Aalto University
School of Electrical
Engineering

Module 5
State Feedback Current Control: Converter Equipped With an LCL Filter

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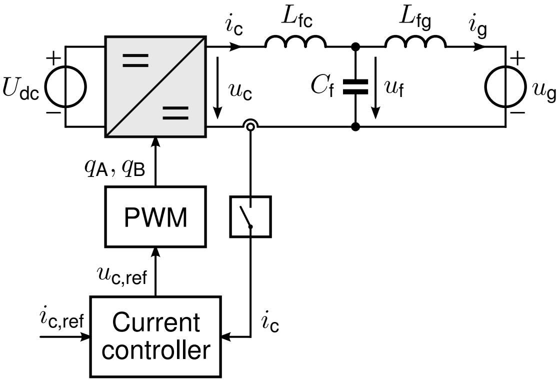
Converter Equipped with an LCL Filter

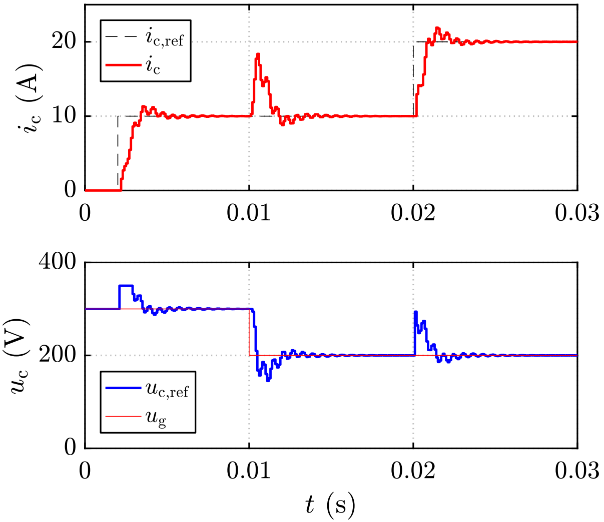
- LCL filters are commonly used in grid converters
- Resonance of the filter should be damped
- Since the resonance frequency is close to the sampling frequency, direct discrete-time control is preferred
- DC system is exemplified here for simplicity, but methods can be extended to 1-phase and 3-phase AC systems



This control problem is fully analogous to speed control of a two-mass mechanical system, where the only speed of the driving motor is measured

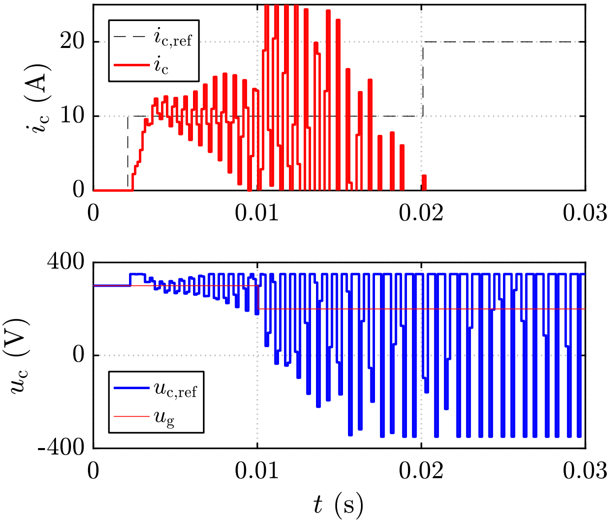
Simulation Example 5.1

- Preliminary example: LCL filter is approximated as an inductor
- Direct discrete-time controller for the LR circuit with disturbance voltage e is applied
- Controller sees the capacitor voltage u_f as the disturbance e
- Plant model is defined by $L = L_\mathrm{fc}$ and $R = 0$
- Maximum voltage $u_\mathrm{max} = 350$ V
- Bandwidth $\alpha_\mathrm{c} = 2\pi \cdot 600$ rad/s
- Sampling period $T_\mathrm{s} = 100$ μs



Simulation Example 5.2

- Sampling period is increased to $T_{\mathrm{s}} = 150\ \mu$s
- Other parameters are same as in the previous case
- For low sampling (switching) frequencies, active resonance damping is clearly needed
- This is a relevant problem especially in high-power converters

