsf{i}}}{s^2 L + (k_1 + R)s + k_{\mathsf{i}}}}_{H(s)} i_{\mathsf{ref}}(s) - \underbrace{\frac{s}{s^2 L + (k_1 + R)s + k_{\mathsf{i}}}}_{Y_e(s)} e(s)$$

- $H(0) = 1$ and $Y_e(0) = 0$ due to the integral action of the controller
- Assuming $\hat{R} = R$ and $\hat{L} = L$, the poles can be arbitrarily placed

$$k_{\mathsf{i}} = \omega_0^2 \hat{L} \qquad k_1 = 2\zeta\omega_0 \hat{L} - \hat{R}$$

- Disturbance rejection $Y_e(s)$ depends only on the poles
- Reference tracking $H(s)$ can be affected via k_{t}

Pole and Zero Placement

- Choosing $\zeta = 1$ gives

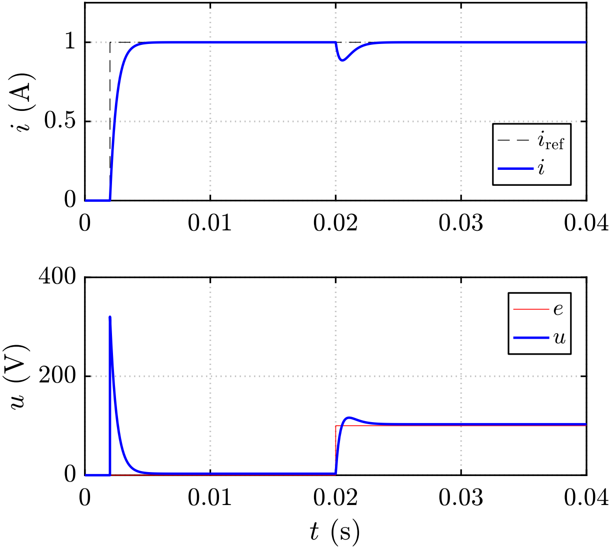
$$k_{\mathsf{i}} = \alpha_{\mathsf{c}}^2 \hat{L} \qquad k_1 = 2\alpha_{\mathsf{c}}\hat{L} - \hat{R} \qquad k_{\mathsf{t}} = \alpha_{\mathsf{c}}\hat{L}$$

- Selected k_{t} places the zero at $s = -\alpha_{\mathsf{c}}$
- Closed-loop current response in the Laplace domain reduces to

$$i(s) = \frac{\alpha_{\mathsf{c}}}{s + \alpha_{\mathsf{c}}} i_{\mathsf{ref}}(s) - \frac{s/L}{(s + \alpha_{\mathsf{c}})^2} e(s)$$

Exercise 1.2

- Modify the PI controller into the state feedback controller
- Simulate your model using similar reference and disturbance as in the figure



Outline

Preliminaries

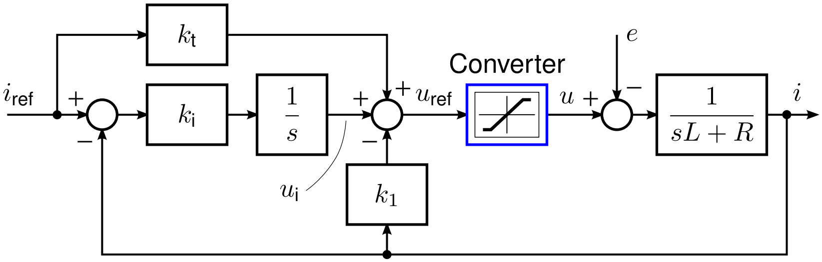
PI Current Control

State Feedback Control

Voltage Saturation and Anti-Windup

Voltage Saturation: Control Loop Becomes Nonlinear

- Maximum converter output voltage is limited: $u_{\text{min}} \leq u \leq u_{\text{max}}$
- u_{ref} may exceed the limits for large i_{ref} steps, especially at large values of $|e|$
- Integral state u_{i} in the controller may wind up

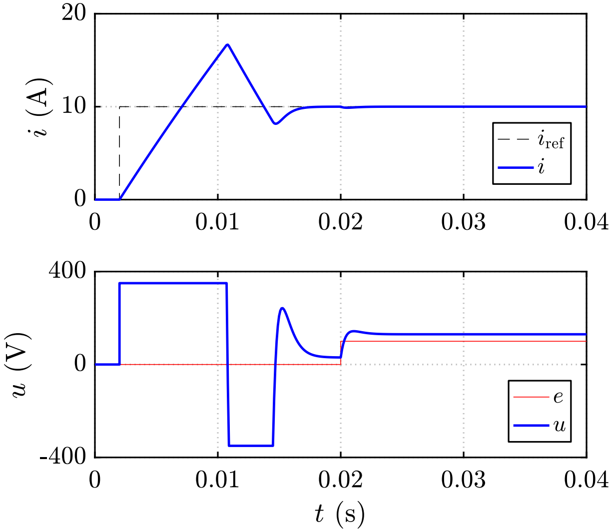


Exercise 1.3

- Include the converter voltage saturation in the simulation model

$$u_{\text{min}} = -350 \text{ V}$$
$$u_{\text{max}} = 350 \text{ V}$$

- Simulate first with the reference step of 1 A
- Change the reference step then to 10 A (in the figure)



