ePL: An Overture

YSC3208: Programming Language Design and Implementation

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1. The Syntax of ePL
2. Dynamic Semantics of ePL Programs
3. Static Semantics for ePL
4. A Virtual Machine for **ePL**
5. Big-Step Semantics for **ePL**

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Syntax of ePL

$$\frac{}{n} \qquad \frac{}{\texttt{true}} \qquad \frac{}{\texttt{false}}$$

$$\frac{E}{p_1[E]} \qquad \frac{E_1 \quad E_2}{p_2[E_1, E_2]}$$

$P_1 = \{\backslash, \texttt{\~{}}\}$,
$P_2 = \{\texttt{|}, \texttt{\&}, \texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{=}, \texttt{>}, \texttt{<}\}$.

Syntactic Conventions

- We can use parentheses in order to group expressions together.
- We use the usual infix and prefix notation for operators. The binary operators are left-associative and the usual precedence rules apply such that `1 + 2 * 3 > 10 - 4` stands for `>[+[1,*[2,3]],-[10,4]]`

Examples

```
42

~15 * (7 + 2)

\ false & true | false

17 < 20 - 4 & 10 = 4 + 11
```

Implementation of Syntax

Lexer/Scanner

Language syntax is typically implemented in a two-step process: (1) lexical analysis, (2) parsing

Lexical analysis

Split program string into sequence of tokens

Parsing

Construct a tree-like data structure that corresponds to the structure of the program

Syntax of ePL Programs (in OCaml)

```
type op_id = string

type ePL_expr =
  | BoolConst of bool
  | IntConst of int
  | UnaryPrimApp of op_id * ePL_expr
  | BinaryPrimApp of op_id * ePL_expr * ePL_expr
```

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Values

Goal of evaluating an expression is to reach a *value*, an expression that cannot be further evaluated.

In ePL, a value is either an integer constant, or a boolean constant.

In the following rules, we denote values by v.

Contraction

$$\frac{}{p_1[v_1] >_{\mathrm{ePL}} v}[\mathrm{OpVals}_1] \qquad \frac{}{p_2[v_1, v_2] >_{\mathrm{ePL}} v}[\mathrm{OpVals}_2]$$

One instance of the second rule is:

$$\frac{}{+[1, 1] \ >_{\mathrm{ePL}} 2}$$

One-Step Evaluation

$$\frac{E >_{\mathrm{ePL}} E'}{E \mapsto_{\mathrm{ePL}} E'}[\mathrm{Contraction}] \qquad \frac{E \mapsto_{\mathrm{ePL}} E'}{p_1[E] \mapsto_{\mathrm{ePL}} p_1[E']}[\mathrm{OpArg}_1]$$

$$\frac{E_1 \mapsto_{\mathrm{ePL}} E_1'}{p_2[E_1, E_2] \mapsto_{\mathrm{ePL}} p_2[E_1', E_2]}[\mathrm{OpArg}_2]$$

$$\frac{E_2 \mapsto_{\mathrm{ePL}} E_2'}{p_2[E_1, E_2] \mapsto_{\mathrm{ePL}} p_2[E_1, E_2']}[\mathrm{OpArg}_3]$$

Evaluation Order

One-step evaluation does not prescribe the order in which a given ePL expression is evaluated. Both of the following statements hold:

$$\texttt{3 * 2 + 4 * 5} \mapsto_{\mathrm{ePL}} \texttt{3 * 2 + 20}$$
$$\texttt{3 * 2 + 4 * 5} \mapsto_{\mathrm{ePL}} \texttt{6 + 4 * 5}$$

Evaluation

Evaluation is the reflexive transitive closure of one-step evaluation.

$$\frac{E_1 \mapsto_{\mathrm{ePL}} E_2}{E_1 \mapsto_{\mathrm{ePL}}{}^{*} E_2}[\mapsto_{\mathrm{ePL}}{}^{*}_{B}] \qquad \frac{}{E \mapsto_{\mathrm{ePL}}{}^{*} E}[\mapsto_{\mathrm{ePL}}{}^{*}_{R}]$$

$$\frac{E_1 \mapsto_{\mathrm{ePL}}{}^{*} E_2 \quad E_2 \mapsto_{\mathrm{ePL}}{}^{*} E_3}{E_1 \mapsto_{\mathrm{ePL}}{}^{*} E_3}[\mapsto_{\mathrm{ePL}}{}^{*}_{T}]$$

Implementation (in OCaml)

Idea

Keep reducing expression until it is irreducible. Similar to small-step semantics.

