PCOM

ASSIGNMENT 02

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D10A 01

Q1. Write mathematical expression and graphical representation of the following signals. Impulse, step, parabolic, ramp, sinusoidal, cosine, traingular, sinc, signum.

→ 1) Impulse Signal

The impulse signal is a signal with infinite magnitude and zero duration, but with an area of A.

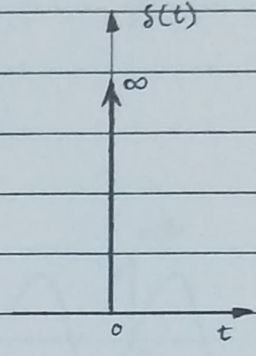
Mathematically,

Impulse signal, $\delta(t) = \infty \; ; t = 0$

$= 0 \; ; t \neq 0$

and

$$\int_{-\infty}^{\infty} \delta(t)\, dt = A$$



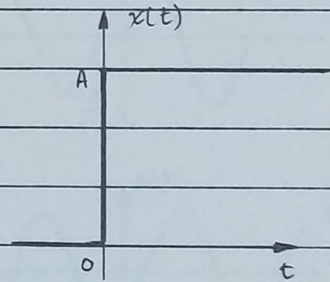
2) Step signal

The signal whose magnitude is greater than zero for time greater than zero

Mathematically,

$x(t) = A \; ; \; t \geq 0$

$= 0 \; ; \; t < 0$



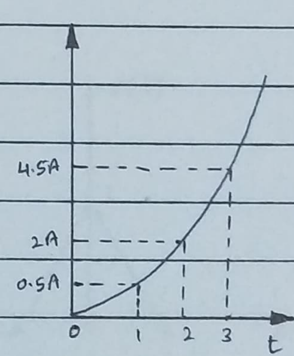
3) Parabolic Signal

The signal, whose magnitude increases with square of the time

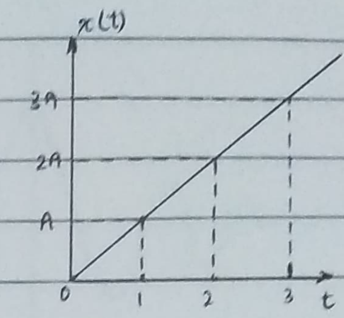
Mathematically,

$x(t) = \frac{At^2}{2} \; ; \text{ for } t \geq 0$

$= 0 \; ; \; t < 0$



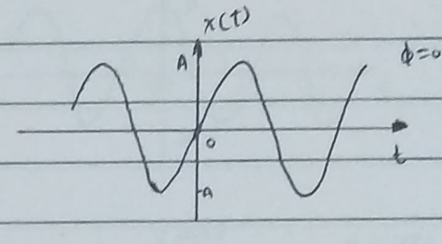
4) Ramp signal

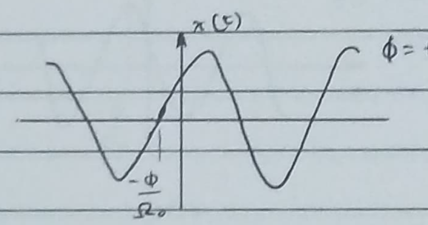


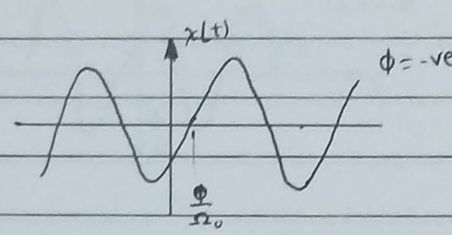
A signal whose magnitude increases same as the time
Mathematically,

$$x(t) = At \quad ; \quad t \geq 0$$
$$= 0 \quad ; \quad t < 0$$

5) Sinusoidal signal







A signal whose magnitude varies interms of sine wave w.r.t tieme, i.e, zero, then +ve max then zero, then -ve max, Mathematically,

