Analog Lab: Impedance, Filters, and Noise

January 20, 2014

1 Week 1

Prerequisites

Reading: Horowitz & Hill, Ch. 1

- Ohm's Law
- Thévenin Equivalent Resistance
- Capacitance and Inductance
- RC Filters
- Diodes

Materials

- breadboard
- misc R, C, L components
- potentiometer ("pot")
- power supply
- function generator w/ external FM reference
- oscilloscope w/ probes
- voltimeter
- speaker w/ amplifier

Some Thoughts

Breadboards

Breadboards are convenient for constructing circuts for this lab. Take some time to get to know yours using a ohmmeter. In particular, figure out what is connected, and what is not.

Grounding

Ground means different things in different contexts (e.g. earth ground, signal ground, chassis ground). For our purposes in this lab, ground simply provides a return path for current. We define it to be 0V (i.e. it is the reference from which we measure voltage differences). The ground of your circuit should be tied to the ground of your power supply. It is good practice to consistently use a color (black, blue, and green are good choices) to indicate ground lines in your circuits.

Capacitor Orientation

Some capacitors, particularly electrolytics, care about which direction voltage is applied across them. Often, one of the leads is longer, and may have a '+' near it on the body of the capacitor. Always connect this lead to the higher voltage. Failure to do so can, for higher voltages, cause them to explode.

Oscilloscopes

'Scopes can be intimidating. The solution is to press buttons until they submit. Seriously, don't be afraid to push buttons. In order to show a static display of a repeating signal, scopes need to trigger on some edge of an input signal. This is always configurable, and you'll need to figure out how. You can adjust the time (X axis) and voltage (Y axis) ranges, and should do so regularly. Many scopes have an "auto" button, which is a totally useful cop-out. Probes that connect to the scope often amplify (10:1), and scopes have a setting to undo this for the display. Finally, there is a difference between attaching a probe and attaching a cable. This difference has to do with termination. We haven't covered yet what termination is (it's next week), but suffice it to say that in order for your voltage scale to mean something, you should select 50Ω if you have connected a cable (BNC, SMA, etc.), and 1MΩ if you are using a probe.

FM Radio

Frequency modulation (FM, see Figure 1) is a clever scheme for encoding a signal. Amplitude modulation (AM) radio directly mixes a carrier tone with a signal, so that the amplitude of the signal creates an envelope inside of which the carrier tone oscillates. FM instead translates the amplitude of the signal into a change in the frequency of the carrier tone (see above). FM radio stations typically transmit between 88 and 108 MHz in the US. However, for the purpose of this lab, owing to lack of radio reception in the lab, and the difficulty in using discrete components at higher frequencies, we will create our own radio station at 1.045 MHz, instead of 104.5 MHz. Typically, FM radio stations are spaced every 200 kHz, and have a frequency deviation of ±75 kHz at maximum scale input.

1.1 Resistive Voltage Divider

1.1.1 Useful Equations

$$V = IR \tag{1}$$

$$P = IV \tag{2}$$

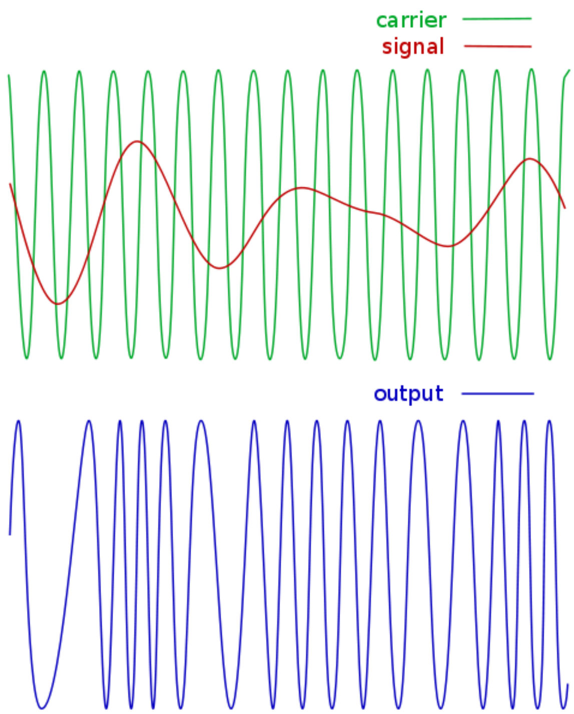

Figure 1: Frequency modulation

1.1.2 Activities

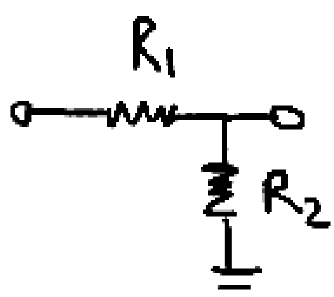

Figure 2: A voltage divider

- build voltage divider (Figure 2) out of two 1k resistors, connect input to +5V DC voltage source
- measure voltage at output
- measure current (think carefully about where before burning out voltimeter fuse!)
- apply 1 MHz sine wave, view input and output on oscilloscope

