

Noise may be defined, in electrical terms, as any unwanted introduction of energy tending to interfere with the proper reception and reproduction of transmitted signals. Many disturbances of an electrical nature produce noise in receivers, modifying the signal in an unwanted manner. In radio receivers, noise may produce hiss in the loudspeaker output. In television receivers "snow" or "confetti" (colored snow) becomes superimposed on the picture. Noise can limit the range of systems, for a given transmitted power. It affects the sensitivity of receivers, by placing a limit on the weakest signals that can be amplified. It may sometimes even force a reduction in the bandwidth of a system.

There are numerous ways of classifying noise. It may be subdivided according to type, source, effect, or relation to the receiver, depending on circumstances. It is most convenient here to divide noise into two broad groups: noise whose sources are external to the receiver, and noise created within the receiver itself. External noise is difficult to treat quantitatively, and there is often little that can be done about it, short of moving the system to another location. Note how radiotelescopes are always located away from industry, whose processes create so much electrical noise. International satellite earth stations are also located in noise-free valleys, where possible. Internal noise is both more quantifiable and capable of being reduced by appropriate receiver design.

2.1 EXTERNAL NOISE

The various forms of noise created outside the receiver come under the heading of external noise and include atmospheric extraterrestrial noise and industrial noise.

2.1.1 Atmospheric Noise

Perhaps the best way to become acquainted with atmospheric noise is to listen to shortwaves on a receiver which is not well equipped to receive them. An astonishing variety of strange sounds will be heard, all tending to interfere with the program. Most of these sounds are the result of spurious radio waves which induce voltages in the antenna. The majority of these radio waves come from natural sources of disturbance. They represent atmospheric noise, generally called *static*.

Static is caused by lightning discharges in thunderstorms and other natural electric disturbances occurring in the atmosphere. It originates in the form of amplitude-modulated impulses, and because such processes are random in nature, it is spread over most of the RF spectrum normally used for broadcasting. Atmospheric noise consists of spurious radio signals with components distributed over a wide range of frequencies. It is propagated over the earth in the same way as ordinary radio waves of the same frequencies, so that at any point on the ground, static will be received from all thunderstorms, local and distant. The static is likely to be more severe but less frequent if the storm is local. Field strength is inversely proportional to frequency, so that this noise will interfere more with the reception of radio than that of television. Such noise consists of impulses, and these nonsinusoidal waves have harmonics whose amplitude falls off with increase in the harmonic. Static from distant sources will vary in intensity according to the variations in propagating conditions. The usual increase in its level takes place at night, at both broadcast and shortwave frequencies.

Atmospheric noise becomes less severe at frequencies above about 30 MHz because of two separate factors. First, the higher frequencies are limited to line-of-sight propagation i.e., less than 80 kilometers or so. Second, the nature of the mechanism generating this noise is such that very little of it is created in the VHF range and above.

2.1.2 Extraterrestrial Noise

It is safe to say that there are almost as many types of space noise as there are sources. For convenience, a division into two subgroups will suffice.

Solar Noise The sun radiates so many things our way that we should not be too surprised to find that noise is noticeable among them, again there are two types. Under normal "quiet" conditions, there is a constant noise radiation from the sun, simply because it is a large body at a very high temperature (over 6000°C on the surface). It therefore radiates over a very broad frequency spectrum which includes the frequencies we use for communication. However, the sun is a constantly changing star which undergoes cycles of peak activity from which electrical disturbances erupt, such as corona flares and sunspots. Even though the additional noise produced comes from a limited portion of the sun's surface, it may still be orders of magnitude greater than that received during periods of quiet sun.

Cosmic Noise Since distant stars are also suns and have high temperatures, they radiate RF noise in the same manner as our sun, and what they lack in nearness they nearly make up in numbers which in combination

can become significant. The noise received is called thermal (or black-body) noise and is distributed fairly uniformly over the entire sky. We also receive noise from the center of our own galaxy (the Milky Way), from other galaxies, and from other virtual point sources such as "quasars" and "pulsars." This galactic noise is very intense, but it comes from sources which are only points in the sky.

