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Chattering free full-order sliding-mode control☆

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ABSTRACT

In conventional sliding-mode control systems, the sliding-mode motion is of reduced order. Two main problems hindering the application of the sliding-mode control are the singularity in terminal sliding-mode control systems and the chattering in both the conventional linear sliding-mode and the terminal sliding-mode control systems. This paper proposes a chattering-free full-order terminal-sliding-mode control scheme. Since the derivatives of terms with fractional powers do not appear in the control law, the control singularities are avoided. A continuous control strategy is developed to achieve the chattering free sliding-mode control. During the ideal sliding-mode motion, the systems behave as a desirable full-order dynamics rather than a desirable reduced-order dynamics. A systematic design method of full-order sliding-mode control for nonlinear systems is presented, which allows both the chattering and singularity problems to be resolved. Simulations validate the proposed chattering free full-order sliding-mode control.

1. Introduction

Sliding-mode control (SMC) has attracted significant amount of interest due to its fast global convergence, simplicity of implementation, order reduction, high robustness to external disturbances and insensitivity to model errors and system parameter variations (Sabanovic, 2011; Utkin, 1992). Thanks to these advantages, SMC has been widely used in many applications, including electrical, mechanical, chemical, industrial, civil, military, aeronautical, and aerospace engineering (Boiko, 2011; Meng, Ren, & You, 2010; Tan, Yu, & Man, 2010).

SMC includes conventional linear sliding-mode (LSM) control and terminal sliding-mode (TSM) control. The former is asymptotically stable, while the latter is finite-time stable. The design of the SMC systems mainly consists of two steps: the choice of the sliding-mode surface, and the design of the sliding-mode controller. The sliding-mode surface is chosen such that a SMC system can behave in a desirable fashion. The controller is designed to guarantee the existence condition of the sliding mode, so the system can be driven to reach the sliding-mode surface in finite-time and remain on it thereafter.

Compared to conventional LSM control, TSM control exhibits various superior properties such as fast and finite-time convergence and smaller steady-state tracking errors (Feng, Zheng, Yu, & Truong, 2009; Feng, Yu, & Man, 2013). However the singularity and chattering problems need to be addressed appropriately. A carefully designed switching scheme to avoid the singularity in TSM control systems was proposed (Man & Yu, 1997). The sliding-mode of the system is switched between the TSM and the LSM, i.e., when a singularity appears, the sliding-mode is switched from TSM to LSM; it is switched back from LSM to TSM as soon as the system trajectory passes the singularity area. The disadvantage of this method is that the convergence time is extended. Transferring the system states to a pre-specified nonsingular open region where the TSM control is executed was proposed (Wu, Yu, & Man, 1998). The aforementioned two methods belong to the indirect approaches. We presented a direct method of avoiding the singularity (Feng, Yu, & Man, 2002). It resolved the singularity problem completely

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via the design of the TSM manifold. However, its application is limited to the second-order systems and a special class of high-order systems. In recent years, backstepping based nonsingular TSM control (Min & Xu, 2009), the derivative and integral TSM control (Chiu, 2012) have been proposed for the purpose of avoiding the singularity. Based on these results, we investigated the singularity problem further (Feng, Yu, & Han, 2013) and showed that for the second-order and higher-order TSM systems, the singularity areas always exist in state space if the TSM manifold is chosen as the reduced-order like in the conventional SMC systems.

Besides the singularity, the chattering is another issue that needs to be addressed in both LSM and TSM control systems. The control in these systems adopts a switching function, which can cause high frequency oscillations in the system states, i.e. chattering. A number of methods for attenuating chattering have been proposed, such as boundary layer method (Utkin, 1992), high-order sliding-mode method (Bartolini, Ferrara, & Usai, 1998; Levant, 1998, 2001, 2005, 2007), and disturbance estimation method (Shtessel, Shkolnikov, & Brown, 2003). The boundary layer method includes the saturation function and the sigmoid function methods. But it can only guarantee the existence condition of the sliding-mode outside a small boundary layer around the sliding-mode manifold, which will increase the steady-state tracking errors. The high-order sliding-mode method is to hide the discontinuity of control in its higher derivatives. The disturbance estimation method can also be used for attenuating chattering. In (Shtessel, Shkolnikov, & Brown, 2003), an asymptotic disturbance observer was designed to compensate the disturbance and a smooth control strategy was developed.

