Aalto University
School of Electrical
Engineering

Module 6

Observer-Based State Feedback Current Control: Converter Equipped With an LCL Filter

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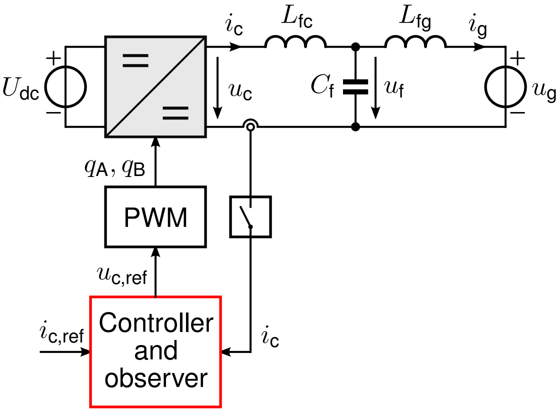
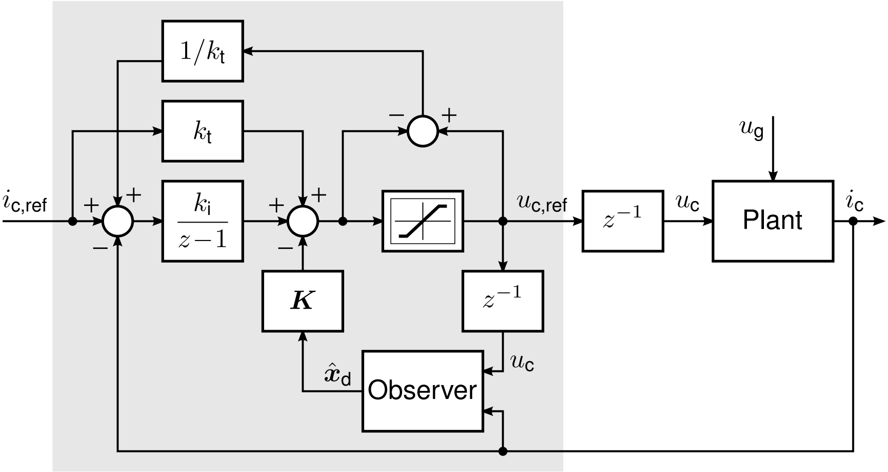
State Observer

- State observer is applied to avoid measuring u_f and i_g
- Observer could also be used to reduce sensitivity to noise and parameter errors
- Full-state feedback control law can be used together with the observed states
- Full-order and reduced-order observers are considered
- Choice depends on the application (measurement noise and disturbance rejection requirements)



Observer-Based State Feedback Control



Output of the observer block: $\hat{\boldsymbol{x}}_d = \begin{bmatrix} \hat{\boldsymbol{x}} \\ u_c \end{bmatrix}$, i.e., signal u_c just runs through the block

Outline

Full-Order Observer

- Sampled-data plant model (as presented earlier)

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{\Phi}\boldsymbol{x}(k) + \boldsymbol{\Gamma}_{\mathsf{c}} u_{\mathsf{c}}(k) + \boldsymbol{\Gamma}_{\mathsf{g}} u_{\mathsf{g}}(k) \\ i_{\mathsf{c}}(k) &= \boldsymbol{C}_{\mathsf{c}}\boldsymbol{x}(k) \end{aligned}$$

- Corresponding observer

$$\hat{\boldsymbol{x}}(k+1) = \boldsymbol{\Phi}\hat{\boldsymbol{x}}(k) + \boldsymbol{\Gamma}_{\mathsf{c}} u_{\mathsf{c}}(k) + \boldsymbol{K}_{\mathsf{o}}\left[i_{\mathsf{c}}(k) - \boldsymbol{C}_{\mathsf{c}}\hat{\boldsymbol{x}}(k)\right]$$

where u_{g} is considered as an unknown disturbance and omitted for simplicity

Estimation-Error Dynamics

- Dynamics of the estimation error $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$

$$\tilde{\boldsymbol{x}}(k+1) = (\boldsymbol{\Phi} - \boldsymbol{K}_{\mathsf{o}}\boldsymbol{C}_{\mathsf{c}})\,\tilde{\boldsymbol{x}}(k) + \boldsymbol{\Gamma}_{\mathsf{g}} u_{\mathsf{g}}(k)$$

