LAB 4: Mapping the Galactic HI Line

Contents

1. Introduction

Interstellar space is filled with diffuse atomic gas, displaying structures that vary greatly in size—from tiny self-gravitating clouds to galactic-scale spiral arms. The morphology of gas in the Milky Way is influenced by shocks from supernovae, by magnetic fields, pressure gradients, and thermal instabilities driven by stellar winds and starlight, and by line cooling in dense, shielded molecular clouds where new stars form. The densest regions contain molecular hydrogen (H_2), and ionized plasma surrounds recent supernova remnants, but almost everywhere else in the galaxy, atomic hydrogen (HI) is prevalent, making the 21-cm line a rich resource for learning about galactic dynamics, star formation, and the evolution of our galaxy as a whole.

In this lab, using the 4.5-m Leuschner dish, you'll embark on projects to map HI structures within our galaxy. The projects are divided into two main categories:
- Large-scale galactic structure and dynamics: These involve obtaining HI spectra along great circles, presenting data as position-velocity images, and analyzing the maps to understand Galactic structure.
- Specific interstellar structures: These involve mapping areas like supernova-produced shells by sampling a grid of spectra, presenting data either as grey-scale images showing velocity-integrated line intensity or as images where brightness indicates integrated intensity and color indicates velocity.

Each group is expected to select one project for completion, focusing on mapping and image processing techniques, including the use of color in images.

For great-circle projects, extensive coverage of great circles is essential for comprehensive data, although geographical limitations may restrict complete coverage. Aim for as broad a coverage as possible to achieve meaningful results.

2. Goals and Instructions for Your Report

The goals of this lab consist of some collection of the following, depending on the project chosen:
- Perform a large-scale observing survey project balancing considerations of telescope time, integration time, sky coverage, data reduction, report writing, and personal time commitment versus research goals and quality of results and their presentation.
- Use the collected data to infer astrophysical phenomenae or quantities such as the properties of supernova-driven interstellar supershells and the Galaxy.
- Present data and/or using greyscale and/or color images.
- Learn about using colors in computer graphics including plots and images.
- Learn about problems in using color for presentations, specifically contrast issues and color-blindness.

3. Schedule

The end of the semester approaches. Here's our schedule.
1. Week 1: Pick a group project. Figure out the RA and Dec boundaries of your field, how you want to sample the area with telescope pointings, and when each pointing is up. Coordinate with other groups to share telescope time. Devise a scheme for scheduling observations of all

the points in your map. Begin the process of obtaining and reducing your map data.

Obtain frequency-switched test spectra, in particular at our canonical comparison position at Galactic coordinates $(\ell, b) = (120°, 0°)$. We will compare the spectra of all groups in class the second week, and if your group doesn't have a spectrum to show this will be considered poor form.

2. Week 2: Get as much data as you can. Reduce the data. Decide what projection you will use to present your maps; create the map plane and populate it to the extent possible with the data you've obtained. You won't have all of your data, but begin the map-making process anyway (§15); this process begins with making a data cube.

3. Week 3: Get the rest of your data. Figure out how you want to present the data as images. Make some first-try images.

4. Week 4: Finalize data. Make final images. Write your report.

4. Software for Controlling the Leuschner Telescope

At this point in the semester, we assume you are familiar with the software used in this class. The major difference for this lab is that, instead of connecting to the `ugastro` subnet on campus, we must log in to a computer at the remote Leuschner observatory and use `screen` to create a persistent session that you can use to conduct your observations, even if your `ssh` connection gets broken.

4.1. Connecting to the Pi at Leuschner

The gateway computer at Leuschner can be accessed with `ssh`:

```
ssh radiolab@leuschner.berkeley.edu
```

Use the standard class password.

From there, you can access the Raspberry Pi connected to the SNAP Spectrometer via:

```
$ ssh pi@192.168.1.154
```

This computer (and password) is identical to the one you used to control the interferometer.