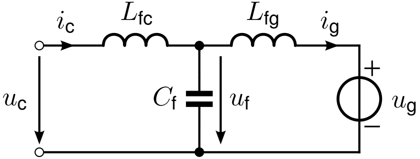


Outline

LCL Filter

- Continuous-time model

$$
\begin{aligned}
L_{\mathrm{fc}}\frac{\mathrm{d}i_{\mathrm{c}}}{\mathrm{d}t} &= u_{\mathrm{c}} - u_{\mathrm{f}} \\
C_{\mathrm{f}}\frac{\mathrm{d}u_{\mathrm{f}}}{\mathrm{d}t} &= i_{\mathrm{c}} - i_{\mathrm{g}} \\
L_{\mathrm{fg}}\frac{\mathrm{d}i_{\mathrm{g}}}{\mathrm{d}t} &= u_{\mathrm{f}} - u_{\mathrm{g}}
\end{aligned}
$$



- Parasitic losses are omitted

- State-space form

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix} i_\mathrm{c} \\ u_\mathrm{f} \\ i_\mathrm{g} \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} 0 & -1/L_\mathrm{fc} & 0 \\ 1/C_\mathrm{f} & 0 & -1/C_\mathrm{f} \\ 0 & 1/L_\mathrm{fg} & 0 \end{bmatrix}}_{\boldsymbol{A}} \begin{bmatrix} i_\mathrm{c} \\ u_\mathrm{f} \\ i_\mathrm{g} \end{bmatrix} + \underbrace{\begin{bmatrix} 1/L_\mathrm{fc} \\ 0 \\ 0 \end{bmatrix}}_{\boldsymbol{B}_\mathrm{c}} u_\mathrm{c} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -1/L_\mathrm{fg} \end{bmatrix}}_{\boldsymbol{B}_\mathrm{g}} u_\mathrm{g}$$

$$i_\mathrm{c} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{\boldsymbol{C}_\mathrm{c}} \boldsymbol{x} \qquad i_\mathrm{g} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{C}_\mathrm{g}} \boldsymbol{x}$$

- Transfer functions can be expressed using system matrices, for example

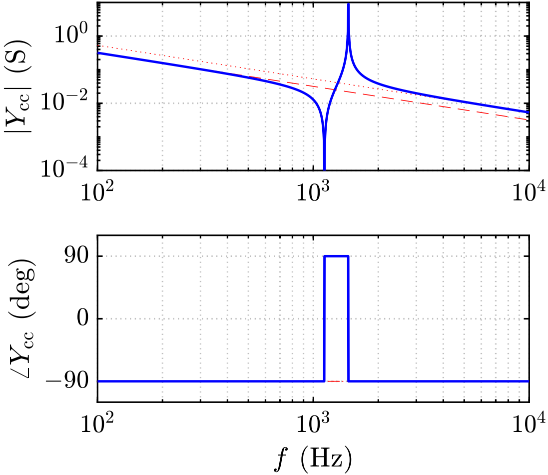
$$\frac{i_\mathrm{c}(s)}{u_\mathrm{c}(s)} = Y_\mathrm{cc}(s) = \boldsymbol{C}_\mathrm{c}(s\mathbf{I} - \boldsymbol{A})^{-1}\boldsymbol{B}_\mathrm{c} \qquad \frac{i_\mathrm{g}(s)}{u_\mathrm{c}(s)} = Y_\mathrm{gc}(s) = \boldsymbol{C}_\mathrm{g}(s\mathbf{I} - \boldsymbol{A})^{-1}\boldsymbol{B}_\mathrm{c}$$

Transfer Function from u_c to i_c

$$\frac{i_\mathsf{c}(s)}{u_\mathsf{c}(s)} = Y_\mathsf{cc}(s) = \frac{1}{sL_\mathsf{fc}} \frac{s^2 + \omega_\mathsf{z}^2}{s^2 + \omega_\mathsf{p}^2}$$

- Antiresonance frequency and resonance frequency

$$\omega_\mathsf{z} = \sqrt{\frac{1}{L_\mathsf{fg} C_\mathsf{f}}} \qquad \omega_\mathsf{p} = \sqrt{\frac{L_\mathsf{fc} + L_\mathsf{fg}}{L_\mathsf{fc} L_\mathsf{fg} C_\mathsf{f}}}$$



