2.1 Introduction

Time is an important independent variable required to measure/monitor any activity. Hence, whatever phenomena we observe in nature are always measured as a function of time.

Time is a continuous independent variable represented by the letter 't'. Any physical observation or a measure is a continuous function of time represented by x(t) and called ***signal.*** The signal x(t) is called the continuous time (CT) signal which is a function of the independent variable, time. In x(t),the unit of time and the unit of the value of x(t) at any time is not considered but only the numerical values are considered.

The signal x(t) can be used to represent any physical quantity, and the start of any observation or a measure of the physical quantity is taken as time t = 0. The time after the start of observation is taken as positive time and the time before the start of observation is taken as negative time.

The physical devices which are all sources of continuous time signals are called ***continuous time systems***. The standard continuous time signals, mathematical operation on continuous time signals and classification of continuous time signals are discussed in this chapter. The mathematical representation of continuous time systems and their analyses are also presented in this chapter. Wherever required, the discussion on LTI systems are presented separately.

2.2 Standard Continuous Time Signals

1. Impulse signal

The impulse signal is a signal with infinite magnitude and zero duration, but with an area of A. Mathematically, impulse signal is defined as,

$$\text{Impulse Signal, } \delta(t) = \infty\ ;\ t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\, dt = A$$
$$= 0\ ;\ t \neq 0$$

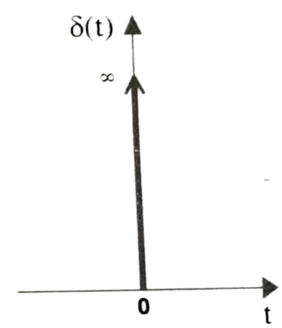

***Fig 2.1 :** Impulse signal (or Unit Impulse signal).*

The unit impulse signal is a signal with infinite magnitude and zero duration, but with unit area. Mathematically, unit impulse signal is defined as,

$$\text{Unit Impulse Signal, } \delta(t) = \infty\ ;\ t = 0 \qquad \text{and} \qquad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0\ ;\ t \neq 0$$

2. Step signal

The step signal is defined as,

$$x(t) = A\ ;\ t \geq 0$$
$$= 0\ ;\ t < 0$$

The unit step signal is defined as,

$$x(t) = u(t) = 1\ ;\ t \geq 0$$
$$= 0\ ;\ t < 0$$

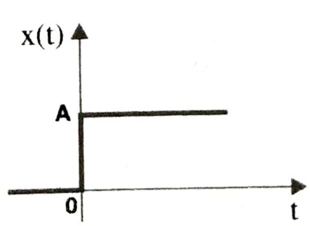

***Fig 2.2 :** Step signal.*

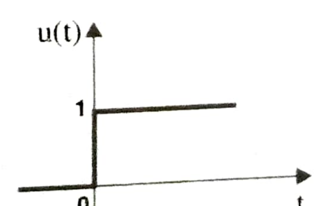

***Fig 2.3 :** Unit step signal.*

3. Ramp signal

The ramp signal is defined as,

$$x(t) = At \;;\; t \geq 0$$
$$= 0 \;;\; t < 0$$

The unit ramp signal is defined as,

$$x(t) = t \;;\; t \geq 0$$
$$= 0 \;;\; t < 0$$

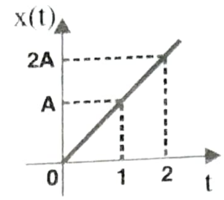

Fig 2.4 : Ramp signal.

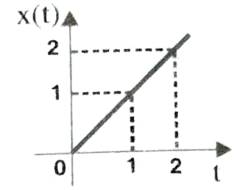

Fig 2.5 : Unit ramp signal.

4. Parabolic signal

The parabolic signal is defined as,

$$x(t) = \frac{At^2}{2} \;;\; \text{for } t \geq 0$$
$$= 0 \;;\; t < 0$$

The unit parabolic signal is defined as,

$$x(t) = \frac{t^2}{2} \;;\; \text{for } t \geq 0$$
$$= 0 \;;\; t < 0$$

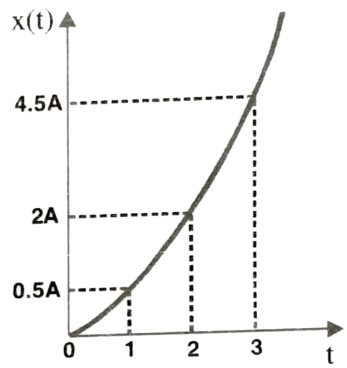

Fig 2.6 : Parabolic signal.

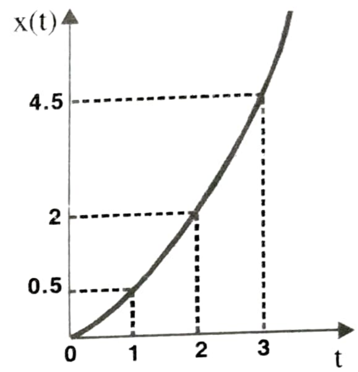

Fig 2.7 :Unit parabolic signal.

5. Unit pulse signal

The unit pulse signal is defined as,

$$x(t) = \Pi(t) = u\left(t + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right)$$

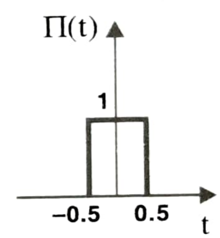

Fig 2.8 : Unit pulse signal.

