Exercise 3: [Plot 2 Ex 1] *Using calculations Shown in Week 3 Part 3 Lecture Slides*

① Define Null hypothesis (H_0)

$\alpha: 0.05$, $z_{0.05/2} = \pm 1.96$ μ(Authentic): -17.65,

σ(Authentic): 0.65, $n=30$ (samples of authentic)

Hypothesis H_0: $\mu = -17.65$ $\sigma = 0.65$

② Estimate interval for population mean

Left Tail: $\bar{x} - z_{0.05/2}\frac{\sigma}{\sqrt{n}} \longrightarrow -17.65 - (1.96)\left(\frac{0.65}{\sqrt{30}}\right) = -17.8825...$

Right Tail: $\bar{x} + z_{0.05/2}\frac{\sigma}{\sqrt{n}} \longrightarrow -17.65 + (1.96)\left(\frac{0.65}{\sqrt{30}}\right) = -17.4174...$

$-17.88 < \mu < -17.42$

③ Evaluate FRR

From our dataset we can see that samples our outside of our interval so $FRR = \frac{FR}{(FR+TA)} = \frac{29}{(29+1)} = \frac{29}{30} = 0.96$ (very large error)

④ Change critical intervals

$-17.88 < \mu < -17.42$ is changed to $-17.30 < \mu < -16.70$

⑤ New z-values which correspond to new critical values:

$\bar{x}_1 = -17.30$, $\bar{x}_2 = -16.70$, $\mu = -17.04$, $\sigma = 0.6$, $n = 30$

$$z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{-17.30 - (-17.04)}{0.65/\sqrt{30}} = -2.1908 \approx -2.2$$

$$z_1 = \frac{\bar{x}_2 - \mu}{\sigma/\sqrt{n}} = \frac{-16.70 - (-17.04)}{0.65/\sqrt{30}} = 2.8650 \approx 2.9$$

Therefore, $FRR = P(z < -2.2) + P(z > 2.9) = 0.0139 + (1 - 0.99813) = 0.01577$

$FRR = 0.01577$

⑥ Formulate Hypothesis H_1 with μ and σ close to 30 impostor Signatures

Hypothesis H_0: $\mu(\text{Authentic}) = -17.6525$ $\sigma(\text{Authentic}) = 0.6513$,

Critical region: $-17.88 < \mu < -17.42$

Hypothesis H_1: $\mu(\text{Impostor}) = -37.6170$ $\sigma(\text{Impostor}) = 4.3575$

$$z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{17.88 - 37.6170}{4.3575/\sqrt{30}} = -24.808720$$

FAR = 0 (almost 0)

→ The distributions for genuine and impostor signatures are well seperated