If you, like me, get tired of manually multi-hopping through computers to get to the one you want, you can streamline the process by editing `.ssh/config` with something like the following:

```
host leuschner
HostName leuschner.berkeley.edu
User radiolab
ServerAliveInterval 120

host leuschpi
HostName 192.168.1.154
User pi
ProxyJump leuschner
ServerAliveInterval 120
```

Then you can ssh (and scp) to `leuschpi` directly. You'll have to enter multiple passwords (unless you set up ssh keys), but otherwise you can act like you are talking directly to the Pi and ignore the intermediary computer.

4.2. Using `screen` at Leuschner

For observing sessions that run automatically for long periods of time, we will use `screen` to serve up a remote desktop. `screen` is a command that allows us to maintain a persistent session on a remote machine even when everyone is logged out. If everyone uses the same `screen` session, we avoid having more than one group observing at a time. We have set up a `screen` window on the Pi at Leuschner.

To attach to this window, run

    screen -r

("r" for "reattach") on heiles. This connects you to the existing session and command-line environment. Please remember to do this as soon as you log in! It can be easy to forget to go into `screen` before running your scripts as you usually would.

While you're in a screen session, all `screen` commands begin with `Ctrl-a`. To detach from the window and return to your login environment, run

    Ctrl-a d

Before you log out of heiles, it's best to first detach the screen window before exiting – otherwise, you may close the entire screen session and accidentally abort running programs.

Lastly, to check whether you are in `screen` or not, run

    screen -ls

This will display any session that exist and tell you whether you are "attached" or "detached" to any of them!

Those commands should be sufficient for this lab, but `screen` is a very useful tool in general. If you're working on a remote machine and would like to run long programs without staying logged in, you can just type in `screen` and it will start the program. You can also create a new window within the `screen` session by running `Ctrl-a c` (for "create"). Once there are multiple windows, you can do `Ctrl-a n` (for "next") to cycle between them. Finally, if you want to close a window, you can do `Ctrl-a k` (for "kill"). And as always, to bring up the full documentation, just do `man screen` at the command line.

4.3. Python Procedures for Controlling the Dish

The `ugradio.leusch` module provides an interface for controlling the Leuschner dish. In particular, the `LeuschTelescope` class should look familiar to you; it is basically the same as the `Interferometer` class you used in the previous lab.

1. `LeuschTelescope.get_pointing()` returns the current (alt,az) of the dish, in degrees.
2. `LeuschTelescop.point(alt,az)` moves the telescope to a specified alt, az in degrees.
3. `LeuschTelescope.maint()` moves the telescope to maintenance position.

4. `LeuschTelescope.stow()` moves the telescope to stow position.

If you find you need the coordinates of the Leuschner Educational Observatory, they are stored in `ugradio.leo`.

The Leuschner dish has a noise diode at the front that you can turn on/off for calibration purposes. It is slightly polarized, so it increments polarization 0 by 79 K and polarization 1 by 58 K.

1. `ugradio.leusch.LeuschNoise()` instantiates an interface to the noise diode (controlled by a Raspberry Pi).
2. `LeuschNoise.on()` turns the noise source on.
3. `LeuschNoise.off()` turns the noise source off.

4.4. Collecting Autocorrelation Spectra

4.4.1. Option 1 (preferred): Using SDRs

Because we are using the Leuschner dish to observe the 21-cm line, we don't need much spectral bandwidth, which means we can use the same SDRs you used in Lab 2. We have plugged two separate SDRs into the same Pi, so if you would like to observe with both feed polarizations, you can build two separate SDR interfaces. Use the `device_index` keyword to select which SDR to target:

```
>>> sdr0 = SDR(device_index=0)
>>> sdr1 = SDR(device_index=1)
```

Recording raw time samples will quickly use up a lot of disk space, so we suggest that, after setting an appropriate `gain`, `center_freq` and `sample_rate`, you compute and record averaged power spectra. Choose an appropriate number of samples to FFT to give yourself plenty of frequency resolution for fitting Gaussian profiles to your HI spectra, but not so much that you waste disk space/sensitivity. Remember that each SDR must be configured separately, and that they are running off of separate oscillators, so they will be slightly out of synchronization. This is okay, because we are not cross-correlating them.