6. Sinusoidal signal

Case i : Cosinusoidal signal

The cosinusoidal signal is defined as,

$$x(t) = A\cos(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

When $\phi = 0$, $\quad x(t) = A\cos\Omega_0 t$

When ϕ = Positive, $\quad x(t) = A\cos(\Omega_0 t + \phi)$

When ϕ = Negative, $\quad x(t) = A\cos(\Omega_0 t - \phi)$

Case ii : Sinusoidal signal

The sinusoidal signal is defined as,

$$x(t) = A \sin(\Omega_0 t + \phi)$$

where, $\Omega_0 = 2\pi F_0 = \dfrac{2\pi}{T}$ = Angular frequency in rad/sec

F_0 = Frequency in cycles/sec or Hz

T = Time period in sec

When $\phi = 0$, $\quad x(t) = A \sin\Omega_0 t$

When ϕ = Positive, $\quad x(t) = A \sin(\Omega_0 t + \phi)$

When ϕ = Negative, $\quad x(t) = A \sin(\Omega_0 t - \phi)$

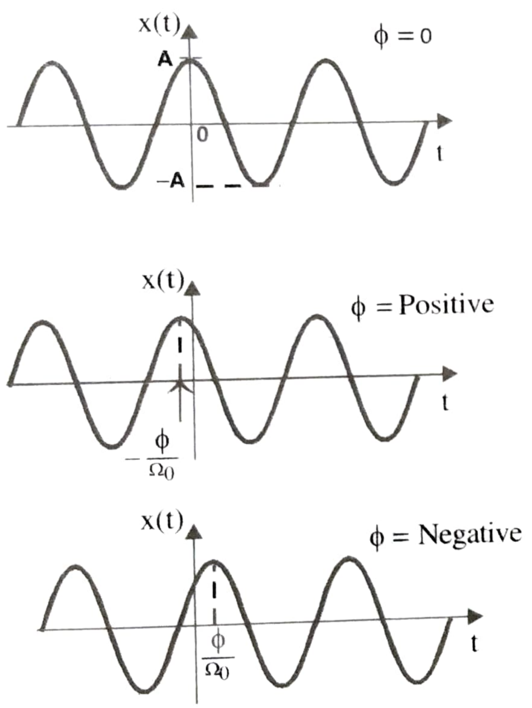

Fig 2.9 : *Cosinusoidal signal.*

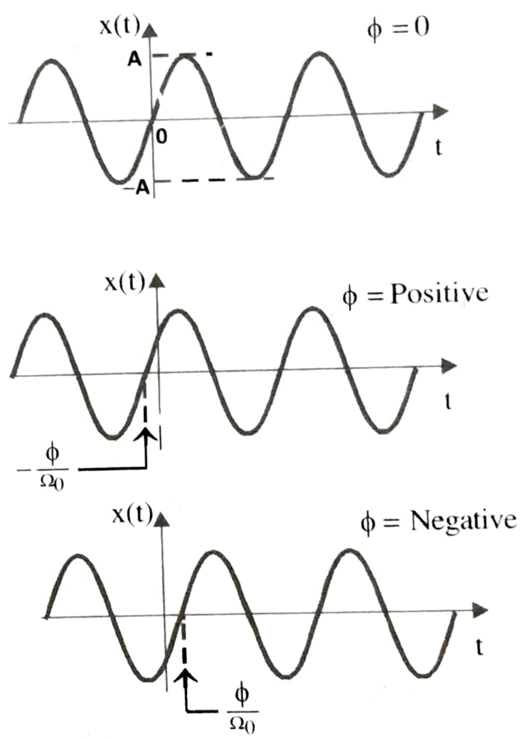

Fig 2.10 : *Sinusoidal signal.*

7. Exponential signal

Case i : Real exponential signal

The real exponential signal is defined as,

$$x(t) = A\, e^{bt}$$

where, A and b are real

Here, when b is positive, the signal x(t) will be an exponentially rising signal; and when b is negative the signal x(t) will be an exponentially decaying signal.

9. Triangular pulse signal

The Triangular pulse signal is defined as

$$x(t) = \Delta_a(t) = 1 - \frac{|t|}{a} \quad ; \quad |t| \le a$$
$$= 0 \quad ; \quad |t| > a$$

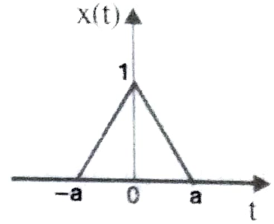

Fig 2.15 : Triangular pulse signal.

10. Signum signal

The Signum signal is defined as the sign of the independent variable t. Therefore, the Signum signal is expressed as,

$$x(t) = \text{sgn}(t) = 1 \quad ; \quad t > 0$$
$$= 0 \quad ; \quad t = 0$$
$$= -1 \quad ; \quad t < 0$$

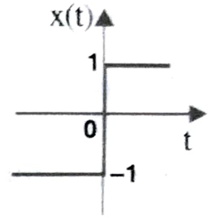

Fig 2.16 : Signum signal.

11. Sinc signal

The Sinc signal is defined as,

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t} \quad ; \quad -\infty < t < \infty$$

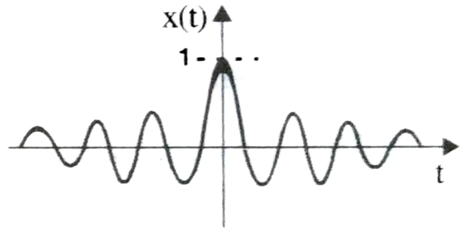

Fig 2.17 : Sinc signal.

12. Gaussian signal

The Gaussian signal is defined as,

$$x(t) = g_a(t) = e^{-a^2t^2} \quad ; \quad -\infty < t < \infty$$

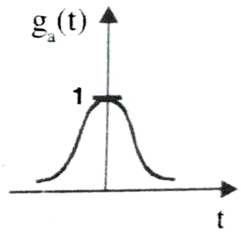

Fig 2.18 : Gaussian signal.

2.3 Classification of Continuous Time Signals

The continuous time signals are classified depending on their characteristics. Some ways of classifying continuous time signals are,

1. Deterministic and Nondeterministic signals
2. Periodic and Nonperiodic signals
3. Symmetric and Antisymmetric signals (Even and Odd signals)
4. Energy and Power signals
5. Causal and Noncausal signals

2.3.1 Deterministic and Nondeterministic Signals

The signal that can be completely specified by a mathematical equation is called a ***deterministic signal***. The step, ramp, exponential and sinusoidal signals are examples of deterministic signals.

Examples of deterministic signals:	$x_1(t) = At$
	$x_2(t) = X_m \sin\Omega_0 t$

The signal whose characteristics are random in nature is called a ***nondeterministic signal.*** The noise signals from various sources like electronic amplifiers, oscillators, radio receivers, etc., are best examples of nondeterministic signals.

2.3.2 Periodic and Nonperiodic Signals

A periodic signal will have a definite pattern that repeats again and again over a certain period of time. Therefore the signal which satisfies the condition,

$\boxed{x(t + T) = x(t)}$ is called a ***periodic signal***.

A signal which does not satisfy the condition, $x(t + T) = x(t)$ is called an ***aperiodic or nonperiodic signal***. In periodic signals, the term T is called the ***fundamental time period*** of the signal. Hence, inverse of T is called the ***fundamental frequency,*** F_0 in cycles/sec or Hz, and $2\pi F_0 = \Omega_0$ is called the ***fundamental angular frequency*** in rad/sec.

The sinusoidal signals and complex exponential signals are always periodic with a periodicity of T, where, $T = \dfrac{1}{F_0} = \dfrac{2\pi}{\Omega_0}$. The proof of this concept is given below.

Example 2.1

Verify whether the following continuous time signals are periodic. If periodic, find the fundamental period.

a) $x(t) = 2\cos\frac{t}{4}$ b) $x(t) = e^{\alpha t}$; $\alpha > 1$ c) $x(t) = e^{\frac{-j2\pi t}{7}}$ d) $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$ e) $x(t) = \cos^2\left(2t - \frac{\pi}{4}\right)$

Solution

a) Given that, $\mathbf{x(t) = 2\cos\frac{t}{4}}$

The given signal is a cosinusoidal signal, which is always periodic.

On comparing x(t) with the standard form "$A\cos 2\pi F_0 t$" we get,

$$2\pi F_0 = \frac{1}{4} \quad \Rightarrow \quad F_0 = \frac{1}{8\pi}$$

$$\text{Period, } T = \frac{1}{F_0} = 8\pi$$

$\therefore$ x(t) is periodic with period, $T = 8\pi$.

---

b) Given that, $\mathbf{x(t) = e^{\alpha t}}$ **;** $\mathbf{\alpha > 1}$

$$\therefore x(t+T) = e^{\alpha(t+T)}$$
$$= e^{\alpha t}\, e^{\alpha T}$$

For any value of α, $e^{\alpha T} \neq 1$ and so $x(t+T) \neq x(t)$

Since $x(t+T) \neq x(t)$, the signal x(t) is non-periodic.

---

c) Given that, $x(t) = e^{\frac{-j2\pi t}{7}}$

The given signal is a complex exponential signal, which is always periodic.

On comparing x(t) with the standard form "$Ae^{-j2\pi F_0 t}$".

We get, $F_0 = \frac{1}{7}$

$\therefore$ Peroid, $T = \frac{1}{F_0} = 7$

$\therefore$ x(t) is periodic with period, T = 7.