- Observer estimates the resonant dynamics properly but nonzero u_{g} causes nonzero steady-state estimation error $\tilde{\boldsymbol{x}}$
- This is not a problem since the controller has an integral action
- If needed, the steady-state estimation error could be avoided by augmenting the observer with a disturbance state for u_{g}

Selection of Observer Pole Locations

- Characteristic polynomial

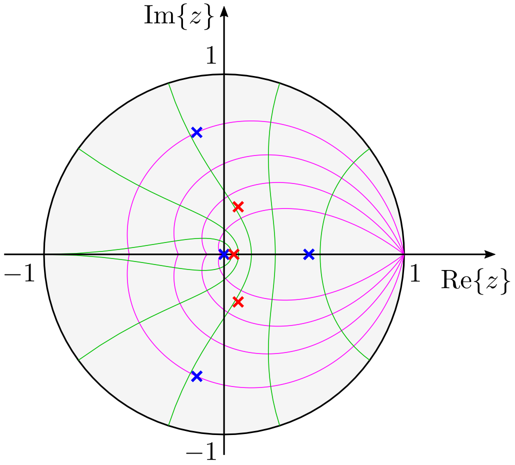
$$D_\mathsf{o}(z) = \mathsf{det}(z\mathbf{I} - \mathbf{\Phi} + \boldsymbol{K}_\mathsf{o}\boldsymbol{C}_\mathsf{c})$$

- Desired observer poles

$$D_\mathsf{o}(z) = (z - p_{\mathsf{o}1})(z - p_{\mathsf{o}2})(z - p_{\mathsf{o}3})$$

- Observer poles can be placed similarly to the control poles
- Resonant poles are damped and dominant observer pole is chosen to be faster than the dominant control poles (typically 2…4 times)
- Observer effectively filters the measurement noise

Control and Observer Poles



× Control pole

× Observer pole

$\alpha_o = 4\alpha_c$

$\zeta_{or} = 0.7$

Control Algorithm With Full-Order Observer

- Compute the controller output

$$u_{\mathsf{c,ref}}(k) = k_{\mathsf{t}} i_{\mathsf{c,ref}}(k) - \boldsymbol{K}\hat{\boldsymbol{x}}_{\mathsf{d}}(k) + k_{\mathsf{i}} x_{\mathsf{i}}(k)$$

  where $\hat{\boldsymbol{x}}_{\mathsf{d}} = [\hat{\boldsymbol{x}}^{\mathsf{T}}, u_{\mathsf{c}}]^{\mathsf{T}}$

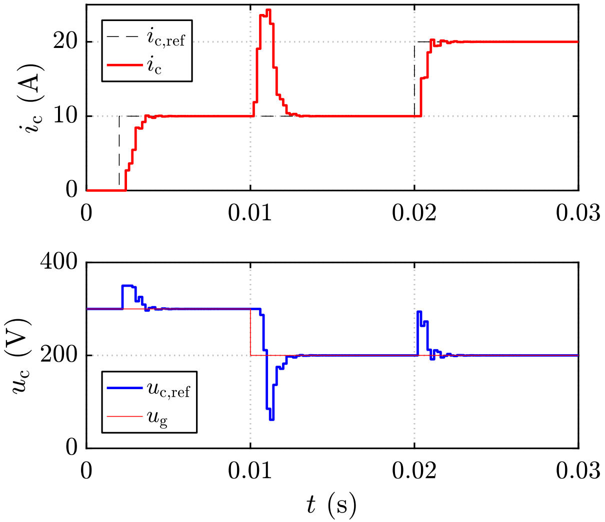
- Update the states for the next sampling period