$$x(t) = A \sin(\Omega_0 t + \phi)$$

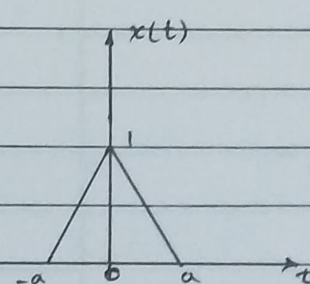
here, $\Omega_0 = 2\pi f_0$

when $\phi = 0$, $x(t) = A \sin \Omega_0 t$

when $\phi = +ve$, $x(t) = A \sin(\Omega_0 t + \phi)$

when $\phi = -ve$ $x(t) = A \sin(\Omega_0 t - \phi)$

6) Triangular Pulse signal



The triangular pulse is defined as,

$$x(t) = \Delta_a t = 1 - |t|/a \quad ; \quad |t| \leq a$$
$$= 0 \quad ; \quad |t| > a$$

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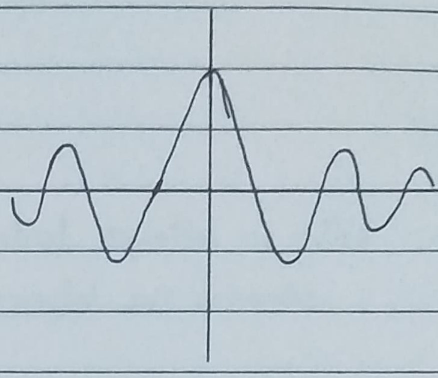
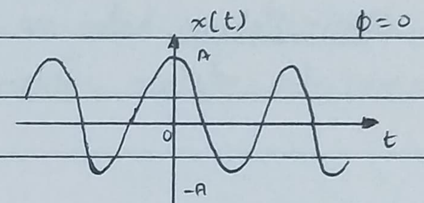
7) Sinc signal



It is a signal with magnitude like decaying sine function. Mathematically,

$$x(t) = \text{sinc}(t) = \frac{\sin t}{t} \quad ; -\infty < t < \infty$$

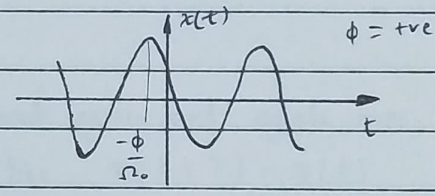
8) Cosine signal



Cosine signal's magnitude varies with cosine function
Mathematically,

$$x(t) = A\cos(\Omega_0 t + \phi)$$

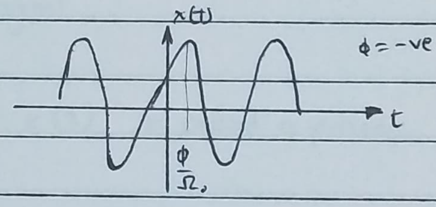
here, $\Omega_0 = 2\pi F_0$



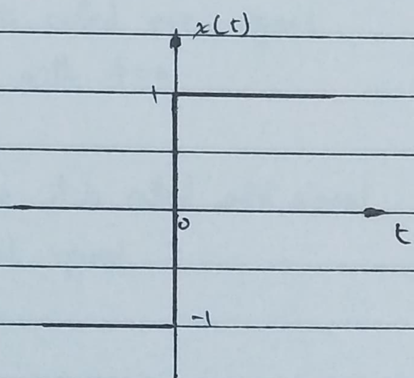
when $\phi = 0$ $\quad x(t) = A\cos\Omega_0 t$

when $\phi = +ve$ $\quad x(t) = A\cos(\Omega_0 t + \phi)$

when $\phi = -ve$ $\quad x(t) = A\cos(\Omega_0 t - \phi)$



9) Signum function



Defined as sign of independant variable t
Mathematically,

$$x(t) = \text{sgn}(t) = 1 \quad ; t > 0$$
$$= 0 \quad ; t = 0$$
$$= -1 \quad ; t < 0$$

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2) Give one example of the following classification of signals.

i) Deterministic and non deterministic signal

The signal that can be completely specified by matematical equation, is called deterministic signal. The step, ramp, expo and sinusoidal are examples.