In conventional SMC systems, the sliding-mode surface is chosen so that it has desirable reduced-order dynamics when constrained to it, i.e., the ideal sliding-mode motion of SMC systems is of reduced order. In this paper, a full-order sliding-mode manifold is utilized, and a chattering free control is proposed. During the ideal sliding-mode motion, the system behaves as a desirable full-order dynamics, not a reduced-order dynamics. The proposed control is smooth and no chattering phenomenon exists in the system response. Both the LSM and TSM based control methods of nonlinear systems are presented. Neither the chattering nor the singularity problems appear in the system's response. In conventional sliding-mode design, e.g. in (Defoort, Floquet, Kokosy, & Perruquetti, 2009), the sliding-mode manifold is either measurable or computable, but the sliding-mode manifold in our paper is neither measurable nor computable. The benefit is that the proposed SMC has no chattering phenomena, while the conventional sliding-mode design has chattering phenomena.

2. Sliding-mode control of the nonlinear systems

Consider a high-order nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(\boldsymbol{x}, t) + d(\boldsymbol{x}, t) + b(\boldsymbol{x}, t)u \end{cases} \tag{1}$$

where n is the order of the system, $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T \in R^n$ represents the system state vector, $f(\boldsymbol{x}, t)$ and $b(\boldsymbol{x}, t) \neq 0$ are two smooth nonlinear functions of $\boldsymbol{x}$, $u \in R$ is the control. The partially known function $d(\boldsymbol{x}, t) : R^n \to R$, which represents the system parameter uncertainties and the external disturbances, is assumed to satisfy the following condition: $|d(\boldsymbol{x}, t)| \le l_d$, where $l_d > 0$ is a bounded constant.

The task of SMC for nonlinear system (1) is to design a control strategy which induces an ideal sliding-mode motion in the prescribed sliding-mode surface and forces system (1) to the origin along the sliding-mode surface asymptotically (for LSM) or in finite-time (for TSM).

It is assumed that all constants are known, as well as functions f and b in system (1), and all coordinates are exactly measurable in real time.

A TSM manifold for system (1) can be selected in the following form:

$$\begin{aligned} s &= x_1^{(n)} + c_n \operatorname{sgn}(x_1^{(n-1)}) \left| x_1^{(n-1)} \right|^{\alpha_n} + \cdots + c_1 \operatorname{sgn}(x_1) |x_1|^{\alpha_1} \\ &= \dot{x}_n + c_n \operatorname{sgn}(x_n) |x_n|^{\alpha_n} + \cdots + c_1 \operatorname{sgn}(x_1) |x_1|^{\alpha_1} \end{aligned} \tag{2}$$

where c_i and α_i $(i = 1, 2, \ldots, n)$ are constants. c_i can be selected such that the polynomial $p^n + c_n p^{n-1} + \cdots + c_2 p + c_1$, which corresponds to system (2), is Hurwitz, i.e., the eigenvalues of the polynomial are all in the left-half side of the complex plane. α_i can be determined based on the following conditions (Bhat & Bernstein, 1997):

$$\begin{cases} \alpha_1 = \alpha, \quad n = 1 \\ \alpha_{i-1} = \dfrac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}, \quad i = 2, \ldots, n \quad \forall n \ge 2 \end{cases} \tag{3}$$

where $\alpha_{n+1} = 1, \alpha_n = \alpha, \alpha \in (1 - \varepsilon, 1), \varepsilon \in (0, 1)$.