$$\begin{aligned}
\hat{\boldsymbol{x}}(k+1) &= \boldsymbol{\Phi}\hat{\boldsymbol{x}}(k) + \boldsymbol{\Gamma}_{\mathsf{c}} u_{\mathsf{c}}(k) + \boldsymbol{K}_{\mathsf{o}}\left[i_{\mathsf{c}}(k) - \boldsymbol{C}_{\mathsf{c}}\hat{\boldsymbol{x}}(k)\right] \\
u_{\mathsf{c}}(k+1) &= u_{\mathsf{c,ref}}(k) \\
x_{\mathsf{i}}(k+1) &= x_{\mathsf{i}}(k) + i_{\mathsf{c,ref}}(k) - i_{\mathsf{c}}(k)
\end{aligned}$$

- Note that the current i_{c} measured at k affects the reference voltage $u_{\mathsf{c,ref}}$ at $k+1$

Simulation Example 6.1

- Bandwidth $\alpha_\mathrm{c} = 2\pi\cdot$ 600 rad/s
- Resonance damping $\zeta_\mathrm{r} = 0.2$
- Sampling period $T_\mathrm{s} = 200$ μs
- Dominant observer pole $\alpha_\mathrm{o} = 4\alpha_\mathrm{c}$
- Observer resonance damping $\zeta_\mathrm{or} = 0.7$
- Reference tracking is similar to the full-state feedback controller but disturbance rejection is poorer



Outline

Splitting the System Model: Measured and Unknown States

- Sampled-data plant model

$$\boldsymbol{x}(k+1) = \boldsymbol{\Phi}\boldsymbol{x}(k) + \boldsymbol{\Gamma}_{\mathsf{c}} u_{\mathsf{c}}(k) + \boldsymbol{\Gamma}_{\mathsf{g}} u_{\mathsf{g}}(k)$$

- Let us split the system: measured state i_{c} and unknown state $\boldsymbol{x}_2$

$$\begin{bmatrix} i_{\mathsf{c}}(k+1) \\ \boldsymbol{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \phi_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{21} & \boldsymbol{\Phi}_{22} \end{bmatrix} \begin{bmatrix} i_{\mathsf{c}}(k) \\ \boldsymbol{x}_2(k) \end{bmatrix} + \begin{bmatrix} \gamma_{\mathsf{c1}} \\ \boldsymbol{\Gamma}_{\mathsf{c2}} \end{bmatrix} u_{\mathsf{c}}(k) + \begin{bmatrix} \gamma_{\mathsf{g1}} \\ \boldsymbol{\Gamma}_{\mathsf{g2}} \end{bmatrix} u_{\mathsf{g}}(k)$$

  where $\boldsymbol{\Phi}_{12} = \begin{bmatrix} \phi_{12} & \phi_{13} \end{bmatrix}$ and

$$\boldsymbol{x}_2 = \begin{bmatrix} u_{\mathsf{f}} \\ i_{\mathsf{g}} \end{bmatrix} \quad \boldsymbol{\Phi}_{21} = \begin{bmatrix} \phi_{21} \\ \phi_{31} \end{bmatrix} \quad \boldsymbol{\Phi}_{22} = \begin{bmatrix} \phi_{22} & \phi_{23} \\ \phi_{32} & \phi_{33} \end{bmatrix} \quad \boldsymbol{\Gamma}_{\mathsf{c2}} = \begin{bmatrix} \gamma_{\mathsf{c2}} \\ \gamma_{\mathsf{c3}} \end{bmatrix} \quad \boldsymbol{\Gamma}_{\mathsf{g2}} = \begin{bmatrix} \gamma_{\mathsf{g2}} \\ \gamma_{\mathsf{g3}} \end{bmatrix}$$

- Only the unknown states are to be estimated

Reduced-Order Observer

- Unknown state

$$\boldsymbol{x}_2(k+1) = \boldsymbol{\Phi}_{22}\boldsymbol{x}_2(k) + \underbrace{\boldsymbol{\Phi}_{21} i_\mathsf{c}(k) + \boldsymbol{\Gamma}_\mathsf{c2} u_\mathsf{c}(k)}_{\text{known input}} + \underbrace{\boldsymbol{\Gamma}_\mathsf{g2} u_\mathsf{g}(k)}_{\text{disturbance input}}$$

