Estimation of uncertainties of ultrasonic velocity measurements

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Introduction

V_{Al} is the velocity of P- (6375 m/s) or S- (3100 m/s) wave in aluminium sample that was used for calibration of traveltimes. These velocities are estimated with uncertainty 25 m/s.

D - the length of the raypath:

- for axial measurements D is the height of the sample (D = 60 mm)
- for radial measurements D is the diameter of cylindrical sample (D = 50 mm).

The uncertainty of length measurements is 0.5 mm;

T_{Al} is the traveltime of P- or S-wave in aluminium sample in axial or radial direction:

- P-waves, axial direction: Tal = 10.03μs
- P-waves, radial direction: Tal = 8.64 μs
- S-waves, axial direction: Tal = 26.00 μs
- S-waves, radial direction: Tal = μs

The uncertainty of P-wave picking is 0.03 μs. The uncertainty of S-wave picking 0.06 μs. The uncertainties were obtained by calculation of standart deviation σ of 100 identical measurements of traveltime (uncertainty was chosen to be $2 \cdot \sigma$, that corresponds to 95% confidence interval). We believe that these picking uncertainties take into account the picking error and the changes in setup including varying thickness of lamination layes (sugar syrup), mechanical adjustment of transducers, and tiny inhomogeneities of aluminium sample.

```
Val  = 3100; % (6375/3100) m/s
dVal = 25; % m/s

Tal = 2.05*8.64e-6; % s (8.64 microseconds)
T   = 2.05*1e-5; % s (usually 10 - 11 microseconds)
dTal = 6e-8;   % s (0.03/0.06 microseconds)
dT   = 6e-8;   % s (0.03/0.06 microseconds)

Dal = 0.05; % m (50/60 mm)
Ds = 0.05; % m (50/60mm)
dDal = 0.0005; % m (0.5 mm)
dDs  = 0.0005; % m (0.5 mm)
```

Step 1: Find time correction

Find a correction time that is the traveltime of the wave in the wires and in the coupling elements:

$$t_c = t_{\mathrm{Al}} - \frac{D_{\mathrm{Al}}}{V_{\mathrm{Al}}}$$

$$\Delta t_c = \Delta t_{\mathrm{Al}} + \frac{\Delta D_{\mathrm{Al}}}{V_{\mathrm{Al}}} + \frac{D_{\mathrm{Al}}}{V_{\mathrm{Al}}} \frac{\Delta V_{\mathrm{Al}}}{V_{\mathrm{Al}}}$$

```
tc = Tal - Dal/Val;
dtc = dTal + dDal/Val + (Dal/Val)*(dVal/Val);
disp(['Time correction: tc = ' num2str(tc*1e6,2) ' microseconds'])
```

```
Time correction: tc = 1.6 microseconds
```

```
disp(['Uncertainty for time correction: dtc = ' num2str(dtc*1e6,2) ' microseconds'])
```

```
Uncertainty for time correction: dtc = 0.35 microseconds
```

Step 2: Apply time correction

Remove correction time from traveltime measurements in the sample:

$$t = \tilde{t} - t_c$$

$$\Delta t = \Delta \tilde{t} + \Delta t_c$$

```
t = T - tc;
dt = dT + dtc;
disp(['Travel time before correction: T = ' num2str(T*1e6,2) ' microseconds'])
```

```
Travel time before correction: T = 20 microseconds
```

```
disp(['Travel time after correction: t = ' num2str(t*1e6,2) ' microseconds'])
```

```
Travel time after correction: t = 19 microseconds
```

```
disp(['Uncertainty for travel time: dt = ' num2str(dt*1e6,2) ' microseconds'])
```

```
Uncertainty for travel time: dt = 0.41 microseconds
```

Step 3: Estimate velocity

Just devide the raypath by traveltime:

$$V = \frac{D_{\mathrm{sample}}}{t}$$

$$\frac{\Delta V}{V} = \frac{\Delta D_{\mathrm{Sample}}}{D_{\mathrm{sample}}} + \frac{\Delta t}{t}$$

```
V = Ds/t;
dVrel = (dDs/Ds) + (dt/t);
disp(['Velocity of wave: V = ' num2str(V,4) ' m/s'])
```

```
Velocity of wave: V = 2643 m/s
```

```
disp(['Relative error of velocity: dV/V = ' num2str(dVrel*100,2) ' %'])
```

```
Relative error of velocity: dV/V = 3.2 %
```

```
disp(['Absolute error of velocity: dV = ' num2str(V*dVrel,3) ' m/s'])
```

```
Absolute error of velocity: dV = 83.9 m/s
```

Conclusions:

- P-wave, radial direction: ΔV_P = 150 m/s , relative error 2.8%
- P-wave, axial direction: ΔV_P = 130 m/s , relative error 2.4%
- S-wave, radial direction: ΔV_P = 80 m/s , relative error 3.2%
- S-wave, axial direction: ΔV_P = 110 m/s , relative error 3.3%