Summary Space noise is observable at frequencies in the range from about 8 MHz to somewhat above 1.43 gigahertz (1.43 GHz), the latter frequency corresponding to the 21-cm hydrogen "line." Apart from man-made noise it is the strongest component over the range of about 20 to 120 MHz. Not very much of it below 20 MHz penetrates down through the ionosphere, while its eventual disappearance at frequencies in excess of 1.5 GHz is probably governed by the mechanisms generating it, and its absorption by hydrogen in interstellar space.

2.1.3 Industrial Noise

Between the frequencies of 1 to 600 MHz (in urban, suburban and other industrial areas) the intensity of noise made by humans easily outstrips that created by any other source, internal or external to the receiver. Under this heading, sources such as automobile and aircraft ignition, electric motors and switching equipment, leakage from high-voltage lines and a multitude of other heavy electric machines are all included. Fluorescent lights are another powerful source of such noise and therefore should not be used where sensitive receiver reception or testing is being conducted. The noise is produced by the arc discharge present in all these operations, and under these circumstances it is not surprising that this noise should be most intense in industrial and densely populated areas.

The nature of industrial noise is so variable that it is difficult to analyze it on any basis other than the statistical. It does, however, obey the general principle that received noise increases as the receiver bandwidth is increased (Section 2.2.1).

2.2 INTERNAL NOISE

Under the heading of internal noise, we discuss noise created by any of the active or passive devices found in receivers. Such noise is generally random, impossible to treat on an individual voltage basis i.e., instantaneous value basis, but easy to observe and describe statistically. Because the noise is randomly distributed over the entire radio spectrum there is, on the average, as much of it at one frequency as at any other. *Random noise power is proportional to the bandwidth over which it is measured.*

2.2.1 Thermal Agitation Noise

The noise generated in a resistance or the resistive component is random and is referred to as *thermal, agitation, white or Johnson noise*. It is due to the rapid and random motion of the molecules (atoms and electrons) inside the component itself.

In thermodynamics, kinetic theory shows that the temperature of a particle is a way of expressing its internal kinetic energy. Thus the "temperature" of a body is the statistical root mean square (rms) value of the velocity of motion of the particles in the body. As the theory states, the kinetic energy of these particles becomes approximately zero (i.e., their motion ceases) at the temperature of absolute zero, which is 0 K (kelvins, formerly called degrees Kelvin) and very nearly equals -273°C . It becomes apparent that the noise generated by a resistor is proportional to its absolute temperature, in addition to being proportional to the bandwidth over which the noise is to be measured.

Therefore

$$P_n \propto T \Delta f = kT \Delta f \quad (2.1)$$

where k = Boltzmann's constant = 1.38×10^{-23} J(joules)/K the appropriate

proportionality constant in this case

$\checkmark T$ = absolute temperature, K = $273 + {}^\circ\text{C}$

$\rightarrow \Delta f$ = bandwidth of interest

$\checkmark P_n$ = maximum noise power output of a resistor

$\checkmark \propto$ = varies directly

Example 2.1

If the resistor is operating at 27°C and the bandwidth of interest is 2 MHz, then what is the maximum noise power output of a resistor?

Solution

$$P_n = k \cdot T \cdot \Delta f = 1.38 \times 10^{-23} \times 300 \times 2 \times 10^6$$

$$\rightarrow P_n = 1.38 \times 10^{-17} \times 600 = 0.138 \times 0.6 \times 10^{-12}$$

$$P_n = 0.0828 \times 10^{-12} \text{ Watts}$$

If an ordinary resistor at the standard temperature of 17°C (290 K) is not connected to any voltage source, it might at first be thought that there is no voltage to be measured across it. That is correct if the measuring instrument is a direct current (dc) voltmeter, but it is incorrect if a very sensitive electronic voltmeter is used. The resistor is a noise generator, and there may even be quite a large voltage across it. Since it is random and therefore has a finite rms value but no dc component, only the alternating current (ac) meter will register a reading. This noise voltage is caused by the random movement of electrons within the resistor, which constitutes a current. It is true that as many electrons arrive at one end of the resistor as at the other over any long period of time. At any instant of time, there are bound to be more electrons arriving at one particular end than at the other because their movement is random. The rate of arrival of electrons at either end of the resistor therefore varies randomly, and so does the potential difference between the two ends. A random voltage across the resistor definitely exists and may be both measured and calculated.

