A Fourier-based Framework for Domain Generalization

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DA vs DG

Data domain shift impairs the performance of networks

- Domain adaptation
 - Bridges the gaps between source domains and a specific target domain with labelled or unlabeled target data
 - There are target domain data in training
- Domain generalization
 - Aims to train model with multiple source domains that can generalize to arbitrary unseen target domains
 - No target domain data in training

Fourier phase & amplitude information

• Phase: high-level semantics

• Amplitude: low-level statistics

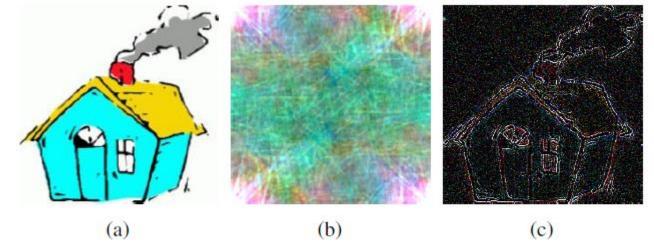


Figure 1. Examples of the amplitude-only and phase-only reconstruction: (a) original image; (b) reconstructed image with amplitude information only by setting the phase component to a constant; (c) reconstructed image with phase information only by setting the amplitude component to a constant.

Motivation

Changing amplitude information for data augmentation

Using augmented data for implementing data generalization

Contribution

- Introduce a novel Fourier-based perspective for domain generalization,
- Develop a novel Fourier-based data augmentation strategy called amplitude mix,
- Utilize a dual-formed consistency loss called co-teacher regularization during training process.

Fourier-based data augmentation

- Amplitude swap
 - Swap central part of amplitude images

- Amplitude mix
 - Linearly interpolate amplitude images

$$\hat{\mathcal{A}}(x_i^k) = (1 - \lambda)\mathcal{A}(x_i^k) + \lambda\mathcal{A}(x_{i'}^{k'}),$$

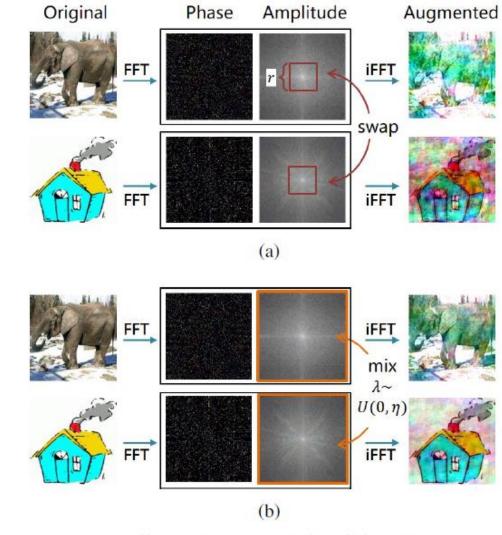
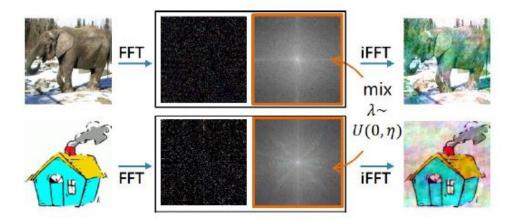


Figure 3. Illustration of (a) AS and (b) AM strategy.

Co-teacher regularization

 The categorical relations predicted from original and augmented images with the same phase information may be different



• Co-teacher regularization is used for alleviate this disagreement

Co-teacher regularization

Dual consistency loss

- Momentum-updated teacher model
 - Teacher model receives parameters from the student model via exponential moving average

$$\theta_{tea} = m\theta_{tea} + (1 - m)\theta_{stu}$$

Objective function

- Classification loss
 - Cross-entropy

$$\mathcal{L}_{cls}^{aug} = -y_i^k \log \left(\sigma(f(\hat{x}_i^k; \theta)) \right)$$

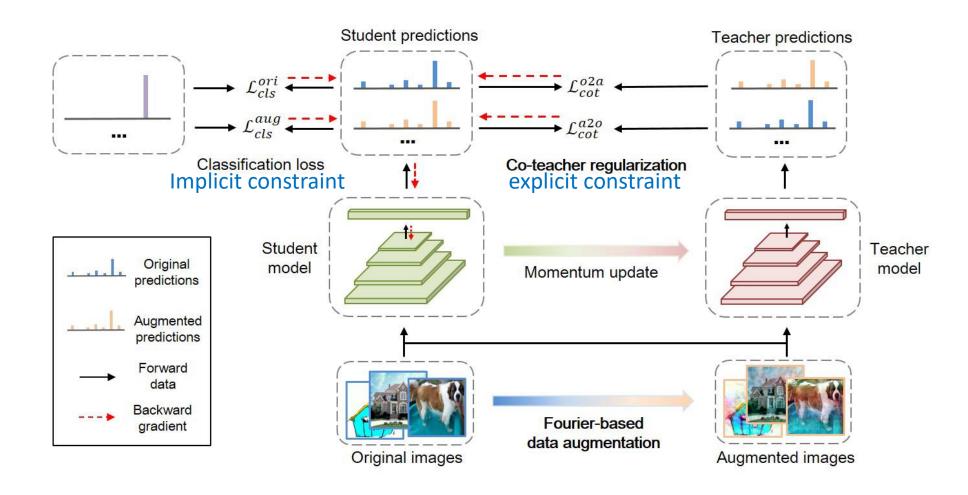
- Co-teacher regularization
 - Softened softmax at temperature T