The signal, whose characteristics are random in nature are called non deterministic signal. Examples are noise signal from sources like amplifier, oscillator, radio reciever

ii) Periodic and non Periodic signal

A periodic signal will have a definite pattern that repeats again and again over certain period of time. ∴ signals which satisfies condition $x(t+T) = x(t)$ are periodic signal, eg sinusoidal signal, exponential signal

A signal which doesn't satisfy the condition $x(t+T) = x(t)$ is called a periodic signal. eg. sinc signal

iii) Symmetric and a symmetric signal

When a signal exhibits symmetry w.r.t $t=0$, then it is called even signal ∴ even signal satisfies $(x(-t) = x(t)$, eg. cosinusoidal with $\phi = 0$

When a signal exhibits anti symmetry with.r.t $t=0$, the it is called odd signal ∴ odd signal satisfies $x(-t) = -x(t)$, eg. Sinusoidal signal

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IV) Energy and power

The signals which have finite energy are called energy signals. The nonperiodic signals like exponential signal will have constant energy, so non periodic signals are Energy signals

The signals which have finite average power are called power signal. The periodic signals like sinusoidal and complex exponential will have constant power, so periodic signals are power signals

V) Casual and non-Casual signal

A signal is said to be casual, if it is defined for $t \geq 0$
$\therefore$ $x(t)$ is casual, then $(x(t) = 0$ for $t < 0$
eg step signal, ramp signal, etc

A signal is said to be non-casual, if it is defined either for $t \leq 0$ or for both $t \leq 0$ and $t > 0$ $\therefore$ $x(t)$ is casul if $x(t) \neq 0$ for $t < 0$
eg Exponential signal, complex exponential signal

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3) Define fourier transform, and inverse of fourier transform

- Fourier transform

let $x(t)$ = Continuous time signal
$X(j\Omega)$ = Fourier transform of $x(t)$
The fourier transform of continuous time signal, $x(t)$ is defined as.

$$X(J\Omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-J\Omega t} \, dt$$

Also, $X(j\Omega)$ is denoted as $F\{x(t)\}$, $F \Rightarrow$ fourier transform

$$F\{x(t)\} = X(j\Omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\Omega t} \, dt$$

- Inverse of fourier transform

The inverse fourier transform of $X(j\Omega)$ is defined as,

$$x(t) = F^{-1}\{X(j\Omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t} \, d\Omega$$

The signals $x(t)$ and $X(J\Omega)$ are called fourier transform pair, expressed as

$$x(t) \underset{F^{-1}}{\overset{F}{\rightleftharpoons}} X(j\Omega)$$

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4) Give fourier tranform.

i) $A \cdot e^{-at}$

$$x(t) = A \cdot e^{-at} \quad ; \text{ for } t \geq 0$$

By deffinition of Fourier transform,

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\Omega t} \, dt = \int_{0}^{\infty} A e^{-at} \cdot e^{-j\Omega t} \, dt$$

$$= \int_{0}^{\infty} A \cdot e^{(-(a+j\Omega)t)} \, dt = \left[\frac{A \cdot e^{-(a+j\Omega)t}}{-(a+j\Omega)} \right]_{0}^{\infty}$$

$$= \left[\frac{A \cdot e^{-\infty}}{-(a+j\Omega)} - \frac{A \cdot e^{0}}{-(a+j\Omega)} \right] = \frac{A}{a+j\Omega}$$

$$\therefore F\{A \cdot e^{-at} u(t)\} = \frac{A}{a+j\Omega}$$

ii) $sgn(t)$

$$x(t) = sgn(t) = 1 \quad ; t > 0$$
$$= -1 \quad ; t < 0$$

The signum function can be represented as a sum of two one sided exponential signal and taking limit "a" tends to zero

$$\therefore sgn(t) = \underset{a \to 0}{Lt} \left[e^{-at} u(t) - e^{at} u(-t) \right]$$

$$\therefore x(t) = sgn(t) = \underset{a \to 0}{Lt} \left[e^{-at} u(t) - e^{at} u(-t) \right]$$

By definition of fourier transform

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\Omega t} \, dt = \int_{-\infty}^{\infty} \underset{a \to 0}{Lt} \left[e^{-at} u(t) - e^{at} u(-t) \right] e^{-j\Omega t} \, dt$$