It must be realized that all formulas referring to random noise are applicable only to the rms value of such noise, not to its instantaneous value, which is quite unpredictable. So far as peak noise voltages are concerned, all that may be stated is that they are unlikely to have values in excess of 10 times the rms value.

Using Equation (2.1), the equivalent circuit of a resistor as a noise generator may be drawn as in Fig. 2.1, and from this the resistor's equivalent noise voltage V_n may be calculated. Assume that R_L is noiseless and is receiving the maximum noise power generated by R ; under these conditions of maximum power transfer, R_L must be equal to R . Then

$$P_n = \frac{V^2}{R_L} = \frac{V^2}{R} = \frac{(V_n/2)^2}{R} = \frac{V_n^2}{4R}$$

$$V_n^2 = 4RP_n = 4RkT \Delta f$$

and

(2.2)

$$V_n = \sqrt{4kT \Delta f R}$$

It is seen from Equation (2.2) that the square of the rms noise voltage associated with a resistor is proportional to the absolute temperature of the resistor, the value of its resistance, and the bandwidth over which the noise is measured. Note especially that the generated noise voltage is quite independent of the frequency at which it is measured. This stems from the fact that it is random and therefore evenly distributed over the frequency spectrum.

$$R_L = R$$

$$\frac{R}{R+R_L} V_{in} \rightarrow$$

$$\frac{1}{2} \frac{R}{R+R_L} V_{in}$$

$$= V_{in}/2$$

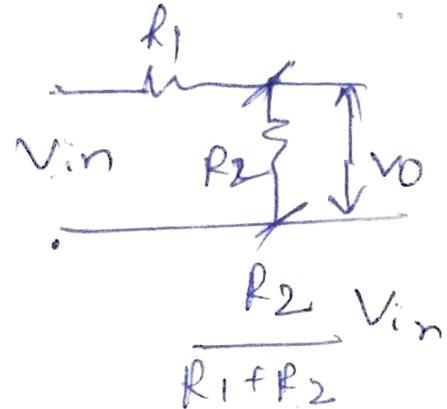
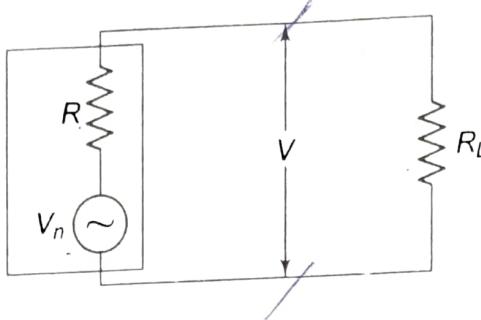


Fig. 2.1 Resistance noise generator.

Example 2.2

An amplifier operating over the frequency range from 18 to 20 MHz has a 10-kilohm (10-kΩ) input resistor. What is the rms noise voltage at the input to this amplifier if the ambient temperature is 27°C?

Solution

$$\begin{aligned} V_n &= \sqrt{4kT \Delta f R} \\ &= \sqrt{4 \times 1.38 \times 10^{-23} \times (27 + 273) \times (20 - 18) \times 10^6 \times 10^4} \\ &= \sqrt{4 \times 1.38 \times 3 \times 2 \times 10^{-11}} = 1.82 \times 10^{-5} \\ &= 18.2 \text{ microvolts (18.2 } \mu\text{V}) \end{aligned}$$

As we can see from this example, it would be futile to expect this amplifier to handle signals unless they were considerably larger than 18.2 μ V. A low voltage fed to this amplifier would be masked by the noise and lost.

2.2.2 Shot Noise

Thermal agitation is by no means the only source of noise in receivers. The most important of all the other sources is the shot effect, which leads to shot noise in all amplifying devices and virtually all active devices.