$$\mathcal{L}_{cot}^{a2o} = \text{KL}(\sigma(f_{stu}(\hat{x}_i^k)/T)||\sigma(f_{tea}(x_i^k)/T))|$$

$$\mathcal{L}_{cot}^{o2a} = \text{KL}(\sigma(f_{stu}(x_i^k)/T)||\sigma(f_{tea}(\hat{x}_i^k)/T))|$$

$$\mathcal{L}_{FACT} = \mathcal{L}_{cls}^{ori} + \mathcal{L}_{cls}^{aug} + \beta(\mathcal{L}_{cot}^{a2o} + \mathcal{L}_{cot}^{o2a})$$

Fourier Augmented Co-Teacher (FACT)



Dataset

- Digits-DG
 - Four datasets
 - MNIST, MNIST-M, SVHN, SYN
- PACS
 - Four domains
 - Photo, art-painting, cartoon, sketch
- Office-Home
 - Four domains
 - Art, clipart, product, real-world

Training strategy

- Leave-one-domain-out
 - Train on data in all domain but one
 - Test on the held-out domain

Table 1. Leave-one-domain-out results on Digits-DG. The best and second-best results are bolded and underlined respectively.

Methods	MNIST	MNIST-M	SVHN	SYN	Avg.
DeepAll [52]	95.8	58.8	61.7	78.6	73.7
CCSA [29]	95.2	58.2	65.5	79.1	74.5
MMD-AAE [24]	96.5	58.4	65.0	78.4	74.6
CrossGrad [38]	96.7	61.1	65.3	80.2	75.8
DDAIG [52]	96.6	64.1	68.6	81.0	77.6
Jigen [2]	96.5	61.4	63.7	74.0	73.9
L2A-OT [53]	96.7	63.9	68.6	83.2	78.1
FACT (ours)	97.9	65.6	72.4	90.3	81.5

Table 3. Leave-one-domain-out results on OfficeHome. The best and second-best results are bolded and underlined respectively.

Methods	Art	Clipart	Product	Real	Avg.
DeepAll	57.88	52.72	73.50	74.80	64.72
CCSA [29]	59.90	49.90	74.10	75.70	64.90
MMD-AAE [24]	56.50	47.30	72.10	74.80	62.70
CrossGrad [38]	58.40	49.40	73.90	75.80	64.40
DDAIG [52]	59.20	52.30	74.60	76.00	65.50
L2A-OT [53]	60.60	50.10	74.80	77.00	65.60
Jigen [2]	53.04	47.51	71.47	72.79	61.20
RSC [17]	58.42	47.90	71.63	74.54	63.12
Jigen (our imple.)	57.95	49.21	72.61	74.90	63.67
RSC (our imple.)	57.67	48.48	72.62	74.16	63.23
FACT (ours)	60.34	54.85	74.48	76.55	66.56

Table 2. Leave-one-domain-out results on PACS. The best and second-best results are bolded and underlined respectively. †: results are reported based on the best models on test splits.

Methods	Art	Cartoon	Photo	Sketch	Avg.				
ResNet18									
DeepAll	77.63	76.77	95.85	69.50	79.94				
MetaReg [1]	83.70	77.20	95.50	70.30	81.70				
JiGen [2]	79.42	75.25	96.03	71.35	80.51				
Epi-FCR [23]	82.10	77.00	93.90	73.00	81.50				
MMLD [27]	81.28	77.16	96.09	72.29	81.83				
DDAIG [52]	84.20	78.10	95.30	74.70	83.10				
CSD [37]	78.90	75.80	94.10	76.70	81.40				
InfoDrop [40]	80.27	76.54	96.11	76.38	82.33				
MASF [4] [†]	80.29	77.17	94.99	71.69	81.04				
L2A-OT [53]	83.30	78.20	96.20	73.60	82.80				
EISNet [46]	81.89	76.44	95.93	74.33	82.15				
RSC [17]	83.43	80.31	95.99	80.85	85.15				
RSC (our imple.)	80.55	78.60	94.43	76.02	82.40				
FACT (ours)	85.37	78.38	95.15	79.15	84.51				
	ResNet50								
DeepAll	84.94	76.98	97.64	76.75	84.08				
MetaReg [1]	87.20	79.20	97.60	70.30	83.60				
MASF [4] [†]	82.89	80.49	95.01	72.29	82.67				
EISNet [46]	86.64	81.53	97.11	78.07	85.84				
RSC [17]	87.89	82.16	97.92	83.35	87.83				
RSC (our imple.)	83.92	79.52	95.15	82.20	85.20				
FACT (ours)	89.63	81.77	96.75	84.46	88.15				

Ablation study: model component

Table 4. Ablation studies on different components of our method on the PACS dataset with ResNet18.