Once the ideal sliding-mode $s = 0$ is established, the nonlinear system (1) will behave in an identical fashion, namely

$$\dot{x}_n + c_n \operatorname{sgn}(x_n) |x_n|^{\alpha_n} + \cdots + c_1 \operatorname{sgn}(x_1) |x_1|^{\alpha_1} = 0 \tag{4}$$

or

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \cdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -c_n \operatorname{sgn}(x_n) |x_n|^{\alpha_n} - \cdots - c_1 \operatorname{sgn}(x_1) |x_1|^{\alpha_1}. \end{cases} \tag{5}$$

If α_i in TSM manifold (2) are selected using (3) and c_i in (2) are determined to guarantee that the polynomial $p^{n-1} + c_{n-1}p^{n-2} + \cdots + c_2 p + c_1$ is Hurwitz, system (4) or (5), which represents the establishment of the ideal sliding-mode $s = 0$ for system (1), can converge to its equilibrium point $\boldsymbol{x} = [x_1, \ldots, x_{n-1}]^T = [0, \ldots, 0]^T$ from any initial condition $\boldsymbol{x}(0) \neq 0$ along the TSM manifold $s = 0$ in finite-time (Bhat & Bernstein, 1997, 2005; Hong, Xu, & Huang, 2002; Hong, Yang, Cheng, & Spurgeon, 2004).

Assumption 2.1. The derivative of $d(\boldsymbol{x}, t)$ in system (1) is bounded:

$$\left| \dot{d}(\boldsymbol{x}, t) \right| \le k_d \tag{6}$$

where $k_d > 0$ is a constant.

Note that this assumption is realistic in practical applications. For example, when a cutting tool or an end mill of a CNC machine tool cuts a work-piece, the load torque may change as the cutting thickness changes, but the change rate of the load torque is always limited.

Theorem 2.2. *The nonlinear system (1) will reach $s = 0$ in finite-time and then converge to zero along $s = 0$ within finite-time, if the sliding-mode surface s is selected as (2) and the control is designed as follows:*

$$u = b^{-1}(\boldsymbol{x}, t) \left(u_{eq} + u_n \right) \tag{7}$$

$$u_{eq} = -f(\boldsymbol{x}, t) - c_n \operatorname{sgn}(x_n) |x_n|^{\alpha_n} - \cdots - c_1 \operatorname{sgn}(x_1) |x_1|^{\alpha_1} \tag{8}$$

$$\dot{u}_n + T u_n = v \tag{9}$$

$$v = -(k_d + k_T + \eta) \operatorname{sgn}(s) \tag{10}$$

where $u_n(0) = 0$; c_i and α_i $(i = 1, \ldots, n)$ are all constants, as defined in (2); η is a positive constant; k_d is a constant defined in (6); two constants, $T \ge 0$ and k_T are selected to satisfy the following condition:

$$k_T \ge T l_d. \tag{11}$$

Proof. From system (1), the sliding-mode manifold (2) can be rewritten as follows:

$$\begin{aligned} s &= \dot{x}_n + c_n \operatorname{sgn}(x_n)\,|x_n|^{\alpha_n} + \cdots + c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1} \\ &= f(\boldsymbol{x},t) + d(\boldsymbol{x},t) + b(\boldsymbol{x},t)u \\ &\quad + c_n \operatorname{sgn}(x_n)\,|x_n|^{\alpha_n} + \cdots + c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1}. \end{aligned}$$

Substituting the control (7) into above equation gives:

$$\begin{aligned} s &= f(\boldsymbol{x},t) + d(\boldsymbol{x},t) + u_{eq} + u_n \\ &\quad + c_n \operatorname{sgn}(x_n)\,|x_n|^{\alpha_n} + \cdots + c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1}. \end{aligned} \tag{12}$$

Further substituting (8) into (12) gives:

$$s = d(\boldsymbol{x},t) + u_n. \tag{13}$$

The solution of (9) is given by

$$\begin{aligned} u_n(t) &= (u_n(t_0) + (1/T)(k_d + k_T + \eta)\operatorname{sgn}(s))\,e^{t-t_0} \\ &\quad - (1/T)(k_d + k_T + \eta)\operatorname{sgn}(s). \end{aligned} \tag{14}$$