---

d) Given that, $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$

The given signal is a cosinusoidal signal, which is always periodic.

$$\therefore x(t+T) = 3\cos\left(5(t+T) + \frac{\pi}{6}\right) = 3\cos\left(5t + 5T + \frac{\pi}{6}\right) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 5T\right)$$

Let $5T = 2\pi$, $\quad \therefore T = \frac{2\pi}{5}$

$$\therefore x(t+T) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 5 \times \frac{2\pi}{5}\right) = 3\cos\left(\left(5t + \frac{\pi}{6}\right) + 2\pi\right)$$

$$= 3\cos\left(5t + \frac{\pi}{6}\right) = x(t)$$

For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$

Since $x(t+T) = x(t)$, the signal x(t) is periodic with period, $T = \frac{2\pi}{5}$

---

e) Given that, $x(t) = \cos^2\left(2t - \frac{\pi}{3}\right)$

$$x(t) = \cos^2\left(2t - \frac{\pi}{3}\right) = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2}$$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$$\therefore x(t+T) = \frac{1 + \cos\left(4(t+T) - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t + 4T - \frac{2\pi}{3}\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4T\right)}{2}$$

Let $4T = 2\pi$, $\therefore T = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\therefore x(t+T) = \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4 \times \frac{\pi}{2}\right)}{2} = \frac{1 + \cos\left(\left(4t - \frac{2\pi}{3}\right) + 2\pi\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \cos^2\left(2t - \frac{\pi}{3}\right) = x(t)$$

Since $x(t+T) = x(t)$, the signal x(t) is periodic with period, $T = \frac{\pi}{2}$

For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$

Example 2.2

Determine the periodicity of the following continuous time signals.

(a) $x(t) = 2\cos\frac{2\pi t}{3} + 3\cos\frac{2\pi t}{7}$ (b) $x(t) = 2\cos 3t + 3\sin 7t$ (c) $x(t) = 5\cos 4\pi t + 3\sin 8\pi t$

Solution

(a) Given that, $x(t) = 2\cos\frac{2\pi t}{3} + 3\cos\frac{2\pi t}{7}$

Let, $x_1(t) = 2\cos\frac{2\pi t}{3}$

Let T_1 be the periodicity of $x_1(t)$. On comparing $x_1(t)$ with the standard form "$A\cos 2\pi F_{01}t$", we get,

$$F_{01} = \frac{1}{3} \; ; \qquad \therefore \text{ Period, } T_1 = \frac{1}{F_{01}} = 3$$

Let, $x_2(t) = 3\cos\frac{2\pi t}{7}$

Let T_2 be the periodicity of $x_2(t)$. On comparing $x_2(t)$ with the standard form "$A\cos 2\pi F_{02}t$", we get,

$$F_{02} = \frac{1}{7} \; ; \qquad \therefore \text{ Period, } T_2 = \frac{1}{F_{02}} = 7$$

Now, $\frac{T_1}{T_2} = \frac{3}{7}$

Since $x_1(t)$ and $x_2(t)$ are periodic, and the ratio of T_1 and T_2 is a rational number, the signal $x(t)$ is also periodic. Let T be the periodicity of $x(t)$. Now the periodicity of $x(t)$ is the LCM (Least Common Multiple) of T_1 and T_2, i.e., LCM of 3 and 7, which is 21.

$\therefore$ Period, T = 21

Proof : $x(t+T) = 2\cos\frac{2\pi(t+T)}{3} + 3\cos\frac{2\pi(t+T)}{7} = 2\cos\left(\frac{2\pi t}{3} + \frac{2\pi T}{3}\right) + 3\cos\left(\frac{2\pi t}{7} + \frac{2\pi T}{7}\right)$

$$= 2\cos\left(\frac{2\pi t}{3} + \frac{2\pi \times 21}{3}\right) + 3\cos\left(\frac{2\pi t}{7} + \frac{2\pi \times 21}{7}\right) \qquad \boxed{\text{Put, } T = 21}$$

$$= 2\cos\left(\frac{2\pi t}{3} + 14\pi\right) + 3\cos\left(\frac{2\pi t}{7} + 6\pi\right)$$

$$= 2\cos\frac{2\pi t}{3} + 3\cos\frac{2\pi t}{7} = x(t) \qquad \boxed{\begin{array}{l}\text{For integer values of M,}\\ \cos(\theta + 2\pi M) = \cos\theta\end{array}}$$

(b) Given that, $x(t) = 2\cos 3t + 3\sin 7t$

Let, $x_1(t) = 2\cos 3t$

Let T_1 be the periodicity of $x_1(t)$. On comparing $x_1(t)$ with the standard form "$A\cos 2\pi F_{01}t$", we get,

$$F_{01} = \frac{3}{2\pi} \; ; \qquad \therefore \text{ Period, } T_1 = \frac{1}{F_{01}} = \frac{2\pi}{3}$$

Let, $x_2(t) = 3\sin 7t$

Let T_2 be the periodicity of $x_2(t)$. On comparing $x_2(t)$ with the standard form "$A\sin 2\pi F_{02}t$", we get,

$$F_{02} = \frac{7}{2\pi} \; ; \qquad \therefore \text{ Period, } T_2 = \frac{1}{F_{02}} = \frac{2\pi}{7}$$

Now, $\frac{T_1}{T_2} = T_1 \times \frac{1}{T_2} = \frac{2\pi}{3} \times \frac{7}{2\pi} = \frac{7}{3}$

Since $x_1(t)$ and $x_2(t)$ are periodic and the ratio of T_1 and T_2 is a rational number, the signal x(t) is also periodic. Let T be the periodicity of x(t). Now the periodicity of x(t) is the LCM (Least Common Multiple) of T_1 and T_2, which is calculated as shown below.

$$T_1 = \frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{21}{2\pi} = 7$$

$$T_2 = \frac{2\pi}{7} = \frac{2\pi}{7} \times \frac{21}{2\pi} = 3$$

Note : *To find LCM, first convert T_1 and T_2 to integers by multiplying by a common number. Find LCM of integer values of T_1 and T_2. Then divide this LCM by the common number.*

Now LCM of 7 and 3 is 21.

$$\therefore \text{ Period, } T = 21 \div \frac{21}{2\pi} = 21 \times \frac{2\pi}{21} = 2\pi$$

Proof : $x(t + T) = 2 \cos 3(t + T) + 3 \sin 7(t + T)$

$= 2 \cos(3t + 3T) + 3 \sin(7t + 7T)$

$= 2 \cos(3t + 3 \times 2\pi) + 3 \sin (7t + 7 \times 2\pi)$

$= 2 \cos(3t + 6\pi) + 3 \sin(7t + 14\pi)$

$= 2 \cos 3t + 3 \sin 7t = x(t)$

Put, $T = 2\pi$

For integer values of M,
$\cos(\theta + 2\pi M) = \cos\theta$
$\sin(\theta + 2\pi M) = \sin\theta$

(c) Given that, $x(t) = 5 \cos 4\pi t + 3 \sin 8\pi t$

Let, $x_1(t) = 5 \cos 4\pi t$

Let T_1 be the periodicity of $x_1(t)$. On comparing $x_1(t)$ with the standard form "$A \cos 2\pi F_{01} t$", we get,

$$F_{01} = 2 \; ; \qquad \therefore \text{ Period, } T_1 = \frac{1}{F_{01}} = \frac{1}{2}$$

Let, $x_2(t) = 3 \sin 8\pi t$

Let T_2 be the periodicity of $x_2(t)$. On comparing $x_2(t)$ with the standard form "$A \sin 2\pi F_{02} t$", we get,

$$F_{02} = 4 \; ; \qquad \therefore \text{ Period, } T_2 = \frac{1}{F_{02}} = \frac{1}{4}$$

$$\text{Now, } \frac{T_1}{T_2} = T_1 \times \frac{1}{T_2} = \frac{1}{2} \times \frac{4}{1} = 2$$