It is caused by random variations in the arrival of electrons (or holes) at the output electrode of an amplifying device and appears as a randomly varying noise current superimposed on the output. When amplified, it is supposed to sound as though a shower of lead shot were falling on a metal sheet. Hence the name shot noise.

Although the average output current of a device is governed by the various bias voltages, at any instant of time there may be more or fewer electrons arriving at the output electrode. In bipolar transistors, this is mainly a result of the random drift of the discrete current carriers across the junctions. The paths taken are random and therefore unequal, so that although the average collector current is constant, minute variations

nevertheless occur. Shot noise behaves in a similar manner to thermal agitation noise, apart from the fact that it has a different source.

Many variables are involved in the generation of this noise in the various amplifying devices, and so it is customary to use approximate equations for it. In addition, shot-noise *current* is a little difficult to add to thermal-noise *voltage* in calculations, so that for all devices with the exception of the diode, shot-noise formulas used are generally simplified.

The most convenient method of dealing with shot noise is to find the value or formula for an *equivalent input-noise resistor*. This precedes the device, which is now assumed to be noiseless, and has a value such that the same amount of noise is present at the output of the equivalent system as in the practical amplifier. The noise current has been replaced by a resistance so that it is now easier to add shot noise to thermal noise. It has also been referred to the input of the amplifier, which is a much more convenient place, as will be seen.

The value of the equivalent shot-noise resistance R_{eq} of a device is generally quoted in the manufacturer's specifications. Approximate formulas for equivalent shot-noise resistances are also available. They all show that such noise is inversely proportional to transconductance and also directly proportional to output current. So far as the use of R_{eq} is concerned, the important thing to realize is that it is a completely fictitious resistance, whose sole function is to simplify calculations involving shot noise. For noise only, this resistance is treated as though it were an ordinary noise-creating resistor, at the same temperature as all the other resistors, and located in series with the input electrode of the device.

2.2.3 Transit-Time Noise

If the time taken by an electron to travel from the emitter to the collector of a transistor becomes significant to the period of the signal being amplified, i.e., at frequencies in the upper VHF range and beyond, the so-called *transit-time effect* takes place, and the noise input admittance of the transistor increases. The minute currents induced in the input of the device by random fluctuations in the output current become of great importance at such frequencies and create random noise (frequency distortion).

Once this high-frequency noise makes its presence felt, it goes on increasing with frequency at a rate that soon approaches 6 decibels (6 dB) per octave, and this random noise then quickly predominates over the other forms. The result of all this is that it is preferable to measure noise at such high frequencies, instead of trying to calculate an input equivalent noise resistance for it. RF transistors are remarkably low-noise. A *noise figure* (see Section 2.4) as low as 1 dB is possible with transistor amplifiers well into the UHF range.

2.3 NOISE CALCULATIONS

2.3.1 Addition of Noise due to Several Sources

Let's assume there are two sources of thermal agitation noise generators in series: $V_{n1} = \sqrt{4kT\Delta f R_1}$ and $V_{n2} = \sqrt{4kT\Delta f R_2}$. The sum of two such rms voltages in series is given by the square root of the sum of their squares, so that we have

$$\begin{aligned}V_{n,tot} &= \sqrt{V_{n1}^2 + V_{n2}^2} = \sqrt{4kT\Delta f R_1 + 4kT\Delta f R_2} \\&= \sqrt{4kT\Delta f (R_1 + R_2)} = \sqrt{4kT\Delta f R_{tot}}\end{aligned}\tag{2.3}$$

(*) sig to noise ratio: In comm system the comparison of sig power with noise power at the same point is important to ensure that point is not excessively large.

It is ratio of sig power to noise power at same point.

$$\therefore S/N = P_s/P_n \text{ pt.}$$

P_s = sig power. P_n = noise power at same point.

It is normally ~~not~~ expressed in dB & typical value of S/N ratio range from about 10dB to 90 dB.