From (11), (13) and (14), the following relationship under the condition $u_n(0) = 0$ can be obtained:

$$k_T \ge T l_d \ge T\,|u_n(t)|_{\max} \ge T\,|u_n(t)|$$

i.e. the following inequality will be kept forever:

$$T\,|u_n(t)| \le k_T. \tag{15}$$

The following Lyapunov function is considered: $V = s^2/2$. For TSM manifold (2), its derivative with respect to time t along system (1) can be obtained from (13) as follows:

$$\begin{aligned} \dot{s} &= \dot{d}(\boldsymbol{x},t) + \dot{u}_n = \dot{d}(\boldsymbol{x},t) + \dot{u}_n + Tu_n - Tu_n \\ &= \dot{d}(\boldsymbol{x},t) + v - Tu_n. \end{aligned}$$

Substituting (9) into above equation gives:

$$\dot{s} = \dot{d}(\boldsymbol{x},t) - (k_d + k_T + \eta)\operatorname{sgn}(s) - Tu_n$$

hence

$$\begin{aligned} s\dot{s} &= \dot{d}(\boldsymbol{x},t)s - (k_d + k_T + \eta)\,|s| - Tu_n s \\ &= \left(\dot{d}(\boldsymbol{x},t)s - k_d\,|s|\right) + (-Tu_n s - k_T\,|s|) - \eta\,|s|. \end{aligned}$$

From (6), (15) and above equation, we have

$$\dot{V} = s\dot{s} \le -\eta\,|s| < 0 \quad \text{for } |s| \ne 0$$

which means that system (1) will reach to $s = 0$ in finite time. On $s = 0$, system (1) behaves in an identical fashion, as shown in (4) or (5), i.e., the system will converge to zero in finite-time along $s = 0$. This completes the proof.

In fact the proposed control provides for the establishment of the $(n+1)$th-order sliding-mode $x_1 = \dot{x}_1 = \cdots = x_1^{(n)} = 0$ in the extended state space with coordinates $t, x_1, x_2, \ldots, x_n, u_n$.

Remark 2.3. In (7)–(10), all variables except s are available since $\dot{x}_n$ is not available in (2). For calculating $\operatorname{sgn}(s)$ in (10), a function $g(t)$ is defined as follows:

$$\begin{aligned} g(t) &= \int_0^t s(t)dt \\ &= x_n + \int_0^t (c_n \operatorname{sgn}(x_n)\,|x_n|^{\alpha_n} + \cdots + c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1})\,dt. \end{aligned} \tag{16}$$

$\operatorname{sgn}(s)$ can be obtained by the following equation:

$$\operatorname{sgn} s = \operatorname{sgn}\left(g(t) - g(t-\tau)\right) \tag{17}$$

where τ is a time delay. Since $s(t) = \lim_{\tau\to 0}\left(g(t) - g(t-\tau)\right)/\tau$, we can choose a fundamental sample time as τ. It should be noted that we do not need the value of s, but only its sign, $\operatorname{sgn}(s)$, i.e., we only need to know whether g increases or decreases. To obtain $\operatorname{sgn}(s)$ is much easier than to obtain s.

Remark 2.4. In Theorem 2.2, the control signal (9) is equivalent to a low-pass filter, where $v(t)$ is the input and $u_n(t)$ is the output of the filter. The Laplace transfer function of the filter (9) is:

$$\frac{u_n(s)}{v(s)} = \frac{1}{s+T} \tag{18}$$

where $\omega = T$ is the bandwidth of the low-pass filter. Although $v(t)$ in (10) is non-smooth because of the switch function, $u_n(t)$ in (7) is the output of the low-pass filter (9) and is softened to be a smooth signal by (9).

In the special case, $T = 0$, (9) and (10) become:

$$\dot{u}_n = v \tag{19}$$

$$v = -(k_d + \eta)\operatorname{sgn}(s). \tag{20}$$

If (9) and (10) are replaced with (19) and (20), Theorem 2.2 holds also and the control u in (7) is continuous as well. In this case, we do not need the condition (11) for Theorem 2.2 and its proof. But (19) is a pure integrator and more difficult for hardware implementation in practical applications than the low-pass filter (9).