Transfer Function from u_{c} to i_{g}

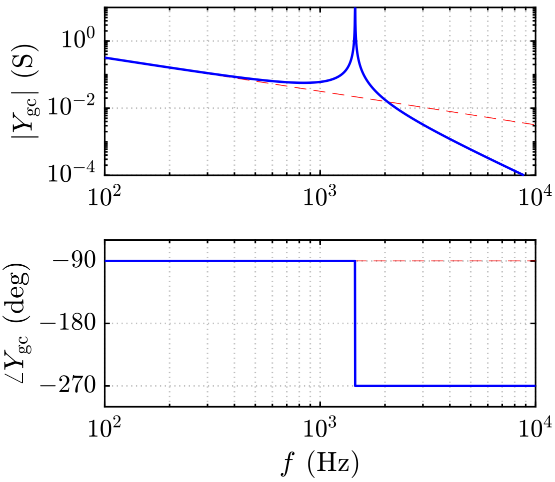
$$\frac{i_{\text{g}}(s)}{u_{\text{c}}(s)} = Y_{\text{gc}}(s) = \frac{1}{sL_{\text{fc}}} \frac{\omega_{\text{z}}^2}{s^2 + \omega_{\text{p}}^2}$$

- Dashed line corresponds to the inductor $L_{\text{fc}} + L_{\text{fg}}$
- Parameters in these examples

$$L_{\text{fc}} = 3 \text{ mH} \quad C_{\text{f}} = 10\ \mu\text{H} \quad L_{\text{fg}} = 2 \text{ mH}$$

- Resonance frequencies

$$f_{\text{z}} = \omega_{\text{z}}/(2\pi) = 1.13 \text{ kHz}$$
$$f_{\text{p}} = \omega_{\text{p}}/(2\pi) = 1.45 \text{ kHz}$$



Outline

Hold-Equivalent Discrete-Time Model

$$\underbrace{\begin{bmatrix} i_\mathsf{c}(k+1) \\ u_\mathsf{f}(k+1) \\ i_\mathsf{g}(k+1) \end{bmatrix}}_{\boldsymbol{x}(k+1)} = \underbrace{\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix}}_{\boldsymbol{\Phi}} \underbrace{\begin{bmatrix} i_\mathsf{c}(k) \\ u_\mathsf{f}(k) \\ i_\mathsf{g}(k) \end{bmatrix}}_{\boldsymbol{x}(k)} + \underbrace{\begin{bmatrix} \gamma_\mathsf{c1} \\ \gamma_\mathsf{c2} \\ \gamma_\mathsf{c3} \end{bmatrix}}_{\boldsymbol{\Gamma}_\mathsf{c}} u_\mathsf{c}(k) + \underbrace{\begin{bmatrix} \gamma_\mathsf{g1} \\ \gamma_\mathsf{g2} \\ \gamma_\mathsf{g3} \end{bmatrix}}_{\boldsymbol{\Gamma}_\mathsf{g}} u_\mathsf{g}(k)$$

$$i_\mathsf{c}(k) = \boldsymbol{C}_\mathsf{c}\boldsymbol{x}(k) \qquad i_\mathsf{g}(k) = \boldsymbol{C}_\mathsf{g}\boldsymbol{x}(k)$$

- Exact hold-equivalent discrete-time matrices

$$\boldsymbol{\Phi} = \mathrm{e}^{\boldsymbol{A}T_\mathsf{s}} \qquad \boldsymbol{\Gamma}_\mathsf{c} = \int_0^{T_\mathsf{s}} \mathrm{e}^{\boldsymbol{A}\tau}\mathrm{d}\tau \boldsymbol{B}_\mathsf{c} \qquad \boldsymbol{\Gamma}_\mathsf{g} = \int_0^{T_\mathsf{s}} \mathrm{e}^{\boldsymbol{A}\tau}\mathrm{d}\tau \boldsymbol{B}_\mathsf{g}$$

- Elements of these matrices can be solved in a closed form, for example

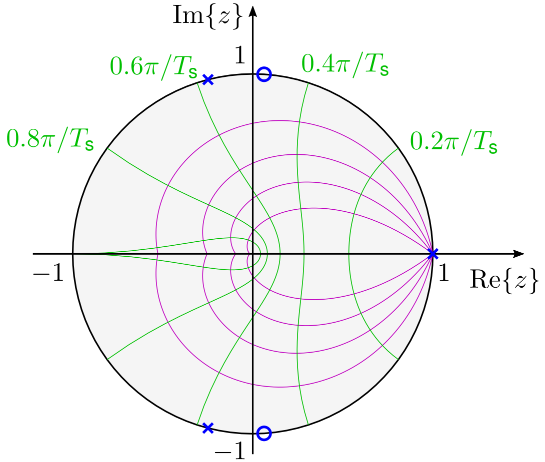
$$\phi_{11} = \frac{L_\mathsf{fc} + L_\mathsf{fg}\cos(\omega_\mathsf{p} T_\mathsf{s})}{L_\mathsf{fc} + L_\mathsf{fg}}$$

