EXPLORING DIGITAL SAMPLING, FOURIER TRANSFORMS, and the DSB MIXER

Contents

The purpose of this lab is to experimentally investigate digital sampling, digital Fourier transforms, and mixers. Mixers are the basis of heterodyne spectroscopy. Heterodyne spectroscopy, in turn, is what you use every day that you listen to a radio, use a cell phone, or watch TV—or do radio astronomy. You will be performing several experiments, analyzing the data and generating a number of different data files. You will need to keep careful notes in your **lab notebook**! Or, pay the penalty, and forget what you did, do things twice, and be completely disorganized. Your choice!

1. GOALS

- Learn how to sample electronic signals—here, one or more sine waves—digitally using our computers.

- Become acquainted with the basic law of sampling: the Nyquist criterion.

- Learn how to use Digital Fourier Transforms (or Discrete Fourier Transform; DFT) to determine the frequency spectrum of a signal.

- Learn about the Fast Fourier Transform (FFT) as a particular, and particularly fast, implementation of the DFT.

- Learn the basics of mixing for frequency conversion (that's the *heterodyne* technique) and for measuring phase.

- Get started with our programming language, IDL, using it for the mathematical analysis, signal processing, and making nice plots.

- Learn enough Latex to write up your results in a formal lab report, including nice plots and graphs.

2. SCHEDULE

There's a lot to do in this lab! If you don't understand the Nyquist criterion by the end of the first week, you're behind. Here's how it should be:

1. *The First Week, 19 Jan to 25 Jan:* Finish §3, §4, §5 below. Be prepared to show your results to the class, making real-time plots in IDL.

2. *The Second Week, 26 Jan to 1 Feb:* Finish §6 and §7 below. Again, be prepared to show your results to the class.

3. *The Third Week, 2 Feb to 8 Feb:* Write your formal report! It should follow the standard format, consisting of an introduction, discussion of experimental activities and results, description of the analysis technique, presentation of analysis results, and discussion/interpretation. With all this, you should hand in a reasonable number of plots together with commentary to illustrate your work, your thought processes, and your conclusions.

3. NYQUIST SAMPLING AND ALIASING: A SINGLE SINE WAVE (First Week)

Here we explore the all-important realms of the Nyquist criterion and aliasing in digital sampling. Clearly, if you sample too slowly the signal won't be well-reproduced. But if you sample really fast, then you generate large data files that take a long time to process. Just how slowly can you sample the signal without completely losing its basic properties (such as, for example, the fact that it oscillates with frequency ν_{sig})?

The fundamental parameter here is the ratio of sampling frequency ν_{smpl} to signal frequency ν_{sig}. With our equipment we can set ν_{smpl} to only selected, quantized values. However, we can set ν_{sig} with almost arbitrarily high precision. So to explore these issues we will pick a sampling frequency ν_{smpl} and take data at several signal frequencies ν_{sig}. Be sure to use a coax T so that you can look at the sampled signal on the oscilloscope. Set the peak-to-peak voltage appropriately so that it doesn't saturate the Analog-to-Digital Converter (known as the ADC).

3.1. Your First Digital Sampling

We want to explore sampling rate issues, so to that end we will begin by...

1. Pick a convenient sampling frequency ν_{smpl}.

2. Set the synthesizer to frequency $\nu_{sig} = (0.1, 0.2, 0.3, \ldots, 0.9)\nu_{smpl}$ and take data.

Take N contiguous samples with N an integral power of 2, say $N = 256$. Throughout the datataking, you should always be monitoring the signal with the oscilloscope. These are sine waves, so it's easy to measure the period by looking at the oscilloscope; each time you digitally sample the signal, you should write down the period (maybe in your *lab notebook?*).

For each dataset, use IDL to plot the digitally sampled waveform versus time. In particular, make the digital plot informative, meaning that you can clearly see the signal shape; if necessary, plot only a part of the data so you can clearly see the signal shape (e.g., a few cycles of the sine

wave); compare this with the oscilloscope trace. Also, for all the datasets derive and plot the Fourier power spectrum. Make sure that you *label the axes* with proper values of time and frequency—and choose convenient units, such as microsec and megaHz, to avoid huge and tiny numbers. In deriving the Fourier spectra, use our homegrown DFT procedure (see §4 below).