Code snippet

```
let rec evaluate (e:ePL_expr): ePL_expr =
  if (reducible e)
  then evaluate (oneStep e)
  else e
}
```

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A Type System for ePL

Not all expressions in ePL make sense. For example,

```
true + 1
```

does not make sense, because `true` is a boolean expression, whereas the operator + to which `true` is passed as first argument, expects integers as arguments.

Typing Relation

The set of well-typed expressions is defined by the binary *typing relation*

$$\text{``:''} : \mathrm{ePL} \rightarrow \{\mathtt{int}, \mathtt{bool}\}$$

We use infix notation for ":", writing $E : t$, which is read as "the expression E has type t.

Examples

- `1+2 : int`
- `false & true : bool`
- `10 < 17-8 : bool`

but:

- `true + 1` : t does not hold for any t
- `3 + 1 * 5` : `bool` does not hold

Typing Relation

$$\frac{}{\mathtt{true} : \mathtt{bool}}\,\textbf{[TrueT]} \qquad \frac{}{\mathtt{false} : \mathtt{bool}}\,\textbf{[FalseT]}$$

$$\frac{}{n : \mathtt{int}}\,\textbf{[NumT]} \qquad \frac{E : \mathtt{int}}{\sim[E] : \mathtt{int}}\,\textbf{[PrimT}_1\textbf{]}$$

$$\frac{E : \mathtt{bool}}{\backslash[E] : \mathtt{bool}}\,\textbf{[PrimT}_1\textbf{]} \qquad \frac{E_1 : t_1 \qquad E_2 : t_2}{p[E_1, E_2] : t}\,\textbf{[PrimT}_2\textbf{]}$$

Types of Primitive Operations

p	t_1	t_2	t
+	int	int	int
-	int	int	int
*	int	int	int
/	int	int	int
&	bool	bool	bool
|	bool	bool	bool
\	bool		bool
~	int		int
=	int	int	bool
<	int	int	bool
>	int	int	bool

Well-typed Expressions

An expression E is well-typed, if there is a type t such that $E : t$.

A Proof

$$\dfrac{\dfrac{\dfrac{}{2 : \texttt{int}}\,[\textbf{NumT}] \qquad \dfrac{}{3 : \texttt{int}}\,[\textbf{NumT}]}{\texttt{2*3} : \texttt{int}}\,[\textbf{PrimT}] \qquad \dfrac{}{7 : \texttt{int}}\,[\textbf{NumT}}{\texttt{2*3>7} : \texttt{bool}}$$

Implementation

Idea

Compute the type of an expression *bottom-up*, starting from the constants at the leaves of the syntax tree

Checking

At each node, check that the components have the right type wrt the operator

Type Safety

Type safety is a property of a given language with a given static and dynamic semantics. In a type-safe language, certain problems are guaranteed not to occur at runtime for well-typed programs.

Problems:

- No progress,
- No preservation

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Motivation

- How do we remember intermediate results?
- How do we know what to do next?

Idea: Translate ePL to a machine language. Execute machine code using an emulator.

Definition of eVML

$$\frac{}{\texttt{DONE}} \qquad \frac{s}{\texttt{LDCI}\ i\ .\ s} \qquad \frac{s}{\texttt{LDCB}\ b\ .\ s}$$

Definition of eVML (cont'd)

$$\frac{s}{\texttt{PLUS}.s} \qquad \frac{s}{\texttt{MINUS}.s} \qquad \frac{s}{\texttt{TIMES}.s} \qquad \frac{s}{\texttt{DIV}.s}$$

$$\frac{s}{\texttt{AND}.s} \qquad \frac{s}{\texttt{OR}.s} \qquad \frac{s}{\texttt{NOT}.s}$$

$$\frac{s}{\texttt{LT}.s} \qquad \frac{s}{\texttt{GT}.s} \qquad \frac{s}{\texttt{EQ}.s}$$

Example

The instruction sequence

$$[\texttt{LDCI}\ 1, \texttt{LDCI}\ 2, \texttt{PLUS}, \texttt{DONE}]$$

represents a valid eVML program.