Open-Loop Poles and Zeros

- Open-loop transfer function

$$\frac{i_c(z)}{u_c(z)} = \boldsymbol{C}_c(z\mathbf{I} - \boldsymbol{\Phi})^{-1}\boldsymbol{\Gamma}_c$$
$$= \frac{\gamma_{c1}(z - z_1)(z - z_2)}{(z - 1)(z - e^{-j\omega_p T_s})(z - e^{j\omega_p T_s})}$$

- Integrator pole at $z = 1$
- Resonant poles at $z = e^{\pm j\omega_p T_s}$
- Resonant zeros at $z = z_{1,2}$



$L_{fc} = 3$ mH $\quad C_f = 10\ \mu$H $\quad L_{fg} = 2$ mH

$T_s = 200\ \mu$s

Inclusion of the Computational Delay

- One-sampling-period computational delay

$$u_{\mathsf{c}}(k) = u_{\mathsf{c,ref}}(k-1)$$

- State vector augmented with the delayed reference

$$\boldsymbol{x}_{\mathsf{d}} = \begin{bmatrix} \boldsymbol{x} \\ u_{\mathsf{c}} \end{bmatrix}$$

- Resulting state-space form

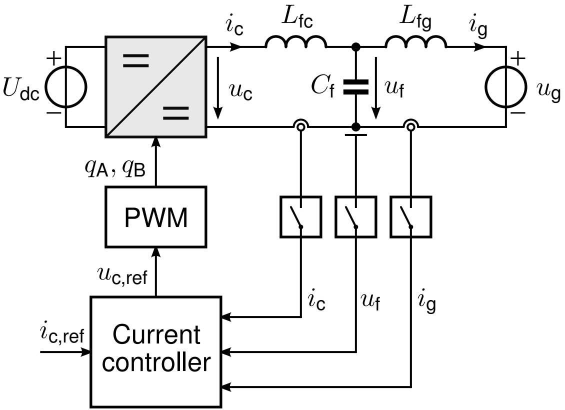
$$\begin{bmatrix} \boldsymbol{x}(k+1) \\ u_{\mathsf{c}}(k+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\Gamma}_{\mathsf{c}} \\ \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(k) \\ u_{\mathsf{c}}(k) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} u_{\mathsf{c,ref}}(k) + \begin{bmatrix} \boldsymbol{\Gamma}_{\mathsf{g}} \\ 0 \end{bmatrix} u_{\mathsf{g}}(k)$$

- Delay causes an extra pole at the origin, $z = 0$

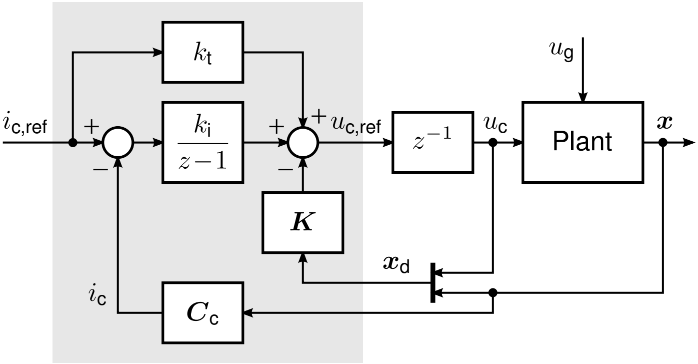
Outline

Full-State Feedback

- First, all the states are assumed to be measured
- State observer is designed later
- Separation principle holds
  - Controller and observer can be designed separately
  - Poles of the complete system: combination of the control and observer poles
- Voltage u_g is considered as an unknown disturbance (to illustrate disturbance rejection)



State Feedback Control System



$$\boldsymbol{x} = \begin{bmatrix} i_c \\ u_f \\ i_g \end{bmatrix}$$

$$\boldsymbol{x}_d = \begin{bmatrix} \boldsymbol{x} \\ u_c \end{bmatrix}$$

- State feedback controller with integral action and reference feedforward
- Plant is modeled as a sampled-data system
- Discrete-time indices k are not marked for simplicity