Since $x_1(t)$ and $x_2(t)$ are periodic and the ratio of T_1 and T_2 is a rational number, the signal x(t) is also periodic. Let T be the periodicity of x(t). Now, the periodicity of x(t) is the LCM (Least Common Multiple) of T_1 and T_2, which is calculated as shown below.

$$T_1 = \frac{1}{2} = \frac{1}{2} \times 4 = 2$$

$$T_2 = \frac{1}{4} = \frac{1}{4} \times 4 = 1$$

Note : *To find LCM, first convert T_1 and T_2 to integers by multiplying by a common number. Find LCM of integer values of T_1 and T_2. Then divide this LCM by the common number.*

Now LCM of 2 and 1 is 2.

$$\therefore \text{ Period, } T = 2 \div 4 = 2 \times \frac{1}{4} = \frac{1}{2}$$

Proof : $x(t + T) = 5 \cos 4\pi(t + T) + 3 \sin 8\pi(t + T)$

$= 5 \cos(4\pi t + 4\pi T) + 3 \sin(8\pi t + 8\pi T)$

$= 5 \cos\left(4\pi t + 4\pi \times \frac{1}{2}\right) + 3 \sin\left(8\pi t + 8\pi \times \frac{1}{2}\right)$

$= 5 \cos (4\pi t + 2\pi) + 3 \sin (8\pi t + 2\pi)$

$= 5 \cos 4\pi t + 3 \sin 8\pi t = x(t)$

Put, $T = \frac{1}{2}$

For integer values of M,
$\cos(\theta + 2\pi M) = \cos\theta$
$\sin(\theta + 2\pi M) = \sin\theta$

2.3.3 Symmetric (Even) and Antisymmetric (Odd) Signals

The signals may exhibit symmetry or antisymmetry with respect to $t = 0$.

When a signal exhibits symmetry with respect to $t = 0$ then it is called an ***even signal.*** Therefore, the even signal satisfies the condition, $\mathbf{x(-t) = x(t)}$.

When a signal exhibits antisymmetry with respect to $t = 0$, then it is called an ***odd signal***. Therefore, the odd signal satisfies the condition, $\mathbf{x(-t) = -x(t)}$.

Since $\cos(-\theta) = \cos\theta$, the cosinusoidal signals are even signals and since $\sin(-\theta) = -\sin\theta$, the sinusoidal signals are odd signals.

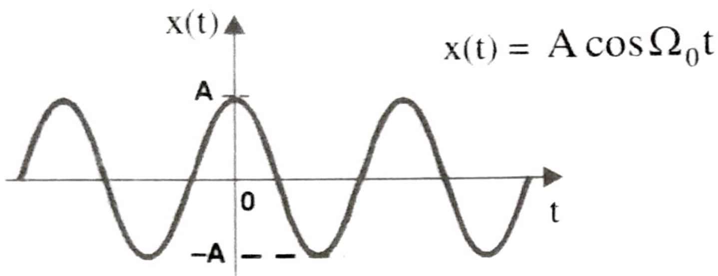

Fig 2.19a : *Symmetric or Even signal.*

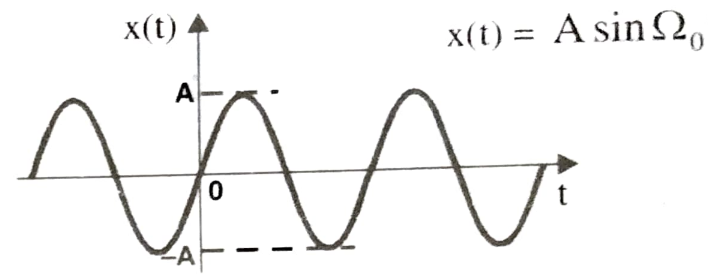

Fig 2.19b : *Antisymmetric or Odd signal.*

Fig 2.19 : *Symmetric and antisymmetric continuous time signals.*

A continuous time signal $x(t)$ which is neither even nor odd can be expressed as a sum of even and odd signal.

Let, $x(t) = x_e(t) + x_o(t)$

where, $x_e(t)$ = Even part of $x(t)$ and $x_o(t)$ = Odd part of $x(t)$

Now, it can be proved that,

$$\boxed{\begin{aligned} x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \\ x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right] \end{aligned}}$$

Example 2.3

Determine the even and odd part of the following continuous time signals.

a) $x(t) = e^t$ b) $x(t) = 3 + 2t + 5t^2$ c) $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

Solution

a) Given that, $x(t) = e^t$

$$\therefore x(-t) = e^{-t}$$

$$\text{Even part, } x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[e^t + e^{-t}]$$

$$\text{Odd part, } x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[e^t - e^{-t}]$$

b) Given that, $x(t) = 3 + 2t + 5t^2$

$$\therefore x(-t) = 3 + 2(-t) + 5(-t)^2$$

$$= 3 - 2t + 5t^2$$

$$\text{Even part, } x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[3 + 2t + 5t^2 + 3 - 2t + 5t^2]$$

$$= \frac{1}{2}[6 + 10t^2] = 3 + 5t^2$$

$$\text{Odd part, } x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[3 + 2t + 5t^2 - 3 + 2t - 5t^2]$$

$$= \frac{1}{2}[4t] = 2t$$

c) Given that, $x(t) = \sin 2t + \cos t + \sin t \cos 2t$

$$\therefore x(-t) = \sin 2(-t) + \cos(-t) + \sin(-t) \cos 2(-t)$$

$$= -\sin 2t + \cos t - \sin t \cos 2t$$

$$\text{Even part, } x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[\sin 2t + \cos t + \sin t \cos 2t - \sin 2t + \cos t - \sin t \cos 2t]$$

$$= \frac{1}{2}[2 \cos t] = \cos t$$

$$\text{Odd part, } x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[\sin 2t + \cos t + \sin t \cos 2t + \sin 2t - \cos t + \sin t \cos 2t]$$

$$= \frac{1}{2}[2 \sin 2t + 2 \sin t \cos 2t] = \sin 2t + \sin t \cos 2t$$

2.3.4 Energy and Power Signals

The signals which have finite energy are called ***energy signals***. The nonperiodic signals like exponential signals will have constant energy and so nonperiodic signals are energy signals.

The signals which have finite average power are called ***power signals***. The periodic signals like sinusoidal and complex exponential signals will have constant power and so periodic signals are power signals.

The ***energy*** E of a continuous time signal x(t) is defined as,

$$\boxed{\text{Energy, } E = \underset{T \to \infty}{\text{Lt}} \int_{-T}^{T} |x(t)|^2 \, dt \text{ in joules}}$$

The average ***power*** of a continuous time signal x(t) is defined as,

$$\boxed{\text{Power, } P = \underset{T \to \infty}{\text{Lt}} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \text{ in watts}}$$

For periodic signals, the average power over one period will be same as average power over an infinite interval.

$$\boxed{\therefore \text{ For periodic signals,} \quad \text{power, } P = \frac{1}{T} \int_{0}^{T} |x(t)|^2 \, dt}$$

For energy signals, the energy will be finite (or constant) and average power will be zero. For power signals the average power is finite (or constant) and energy will be infinite.

i.e., For energy signal, E is constant (i.e., $0 < E < \infty$) and $P = 0$.

For power signal, P is constant (i.e., $0 < P < \infty$) and $E = \infty$.

