Exercise #1:

Given: Influenza is true and then it is false, Given the evidence that Fever is true

Use Bayes Role: (with Influenza true)

$$Bel(I=T) = P(I=T \mid F=T) = \frac{P(F=T \mid I=T) \times P(I=T)}{P(F=T)} = \frac{P(F=T, I=T)}{P(F=T)}$$

Joint probability use chain role:

$$P(F=T, I=T) = P(\underbrace{W, I=T, C}_{\text{Not defined}}, F=T)$$

$$\sum_{i,j} = \{ \underset{1}{P(W=i)} \times \underset{2}{P(I=T \mid W=i)} \times \underset{3}{P(C=j \mid W=i)} \times \underset{4}{P(F=T \mid I=T, C=j)} \}$$

W=i	C=j	1	2	3	4	$\Pi(1,2,3,4)$
0	0	0.5	0.5	0.8	0.9	0.18
1	1	0.5	0.9	0.8	0.99	0.3564
0	1	0.5	0.5	0.2	0.99	0.0495
1	0	0.5	0.9	0.2	0.9	0.081

0.6669 ↲ Sum of all products

$$P(F=T, I=T) = 0.6669$$

Joint probability use chain role:

$$P(F=T) = P(\underbrace{W, I, C}_{\text{Not defined}}, F=T)$$

$$\sum_{i,j,k} = \{ \underset{1}{P(W=i)} \times \underset{2}{P(I=k \mid W=i)} \times \underset{3}{P(C=j \mid W=i)} \times \underset{4}{P(F=T \mid I=k, C=j)} \}$$

W=i	C=j	I=k	1	2	3	4	$\Pi(1,2,3,4)$
0	0	0	0.5	0.5	0.8	0	0
1	1	1	0.5	0.9	0.8	0.99	0.3564
0	1	0	0.5	0.5	0.2	0.9	0.045
1	0	1	0.5	0.9	0.2	0.9	0.081
0	0	1	0.5	0.5	0.8	0.9	0.18
1	1	0	0.5	0.1	0.8	0.9	0.036
0	1	1	0.5	0.5	0.2	0.99	0.0495
1	0	0	0.5	0.1	0.2	0	0
							0.7479

$P(F=T) = 0.7479$

Now we plug it in: $\frac{P(F=T, I=T)}{P(F=T)} = \frac{0.6669}{0.7479} = 0.81697509$

$\boxed{\text{0.8917 or 89.17\% is the probability that given the evidence Fever is true then Influenza is true}}$

Use Bayes Role: (with Influenza false)

$$Bel(I=F) = P(I=F \mid F=T) = \frac{P(F=T \mid I=F) \times P(I=F)}{P(F=T)} = \frac{P(F=T, I=F)}{P(F=T)}$$

Joint probability use chain role:

$P(F=T, I=F) = P(\underbrace{W, I=F, C}_{\text{Not defined}}, F=T)$

$$\sum_{i,j} = \{\underset{1}{P(W=i)} \times \underset{2}{P(I=F \mid W=i)} \times \underset{3}{P(C=j \mid W=i)} \times \underset{4}{P(F=T \mid I=F, C=j)}\}$$

W=i	C=j	1	2	3	4	$\Pi(1,2,3,4)$
0	0	0.5	0.5	0.8	0	0
1	1	0.5	0.1	0.8	0.9	0.036
0	1	0.5	0.5	0.2	0.9	0.045
1	0	0.5	0.1	0.2	0	0

0.081 ↲ Sum of all products

$P(F=T, I=F) = 0.081$

$P(F=T) = 0.7479$ from above

Now we plug it in: $\frac{P(F=T, I=T)}{P(F=T)} = \frac{0.081}{0.7479} = 0.1083032491$

0.1083 or 10.83% is the probability that given the evidence Fever is true then Influenza is false