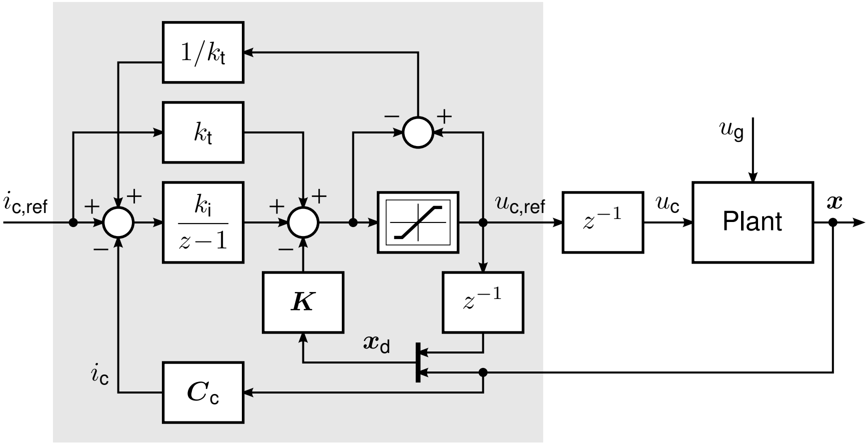
- Control law

$$x_\mathrm{i}(k+1) = x_\mathrm{i}(k) + i_\mathrm{c,ref}(k) - i_\mathrm{c}(k)$$
$$u_\mathrm{c,ref}(k) = k_\mathrm{t} i_\mathrm{c,ref}(k) - k_1 i_\mathrm{c}(k) - k_2 u_\mathrm{f}(k) - k_3 i_\mathrm{g}(k) - k_4 u_\mathrm{c}(k) + k_\mathrm{i} x_\mathrm{i}(k)$$

- Choosing $k_1 = k_\mathrm{t}$ and $k_2 = k_3 = k_4 = 0$ yields the PI controller
  - PI controller is not sufficient alone (unless passive resonance damping is used)
  - Some augmentation, such as a notch filter in the reference path, is needed
  - Filter and PI controller tuning is often a complicated and iterative process
- Resonance damping by means of a state feedback controller is simpler
- Direct discrete-time methods may look complicated but they easy to implement, since neither Tustin nor other approximations are needed

Compared to the previous LR circuit examples, the integrator state is chosen in a different manner here. This change is done in order to be able to easily use the state-space form for the integral gain calculation. If needed, this form can be easily transformed back to the previous one (which suits better for gain-scheduled k_i).

Controller With Antiwindup



Outline

Inclusion of the Integrator

- For placing the poles, the integrator is included in the state-space form

$$\underbrace{\begin{bmatrix} \boldsymbol{x}(k+1) \\ u_{\mathsf{c}}(k+1) \\ x_{\mathsf{i}}(k+1) \end{bmatrix}}_{\boldsymbol{x}_{\mathsf{a}}(k+1)} = \underbrace{\begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\Gamma}_{\mathsf{c}} & \boldsymbol{0} \\ \boldsymbol{0} & 0 & 0 \\ -\boldsymbol{C}_{\mathsf{c}} & 0 & 1 \end{bmatrix}}_{\boldsymbol{\Phi}_{\mathsf{a}}} \underbrace{\begin{bmatrix} \boldsymbol{x}(k) \\ u_{\mathsf{c}}(k) \\ x_{\mathsf{i}}(k) \end{bmatrix}}_{\boldsymbol{x}_{\mathsf{a}}(k)} + \underbrace{\begin{bmatrix} \boldsymbol{0} \\ 1 \\ 0 \end{bmatrix}}_{\boldsymbol{\Gamma}_{\mathsf{a}}} u_{\mathsf{c,ref}}(k) + \underbrace{\begin{bmatrix} \boldsymbol{0} \\ 0 \\ 1 \end{bmatrix}}_{\boldsymbol{\Gamma}_{\mathsf{t}}} i_{\mathsf{c,ref}}(k)$$

$$i_{\mathsf{c}}(k) = \underbrace{\begin{bmatrix} \boldsymbol{C}_{\mathsf{c}} & 0 & 0 \end{bmatrix}}_{\boldsymbol{C}_{\mathsf{a}}} \boldsymbol{x}_{\mathsf{a}}(k)$$