Method	AM	\mathcal{L}_{cot}^{a2o}	\mathcal{L}_{cot}^{o2a}	Teacher	Art	Cartoon	Photo	Sketch Av	g.
Baseline	-	-	-	-	77.63±0.84	76.77±0.33	95.85±0.20	69.50±1.26 79.9	94
Model A	✓	-	-	-	83.90±0.50	76.95±0.45	95.55±0.12	77.36±0.71 83.4	44
Model B	✓	\checkmark	\checkmark	-	83.71±0.30	77.84±0.49	94.73±0.12	78.55±0.46 83.7	71
Model C	-	\checkmark	\checkmark	✓	82.68±0.44	78.06±0.39	95.35±0.44	74.76±0.67 82.7	71
Model D	✓	\checkmark	-	✓	83.97±0.77	77.04±0.86	94.59±0.03	79.08±0.56 83.6	67
Model E	✓	-	\checkmark	✓	84.07±0.43	77.70±0.65	95.28±0.34	78.29±0.61 83.8	84
FACT	✓	✓	✓	✓	85.37±0.29	78.38±0.29	95.15±0.26	79.15±0.69 84.5	51

Ablation study: data augmentation

Table 5. Ablation studies of different choices of the Fourier data augmentation on the PACS dataset with ResNet18.

Methods	Art	Cartoon	Photo	Sketch	Avg.			
	DeepAll with							
AS-partial	82.00	76.19	93.89	77.27	82.34			
AS-full	83.50	76.07	94.49	77.13	82.80			
AM	83.90	76.95	95.55	77.36	83.44			
	FACT with							
AS-partial	81.61	76.95	93.83	78.30	82.67			
AS-full	83.46	77.37	94.10	78.63	83.39			
AM	85.37	78.38	95.15	79.15	84.51			

Discussion

 Phase information contains meaningful semantics and helps generalization

Table 6. The performance changes of training with phase-only reconstructed images and amplitude-only reconstructed images when compared with original images. The values greater than zero (meaning an improvement) are in bold.

Data	Test Train	Photo	Art	Cartoon	Sketch
	Photo	-4.68	3.16	4.07	2.38
Phase	Art	-5.35	1.28	5.97	15.87
only	Cartoon	-11.53	0.29	-4.08	18.55
	Sketch	10.66	14.56	21.26	-1.09
	Photo	-14.03	-4.15	-4.41	-0.08
Amplitude	Art	-18.40	-21.96	-5.59	-10.72
only	Cartoon	-13.95	-7.48	-15.89	1.36
	Sketch	-4.79	-0.73	-1.99	-13.99

Discussion

Amplitude perturbation constrains the model to focus more on phase information. Our Fourier-based data augmentation are implemented via perturbing the amplitude information. Here we present a brief theoretical analysis to demonstrate that amplitude perturbation does make the model to focus more on phase information. For simplicity, we consider the case of a linear softmax classifier together with a feature extractor \mathbf{h} . Suppose the distribution of Fourier-based data augmentation is $g \sim \mathcal{G}$, and the risk of training on the augmented data is:

$$\hat{R}_{\text{aug}} = \frac{1}{N} \sum_{n=1}^{N} \mathbb{E}_{g \sim \mathcal{G}} \left[\ell \left(\mathbf{W}^{\top} \mathbf{h} \left(g \left(x \right) \right), y \right) \right]$$
 (10)

Similar as in [3, 14], we expand \hat{R}_{aug} with Taylor expansion:

$$\mathbb{E}_{g \sim \mathcal{G}} \left[\ell \left(\mathbf{W}^{\top} \mathbf{h}(g(x)), y \right) \right] = \ell \left(\mathbf{W}^{\top} \overline{\mathbf{h}}, y \right) + \frac{1}{2} \mathbb{E}_{g \sim \mathcal{G}} \left[\Delta^{\top} \mathbf{H}(\tau, y) \Delta \right]$$
(11)

where $\overline{\mathbf{h}} = \mathbb{E}_{g \sim \mathcal{G}}[\mathbf{h}(g(x))]$, $\Delta = \mathbf{W}^{\top}(\overline{\mathbf{h}} - \mathbf{h}(g(x)))$ and \mathbf{H} is the Hessian matrix. For cross-entropy loss with softmax, \mathbf{H} is semi-definite and independent of y. Then, minimizing the second-order term in Eq. 11 requires that for some feature h_d , if its variance $h_d(g(x))$ is large, the weight $w_{i,d}$ will approach 0. Suppose that the features induced from phase information and amplitude information is h_p and h_a respectively, since we only perturb the amplitude information and keep the phase information unchanged, it is reasonable that:

$$\begin{cases} |h_p(g(x)) - h_p(x)| < \zeta \\ |h_a(g(x)) - h_a(x)| > \epsilon \end{cases}$$
 (12)

where $\zeta > 0$ is a small value, and $\epsilon \geq \zeta$. Therefore, minimizing \hat{R}_{aug} restricts $w_{i,a} \to 0$ for those features h_a derived from the amplitude information. As a result, the classifier would pay more attention to the features h_p that derived from the phase information when making decisions.