4.4.2. Option 2: Using the SNAP

A second option is to use the same SNAP board we used in Lab 3, but as a spectrometer. This spectrometer is the same FPGA design as the interferometer correlator, but we will initialize it with `mode='spec'` to deliver us two auto-correlations instead of a cross-correlation. First, though, we need wo mix the 21-cm line down to be sampled by the SNAP. We have provided an LO you can control remotely with *ugradio.agilent*.

1. `ugradio.agilent.SynthClient()` instantiates an interface to the LO
2. `SynthClient.set_frequency` changes the LO frequency.

The output of this LO goes through a 30-MHz bandpass filter centered at 327 MHz, so set your LO frequency accordingly.

Next, we need configure the spectrometer to operate with a slower sample clock, so that we can spread our 1024 channels over a narrow bandwidth to get better (but still probably too coarse) frequency resolution. This is done as follows: `mode='spec'` to deliver us two auto-correlations

instead of the cross-correlation, e.g.:

```
>>> import snap_spec
>>> spec = snap_spec.snap.UGRadioSnap(is_discover=True, acc_len=6000)
>>> target_sample_rate = 77 # MHz
>>> spec.initialize(mode='spec', sample_rate=target_sample_rate)
>>> actual_sample_rate = spec.adc.lmx.getFreq()
```

You will see, running the commands above, that the actual sample rate is not quite equal to the target sample rate. Make sure to use the actual one in you frequency calculations. Note that, for a ~ 77 MHz sample rate, we will be sampling a 312–342 MHz band in the *9th Nyquist Zone*. We are aliasing our signal on purpose, and you will need to use the actual sample rate to accurately calculate the edges of the Nyquist window you are operating in.

Above, w also lowered the accumuation length in the SNAP spectrometer with `acc_len` so that, with samples coming less frequently relative to the 500-MHz sampling we used in Lab 3, we aren't waiting for too long for new accumulations to show up. After initialization, the only difference from the code you used in Lab 3 will be that `spec.read_data()` returns a dictionary with different keys, and your IF frequencies will correspond to different RF frequencies, depending on how you set your LO.

5. Mapping: How to Spatially Sample

All of the HI projects examine the angular dependence of the 21-cm line profile, so you need closely spaced observations to make a map or an image. How closely should the observations be spaced? The usual criterion is two samples per FWHM (Full Width, Half Max) of the beam, because observing at more closely spaced angles just gives redundant information and takes extra time, while more widely spaced angles loses small-scale structural information. (This is a re-statement of the Nyquist criterion for sampling sky structure versus angle). The FWHM is almost $4°$; so a spacing of $2°$ is about right.

For the mapping projects (not the great-circle ones), we need to cover areas of 1000 to 2000 square degrees. Consider the Orion-Eridanus Superbubble (§10 below), which aims to map a region covering roughly $(\ell = 160° \to 220°, b = -70° \to -10°)$. Suppose you set up a regular grid in (ℓ, b) with $(\Delta\ell, \Delta b)$ both equal to $2°$. That's fine for Δb, but you'd be sampling too closely in ℓ because the true (great-circle) angular distance between two points is $\Delta\ell \cdot \cos(b)$; there's a foreshortening for $b \neq 0°$ (e.g.: when you're at the Earth's North Pole, you can go completely around a small circle of constant b with just a few steps.). So you can save lots of observing time by observing at $\Delta\ell = \frac{2°}{\cos(b)}$ instead of $\Delta\ell = 2°$.

The true angular area is not the total extent in ℓ times the total extent in b because of the foreshortening discussed in the above paragraph. So the actual angular area of the Orion-Eridanus map isn't the range in ℓ times the range in b, i.e. $60° \times 60° = 3600$ square degrees (which would require 900 pointings), but rather only about 2800 square degrees. At $2°$ sampling, that's about 700 pointings.

These projects require a lot of pointings, but fortunately, the telescope is automated. With a computer-controlled telescope you can do these observations in one or two days, but that requires planning and coordination with other groups.