Now, *look at both sets of these plots* and note any funny business. Think about your results and draw your own conclusion: just what is the minimum sampling rate that you can get away with? (That's *Nyquist's criterion).*

3.2.   Let's Go to Extremes. . .

By now you might have an idea of what's going on. Test yourself: try the following two experiments. But before analyzing the results, *predict* to yourself what they will look like. How? Use good old-fashioned paper and pencil to make some diagrams. If you can successfully predict the following, then you *really* understand the Nyquist criterion! So here we go:

1. Repeat the above for $\nu_{sig} = \nu_{smpl}$.

2. Now make $\frac{\nu_{sig}}{\nu_{smpl}}$ *really large!* In other words, *blatantly violate* Nyquist's criterion! Our oscillators won't run faster than 30 MHz, so to accomplish this you'll have to use not only a large ν_{sig} but also change ν_{smpl} to be very slow. Use as large as a ratio as you can, but make sure that the ratio is *not* an integral or half-integral number. Take lots of samples. Look at the sampled waveform. What do you get? Why?

3.3.   For your Lab Report

In your report, select a well-considered set of plots to illustrate what you've learned, and compose a well-written commentary that convinces me that you really understand what's going on. Also clearly state what you have concluded regarding Nyquist's criterion.

With your plots, you can save paper by fitting multiple plots on a page by using IDL's `!p.multi` system variable; for example, if you wanted 4 plots running vertically down the page you'd set `!p.multi=[0,1,4]`; if you wanted two columns of 4 plots down the page, you'd set `!p.multi=[0,2,4]`. To get back to one plot per page, set `!p.multi=0`.

4. ON USING DFT AND FFT TO CALCULATE A POWER SPECTRUM (First Week)

4.1. The Analytic Fourier Transform

The input to the Fourier transform is voltage versus time, say $E(t)$; the output is voltage versus frequency, say $E(\nu)$. The Fourier transform is the integral

$$E(\nu) = \frac{1}{T} \int_{-T/2}^{T/2} E(t) e^{2\pi j\nu t} dt \ . \tag{1}$$

The input voltage is real; it is multiplied by the complex exponential and integrated, so the output is complex. Of particular importance is that he Fourier Transform is *invertable*: you can go from the time to the frequency domain, and from the frequency domain you can get back to the time domain using the inverse transform

$$E(t) = \frac{1}{F} \int_{-F/2}^{F/2} E(\nu) e^{-2\pi j\nu t} dt \ . \tag{2}$$

Note: If you're paying attention, you would wonder how F and T are defined above. In the proper analytic formulation, they are both infinity. We emphasize their boundedness here because, in practice, i.e. when you do numerical calculations, neither can be infinity!

4.2. The Discrete Fourier Transform (DFT)

Our voltage versus time is not continuous, but rather it is discrete samples. With the digital transform, the integral becomes a sum. In this sum, you need to specify:

1. The set of sample times. I *strongly* suggest:

    (a) Using N samples, where N is even (and even better: a power of 2).
    (b) Use the center channel as the zero point. With N even, there is no center channel, so make the times run from $\frac{-N}{2}/\nu_{smpl}$ to $(\frac{N}{2} - 1)/\nu_{smpl}$.

2. The output is a function of frequency, so you have to specify the frequencies for which you want the output $E(\nu)$. I *strongly* suggest that, *at first*, you calculate the the output for N frequencies running from $-\frac{\nu_{smpl}}{2}$ to $+\frac{\nu_{smpl}}{2}\left(1 - \frac{2}{N}\right)$. This makes the frequency increment equal to $\Delta\nu = \nu_{smpl}/N$. Thus, you calculate a *voltage* spectrum running from $-\frac{\nu_{smpl}}{2}$ to about $\frac{\nu_{smpl}}{2}$ using our in-house DFT procedure. To find out how to use DFT, use the `doc_library` or `doc` function; in IDL, type: `doc, 'dft'`.

Later on, if you are intellectually daring and curious, try doubling or tripling the frequency range, keeping the separation $\Delta\nu$ the same (i.e., by increasing the number of output frequencies to $2N$ or $3N$).