- append a second voltage divider to the output of the first (choose resistor values carefully)
- for a voltage divider, is it better to have high impedances or low? what considerations might drive you in either direction?
- from the perspective of the second voltage divider, what is the Thèvenin equivalent resistance of the first divider, as seen from its output?

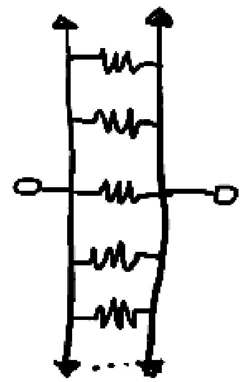

Figure 3: An infinite series of resistors

- if each resistor in Figure 3 is the same value, what is the equivalent resistance between the two terminals? (BTW, I still haven't solved http://xkcd.com/356, but don't get nerd sniped while doing this lab!)

1.2 Capacitive Voltage Divider

1.2.1 Useful Equations

$$I = C\frac{dV}{dt} \tag{3}$$

$$Z_C = \frac{1}{j\omega C} \tag{4}$$

1.2.2 Activities

- build a voltage divider out of two 1μF capacitors, connect input to 1 MHz sine wave
- compare input and output on oscilloscope. is it what you expected? what is the DC level at the output? resistor?
- add a 1k resistor to ground at the output, and then for your modified circuit, graph the expected V_{out}/V_{in} versus frequency

1.3 RC filter

1.3.1 Activities

- design a high-pass filter with cutoff (-3dB) at 100 kHz
- design a low-pass filter with the same cutoff
- plot the expected responses of each. spot-check with oscilliscope measurements.
- what could you do to get steeper transitions in RC filters such as these?

1.4 LC filter

1.4.1 Useful Equations

$$Z_L = j\omega L \tag{5}$$

$$Q = \omega_0 RC = \frac{f_0}{\Delta f_{3dB}} \tag{6}$$

1.4.2 Activities

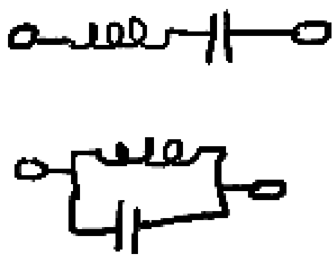

Figure 4: An inductor in series and in parallel with a capacitor

- plot the theoretical impedance versus frequency of a 1 μH inductor in parallel with and in series with a 1 μF capacitor (Figure 4).
- design and build an LC band-pass filter (Figure 5) tuned to 1 MHz
- choose an R to select a quality factor appropriate for $\Delta f_{3dB} = 200kHz$.
- for a moment, swap out your C for something quite small (say, 100 pF) and measure empirically the frequency of peak response. assuming your inductor value is correct, what is your inferred capacitance? if this doesn't agree with your C, can you explain why?

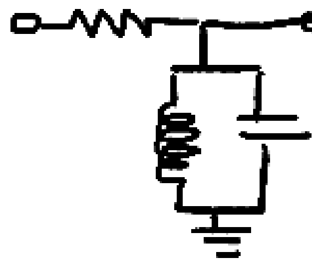

Figure 5: A bandpass RLC filter

1.5 Diodes

- measure the voltage drop over a conducting diode (important: limit current flow with a resistor!)
- use different resistor values (or a pot) and measure the voltage drop as a function of current
- use a large ($\pm$ 1V) oscillating signal into the diode and view the original and rectified signal on an oscilloscope
- apply a low-pass filter with time constant of order the input frequency, and view the output on an oscilloscope. generate a plot of what is going on here.
- suppose you have an oscillating signal whose amplitude is less than the voltage drop across your conducting diode. can you come up with a circuit that re-centers your oscillating signal on the threshold of diode conduction?