5.1. Observing Procedure

For taking the data at a particular position, we recommend thinking through the procedure for each pointing:

1. Select the (ra, dec) of your position. You can use `astropy` if you wish.
2. Decide the number of spectra to obtain, based on the desired integration time.
3. Plan how often to take calibration data (with the noise diode on) and any LO frequency shifts you want to use to separate the astrophysical signal from the instrumental response.
4. Choose a filename and metadata format that encodes enough useful information (JD, pointing, LO, noise diode, etc.) that you can find what you need after making many observations.
5. Use `telescope.point(alt,az)` to point (and track, if necessary).
6. Use `sdr.capture_data(...)` or `spec.read_data(...)` to take and record data to a file for each pointing/observing mode, along with all of the appropriate metadata for this position. Make sure you can easily reconstruct which pointing this file corresponds to, along with all of the relevant LO/noise settings you used.

5.2. Beware Operating near Azimuth 0/360

If the azimuth of the current position is $6°$ and of the next one is $354°$, the telescope has to rotate nearly $360°$ in azimuth, which takes a long time. You don't want to make this transition more than once (or at all, if you can manage it).

5.3. Beware Operating Above Altitude 85 and Below/Near 14

The telescope cannot point above altitude $85°$ or below $14°$. When you are at low altitude, you might be seeing the ground.

6. Project: The Galactic Plane ($b = 0°$)

Point the telescope at a series of Galactic longitudes (ℓ) along the Galactic plane (Galactic latitude $b = 0°$), over the full longitude range that is observable from Leuschner. Getting nice results requires data over the largest section of the Galactic plane you can get, which is roughly (or somewhat less than) $\sim -10° \lesssim \ell \lesssim 250°$. This observation has two science goals:

1. *The Galactic rotation curve.* Determine the rotation curve of the Galaxy for the portion of the Galaxy inside the Sun and use the result to estimate the *gravitational* mass M_{grav} of the Galaxy that lies inside the Solar circle; for this, you just assume the usual $v^2 = \frac{GM_{grav}}{R}$. In fact, you can even get the radial dependence $M_{grav}(R)$. Also, use your observations to estimate the *gaseous* mass $M_{gas}(R)$ of the Galaxy. What fraction of the total Galactic mass comes from interstellar gas? For this you'll need to know the radius of the Solar circle: it's 8.5 kpc. See *Commentary* below for a discussion.
2. *The Galactic center.* Our Galactic center, like others, has a big black hole, with infalling matter and jets and lots of anomalous velocities. You can see a lot of this activity in your map. Using the spatial extents and the velocities, you can estimate a black hole mass. To get an accurate estimate, you need to look at motions of matter lying really close to the black

hole, which you can't do because you don't have the angular resolution. But you can at least make an estimate.

3. *Spiral Structure of the Galaxy.* All your life, you've been told that we live in a spiral galaxy. Are they lying? In principle, to detect spiral structure you make a position-velocity plot along the Galactic plane [that's the image $T_A(\ell, V_{LSR})$ for $b = 0°$]. Bright regions are regions of excess density—or regions in which lots of HI is packed into a narrow velocity range (see §14.1 below).

Do you see spirals in your position-velocity plot? How do you tell? There's only one easy way. Make a model of a spiral arm and see how it projects onto position-velocity space. To do this, you need to know the rotation curve. You've measured it for the inner Galaxy; for the outer Galaxy (outside the Solar circle) assume that the rotation curve is constant beyond the solar circle [$V(R)$=220 km/s for $R > 8.5$ kpc]. In polar coordinates, the equation for a spiral arm is

$$R_{arm} = R_0 e^{\kappa(\phi - \phi_0)} \tag{1}$$

where R_0, κ, and ϕ_0 are free parameters; κ is the tangent of the *pitch angle.* You project this into position-velocity space using equation 8 and your knowledge of the Galactic geometry.

Comment: Finding spiral arms inside the Solar circle is very difficult, and has thwarted efforts by astronomers over the past several decades. Finding them in the outer Galaxy is not as difficult.