Remark 2.5. We prevent differentiating terms $c_i \operatorname{sgn}(x_i)\,|x_i|^{\alpha_i}$ in the TSM manifold (2) from deriving the control laws. So singularity can be avoided, and the ideal TSM, $s = 0$, is nonsingular.

Below are several controller design examples.

The first-order systems

$$\dot{x}_1 = f(x_1,t) + d(x_1,t) + b(x_1,t)u. \tag{21}$$

Based on (2), a TSM manifold for system (21) can be selected in the following form:

$$s = \dot{x}_1 + c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1} \tag{22}$$

where $0 < \alpha_1 < 1$. The control strategy can be designed based on (7)–(10), and the control signal u_{eq} is:

$$u_{eq} = -f(x_1,t) - c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1}. \tag{23}$$

The second-order systems

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(\boldsymbol{x},t) + d(\boldsymbol{x},t) + b(\boldsymbol{x},t)u \end{cases} \tag{24}$$

where $\boldsymbol{x} = [x_1, x_2]^T$, $u \in R$.

Based on (2) and (8), a TSM manifold and the control signal u_{eq} for system (24) can be designed as follows:

$$s = \ddot{x}_1 + c_2 \operatorname{sgn}(\dot{x}_1)\,|\dot{x}_1|^{\alpha_2} + c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1} \tag{25}$$

$$u_{eq} = -f(\boldsymbol{x},t) - c_2 \operatorname{sgn}(x_2)\,|x_2|^{\alpha_2} - c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1}. \tag{26}$$

The whole control strategies for system (24) are the same as those in (7)–(10).

The third-order systems

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = f(\boldsymbol{x},t) + d(\boldsymbol{x},t) + b(\boldsymbol{x},t)u \end{cases} \tag{27}$$

where $\boldsymbol{x} = [x_1, x_2, x_3]^T$, $u \in R$.

Based on (2) and (8), a TSM manifold and the control signal u_{eq} can be designed respectively as follows:

$$s = x_1 + c_3 \operatorname{sgn}(\ddot{x}_1)\,|\ddot{x}_1|^{\alpha_3} + c_2 \operatorname{sgn}(\dot{x}_1)\,|\dot{x}_1|^{\alpha_2} + c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1} \tag{28}$$

$$\begin{aligned} u_{eq} &= -f(\boldsymbol{x},t) - c_3 \operatorname{sgn}(x_3)\,|x_3|^{\alpha_3} - c_2 \operatorname{sgn}(x_2)\,|x_2|^{\alpha_2} \\ &\quad - c_1 \operatorname{sgn}(x_1)\,|x_1|^{\alpha_1}. \end{aligned} \tag{29}$$

In these examples, only v in (10) contains switching terms, while the actual control u does not contain these terms. Therefore the proposed control is chattering-free.

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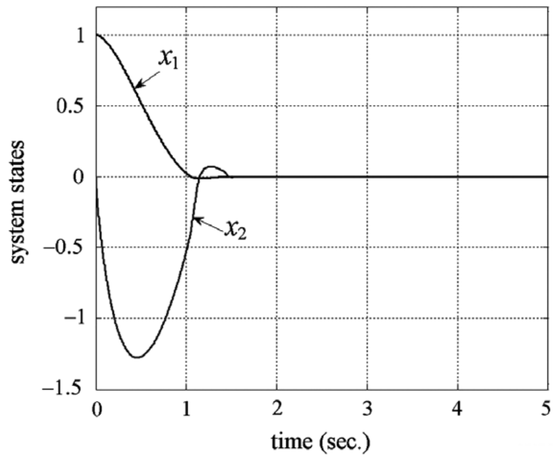

Fig. 1. System states of the second-order system.