1.6 FM demodulation

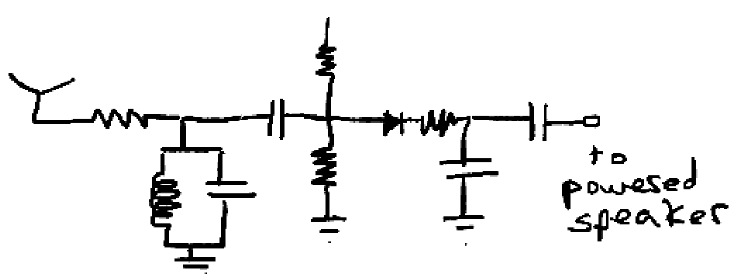

Figure 6: An FM demodulation circuit

- use your LC circuit to convert a frequency-modulated signal into an amplitude modulated signal

- use your diode circuit as an ”envelope detector” that filters out the carrier frequency and extracts the amplitude modulation. Use a 100 kHz low-pass filter in the envelope detection. (Humans can only hear up to 40 kHz or so). You should have something like Figure 6.
- connect in our faked FM station at 1.045 MHz
- connect the output through a coupling capacitor (to remove the DC bias) to amplifying audio speakers
- enjoy listening to the music
- fun part: which components of your circuit are strictly necessary? try removing some of them and see if it still works? Draw your circuit as it worked before and after “contracting” it.
- if our station is at 1.045 MHz, why is your LC tuned to 1.0 MHz? what was relevance of the quality factor?

2 Week 2

Prerequisites

Reading: Horowitz & Hill, Ch. 2

- Transmission Lines
- Transistors
- Amplifier Circuits

Materials

- rope and string of various thicknesses (pretty long)
- breadboard
- misc R, C, transistor components
- power supply
- function generator w/ external FM reference
- oscilloscope w/ probes
- voltimeter
- speaker w/o amplifier
- long cable (as long as possible, longer than 100m would be great)

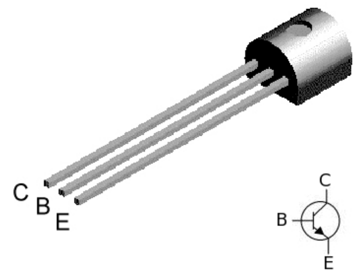

Figure 7: Pinout for a generic NPN transistor (e.g. 2N3904)

Some Thoughts

Transistors

We'll be using an NPN bipolar junction transistor in this lab (see Figure 7). With the flat face to you, pins typically read, from left to right, emitter, base, collector. Now you know.

Speakers

Last week, we just stuck our signal into an amplifying speaker and didn't think much about it. This weak, since we are building our own amplifier, you might be wondering what's left that makes a "speaker." Speakers aren't that complicated. At their simplest, they are solenoids that push/pull on a small magnet attached to a diaphram. Depending on which direction current is made to flow through the solenoid by an alternating voltage signal, a magnetic field is set up that attracts or repels the magnet on the diaphram. The diaphram translates the resulting motion into pressure waves that propagate through the air.

Termination

At the end of this lab, we'll be playing with some 50Ω (and maybe some 75Ω) cable, examining how waves are reflected and transmitted. Of course, now you also have a better idea of why the oscilloscope has selectable termination, and why it might be different depending on whether you connect as scope probe or a BNC cable to the input. Now that you know why and how, you have no excuse not to exercise good termination practices!

2.1 Impedance Mismatches on Rope

- Get several lengths of ropes of different thicknesses. Tie them together and tie the end of the thinner rope to a doorknob or some other stable structure. Hold the untied end, pull the rope relatively taut, and then, with a quick up-down movement of the hand, send a pulse down the rope. Observe what happens at the interface between the ropes, and at the doorknob.
- Switch the rope around so that the thicker end is now tied to the doorknob. Repeat the above. Any difference?
- What would happen, do you think, if the far end of the rope, instead of being tied to a fixed point, was tied to a ring that could move up and down on a pole?

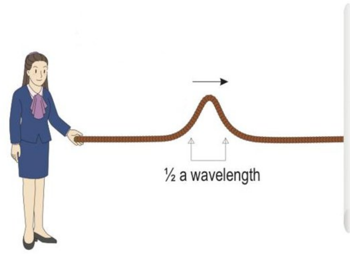

Figure 8: Sending a pulse down a rope

- Any idea how to stop the wave from reflecting off the far end of the rope?
- If you have two thick ropes joined by a very short thin piece (small with respect to the wavelength of the signal), what happens at that interface?
- What about vice versa (thin ropes joined by a short thick piece)?

2.2 Impedance Mismatches in a Transmission Line

- Get a function generator that can output square waves. Get a long stretch of cable and connect the function generator output to one end of the cable. Leave a point that you can probe with an oscilloscope.
- Use an oscilloscope to try to detect the reflected wave. Draw the superposition of transmitting and reflecting waveforms that produces your output.
- How long is your cable? How fast did your pulse travel? (BTW, it can be helpful to know that, round-about, light travels 1 ft in 1 ns, and that waves on a cable can be up to a factor of 2 slower than that.)
- Use a 10Ω resistor to terminate the far end of the cable. Observe the change.
- Now properly terminate the far end, and recover your waveform! What was the impedance of the cable?