- Disturbance input u_{g} is omitted for simplicity

Closed-Loop System

- Augmented open-loop system model

$$\boldsymbol{x}_{\mathrm{a}}(k+1) = \boldsymbol{\Phi}_{\mathrm{a}}\boldsymbol{x}_{\mathrm{a}}(k) + \boldsymbol{\Gamma}_{\mathrm{a}}u_{\mathrm{c,ref}}(k) + \boldsymbol{\Gamma}_{\mathrm{t}}i_{\mathrm{c,ref}}(k)$$

- Control law

$$u_{\mathrm{c,ref}}(k) = k_{\mathrm{t}}i_{\mathrm{c,ref}}(k) - \boldsymbol{K}_{\mathrm{a}}\boldsymbol{x}_{\mathrm{a}}(k)$$

  where

$$\boldsymbol{K}_{\mathrm{a}} = \begin{bmatrix} \boldsymbol{K} & -k_{\mathrm{i}} \end{bmatrix}$$

- Resulting closed-loop transfer function

$$\frac{i_{\mathrm{c}}(z)}{i_{\mathrm{c,ref}}(z)} = \boldsymbol{C}_{\mathrm{a}}(z\mathbf{I} - \boldsymbol{\Phi}_{\mathrm{a}} + \boldsymbol{\Gamma}_{\mathrm{a}}\boldsymbol{K}_{\mathrm{a}})^{-1}(k_{\mathrm{t}}\boldsymbol{\Gamma}_{\mathrm{a}} + \boldsymbol{\Gamma}_{\mathrm{t}}) = \frac{N(z)}{D(z)}$$

- Numerator polynomial

$$N(z) = \gamma_{\mathsf{c1}} k_{\mathsf{t}} (z - 1 + k_{\mathsf{i}}/k_{\mathsf{t}})(z - z_1)(z - z_2)$$

- $z_{1,2}$ are the original open-loop zeros and the feedforward k_{t} adds one zero
- Denominator (characteristic) polynomial

$$D(z) = \det(z\mathbf{I} - \mathbf{\Phi}_{\mathsf{a}} + \mathbf{\Gamma}_{\mathsf{a}} \boldsymbol{K}_{\mathsf{a}})$$

- Poles can be freely placed: let the desired characteristic polynomial be

$$D(z) = (z - p_1)(z - p_2)(z - p_3)(z - p_4)(z - p_5)$$

- Gain $\boldsymbol{K}_{\mathsf{a}}$ leading to the desired poles $p_1 \dots p_5$ can be solved in a closed form or numerically (e.g. `acker.m`)

Outline

LCL Filter

Sampled-Data System Model

Control Law

Pole Placement

Selection of Pole Locations

Simulation Examples

Selection of Pole Locations

- Closed-loop poles can be arbitrarily set
- Poles can be first specified in the continuous-time domain and then mapped to the discrete-time domain via $z = \exp(sT_\mathsf{s})$
- Large control effort (output voltages) can be needed if the closed-loop poles are set far from the open-loop poles
- Resonant poles can just be damped while their natural frequency is not altered
- Selecting pole locations involves compromise between the robustness and dynamic performance
- Integrator pole can be cancelled with the reference-feedforward zero
- Delays cause open-loop poles at $z = 0$, just let them be there

Example Design

$$p_{1,2} = \exp\left[\left(-\zeta_\mathsf{r} \pm \mathsf{j}\sqrt{1-\zeta_\mathsf{r}^2}\right)\omega_\mathsf{p} T_\mathsf{s}\right] \qquad p_{3,4} = \beta = \exp(-\alpha_\mathsf{c} T_\mathsf{s}) \qquad p_5 = 0$$

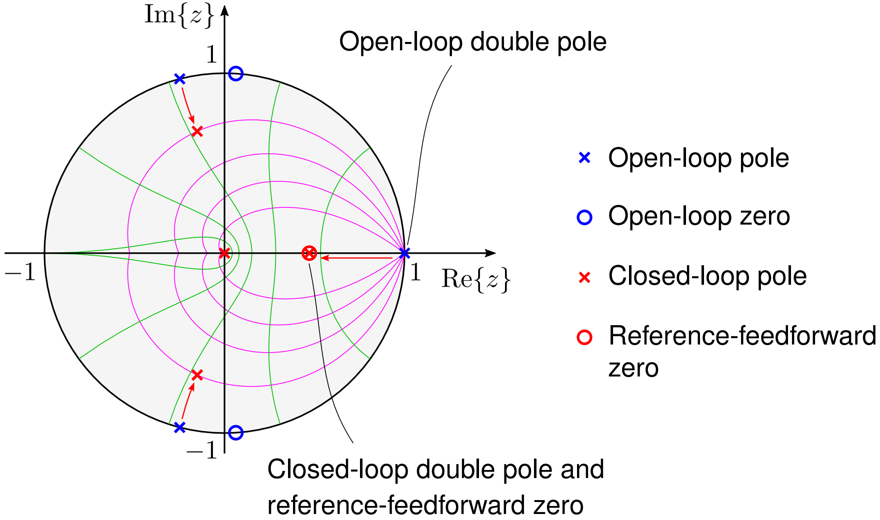
- ζ_r is the desired damping for resonant poles
- α_c is the desired bandwidth
- Reference-feedforward zero placed at $z = \beta$