We are interested in the output *power spectrum*, say $P(\nu)$. Power is voltage squared. For complex quantities, the squaring operation means we want the sum of the squares of the real and imaginary parts. We obtain this by multiplying the voltage by its complex conjugate,

$$P(\nu) = E(\nu)E(\nu)^* . \tag{3}$$

In IDL, there are two ways to get this product. One is to use the `conj` function, i.e. `PF = EF * conj(EF)`. Should the imaginary part of `PF` be zero? (answer: yes! Why is this?) Is it? (answer: no! Why not?) To get rid of this annoying and extraneous imaginary part, you can use the `float` function: `PF = float(PF)`.

The other (more convenient and suggested) way is to square the length of the complex vector, i.e. `PF = (abs(EF))^2`. The result is automatically real.

4.3. OPTIONAL: The Fast Fourier Transform (FFT)

Above in §4.2, you had N time samples and evaluated the DFT for N well-chosen frequencies. These were "well-chosen" because for these particular values of frequency—and *only* these particular values—you can get back to the time domain by using the inverse transform (in IDL using `dft`, you accomplish this by setting the `inverse` keyword). *If you have the time and energy*, try this and make it work!

It so happens that, for these particular combinations of frequency and time, there is a very fast algorithmic implementation called the *Fast Fourier Transform*, the FFT. What do we mean by "Fast"? Well, normally when you do a DFT, you have N input numbers and N output numbers and the number of calculations $\propto N^2$. When N gets large, this takes a long time to calculate! For the FFT, on the contrary, the number of cdalculations $\propto N\ln_2(N)$, and this makes it possible to do large-N transforms.

If you have the time and energy, try IDL's FFT and compare it to your DFT calculation above. The FFT output is ordered in what you might think is a funny and awkward way; however, it's really *not* awkward for most applications. See our "DFT's with DFT's" handout for details.

5. LEAKAGE POWER AND FREQUENCY RESOLUTION (First Week)

5.1. Leakage Power

Above, you calculated a power spectrum for each input signal at N distinct frequencies separated by $\Delta\nu = \nu_{smpl}/N$. In each, you found a spike corresponding to the input signal's frequency. Here, focus on just one of the properly-sampled signals ν_{sig}. Calculate the power spectrum for many more than N output frequencies over the Nyquist range $\left[-\frac{\nu_{smpl}}{2} \text{ to } +\frac{\nu_{smpl}}{2}\left(1 - \frac{2}{N}\right)\right]$; i.e., make the frequency increment much smaller than $\Delta\nu = \nu_{smpl}/N$. Making the output frequencies closer together gives a more nearly continuous frequency covereage in the plot of the output spectrum. Turn up the vertical scale a lot to see if there is any nonzero power at frequencies other than ν_{sig}. You *do* see such power! This is *Spectral Leakage*. It affects all power spectra calculated using Fourier techniques.

Can you understand what's going on from a mathematical viewpoint?

5.2. Frequency Resolution

If you had two sharp spectral lines, how closely spaced in frequency could they be and still resolve them? Roughly, this is just the apparent width of the line when plotted against frequency. Look at the width of the line for your plot of §5 above. Compare this width to $\frac{1}{T}$, where T is the total time span over which the samples were taken. If you have the inclination, try taking other time series with varying number of samples (and thus, varying T) and confirm any relationship between line width (that is, frequency resolution) and T.

Can you understand this from a mathematical viewpoint?

6. BASIC DOUBLE SIDEBAND (DSB) MIXER OPERATION: UPPER AND LOWER SIDEBANDS (Second Week)

For this, use two SRS synthesizer oscillators as inputs to a mixer to explore the spectra and waveforms in the DSB mixing process. The SRS synthesizers work up to 30 MHz. Assign one of the SRS synthesizers to be your "local oscillator" (*lo*) with frequency ν_{lo}, and the other your "signal" with frequencies $\nu_{sig} = \nu_{lo} \pm \delta\nu$. Here, you choose the frequency difference $\delta\nu$ and you set the two synthesizers, one to the lo frequency and the other to the signal frequency. There are two cases for the signal freqquency, $\nu_{sig} = \nu_{lo} + \delta\nu$ and $\nu_{sig} = \nu_{lo} - \delta\nu$. Make $\delta\nu$ somewhat small compared to ν_{lo}, maybe 5% of ν_{lo}. For the input power level, a good choice is 0 dbm[1] for both synthesizers.