7. Project: Is the Galactic Plane Flat?

External galaxies are often distorted (like the brim of a Fedora). These distortions are probably produced primarily by nearby neighbors. How about our own Galaxy? To determine this we want to look at the vertical structure of the Galactic plane and see where the peaks occur; a flat Galaxy would everywhere have the peaks at $b = 0°$.

The most comprehensive way to observe and display these distortions is with a map that covers the biggest possible swath in Galactic longitude ℓ for the range of Galactic latitude (b) from $-20°$ to $+20°$. This is a huge area—almost 10^4 square degrees. At $2°$ sampling, you'd need ~ 2500 profiles. To make the project more manageable, you can increase the increment in ℓ, say to 4 degrees, while keeping the same, fully-sampled increment in b. Resolution is l is less important because—like a fedora—the warp doesn't change rapidly with l.

This project affords a great opportunity to make a spectacular color image in (ℓ, b) coordinates: brightness showing the amount of gas, color the Galactic rotation. Officially, the science goals here are to estimate the thicknesses of the Galactic disk and, also, to obtain an approximate Fourier representation of the warp—which is not so straightforward given the absence of data in the "Southern" Galaxy.

8. Project: The Galactic Poles—Great Circles at Constant Longitude ℓ

How is the interstellar gas distributed within the Galaxy—for example, is it "disk-like" or more spherically distributed? Are there any systematic motions other than just "Galactic rotation"?

(You'd be surprised). To answer these questions, determine the column density and mean velocity of interstellar gas as a function of Galactic latitude b.

To answer this completely you'd need to do an all-sky survey. That's a big job. Let's do something easier. We have one project (§6) that looks around a great circle, the Galactic equator (constant b). Other projects could check out one or more orthogonal great circles (at constant longitudes ℓ) going from one Galactic pole to the other and back again (or, if not the whole 360 degree circle, as much as possible). Particularly appropriate sets of constant-longitude great circles are the pairs $\ell = (220°, 40°)$ (partly because you can get the whole great circle) and $\ell = (130°, 310°)$ (because at positive latitudes there's weak high-velocity gas).

The science goals here are to characterize the thickness of the gas layer and to determine the vertical kinematics of the gas, and if possible an estimate of the energy involved in the vertical motions.

9.   Project: Mapping the North Celestial Pole

The region near the North Celestial pole contains a large shell, probably produced by one or more supernovae, and also to has angular scales that are well-matched to our telescope and the available time for your project. For this, you'd map the region covering roughly $(\ell = 105°$ to $160°), (b = 15°$ to $50°)$. Interesting features should produce antenna temperatures ~ 10 K. This is about 1600 square degrees and, with 2 degree spacing, requires about 400 profiles. With our telescope, this is probably the most spectacular and contrasty object we can map in the sky.

10.   Project: Mapping the Orion-Eridanus Superbubble

The "Orion-Eridanus Superbubble" was, and continues to be, produced by energetic stellar winds and supernovae that were located in immediate vicinity of the Orion nebula. For this, you'd map the region covering roughly $(\ell = 160° \rightarrow 220°, b = -70° \rightarrow -10°)$. Interesting features should produce antenna temperatures ~ 20 K. This is almost 2800 square degrees and, with 2 degree spacing, requires almost 700 profiles. See §5.

The goal here is to map the HI in the 3-d space of (l, b, v) (that's Galactic longitude, latitude, and velocity) and then present the results in one or more color images to show the hollowed-out shell with the swept-up gas piled up at the edges.

11.   Project: Mapping the North Polar Spur's Expanding HI Shell

This is a huge shell produced (and continuing) by several energetic stellar winds and supernovae that were located in the Sco/Oph association of stars. The shell contains not only HI but also relativistic electrons, so the shell is visible in both the 21-cm line and synchrotron emission. We can't easily see the latter, but we can see the HI—and the pattern of its velocity follows that of an expanding shell.

