

Exercise 3: [Plot 2 Ex 1] * Using calculations Shown in Week 3 Part 3 Lecture Slides *

① Define Null hypothesis (H_0)

$$\alpha: 0.05, Z_{0.05/2} = \pm 1.96 \quad \mu(\text{Authentic}): -17.65,$$

$$\sigma(\text{Authentic}): 0.65, n=30 (\text{samples of authentic})$$

$$\text{Hypothesis } H_0: \mu = -17.65 \quad \sigma = 0.65$$

② Estimate interval for population mean

$$\text{Left Tail: } \bar{X} - Z_{0.05/2} \frac{\sigma}{\sqrt{n}} \rightarrow -17.65 - (1.96) \left(\frac{0.65}{\sqrt{30}} \right) = -17.8825...$$

$$\text{Right Tail: } \bar{X} + Z_{0.05/2} \frac{\sigma}{\sqrt{n}} \rightarrow -17.65 + (1.96) \left(\frac{0.65}{\sqrt{30}} \right) = -17.4174...$$

$$-17.88 < \mu < -17.42$$

③ Evaluate FRR

From our dataset we can see that samples our outside of our interval so $FRR = \frac{FR}{(FR+TA)} = \frac{29}{(29+1)} = \frac{29}{30} = 0.96$ (very large error)

④ Change critical intervals

$$-17.88 < \mu < -17.42 \text{ is changed to } -17.30 < \mu < -16.70$$

⑤ New z-values which correspond to new critical values:

$$\bar{X}_1 = -17.30, \bar{X}_2 = -16.70, \mu = -17.04, \sigma = 0.6, n = 30$$

$$Z_1 = \frac{\bar{X}_1 - \mu}{\sigma/\sqrt{n}} = \frac{-17.30 - (-17.04)}{0.65/\sqrt{30}} = -2.1908 \approx -2.2$$

$$Z_2 = \frac{\bar{X}_2 - \mu}{\sigma/\sqrt{n}} = \frac{-16.70 - (-17.04)}{0.65/\sqrt{30}} = 2.8650 \approx 2.9$$

$$\text{Therefore, } FRR = P(z < -2.2) + P(z > 2.9) = 0.0139 + (1 - 0.9981) = 0.0158$$

$$FRR = 0.0158$$

⑥ Formulate Hypothesis H_1 with μ and σ close to 30 impostor Signatures

Hypothesis H_0 : $\mu(\text{Authentic}) = -17.6525$ $\sigma(\text{Authentic}) = 0.6513$,

Critical region: $-17.88 < \mu < -17.42$

Hypothesis H_1 : $\mu(\text{Impostor}) = -37.6170$ $\sigma(\text{Impostor}) = 4.3575$

$$Z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{17.88 - 37.6170}{4.3575/\sqrt{30}} = -24.808720$$

FAR = 0 (almost 0)

→ The distributions for genuine and impostor signatures are well separated