2.3 Building an Follower

- Build the emitter-follower circuit shown in Figure 9, choosing approprite values for resistors and capacitors. Use a 2N3904 transistor, or a similar substitute NPN BJT. You will power the circuit with $V_{cc} = +5V$, and will want to amplify signals in the range 100Hz to 50kHz. Mind your R's and C's!
- Measure the bias voltage V_{BE}.
- What is the purpose of the emitter resistor (that is, the resistor most immediately connected to the emitter)?
- Input a 10 kHz sine wave with 1V amplitude. What is the relationship between the input and output voltages?

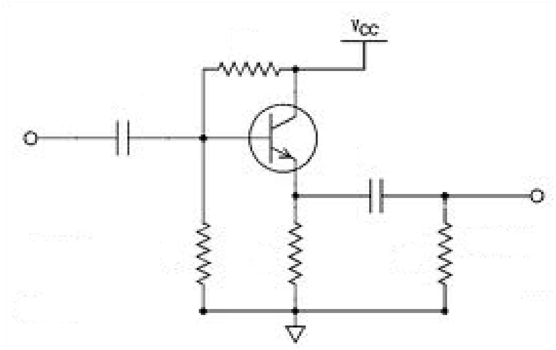

Figure 9: An emitter follower that uses a biasing circuit.

- Predict first, and then measure, the maximum sine amplitude for which this circuit correctly operates.
- Measure the bias voltage at the base. How small can you make the emitter resistor before it changes? You might need to make the resistors in your bias circuit large so that you don't burn up the emitter resistor.

2.4 Building an Amplifier for our FM radio

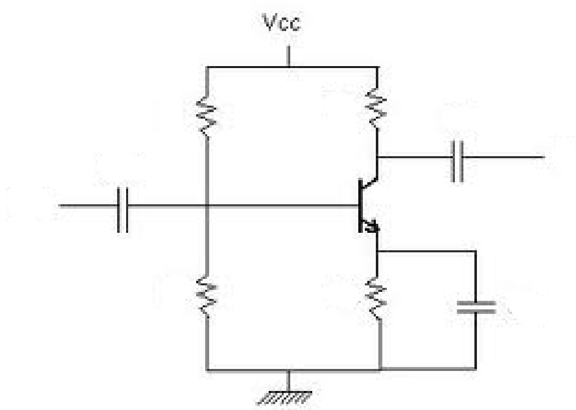

Figure 10: An amplifier based on an NPN BJT.

- Build the amplifier circuit shown in Figure 10, choosing appropriate values for resistors and capacitors, using the same constraints as for the follower circuit above, and choosing a DC gain of 2. For the time being, omit the emitter capacitor.
- What is the voltage at the collector?
- Input a 10 kHz sine wave with 100mV amplitude. What is the relationship between the input and output voltages? If you did not achieve a gain of 2, why not?

- Predict first, and then measure, the maximum sine amplitude for which this circuit correctly operates.
- Now add in the emitter capacitor to select for a gain of 10 at 10kHz. Again, mind your R's and C's!
- Input a 10 kHz sine wave with 100mV amplitude. What is the relationship between the input and output voltages? If you did not achieve a gain of 10, why not?
- Predict first, and then measure, the gain at 20 kHz.
- Can you think of a way to make this circuit have the same gain across our whole band of interest?
- Finally, attach a passive speaker to the output, your passive FM demodulator to the input, and bask in the sound of your success. You'll need to pay attention to the impedance of the speaker, and plan accordingly.

3 Week 3

Prerequisites

- Central Limit Theorem
- Johnson-Nyquist Noise
- Receiver Temperature
- Radiometer Equation

Materials

- 2W 50Ω resistor
- power supply
- oscilloscope w/ probes
- minicircuits amplifiers
- voltimeter
- infrared (remote) thermometer

Some Thoughts

Minicircuits Amplifiers

Now that you've built your own amplifier, you have a feel for what goes on inside an amplifier. We're now going to use pre-packaged amplifiers that are commercially available (Minicircuits is a typical vendor). These will be SMA in, SMA out, with two leads for V_{cc} (+15V, for the parts we're using) and ground. These amplifiers specify their gain in dB, and you do have to be a bit careful about signal levels. As a general rule, the output of amplifiers should be connected (to a 50Ω load) before they are powered on, so that they don't have to dissipate their output power internally.