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$$= \lim_{a\to 0}\left[\int_0^{\infty} e^{-at} e^{-j\Omega t}\,dt - \int_{-\infty}^{0} e^{at}\, e^{-j\Omega t}\,dt\right]$$

$$= \lim_{a\to 0}\left[\int_0^{\infty} e^{-(a+j\Omega)t}\,dt - \int_{-\infty}^{0} e^{(a-j\Omega)t}\,dt\right]$$

$$= \lim_{a\to 0}\left[\left[\frac{e^{-(a+j\Omega)t}}{-(a+j\Omega)}\right]_0^{\infty} - \left[\frac{e^{(a-j\Omega)t}}{(a-j\Omega)}\right]_{-\infty}^{0}\right]$$

$$= \lim_{a\to 0}\left[\frac{1}{a+j\Omega} - \frac{1}{a-j\Omega}\right] = \frac{1}{j\Omega} + \frac{1}{j\Omega}$$

$$\therefore F\{sgn(t)\} = \frac{2}{j\Omega}$$

iii) $u(t)$

Unit step signal is defined as

$u(t) = 1 \; ; t \geq 0$

$\quad = 0 \; ; t < 0$

If can be be proved that, $sgn(t) = 2u(t) - 1$

$\therefore x = u(t) = \frac{1}{2}[1 + sgn(t)]$

· on taking fourier transform of above equation

$$F\{x(t)\} = F\{1/2[1 + sgn(t)]\} = 1/2\, F\{1\} + 1/2\, F\{sgn(t)\}$$

$$= 1/2\,[2\pi\,\delta(\Omega)] + 1/2\,[2/j\Omega] = \pi\,\delta(\Omega) + 1/j\Omega$$

$$F\{u(t)\} = \pi\,\delta(\Omega) + \frac{1}{j\Omega}$$

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iv) $A \sin \Omega t$

The sinusoidal signal is defined as

$$x(t) = A \sin \Omega_0 t = \frac{A}{2j}\left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right)$$

on taking fourier transform, we get

$$F\{x(t)\} = F\left\{\frac{A}{2j}\left(e^{j\Omega_0 t} - e^{-j\Omega_0 t}\right)\right\}$$

$$= \frac{A}{2j}\left[F\{e^{j\Omega_0 t}\} - F\{e^{-j\Omega_0 t}\}\right]$$

$$= \frac{A}{2j}\left\{2\pi\,\delta(\Omega-\Omega_0) \cancel{+} - 2\pi\,\delta(\Omega+\Omega_0)\right\}$$

$$= \frac{A\pi}{j}\left[\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)\right]$$

$$\therefore F\{A \sin \Omega_0 t\} = \frac{A\pi}{j}\left[\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)\right]$$

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5) Define Noise figure and noise factor

- Noise figure

Sometimes noise factor expressed in db, when it is expressed in the form of db, they are called noise figure.

$$\text{Noise figure} = F_{db} = 10 \log_{10} F$$

putting in expression of F,

$$\text{Noise figure} = 10 \log \left\{ \frac{S/N \text{ at } i/p}{S/N \text{ at } o/p} \right\}$$

$$\therefore \text{Noise figure} = (S/N)_i \, db - (S/N)_o \, db$$

- Noise factor

If noise is expressed in terms of sig to noise ratio at i/p and o/p of system.

$$F = \frac{S/N \text{ ratio at } i/p}{S/N \text{ ratio at } o/p}$$

$$\therefore \text{Noise factor } F = \frac{P_{Si}}{P_{Ni}} \times \frac{P_{No}}{P_{So}}$$

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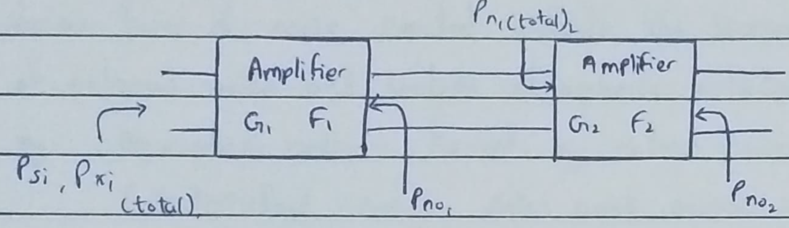
6) Explain friss transformation formula