Example 2.4

Determine the power and energy for the following continuous time signals.

a) $x(t) = e^{-2t}\,u(t)$

b) $x(t) = e^{j\left(2t+\frac{\pi}{4}\right)}$

c) $x(t) = 3\cos 5\Omega_0 t$

Solution

a) Given that, $x(t) = e^{-2t}\,u(t)$

Here, $x(t) = e^{-2t}\,u(t)$; for all t

$\therefore\ x(t) = e^{-2t}$; for $t \geq 0$

$$\therefore \int_{-T}^{T} |x(t)|^2\, dt = \int_{0}^{T} \left(\left|e^{-2t}\right|\right)^2 dt = \int_{0}^{T} \left(e^{-2t}\right)^2 dt = \int_{0}^{T} e^{-4t}\, dt = \left[\frac{e^{-4t}}{-4}\right]_0^T$$

$$= \left[\frac{e^{-4T}}{-4} - \frac{e^{0}}{-4}\right] = \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$\text{Energy, } E = \underset{T\to\infty}{\mathrm{Lt}} \int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{\mathrm{Lt}} \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$= \frac{1}{4} - \frac{e^{-\infty}}{4} = \frac{1}{4} - \frac{0}{4} = \frac{1}{4} \text{ joules}$$

$$\text{Power, } P = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T}\left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$= \frac{1}{\infty}\left[\frac{1}{4} - \frac{e^{-\infty}}{4}\right] = 0 \times \left[\frac{1}{4} - 0\right] = 0$$

Since energy is constant and power is zero, the given signal is an energy signal.

b) Given that, $x(t) = e^{j\left(2t+\frac{\pi}{4}\right)}$

Here, $x(t) = e^{j\left(2t+\frac{\pi}{4}\right)} = 1 \angle\left(2t + \frac{\pi}{4}\right)$

$\therefore\ |x(t)| = 1$

$$\int_{-T}^{T} |x(t)|^2\, dt = \int_{-T}^{T} 1 \times dt = [t]_{-T}^{T} = T + T = 2T$$

$$\text{Energy, } E = \underset{T\to\infty}{\mathrm{Lt}} \int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{\mathrm{Lt}}\ 2T = \infty$$

$$\text{Power, } P = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{2T} \times 2T = 1 \text{ watt}$$

Since power is constant and energy is infinite, the given signal is a power signal.

c) Given that, $x(t) = 3\cos 5\Omega_0 t$

$$\therefore \int_{-T}^{T} |x(t)|^2\, dt = \int_{-T}^{T} \left(|3\cos 5\Omega_0 t|\right)^2 dt = \int_{-T}^{T} \left|(3\cos 5\Omega_0 t)^2\right| dt = \int_{-T}^{T} (3\cos 5\Omega_0 t)^2\, dt$$

$$= \int_{-T}^{T} 9\cos^2 5\Omega_0 t\, dt = 9\int_{-T}^{T} \left(\frac{1+\cos 10\Omega_0 t}{2}\right) dt$$

$$\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$= \frac{9}{2}\int_{-T}^{T} (1+\cos 10\Omega_0 t)\, dt = \frac{9}{2}\left[t + \frac{\sin 10\Omega_0 t}{10\Omega_0}\right]_{-T}^{T}$$

$$= \frac{9}{2}\left[T + \frac{\sin 10\Omega_0 T}{10\Omega_0} - \left(-T + \frac{\sin 10\Omega_0(-T)}{10\Omega_0}\right)\right]$$

$$= \frac{9}{2}\left[2T + 2\frac{\sin 10\Omega_0 T}{10\Omega_0}\right] = \frac{9}{2}\left[2T + 2\frac{\sin 10\frac{2\pi}{T}T}{10\frac{2\pi}{T}}\right]$$

$$\boxed{\sin(-\theta) = -\sin\theta}$$

$$\boxed{\Omega_0 = 2\pi F_0 = \frac{2\pi}{T}}$$

$$= \frac{9}{2}\left[2T + \frac{T}{10\pi}\sin 20\pi\right] = \frac{9}{2}\left[2T + \frac{T}{10\pi}\times 0\right] = 9T$$

$$\boxed{\begin{array}{l}\text{For integer M,}\\ \sin\pi M = 0\end{array}}$$

$$\text{Energy, } E = \underset{T\to\infty}{Lt}\int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{Lt}\; 9T = \infty$$

$$\text{Power, } P = \underset{T\to\infty}{Lt}\frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{Lt}\frac{1}{2T}\times 9T = \underset{T\to\infty}{Lt}\frac{9}{2} = \frac{9}{2} = 4.5 \text{ watts}$$

Since energy is infinite and power is constant, the given signal is a power signal.

2.3.5 Causal, Noncausal and Anticausal Signals

A signal is said to be ***causal***, if it is defined for $t \geq 0$.

Therefore if x(t) is causal, then $x(t) = 0$, for $t < 0$.

A signal is said to be ***noncausal***, if it is defined for either $t \leq 0$, or for both $t \leq 0$ and $t > 0$.

Therefore if x(t) is noncausal, then $x(t) \neq 0$, for $t < 0$.

When a noncausal signal is defined only for $t \leq 0$, it is called ***anticausal signal***.

Examples of causal and noncausal signals

Step signal,	$x(t) = A \quad ; t \geq 0$	Causal signals
Unit step signal,	$x(t) = u(t) = 1 \quad ; t \geq 0$	Causal signals
Exponential signal,	$x(t) = A\, e^{bt}\, u(t)$	Causal signals
Complex exponential signal,	$x(t) = A\, e^{j\Omega_0 t}\, u(t)$	Causal signals
Exponential signal,	$x(t) = A\, e^{bt}$; for all t	Noncausal signals
Complex exponential signal,	$x(t) = A\, e^{j\Omega_0 t}$; for all t	Noncausal signals

Note : On multiplying a noncausal signal by u(t), it becomes causal.

Definition of Fourier Transform

Let, $x(t)$ = Continuous time signal

$X(j\Omega)$ = Fourier transform of $x(t)$

The Fourier transform of continuous time signal, $x(t)$ is defined as,

$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt$$

Also, $X(j\Omega)$ is denoted as $\mathcal{F}\{x(t)\}$ where "$\mathcal{F}$" is the symbol used to denote the Fourier transform operation.

$$\boxed{\therefore\ \mathcal{F}\{x(t)\} = X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt} \quad .....(4.35)$$

Note : *Sometimes the Fourier transform is expressed as a function of cyclic frequency F, rather than radian frequency Ω. The Fourier transform as a function of cyclic frequency F, is defined as,*

$$X(jF) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

Condition for Existence of Fourier Transform

The Fourier transform of $x(t)$ exists if it satisfies the following Dirichlet condition.

1. The $x(t)$ be absolutely integrable.

i.e., $$\int_{-\infty}^{+\infty} x(t)\, dt < \infty$$

2. The $x(t)$ should have a finite number of maxima and minima within any finite interval.
3. The $x(t)$ can have a finite number of discontinuities within any interval.