Higher value of S/N ratio better the system performance in presence of noise.

$$S/N(\text{dB}) = 10 \log_{10}(P_s/P_n)$$

Power can be expressed in terms of sig & noise vlg.

$$P_s = \frac{V_s^2}{R} \quad \& \quad P_n = \frac{V_n^2}{R}$$

$$V_s = \text{sig vlg} \quad V_n = \text{noise vlg.}$$

$$S/N = \frac{V_s^2/R}{V_n^2/R} = \left(\frac{V_s}{V_n} \right)^2$$

(*) Sig to noise ratio in dB,

$$(S/N)_{\text{dB}} = 10 \log_{10} \left(\frac{V_s}{V_n} \right)^2 = 20 \log_{10} \left(\frac{V_s}{V_n} \right)$$

All the possible efforts are made to keep the sig to noise ratio high as possible under all operating condition.

It is a fundamental chara. of comm sys it is often difficult to measure. In practice instead of measuring S/N another ratio called $(S+N)/N$ is measured.

~~sig to noise ratio at i/p~~, $SNR_i = S_i / N_i$

$$S_i = i/p \text{ sig power} = V_s^2$$

$N_i = i/p \text{ noise power} = V_n^2 = 4 K T_0 B (R_p + R_n)$

$$SNR_i = V_s^2$$

$$\text{but } R_p = P_s R_i$$

$$4 K T_0 B (R_p + R_n) \cdot \frac{R_s + R_i}{R_s + R_i}$$

~~sig to noise ratio at o/p, $SNR_o = SNR_i$~~

\therefore Amp is noiseless \therefore SNR at i/p & o/p is same



Noise factor

F is defined in terms of sig to noise ratio at i/p & o/p of system,

$$F = \frac{\text{S/N ratio at i/p}}{\text{S/N ratio at o/p}}$$

$$= \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$

P_{si} & P_{ni} = sig to noise power at i/p.

P_{so} & P_{no} = ~~at o/p~~ at o/p

S/N at i/p will always be greater than o/p.

This is due to noise added by any \therefore

Noise factor is the means to measure amount of noise added & it will always be ~~more~~ greater than one. Ideal value of 'F' is unity.

(b) Noise o/p power in terms of 'F'.

$$\text{Power gain, } G_1 \equiv \frac{P_{so}}{P_{si}}$$

$$\therefore F = \frac{P_{no}}{G_1 P_{ni}}$$

$$\therefore F = \frac{P_{no}}{\frac{P_{so}}{P_{si}}} \times \frac{P_{no}}{P_{so}}$$

$$\therefore P_{no} = F G_1 P_{ni}$$

But $P_{n1} = kT_0B$ at room temp.

$$\therefore P_{n2} = F G k T_0 B$$

This \uparrow se in noise factor, Noise power at OLP will \uparrow se. Higher the noise factor value is more will be noise contributed by the amp.

$$F = \frac{R_p + R_n}{R_p}$$

N Noise Figure

Sometimes Noise factor expressed in dB. When it is expressed in dB it is called noise figure. $= F_{dB} = 10 \log_{10} F$

Put in expression of 'F'.

$$\therefore \text{Noise figure} = 10 \log_{10} \frac{S/I_p \text{ at } i.p.}{S/I_p \text{ at } o.p.}$$

$$= 10 \log_{10} (S/I) - 10 \log_{10} (S/I)$$

$$\therefore \text{Noise figure } F_{dB} = (S/I)_i dB - (S/I)_o dB$$

To improve noise figure, device used for amp's & mixer stage must produce low noise. The diodes & FET, are \therefore preferred. The Receiver can operate at low temperature. This is done in satellite low noise receiver. Use High gain Amp.

① Noise Temperature

This is defined as temp. at which noisy resistor has to be maintained so that by connecting this resistor to the IIP of noiseless version of the system, it will produce the same amount of noise power at the system output as that produced by the actual system.

$$\text{Equivalent noise power } P_n = (F-1) k T_0 B.$$

Temp. T_{eq}

$$k T_{eq} B = (F-1) k T_0 B$$

$$T_{eq} = (F-1) T_0.$$

This eq shows that T_{eq} is just an alternative measure of F .