- Known measurements

$$i_\mathsf{c}(k+1) - \phi_{11} i_\mathsf{c}(k) - \gamma_\mathsf{c1} u_\mathsf{c}(k) - \gamma_\mathsf{g1} u_\mathsf{g}(k) = \boldsymbol{\Phi}_{12}\boldsymbol{x}_2(k)$$

- Reduced-order observer can be formulated

$$\begin{aligned} \hat{\boldsymbol{x}}_2(k+1) = &\boldsymbol{\Phi}_{22}\hat{\boldsymbol{x}}_2(k) + \boldsymbol{\Phi}_{21} i_\mathsf{c}(k) + \boldsymbol{\Gamma}_\mathsf{c2} u_\mathsf{c}(k) \\ &+ \boldsymbol{K}_\mathsf{o2}\left[i_\mathsf{c}(k+1) - \phi_{11} i_\mathsf{c}(k) - \gamma_\mathsf{c1} u_\mathsf{c}(k) - \boldsymbol{\Phi}_{12}\hat{\boldsymbol{x}}_2(k)\right] \end{aligned}$$

where the disturbance input u_g has been omitted

Pole Placement

- Dynamics of the estimation error $\tilde{\boldsymbol{x}}_2 = \boldsymbol{x}_2 - \hat{\boldsymbol{x}}_2$

$$\tilde{\boldsymbol{x}}_2(k+1) = \left(\boldsymbol{\Phi}_{22} - \boldsymbol{K}_{\mathsf{o2}}\boldsymbol{\Phi}_{12}\right)\tilde{\boldsymbol{x}}_2(k) + \left(\boldsymbol{\Gamma}_{\mathsf{g2}} - \boldsymbol{K}_{\mathsf{o2}}\gamma_{\mathsf{g1}}\right)u_{\mathsf{g}}(k)$$

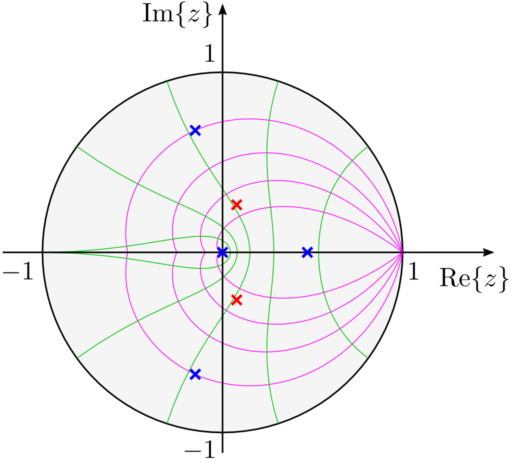
- Characteristic polynomial

$$D_{\mathsf{o2}}(z) = \det(z\mathbf{I} - \boldsymbol{\Phi}_{22} + \boldsymbol{K}_{\mathsf{o2}}\boldsymbol{\Phi}_{12})$$

- Desired poles

$$D_{\mathsf{o2}}(z) = (z - p_{\mathsf{o1}})(z - p_{\mathsf{o2}})$$

Control and Observer Poles



× Control pole

× Observer pole

$\zeta_{\text{or}} = 0.7$

Control Algorithm With Reduced-Order Observer

- First compute the state estimate

$$\begin{aligned}\hat{\boldsymbol{x}}_2(k) = \boldsymbol{\Phi}_{22}\hat{\boldsymbol{x}}_2(k-1) + \boldsymbol{\Phi}_{21} i_\mathsf{c}(k-1) + \boldsymbol{\Gamma}_{\mathsf{c2}} u_\mathsf{c}(k-1) \\ + \boldsymbol{K}_{\mathsf{o2}}\left[i_\mathsf{c}(k) - \phi_{11} i_\mathsf{c}(k-1) - \gamma_{\mathsf{c1}} u_\mathsf{c}(k-1) - \boldsymbol{\Phi}_{12}\hat{\boldsymbol{x}}_2(k-1)\right]\end{aligned}$$