Definition of Inverse Fourier Transform

The ***inverse Fourier transform*** of $X(j\Omega)$ is defined as,

$$\boxed{x(t) = \mathcal{F}^{-1}\{X(j\Omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega} \quad .....(4.36)$$

The signals $x(t)$ and $X(j\Omega)$ are called ***Fourier transform pair*** and can be expressed as shown below,

$$\boxed{x(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\longleftrightarrow}} X(j\Omega)}$$

Note : *When Fourier transform is expressed as a function of cyclic frequency F, the inverse Fourier transform is defined as,*

$$x(t) = F^{-1}\{X(jF)\} = \int_{-\infty}^{+\infty} X(jF)\, e^{j2\pi Ft}\, dF$$

Table 4.3 : Summary of Properties of Fourier Transform

Let, $\mathcal{F}\{x(t)\} = X(j\Omega)$; $\mathcal{F}\{x_1(t)\} = X_1(j\Omega)$; $\mathcal{F}\{x_2(t)\} = X_2(j\Omega)$

Property	Time domain signal	Frequency domain signal
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(j\Omega) + a_2 X_2(j\Omega)$
Time shifting	$x(t - t_0)$	$e^{-j\Omega t_0} X(j\Omega)$
Time scaling	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{j\Omega}{a}\right)$
Time reversal	$x(-t)$	$X(-j\Omega)$
Conjugation	$x^*(t)$	$X^*(-j\Omega)$
Frequency shifting	$e^{j\Omega_0 t} x(t)$	$X(j(\Omega - \Omega_0))$
Time differentiation	$\frac{d}{dt} x(t)$	$j\Omega X(j\Omega)$
Time integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{X(j\Omega)}{j\Omega} = \pi X(0)\, \delta(\Omega)$
Frequency differentiation	$t\, x(t)$	$j \frac{d}{d\Omega} X(j\Omega)$
Time convolution	$x_1(t) * x_2(t) = \int_{-\infty}^{+\infty} x_1(\tau)\, x_2(t - \tau) d\tau$	$X_1(j\Omega)\, X_2(j\Omega)$
Frequency convolution (or Multiplication)	$x_1(t)\, x_2(t)$	$\frac{1}{2\pi} \int_{\lambda=-\infty}^{\lambda=+\infty} X_1(j\lambda)\, X_2(j(\Omega - \lambda))\, d\lambda$

.11 Fourier Transform of Some Important Signals

ourier Transform of Unit Impulse Signal

The impulse signal is defined as,

$$x(t) = \delta(t) = \infty \quad ; \quad t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$
$$= 0 \quad ; \quad t \neq 0$$

By definition of Fourier transform,

$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\Omega t}\, dt$$

$$= 1 \times e^{-j\Omega t}\Big|_{t=0} = 1 \times e^{0} = 1 \qquad \boxed{\delta(t) \text{ exists only for } t = 0}$$

$$\therefore \boxed{\mathcal{F}\{x(t)\} = 1}$$

The plot of impulse signal and its magnitude spectrum are shown in fig 4.18 and fig 4.19 respectively.

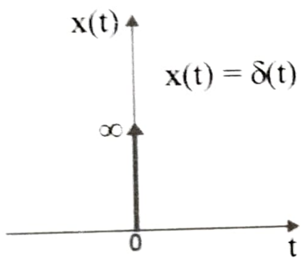

Fig 4.18 : Impulse signal.

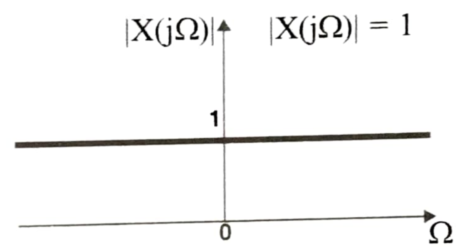

Fig 4.19 : Magnitude spectrum of impulse signal.

Fourier Transform of Single Sided Exponential Signal

The single sided exponential signal is defined as,

$$x(t) = A\, e^{-at} \quad ; \quad \text{for } t \geq 0$$

By definition of Fourier transform,

$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt = \int_{0}^{+\infty} A\, e^{-at}\, e^{-j\Omega t}\, dt$$

$$= \int_{0}^{+\infty} A\, e^{-(a+j\Omega)t}\, dt = \left[\frac{A\, e^{-(a+j\Omega)t}}{-(a+j\Omega)}\right]_{0}^{+\infty}$$

$$= \left[\frac{A\, e^{-\infty}}{-(a+j\Omega)} - \frac{A\, e^{0}}{-(a+j\Omega)}\right] = \frac{A}{a+j\Omega} \qquad \boxed{e^{-\infty} = 0}$$

$$\therefore \boxed{\mathcal{F}\left\{A\, e^{-at}\, u(t)\right\} = \frac{A}{a+j\Omega}} \qquad .....(4.56)$$

The plot of exponential signal and its magnitude spectrum are shown in fig 4.20 and fig 4.21 respectively.

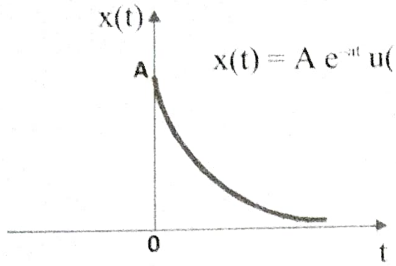

Fig 4.20: Single sided exponential signal.

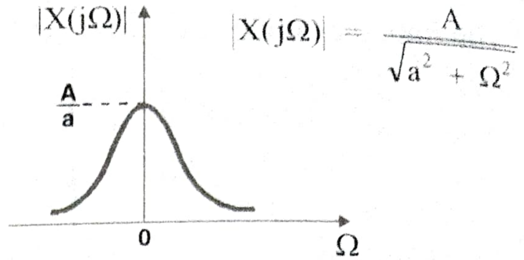

Fig 4.21 : Magnitude spectrum of single sided exponential signal.

Fourier Transform of Double Sided Exponential Signal

The double sided exponential signal is defined as,

$$x(t) = A\,e^{-a|t|} \quad ; \text{ for all } t$$

$$\therefore \; x(t) = A\,e^{+at} \quad ; \text{ for } t = -\infty \text{ to } 0$$

$$= A\,e^{-at} \quad ; \text{ for } t = 0 \text{ to } +\infty$$

By definition of Fourier transform,

$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\,e^{-j\Omega t}\,dt = \int_{-\infty}^{0} A\,e^{at}\,e^{-j\Omega t}\,dt + \int_{0}^{+\infty} A\,e^{-at}\,e^{-j\Omega t}\,dt$$

$$= \int_{-\infty}^{0} A\,e^{(a-j\Omega)t}\,dt + \int_{0}^{\infty} A\,e^{-(a+j\Omega)t}\,dt = \left[\frac{A\,e^{(a-j\Omega)t}}{a-j\Omega}\right]_{-\infty}^{0} + \left[\frac{A\,e^{-(a+j\Omega)t}}{-(a+j\Omega)}\right]_{0}^{\infty}$$

$$= \frac{A\,e^{0}}{a-j\Omega} - \frac{A\,e^{-\infty}}{a-j\Omega} + \frac{A\,e^{-\infty}}{-(a+j\Omega)} - \frac{A\,e^{0}}{-(a+j\Omega)} = \frac{A}{a-j\Omega} + \frac{A}{a+j\Omega}$$

$$\boxed{e^{-\infty} = 0}$$

$$= \frac{A(a+j\Omega) + A(a-j\Omega)}{(a-j\Omega)\,(a+j\Omega)} = \frac{2aA}{a^2+\Omega^2}$$

$$\boxed{(a+b)(a-b) = a^2 - b^2 \quad j^2 = -1}$$

$$\therefore \; \boxed{\mathcal{F}\{A\,e^{-a|t|}\} = \frac{2aA}{a^2+\Omega^2}} \qquad .....(4.57)$$

The plot of double sided exponential signal and its magnitude spectrum are shown in fig 4.22 and fig 4.23 respectively.

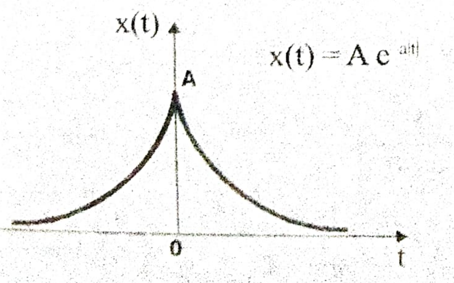

Fig 4.22 : Double sided exponential signal.