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[1]What does this "dbm" mean? It's the power relative to 1 milliwatt, expressed in decibels (db). For our system the cable impedence is 50 ohms; what's the rms voltage for a signal with power level 0 dbm?

We will want to digitally sample the mixer output and explore both the sum and difference frequencies. As you learned in the Fourier lab, there are extremely important issues regarding sampling rate. The most basic is the Nyquist criterion. For this lab, we also want enough samples per period to give you a reasonable facsimile of the sine wave when you plot it; from this standpoint, it's not unreasonable to sample at twice Nyquist, or even faster. Another issue is the *number of points* you sample, which must be large enough to give you at least a few periods of the slowest sine wave.

From what you know about mixers, what is the fastest sine wave in the output? This, combined with our above comment and the upper limit on our sampling frequency (20 MHz for single channel, 10 MHz for dual channel), would determine the upper limit on ν_{lo}.

6.1. The Mixer

Combine the two signals, ν_{lo} and ν_{sig}, in a mixer for the two values of ν_{sig}. For the mixer use a Mini-Circuits ZAD-1, which has three BNC connectors (three *ports*) and works well at these frequencies. The ZAD-1, like nearly all mixers, has its ports labeled "R" (the "RF" or "signal"); "L" (the "local oscillator"); and "X" (the "mixing product") or "I" (the "intermediate frequency"). However, as we explain below, these labels are misnomers. They are based on the usual use for a mixer, which is to take two high frequency signals as the inputs to the R and L ports and produce a low frequency difference frequency as the output at the I port.

The ZAD-1 is a balanced mixer, so the "R" and "L" ports are identical, and in particular will not couple to DC or very low frequencies. In contrast, the "I" port is coupled differently and will handle voltages all the way down to, and including, DC. The mixing process functions no matter which two ports are used as inputs. For example, if you are using a mixer to modulate a high frequency (say, a few MHz) with a low frequency (say, a few kHz), you should use the "I" port for the low frequency and either of the other two for the high frequency; take the output from the third port.

We will want to look at the output, which consists of both the sum and difference frequencies, so choose the ports appropriately. Digitally sample the mixer output for both cases ($\nu_{sig} = \nu_{lo} \pm \delta\nu$).

6.2. For Your Lab Report

For the two cases, plot the power spectra versus frequency. Explain why the plots look the way they do. In your explanation include the terms "upper sideband" and "lower sideband".

For one of the cases, plot the waveform. Does it look like the oscilloscope trace? Also, take the Fourier transform (not the power spectrum) of the waveform and remove the sum frequency component by zeroing both the real and imaginary portions (this is "Fourier filtering"). Recreate

the signal from the filtered transform by taking the inverse transform; see §6.3 to see how this is done. Plot the filtered signal versus time. Explain what you see.

6.3.   On Fourier Filtering

When you use DFT to go from the time to the frequency domain, you specify the times and the sampled voltages as input and calculate the output for a well-chosen set of frequencies. These times and frequencies should be symmetric around zero, as we strongly suggested above in §4. To filter out the high-frequency mixed signal, you have to zero both the real and imaginary high-frequency components, and you must zero both the negative and positive frequencies. In frequency space, these zeroed values *must* be symmetric.

To go back from the frequency domain to the time domain, use the filtered frequency Fourier components (which are complex) and their associated frequencies as *inputs* and calculate the output for the *original times*. You should also set the `inverse` keyword in DFT, which keeps the amplitude scale correct (check the documentation for DFT using `doc_library` or `doc`). The output will be a time series, and because you've eliminated the sum frequency component, the only thing that remains should be the difference frequency component.

One more thing. The output of the inverse transform had better be real—after all, your original input was real! You'd better check this! If it isn't real, then either (1) you didn't treat the negative and positive frequencies symmetrically when you zeroed the signal, or (2) you didn't use our suggested input times and output frequencies. If you're having trouble, check your basic technique by doing the inverse transform on the non-filtered Fourier components; you should recover the original time samples.

7.   DSB MIXER AS A PHASE DETECTOR (Second Week)

Use a mixer as a phase detector by setting the two synthesizers to the same frequency, i.e. setting $\delta\nu = 0$. Look at the mixer output on the oscilloscope. Vary the phase of one of the synthesizers. What happens to the mixer output as shown on the oscilloscope trace? What's going on here?