For this, you'd map the region covering roughly $(\ell = 210° \rightarrow 20°, b = 0° \rightarrow 90°)$. (The longitude range is 170°: $210° \rightarrow 360°$ plus $0° \rightarrow 20°$). This is roughly 1/4 of the whole sky, so this is a huge area—almost 10000 square degrees. (This shell has an angular diameter of 120°). But (unfortunately) you can't measure the equivalent 2000 HI profiles because most of this area is too

far south. On the plus side, this makes the project do-able. In doing your observations, you need to cover as far south as you possibly can. See §5.

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then present the results in one or more color images to show the hollowed-out shell with the expanding swept-up gas piled up at the edges.

12. Project: Mapping a Big High-Velocity Cloud

When we look up, away from the Galactic plane, we see infalling gas—some a very high velocities. It's called "High-Velocity Gas". The line is weak (about 1 to 1.5 K), so you need high sensitivity— you need to use much longer integration times than for the above projects, at least a few minutes per point. This project needs to map the region bounded roughly by $(\ell = 60° \rightarrow 180°), b = 20° \rightarrow 60°$). This is about 3700 square degrees and needs about 900 profiles at 2° spacing. See §5.

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then present the results in one or more color images.

13. Project: Mapping the Magellanic Stream

The Magellanic Stream is an intergalactic tidal stream produced by gravitational interaction between the Magellanic Clouds and the Milky Way. The signal is weak ($\sim$ a few tenths K), so you need to use much longer integration times than for the above projects—a minimum of 10 minutes per point. You need to map the pie-shaped area bounded roughly by $(\ell = 60° \rightarrow 110°), b = -90° \rightarrow -30°$). This is about 1250 square degrees and needs about 310 profiles at 2° spacing. See §5. Also, the LSR velocity of this gas ranges from roughly -400 to -100 km/s along its length, so you need frequency switch by a large enough interval.

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then present the results in one or more color images.

14. Basics of the 21-CM Line

The 21-cm line, with frequency 1420.405751786 MHz, comes from atomic hydrogen (HI). In the terrestrial environment, H atoms quickly become H_2 molecules; in interstellar space, where densities are far lower than in the best vacuum systems on Earth and there are "lots" of UV photons that dissociate H_2, H remains atomic unless it resides inside dark clouds where it is shielded from starlight.

14.1. Column Density and Mass

The intensity of the 21-cm line is directly proportional to the column density of H atoms as long as the opacity of the line is small; this is a reasonably good approximation (but not perfect, particularly in the Galactic plane where the line is strong). With this approximation,

$$N_{HI} = 1.8 \times 10^{18} \int T_B(v) dv \text{ cm}^{-2}. \tag{2}$$

Here N_{HI} is the column density of H atoms—the number of atoms in a 1 cm^2 column along the line of sight; $T_B(v)$ is the brightness temperature of the 21-cm line, which is a function of velocity v; and v is the velocity in km s^{-1}. The velocity v is produced by the Doppler effect, so a frequency shift $\Delta\nu$ from line center corresponds to velocity $v = -c\frac{\Delta\nu}{\nu}$ or, for the 21-cm line, $v = -\frac{\Delta\nu}{4.73\text{kHz}}$ km/s. Note the minus sign. It means that positive velocities are receding—a very convenient convention for astronomy, because of the expansion of the Universe.

Note an important corollary to equation 2: If we consider a small velocity interval Δv, then the number of H atoms in the column in that velocity range is just

$$N_{HI}(v \rightarrow v + \Delta v) = 1.8 \times 10^{18} T_B(v)\Delta v \text{ cm}^{-2} . \tag{3}$$

This is very important, because it gives us the opportunity to make maps of the interstellar gas at different velocities and to determine how it moves.

To get the column density we need to get the brightness temperature T_B, which is not the same as our antenna temperature T_A. The relationship between these depends on the relative size of the source and the beam. This is equation (9) in the "Fount of All Knowledge!" handout, namely

$$T_A \approx T_B\frac{\Omega_s}{\Omega_s + \Omega_b} , \tag{4}$$

where Ω_s and Ω_b are the solid angles of the source and beam, respectively. From this, we see that if the source fills the beam ($\Omega_s \gtrsim \Omega_b$) then $T_A \approx \langle T_B \rangle$ (the brackets denote the average over solid angle); while if it is much smaller ($\Omega_s \ll \Omega_b$) then $T_A \approx \frac{\langle T_B\Omega_s \rangle}{\Omega_b}$, i.e. T_A is smaller by the ratio of the solid angles.

