R-factor: Compare surface subsidence computed numerically with Geertsmaa's solutions and with Dyskin's solutions

Introduction

Dyskin's solutions

see formulas 20, 21, 32 and 33 in Dyskin, A., Pasternak, E., and Shapiro, S. A. (2020).

Subsidence, uplift and shift due to fluid extraction and production in a

finite reservoir. International Journal for Numerical and Analytical Methods in

Geomechanics (in review)

Geertsmaa's solutions

see formulas 6, 7 in Geertsma, J. (1973). Land subsidence above compacting oil

and gas reservoirs. Journal of Petroleum Technology, 25(06), 734-744.

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```
clear; close all; clc;
mlibfolder = '/home/ivan/Desktop/MLIB';
path(path, mlibfolder);
add_mlib_path;
```

Load G-file and comsol data

```
% Model 5 - constant overburden model
% Model 6 - Groningen model
% Model 7 - Groningen model, but different permeability of basement
Results = MLD('/home/ivan/Desktop/Comsol/My_model_5_results.mat');
G = MLD('/home/ivan/Desktop/Comsol/My_model_5_G_file.mat');
```

Plot pressure vs displacement

We consider land surface subsidence due to compaction of the disc-shaped reservoir of radius R and thickness H with the upper boundary at depth D embedded in an elastic half-space.

1. Introduce the main parameters:

```
D = 3000; % m - depth of the upper boundary
R = 3000; % m - radius of the reservoir
H = 300;  % m - thickness of the reservoir
dP = 16.52*1e6; % Pa - pore pressure change
```

```
nu = 0.2; % Poisson's ratio of the surrounding medium

ov.K  = 12.0*1e9; % bulk modulus (dr) of the overburden
ov.G  =  9.0*1e9; % shear modulus (dr) of the overburden
ov.alpha = 0.3;   % Biot coefficient (overburden)
res.K = 13.3*1e9; % bulk modulus (dr) of the reservoir
res.G = 10.0*1e9; % shear modulus (dr) of the reservoir
res.alpha = 0.95;   % Biot coefficient (reservoir)
```

2. Define uniaxisal compaction coefficient

$c_m = 1/3(\frac{1+\nu}{1-\nu})\frac{\alpha}{K}$

$c_m = 1/2(\frac{1-2\nu}{1-\nu})\frac{\alpha}{G}$

$c_m = \frac{\alpha}{K+4/3G}$

$\alpha = 1 - K_{dr}/K_{gr}$- the poroelastic Bio coefficient,

K_{dr} - bulk modulus of rock matrix (drained skeleton)

K_{gr} - bulk modulus of the grain meterial

$\nu = \frac{3K-2G}{2(3K+G)}$ - Poisson's ratio

```
res.nu = (3*res.K - 2*res.G)/(3*res.K + res.G)/2
```

```
res = struct with fields:
        K: 1.3300e+10
        G: 1.0000e+10
    alpha: 0.9500
       nu: 0.1994
```

```
cm = (1+res.nu)/(1-res.nu)*res.alpha/res.K/3;
disp(['Cm, definition 1: cm = ' num2str(cm)])
```

```
Cm, definition 1: cm = 3.567e-11
```

```
cm = (1-2*res.nu)/(1-res.nu)*res.alpha/res.G/2;
disp(['Cm, definition 2: cm = ' num2str(cm)])
```

```
Cm, definition 2: cm = 3.567e-11
```

```
cm = res.alpha/(res.K+4/3*res.G);
```

```
disp(['Cm, definition 3: cm = ' num2str(cm)])
```

```
Cm, definition 3: cm = 3.567e-11
```

```
res.cm = cm;
```

3. Define reservoir compaction:

$b = c_m \cdot \delta P \cdot H$

```
b_cal = res.cm*dP*H;

disp(['Estimated reservoir compaction = ' num2str(b_cal)])
```

```
Estimated reservoir compaction = 0.17678
```

```
b_mod = 0.159;
disp(['Modeled reservoir compaction = ' num2str(b_mod)])
```

```
Modeled reservoir compaction = 0.159
```

```
% FEM subsidence
Uz.model5 = Results.disp_y(:,1,101)'-Results.disp_y(:,1,1)';
Ur.model5 = Results.disp_x(:,1,101)'-Results.disp_x(:,1,1)';

% Geertsma
uz = get_subsidence_vertical_Geertsma(D,R,G.xx);
Uz.geertsma = -2*b_cal*(1-res.nu)*uz;

ur = get_subsidence_radial_Geertsma(D,R,G.xx);
Ur.geertsma = +2*b_cal*(1-res.nu)*ur;

% Dyskin, integral solution
uz = get_subsidence_vertical_Dyskin_integral(D,R,G.xx);
Uz.dyskin_int = b_mod*uz;

ur = get_subsidence_radial_Dyskin_integral(D,R,G.xx);
Ur.dyskin_int = -b_mod*(3-2*res.nu)/(1-res.nu)/2*ur;

% Dyskin, approximate expressions
r = 0:500:10000;
uz = get_subsidence_vertical_Dyskin(D,R,r);
Uz.dyskin_app = b_mod*uz;

ur = get_subsidence_radial_Dyskin(D,R,r);
Ur.dyskin_app = b_mod*(3-2*res.nu)/(1-res.nu)*ur;

% Plot results
```

```
figure(433)
fig = figure('Position', [1 1 1000 500]);
subplot(1,2,1)
plot(G.xx/1e3, Uz.model5+0.0073, 'k')
hold on
plot(G.xx/1e3,  Uz.geertsma, 'r')
plot(G.xx/1e3, -Uz.dyskin_int, 'b')
plot(r/1e3,    -Uz.dyskin_app, 'b*')
legend('FEM', 'Geertsma', 'Dyskin, integral', 'Dyskin, approx.', 'Location', 'Best')
xlabel('Distance [km]')
ylabel('Vertical displacement [m]')
grid on
text(-2, 0, 'a)', 'FontSize', 16)
axis([0 10 -0.12 0])

subplot(1,2,2)
plot(G.xx/1e3, Ur.model5, 'k')
hold on
plot(G.xx/1e3, -Ur.geertsma, 'r')
plot(G.xx/1e3, Ur.dyskin_int, 'b')
plot(r/1e3,    Ur.dyskin_app, 'b*')
legend('FEM', 'Geertsma', 'Dyskin, integral', 'Dyskin, approx.', 'Location', 'Best')
xlabel('Distance [km]')
ylabel('Radial displacement [m]')
grid on
text(-2, 0, 'b)', 'FontSize', 16)
axis([0 10 -0.08 0])
```