Now measure the mixer output with the HP rack-mounted digital meter in its d.c. mode, reading it with the computer. The d.c. mode is akin to a low pass filter because the sum frequency, which is a fast sine wave, contains no d.c. Generate a file that contains both the phase and the d.c. voltage and plot (to hand in!) the result. If you're clever, you can write an IDL script to take the data automatically. How does this correspond to the interferometer fringe?

Now displace one of the synthesizer frequencies by some small amount—say, 1 Hz. What happens to the output? This should remind you of the interferometer fringe. . .

For your lab report, hand in appropriate plots and discussion.

8.   SOME COMMENTARY

Mixers are important because they allow us to shift the frequency of the whole input spectrum by a uniform amount. They do this by multiplying the input signal by the "local oscillator" (l.o.) with frequency ν_{lo}; this shifts the frequencies by ν_{lo}. In radio reception, this is very important because nearly always our *detectors* work best in a fixed frequency range, but our *signals* come in at many different frequencies. For example, for an AM station playing rock music, the ultimate detector is our ear, which works only at audio frequencies; however, the AM stations transmit at much higher frequency, nearly 1 MHz. A mixer is used to shift the frequencies of the AM station down to the audio region. Such receivers are called *heterodyne* receivers, and this principle is used universally not only in consumer radios, TV's, and cellphones but also in many other applications including radio astronomy.

For example, later in the course we have the 21-cm line at 1420.4 MHz and use an l.o. at 1270.4 MHz, the difference frequency is 150.0 MHz and the sum frequency is 2690.8 MHz. We use a low-pass filter to obliterate the sum frequency, so we are left with a replica of the 21-cm line that is centered at 150.0 MHz instead of 1420.4 MHz. If the Doppler shift from the Earth's orbit (for example) shifts the 21-cm line from 1420.4 to 1420.0 MHz, we change the l.o. frequency to 1270.0 MHz so that the line remains centered at 150 MHz. In this way, our spectrometer can always work at the same frequency, 150 MHz.

Mixers are crucially central to the important radioastronomical technique called interferometry. In this technique, two widely-separated telescopes point at the same source. Usually, the radio source is further from one telescope than the other, which produces a phase difference ϕ between the signals from the two dishes. The two received signals from the source are combined in a mixer, which produces a d.c. voltage that is proportional to $\sin\phi$. The phase difference changes with time, which leads to d.c. voltage proportional to $\sin 2\pi f_{fringe} t$, where f_{fringe} is the local fringe fringe frequency and is equal to $\frac{d\phi}{dt}$. You can also think of of this sinusoidal phase resulting from the differing velocities Δv of the two telescopes with respect to the source, with f_{fringe} being the Doppler shift, equal to the observing frequency times $\frac{\Delta v}{c}$.

9.   SOME THEORY: THE ORDINARY DOUBLE SIDEBAND (DSB) MIXER

We now turn to the basic theory of the ordinary DSB mixer, which is very straightforward. An ideal mixer multiplies the two input signals together; this multiplication makes the output signal have the sum and difference frequencies. Usually, one of these is eliminated by using a filter.

Suppose for simplicity that the mixer is ideal and that the two input signals are the following:

(1) the "local oscillator" with voltage equal to unity (for convenience) and frequency ω_0; and **(2)** two "signals" with voltage E_s and frequencies $\omega_{s-} = (\omega_0 - |\delta\omega|)$ and $\omega_{s+} = (\omega_0 + |\delta\omega|)$. We handle the two signal case simultaneously by writing $\omega_{s\pm} = (\omega_0 \pm |\delta\omega|)$.[2]

The mixer outputs in the two cases are MO_- and MO_+; similarly, we write $MO_\pm$. MO is the product of the signal and l.o. signals, and can be expressed in terms of the sum and difference frequencies by the usual trig identity

$$MO_\pm = E_s \underbrace{\cos[\omega_{s\pm}t]\cos[\omega_0 t]}_{product} = \frac{E_s}{2}(\underbrace{\cos[(\omega_{s\pm} - \omega_0)t]}_{diff} + \underbrace{\cos[(\omega_{s\pm} + \omega_0)t]}_{sum}) \tag{4a}$$