To get the mass of HI from the column density, you need to know the area of the region in *linear* measure, which is just $\Omega\, d^2$, where d is the distance. Each HI atom has mass m_H, so the total mass of HI in a region of size Ω is just

$$M_{HI} = m_H\, d^2\, \langle N_{HI}\Omega \rangle . \tag{5a}$$

Here $\langle N_{HI}\Omega \rangle$ is shorthand for the general relation, which uses an integral:

$$\langle N_{HI}\Omega \rangle = \int_{\text{region}} N_{HI}d\Omega \tag{5b}$$

Substituting for N_{HI} from equation 3, we have

$$M_{HI}(v) = 1.8 \times 10^{18}\Delta v\, d^2\, m_H\, \langle T_B\Omega \rangle \text{ gm} . \tag{6}$$

Units are cgs. Sometimes there is a well-defined source with boundaries; in this case, to get the mass of the whole source, we use the solid angle occupied by the source, Ω_s.

Sometimes (in fact, often) we want to know the mass seen *by the telescope* for some particular observation. For an extended region with $\Omega \gg \Omega_b$, $T_A = T_B$ and the telescope sees the solid angle Ω_b, so the product $\langle T_B\Omega \rangle = T_A\Omega_b$ because angular area seen by the telescope is limited by its beam size. In contrast, if the HI emission is limited to a small region—a small source with $\Omega_s \ll \Omega_b$—then $T_A \approx \frac{\langle T_B\Omega_b \rangle}{\Omega_s}$ so the product $\langle T_B\Omega_s \rangle = T_A\Omega_b$; the telescope sees the whole source.

For both extremes, the product $T_B\Omega = T_A\Omega_b$. It's not only these two extremes, but for *all* sources the product $\langle T_B\Omega \rangle = T_A\Omega_b$. Thus, it is always true that the mass seen by the telescope is

$$M_{HI}(v) = 1.8 \times 10^{18} \Delta v \ d^2 \ m_H \ T_A \Omega_b \ \text{gm} \ . \tag{7}$$

The distance d is usually a function of velocity v, especially in the Galactic plane.

$T_A(v)$ is directly measured and Ω_b is the telescope beam area, which is known, so it's easy to calculate $M_{HI}(v)$—but only if you know the distance. Often you don't know the distance, but you might have a hunch for a reasonable value for the distance. Suppose this is 100 parsecs. In this case, people usually evaluate the mass for this distance and, when giving the mass, say "The mass is $xxx \times d_{100}^2$, where d_{100} is the distance in units of 100 pc".

14.2. Converting Galactic Rotation to Doppler Velocity

Measurement of the Doppler shift caused by differential Galactic rotation for $0° < \ell < 90°$ allows a direct determination of the Galactic rotation curve inside the Solar circle, using the "tangent point" method. See Burton's article "Structure of our Galaxy from Observations of HI" in *Galactic and Extragalactic Radio Astronomy, second edition* (editors: G.L. Verschuur and K.I. Kellermann). In essence, you use the following equation.

We won't go through the derivation here; we cover it in class, and it is also done in Burton's article, page 303-304. Let V_{Dopp} be the Doppler velocity (this is also the observed LSR velocity), $V(R)$ be the Galactic rotation velocity at Galactocentric distance R, and let the subscript $\odot$ denote values at the Solar circle. Then we have

$$V_{Dopp} = \left[\frac{V(R)}{R} - \frac{V(R_\odot)}{R_\odot} \right] R_\odot \sin(\ell) \tag{8}$$

The values at the solar circle are $V(R_\odot) \approx 220$ km/s, $R_\odot \approx 8.5$ kpc.

15. Displaying Your Data

Before actually working with images on the computer, you may need to familiarize yourself with our imaging handouts:

1. Map Projections: Representing a Spherical Surface on a Flat Screen
2. `jupyter_tutorials/lab4` has examples of making images, colorbars, and map projections.