Noise factor of amplifier in cascade



$$P_n = F_1 KTB \; ; \qquad P_{no_1} = G_1 \, P_{ni} \, (total)_1$$

$$P_{no_1} = G_1 F_1 \, KTB$$

$$P_{ni} \, (total)_2 = P_{ni_2} + P_{na_2}$$

$$= P_{no_1} + P_{na_2}$$

$$P_{ni} \, (total)_2 = F_1 G_1 \, KTB + (F_2 - 1) \, KTB$$

$$P_{No_2} = G_2 \times P_{n_i} (total)_2$$

$$P_{no_2} = F_1 G_1 G_1 \, KTB + (F_2 - 1) \, G_2 \, KTB$$

Overall power gain $G = G_1 G_2$

Overall Noise factor $F = \dfrac{P_{no}}{G P_{ni}} = \dfrac{P_{no}}{G_1 G_2 P_{ni}}$

$$\therefore F = \frac{F_1 G_1 G_2 \, KTB + (F_2 - 1) \, G_2 \, KTB}{G_1 G_2 \, KTB}$$

$$\therefore F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \cdots$$

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7) What are types of internal noises and external noises.

• External noise

Various form of noise created outside the receiver come under the heading of external noise and include atmospheric extaterrestrial noise and industrial noise

eg. Atmospheric noise :- formed by natural sources of disturbance

Extraterristrial noise :- solar noise, cosmic noise

Industrial noise :- formed by artificial source of industry

• Internal noise

Under the heding of internal noise, we disuss noise created by any of the active or passive devices found in reciever. Such noise is genarally random, impossible to treat on an individual voltage basis, i.e instantaneous reciever. Such noise but easy to observe and describe statistically. Because the noise is Randomly distributed over the radio spectrum there is, on average, as much of it at one frequency as at any other.

eg. Thermal agitation noise :- The noise generated in ressistance or ressistive component

Shot noise :- It is caused by randome variation in arrival of e^- at output eletrode

Transit-time noise :- If time taken by e^- to travel from colletor of trans beomes significant, to the period for amplification, transit time noise is formed

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8) Summarise :- Equivalent noise temperature

Noise temperature is defined as temperature at which noisy ressistor has to be maintained so that by conneting this ressistor to the input of noiseless version of the system it will produce the same amount of noise power at the system output as that produced by the actual system

$$P_n = (F-1)\ kT_oB$$

$$k\,T_{eq}\,B = (F-1)\ kT_oB$$

$$\therefore T_{eq} = (F-1)\,T_o$$

This shows that T_{eq} is an alternative measure for F

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9) State and prove The following property

i) Time shifting property

Time shifting property states that if $x(t)$ and $x(\omega)$ form a fourier transform parir, then

$$x(t-t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} x(\omega) \text{ or } e^{-j\Omega t_0} \cdot X(j\Omega)$$

$$F\{x(t)\} = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\Omega t} \, dt$$

$$F\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(t-t_0) e^{-j\Omega t} \, dt$$

let $t - t_0 = a$

$\therefore t = a + t_0$

$\cdot \, dt = da$

$$\therefore F\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(a) \cdot e^{-j\Omega(a+t_0)} \, da$$

$$= \int_{-\infty}^{\infty} x(a) \cdot e^{-j\Omega a} \cdot e^{-j\omega t_0} \, da$$

$$= e^{-j\Omega t_0} \cdot \int_{-\infty}^{\infty} x(a) \, e^{-j\Omega a} \, da$$

$$= e^{-j\Omega t_0} \, X(j\Omega)$$

Hence proved

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- **Frequency shifting property**

Frequency shifting property of fourier transform states that,

$$x(t) \cdot e^{j\Omega_0 t} \xleftrightarrow{F.T} X(j(\Omega - \Omega_0))$$

$$F\{x(t)\} = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\Omega t} dt = X(j\Omega)$$

$$F\{x(t) \cdot e^{j\Omega_0 t}\} = \int_{-\infty}^{\infty} x(t) \cdot e^{j\Omega_0 t} \cdot e^{-j\Omega t} dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot e^{-j(\Omega - \Omega_0)t} dt$$

$$\therefore \underline{F\{x(t) \cdot e^{j\Omega_0 t}\} = X(j(\Omega - \Omega_0))}$$

Hence proved

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Q) Explain Short noise and equivalent temprature, discuss Friss formula