Compiling **ePL** to eVML

$$\Rightarrow : \textbf{ePL} \rightarrow \text{eVML}$$

$$\frac{E \hookrightarrow s}{E \Rightarrow s.\texttt{DONE}}$$

Compiling **ePL** to eVML (cont'd)

$$\frac{}{n \hookrightarrow \texttt{LDCI}\ n}$$

$$\frac{}{\texttt{true} \hookrightarrow \texttt{LDCB true}}$$

$$\frac{}{\texttt{false} \hookrightarrow \texttt{LDCB false}}$$

Compiling **ePL** to eVML (cont'd)

$$\frac{E_1 \hookrightarrow s_1 \qquad E_2 \hookrightarrow s_2}{E_1 + E_2 \hookrightarrow s_1.s_2.\mathtt{PLUS}}$$

$$\frac{E_1 \hookrightarrow s_1 \qquad E_2 \hookrightarrow s_2}{E_1 * E_2 \hookrightarrow s_1.s_2.\mathtt{TIMES}}$$

$$\frac{E_1 \hookrightarrow s_1 \qquad E_2 \hookrightarrow s_2}{E_1 / E_2 \hookrightarrow s_1.s_2.\mathtt{DIV}}$$

...

Examples

```
(1 + 2) * 3
[LDCI 1, LDCI 2, PLUS, LDCI 3, TIMES, DONE]
```

```
1 + (2 * 3)
[LDCI 1, LDCI 2, LDCI 3, TIMES, PLUS, DONE].
```

Executing eVML Code

Registers:

- *pc*: program counter,
- *os*: operand stack

Example

$pc = 2$

$s = [\texttt{LDCI 1}, \texttt{LDCI 2}, \texttt{PLUS}, \texttt{LDCI 3}, \texttt{TIMES}, \texttt{DONE}]$

$s(pc) = \texttt{PLUS}$

Transition Function

$$\frac{s(pc) = \texttt{LDCI}\ i}{(os, pc) \rightrightarrows_s (i.os, pc + 1)}$$

$$\frac{s(pc) = \texttt{LDCB}\ b}{(os, pc) \rightrightarrows_s (b.os, pc + 1)}$$

Transition Function (cont'd)

$$\frac{s(pc) = \mathrm{PLUS}}{(i_2.i_1.os, pc) \rightrightarrows_s (i_1 + i_2.os, pc + 1)}$$

End Configuration

$s(pc) = \texttt{DONE}$

The result of the computation can be found on top of the operand stack of the end configuration.

$$R(M_s) = v, \text{ where } (\langle\rangle, 0) \rightrightarrows_s^* (\langle v.os\rangle, pc), \text{ and } s(pc) = \texttt{DONE}$$

Example

$$[\mathtt{LDCI}\ 10, \mathtt{LDCI}\ 20, \mathtt{PLUS}, \mathtt{LDCI}\ 6, \mathtt{TIMES}, \mathtt{DONE}]$$

$$(\langle\rangle, 0) \rightrightarrows (\langle 10\rangle, 1) \rightrightarrows (\langle 20, 10\rangle, 2) \rightrightarrows$$

$$(\langle 30\rangle, 3) \rightrightarrows (\langle 6, 30\rangle, 4) \rightrightarrows (\langle 180\rangle, 5)$$

Implementation

Registers

Keep registers in local variables (or objects)

Execution

Use a switch statement (or match construct) to interpret each instruction

Implementation in OCaml

Instructions

Capture simple bytecode instructions.

```
type eVML_inst =
  | LDCI of int
  | LDCB of int (* 0 - false; 1 - true *)
  | PLUS | MINUS | TIMES | DIV | AND | NEG
  | NOT | OR | LT | GT | EQ | DONE

type eVML_prog = eVML_inst list
```