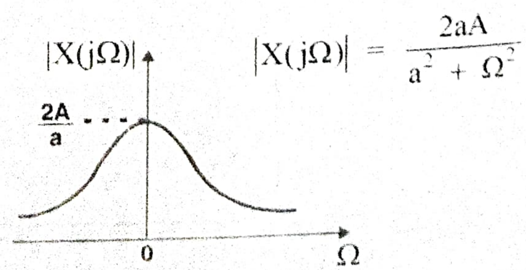

Fig 4.23 : Magnitude spectrum of double sided exponential signal.

Fourier Transform of a Constant

Let, $x(t) = A$, where A is a constant.

If definition of Fourier transform is directly applied, the constant will not satisfy the condition,

$$\int_{-\infty}^{+\infty} |x(t)|\, dt < \infty$$

Hence the constant can be viewed as a double sided exponential with limit "a" tends to 0 as shown below.

Let $x_1(t)$ = Double sided exponential signal.

The double sided exponential signal is defined as,

$$x_1(t) = A\, e^{-a|t|}$$

$$\text{i.e,} \quad x_1(t) = A\, e^{at} \quad ; \quad \text{for } t = -\infty \text{ to } 0$$
$$= A\, e^{-at} \quad ; \quad \text{for } t = 0 \text{ to } +\infty$$

$$\therefore\ x(t) = A \underset{a \to 0}{\text{Lt}}\ x_1(t)$$

On taking Fourier transform of the above equation we get,

$$\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\underset{a \to 0}{\text{Lt}}\ x_1(t)\right\}$$

$$\mathcal{F}\{x(t)\} = \underset{a \to 0}{\text{Lt}}\ \mathcal{F}\{x_1(t)\}$$

$$\boxed{\mathcal{F}\{x(t)\} = X(j\Omega) \quad \mathcal{F}\{x_1(t)\} = X_1(j\Omega)}$$

$$X(j\Omega) = \underset{a \to 0}{\text{Lt}}\ [X_1(j\Omega)]$$

$$= \underset{a \to 0}{\text{Lt}}\ \frac{2aA}{\Omega^2 + a^2}$$

$\boxed{\text{Using equation (4.57)}}$

The above equation is 0 for all values of Ω except at $\Omega = 0$.

At $\Omega = 0$, the above equation represents an impulse of magnitude "k".

$$\therefore\ X(j\Omega) = k\,\delta(\Omega) \quad ; \quad \Omega = 0$$
$$= 0 \quad ; \quad \Omega \neq 0$$

The magnitude "k" can be evaluated as shown below.

$$\boxed{\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}}$$

$$k = \int_{-\infty}^{+\infty} \frac{2aA}{\Omega^2 + a^2}\, d\Omega = 2aA \int_{-\infty}^{+\infty} \frac{1}{\Omega^2 + a^2}\, d\Omega$$

$$= 2aA \left[\frac{1}{a}\tan^{-1}\left(\frac{\Omega}{a}\right)\right]_{-\infty}^{+\infty} = 2aA\left[\frac{1}{a}\tan^{-1}(+\infty) - \frac{1}{a}\tan^{-1}(-\infty)\right]$$

$$= 2aA\left[\frac{1}{a}\frac{\pi}{2} - \frac{1}{a}\left(-\frac{\pi}{2}\right)\right] = 2aA\left(\frac{\pi}{a}\right) = 2\pi A \qquad \text{.....(4.58)}$$

$$\therefore\ \boxed{\mathcal{F}\{A\} = 2\pi A\,\delta(\Omega)}$$

The plot of constant and its magnitude spectrum are shown in fig 4.24 and fig 4.25 respectively.

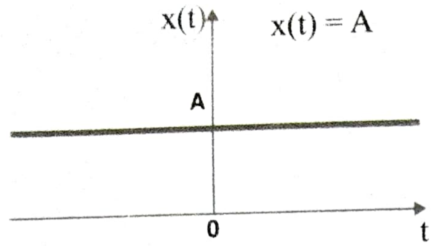

Fig 4.24 : Constant.

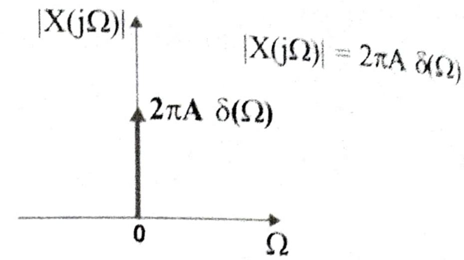

Fig 4.25 : Magnitude spectrum of constant.

Fourier Transform of Signum Function

The signum function is defined as,

$$x(t) = \mathrm{sgn}(t) = \begin{cases} 1 & ; \ t > 0 \\ -1 & ; \ t < 0 \end{cases}$$

The signum function can be expressed as a sum of two one sided exponential signal and taking limit "a" tends to 0 as shown below.

$$\therefore\ \mathrm{sgn}(t) = \underset{a \to 0}{\mathrm{Lt}} \left[e^{-at}\, u(t) - e^{at}\, u(-t)\right]$$

$$\therefore\quad x(t) = \mathrm{sgn}(t) = \underset{a \to 0}{\mathrm{Lt}} \left[e^{-at}\, u(t) - e^{at}\, u(-t)\right]$$

By definition of Fourier transform,

$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\Omega t}\, dt = \int_{-\infty}^{+\infty} \underset{a \to 0}{\mathrm{Lt}} \left[e^{-at}\, u(t) - e^{at}\, u(-t)\right] e^{-j\Omega t}\, dt$$

$$= \underset{a \to 0}{\mathrm{Lt}} \left[\int_{0}^{+\infty} e^{-at}\, e^{-j\Omega t}\, dt - \int_{-\infty}^{0} e^{at}\, e^{-j\Omega t}\, dt\right]$$

$$= \underset{a \to 0}{\mathrm{Lt}} \left[\int_{0}^{+\infty} e^{-(a + j\Omega)t}\, dt - \int_{-\infty}^{0} e^{+(a - j\Omega)t}\, dt\right]$$

$$= \underset{a \to 0}{\mathrm{Lt}} \left[\left[\frac{e^{-(a + j\Omega)t}}{-(a + j\Omega)}\right]_{0}^{\infty} - \left[\frac{e^{(a - j\Omega)t}}{(a - j\Omega)}\right]_{-\infty}^{0}\right] \qquad \boxed{e^{0} = 1\ ;\ e^{-\infty} = 0}$$

$$= \underset{a \to 0}{\mathrm{Lt}} \left[\frac{e^{-\infty}}{-(a + j\Omega)} - \frac{e^{0}}{-(a + j\Omega)} - \frac{e^{0}}{a - j\Omega} + \frac{e^{-\infty}}{a - j\Omega}\right]$$

$$= \underset{a \to 0}{\mathrm{Lt}} \left[\frac{1}{a + j\Omega} - \frac{1}{a - j\Omega}\right] = \frac{1}{j\Omega} + \frac{1}{j\Omega} = \frac{2}{j\Omega}$$

$$\therefore\ \boxed{\mathcal{F}\{\mathrm{sgn}(t)\} = \frac{2}{j\Omega}} \qquad .....(4.59)$$

The plot of signum function and its magnitude spectrum are shown in fig 4.26 and fig 4.27 respectively.

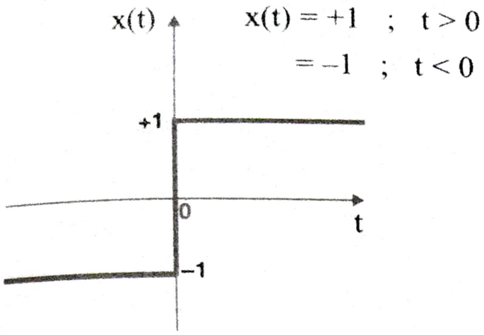

Fig 4.26 : Signum function.