★ Sig to Noise Ratio

$$SNR = P_s / P_n = \frac{V_s^2}{V_n^2}$$

① Noise Factor

$$x_i = x_s + x_n \rightarrow \left[\begin{array}{l} \text{Amp} \\ \text{noise} \end{array} \right] \rightarrow x_o = A_v(x_s + x_n)$$

$$x_{ni} = x_n$$

$$SNR_i = \frac{x_s^2}{x_n^2}$$

$$x_{no} = A_v x_n + \underline{x_{na}}$$

$$SNR_o = \frac{(A_v x_s)^2}{(A_v x_n)^2} = \frac{x_s^2}{x_n^2} \quad \left. \begin{array}{l} \text{ideal} \\ \text{noise} \end{array} \right\}$$

$$SNR_o < SNR_i$$

Noise factor = $\frac{SNR \text{ at i/p}}{SNR \text{ at o/p}}$ for noise less Amp
 $F = 1$

practically $F > 1$

$$F = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}} = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$

* Noise figure

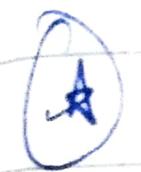
Noise figure

NF = Noise factor in dB.

$$\text{Noise Figure} = 10 \log_{10}(F) = 10 \log_{10} \left[\frac{\text{SNR}_i}{\text{SNR}_o} \right]$$

$$= 10 \log_{10}(\text{SNR}_i) - 10 \log_{10}(\text{SNR}_o)$$

$$\text{Noise figure} = (\text{SNR}_i)_{\text{dB}} - (\text{SNR}_o)_{\text{dB}}$$



Noise Power at O/P in terms of 'F'

$$F = \frac{P_{Si^*}}{P_{Ni}} \times \frac{P_{No}}{P_{SO}}$$

Power gain $\rightarrow G_I = \frac{P_{SO}}{P_{Si^*}} \therefore F = \frac{P_{No}}{G_I P_{Ni}}$

$$\therefore P_{No} = FG_I P_{Ni} \quad P_{Ni} = KTB$$

$$\therefore \boxed{P_{No} = FG_I KTB}$$

$$P_{Ni(\text{total})} = \frac{FG_I KTB}{G_I}$$

$$\therefore \boxed{P_{Ni(\text{total})} = F KTB}$$

$\rightarrow \Psi_i \rightarrow \boxed{\text{Amp}^r} \rightarrow P_{S0} = G_1 \Psi_i$

$P_{Ni(\text{total})}$ $P_{No} = G_1 P_{Ni(\text{total})}$

$P_{No} = G_1 P_{Ni} + \text{Internal noise}$

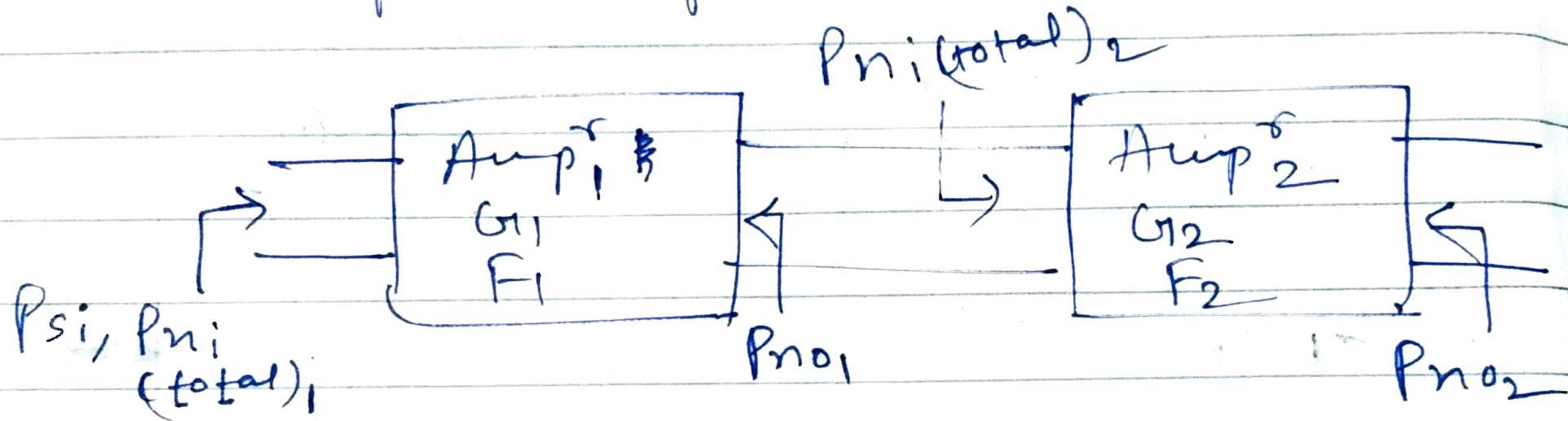
$P_{Ni(\text{total})} = P_{Ni} + P_{Na}$