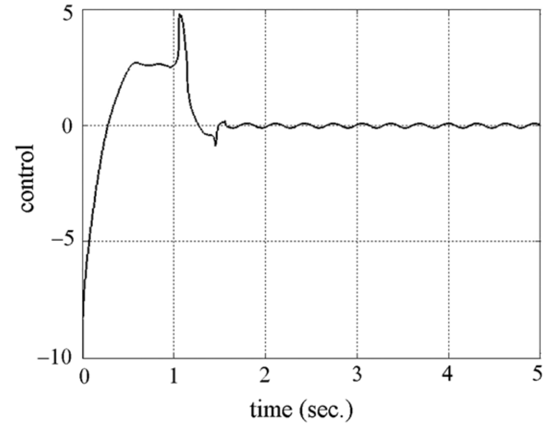

Fig. 2. Control of the second-order system.

Remark 2.6. Theorem 2.2 can be extended to the LSM control. Unlike TSM manifold (2), the LSM manifold for the nth-order systems (1) can be chosen as follows:

$$s = x_1^{(n)} + c_n x_1^{(n-1)} + \cdots + c_1 x_1 = \dot{x}_n + c_n x_n + \cdots + c_1 x_1 \tag{30}$$

where $c_i > 0$ ($i = 1, 2, \ldots, n$) can be selected such that the polynomial $p^n + c_n p^{n-1} + \cdots + c_2 p + c_1$, which corresponds to system (30), is Hurwitz. The difference between LSM (30) and TSM (2) is that the system will converge to zero in finite-time along TSM (2), but asymptotically converge to zero along LSM (30).

The LSM based controller for nonlinear system (1) can also be designed based on Theorem 2.2 by assuming the parameters, $\alpha_i = 1, i = 1, 2, \ldots, n$.

3. Simulations

To evaluate the effectiveness of the proposed method, two examples are given below.

Example 1 (*TSM Control of a Second-order System*)**.**

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_2^3 + 0.1\sin(20t) + u. \end{cases} \tag{31}$$

A TSM manifold is chosen from (2) as follows:

$$\begin{aligned} s &= \ddot{x}_1 + 7\,\mathrm{sgn}\dot{x}_1\,|\dot{x}_1|^{9/16} + 10x_1^{9/23} \\ &= \dot{x}_2 + 7\,\mathrm{sgn}x_2\,|x_2|^{9/16} + 10x_1^{9/23} \end{aligned}$$

where the parameters, 9/16 and 9/23 are chosen by using (3), and the Hurwitz polynomial is selected as $p^2+7p+10 = (p+2)(p+5)$.

Based on Theorem 2.2, $u = u_{eq} + u_n$ are designed as:

$$u_{eq} = -x_2^3 - 7\,\mathrm{sgn}x_2\,|x_2|^{9/16} - 10x_1^{9/23} \tag{32}$$

$$\dot{u}_n + 0.1u_n = v \tag{33}$$

$$v = -10\,\mathrm{sgn}(s) \tag{34}$$

where $k_d + k_T + \eta = 10$. In system (31), $k_d = 2$ because of $\left|\dot{d}(\boldsymbol{x}, t)\right| \le 2$. T is selected as 0.1, $k_T + \eta$ is selected as $k_T + \eta = 8$. sgn(s) can be calculated using (16) and (17).

The simulation results are shown in Figs. 1 and 2. Two state variables of the system are depicted in Fig. 1. The actual control signal of the system u in (31) is displayed in Fig. 2. It can be seen that although there is a switching function in signal v (34), the actual control signal u is soften to be smooth because of the low-pass filter (33). In addition, no singularity occurs in the signal u. Therefore the proposed full-order sliding-mode control method in the paper can resolve two main problems in the second-order TSM systems: the singularity and the chattering.

Example 2 (*TSM Control of a Third-order System*)**.**

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_2^3 + 0.1\sin(20t) + u. \end{cases} \tag{35}$$

Based on (2), a TSM manifold is designed as follows:

$$\begin{aligned} s &= x_1 + 15\,\mathrm{sgn}\ddot{x}_1\,|\ddot{x}_1|^{7/10} + 66\dot{x}_1^{7/13} + 80\,\mathrm{sgn}x_1\,|x_1|^{7/16} \\ &= \dot{x}_3 + 15\,\mathrm{sgn}x_3\,|x_3|^{7/10} + 66x_2^{7/13} + 80\,\mathrm{sgn}x_1\,|x_1|^{7/16} \end{aligned} \tag{36}$$

where the parameters, 7/10, 7/13 and 7/16 are chosen by using (3), and the Hurwitz polynomial is selected as $p^3 + 15p^2 + 66p + 80 = (p+2)\,(p+5)(p+8)$.