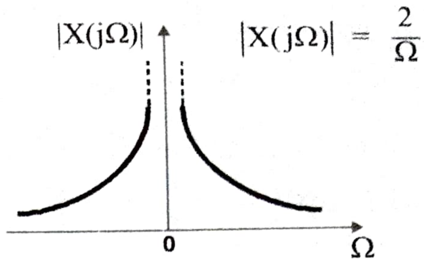

Fig 4.27 : Magnitude spectrum of signum function.

Fourier Transform of Unit Step Signal

The unit step signal is defined as,

$$u(t) = 1 \quad ; \quad t \geq 0$$
$$\quad\; = 0 \quad ; \quad t < 0$$

If can be proved that, $\text{sgn}(t) = 2u(t) - 1 \quad \Rightarrow \quad u(t) = \frac{1}{2}\left[1 + \text{sgn}(t)\right]$

$$\therefore\; x(t) = u(t) = \frac{1}{2}\left[1 + \text{sgn}(t)\right]$$

On taking Fourier transform of the above equation we get,

$$\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\frac{1}{2}\left[1 + \text{sgn}(t)\right]\right\}$$

$$\therefore\; X(j\Omega) = \mathcal{F}\left\{\frac{1}{2}\right\} + \mathcal{F}\left\{\frac{1}{2}\,\text{sgn}(t)\right\} = \frac{1}{2}\,\mathcal{F}\{1\} + \frac{1}{2}\,\mathcal{F}\{\text{sgn}(t)\}$$

$$= \frac{1}{2}\left[2\pi\,\delta(\Omega)\right] + \frac{1}{2}\left[\frac{2}{j\Omega}\right] = \pi\,\delta(\Omega) + \frac{1}{j\Omega}$$

Using equations (4.58) and (4.59)

$$\therefore\; \boxed{\mathcal{F}\{u(t)\} = \pi\,\delta(\Omega) + \frac{1}{j\Omega}} \qquad .....(4.60)$$

The plot of unit step signal and its magnitude spectrum are shown in fig 4.28 and fig 4.29 respectively.

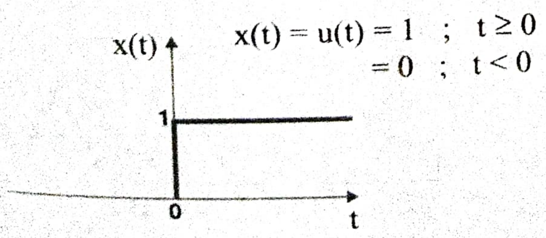

Fig 4.28 : Unit step signal.

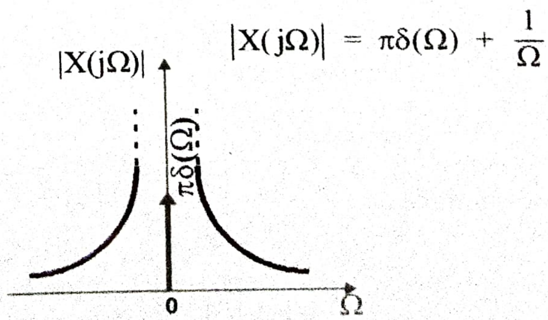

Fig 4.29 : Magnitude spectrum of unit step signal.

Fourier Transform of Sinusoidal Signal

The sinusoidal signal is defined as,

$$x(t) = A \sin\Omega_0 t = \frac{A}{2j}\left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right)$$

$$\boxed{\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}}$$

On taking Fourier transform we get,

$$\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\frac{A}{2j}\left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right)\right\} = \frac{A}{2j}\left[\mathcal{F}\{e^{j\Omega_0 t}\} - \mathcal{F}\{e^{-j\Omega_0 t}\}\right]$$

Using equations (4.61) and (4.62).

$$= \frac{A}{2j}\left[2\pi\,\delta(\Omega - \Omega_0) - 2\pi\,\delta(\Omega + \Omega_0)\right] = \frac{A\pi}{j}\left[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)\right]$$

$$\therefore \boxed{\mathcal{F}\{A \sin\Omega_0 t\} = \frac{A\pi}{j}\left[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)\right]} \qquad .....(4.63)$$

The plot of sinusoidal signal and its spectrum are shown in fig 4.34 and fig 4.35.

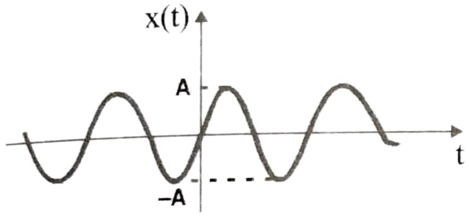

Fig 4.34 : Sinusoidal signal.

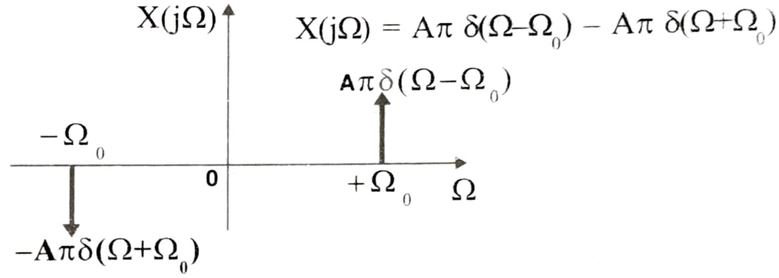

Fig 4.35 : Spectrum of sinusoidal signal.

Fourier Transform of Cosinusoidal Signal

The cosinusoidal signal is defined as,

$$x(t) = A \cos\Omega_0 t = \frac{A}{2}\left(e^{j\Omega_0 t} + e^{-j\Omega_0 t}\right)$$

$$\boxed{\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}}$$

On taking Fourier transform we get,

$$\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\frac{A}{2}\left(e^{j\Omega_0 t} + e^{-j\Omega_0 t}\right)\right\} = \frac{A}{2}\left[\mathcal{F}\{e^{j\Omega_0 t}\} + \mathcal{F}\{e^{-j\Omega_0 t}\}\right]$$

Using equations (4.61) and (4.62).

$$= \frac{A}{2}\left[2\pi\,\delta(\Omega - \Omega_0) + 2\pi\,\delta(\Omega + \Omega_0)\right] = A\pi\left[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)\right]$$

$$\therefore \boxed{\mathcal{F}\{A \cos\Omega_0 t\} = A\pi\left[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)\right]} \qquad .....(4.64)$$

The plot of cosinusoidal signal and its magnitude spectrum are shown in fig 4.36 and fig 4.37.

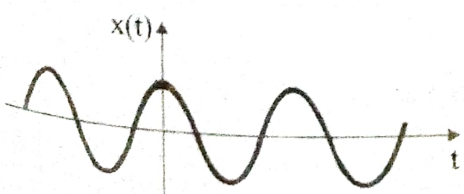

Fig 4.36 : Cosinusoidal signal.

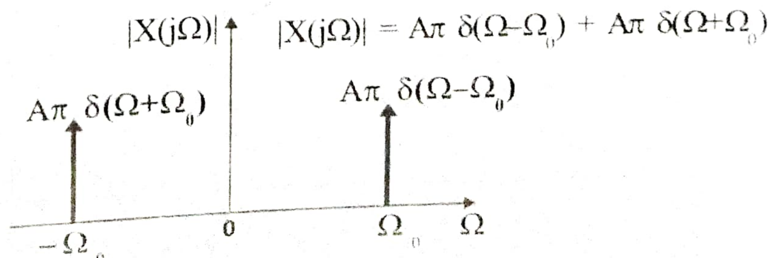

Fig 4.37 : Magnitude spectrum of cosinusoidal signal.