$\therefore P_{Na} = P_{Ni(\text{total})} - P_{Ni}$

$\therefore P_{Na} = FFTB - FTB = [(F-1)FTB]$

 (FRISS Formula)

→ Noise factor of Amp^r in Cascaded



$$P_{ni_1} = F_1 KTB ; \quad P_{no_1} = G_1 P_{ni(\text{total})_1}$$

$$P_{no_1} = G_1 F_1 KTB$$

$$P_{ni(\text{total})_2} = P_{ni_2} + P_{na_2}$$

$$= P_{no_1} + P_{na_2}$$

$$P_{ni(\text{total})_2} = F_1 G_1 KTB + (F_2 - 1) KTB$$

$$P_{n02} = G_2 \times P_{ni(\text{total})_2}$$

$$P_{n02} = F_1 G_1 G_2 KTB + (F_2 - 1) G_2 KTB.$$

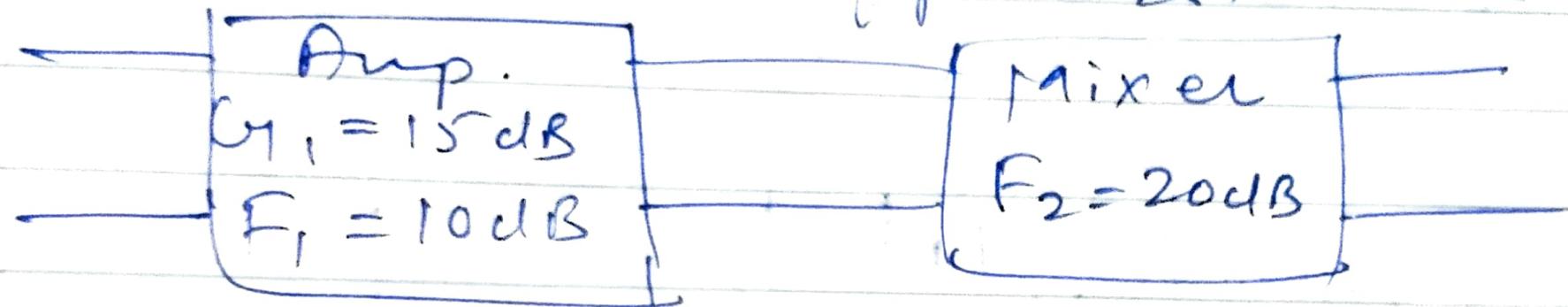
Overall Power gain $G_1 = G_1, G_2$

$$\text{Overall Noise Factor} \rightarrow F = \frac{P_{n0}}{G_1 P_{ni}} = \frac{P_{n0}}{G_1, G_2 P_{ni}}$$

$$\therefore F = \frac{F_1 G_1 G_2 KTB + (F_2 - 1) G_2 KTB}{G_1, G_2 KTB}$$

$$\therefore F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \dots$$

★ Example on FRIIS formula.
→ Find overall Noise figure & noise factor.