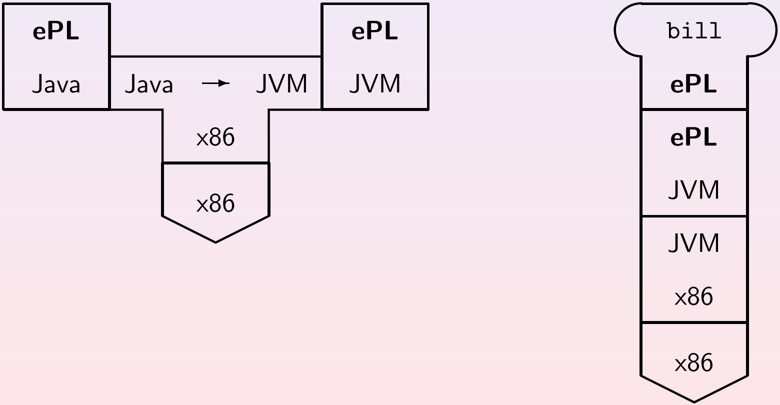
Virtual Machine in OCaml

Execution

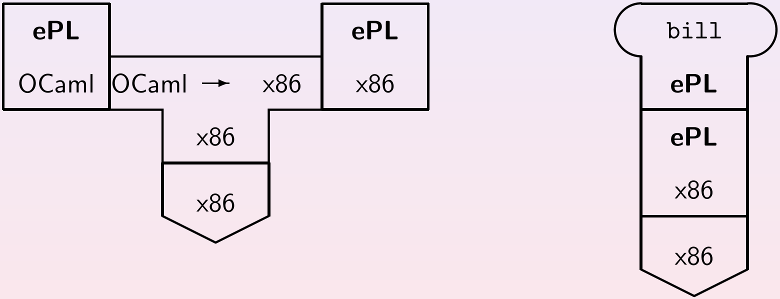
Use an array to store the instructions.

```
let eVML_mc (instArr:eVML_inst array) =
  let stk = Stack.create () in
  let rec execute pc =
    let c = Array.get instArr pc in
    match c with
      | DONE -> Stack.pop stk
      | _ -> (proc_inst stk c; execute (pc+1))
  in execute 0
```

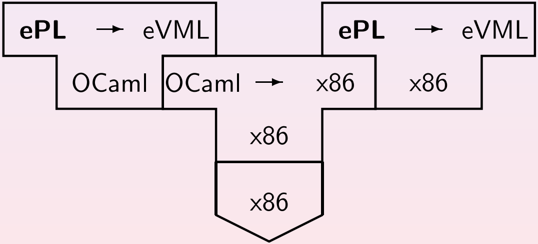
Interpreter for ePL in Java



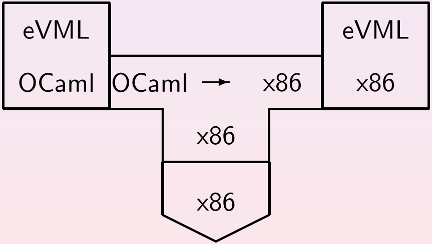
Interpreter for ePL in OCaml



Compiling the ePL Compiler in OCaml



Compiling the eVML Emulator



Simple Exercise

- Draw T-diagram for ePL programs compiled using an ePL compiler produced by the following steps.

```
> ocamlopt eplc.ml -o eplc
> ocamlopt eplm.ml -o eplm
> ./eplc bill.epl
bill.epl bill.evml
> ./eplm bill.evml
249
```

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$$\frac{}{+[1,1] \;>_{\mathrm{ePL}} 2}$$

One-Step Evaluation

$$\frac{E >_{\mathrm{ePL}} E'}{E \mapsto_{\mathrm{ePL}} E'}\,[\mathrm{Contraction}] \qquad \frac{E \mapsto_{\mathrm{ePL}} E'}{p_1[E] \mapsto_{\mathrm{ePL}} p_1[E']}\,[\mathrm{OpArg}_1]$$

$$\frac{E_1 \mapsto_{\mathrm{ePL}} E_1'}{p_2[E_1, E_2] \mapsto_{\mathrm{ePL}} p_2[E_1', E_2]}\,[\mathrm{OpArg}_2]$$

$$\frac{E_2 \mapsto_{\mathrm{ePL}} E_2'}{p_2[E_1, E_2] \mapsto_{\mathrm{ePL}} p_2[E_1, E_2']}\,[\mathrm{OpArg}_3]$$

Big-Step Semantics

- One-step reduction approach to dynamic semantics is often referred to as *small-step* semantics.
- If you are interested only in the final outcome, rather than the intermediate steps, you can use *big-step* semantics.
- *Big-step* semantics will reduce each expression directly into its final value (not an intermediate value).
- For ePL, we denote it using $E \mapsto^*_{\mathrm{ePL}} v_1$.

Big-Step Evaluation

$$\frac{E \mapsto^*_{\mathrm{ePL}} v_1 \qquad p_1[v_1] >_{\mathrm{ePL}} v_2}{p_1[E] \mapsto^*_{\mathrm{ePL}} v_2}\,[\mathrm{OpArg}_1]$$

$$\frac{E_1 \mapsto^*_{\mathrm{ePL}} v_1 \qquad E_2 \mapsto^*_{\mathrm{ePL}} v_2 \qquad p_2[v_1, v_2] >_{\mathrm{ePL}} v}{p_2[E_1, E_2] \mapsto^*_{\mathrm{ePL}} v}\,[\mathrm{OpArg}_2]$$

Next Week

- Introduction of simPL