Replacing $\omega_{s\pm}$ by $\omega_0 \pm |\delta\omega|$, we get

$$MO_\pm = E_s \underbrace{\cos[(\omega_0 \pm |\delta\omega|)t] \times \cos[\omega_0 t]}_{product} = \frac{E_s}{2}(\underbrace{\cos[\pm|\delta\omega|t]}_{diff} + \underbrace{\cos[(2\omega_0 \pm |\delta\omega|)t]}_{sum}) \ . \tag{4b}$$

Thus, the signal frequency has been shifted to *two* frequencies by an amount equal to the l.o. frequency: downward in the first term (to $\delta\omega$, the difference frequency) and upward in the second term (to $2\omega_0 \pm |\delta\omega|$, the sum frequency). The output contains *both* the difference *and* the sum frequency terms.

Now we insert a low pass filter to eliminate the sum term. We do this because we are observing at some very high frequency, say the 21-cm line at 1.4 GHz, and need to convert to much lower frequencies where our backend equipment works. (In the TV case it's the same: the signals are at hundreds of MHz and the picture processing circuitry operates below 10 MHz). This low-pass filter removes the second term.

So with the low-pass filter we drop the sum frequency term. This leaves the following, in which we add the subscript *cos* to indicate that we began with the signal represented by a cosine:

$$MO_{cos,\pm} = \frac{E_s}{2}\cos[\pm|\delta\omega|t] = \frac{E_s}{2}\cos[|\delta\omega|t] \ . \tag{5a}$$

If we had begun by representing the signal with a sine instead of a cosine (while retaining the cosine for the l.o.) we would have obtained

$$MO_{sin,\pm} = \frac{E_s}{2}\sin[\pm|\delta\omega|t] = \pm\frac{E_s}{2}\sin[|\delta\omega|t] \ . \tag{5b}$$

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[2]In real life, e.g. a radio station, the "signal" is speech or music with a broad range of δf. In astronomical life, e.g. the 21-cm line, the "signal" is a Doppler broadened line, which again has a broad range of δf. The system is linear, so signals add without mutual interaction, so the discussion for a single δf also applies to these broad spectra.

Four things are important here:

1. *The two sidebands—the two different* input frequencies ($[\omega_{s-} = \omega_0 - \delta\omega]$ and $[\omega_{s+} = \omega_0 + \delta\omega]$)— produce the *same symmetric-around-zero pair of final output frequency* $\pm|\delta\omega|$. The DSB mixer cannot distinguish between the two input frequencies.

2. Consider how $|\delta\omega|$ depends on ω_0: for the upper sideband, $\frac{d|\delta\omega|}{d\omega_0} = 1$, while for the lower $d\frac{|\delta\omega|}{d\omega_0} = -1$. We hope that Figure 1 elucidates the situation.

3. A value of E_s for one sideband produces a certain mixer output *power*; the same value of E_s for the other sideband produces the same power. With regard to power, the sidebands are *identical*.

4. But the sidebands are not *completely* identical; we will discuss this in detail later in the course. The sine term changes changes sign for the lower sideband. Consider representing the output voltage in the complex plane. For the upper sideband, which has identical signs for the (cosine, sine) term, the vector rotates counterclockwise. For the lower sideband, it rotates clockwise. Later, this distinction will allow us to construct a *single-sideband* mixer, which provides *two* outputs, one for each sideband.