Infrared Thermometer

If you assume that objects radiate heat as a blackbodies (which we'll discuss in later labs), you can measure their temperature remotely just by their infrared radiation. The infrared thermometer we will use in this lab is an infrared telescope (with a well-defined beam size) that is calibrated to output a voltage in mV that is the temperature in Fahrenheit. Use it wisely.

Interference

In this lab, we're going to attempt to make a relatively sensitive measurement of the Johnson noise on a 50Ω resistor. It turns out that lots of things can produce signals at this level that can interfere with your measurement. For one, if you make a loop that electrons can flow around (which, after all, is what a circuit is), then a changing magnetic flux through that loop produces an EMF that drives a current that sets up a voltage across the resistor. This may be the very first radio antenna we build in this class. Unfortunately, that's not what we were going for, so we need ways to avoid this. Since the area inside the loop matters, you can try to make your circuits physically narrow. You might also try making a Faraday cage.

Writing Code for Lab

This is the first lab where a significant portion of the lab is writing code. We are going to keep everyone's code in a class Git repository. So the first thing you should do set up a github account and fork the class repo: `https://github.com/AaronParsons/astro250.git` . Next, clone your repo onto your computer. (If you haven't yet, you should get Git, Python, Numpy, and Pylab installed on your computer). Inside astro250, add a directory with your name, and create two files: central_limit.py and radiometer_test.py. Commit, and push your changes back up to your github account. I'll pull your fork of astro250 into my repo, along with everyone else's work. From time to time, you should pull from my repo (you can add a shorthand for it to your working copy on your computer with *git remote add ...*) so you can see what everyone else is doing, too.

3.1 Calculating the Noise Figure of a Reciever

Determine Amount of Gain Needed

- Calculate how much amplification is needed to bring the Johnson noise on a 50Ω resistor up to at least the ∼1mV RMS level that you can measure on a scope. You'll have to pick an appropriate bandwidth, and you might want to check that you can get a filter that supplies that bandwidth.

- Looking at the data sheets for the minicircuits amplifiers, choose a number of gain stages.

- Connect the gain stages (and the filter at the end), and power them at +15V. IMPORTANT: Most amplifiers want to have the output connected first, so that the output power has some place to go.

Estimate and Measure the Resistor Temperature

$$P = A\epsilon\sigma T^4 \tag{7}$$

- Set a power supply to the appropriate voltage to dissipate $\sim$2W in a 50Ω resistor (make sure it is a beefy resistor rated to 2W, and do not exceed that threshold, or meltdown will ensue).
- Using the Stefan-Boltzmann Law, calculate what temperature you think the resistor will hit.
- Use an infrared thermometer to measure the temperature. Were you close?

Measure the Noise Figure

- Determine the power out of the receiver for the input resistor at room temperature. Ideally, you'd do this with a power meter or a spectrum analyzer, but today, we'll do the poor man's method of eyeballing one standard deviation from the mean on an oscilloscope. This isn't easy, and I apologize for that. To keep your sanity, you should estimate errorbars on this measurement, so you can see the effect of error on your bottom line.
- As in the previous step, set the resistor dissipating 2W, and measure the power out of the receiver. IMPORTANT: Use a blocking cap to prevent a large DC voltage from entering the amplifier chain. Otherwise, you could damage your amplifiers. When in doubt, ask.
- Since you already know the temperature the resistor hits when dissipating 2W, you should be able to solve for the gain and the receiver temperature (and if you want, you can calculate your Y factor). This would be a good time to use your error bars to determine your confidence interval on your measurement of the gain and receiver temperature. Make a plot and draw some lines!
- Convert your receiver temperature to a noise figure.
- How much is your error on the reciever temperature reduced if you have a prior on the gain (which you should look up from the data sheet)?
- How does your measured receiver temperature compare to the noise temperature you'd expect given your chain of amplifiers?

3.2 Demonstrate the Central Limit Theorem

- Write a program (in the central_limit.py file that you created in the astro250 repo under your name) that shows that, in the large-N limit, adding samples drawn from non-Gaussian random distributions converges to a Gaussian distribution.
- Also show that the standard deviation of the mean of N Gaussian-random samples decreases as $\sqrt{N}$. If you are trying to characterize samples of an unknown character to show that they are noise-like, this is called an Allen variance test.

3.3 Numerical Demonstration of the Radiometer Equation

- Write a program (in the radiometer_test.py file in the astro250 repo under your name) that simulates random noise corresponding to a noise temperature, and shows that the radiometer equation works. In particular, prove (numerically) that it is, in fact, $\sqrt{Bt}$ and not $\sqrt{2BT}$ in the denominator of the radiometer equation.