$$k_\mathsf{t} = k_\mathsf{i}/(1-\beta)$$

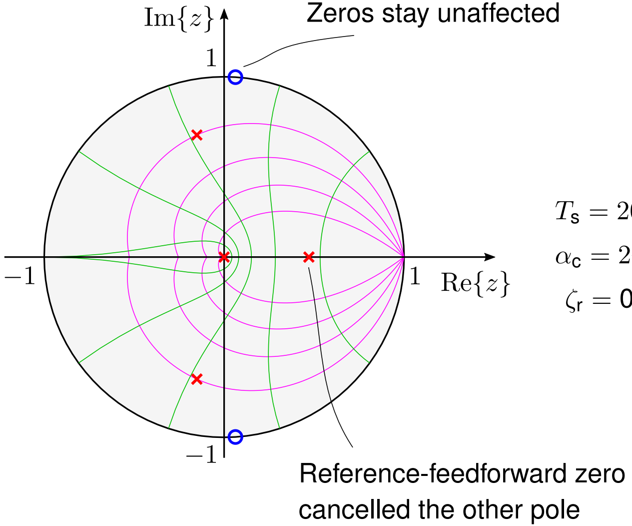
- Resulting closed-loop reference-tracking transfer function

$$\frac{i_\mathsf{c}(z)}{i_\mathsf{c,ref}(z)} = H(z) = \underbrace{\frac{1-\beta}{z(z-\beta)}}_{\text{Dominant dynamics}} \cdot \underbrace{\frac{\gamma_\mathsf{c1} k_\mathsf{t}}{1-\beta} \cdot \frac{(z-z_1)(z-z_2)}{(z-p_1)(z-p_2)}}_{\text{Damped resonant dynamics}}$$

Effect of the State Feedback on the Pole Locations



Closed-Loop Poles and Zeros



$T_\mathsf{s} = 200\ \mu\mathsf{s}$

$\alpha_\mathsf{c} = 2\pi \cdot 600$ rad/s

$\zeta_\mathsf{r} = 0.2$

Transfer Function from $i_{\mathsf{c,ref}}$ to i_{c}

- Closed-loop transfer function

$$H(z) = \frac{i_{\mathsf{c}}(z)}{i_{\mathsf{c,ref}}(z)}$$

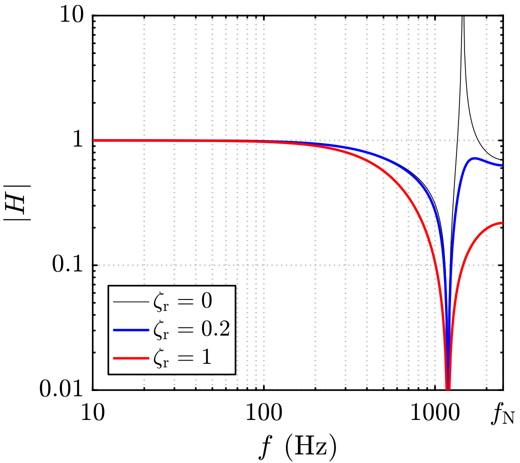
- Same parameters as before

$$T_{\mathsf{s}} = 200\ \mu\mathsf{s} \quad \alpha_{\mathsf{c}} = 2\pi \cdot 600\ \mathsf{rad/s}$$