15.1. Map Projections: Representing a Spherical Surface

Computer screens consist of an array of dots that can glow in various colors. The dots are arranged in a square array and are called *pixels*. Images are the same, with the added feature that the pixels need not be square; the horizontal distance can differ from the vertical one. To display our data as an image on the screen, we must populate these pixels with observed data.

Our observed positions do not lie on this grid. Rather, our observations lie on the celestial sphere. We need to represent this spherical surface on our 2d pixellated computer screen—or as a 2d image on a flat piece of paper. There are innumerable ways to do this *projection*, each of which has its own desirable properties. No projection is perfect; each is a compromise among various desirable and undesirable properties.

Here are two examples:

1. The simplest is the *cylindrical equidistant projection*, for which the entire sky is represented by longitude horizontally and latitude vertically with square pixels each having the same size in $(\Delta\ell, \Delta b)$. Near the equator this is a very nice projection because it has many desirable qualities: it is nearly conformal, the pixels are nearly of equal solid angle. But as you move away from the equator, these properties decay with increasing severity because of foreshortening.

2. Suppose we want to be as conformal as possible, particularly with respect to representing small circles (supernova remnants or supershell walls). This is the *stereographic projection*, which is centered on its own defined pole and for which the conversion between its $(long, lat)$ and the pixel values (x, y) is

$$R = \tan[0.5 \times (90. - lat)] \tag{9a}$$

$$x = R \ \cos(lon) \tag{9b}$$

$$y = R \ \sin(lon) \tag{9c}$$

For other options, we refer you to the `cartopy` package.

15.2. Regridding

How do we populate this (x, y) grid with our data? This process is called *regridding*. There are several ways to regrid; we describe the one that loses least information (but requires the most computer resources—our philosophy is 'computers are cheap').

To regrid proceed as follows:

1. Set the angular (x, y) pixel size. The size must be smaller than the smallest great-circle distance between observed points. For example, if the separation between observed points is 2 degrees, a pixel size of 1 degree (or 0.1 degree) is OK. For the cylindrical equidistant projection with a pixel size of 1 degree there are 360 horizontal pixels and 180 vertical ones, for a total of 64800 pixels.

2. Create an empty 3-d data cube of the appropriate dimensions. The first two dimensions are (x, y) and the third is velocity. If you have 256 velocity channels, the above cylindrical equidistant cube would have dimensions $360 \times 180 \times 256$. The cells in this cube are 3-d and are called *voxels*.

3. For each pixel, find the nearest observed position. For the angles on the sky, a suitable approximation for the distance squared is $[(\cos(b)\Delta\ell)^2 + \Delta b^2]$. Populate that pixel with the nearest observed data value. For the velocity, use linear interpolation. (`scipy.interpolate` has many convenient options for this purpose).

15.3. Displaying Your Data—The Exploratory Phase

For live presentations, you may want to display your data cube in its full 3d glory with a 2d image where brightness represents 'amount' (e.g., the integrated area of the HI line) and color the mean velocity. Or by describing each pixel by 3 numbers (e.g., the integrated intensity within 3 different velocity ranges) and writing them on the same image plane as the 3 independent colors red, green, blue.

However, making effective use of color requires defining what color means. Set up an `matplotlib` imaging window and look at successive (x, y) scalar images at different velocities.

That is, make a 'movie'. For our above example $360 \times 180 \times 256$ data cube, a quick and dirty way is:

```
plt.ion() # makes matplotlib interactive, so you can replot data in the same figure
fig = plt.imshow(data[...,0], vmax=max, vmin=min, cmap=cmap)
plt.draw() # not necessary if run within IPython
for i in xrange(1, data.shape[-1]):
    time.sleep(1) # change data every 1 second
    fig.set_data(data[...,i])
    plt.draw()
```

Here, colors are chosen from the `cmap` (there are bazillions of options). The keywords (`vmin`, `vmax`) mean a data value of `vmin` gives the minimum color and `vmax` gives the maximum.