Based on Theorem 2.2, $u = u_{eq} + u_n$ are designed as:

$$u_{eq} = -x_2^3 - 15\,\mathrm{sgn}x_3\,|x_3|^{7/10} - 66x_2^{7/13} - 80\,\mathrm{sgn}x_1\,|x_1|^{7/16} \tag{37}$$

$$\dot{u}_n + 0.1u_n = v \tag{38}$$

$$v = -10\,\mathrm{sgn}(s) \tag{39}$$

where $k_d + k_T + \eta = 10$. In system (35), $k_d = 2$ because of $\left|\dot{d}(\boldsymbol{x}, t)\right| \le 2$. T is selected as 0.1, $k_T + \eta$ is selected as $k_T + \eta = 8$. sgn(s) can be calculated using (16) and (17).

The simulation results are shown in Figs. 3 and 4. Fig. 3 depicts the three system states, x_1, x_2, and x_3 respectively. The actual control of the system u in (35) is shown in Fig. 4. It can be seen that although there is a switching function in v (39), the actual control u is smooth, and no singularity occurs in u.

From the two examples above, it can be observed that the proposed method can realize the globally finite-time stability of the nth-order systems, and the singularity problem can be avoided. In addition, the control signals are smooth, as shown in Figs. 2 and 4, which means that the chattering problem in SMC systems has been eliminated.

4. Conclusions

In this paper, the chattering free full-order sliding-mode control has been proposed. The main contributions can be summarized as follows: (1) the singularity problem in TSM systems has been avoided by preventing differentiating terms with fractional power in sliding-mode manifold from deriving the control laws. (2) The chattering in both LSM and TSM systems has also been resolved by applying a continuous control strategy. During the

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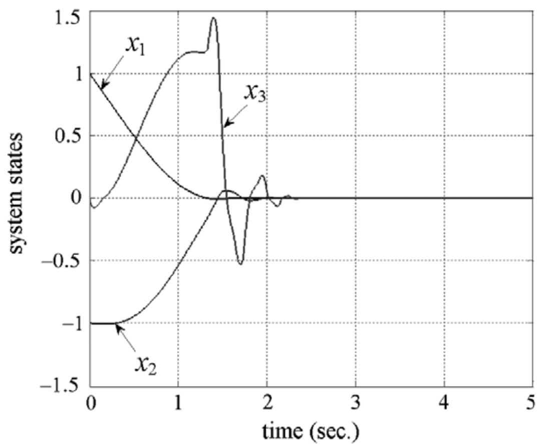

Fig. 3. System states of the third-order system.

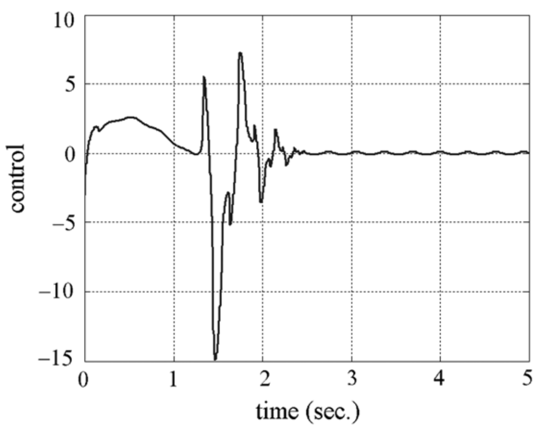

Fig. 4. Control of the third-order system.

ideal sliding-mode motion, the system behaves as a desirable full-order dynamics rather than a desirable reduced-order dynamics with the proposed control strategy in place. The proposed control is smooth and no chattering phenomenon exists. The proposed design method can be used for both LSM- and TSM-based nth-order systems.