- Shot noise

Shot effect leads to shot noise in all amplifying devices and virtually all active devices it is caused by random variation in arrival of e^- at the output electrode of amplifying device and appears as randomly varying noise current superimposed on the output. When amplified, its supposed to sound as though a shower of lead shot were falling on metal sheet Hence the name shot noise

Although the average output of device is governed by various bias voltages at any instant of time, there may be more or fewer e^- arriving at output electrode

- Noise - equivalent temperature

It is a measure of sensitivity detetor of thermal radiation in IR, terahets or microwave portion. It is the amount of signal temperature that would be needed to match the internal noise of the detector such that the signal to noise ratio is equal to one

For a particular temperature, there is a fundamental limit to noise equivalent temp given by the natural thermodynamic fluctuations of the photon flux from the source under investigation

- Friis formula

Friis formula is used to calculate the total noise factor of cascade of stages each with its own noise factor and power gain. The total noise factor can be used to study total noise figure

$$F_{dB} = 10 \log_{10}(F)$$

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11) What is meant by signal to noise ratio. Discuss importance of SNR in radio reciever

In communication system, th comparison of s/g power with noise power at the same point is important to ensure that the point is not excessively large.
It is ratio of s/g power to noise power at the same time

$$S/N = P_s / P_n$$

where, P_s = s/g power, P_N = noise power at the same point

Signal to noise ratio is one of the most straightforward method of measuring radio reciever sensitivity
As with any sensitivity measurement the performance of overall radio reciever is determined by the performance of the front of RF amplifier stage
Fist amplifier of any radio reciever should be a low noise amplifier

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12) Define i) Noise Figure ii) Noise temperature

Noise figure

Some time noise factor is expressed in dB then it is called noise figure

Noise figure = $F_{dB} = 10 \log_{10} F$

$\therefore$ Noise figure = $10 \log_{10} \left[\frac{S/N \text{ at input}}{S/N \text{ at output}} \right]$

$\therefore$ Noise figure $F_{dB} = (S/N)_{i\,dB} - (S/N)_{o\,dB}$

To improve noise figure, device used for amplifier must be producing low noise

Noise temperature

It is defined at which noisy ressistor has to be maintained so that by conneting this ressistor to the i/p of noiseless version of the system, it will produce the same amount of noise power at the system o/p as that produced by the actual system

$$P_n = (F-1)\, K T_0 B$$

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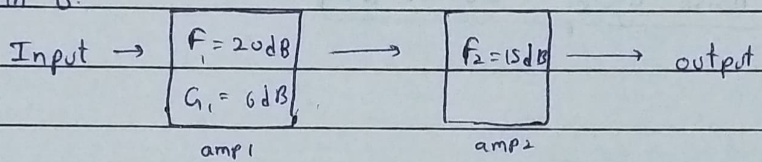
13) Signal to noise ratio

In communication system, comparison of signal power with noise power at the same point is important to ensure that the point is not excessively large it is the ratio of signal power to noise power at same point

$$S/N = P_s / P_n$$

when amplifiers are cascaded together in order to amplify very weak signals, it is generally the first amplifier in the chain which will have the greatest influence upon the signal to noise ratio becase the noise floor is lowest at that point in chain

Given system is:-



Converting dB into equivalent ratio,

$F_{1dB} = 20dB$ $\quad \therefore F_{1dB} = 10 \log_{10} F_1$

$\therefore 20 = 10 \log_{10} F_1$

$\therefore F_1 = 100$

$F_{2dB} = 15 dB$ $\quad \therefore F_2 dB = 10 \log_{10} F_2$

$\therefore 15 = 10 \log_{10} F_2$

$\therefore F_2 = 31.62$

$G_{1dB} = 6dB$ $\quad \therefore G_{1dB} = 10 \log_{10} G_1$

$\therefore G_{dB} = 10 \log_{10} G_1$

$\therefore G_1 = 3.98$

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Using Friis formula,

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1}$$

$$\therefore F_{total} = 100 + \frac{31.62 - 1}{3.98}$$

$$\therefore F_{total} = 107.69$$

$$\therefore \text{Overall noise figure} = 10 \log_{10}(F_{Total})$$

$$= 10 \log_{10}(107.69)$$

$$\therefore \boxed{\text{Overall noise figure} = 20.32 \text{ dB}}$$