Concept of the Realizable Reference

- Ideal (unlimited) voltage reference will be denoted by

$$u'_{\mathrm{ref}} = k_{\mathrm{t}} i_{\mathrm{ref}} - k_1 i + u_{\mathrm{i}}$$

- If the realizable reference i'_{ref} were applied to the controller instead of i_{ref}, the unlimited output u'_{ref} would equal the real plant input u obtained with i_{ref}

$$u = k_{\mathrm{t}} i'_{\mathrm{ref}} - k_1 i + u_{\mathrm{i}}$$

- Realizable reference can be solved

$$i'_{\mathrm{ref}} = i_{\mathrm{ref}} + \frac{u - u'_{\mathrm{ref}}}{k_{\mathrm{t}}}$$

- To prevent the integrator windup, we apply i'_{ref} for the integrator

Realizable Reference Anti-Windup

- Maximum voltage is known (typically U_{dc} is measured)
- Limited voltage reference

$$u_{\mathrm{ref}} = \mathrm{sat}(u'_{\mathrm{ref}}) = \begin{cases} u_{\mathrm{max}}, & \text{if } u'_{\mathrm{ref}} > u_{\mathrm{max}} \\ u_{\mathrm{min}}, & \text{if } u'_{\mathrm{ref}} < u_{\mathrm{min}} \\ u'_{\mathrm{ref}}, & \text{otherwise} \end{cases}$$

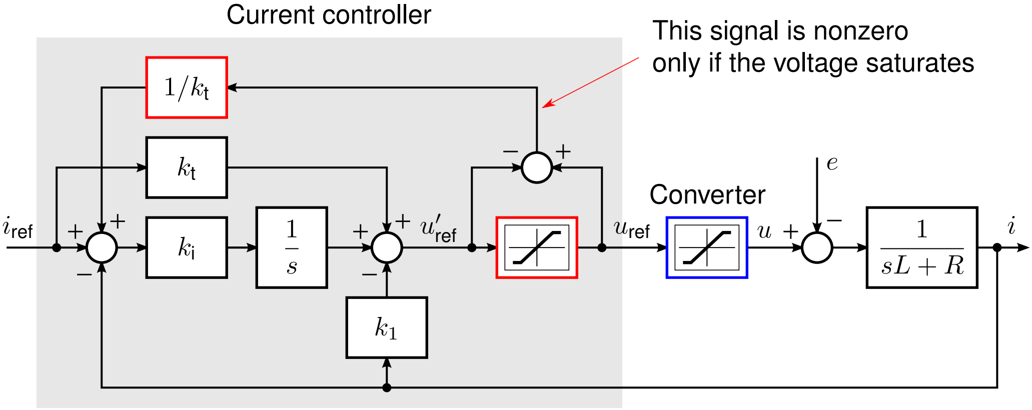
- Resulting control law

$$\frac{\mathrm{d}u_{\mathrm{i}}}{\mathrm{d}t} = k_{\mathrm{i}}\left(i_{\mathrm{ref}} - i + \frac{u_{\mathrm{ref}} - u'_{\mathrm{ref}}}{k_{\mathrm{t}}}\right)$$

$$u'_{\mathrm{ref}} = k_{\mathrm{t}} i_{\mathrm{ref}} - k_1 i + u_{\mathrm{i}}$$

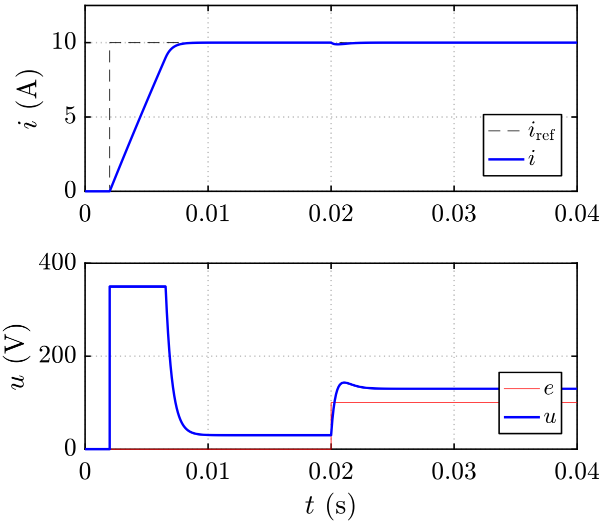
$$u_{\mathrm{ref}} = \mathrm{sat}(u'_{\mathrm{ref}})$$

Realizable Reference Anti-Windup



Exercise 1.4

- Implement the anti-windup in the controller
- Simulate the previous sequence
- No overshoot
- Rise time is longer than the specified one (due to voltage saturation)



Further Reading

- Y. Peng, D. Vrancic, and R. Hanus, “Anti-windup, bumpless, and conditioned transfer techniques for PID controllers,” *IEEE Control Syst. Mag.*, 1996.
- L. Harnefors and H.-P. Nee, “Model-based current control of ac machines using the internal model control method,” *IEEE Trans. Ind. Appl.*, 1998.
- F. Briz, M. W. Degner, and R. D. Lorenz, “Analysis and design of current regulators using complex vectors,” *IEEE Trans. Ind. Appl.*, 2000.
- M. Hinkkanen, H. A. A. Awan, Z. Qu, T. Tuovinen, and F. Briz, “Current control for synchronous motor drives: Direct discrete-time pole-place- ment design,” *IEEE Trans. Ind. Applicat.*, 2016.