$$F_1 \text{ dB} = 10 \text{ dB} \quad \therefore F_1 \text{ dB} = 10 \log_{10} (F_1) \quad \text{cancel}$$

$$\therefore 10 = 10 \log_{10} (F_1)$$

$$\therefore F_1 = 10^{\frac{10}{10}}$$

$$F_2 \text{ dB} = 20 \text{ dB} \quad \therefore F_2 \text{ dB} = 10 \log_{10} (F_2)$$

$$\therefore 20 = 10 \log_{10} (F_2)$$

$$\therefore F_2 = 10^2 = 100 \mu$$

$$G_{1, \text{dB}} = 15 \text{ dB} \therefore G_{1, \text{UB}} = 10 \log_{10}(G_{1,1})$$

$$\therefore 15 = 10 \log_{10}(G_{1,1})$$

$$\therefore G_{1,1} = 10^{1.5} = 31.62,$$

Overall

$$\text{Noise Factor} = F = F_1 + \frac{F_2 - 1}{G_{1,1}} = 10 + \frac{100 - 1}{31.62}$$

$$\therefore F = 13.13,$$

$$\text{Noise figure} = 10 \log_{10}(F)$$

$$\text{Noise Figure} = 10 \log_{10}(13.13) = 11.18 \text{ dB}$$

The Friis Equation

we introduce one of the most fundamental equations in antenna theory, the **Friis Transmission Equation**. The Friis Transmission Equation is used to calculate the power received from one antenna (with gain G_1), when transmitted from another antenna (with gain G_2), separated by a distance R , and operating at frequency f or wavelength λ . This page is worth reading a couple times and should be fully understood.

Derivation of Friis Transmission Formula

To begin the derivation of the Friis Equation, consider two antennas in free space (no obstructions nearby) separated by a distance R :

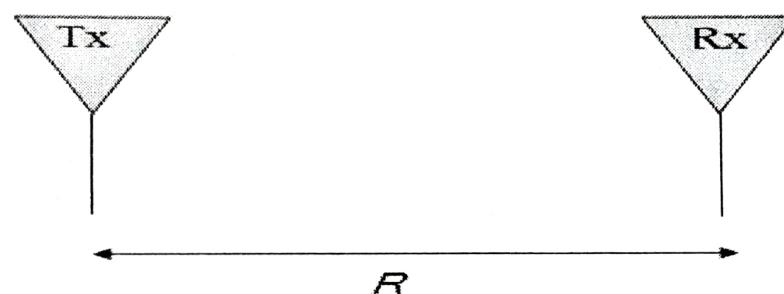


Figure 1. Transmit (Tx) and Receive (Rx) Antennas separated by R .

Assume that P_T Watts of total power are delivered to the transmit antenna. For the moment, assume that the transmit antenna is omnidirectional, lossless, and that the receive antenna is in the far field of the transmit antenna. Then the power density p (in Watts per square meter) of the plane wave incident on the receive antenna a distance R from the transmit antenna is given by:

$$p = \frac{P_T}{4\pi R^2}$$

If the transmit antenna has an antenna gain in the direction of the receive antenna given by G_T , then the power density equation above becomes:

$$p = \frac{P_T}{4\pi R^2} G_T$$

The gain term factors in the directivity and losses of a real antenna. Assume now that the receive antenna has an effective aperture given by A_{ER} . Then the power received by this antenna () is given by:

$$P_R = \frac{P_T}{4\pi R^2} G_T A_{ER}$$

Since the effective aperture for any antenna can also be expressed as:

$$A_e = \frac{\lambda^2}{4\pi} G$$

The resulting received power can be written as:

$$P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi R)^2} \quad [\text{Equation 1}]$$

This is known as the ***Friis Transmission Formula***. It relates the free space path loss, antenna gains and wavelength to the received and transmit powers. This is one of the fundamental equations in antenna theory, and should be remembered (as well as the derivation above).

Another useful form of the Friis Transmission Equation is given in Equation [2]. Since wavelength and frequency f are related by the speed of light c , we have the Friis Transmission Formula in terms of frequency: its similar to equation 1 , only replace $\lambda = c/f$ [Equation 2]

Equation [2] shows that more power is lost at higher frequencies. This is a fundamental result of the Friis Transmission Equation. This means that for antennas with specified gains, the energy transfer will be highest at lower frequencies. The difference between the power received and the power transmitted is known as *path loss*. Said in a different way, Friis Transmission Equation says that the path loss is higher for higher frequencies.