

Estimation of uncertainties of ultrasonic velocity measurements

Author: Martin Rühlmann, Abakumov Ivan

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E-mail: abakumov_ivan@mail.ru

Introduction

V_{Al} is the velocity of P- (6375 m/s) or S- (3105 m/s) wave in aluminium sample that was used for calibration of traveltimes. These velocities are estimated with uncertainty 25 m/s.

D - the length of the raypath:

- for axial measurements D is the height of the sample (D = 60 mm)
- for radial measurements D is the diameter of cylindrical sample (D = 50 mm).

The uncertainty of length measurements is 0.5 mm;

T_{Al} is the travelttime of P- or S-wave in aluminium sample in axial or radial direction:

- P-waves, axial direction: $T_{al} = 18.52 \mu s$
- P-waves, radial direction: $T_{al} = 8.64 \mu s$
- S-waves, axial direction: $T_{al} = 37.95 \mu s$
- S-waves, radial direction: $T_{al} = 17.72 \mu s$

The uncertainty of P-wave picking is $0.03 \mu s$. The uncertainty of S-wave picking $0.06 \mu s$. The uncertainties were obtained by calculation of standart deviation σ of 100 identical measurements of travelttime (uncertainty was chosen to be $2 \cdot \sigma$, that corresponds to 95% confidence interval). We believe that these picking uncertainties take into account the picking error and the changes in setup including varying thickness of lamination layes (sugar syrup), mechanical adjustment of transducers, and tiny inhomogeneities of aluminium sample.

In our samples:

- Typical values for P-wave traveltimes in axial direction are about $21 \mu s$
- Typical values for S-wave traveltimes in axial direction are about $38-39 \mu s$
- Typical values for P-wave traveltimes in radial direction are about $11-12 \mu s$
- Typical values for S-wave traveltimes in radial direction are about $22-24 \mu s$

```
Val = 3105; % (6375/3100) m/s
dVal = 25; % m/s
```

```
Tal = 37.95e-6; % s (8.64 microseconds)
T = 39e-6; % s (P waves: radial 11-12 mus, axial 21-22 mus, S-waves: radial 22-24, ax
dTal = 6e-8; % s (P/S waves: 0.03/0.06 microseconds)
dT = 6e-8; % s (P/S waves: 0.03/0.06 microseconds)
```

```
Dal = 0.06; % m (50/60 mm)
Ds = 0.06; % m (50/60mm)
```

```
dDal = 0.0005; % m (0.5 mm)
dDs = 0.0005; % m (0.5 mm)
```

Step 1: Find time correction

Find a correction time that is the travelttime of the wave in the wires and in the coupling elements:

$$t_c = t_{Al} - \frac{D_{Al}}{V_{Al}}$$

$$\Delta t_c = \Delta t_{Al} + \frac{\Delta D_{Al}}{V_{Al}} + \frac{D_{Al}}{V_{Al}} \frac{\Delta V_{Al}}{V_{Al}}$$

```
tc = Tal - Dal/Val;
dtc = dTal + dDal/Val + (Dal/Val)*(dVal/Val);
disp(['Time correction: tc = ' num2str(tc*1e6,4) ' microseconds'])
```

Time correction: tc = 18.63 microseconds

```
disp(['Uncertainty for time correction: dtc = ' num2str(dtc*1e6,2) ' microseconds'])
```

Uncertainty for time correction: dtc = 0.38 microseconds

Step 2: Apply time correction

Remove correction time from travelttime measurements in the sample:

$$t = \tilde{t} - t_c$$

$$\Delta t = \Delta \tilde{t} + \Delta t_c$$

```
t = T - tc;
dt = dT + dtc;
disp(['Travel time before correction: T = ' num2str(T*1e6,2) ' microseconds'])
```

Travel time before correction: T = 39 microseconds

```
disp(['Travel time after correction: t = ' num2str(t*1e6,2) ' microseconds'])
```

Travel time after correction: t = 20 microseconds

```
disp(['Uncertainty for travel time: dt = ' num2str(dt*1e6,2) ' microseconds'])
```

Uncertainty for travel time: dt = 0.44 microseconds

Step 3: Estimate velocity

Just devide the raypath by travelttime:

$$V = \frac{D_{\text{sample}}}{t}$$

$$\frac{\Delta V}{V} = \frac{\Delta D_{\text{Sample}}}{D_{\text{sample}}} + \frac{\Delta t}{t}$$

```
V = Ds/t;
dVrel = (dDs/Ds) + (dt/t);
disp(['Velocity of wave: V = ' num2str(V,4) ' m/s'])
```

Velocity of wave: V = 2945 m/s

```
disp(['Relative error of velocity: dV/V = ' num2str(dVrel*100,2) ' %'])
```

Relative error of velocity: dV/V = 3 %

```
disp(['Absolute error of velocity: dV = ' num2str(V*dVrel,3) ' m/s'])
```

Absolute error of velocity: dV = 87.7 m/s

Conclusions:

- P-wave, radial direction: $\Delta V_p = 150$ m/s , relative error 2.8%
- P-wave, axial direction: $\Delta V_p = 130$ m/s , relative error 2.4%
- S-wave, radial direction: $\Delta V_s = 70$ m/s , relative error 2.9%
- S-wave, axial direction: $\Delta V_s = 90$ m/s , relative error 3.0%

t_c for P waves is equal $9.10 \mu\text{s}$ in axial direction

t_c for S waves is equal $18.62 \mu\text{s}$ in axial direction

t_c for P waves is equal $0.80 \mu\text{s}$ in radial direction

t_c for P waves is equal $1.62 \mu\text{s}$ in radial direction