- Nyquist frequency

$$f_{\mathsf{N}} = \frac{1}{2T_{\mathsf{s}}} = 2.5\ \mathsf{kHz}$$

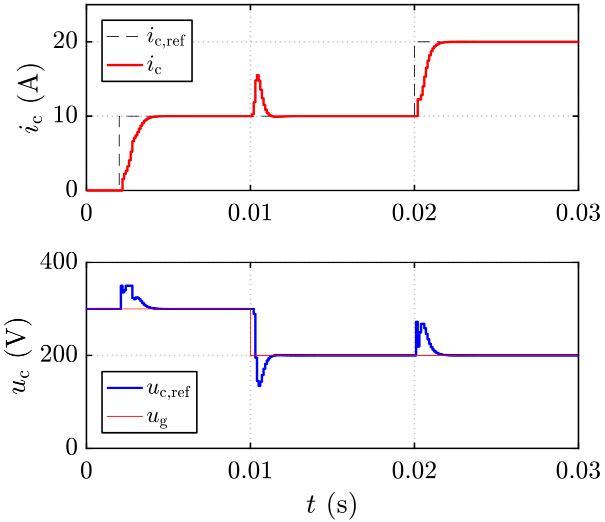
and resonant zeros set the fundamental upper limit for the achievable bandwidth



Outline

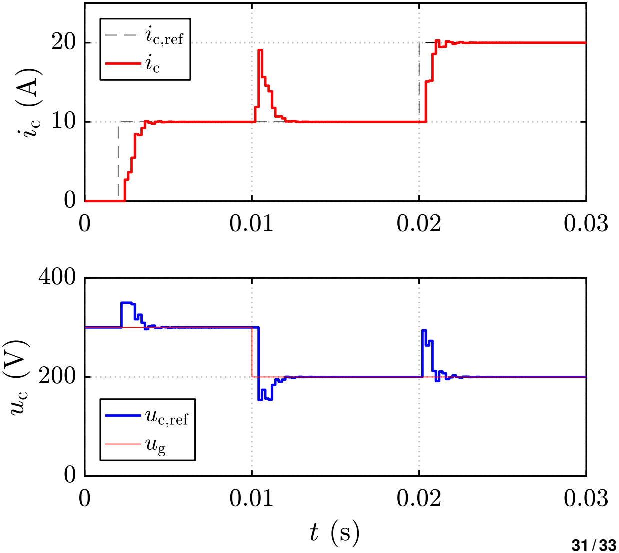
Simulation Example 5.3

- Bandwidth $\alpha_\text{c} = 2\pi\cdot$ 600 rad/s
- Resonance damping $\zeta_\text{r} = 1$
- Sampling period $T_\text{s} = 100\ \mu$s
- Effect of the resonant zeros can be seen in the step responses



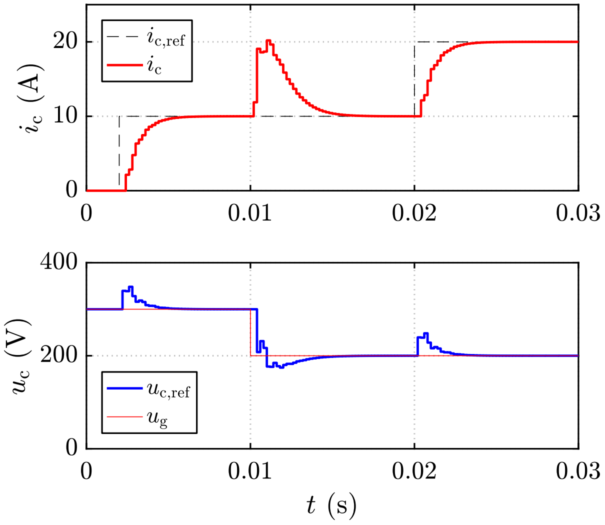
Simulation Example 5.4

- Bandwidth $\alpha_\mathrm{c} = 2\pi\cdot$ 600 rad/s
- Resonance damping $\zeta_\mathrm{r} = 0.2$
- Sampling period $T_\mathrm{s} = 200\ \mu$s
- Step in the unkown disturbance u_g causes a larger spike in i_c, due to longer T_s
- Control effort $u_\mathrm{c,ref}$ immediately after the steps in $i_\mathrm{c,ref}$ is lazier, due to the smaller ζ_r



Simulation Example 5.5

- Bandwidth dropped down to $\alpha_\mathrm{c} = 2\pi \cdot$ 200 rad/s in order to illustrate the desired step response
- Other parameters are the same as in the previous case
- Get familiar with the given simulation model and study also robustness aspects
- How could we get rid of measurements for u_f and i_g?



Further Reading

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- J. Koppinen, J. Kukkola, and M. Hinkkanen, “Parameter estimation of an LCL filter for control of grid converters,” in *Proc. ICPE 2015-ECCE Asia*, Seoul, South Korea, 2015.
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