- Then compute the controller output

$$u_{\mathsf{c,ref}}(k) = k_\mathsf{t} i_{\mathsf{c,ref}}(k) - \boldsymbol{K}\hat{\boldsymbol{x}}_\mathsf{d}(k) + k_\mathsf{i} x_\mathsf{i}(k)$$

  where $\hat{\boldsymbol{x}}_\mathsf{d} = [i_\mathsf{c}, \hat{\boldsymbol{x}}_2^\mathsf{T}, u_\mathsf{c}]^\mathsf{T}$

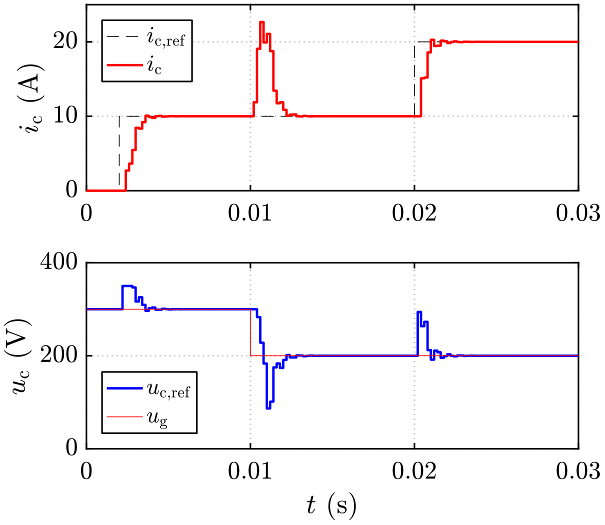
- Update the states for the next sampling period

$$\begin{aligned}u_\mathsf{c}(k+1) &= u_{\mathsf{c,ref}}(k) \\ x_\mathsf{i}(k+1) &= x_\mathsf{i}(k) + i_{\mathsf{c,ref}}(k) - i_\mathsf{c}(k)\end{aligned}$$

- Note that the current i_c measured at k affects the reference voltage $u_{\mathsf{c,ref}}$ at k

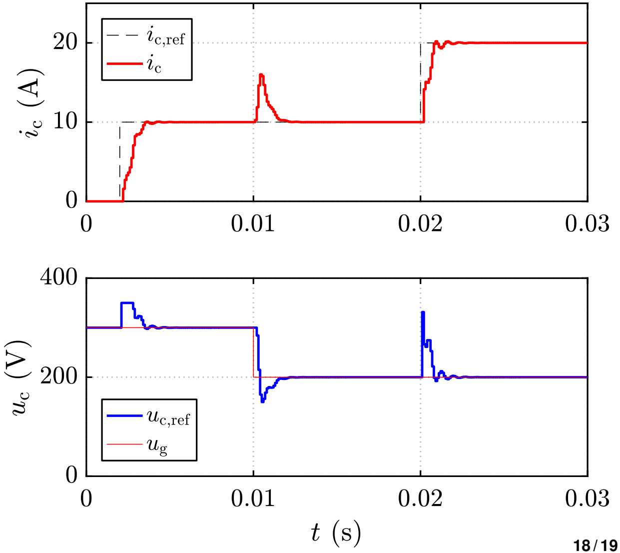
Simulation Example 6.2

- Bandwidth $\alpha_{\mathrm{c}} = 2\pi\cdot$ 600 rad/s
- Resonance damping $\zeta_{\mathrm{r}} = 0.2$
- Sampling period $T_{\mathrm{s}} = 200$ μs
- Observer resonance damping $\zeta_{\mathrm{or}} = 0.7$
- Better disturbance rejection than in the case of the full-order observer
- More sensitive to noise than the full-order observer



Simulation Example 6.3

- Sampling period $T_s = 100\ \mu$s
- Otherwise the same parameters as in the previous case
- Only three parameters are needed for tuning the control system: α_c, ζ_r, and ζ_{or}



Further Reading

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