Figure 1 illustrates these mathematical results. The top panel shows the original RF spectrum. The second panel shows the USB after DSB mixing: it appears at negative and positive frequencies and the spectrum is symmetric, meaning that the negative frequencies give exactly the same result as the positive ones. The third panel shows the same for the LSB. You can achieve the rejection of either the LSB or the USB by using an appropriate *bandpass* filter. The fourth panel shows what happens without a bandpass filter: the LSB and USB are inextricably mixed and you get the sum of the power spectra. The bottom panel shows that SSB (Single Sideband) mixing retains the sideband separation and identity.
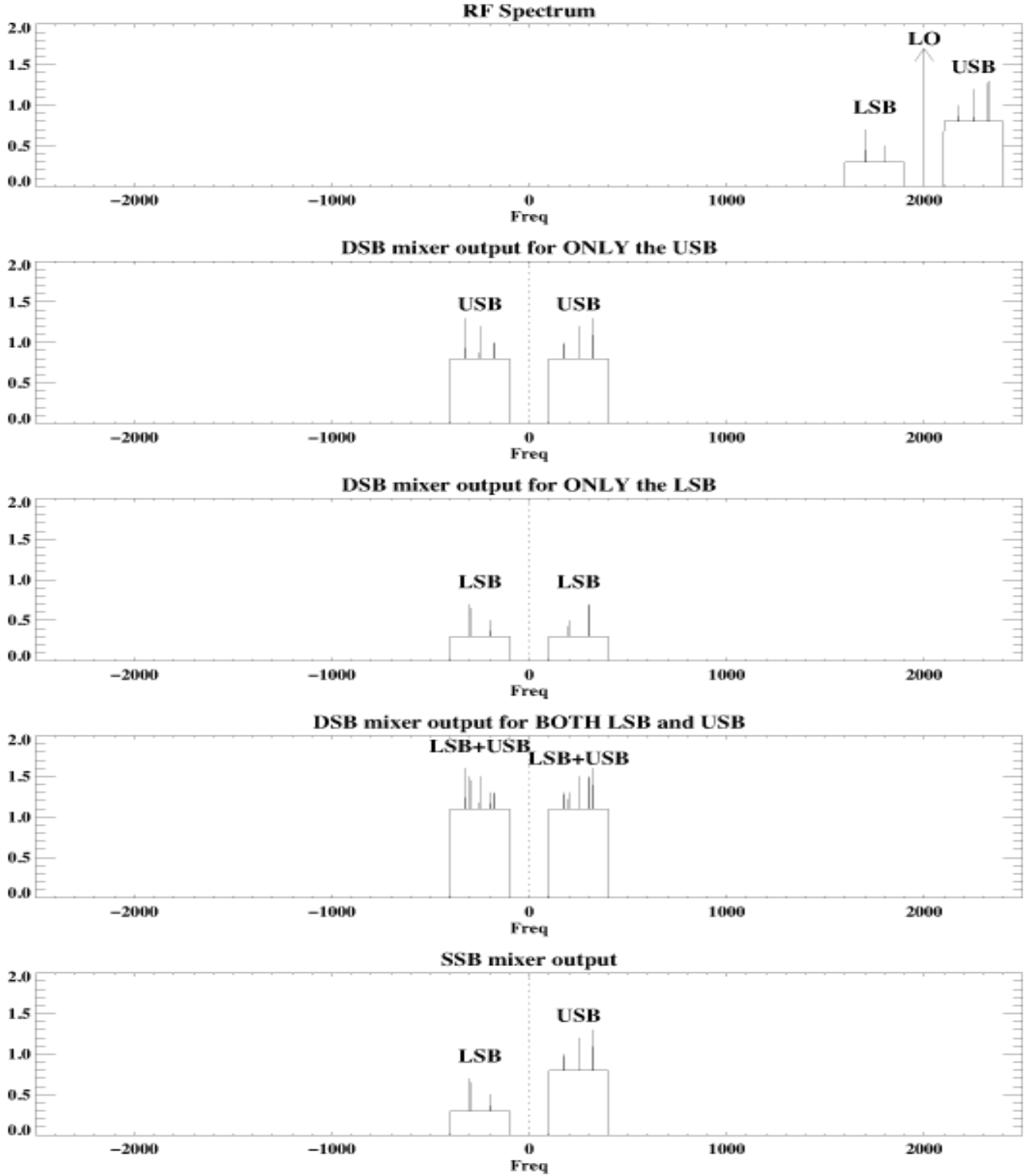
RF Spectrum

LO

USB

LSB

Freq

DSB mixer output for ONLY the USB

USB   USB

Freq

DSB mixer output for ONLY the LSB

LSB   LSB

Freq

DSB mixer output for BOTH LSB and USB

LSB+USB
LSB+USB

Freq

SSB mixer output

USB

LSB

Freq

Fig. 1.— Upper and lower sidebands in DSB and SSB mixers for a set of δ-function test signals on top of broad level noise spectra. Top: the RF spectrum. The next two show the USB and LSB *individually* when they undergo the DSB mixing process; panel 4 shows how they both add together. The bottom panel shows the SSB mixer, which keeps them separate.