References

Bartolini, G., Ferrara, A., & Usai, E. (1998). Chattering avoidance by second-order sliding mode control. *IEEE Transactions on Automatic Control*, *43*(2), 241–246.

Bhat, S. P., & Bernstein, D. S. (1997). Finite-time stability of homogeneous systems. In *Proc. American control conf* (pp. 1073–1078).

Bhat, S. P., & Bernstein, D. S. (2005). Geometric homogeneity with applications to finite-time stability. *Mathematics of Control, Signals and Systems*, *17*(2), 101–127.

Boiko, M. (2011). On frequency-domain criterion of finite-time convergence of second-order sliding mode control algorithms. *Automatica*, *47*(9), 1969–1973.

Chiu, C. S. (2012). Derivative and integral terminal sliding mode control for a class of MIMO nonlinear systems. *Automatica*, *48*(2), 316–326.

Defoort, M., Floquet, T., Kokosy, A., & Perruquetti, W. (2009). A novel higher order sliding mode control scheme. *Systems & Control Letters*, *58*(2), 102–108.

Feng, Y., Yu, X., & Man, Z. (2002). Non-singular terminal sliding mode control of rigid manipulators. *Automatica*, *28*(12), 2159–2167.

Feng, Y., Zheng, J., Yu, X., & Truong, N. V. (2009). Hybrid terminal sliding-mode observer design method for a permanent-magnet synchronous motor control system. *IEEE Transactions on Industrial Electronics*, *56*(9), 3424–3431.

Feng, Y., Yu, X., & Man, Z. (2013). On nonsingular terminal sliding mode control of nonlinear systems. *Automatica*, *49*(6), 1715–1722.

Feng, Y., Yu, X., & Han, F. (2013). High-order terminal sliding-mode observer for parameter estimation of a permanent-magnet synchronous motor. *IEEE Transactions on Industrial Electronics*, *60*(10), 4272–4280.

Hong, Y., Xu, Y., & Huang, J. (2002). Finite-time control for robot manipulators. *Systems & Control Letters*, *46*(4), 243–253.

Hong, Y., Yang, G., Cheng, D., & Spurgeon, S. (2004). Finite time convergent control using terminal sliding mode. *Journal of Control Theory and Applications*, *2*(1), 69–74.

Levant, A. (1998). Robust exact differentiation via sliding mode technique. *Automatica*, *34*(3), 379–384.

Levant, A. (2001). Universal SISO sliding-mode controllers with finite-time convergence. *IEEE Transactions on Automatic Control*, *46*(9), 1447–1451.

Levant, A. (2005). Homogeneity approach to high-order sliding mode design. *Automatica*, *41*(5), 823–830.

Levant, A. (2007). Principles of 2-sliding mode design. *Automatica*, *43*(4), 576–586.

Man, Z., & Yu, X. (1997). Terminal sliding mode control of MIMO linear systems. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, *44*(11), 1065–1070.

Meng, Z., Ren, W., & You, Z. (2010). Distributed finite-time attitude containment control for multiple rigid bodies. *Automatica*, *46*(12), 2092–2099.

Min, J., & Xu, Z. (2009) Backstepping control for a class of uncertain systems based on non-singular terminal sliding mode. In *Proc. int. conf. ind. mechatronics autom.* (pp. 169–172).

Sabanovic, A. (2011). Variable structure systems with sliding modes in motion control—a survey. *IEEE Transactions on Industrial Informatics*, *7*(2), 212–223.

Shtessel, Y. B., Shkolnikov, I. A., & Brown, M. D. J. (2003). An asymptotic second-order smooth sliding mode control. *Asian Journal of Control*, *5*(4), 498–504.

Tan, C. P., Yu, X., & Man, Z. (2010). Terminal sliding mode observers for a class of nonlinear systems. *Automatica*, *46*(8), 1401–1404.

Utkin, V. (1992). *Sliding modes in control and optimization*. Berlin, Germany: Springer-Verlag.

Wu, Y., Yu, X., & Man, Z. (1998). Terminal sliding mode control design for uncertain dynamic systems. *Systems & Control Letters*, *34*, 281–288.